Computer algebra independent integration tests

Summer 2022 edition

6-Hyperbolic-functions/6.1-Hyperbolic-sine/164-6.1.7-hyper^m-a+b-sinh^n-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [525]. This is test number [164].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (525)	0.00 (0)
Mathematica	95.43 (501)	4.57 (24)
Maple	92.95 (488)	7.05 (37)
Fricas	84.57 (444)	15.43 (81)
Giac	55.24 (290)	44.76 (235)
Mupad	47.05 (247)	52.95 (278)
Maxima	37.33 (196)	62.67 (329)
Sympy	14.67 (77)	85.33 (448)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

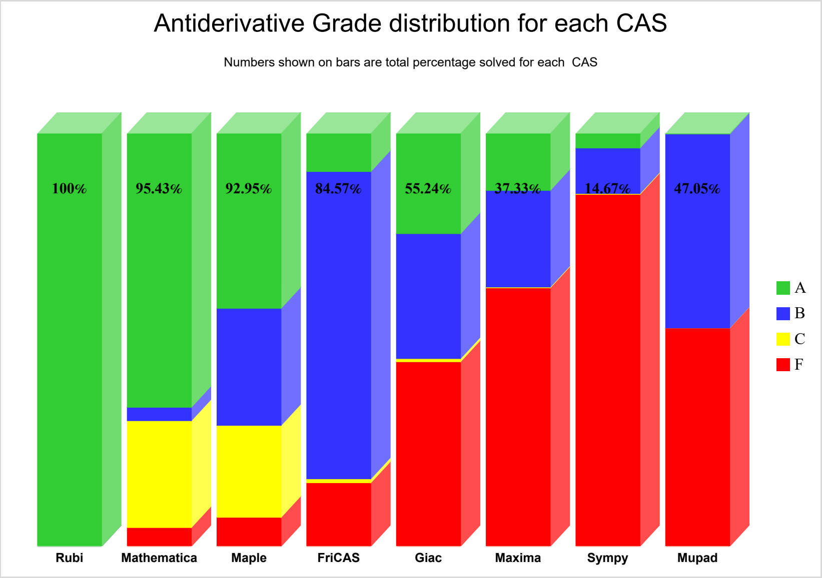
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

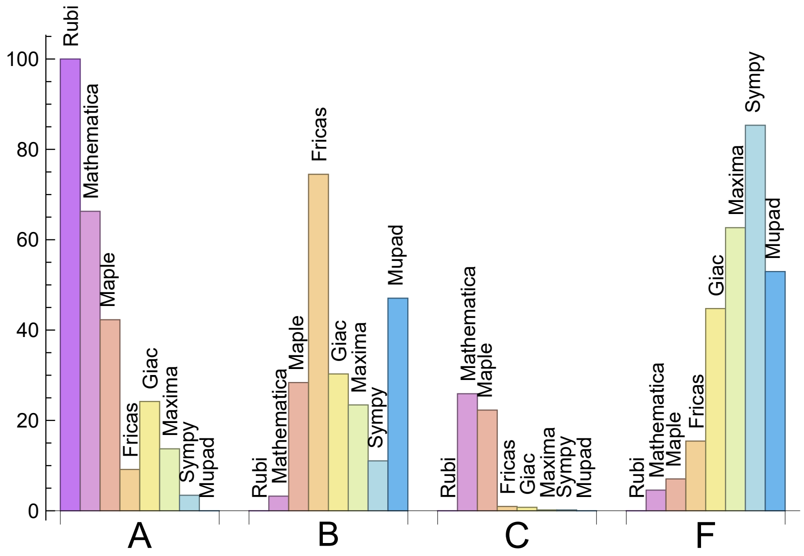
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	66.29	3.24	25.90	4.57
Maple	42.29	28.38	22.29	7.05
Giac	24.19	30.29	0.76	44.76
Maxima	13.71	23.43	0.19	62.67
Fricas	9.14	74.48	0.95	15.43
Sympy	3.43	11.05	0.19	85.33
Mupad	N/A	47.05	0.00	52.95

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	24	95.83 %	4.17 %	0.00 %
Maple	37	100.00 %	0.00 %	0.00 %
Fricas	81	97.53 %	0.00 %	2.47 %
Giac	235	29.79 %	0.00 %	70.21 %
Maxima	329	89.67 %	0.30 %	10.03 %
Sympy	448	40.62 %	46.21 %	13.17 %
Mupad	278	96.76 %	3.24 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

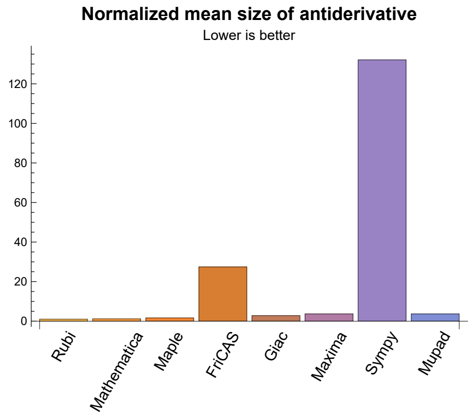
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

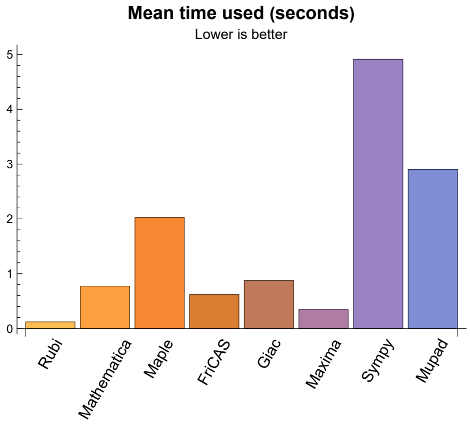
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	133.48	1.00	114.00	1.00
Mathematica	0.77	154.48	1.19	117.00	0.99
Maple	2.03	212.56	1.65	150.00	1.51
Maxima	0.35	337.77	3.69	191.00	2.36
Fricas	0.62	5263.77	27.47	1454.50	14.14
Sympy	4.91	5507.10	132.13	221.00	2.66
Giac	0.88	398.94	2.79	196.50	1.94
Mupad	2.90	423.59	3.68	189.00	2.33

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
20
40
60
80
100
120
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{299, 301, 303, 310, 312, 314, 356, 367, 376, 384, 385, 394\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

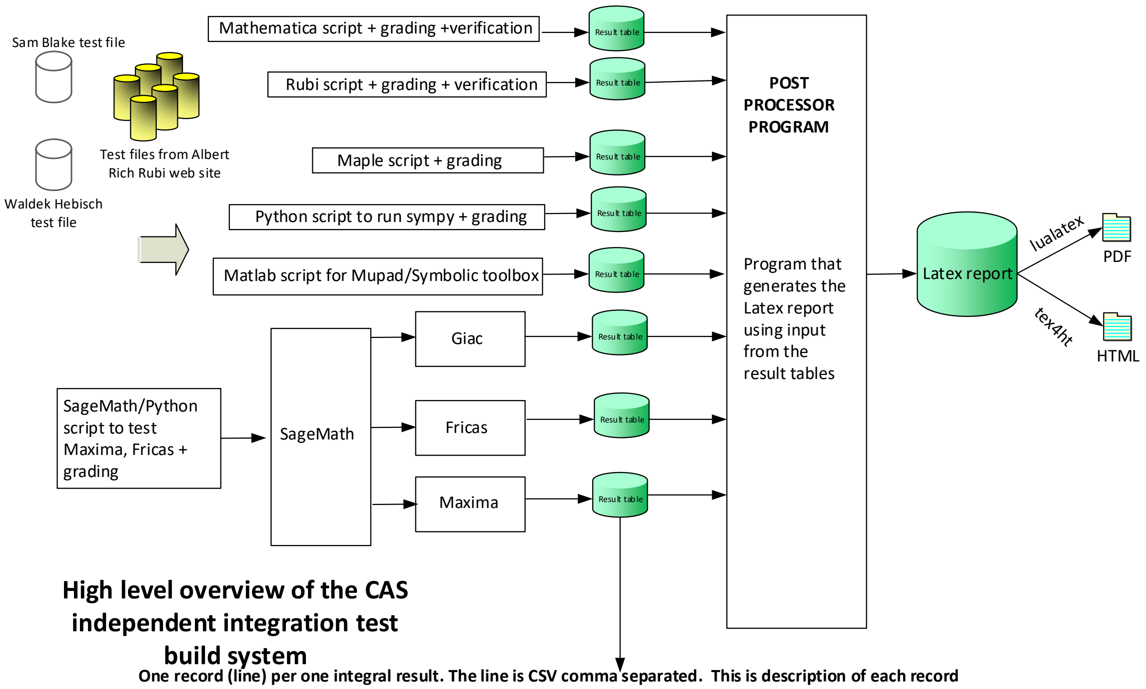
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 37, 39, 41, 42, 44, 46, 48, 50, 51, 53, 55, 57, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 72, 73, 76, 77, 78, 79, 80, 81, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 107, 108, 109, 110, 113, 114, 116, 117, 118, 119, 122, 123, 125, 126, 127, 128, 129, 133, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 235, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 265, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 300, 302, 304, 305, 306, 307, 308, 309, 311, 313, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 359, 362, 363, 364, 365, 370, 373, 374, 375, 379, 382, 383, 389, 391, 392, 393, 398, 402, 403, 411, 412, 413, 415, 416, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 458, 459, 460, 461, 462, 465, 468, 469, 470, 471, 472, 473, 476, 479, 480, 481, 482, 483, 484, 487, 498, 509, 513, 514, 515, 516, 521, 522, 523, 524, 525 }

B grade: { 6, 8, 17, 18, 26, 138, 159, 193, 195, 206, 225, 226, 227, 228, 315, 345, 355 }

C grade: { 28, 30, 32, 34, 36, 38, 40, 43, 45, 47, 49, 52, 54, 56, 58, 71, 74, 75, 82, 85, 86, 102, 105, 106, 111, 112, 115, 120, 121, 124, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 229, 230, 231, 232, 233, 234, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 258, 266, 267, 268, 269, 270, 271, 272, 273, 274, 299, 301, 303, 310, 312, 314, 356, 357, 358, 360, 361, 366, 367, 368, 369, 371, 372, 376, 377, 378, 380, 381, 384, 385, 386, 387, 388, 390, 394, 395, 396, 397, 399, 414, 418, 453, 463, 464, 466, 467, 474, 475, 477, 478, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 510, 511 }

F grade: { 63, 130, 131, 132, 134, 135, 136, 137, 139, 140, 400, 401, 404, 405, 406, 407, 408, 409, 410, 512, 517, 518, 519, 520 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 25, 27, 32, 34, 36, 38, 40, 43, 47, 49, 58, 71, 72, 73, 74, 75, 82, 83, 84, 85, 86, 88, 89, 91, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 129, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 184, 185, 186, 187, 188, 190, 192, 196, 197, 198, 199, 201, 203, 205, 207, 208, 209, 217, 218, 220, 221, 222, 230, 231, 232, 233, 234, 245, 246, 258, 278, 280, 282, 283, 284, 285, 286, 287, 290, 292, 293, 294, 295, 296, 297, 304, 305, 306, 307, 308, 322, 334, 344, 353, 357, 358, 359, 360, 361, 363, 368, 369, 370, 371, 372, 374, 377, 378, 379, 380, 381, 383, 386, 387, 388, 389, 390, 393, 398, 428, 431, 432, 433, 434, 435, 436, 439, 442, 443, 444, 445, 446, 449, 452, 453, 454, 455, 456, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 485, 486, 487, 488, 489, 496, 497, 498, 499, 500, 507, 509, 510, 522, 523, 524, 525 }

B grade: { 8, 15, 17, 24, 26, 28, 29, 30, 31, 33, 35, 37, 39, 41, 42, 44, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 76, 77, 78, 79, 80, 81, 87, 90, 92, 97, 98, 99, 100, 101, 109, 110, 120, 121, 122, 128, 154, 156, 158, 189, 191, 193, 194, 195, 200, 202, 204, 206, 210, 211, 212, 213, 214, 215, 216, 219, 223, 224, 225, 226, 227, 228, 229, 241, 242, 243, 244, 253, 254, 255, 256, 257, 265, 276, 277, 281, 288, 298, 300, 302, 309, 311, 313, 315, 316, 317, 318, 319, 320, 321, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 395, 396, 397, 399, 413, 417, 425, 508, 511 }

C grade: { 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 235, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 279, 289, 291, 299, 301, 303, 310, 312, 314, 352, 354, 355, 356, 362, 364, 365, 366, 367, 373, 375, 376, 382, 384, 385, 391, 392, 394, 411, 412, 414, 415, 416, 418, 426, 427, 429, 430, 437, 438, 440, 441, 447, 448, 450, 451, 457, 458, 459, 460, 461, 462, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 495, 501, 502, 503, 504, 505, 506, 521 }

F grade: { 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 419, 420, 421, 422, 423, 424, 512, 513, 514, 515, 516, 517, 518, 519, 520 }

2.1.4 Maxima

A grade: { 1, 3, 5, 6, 7, 10, 12, 14, 16, 19, 21, 23, 25, 27, 88, 93, 125, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 161, 162, 163, 164, 165, 166, 168, 184, 186, 188, 189, 190, 197, 199, 201, 203, 217, 218, 219, 220, 277, 279, 280, 283, 285, 286, 287, 293, 295, 296, 304, 306, 307, 428, 429, 430, 433, 439, 440, 441, 450, 451 }

B grade: { 2, 4, 8, 9, 11, 13, 15, 17, 18, 20, 22, 24, 26, 60, 61, 62, 63, 64, 65, 89, 94, 108, 117, 118, 148, 149, 156, 157, 158, 159, 160, 167, 169, 170, 185, 187, 191, 192, 193, 194, 195, 196, 198, 200, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 221, 222, 223, 224, 225, 226, 227, 228, 265, 276, 278, 281, 282, 284, 288, 289, 290, 291, 292, 294, 297, 298, 299, 300, 301, 302, 303, 305, 308, 309, 310, 311, 312, 313, 314, 315, 349, 350, 351, 383, 392, 393, 425, 426, 427, 431, 432, 434, 435, 436, 437, 438, 442, 443, 444, 445, 446, 447, 448, 449, 452, 453, 454, 455, 456 }

C grade: { 128 }

F grade: { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 391, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 5, 10, 12, 14, 16, 19, 21, 23, 25, 88, 93, 125, 127, 141, 142, 143, 144, 145, 147, 151, 152, 153, 155, 162, 184, 186, 188, 190, 199, 276, 277, 278, 279, 280, 283, 284, 285, 286, 293, 295, 298, 304, 306, 524, 525 }

B grade: { 2, 6, 7, 8, 9, 11, 13, 15, 17, 18, 20, 22, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 86, 98, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 129, 146, 148, 149, 150, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 187, 189, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 269, 270, 271, 272, 273, 274, 275, 281, 282, 287, 288, 289, 290, 291, 292, 294, 296, 297, 299, 300, 301, 302, 303, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 360, 361, 362, 363, 364, 365, 366, 367, 372, 373, 374, 375, 376, 379, 380, 381, 382, 383, 384, 385, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 411, 412, 413, 414, 415, 416, 417, 418, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 523 }

C grade: { 89, 94, 128, 268, 521 }

F grade: { 71, 72, 73, 82, 83, 84, 85, 87, 90, 91, 92, 95, 96, 97, 102, 103, 111, 112, 120, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 267, 357, 358, 359, 368, 369, 370, 371, 377, 378, 386, 387, 395, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 419, 420, 421, 422, 423, 424, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522 }

2.1.6 Sympy

A grade: { 5, 141, 143, 145, 150, 151, 152, 153, 161, 162, 163, 188, 279, 280, 286, 296, 307, 413 }

B grade: { 1, 2, 3, 4, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 34, 35, 60, 61, 62, 63, 64, 65, 142, 144, 182, 184, 185, 186, 187, 196, 197, 198, 199, 207, 208, 209, 217, 218, 265, 276, 277, 278, 283, 284, 285, 293, 294, 295, 304, 305, 306, 322, 334, 344, 349, 350, 351, 417 }

C grade: { 183 }

F grade: { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 146, 147, 148, 149, 154, 155, 156, 157, 158, 159, 160, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 189, 190, 191, 192, 193, 194, 195, 200, 201, 202, 203, 204, 205, 206, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 281, 282, 287, 288, 289, 290, 291, 292, 297, 298, 299, 300, 301, 302, 303, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525 }

2.1.7 Giac

A grade: { 1, 3, 5, 6, 7, 9, 10, 12, 14, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 42, 44, 46, 48, 50, 57, 59, 61, 62, 65, 88, 93, 105, 125, 141, 142, 143, 144, 145, 146, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 182, 183, 184, 186, 188, 189, 192, 197, 199, 201, 203, 204, 217, 218, 220, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 268, 269, 271, 272, 275, 276, 277, 279, 280, 283, 285, 287, 288, 290, 292, 293, 295, 297, 304, 306, 319, 321, 324, 326, 331, 333, 375, 380, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 521 }

B grade: { 2, 4, 8, 11, 13, 15, 16, 17, 18, 20, 22, 24, 26, 41, 51, 53, 55, 60, 63, 64, 66, 76, 77, 80, 81, 100, 101, 102, 106, 108, 109, 111, 117, 118, 119, 147, 148, 156, 157, 158, 166, 167, 168, 169, 170, 185, 187, 190, 191, 193, 194, 195, 196, 198, 200, 202, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 258, 265, 270, 273, 274, 278, 281, 282, 284, 286, 289, 291, 294, 296, 298, 299, 300, 301, 302, 303, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 328, 329, 336, 338, 339, 341, 343, 346, 348, 349, 350, 351, 352, 362, 363, 366, 367, 372, 376, 377, 381, 383, 384, 385, 386, 392, 393, 394, 479, 480, 485, 490, 501, 502 }

C grade: { 89, 94, 128, 266 }

F grade: { 28, 30, 32, 34, 36, 38, 40, 43, 45, 47, 49, 52, 54, 56, 58, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 96, 97, 98, 99, 103, 104, 107, 110, 112, 113, 114, 115, 116, 120, 121, 122, 123, 124, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 235, 267, 316, 318, 320, 322, 323, 325, 327, 330, 332, 334, 335, 337, 340, 342, 344, 345, 347, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 368, 369, 370, 371, 373, 374, 378, 379, 382, 387, 388, 389, 390, 391, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522, 523, 524, 525 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 60, 61, 62, 63, 64, 65, 88, 89, 108, 117, 118, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 265, 266, 268, 271, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 334, 344, 349, 350, 351, 353, 363, 374, 383, 392, 393, 403, 413, 417, 421, 424, 425, 426, 427, 428, 434, 435, 436, 437, 438, 439, 444, 445, 446, 447, 448, 449, 454, 455, 456, 521 }

C grade: { }

F grade: { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 267, 269, 270, 272, 273, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 391, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 418, 419, 420, 422, 423, 429, 430, 431, 432, 433, 440, 441, 442, 443, 450, 451, 452, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522, 523, 524, 525 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	68	67	150	122	258	125	76
N.S.	1	1.00	0.76	0.75	1.69	1.37	2.90	1.40	0.85
time (sec)	N/A	0.041	0.084	1.273	0.269	0.388	0.451	0.431	0.784

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	77	55	141	102	105	112	57
N.S.	1	1.00	1.45	1.04	2.66	1.92	1.98	2.11	1.08
time (sec)	N/A	0.040	0.024	0.647	0.262	0.415	0.274	0.430	0.641

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	46	97	64	158	79	50
N.S.	1	1.00	0.77	0.75	1.59	1.05	2.59	1.30	0.82
time (sec)	N/A	0.032	0.064	0.765	0.269	0.530	0.192	0.415	0.109

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	53	32	67	48	56	70	34
N.S.	1	1.00	1.66	1.00	2.09	1.50	1.75	2.19	1.06
time (sec)	N/A	0.021	0.021	0.553	0.274	0.421	0.119	0.423	0.592

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	36	32	38	30	51	38	23
N.S.	1	1.00	1.20	1.07	1.27	1.00	1.70	1.27	0.77
time (sec)	N/A	0.013	0.024	0.743	0.271	0.411	0.072	0.413	0.074

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	62	24	43	126	0	50	66
N.S.	1	1.00	2.48	0.96	1.72	5.04	0.00	2.00	2.64
time (sec)	N/A	0.024	0.025	0.767	0.284	0.412	0.000	0.408	0.135

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	36	0	28	23
N.S.	1	1.00	1.00	1.50	1.44	2.25	0.00	1.75	1.44
time (sec)	N/A	0.019	0.017	1.015	0.265	0.382	0.000	0.428	0.566

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	99	97	125	484	0	96	131
N.S.	1	1.00	2.48	2.42	3.12	12.10	0.00	2.40	3.28
time (sec)	N/A	0.031	0.028	1.077	0.265	0.403	0.000	0.442	0.127

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	49	62	113	159	0	61	61
N.S.	1	1.00	1.14	1.44	2.63	3.70	0.00	1.42	1.42
time (sec)	N/A	0.029	0.026	0.993	0.266	0.415	0.000	0.424	0.602

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	133	118	267	238	490	215	149
N.S.	1	1.00	0.91	0.81	1.83	1.63	3.36	1.47	1.02
time (sec)	N/A	0.130	0.144	1.342	0.271	0.392	1.041	0.468	0.902

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	154	97	247	213	204	196	112
N.S.	1	1.00	1.81	1.14	2.91	2.51	2.40	2.31	1.32
time (sec)	N/A	0.069	0.036	0.701	0.270	0.419	0.680	0.427	0.234

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	117	99	89	189	149	332	159	108
N.S.	1	1.06	0.90	0.81	1.72	1.35	3.02	1.45	0.98
time (sec)	N/A	0.077	0.133	0.971	0.273	0.396	0.494	0.437	0.234

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	111	66	157	122	128	138	76
N.S.	1	1.00	1.95	1.16	2.75	2.14	2.25	2.42	1.33
time (sec)	N/A	0.043	0.034	0.653	0.268	0.388	0.280	0.427	0.647

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	58	105	80	168	101	67
N.S.	1	1.00	0.83	0.81	1.46	1.11	2.33	1.40	0.93
time (sec)	N/A	0.016	0.089	0.809	0.278	0.376	0.209	0.436	0.098

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	104	112	102	492	0	110	116
N.S.	1	1.00	2.00	2.15	1.96	9.46	0.00	2.12	2.23
time (sec)	N/A	0.046	0.030	1.162	0.268	0.461	0.000	0.431	0.163

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	64	56	68	63	89	0	135	67
N.S.	1	1.28	1.12	1.36	1.26	1.78	0.00	2.70	1.34
time (sec)	N/A	0.056	0.115	1.288	0.273	0.489	0.000	0.430	0.662

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	134	137	157	902	0	125	179
N.S.	1	1.00	2.39	2.45	2.80	16.11	0.00	2.23	3.20
time (sec)	N/A	0.063	0.046	1.410	0.274	0.405	0.000	0.431	0.666

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	85	69	121	174	0	81	166
N.S.	1	1.00	2.12	1.72	3.02	4.35	0.00	2.02	4.15
time (sec)	N/A	0.054	0.485	1.355	0.277	0.421	0.000	0.435	0.619

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	162	177	405	406	777	325	239
N.S.	1	1.00	0.62	0.68	1.55	1.56	2.98	1.25	0.92
time (sec)	N/A	0.306	0.286	1.506	0.281	0.413	2.037	0.459	1.203

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	127	147	376	373	330	296	185
N.S.	1	1.00	1.10	1.28	3.27	3.24	2.87	2.57	1.61
time (sec)	N/A	0.090	0.526	0.796	0.272	0.408	1.429	0.449	0.430

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	130	140	306	269	561	251	181
N.S.	1	1.00	0.72	0.77	1.69	1.49	3.10	1.39	1.00
time (sec)	N/A	0.132	0.213	1.188	0.279	0.404	1.050	0.447	0.967

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	94	108	263	234	221	222	129
N.S.	1	1.00	1.19	1.37	3.33	2.96	2.80	2.81	1.63
time (sec)	N/A	0.060	0.202	0.684	0.295	0.433	0.680	0.463	0.258

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	95	102	197	165	350	177	123
N.S.	1	1.00	0.74	0.80	1.54	1.29	2.73	1.38	0.96
time (sec)	N/A	0.069	0.174	0.910	0.282	0.406	0.492	0.421	0.750

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	224	193	1128	0	202	184
N.S.	1	1.00	1.00	2.70	2.33	13.59	0.00	2.43	2.22
time (sec)	N/A	0.059	0.150	1.114	0.277	0.420	0.000	0.441	0.298

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	113	147	130	169	0	177	121
N.S.	1	1.00	0.82	1.07	0.95	1.23	0.00	1.29	0.88
time (sec)	N/A	0.138	1.243	1.138	0.275	0.397	0.000	0.460	0.758

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	210	208	217	1814	0	174	229
N.S.	1	1.00	2.53	2.51	2.61	21.86	0.00	2.10	2.76
time (sec)	N/A	0.074	3.041	1.297	0.275	0.460	0.000	0.455	0.216

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	107	113	161	281	0	154	222
N.S.	1	1.00	0.95	1.00	1.42	2.49	0.00	1.36	1.96
time (sec)	N/A	0.108	1.624	1.355	0.277	0.389	0.000	0.456	0.138

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	165	296	0	3242	0	0	415
N.S.	1	1.00	1.51	2.72	0.00	29.74	0.00	0.00	3.81
time (sec)	N/A	0.107	0.609	1.177	0.000	0.527	0.000	0.000	1.641

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	97	417	0	1725	0	208	266
N.S.	1	1.00	0.80	3.45	0.00	14.26	0.00	1.72	2.20
time (sec)	N/A	0.160	0.322	1.265	0.000	0.488	0.000	2.677	1.070

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	134	179	0	1668	0	0	348
N.S.	1	1.00	1.70	2.27	0.00	21.11	0.00	0.00	4.41
time (sec)	N/A	0.073	0.282	1.088	0.000	0.460	0.000	0.000	1.381

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	301	0	859	0	126	216
N.S.	1	1.00	0.90	3.81	0.00	10.87	0.00	1.59	2.73
time (sec)	N/A	0.086	0.166	1.042	0.000	0.462	0.000	1.680	0.938

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	107	93	0	746	0	0	293
N.S.	1	1.00	1.91	1.66	0.00	13.32	0.00	0.00	5.23
time (sec)	N/A	0.059	0.166	0.953	0.000	0.451	0.000	0.000	1.174

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	216	0	464	0	64	473
N.S.	1	1.00	1.00	4.32	0.00	9.28	0.00	1.28	9.46
time (sec)	N/A	0.061	0.088	1.010	0.000	0.438	0.000	1.268	1.224

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	91	51	0	502	367433	0	116
N.S.	1	1.00	2.28	1.28	0.00	12.55	9185.82	0.00	2.90
time (sec)	N/A	0.034	0.091	0.825	0.000	0.444	117.790	0.000	0.987

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	180	0	430	15870	47	146
N.S.	1	1.00	1.00	4.50	0.00	10.75	396.75	1.18	3.65
time (sec)	N/A	0.020	0.050	1.142	0.000	0.435	17.099	0.561	0.450

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	124	72	0	586	0	0	323
N.S.	1	1.00	2.07	1.20	0.00	9.77	0.00	0.00	5.38
time (sec)	N/A	0.053	0.153	1.390	0.000	0.424	0.000	0.000	1.059

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	208	0	675	0	72	176
N.S.	1	1.00	1.00	3.65	0.00	11.84	0.00	1.26	3.09
time (sec)	N/A	0.057	0.186	1.399	0.000	0.469	0.000	0.633	0.467

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	201	113	0	1837	0	0	571
N.S.	1	1.00	2.28	1.28	0.00	20.88	0.00	0.00	6.49
time (sec)	N/A	0.088	0.473	1.653	0.000	0.452	0.000	0.000	1.396

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	126	266	0	1972	0	118	350
N.S.	1	1.00	1.62	3.41	0.00	25.28	0.00	1.51	4.49
time (sec)	N/A	0.086	0.442	1.559	0.000	0.478	0.000	0.695	1.183

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	295	156	0	5809	0	0	1639
N.S.	1	1.00	2.27	1.20	0.00	44.68	0.00	0.00	12.61
time (sec)	N/A	0.147	5.252	5.524	0.000	0.550	0.000	0.000	5.659

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	155	347	0	4540	0	213	479
N.S.	1	1.00	1.41	3.15	0.00	41.27	0.00	1.94	4.35
time (sec)	N/A	0.096	0.947	1.582	0.000	0.476	0.000	0.685	1.209

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	99	319	0	1772	0	168	-1
N.S.	1	1.00	0.97	3.13	0.00	17.37	0.00	1.65	-0.01
time (sec)	N/A	0.123	0.580	1.088	0.000	0.486	0.000	1.361	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	141	155	0	1889	0	0	-1
N.S.	1	1.00	1.57	1.72	0.00	20.99	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.427	1.043	0.000	0.439	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	81	275	0	1523	0	135	-1
N.S.	1	1.00	0.96	3.27	0.00	18.13	0.00	1.61	-0.01
time (sec)	N/A	0.066	0.291	0.990	0.000	0.442	0.000	1.196	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	130	146	0	1628	0	0	-1
N.S.	1	1.00	1.60	1.80	0.00	20.10	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.250	0.895	0.000	0.436	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	96	285	0	1617	0	144	-1
N.S.	1	1.00	1.01	3.00	0.00	17.02	0.00	1.52	-0.01
time (sec)	N/A	0.048	0.198	1.195	0.000	0.425	0.000	0.568	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	176	171	0	2529	0	0	-1
N.S.	1	1.00	1.60	1.55	0.00	22.99	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.451	1.500	0.000	0.507	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	170	316	0	2988	0	229	-1
N.S.	1	1.00	1.20	2.23	0.00	21.04	0.00	1.61	-0.01
time (sec)	N/A	0.113	0.519	1.512	0.000	0.426	0.000	0.679	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	350	213	0	8059	0	0	-1
N.S.	1	1.00	2.17	1.32	0.00	50.06	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.884	1.777	0.000	0.531	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	210	371	0	7110	0	220	-1
N.S.	1	1.00	1.21	2.13	0.00	40.86	0.00	1.26	-0.01
time (sec)	N/A	0.175	0.834	1.611	0.000	0.467	0.000	0.717	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	104	357	0	5186	0	282	-1
N.S.	1	1.00	0.84	2.88	0.00	41.82	0.00	2.27	-0.01
time (sec)	N/A	0.092	0.933	1.017	0.000	0.649	0.000	3.796	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	170	281	0	6087	0	0	-1
N.S.	1	1.00	1.26	2.08	0.00	45.09	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.897	1.414	0.000	0.462	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	121	401	0	5519	0	277	-1
N.S.	1	1.00	0.87	2.88	0.00	39.71	0.00	1.99	-0.01
time (sec)	N/A	0.111	0.897	1.324	0.000	0.455	0.000	1.848	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	149	277	0	5152	0	0	-1
N.S.	1	1.00	1.26	2.35	0.00	43.66	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.531	0.950	0.000	0.469	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	132	418	0	5925	0	302	-1
N.S.	1	1.00	0.86	2.71	0.00	38.47	0.00	1.96	-0.01
time (sec)	N/A	0.115	0.811	1.431	0.000	0.516	0.000	0.813	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	237	308	0	9815	0	0	-1
N.S.	1	1.00	1.43	1.86	0.00	59.13	0.00	0.00	-0.01
time (sec)	N/A	0.179	2.231	1.744	0.000	0.526	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	225	442	0	9102	0	331	-1
N.S.	1	1.00	1.05	2.06	0.00	42.33	0.00	1.54	-0.00
time (sec)	N/A	0.217	1.175	1.667	0.000	0.480	0.000	0.810	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	419	350	0	22563	0	0	-1
N.S.	1	1.00	1.87	1.56	0.00	100.73	0.00	0.00	-0.00
time (sec)	N/A	0.291	1.815	1.943	0.000	0.692	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	167	497	0	17294	0	378	-1
N.S.	1	1.00	0.64	1.92	0.00	66.77	0.00	1.46	-0.00
time (sec)	N/A	0.258	1.865	1.869	0.000	0.540	0.000	0.816	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	17	10	20	14	10	10
N.S.	1	1.00	1.00	8.50	5.00	10.00	7.00	5.00	5.00
time (sec)	N/A	0.011	0.005	0.382	0.274	0.418	0.281	0.415	0.039

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	17	36	49	84	104	18	18
N.S.	1	1.00	1.55	3.27	4.45	7.64	9.45	1.64	1.64
time (sec)	N/A	0.012	0.003	0.388	0.268	0.484	0.711	0.418	0.588

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	27	52	111	185	260	24	24
N.S.	1	1.00	1.42	2.74	5.84	9.74	13.68	1.26	1.26
time (sec)	N/A	0.014	0.003	0.398	0.260	0.416	1.749	0.403	0.609

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	B	B	B	B	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	0	40	61	66	209	37	50
N.S.	1	1.00	0.00	2.67	4.07	4.40	13.93	2.47	3.33
time (sec)	N/A	0.009	0.018	0.395	0.469	0.453	0.834	0.416	0.154

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	92	87	216	2052	62	77
N.S.	1	1.00	0.95	2.49	2.35	5.84	55.46	1.68	2.08
time (sec)	N/A	0.018	0.094	0.408	0.479	0.404	4.601	0.411	0.693

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	124	111	575	5666	74	112
N.S.	1	1.00	0.93	2.25	2.02	10.45	103.02	1.35	2.04
time (sec)	N/A	0.040	0.133	0.406	0.476	0.427	11.807	0.412	0.610

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	114	339	0	3037	0	890	-1
N.S.	1	1.00	0.88	2.61	0.00	23.36	0.00	6.85	-0.01
time (sec)	N/A	0.102	0.463	1.826	0.000	0.523	0.000	0.702	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	97	200	0	2130	0	0	-1
N.S.	1	1.00	1.18	2.44	0.00	25.98	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.104	0.945	0.000	0.552	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	97	174	0	4423	0	0	-1
N.S.	1	1.00	1.15	2.07	0.00	52.65	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.089	1.151	0.000	0.510	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	104	230	0	1277	0	0	-1
N.S.	1	1.00	1.18	2.61	0.00	14.51	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.205	1.111	0.000	0.446	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	129	381	0	3395	0	0	-1
N.S.	1	1.00	0.90	2.65	0.00	23.58	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.376	103.824	0.000	0.610	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	210	512	0	25	0	0	-1
N.S.	1	1.00	0.70	1.71	0.00	0.08	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.963	2.565	0.000	0.094	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	170	353	0	25	0	0	-1
N.S.	1	1.00	0.96	1.99	0.00	0.14	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.588	1.160	0.000	0.120	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	69	140	0	16	0	0	-1
N.S.	1	1.00	1.15	2.33	0.00	0.27	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.069	0.946	0.000	0.106	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	151	166	0	542	0	0	-1
N.S.	1	1.00	0.76	0.83	0.00	2.72	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.426	0.986	0.000	0.101	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	208	436	0	2144	0	0	-1
N.S.	1	1.00	0.75	1.58	0.00	7.77	0.00	0.00	-0.00
time (sec)	N/A	0.196	2.012	5.329	0.000	0.129	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	151	483	0	4608	0	1585	-1
N.S.	1	1.00	0.85	2.73	0.00	26.03	0.00	8.95	-0.01
time (sec)	N/A	0.130	0.410	2.416	0.000	0.551	0.000	1.065	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	111	336	0	2977	0	897	-1
N.S.	1	1.00	0.92	2.78	0.00	24.60	0.00	7.41	-0.01
time (sec)	N/A	0.065	0.213	0.928	0.000	0.517	0.000	0.763	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	136	268	0	5565	0	0	-1
N.S.	1	1.00	1.07	2.11	0.00	43.82	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.379	1.251	0.000	0.574	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	143	297	0	6622	0	0	-1
N.S.	1	1.00	1.10	2.28	0.00	50.94	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.458	1.256	0.000	0.633	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	123	379	0	3133	0	2145	-1
N.S.	1	1.00	0.91	2.81	0.00	23.21	0.00	15.89	-0.01
time (sec)	N/A	0.100	0.385	1.387	0.000	0.610	0.000	0.897	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	174	569	0	7369	0	4684	-1
N.S.	1	1.00	0.87	2.86	0.00	37.03	0.00	23.54	-0.01
time (sec)	N/A	0.135	0.704	1.485	0.000	0.965	0.000	1.692	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	262	743	0	38	0	0	-1
N.S.	1	1.00	0.71	2.02	0.00	0.10	0.00	0.00	-0.00
time (sec)	N/A	0.316	1.906	1.405	0.000	0.113	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	213	535	0	38	0	0	-1
N.S.	1	1.00	0.90	2.27	0.00	0.16	0.00	0.00	-0.00
time (sec)	N/A	0.233	0.942	1.175	0.000	0.115	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	169	428	0	16	0	0	-1
N.S.	1	1.00	0.97	2.46	0.00	0.09	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.537	1.101	0.000	0.100	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	155	243	0	46	0	0	-1
N.S.	1	1.00	0.76	1.19	0.00	0.23	0.00	0.00	-0.00
time (sec)	N/A	0.149	0.738	1.143	0.000	0.104	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	213	454	0	2222	0	0	-1
N.S.	1	1.00	0.80	1.70	0.00	8.32	0.00	0.00	-0.00
time (sec)	N/A	0.210	2.662	1.338	0.000	0.135	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	208	609	0	45	0	0	-1
N.S.	1	1.00	0.90	2.62	0.00	0.19	0.00	0.00	-0.00
time (sec)	N/A	0.214	0.953	1.395	0.000	0.146	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	2	0	11	2
N.S.	1	1.00	1.00	1.27	1.00	0.18	0.00	1.00	0.18
time (sec)	N/A	0.017	0.005	0.806	0.482	0.413	0.000	0.413	0.066

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	25	14	0	11	5
N.S.	1	1.00	1.00	1.15	1.92	1.08	0.00	0.85	0.38
time (sec)	N/A	0.019	0.004	0.764	0.481	0.413	0.000	0.430	0.169

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	51	0	12	0	0	-1
N.S.	1	1.00	1.00	4.64	0.00	1.09	0.00	0.00	-0.09
time (sec)	N/A	0.009	0.018	1.000	0.000	0.092	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	61	0	10	0	0	-1
N.S.	1	1.00	1.00	1.85	0.00	0.30	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.023	0.774	0.000	0.092	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	54	109	0	12	0	0	-1
N.S.	1	1.00	1.29	2.60	0.00	0.29	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.038	1.179	0.000	0.096	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	21	23	17	0	25	-1
N.S.	1	1.00	0.79	0.72	0.79	0.59	0.00	0.86	-0.03
time (sec)	N/A	0.022	0.015	0.763	0.484	0.432	0.000	0.424	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	25	21	53	26	0	25	-1
N.S.	1	1.00	0.76	0.64	1.61	0.79	0.00	0.76	-0.03
time (sec)	N/A	0.026	0.006	0.713	0.498	0.550	0.000	0.419	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	103	0	12	0	0	-1
N.S.	1	1.00	1.00	2.29	0.00	0.27	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.051	0.984	0.000	0.082	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	78	106	0	10	0	0	-1
N.S.	1	1.00	0.90	1.22	0.00	0.11	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.087	0.974	0.000	0.088	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	132	333	0	12	0	0	-1
N.S.	1	1.00	1.07	2.71	0.00	0.10	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.282	1.138	0.000	0.099	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	98	204	0	2116	0	0	-1
N.S.	1	1.00	1.18	2.46	0.00	25.49	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.197	1.160	0.000	0.498	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	49	108	0	1654	0	0	-1
N.S.	1	1.00	1.20	2.63	0.00	40.34	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.082	0.824	0.000	0.493	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	49	113	0	572	0	78	-1
N.S.	1	1.00	1.17	2.69	0.00	13.62	0.00	1.86	-0.02
time (sec)	N/A	0.050	0.126	1.146	0.000	0.618	0.000	0.489	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	102	234	0	1285	0	669	-1
N.S.	1	1.00	1.15	2.63	0.00	14.44	0.00	7.52	-0.01
time (sec)	N/A	0.078	0.235	2.144	0.000	0.474	0.000	0.564	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	168	356	0	25	0	804	-1
N.S.	1	1.00	0.73	1.55	0.00	0.11	0.00	3.51	-0.00
time (sec)	N/A	0.148	0.691	1.165	0.000	0.094	0.000	1.146	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	89	113	0	25	0	0	-1
N.S.	1	1.00	0.70	0.88	0.00	0.20	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.193	0.905	0.000	0.095	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	68	86	0	147	0	0	-1
N.S.	1	1.00	1.13	1.43	0.00	2.45	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.062	0.879	0.000	0.101	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	150	189	0	541	0	276	-1
N.S.	1	1.00	1.12	1.41	0.00	4.04	0.00	2.06	-0.01
time (sec)	N/A	0.103	0.397	1.918	0.000	0.127	0.000	0.484	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	201	456	0	2131	0	1118	-1
N.S.	1	1.00	0.75	1.71	0.00	7.98	0.00	4.19	-0.00
time (sec)	N/A	0.200	2.388	2.082	0.000	0.186	0.000	1.277	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	98	146	0	3038	0	0	-1
N.S.	1	1.00	1.18	1.76	0.00	36.60	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.300	1.098	0.000	0.570	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	43	32	246	296	0	99	191
N.S.	1	1.00	1.19	0.89	6.83	8.22	0.00	2.75	5.31
time (sec)	N/A	0.036	0.104	0.700	0.504	0.455	0.000	0.624	0.921

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	98	154	0	1641	0	198	-1
N.S.	1	1.00	1.17	1.83	0.00	19.54	0.00	2.36	-0.01
time (sec)	N/A	0.075	0.267	1.233	0.000	0.475	0.000	0.572	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	134	251	0	4441	0	0	-1
N.S.	1	1.00	0.96	1.81	0.00	31.95	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.500	5.719	0.000	0.640	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	211	500	0	55	0	1143	-1
N.S.	1	1.00	0.62	1.47	0.00	0.16	0.00	3.35	-0.00
time (sec)	N/A	0.232	0.840	1.273	0.000	0.114	0.000	2.870	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	156	313	0	55	0	0	-1
N.S.	1	1.00	0.61	1.22	0.00	0.21	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.718	1.181	0.000	0.125	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	151	127	0	1155	0	0	-1
N.S.	1	1.00	0.87	0.73	0.00	6.68	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.318	1.008	0.000	0.104	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	100	253	0	1464	0	0	-1
N.S.	1	1.00	0.87	2.20	0.00	12.73	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.116	1.175	0.000	0.106	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	185	284	0	2829	0	0	-1
N.S.	1	1.00	0.64	0.98	0.00	9.76	0.00	0.00	-0.00
time (sec)	N/A	0.214	0.901	2.413	0.000	0.137	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	130	230	0	7938	0	0	-1
N.S.	1	1.00	0.91	1.61	0.00	55.51	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.599	1.404	0.000	0.730	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	67	64	955	1214	0	276	148
N.S.	1	1.00	0.77	0.74	10.98	13.95	0.00	3.17	1.70
time (sec)	N/A	0.069	0.223	1.016	0.558	0.550	0.000	1.136	1.618

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	63	57	499	1186	0	254	133
N.S.	1	1.00	0.80	0.72	6.32	15.01	0.00	3.22	1.68
time (sec)	N/A	0.045	0.121	1.394	0.521	0.707	0.000	0.886	1.285

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	130	236	0	5342	0	404	-1
N.S.	1	1.00	0.96	1.74	0.00	39.28	0.00	2.97	-0.01
time (sec)	N/A	0.116	0.544	2.125	0.000	0.658	0.000	0.658	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	207	868	0	71	0	0	-1
N.S.	1	1.00	0.60	2.52	0.00	0.21	0.00	0.00	-0.00
time (sec)	N/A	0.251	1.402	1.445	0.000	0.131	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	198	659	0	4985	0	0	-1
N.S.	1	1.00	0.81	2.70	0.00	20.43	0.00	0.00	-0.00
time (sec)	N/A	0.174	1.125	1.155	0.000	0.182	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	187	601	0	4684	0	0	-1
N.S.	1	1.00	0.78	2.49	0.00	19.44	0.00	0.00	-0.00
time (sec)	N/A	0.216	0.961	1.534	0.000	0.185	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	190	406	0	5442	0	0	-1
N.S.	1	1.00	0.76	1.62	0.00	21.68	0.00	0.00	-0.00
time (sec)	N/A	0.198	0.929	1.812	0.000	0.199	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	234	747	0	8769	0	0	-1
N.S.	1	1.00	0.61	1.94	0.00	22.78	0.00	0.00	-0.00
time (sec)	N/A	0.328	1.607	1.914	0.000	0.313	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	19	15	5	8	0	5	-1
N.S.	1	1.00	1.36	1.07	0.36	0.57	0.00	0.36	-0.07
time (sec)	N/A	0.015	0.009	0.790	0.500	0.445	0.000	0.410	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	0	1	0	0	-1
N.S.	1	1.00	1.00	3.73	0.00	0.09	0.00	0.00	-0.09
time (sec)	N/A	0.008	0.026	0.769	0.000	0.089	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	61	0	42	0	0	-1
N.S.	1	1.00	1.00	1.85	0.00	1.27	0.00	0.00	-0.03
time (sec)	N/A	0.015	0.032	0.823	0.000	0.117	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	34	5	13	0	5	-1
N.S.	1	1.00	1.31	2.12	0.31	0.81	0.00	0.31	-0.06
time (sec)	N/A	0.016	0.007	0.760	0.534	0.448	0.000	0.415	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	53	63	0	136	0	0	-1
N.S.	1	1.00	1.26	1.50	0.00	3.24	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.038	0.895	0.000	0.104	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	0	0	0	27	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.21	0.00	0.00	-0.01
time (sec)	N/A	0.087	6.253	1.813	0.000	0.499	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.11	0.00	0.00	-0.00
time (sec)	N/A	0.182	7.565	1.720	0.000	0.427	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.18	0.00	0.00	-0.01
time (sec)	N/A	0.087	9.466	1.913	0.000	0.417	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	0	0	23	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.158	0.698	0.000	0.402	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	23	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.062	2.321	1.236	0.000	0.406	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.064	81.551	1.270	0.000	0.446	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.066	180.002	1.283	0.000	0.425	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.24	0.00	0.00	-0.01
time (sec)	N/A	0.075	6.753	1.990	0.000	0.420	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	250	0	0	25	0	0	-1
N.S.	1	1.00	2.48	0.00	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.551	1.839	0.000	0.414	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.070	2.939	1.082	0.000	0.439	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.24	0.00	0.00	-0.01
time (sec)	N/A	0.070	5.454	1.202	0.000	0.431	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	81	93	164	188	192	182	85
N.S.	1	1.00	0.76	0.88	1.55	1.77	1.81	1.72	0.80
time (sec)	N/A	0.068	0.082	1.326	0.286	0.428	0.652	0.407	0.260

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	66	78	143	135	194	152	67
N.S.	1	1.00	0.67	0.79	1.44	1.36	1.96	1.54	0.68
time (sec)	N/A	0.076	0.098	1.312	0.279	0.426	0.437	0.435	0.454

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	79	63	120	105	117	122	55
N.S.	1	1.00	1.13	0.90	1.71	1.50	1.67	1.74	0.79
time (sec)	N/A	0.057	0.073	0.982	0.280	0.445	0.267	0.417	0.116

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	45	47	74	63	121	92	42
N.S.	1	1.00	0.75	0.78	1.23	1.05	2.02	1.53	0.70
time (sec)	N/A	0.046	0.096	1.056	0.283	0.467	0.189	0.404	0.198

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	33	59	47	41	59	29
N.S.	1	1.00	1.06	1.03	1.84	1.47	1.28	1.84	0.91
time (sec)	N/A	0.014	0.011	0.853	0.268	0.452	0.115	0.412	0.625

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	72	40	50	258	0	62	73
N.S.	1	1.00	1.80	1.00	1.25	6.45	0.00	1.55	1.82
time (sec)	N/A	0.039	0.047	1.552	0.264	0.562	0.000	0.406	0.691

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	48	47	40	0	59	47
N.S.	1	1.00	1.46	2.00	1.96	1.67	0.00	2.46	1.96
time (sec)	N/A	0.037	0.027	1.833	0.276	0.437	0.000	0.410	0.094

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	63	71	91	521	0	73	102
N.S.	1	1.00	1.62	1.82	2.33	13.36	0.00	1.87	2.62
time (sec)	N/A	0.045	0.014	2.059	0.284	0.453	0.000	0.417	0.105

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	76	63	131	652	0	62	110
N.S.	1	1.00	1.85	1.54	3.20	15.90	0.00	1.51	2.68
time (sec)	N/A	0.039	0.025	2.052	0.275	0.466	0.000	0.410	0.686

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	125	152	272	355	325	301	149
N.S.	1	1.00	0.65	0.79	1.42	1.85	1.69	1.57	0.78
time (sec)	N/A	0.128	0.500	1.422	0.286	0.439	1.401	0.453	0.932

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	133	135	237	274	340	260	126
N.S.	1	1.00	0.74	0.75	1.32	1.52	1.89	1.44	0.70
time (sec)	N/A	0.120	0.139	1.534	0.275	0.423	0.977	0.434	1.627

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	92	109	180	220	219	219	104
N.S.	1	1.00	0.71	0.84	1.38	1.69	1.68	1.68	0.80
time (sec)	N/A	0.086	0.305	1.176	0.273	0.409	0.698	0.433	0.247

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	94	93	151	160	212	178	85
N.S.	1	1.00	0.82	0.82	1.32	1.40	1.86	1.56	0.75
time (sec)	N/A	0.062	0.107	1.316	0.267	0.416	0.437	0.418	0.461

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	96	171	140	1052	0	154	177
N.S.	1	1.00	1.09	1.94	1.59	11.95	0.00	1.75	2.01
time (sec)	N/A	0.071	0.232	1.998	0.263	0.460	0.000	0.432	0.214

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	92	124	113	142	0	149	123
N.S.	1	1.00	1.12	1.51	1.38	1.73	0.00	1.82	1.50
time (sec)	N/A	0.074	0.200	1.991	0.283	0.444	0.000	0.455	0.738

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	105	144	152	1616	0	162	175
N.S.	1	1.00	1.36	1.87	1.97	20.99	0.00	2.10	2.27
time (sec)	N/A	0.075	0.029	2.155	0.267	0.478	0.000	0.438	0.705

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	81	108	170	1748	0	151	163
N.S.	1	1.00	1.07	1.42	2.24	23.00	0.00	1.99	2.14
time (sec)	N/A	0.064	0.291	2.249	0.290	0.483	0.000	0.438	0.133

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	149	171	188	2119	0	172	355
N.S.	1	1.00	1.66	1.90	2.09	23.54	0.00	1.91	3.94
time (sec)	N/A	0.097	0.037	2.227	0.266	0.592	0.000	0.464	0.143

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	197	130	303	2310	0	141	351
N.S.	1	1.00	2.24	1.48	3.44	26.25	0.00	1.60	3.99
time (sec)	N/A	0.074	0.669	2.278	0.284	0.467	0.000	0.452	0.652

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	235	209	316	3607	0	204	434
N.S.	1	1.00	1.77	1.57	2.38	27.12	0.00	1.53	3.26
time (sec)	N/A	0.124	0.044	2.274	0.280	0.474	0.000	0.500	0.157

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	194	220	387	568	498	431	231
N.S.	1	1.00	0.67	0.76	1.33	1.95	1.71	1.48	0.79
time (sec)	N/A	0.154	0.368	1.764	0.271	0.434	3.054	0.471	1.086

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	184	192	318	453	496	379	189
N.S.	1	1.00	0.69	0.72	1.19	1.70	1.86	1.42	0.71
time (sec)	N/A	0.166	0.316	1.788	0.266	0.446	2.281	0.476	2.953

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	159	167	280	380	340	327	164
N.S.	1	1.00	0.78	0.82	1.37	1.86	1.67	1.60	0.80
time (sec)	N/A	0.094	0.180	1.332	0.281	0.402	1.532	0.429	0.931

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	158	325	257	2609	0	279	315
N.S.	1	1.00	0.79	1.62	1.28	12.98	0.00	1.39	1.57
time (sec)	N/A	0.135	0.192	2.079	0.291	0.447	0.000	0.481	0.504

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	140	268	220	302	0	276	252
N.S.	1	1.00	0.92	1.76	1.45	1.99	0.00	1.82	1.66
time (sec)	N/A	0.101	0.766	1.898	0.285	0.489	0.000	0.496	0.396

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	150	258	244	3627	0	289	290
N.S.	1	1.00	0.96	1.65	1.56	23.25	0.00	1.85	1.86
time (sec)	N/A	0.124	2.287	1.999	0.289	0.495	0.000	0.516	0.929

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	169	214	260	3801	0	285	267
N.S.	1	1.00	1.31	1.66	2.02	29.47	0.00	2.21	2.07
time (sec)	N/A	0.086	0.313	2.082	0.272	0.468	0.000	0.502	0.880

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	218	249	255	4541	0	329	451
N.S.	1	1.00	1.47	1.68	1.72	30.68	0.00	2.22	3.05
time (sec)	N/A	0.129	6.107	2.105	0.277	0.484	0.000	0.535	0.868

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	225	204	365	4629	0	270	432
N.S.	1	1.00	1.72	1.56	2.79	35.34	0.00	2.06	3.30
time (sec)	N/A	0.093	1.274	2.081	0.283	0.478	0.000	0.508	0.807

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	236	254	355	6210	0	327	486
N.S.	1	1.00	1.42	1.53	2.14	37.41	0.00	1.97	2.93
time (sec)	N/A	0.148	1.125	2.126	0.279	0.506	0.000	0.524	0.294

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	168	236	0	28816	0	0	1579
N.S.	1	1.00	0.51	0.72	0.00	87.85	0.00	0.00	4.81
time (sec)	N/A	0.625	0.256	2.303	0.000	5.587	0.000	0.000	10.805

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	299	190	0	28427	0	0	1114
N.S.	1	1.00	1.01	0.64	0.00	96.36	0.00	0.00	3.78
time (sec)	N/A	0.418	0.233	2.210	0.000	3.343	0.000	0.000	11.481

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	214	123	0	20941	0	0	906
N.S.	1	1.00	0.71	0.41	0.00	69.11	0.00	0.00	2.99
time (sec)	N/A	0.480	0.241	2.145	0.000	1.505	0.000	0.000	23.502

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	145	124	0	27931	0	0	1498
N.S.	1	1.00	0.49	0.42	0.00	95.00	0.00	0.00	5.10
time (sec)	N/A	0.405	0.146	2.099	0.000	1.242	0.000	0.000	10.762

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	275	78	0	24063	0	0	932
N.S.	1	1.00	1.05	0.30	0.00	91.84	0.00	0.00	3.56
time (sec)	N/A	0.216	0.113	2.025	0.000	1.675	0.000	0.000	11.126

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	199	82	0	18312	0	0	857
N.S.	1	1.00	0.69	0.28	0.00	63.14	0.00	0.00	2.96
time (sec)	N/A	0.320	0.142	2.187	0.000	1.258	0.000	0.000	21.839

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	131	87	0	24084	0	0	1261
N.S.	1	1.00	0.47	0.31	0.00	86.01	0.00	0.00	4.50
time (sec)	N/A	0.237	0.106	1.958	0.000	1.219	0.000	0.000	9.486

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	295	98	0	28005	0	0	2500
N.S.	1	1.00	1.03	0.34	0.00	97.92	0.00	0.00	8.74
time (sec)	N/A	0.356	0.163	2.592	0.000	2.617	0.000	0.000	55.942

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	230	118	0	21133	0	0	1293
N.S.	1	1.00	0.76	0.39	0.00	69.52	0.00	0.00	4.25
time (sec)	N/A	0.437	0.299	2.660	0.000	1.239	0.000	0.000	23.059

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	178	138	0	29179	0	0	2500
N.S.	1	1.00	0.55	0.43	0.00	90.62	0.00	0.00	7.76
time (sec)	N/A	0.428	0.362	2.870	0.000	8.963	0.000	0.000	89.246

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	370	164	0	30233	0	0	2500
N.S.	1	1.00	1.17	0.52	0.00	95.37	0.00	0.00	7.89
time (sec)	N/A	0.364	3.915	2.804	0.000	6.678	0.000	0.000	59.687

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	156	82	0	185	5697	102	203
N.S.	1	1.00	1.12	0.59	0.00	1.33	40.99	0.73	1.46
time (sec)	N/A	0.140	1.046	0.638	0.000	0.443	32.874	0.412	1.772

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	156	80	0	180	5697	106	225
N.S.	1	1.00	1.17	0.60	0.00	1.35	42.83	0.80	1.69
time (sec)	N/A	0.135	0.909	0.636	0.000	0.443	40.704	0.409	2.073

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	82	82	175	174	306	155	88
N.S.	1	1.00	0.74	0.74	1.58	1.57	2.76	1.40	0.79
time (sec)	N/A	0.106	0.110	1.133	0.275	0.438	1.222	0.443	0.916

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	93	70	157	155	128	142	66
N.S.	1	1.00	1.39	1.04	2.34	2.31	1.91	2.12	0.99
time (sec)	N/A	0.045	0.028	0.698	0.271	0.407	0.629	0.433	0.794

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	63	61	122	109	206	113	64
N.S.	1	1.00	0.76	0.73	1.47	1.31	2.48	1.36	0.77
time (sec)	N/A	0.068	0.065	1.033	0.269	0.418	0.460	0.424	0.159

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	69	47	97	91	80	100	46
N.S.	1	1.00	1.50	1.02	2.11	1.98	1.74	2.17	1.00
time (sec)	N/A	0.023	0.022	0.707	0.264	0.569	0.275	0.420	0.101

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	39	66	59	100	66	38
N.S.	1	1.00	0.94	0.75	1.27	1.13	1.92	1.27	0.73
time (sec)	N/A	0.022	0.043	1.037	0.264	0.380	0.174	0.417	0.700

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	70	88	71	395	0	78	96
N.S.	1	1.00	1.67	2.10	1.69	9.40	0.00	1.86	2.29
time (sec)	N/A	0.031	0.024	1.095	0.260	0.404	0.000	0.430	0.133

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	45	55	54	70	0	88	54
N.S.	1	1.00	1.15	1.41	1.38	1.79	0.00	2.26	1.38
time (sec)	N/A	0.037	0.094	1.089	0.271	0.391	0.000	0.430	0.723

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	82	95	115	690	0	107	126
N.S.	1	1.00	1.74	2.02	2.45	14.68	0.00	2.28	2.68
time (sec)	N/A	0.038	0.025	1.247	0.269	0.410	0.000	0.435	0.722

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	40	37	97	129	0	45	81
N.S.	1	1.00	1.29	1.19	3.13	4.16	0.00	1.45	2.61
time (sec)	N/A	0.036	0.014	1.387	0.263	0.419	0.000	0.447	0.702

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	139	121	174	1476	0	124	242
N.S.	1	1.00	2.17	1.89	2.72	23.06	0.00	1.94	3.78
time (sec)	N/A	0.044	0.026	1.278	0.279	0.466	0.000	0.430	0.743

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	71	98	228	333	0	97	337
N.S.	1	1.00	1.51	2.09	4.85	7.09	0.00	2.06	7.17
time (sec)	N/A	0.029	0.031	1.204	0.270	0.594	0.000	0.430	0.722

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	199	213	268	3115	0	207	472
N.S.	1	1.00	2.16	2.32	2.91	33.86	0.00	2.25	5.13
time (sec)	N/A	0.063	0.033	1.286	0.285	0.417	0.000	0.451	0.774

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	207	138	307	404	280	278	150
N.S.	1	1.00	1.72	1.15	2.56	3.37	2.33	2.32	1.25
time (sec)	N/A	0.089	0.052	0.849	0.274	0.382	2.993	0.434	0.357

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	139	130	260	305	484	241	149
N.S.	1	1.00	0.86	0.81	1.61	1.89	3.01	1.50	0.93
time (sec)	N/A	0.193	0.273	1.523	0.267	0.433	2.246	0.447	0.405

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	164	107	226	279	204	220	111
N.S.	1	1.00	1.78	1.16	2.46	3.03	2.22	2.39	1.21
time (sec)	N/A	0.058	0.036	0.777	0.268	0.407	1.544	0.479	0.883

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	92	100	183	205	332	183	108
N.S.	1	1.00	0.74	0.80	1.46	1.64	2.66	1.46	0.86
time (sec)	N/A	0.121	0.121	1.120	0.274	0.406	1.041	0.424	0.289

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	146	234	177	1575	0	196	198
N.S.	1	1.00	1.59	2.54	1.92	17.12	0.00	2.13	2.15
time (sec)	N/A	0.065	0.038	1.316	0.274	0.429	0.000	0.460	0.344

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	77	168	146	217	0	179	148
N.S.	1	1.00	0.75	1.63	1.42	2.11	0.00	1.74	1.44
time (sec)	N/A	0.135	0.230	1.246	0.274	0.385	0.000	0.456	0.232

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	200	204	2272	0	182	214
N.S.	1	1.00	1.57	2.17	2.22	24.70	0.00	1.98	2.33
time (sec)	N/A	0.089	0.051	1.524	0.282	0.511	0.000	0.464	0.834

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	115	165	300	0	142	164
N.S.	1	1.00	0.75	1.26	1.81	3.30	0.00	1.56	1.80
time (sec)	N/A	0.129	0.240	1.323	0.282	0.402	0.000	0.470	0.172

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	186	195	234	3356	0	179	328
N.S.	1	1.00	1.84	1.93	2.32	33.23	0.00	1.77	3.25
time (sec)	N/A	0.103	0.041	1.523	0.274	0.441	0.000	0.588	0.215

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	67	140	267	457	0	166	397
N.S.	1	1.00	0.80	1.67	3.18	5.44	0.00	1.98	4.73
time (sec)	N/A	0.109	0.602	1.563	0.281	0.381	0.000	0.485	0.746

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	240	250	299	4500	0	243	535
N.S.	1	1.00	2.16	2.25	2.69	40.54	0.00	2.19	4.82
time (sec)	N/A	0.121	0.038	1.537	0.273	0.417	0.000	0.506	0.818

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	288	259	600	1030	592	520	319
N.S.	1	1.00	1.31	1.18	2.73	4.68	2.69	2.36	1.45
time (sec)	N/A	0.159	1.611	1.351	0.275	0.374	17.337	0.542	1.762

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	185	222	501	795	484	446	266
N.S.	1	1.00	1.01	1.21	2.74	4.34	2.64	2.44	1.45
time (sec)	N/A	0.129	1.739	1.127	0.286	0.542	9.950	0.520	1.326

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	157	183	399	594	377	372	211
N.S.	1	1.00	1.10	1.28	2.79	4.15	2.64	2.60	1.48
time (sec)	N/A	0.112	0.739	0.956	0.270	0.392	5.433	0.511	1.069

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	139	533	327	3824	0	377	326
N.S.	1	1.00	0.88	3.37	2.07	24.20	0.00	2.39	2.06
time (sec)	N/A	0.095	0.276	1.354	0.282	0.425	0.000	0.536	0.652

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	155	379	334	4895	0	300	326
N.S.	1	1.00	1.05	2.56	2.26	33.07	0.00	2.03	2.20
time (sec)	N/A	0.153	0.280	1.483	0.286	0.423	0.000	0.579	1.189

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	173	336	340	6441	0	271	421
N.S.	1	1.00	1.22	2.37	2.39	45.36	0.00	1.91	2.96
time (sec)	N/A	0.190	0.290	1.549	0.280	0.464	0.000	0.624	1.154

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	223	358	390	8547	0	321	633
N.S.	1	1.00	1.43	2.29	2.50	54.79	0.00	2.06	4.06
time (sec)	N/A	0.211	0.517	1.520	0.285	0.486	0.000	0.592	1.135

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	219	375	463	10848	0	335	759
N.S.	1	1.00	1.28	2.19	2.71	63.44	0.00	1.96	4.44
time (sec)	N/A	0.235	1.273	1.520	0.298	0.478	0.000	0.601	1.119

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	265	548	573	13503	0	477	1194
N.S.	1	1.00	1.40	2.90	3.03	71.44	0.00	2.52	6.32
time (sec)	N/A	0.263	1.717	1.539	0.287	0.481	0.000	0.640	1.114

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	246	632	720	17811	0	537	1314
N.S.	1	1.00	1.12	2.87	3.27	80.96	0.00	2.44	5.97
time (sec)	N/A	0.280	1.485	1.502	0.299	0.532	0.000	0.619	1.103

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	189	215	442	627	877	401	393
N.S.	1	1.00	0.74	0.84	1.73	2.46	3.44	1.57	1.54
time (sec)	N/A	0.399	0.506	1.888	0.283	0.386	7.778	0.567	0.788

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	156	177	344	461	666	327	210
N.S.	1	1.00	0.74	0.84	1.63	2.18	3.16	1.55	1.00
time (sec)	N/A	0.283	0.314	1.478	0.275	0.385	4.386	0.425	0.563

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	134	357	284	474	0	355	265
N.S.	1	1.00	0.74	1.97	1.57	2.62	0.00	1.96	1.46
time (sec)	N/A	0.312	0.566	1.250	0.290	0.380	0.000	0.559	1.086

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	131	264	282	567	0	285	269
N.S.	1	1.00	0.81	1.64	1.75	3.52	0.00	1.77	1.67
time (sec)	N/A	0.268	0.549	1.419	0.290	0.387	0.000	0.587	1.069

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	110	253	359	768	0	286	511
N.S.	1	1.00	0.74	1.71	2.43	5.19	0.00	1.93	3.45
time (sec)	N/A	0.256	0.807	1.552	0.275	0.382	0.000	0.572	1.033

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	106	207	537	928	0	253	749
N.S.	1	1.00	0.80	1.56	4.04	6.98	0.00	1.90	5.63
time (sec)	N/A	0.195	0.551	1.539	0.281	0.424	0.000	0.581	1.032

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	115	322	842	1314	0	360	1500
N.S.	1	1.00	0.82	2.30	6.01	9.39	0.00	2.57	10.71
time (sec)	N/A	0.153	0.453	1.569	0.274	0.398	0.000	0.590	1.077

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	239	328	1291	1607	0	359	1955
N.S.	1	1.00	1.63	2.23	8.78	10.93	0.00	2.44	13.30
time (sec)	N/A	0.102	6.086	1.543	0.285	0.393	0.000	0.590	0.979

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	350	564	1916	2323	0	563	2500
N.S.	1	1.00	2.43	3.92	13.31	16.13	0.00	3.91	17.36
time (sec)	N/A	0.094	2.207	1.495	0.300	0.398	0.000	0.598	1.141

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	404	622	2731	2967	0	621	2500
N.S.	1	1.00	2.22	3.42	15.01	16.30	0.00	3.41	13.74
time (sec)	N/A	0.114	3.118	1.611	0.302	0.387	0.000	0.628	1.222

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	458	680	3719	3585	0	679	2500
N.S.	1	1.00	2.07	3.08	16.83	16.22	0.00	3.07	11.31
time (sec)	N/A	0.139	4.628	1.638	0.298	0.380	0.000	0.642	1.310

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	512	738	4883	4259	0	737	2500
N.S.	1	1.00	2.06	2.98	19.69	17.17	0.00	2.97	10.08
time (sec)	N/A	0.156	5.992	1.682	0.331	0.562	0.000	0.649	1.416

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	390	263	0	1617	0	753	1124
N.S.	1	1.00	2.64	1.78	0.00	10.93	0.00	5.09	7.59
time (sec)	N/A	0.187	0.326	1.691	0.000	0.445	0.000	0.590	9.796

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	235	174	0	1247	0	525	1046
N.S.	1	1.00	1.69	1.25	0.00	8.97	0.00	3.78	7.53
time (sec)	N/A	0.159	0.199	1.616	0.000	0.425	0.000	0.581	7.852

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	365	148	0	975	0	314	975
N.S.	1	1.00	3.17	1.29	0.00	8.48	0.00	2.73	8.48
time (sec)	N/A	0.098	0.127	1.411	0.000	0.406	0.000	0.543	6.158

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	221	132	0	979	0	332	1007
N.S.	1	1.00	1.77	1.06	0.00	7.83	0.00	2.66	8.06
time (sec)	N/A	0.083	0.120	1.710	0.000	0.397	0.000	0.543	8.151

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	385	164	0	1067	0	419	1243
N.S.	1	1.00	2.83	1.21	0.00	7.85	0.00	3.08	9.14
time (sec)	N/A	0.143	0.181	1.978	0.000	0.431	0.000	0.466	9.193

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	265	181	0	1954	0	467	1517
N.S.	1	1.00	1.44	0.98	0.00	10.62	0.00	2.54	8.24
time (sec)	N/A	0.159	0.276	2.159	0.000	0.443	0.000	0.506	12.211

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	158	206	0	1441	0	0	2191
N.S.	1	1.00	0.90	1.18	0.00	8.23	0.00	0.00	12.52
time (sec)	N/A	0.180	0.657	1.687	0.000	0.435	0.000	0.000	11.239

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	143	139	0	1009	0	13	1861
N.S.	1	1.00	1.13	1.09	0.00	7.94	0.00	0.10	14.65
time (sec)	N/A	0.146	0.334	1.522	0.000	0.420	0.000	0.576	11.048

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	127	94	0	975	0	1	1859
N.S.	1	1.00	1.02	0.75	0.00	7.80	0.00	0.01	14.87
time (sec)	N/A	0.088	0.261	1.785	0.000	0.406	0.000	0.528	12.901

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	128	102	0	975	0	1	1787
N.S.	1	1.00	1.11	0.89	0.00	8.48	0.00	0.01	15.54
time (sec)	N/A	0.074	0.167	1.674	0.000	0.418	0.000	0.426	10.105

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	143	130	0	1305	0	21	2128
N.S.	1	1.00	1.03	0.94	0.00	9.39	0.00	0.15	15.31
time (sec)	N/A	0.129	0.585	1.971	0.000	0.431	0.000	0.453	11.285

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	165	168	0	2206	0	34	2178
N.S.	1	1.00	1.11	1.14	0.00	14.91	0.00	0.23	14.72
time (sec)	N/A	0.152	1.714	2.072	0.000	0.442	0.000	0.464	12.333

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	615	373	0	7664	0	1082	-1
N.S.	1	1.00	2.62	1.59	0.00	32.61	0.00	4.60	-0.00
time (sec)	N/A	0.372	0.695	5.933	0.000	0.577	0.000	0.875	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	737	434	0	6266	0	1009	-1
N.S.	1	1.00	3.51	2.07	0.00	29.84	0.00	4.80	-0.00
time (sec)	N/A	0.271	0.486	5.594	0.000	0.559	0.000	0.730	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	597	347	0	6250	0	984	-1
N.S.	1	1.00	2.75	1.60	0.00	28.80	0.00	4.53	-0.00
time (sec)	N/A	0.223	0.409	5.131	0.000	0.558	0.000	0.691	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	422	322	0	5238	0	861	-1
N.S.	1	1.00	2.27	1.73	0.00	28.16	0.00	4.63	-0.01
time (sec)	N/A	0.160	0.352	4.101	0.000	0.492	0.000	0.631	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	597	339	0	6018	0	1054	-1
N.S.	1	1.00	2.70	1.53	0.00	27.23	0.00	4.77	-0.00
time (sec)	N/A	0.222	0.270	5.014	0.000	0.538	0.000	0.600	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	761	364	0	7793	0	1116	-1
N.S.	1	1.00	2.34	1.12	0.00	23.98	0.00	3.43	-0.00
time (sec)	N/A	0.306	0.628	2.797	0.000	0.685	0.000	0.548	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	262	331	0	6944	0	149	-1
N.S.	1	1.00	0.82	1.03	0.00	21.70	0.00	0.47	-0.00
time (sec)	N/A	0.343	3.388	2.121	0.000	0.666	0.000	1.046	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	238	305	0	6045	0	153	-1
N.S.	1	1.00	1.02	1.31	0.00	25.94	0.00	0.66	-0.00
time (sec)	N/A	0.231	1.838	1.950	0.000	0.635	0.000	1.099	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	225	263	0	5658	0	128	-1
N.S.	1	1.00	1.15	1.35	0.00	29.02	0.00	0.66	-0.01
time (sec)	N/A	0.177	3.029	1.653	0.000	0.527	0.000	0.828	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	253	297	0	6525	0	152	-1
N.S.	1	1.00	1.15	1.35	0.00	29.66	0.00	0.69	-0.00
time (sec)	N/A	0.219	1.469	2.211	0.000	0.641	0.000	0.655	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	230	307	0	6522	0	127	-1
N.S.	1	1.00	1.10	1.46	0.00	31.06	0.00	0.60	-0.00
time (sec)	N/A	0.181	2.054	2.046	0.000	0.636	0.000	0.456	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	272	326	0	8824	0	238	-1
N.S.	1	1.00	1.15	1.38	0.00	37.23	0.00	1.00	-0.00
time (sec)	N/A	0.376	1.396	2.408	0.000	0.739	0.000	0.488	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	1021	652	0	21541	0	1091	-1
N.S.	1	1.00	3.24	2.07	0.00	68.38	0.00	3.46	-0.00
time (sec)	N/A	0.424	1.223	10.764	0.000	0.850	0.000	0.982	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	802	585	0	20362	0	1515	-1
N.S.	1	1.00	2.77	2.02	0.00	70.21	0.00	5.22	-0.00
time (sec)	N/A	0.357	0.962	11.002	0.000	0.736	0.000	1.026	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	1019	650	0	22506	0	1797	-1
N.S.	1	1.00	3.26	2.08	0.00	71.90	0.00	5.74	-0.00
time (sec)	N/A	0.393	1.358	10.677	0.000	0.832	0.000	0.927	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	802	593	0	20961	0	1580	-1
N.S.	1	1.00	2.78	2.06	0.00	72.78	0.00	5.49	-0.00
time (sec)	N/A	0.384	0.885	10.731	0.000	0.736	0.000	0.753	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	1018	641	0	22332	0	1127	-1
N.S.	1	1.00	3.25	2.05	0.00	71.35	0.00	3.60	-0.00
time (sec)	N/A	0.377	0.976	9.996	0.000	0.828	0.000	0.705	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	1189	647	0	28586	0	1783	-1
N.S.	1	1.00	1.93	1.05	0.00	46.33	0.00	2.89	-0.00
time (sec)	N/A	0.655	3.849	4.046	0.000	1.429	0.000	0.643	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	331	535	0	20486	0	389	-1
N.S.	1	1.00	1.04	1.68	0.00	64.22	0.00	1.22	-0.00
time (sec)	N/A	0.378	2.889	2.504	0.000	0.907	0.000	1.485	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	351	594	0	22729	0	451	-1
N.S.	1	1.00	1.02	1.72	0.00	65.88	0.00	1.31	-0.00
time (sec)	N/A	0.521	2.333	2.800	0.000	1.204	0.000	1.274	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	316	523	0	21932	0	362	-1
N.S.	1	1.00	1.01	1.67	0.00	69.85	0.00	1.15	-0.00
time (sec)	N/A	0.475	3.414	2.319	0.000	0.911	0.000	0.984	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	343	583	0	23355	0	449	-1
N.S.	1	1.00	0.99	1.68	0.00	67.11	0.00	1.29	-0.00
time (sec)	N/A	0.487	3.403	2.983	0.000	1.330	0.000	0.711	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	333	577	0	23125	0	391	-1
N.S.	1	1.00	1.04	1.80	0.00	72.27	0.00	1.22	-0.00
time (sec)	N/A	0.472	2.182	2.780	0.000	1.378	0.000	0.483	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	357	608	0	28429	0	486	-1
N.S.	1	1.00	0.99	1.69	0.00	79.19	0.00	1.35	-0.00
time (sec)	N/A	0.844	2.409	3.183	0.000	1.485	0.000	0.593	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	55	69	113	908	48	63
N.S.	1	1.00	0.96	2.20	2.76	4.52	36.32	1.92	2.52
time (sec)	N/A	0.012	0.094	0.429	0.468	0.373	2.853	0.405	0.156

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	45	44	0	596	0	281	205
N.S.	1	1.00	0.26	0.25	0.00	3.39	0.00	1.60	1.16
time (sec)	N/A	0.106	0.059	0.536	0.000	0.385	0.000	0.448	1.152

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	141	113	0	0	0	0	-1
N.S.	1	1.00	0.32	0.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.683	0.220	1.352	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	134	128	0	16401	0	1	857
N.S.	1	1.00	0.77	0.73	0.00	93.72	0.00	0.01	4.90
time (sec)	N/A	0.197	0.127	0.859	0.000	1.423	0.000	0.457	58.564

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	160	162	0	661332	0	1	-1
N.S.	1	1.00	0.65	0.66	0.00	2699.31	0.00	0.00	-0.00
time (sec)	N/A	0.377	0.195	1.091	0.000	3.538	0.000	0.534	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	439	124	0	3507	0	4946	-1
N.S.	1	1.00	1.81	0.51	0.00	14.49	0.00	20.44	-0.00
time (sec)	N/A	0.354	0.689	0.671	0.000	0.532	0.000	2.250	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	87	61	0	692	0	10	325
N.S.	1	1.00	1.23	0.86	0.00	9.75	0.00	0.14	4.58
time (sec)	N/A	0.086	0.153	0.592	0.000	0.411	0.000	0.408	4.209

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	127	64	0	3773	0	1	-1
N.S.	1	1.00	0.98	0.50	0.00	29.25	0.00	0.01	-0.01
time (sec)	N/A	0.127	0.097	0.606	0.000	0.492	0.000	0.422	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	437	124	0	3500	0	4948	-1
N.S.	1	1.00	1.92	0.54	0.00	15.35	0.00	21.70	-0.00
time (sec)	N/A	0.299	0.687	0.705	0.000	0.536	0.000	2.251	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	70	160	0	155	0	143	285
N.S.	1	1.00	0.84	1.93	0.00	1.87	0.00	1.72	3.43
time (sec)	N/A	0.075	0.340	0.624	0.000	0.413	0.000	0.429	2.707

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	64	99	0	708	0	48	273
N.S.	1	1.00	0.93	1.43	0.00	10.26	0.00	0.70	3.96
time (sec)	N/A	0.052	0.367	0.636	0.000	0.412	0.000	0.501	4.814

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	67	34	20	124	29	35
N.S.	1	1.00	1.06	3.72	1.89	1.11	6.89	1.61	1.94
time (sec)	N/A	0.032	0.004	0.422	0.260	0.368	2.757	0.410	1.364

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	65	25	12	153	28	25
N.S.	1	1.00	0.90	3.25	1.25	0.60	7.65	1.40	1.25
time (sec)	N/A	0.030	0.003	0.431	0.262	0.373	1.563	0.419	1.325

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	17	6	17	14	6
N.S.	1	1.00	1.00	1.17	2.83	1.00	2.83	2.33	1.00
time (sec)	N/A	0.028	0.003	0.373	0.274	0.382	0.895	0.406	1.287

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	11	5	5	2	5	5
N.S.	1	1.00	1.00	2.20	1.00	1.00	0.40	1.00	1.00
time (sec)	N/A	0.025	0.000	0.507	0.288	0.483	0.475	0.408	1.261

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	12	8	10	11	5	8	7
N.S.	1	1.00	1.71	1.14	1.43	1.57	0.71	1.14	1.00
time (sec)	N/A	0.017	0.005	0.321	0.468	0.387	0.079	0.396	0.072

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	40	40	151	0	52	54
N.S.	1	1.00	0.91	1.82	1.82	6.86	0.00	2.36	2.45
time (sec)	N/A	0.028	0.004	0.612	0.497	0.404	0.000	0.408	1.303

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	56	69	488	0	67	118
N.S.	1	1.00	0.97	1.60	1.97	13.94	0.00	1.91	3.37
time (sec)	N/A	0.041	0.004	0.590	0.492	0.369	0.000	0.418	1.288

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	63	67	152	117	250	121	76
N.S.	1	1.00	0.71	0.75	1.71	1.31	2.81	1.36	0.85
time (sec)	N/A	0.039	0.133	1.697	0.267	0.368	0.456	0.419	1.425

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	48	55	136	82	85	108	48
N.S.	1	1.00	1.04	1.20	2.96	1.78	1.85	2.35	1.04
time (sec)	N/A	0.026	0.107	1.645	0.264	0.382	0.283	0.422	1.340

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	43	70	76	59	150	71	38
N.S.	1	1.00	0.70	1.15	1.25	0.97	2.46	1.16	0.62
time (sec)	N/A	0.031	0.060	1.038	0.297	0.364	0.193	0.416	0.098

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	39	25	26	41	36	70	25
N.S.	1	1.00	1.39	0.89	0.93	1.46	1.29	2.50	0.89
time (sec)	N/A	0.016	0.012	0.569	0.280	0.581	0.108	0.414	0.091

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	37	34	56	101	0	40	88
N.S.	1	1.00	1.32	1.21	2.00	3.61	0.00	1.43	3.14
time (sec)	N/A	0.025	0.016	0.832	0.493	0.388	0.000	0.433	1.780

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	36	43	47	41	0	32	27
N.S.	1	1.00	1.89	2.26	2.47	2.16	0.00	1.68	1.42
time (sec)	N/A	0.024	0.015	1.809	0.296	0.376	0.000	0.411	0.797

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	71	109	136	324	0	105	127
N.S.	1	1.00	1.69	2.60	3.24	7.71	0.00	2.50	3.02
time (sec)	N/A	0.026	0.022	1.996	0.473	0.385	0.000	0.425	0.122

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	44	48	185	159	0	47	47
N.S.	1	1.00	1.38	1.50	5.78	4.97	0.00	1.47	1.47
time (sec)	N/A	0.024	0.009	1.537	0.275	0.365	0.000	0.405	0.821

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	172	228	1046	0	153	280
N.S.	1	1.00	0.86	2.46	3.26	14.94	0.00	2.19	4.00
time (sec)	N/A	0.032	0.114	1.288	0.474	0.379	0.000	0.440	0.827

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	102	84	486	343	0	83	298
N.S.	1	1.00	1.89	1.56	9.00	6.35	0.00	1.54	5.52
time (sec)	N/A	0.033	0.044	1.439	0.271	0.479	0.000	0.411	0.820

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	98	105	225	212	481	191	121
N.S.	1	1.00	0.62	0.66	1.42	1.33	3.03	1.20	0.76
time (sec)	N/A	0.118	0.226	1.918	0.268	0.382	1.109	0.414	0.359

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	83	97	242	188	136	196	80
N.S.	1	1.00	1.12	1.31	3.27	2.54	1.84	2.65	1.08
time (sec)	N/A	0.050	0.162	1.839	0.268	0.368	0.673	0.424	0.957

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	79	134	171	143	314	149	95
N.S.	1	1.00	0.66	1.13	1.44	1.20	2.64	1.25	0.80
time (sec)	N/A	0.095	0.207	1.246	0.266	0.377	0.488	0.415	0.217

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	41	45	106	58	134	42
N.S.	1	1.00	1.00	0.84	0.92	2.16	1.18	2.73	0.86
time (sec)	N/A	0.025	0.020	0.717	0.262	0.379	0.276	0.411	0.829

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	70	70	133	446	0	102	182
N.S.	1	1.00	1.27	1.27	2.42	8.11	0.00	1.85	3.31
time (sec)	N/A	0.040	0.187	0.980	0.490	0.388	0.000	0.423	0.172

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	109	119	97	0	131	75
N.S.	1	1.00	0.94	2.06	2.25	1.83	0.00	2.47	1.42
time (sec)	N/A	0.062	0.227	1.587	0.271	0.385	0.000	0.425	0.879

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	253	190	234	759	0	163	220
N.S.	1	1.00	3.95	2.97	3.66	11.86	0.00	2.55	3.44
time (sec)	N/A	0.054	11.263	1.467	0.505	0.377	0.000	0.428	0.885

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	57	92	267	200	0	98	194
N.S.	1	1.00	1.21	1.96	5.68	4.26	0.00	2.09	4.13
time (sec)	N/A	0.043	0.275	1.669	0.318	0.375	0.000	0.434	0.844

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F(-2)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	303	276	347	1472	0	218	327
N.S.	1	1.00	3.16	2.88	3.61	15.33	0.00	2.27	3.41
time (sec)	N/A	0.062	4.522	1.669	0.496	0.386	0.000	0.442	0.893

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	69	129	698	403	0	128	464
N.S.	1	1.00	1.21	2.26	12.25	7.07	0.00	2.25	8.14
time (sec)	N/A	0.041	0.324	1.695	0.270	0.391	0.000	0.440	0.863

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F(-2)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	715	358	483	2824	0	291	582
N.S.	1	1.00	5.46	2.73	3.69	21.56	0.00	2.22	4.44
time (sec)	N/A	0.088	9.205	1.772	0.479	0.402	0.000	0.453	0.922

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	144	165	363	376	774	293	209
N.S.	1	1.00	0.61	0.69	1.53	1.58	3.25	1.23	0.88
time (sec)	N/A	0.219	0.386	2.356	0.273	0.413	1.993	0.441	0.584

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	125	142	349	324	182	286	112
N.S.	1	1.00	1.28	1.45	3.56	3.31	1.86	2.92	1.14
time (sec)	N/A	0.058	0.409	2.200	0.267	0.376	1.378	0.465	0.294

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	120	216	287	257	559	231	166
N.S.	1	1.00	0.59	1.06	1.41	1.27	2.75	1.14	0.82
time (sec)	N/A	0.186	0.244	1.458	0.275	0.391	1.039	0.433	0.419

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	56	63	209	75	222	58
N.S.	1	1.00	1.00	0.84	0.94	3.12	1.12	3.31	0.87
time (sec)	N/A	0.029	0.033	0.816	0.270	0.508	0.630	0.431	0.154

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	100	114	233	1114	0	204	294
N.S.	1	1.00	1.16	1.33	2.71	12.95	0.00	2.37	3.42
time (sec)	N/A	0.050	0.394	1.003	0.470	0.393	0.000	0.443	1.041

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	78	212	215	178	0	197	141
N.S.	1	1.00	0.85	2.30	2.34	1.93	0.00	2.14	1.53
time (sec)	N/A	0.089	0.399	1.627	0.269	0.378	0.000	0.464	0.984

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F(-2)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	347	311	357	1679	0	247	308
N.S.	1	1.00	3.81	3.42	3.92	18.45	0.00	2.71	3.38
time (sec)	N/A	0.063	5.734	1.495	0.510	0.424	0.000	0.444	2.278

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	84	177	382	321	0	208	273
N.S.	1	1.00	1.02	2.16	4.66	3.91	0.00	2.54	3.33
time (sec)	N/A	0.074	0.752	1.810	0.291	0.414	0.000	0.460	0.160

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F(-2)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	472	409	489	2245	0	301	430
N.S.	1	1.00	4.58	3.97	4.75	21.80	0.00	2.92	4.17
time (sec)	N/A	0.089	9.395	1.723	0.483	0.426	0.000	0.457	0.186

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	86	207	824	530	0	213	563
N.S.	1	1.00	1.16	2.80	11.14	7.16	0.00	2.88	7.61
time (sec)	N/A	0.051	0.552	1.752	0.291	0.378	0.000	0.482	0.843

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	1192	495	646	3675	0	383	601
N.S.	1	1.00	7.74	3.21	4.19	23.86	0.00	2.49	3.90
time (sec)	N/A	0.100	13.523	1.775	0.500	0.441	0.000	0.456	0.196

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	163	261	1754	814	0	260	994
N.S.	1	1.00	2.04	3.26	21.92	10.18	0.00	3.25	12.42
time (sec)	N/A	0.049	0.600	1.761	0.301	0.544	0.000	0.475	0.875

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	117	444	0	3066	0	0	954
N.S.	1	1.00	1.08	4.11	0.00	28.39	0.00	0.00	8.83
time (sec)	N/A	0.076	0.357	1.787	0.000	0.471	0.000	0.000	1.514

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	106	438	0	1817	0	226	264
N.S.	1	1.00	0.88	3.62	0.00	15.02	0.00	1.87	2.18
time (sec)	N/A	0.142	0.245	1.822	0.000	0.454	0.000	2.421	1.372

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	79	318	0	1490	0	0	668
N.S.	1	1.00	1.03	4.13	0.00	19.35	0.00	0.00	8.68
time (sec)	N/A	0.062	0.186	1.776	0.000	0.446	0.000	0.000	1.283

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	316	0	875	0	138	300
N.S.	1	1.00	0.99	3.90	0.00	10.80	0.00	1.70	3.70
time (sec)	N/A	0.091	0.116	1.638	0.000	0.419	0.000	1.694	1.612

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	230	0	659	0	0	426
N.S.	1	1.00	0.96	4.42	0.00	12.67	0.00	0.00	8.19
time (sec)	N/A	0.047	0.038	1.488	0.000	0.404	0.000	0.000	1.144

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	219	0	443	0	68	166
N.S.	1	1.00	1.00	4.38	0.00	8.86	0.00	1.36	3.32
time (sec)	N/A	0.052	0.068	1.514	0.000	0.448	0.000	1.100	0.483

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	0	459	107	0	23
N.S.	1	1.00	1.00	0.75	0.00	14.34	3.34	0.00	0.72
time (sec)	N/A	0.026	0.009	0.531	0.000	0.477	0.857	0.000	0.870

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	209	0	511	0	0	648
N.S.	1	1.00	0.92	3.54	0.00	8.66	0.00	0.00	10.98
time (sec)	N/A	0.050	0.101	1.646	0.000	0.395	0.000	0.000	1.358

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	219	0	709	0	80	265
N.S.	1	1.00	1.00	3.65	0.00	11.82	0.00	1.33	4.42
time (sec)	N/A	0.053	0.126	1.521	0.000	0.411	0.000	0.642	1.471

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	91	272	0	1644	0	0	2797
N.S.	1	1.00	0.99	2.96	0.00	17.87	0.00	0.00	30.40
time (sec)	N/A	0.071	0.173	2.016	0.000	0.480	0.000	0.000	6.137

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	84	268	0	2444	0	138	710
N.S.	1	1.00	0.95	3.05	0.00	27.77	0.00	1.57	8.07
time (sec)	N/A	0.076	0.513	1.835	0.000	0.395	0.000	0.710	2.456

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	139	352	0	5500	0	0	2500
N.S.	1	1.00	1.01	2.55	0.00	39.86	0.00	0.00	18.12
time (sec)	N/A	0.112	0.560	2.125	0.000	0.467	0.000	0.000	12.047

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	119	346	0	6046	0	253	1152
N.S.	1	1.00	0.94	2.75	0.00	47.98	0.00	2.01	9.14
time (sec)	N/A	0.102	0.675	1.835	0.000	0.451	0.000	0.687	2.891

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	118	425	0	3629	0	305	-1
N.S.	1	1.00	0.75	2.69	0.00	22.97	0.00	1.93	-0.01
time (sec)	N/A	0.188	0.420	1.997	0.000	0.423	0.000	1.654	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	106	337	0	2739	0	0	-1
N.S.	1	1.00	1.02	3.24	0.00	26.34	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.216	1.999	0.000	0.552	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	108	320	0	1527	0	178	-1
N.S.	1	1.00	1.08	3.20	0.00	15.27	0.00	1.78	-0.01
time (sec)	N/A	0.092	0.485	1.835	0.000	0.441	0.000	1.887	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	75	280	0	1615	0	0	-1
N.S.	1	1.00	0.97	3.64	0.00	20.97	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.252	1.696	0.000	0.432	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	252	0	1421	0	126	-1
N.S.	1	1.00	0.99	3.19	0.00	17.99	0.00	1.59	-0.01
time (sec)	N/A	0.053	0.149	1.490	0.000	0.410	0.000	1.357	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	55	0	1320	377	0	54
N.S.	1	1.00	0.97	0.83	0.00	20.00	5.71	0.00	0.82
time (sec)	N/A	0.032	0.032	0.550	0.000	0.436	6.524	0.000	0.914

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	174	305	0	2143	0	0	-1
N.S.	1	1.00	1.64	2.88	0.00	20.22	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.292	1.770	0.000	0.449	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	105	307	0	3147	0	221	-1
N.S.	1	1.00	0.92	2.69	0.00	27.61	0.00	1.94	-0.01
time (sec)	N/A	0.119	0.669	1.630	0.000	0.462	0.000	0.762	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	230	368	0	6548	0	0	-1
N.S.	1	1.00	1.46	2.34	0.00	41.71	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.787	2.060	0.000	0.705	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	130	356	0	7894	0	270	-1
N.S.	1	1.00	0.91	2.49	0.00	55.20	0.00	1.89	-0.01
time (sec)	N/A	0.143	1.472	1.967	0.000	0.477	0.000	0.729	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	164	429	0	5511	0	353	-1
N.S.	1	1.00	1.02	2.68	0.00	34.44	0.00	2.21	-0.01
time (sec)	N/A	0.162	1.329	2.053	0.000	0.456	0.000	4.081	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	149	400	0	5844	0	0	-1
N.S.	1	1.00	1.12	3.01	0.00	43.94	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.259	1.901	0.000	0.429	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	102	300	0	4486	0	244	-1
N.S.	1	1.00	0.89	2.63	0.00	39.35	0.00	2.14	-0.01
time (sec)	N/A	0.065	0.491	1.526	0.000	0.428	0.000	1.752	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	114	354	0	4907	0	0	-1
N.S.	1	1.00	0.97	3.03	0.00	41.94	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.506	1.714	0.000	0.422	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	124	376	0	5183	0	269	-1
N.S.	1	1.00	0.87	2.63	0.00	36.24	0.00	1.88	-0.01
time (sec)	N/A	0.086	0.994	1.599	0.000	0.467	0.000	1.721	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	79	86	0	3934	835	0	87
N.S.	1	1.00	0.82	0.90	0.00	40.98	8.70	0.00	0.91
time (sec)	N/A	0.038	0.131	0.559	0.000	0.633	30.098	0.000	0.944

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	321	414	0	8083	0	0	-1
N.S.	1	1.00	2.02	2.60	0.00	50.84	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.550	1.921	0.000	0.493	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	165	394	0	9442	0	367	-1
N.S.	1	1.00	0.96	2.29	0.00	54.90	0.00	2.13	-0.01
time (sec)	N/A	0.200	1.012	1.691	0.000	0.487	0.000	1.327	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	222	477	0	18765	0	0	-1
N.S.	1	1.00	1.02	2.20	0.00	86.47	0.00	0.00	-0.00
time (sec)	N/A	0.207	1.304	2.292	0.000	0.643	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	169	445	0	19032	0	436	-1
N.S.	1	1.00	0.83	2.19	0.00	93.75	0.00	2.15	-0.00
time (sec)	N/A	0.249	2.015	2.083	0.000	0.562	0.000	0.942	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	54	64	70	238	41	56
N.S.	1	1.00	1.26	2.84	3.37	3.68	12.53	2.16	2.95
time (sec)	N/A	0.029	0.065	0.536	0.461	0.437	3.456	0.415	0.142

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	14	50	39	71	129	37	39
N.S.	1	1.00	1.40	5.00	3.90	7.10	12.90	3.70	3.90
time (sec)	N/A	0.025	0.009	0.522	0.253	0.446	0.622	0.397	0.058

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	98	75	163	2431	61	66
N.S.	1	1.00	1.07	3.27	2.50	5.43	81.03	2.03	2.20
time (sec)	N/A	0.039	0.057	0.545	0.476	0.569	8.290	0.406	0.836

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	124	52	0	3281	0	871	-1
N.S.	1	1.00	1.06	0.44	0.00	28.04	0.00	7.44	-0.01
time (sec)	N/A	0.077	0.507	1.322	0.000	0.462	0.000	0.669	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	96	60	0	2419	0	0	61
N.S.	1	1.00	1.33	0.83	0.00	33.60	0.00	0.00	0.85
time (sec)	N/A	0.036	0.181	0.512	0.000	0.450	0.000	0.000	0.961

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	130	51	0	5139	0	0	-1
N.S.	1	1.00	1.53	0.60	0.00	60.46	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.650	1.105	0.000	0.501	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	175	35	0	1327	0	0	-1
N.S.	1	1.00	2.03	0.41	0.00	15.43	0.00	0.00	-0.01
time (sec)	N/A	0.063	1.126	1.148	0.000	0.435	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	684	35	0	3727	0	0	-1
N.S.	1	1.00	4.53	0.23	0.00	24.68	0.00	0.00	-0.01
time (sec)	N/A	0.098	10.145	60.866	0.000	0.553	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	211	521	0	25	0	0	-1
N.S.	1	1.00	0.70	1.73	0.00	0.08	0.00	0.00	-0.00
time (sec)	N/A	0.199	1.014	1.752	0.000	0.097	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	168	351	0	25	0	0	-1
N.S.	1	1.00	0.75	1.57	0.00	0.11	0.00	0.00	-0.00
time (sec)	N/A	0.142	0.832	1.447	0.000	0.097	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	69	140	0	16	0	0	-1
N.S.	1	1.00	1.15	2.33	0.00	0.27	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.064	0.950	0.000	0.126	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	148	177	0	535	0	0	-1
N.S.	1	1.00	2.11	2.53	0.00	7.64	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.346	1.464	0.000	0.104	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	204	318	0	2257	0	0	-1
N.S.	1	1.00	0.99	1.54	0.00	10.96	0.00	0.00	-0.00
time (sec)	N/A	0.122	2.421	3.153	0.000	0.117	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	149	77	0	4603	0	1546	-1
N.S.	1	1.00	0.95	0.49	0.00	29.32	0.00	9.85	-0.01
time (sec)	N/A	0.090	0.880	1.293	0.000	0.503	0.000	1.034	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	93	86	0	3161	0	791	60
N.S.	1	1.00	0.89	0.83	0.00	30.39	0.00	7.61	0.58
time (sec)	N/A	0.046	0.384	0.496	0.000	0.463	0.000	0.775	1.024

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	142	63	0	6337	0	0	-1
N.S.	1	1.00	1.14	0.50	0.00	50.70	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.347	0.948	0.000	0.573	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	150	63	0	7350	0	0	-1
N.S.	1	1.00	1.13	0.47	0.00	55.26	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.775	1.145	0.000	0.788	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	66	63	0	3089	0	2017	-1
N.S.	1	1.00	0.52	0.50	0.00	24.52	0.00	16.01	-0.01
time (sec)	N/A	0.080	0.090	1.368	0.000	0.552	0.000	0.903	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	959	63	0	7633	0	4555	-1
N.S.	1	1.00	4.68	0.31	0.00	37.23	0.00	22.22	-0.00
time (sec)	N/A	0.117	11.376	1.213	0.000	0.954	0.000	1.725	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	256	730	0	46	0	0	-1
N.S.	1	1.00	0.72	2.04	0.00	0.13	0.00	0.00	-0.00
time (sec)	N/A	0.278	2.054	1.775	0.000	0.097	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	213	535	0	46	0	0	-1
N.S.	1	1.00	0.71	1.79	0.00	0.15	0.00	0.00	-0.00
time (sec)	N/A	0.196	1.079	1.473	0.000	0.110	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	169	428	0	16	0	0	-1
N.S.	1	1.00	0.97	2.46	0.00	0.09	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.526	1.053	0.000	0.093	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	160	334	0	46	0	0	-1
N.S.	1	1.00	0.76	1.59	0.00	0.22	0.00	0.00	-0.00
time (sec)	N/A	0.133	0.708	1.394	0.000	0.092	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	197	324	0	2071	0	1574	-1
N.S.	1	1.00	1.02	1.68	0.00	10.73	0.00	8.16	-0.01
time (sec)	N/A	0.127	1.462	1.699	0.000	0.152	0.000	4.652	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	77	35	0	2479	0	0	-1
N.S.	1	1.00	0.97	0.44	0.00	31.38	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.073	1.274	0.000	0.452	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	34	0	1990	0	0	33
N.S.	1	1.00	1.00	0.89	0.00	52.37	0.00	0.00	0.87
time (sec)	N/A	0.030	0.013	0.418	0.000	0.507	0.000	0.000	1.016

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	0	598	0	80	-1
N.S.	1	1.00	1.00	0.76	0.00	13.00	0.00	1.74	-0.02
time (sec)	N/A	0.047	0.025	1.145	0.000	0.451	0.000	0.470	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	443	35	0	1503	0	693	-1
N.S.	1	1.00	4.57	0.36	0.00	15.49	0.00	7.14	-0.01
time (sec)	N/A	0.071	8.692	1.947	0.000	0.478	0.000	0.553	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	179	356	0	25	0	806	-1
N.S.	1	1.00	0.74	1.48	0.00	0.10	0.00	3.34	-0.00
time (sec)	N/A	0.148	0.614	1.687	0.000	0.089	0.000	1.110	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	95	86	0	25	0	0	-1
N.S.	1	1.00	0.54	0.49	0.00	0.14	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.177	1.085	0.000	0.095	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	68	86	0	147	0	0	-1
N.S.	1	1.00	1.13	1.43	0.00	2.45	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.055	0.804	0.000	0.093	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	159	131	0	576	0	282	-1
N.S.	1	1.00	0.99	0.82	0.00	3.60	0.00	1.76	-0.01
time (sec)	N/A	0.123	0.716	2.082	0.000	0.103	0.000	0.483	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	219	343	0	2443	0	1142	-1
N.S.	1	1.00	1.00	1.57	0.00	11.16	0.00	5.21	-0.00
time (sec)	N/A	0.136	1.538	2.281	0.000	0.183	0.000	1.302	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	89	35	0	3126	0	0	-1
N.S.	1	1.00	1.16	0.45	0.00	40.60	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.126	1.145	0.000	0.501	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	246	245	0	118	191
N.S.	1	1.00	1.00	0.97	8.48	8.45	0.00	4.07	6.59
time (sec)	N/A	0.033	0.021	0.434	0.507	0.419	0.000	0.595	1.133

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	315	101	0	1717	0	294	-1
N.S.	1	1.00	3.71	1.19	0.00	20.20	0.00	3.46	-0.01
time (sec)	N/A	0.066	7.514	0.959	0.000	0.458	0.000	0.544	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	231	95	0	4845	0	984	-1
N.S.	1	1.00	1.63	0.67	0.00	34.12	0.00	6.93	-0.01
time (sec)	N/A	0.112	4.193	6.678	0.000	0.687	0.000	0.885	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	196	498	0	55	0	1201	-1
N.S.	1	1.00	0.60	1.53	0.00	0.17	0.00	3.70	-0.00
time (sec)	N/A	0.207	1.098	1.680	0.000	0.104	0.000	2.830	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	155	334	0	55	0	0	-1
N.S.	1	1.00	0.64	1.37	0.00	0.23	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.451	1.720	0.000	0.097	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	143	181	0	1068	0	0	-1
N.S.	1	1.00	1.57	1.99	0.00	11.74	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.232	1.467	0.000	0.164	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	100	253	0	1464	0	0	-1
N.S.	1	1.00	0.87	2.20	0.00	12.73	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.118	1.151	0.000	0.109	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	178	342	0	2786	0	0	-1
N.S.	1	1.00	0.82	1.58	0.00	12.84	0.00	0.00	-0.00
time (sec)	N/A	0.140	1.059	2.335	0.000	0.144	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	126	65	0	6774	0	0	-1
N.S.	1	1.00	0.94	0.49	0.00	50.55	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.641	1.400	0.000	0.693	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	50	65	955	945	0	333	144
N.S.	1	1.00	0.68	0.89	13.08	12.95	0.00	4.56	1.97
time (sec)	N/A	0.062	0.077	1.358	0.518	0.492	0.000	1.131	1.831

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	47	56	499	912	0	333	129
N.S.	1	1.00	0.72	0.86	7.68	14.03	0.00	5.12	1.98
time (sec)	N/A	0.038	0.034	1.122	0.502	0.502	0.000	0.815	1.486

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	1331	169	0	5396	0	1177	-1
N.S.	1	1.00	9.93	1.26	0.00	40.27	0.00	8.78	-0.01
time (sec)	N/A	0.103	8.499	1.926	0.000	0.644	0.000	0.731	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	206	812	0	71	0	0	-1
N.S.	1	1.00	0.62	2.46	0.00	0.22	0.00	0.00	-0.00
time (sec)	N/A	0.229	1.602	1.851	0.000	0.104	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	178	597	0	4355	0	0	-1
N.S.	1	1.00	0.80	2.68	0.00	19.53	0.00	0.00	-0.00
time (sec)	N/A	0.143	1.200	1.754	0.000	0.161	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	193	662	0	4770	0	0	-1
N.S.	1	1.00	0.85	2.90	0.00	20.92	0.00	0.00	-0.00
time (sec)	N/A	0.141	1.386	1.849	0.000	0.173	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	190	406	0	5442	0	0	-1
N.S.	1	1.00	0.76	1.62	0.00	21.68	0.00	0.00	-0.00
time (sec)	N/A	0.198	1.085	1.757	0.000	0.187	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	260	1002	0	8928	0	0	-1
N.S.	1	1.00	0.89	3.43	0.00	30.58	0.00	0.00	-0.00
time (sec)	N/A	0.220	2.764	2.025	0.000	0.287	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	0	0	0	27	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.23	0.00	0.00	-0.01
time (sec)	N/A	0.078	6.706	1.573	0.000	0.454	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.12	0.00	0.00	-0.00
time (sec)	N/A	0.147	7.998	2.030	0.000	0.407	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	119	120	0	0	25	0	0	-1
N.S.	1	0.95	0.96	0.00	0.00	0.20	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.232	2.168	0.000	0.399	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	23	0	0	64
N.S.	1	1.00	1.00	0.00	0.00	0.34	0.00	0.00	0.96
time (sec)	N/A	0.033	0.022	0.756	0.000	0.501	0.000	0.000	1.503

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	23	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.053	2.480	1.113	0.000	0.432	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.32	0.00	0.00	-0.01
time (sec)	N/A	0.057	4.454	1.174	0.000	0.468	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.057	7.309	1.678	0.000	0.463	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.056	7.142	1.874	0.000	0.459	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	16	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.17	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.857	0.969	0.000	0.412	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.056	3.215	1.009	0.000	0.472	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.056	6.590	1.119	0.000	0.416	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	220	435	0	2595	0	0	-1
N.S.	1	1.00	0.85	1.68	0.00	10.02	0.00	0.00	-0.00
time (sec)	N/A	0.200	0.411	2.510	0.000	1.323	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	117	245	0	879	0	0	-1
N.S.	1	1.00	0.86	1.80	0.00	6.46	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.110	2.224	0.000	0.920	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	80	0	225	68	0	39
N.S.	1	1.00	0.95	1.86	0.00	5.23	1.58	0.00	0.91
time (sec)	N/A	0.033	0.021	1.000	0.000	0.856	1.110	0.000	1.008

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	229	195	0	17277	0	0	-1
N.S.	1	1.00	0.80	0.68	0.00	60.41	0.00	0.00	-0.00
time (sec)	N/A	0.334	0.190	1.808	0.000	5.384	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	288	525	0	5181	0	0	-1
N.S.	1	1.00	1.07	1.94	0.00	19.19	0.00	0.00	-0.00
time (sec)	N/A	0.227	0.435	3.891	0.000	1.030	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	123	327	0	2137	0	0	-1
N.S.	1	1.00	0.87	2.30	0.00	15.05	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.464	4.069	0.000	0.988	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	127	0	564	151	0	45
N.S.	1	1.00	0.86	2.59	0.00	11.51	3.08	0.00	0.92
time (sec)	N/A	0.039	0.064	1.046	0.000	0.508	4.458	0.000	1.406

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	280	376	0	30856	0	0	-1
N.S.	1	1.00	0.73	0.98	0.00	80.35	0.00	0.00	-0.00
time (sec)	N/A	0.443	0.814	3.293	0.000	21.638	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	119	0	0	25	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.19	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.103	1.519	0.000	0.428	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	25	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.044	1.297	0.000	0.458	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	23	0	0	38
N.S.	1	1.00	1.00	0.00	0.00	0.62	0.00	0.00	1.03
time (sec)	N/A	0.031	0.009	0.417	0.000	0.452	0.000	0.000	1.130

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	119	0	0	43	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.33	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.125	2.340	0.000	0.454	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	43	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.040	1.635	0.000	0.479	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	41	0	0	38
N.S.	1	1.00	1.00	0.00	0.00	1.11	0.00	0.00	1.03
time (sec)	N/A	0.030	0.010	1.579	0.000	0.472	0.000	0.000	0.948

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	23	41	45	47	0	25	27
N.S.	1	1.00	1.35	2.41	2.65	2.76	0.00	1.47	1.59
time (sec)	N/A	0.022	0.012	0.609	0.261	0.476	0.000	0.429	0.084

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	42	316	875	0	70	252
N.S.	1	1.00	0.81	0.67	5.02	13.89	0.00	1.11	4.00
time (sec)	N/A	0.088	0.074	1.372	0.523	0.475	0.000	0.455	0.940

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	42	114	311	0	44	67
N.S.	1	1.00	0.76	1.11	3.00	8.18	0.00	1.16	1.76
time (sec)	N/A	0.081	0.069	1.064	0.531	0.462	0.000	0.430	0.910

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	34	139	0	24	18
N.S.	1	1.00	1.00	1.06	1.89	7.72	0.00	1.33	1.00
time (sec)	N/A	0.050	0.029	0.547	0.500	0.538	0.000	0.406	0.922

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	42	72	200	0	47	-1
N.S.	1	1.00	0.84	0.84	1.44	4.00	0.00	0.94	-0.02
time (sec)	N/A	0.067	0.045	0.987	0.519	0.494	0.000	0.420	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	54	134	764	0	108	-1
N.S.	1	1.00	0.89	0.62	1.54	8.78	0.00	1.24	-0.01
time (sec)	N/A	0.098	0.181	1.100	0.511	0.519	0.000	0.428	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	75	85	955	1645	0	124	-1
N.S.	1	1.00	0.62	0.71	7.96	13.71	0.00	1.03	-0.01
time (sec)	N/A	0.087	0.253	1.664	0.513	0.474	0.000	0.454	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	69	413	742	0	100	-1
N.S.	1	1.00	0.60	0.76	4.54	8.15	0.00	1.10	-0.01
time (sec)	N/A	0.087	0.143	1.243	0.529	0.491	0.000	0.460	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	40	41	53	182	0	35	-1
N.S.	1	1.00	0.70	0.72	0.93	3.19	0.00	0.61	-0.02
time (sec)	N/A	0.074	0.039	1.102	0.480	0.474	0.000	0.417	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	35	42	133	317	0	48	67
N.S.	1	1.00	0.62	0.75	2.38	5.66	0.00	0.86	1.20
time (sec)	N/A	0.080	0.056	0.931	0.504	0.468	0.000	0.428	0.927

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	47	55	521	885	0	74	281
N.S.	1	1.00	0.52	0.60	5.73	9.73	0.00	0.81	3.09
time (sec)	N/A	0.085	0.054	0.947	0.506	0.477	0.000	0.416	0.942

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	67	65	1126	1696	0	96	427
N.S.	1	1.00	0.54	0.52	9.08	13.68	0.00	0.77	3.44
time (sec)	N/A	0.087	0.169	1.322	0.549	0.497	0.000	0.453	0.906

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	43	41	476	1387	0	0	381
N.S.	1	1.00	0.65	0.62	7.21	21.02	0.00	0.00	5.77
time (sec)	N/A	0.087	0.113	1.264	0.536	0.593	0.000	0.000	0.888

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	31	41	196	641	0	0	82
N.S.	1	1.00	0.74	0.98	4.67	15.26	0.00	0.00	1.95
time (sec)	N/A	0.081	0.089	1.121	0.523	0.474	0.000	0.000	0.118

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	35	168	0	0	30
N.S.	1	1.00	1.00	1.05	1.84	8.84	0.00	0.00	1.58
time (sec)	N/A	0.052	0.036	0.604	0.522	0.466	0.000	0.000	0.847

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	49	33	42	174	0	0	-1
N.S.	1	1.00	1.58	1.06	1.35	5.61	0.00	0.00	-0.03
time (sec)	N/A	0.063	0.058	0.951	0.511	0.454	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	42	106	529	0	0	-1
N.S.	1	1.00	0.98	0.64	1.61	8.02	0.00	0.00	-0.02
time (sec)	N/A	0.091	0.149	1.231	0.542	0.457	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	66	68	672	1328	0	0	-1
N.S.	1	1.00	0.73	0.75	7.38	14.59	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.100	1.435	0.553	0.408	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	44	51	231	504	0	0	-1
N.S.	1	1.00	0.71	0.82	3.73	8.13	0.00	0.00	-0.02
time (sec)	N/A	0.082	0.043	1.325	0.508	0.445	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	107	170	0	0	76
N.S.	1	1.00	1.00	1.28	4.28	6.80	0.00	0.00	3.04
time (sec)	N/A	0.073	0.028	0.987	0.497	0.457	0.000	0.000	0.112

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	37	44	592	647	0	0	95
N.S.	1	1.00	0.61	0.72	9.70	10.61	0.00	0.00	1.56
time (sec)	N/A	0.080	0.048	1.209	0.533	0.443	0.000	0.000	0.889

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	49	54	1315	1399	0	0	381
N.S.	1	1.00	0.51	0.56	13.70	14.57	0.00	0.00	3.97
time (sec)	N/A	0.085	0.067	1.577	0.572	0.454	0.000	0.000	0.899

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	44	626	2507	0	0	457
N.S.	1	1.00	0.75	0.65	9.21	36.87	0.00	0.00	6.72
time (sec)	N/A	0.099	0.080	1.252	0.568	0.454	0.000	0.000	0.163

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	34	44	286	1400	0	0	305
N.S.	1	1.00	0.77	1.00	6.50	31.82	0.00	0.00	6.93
time (sec)	N/A	0.093	0.092	1.182	0.524	0.428	0.000	0.000	0.926

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	65	608	0	0	58
N.S.	1	1.00	1.00	0.95	3.10	28.95	0.00	0.00	2.76
time (sec)	N/A	0.057	0.030	0.617	0.556	0.413	0.000	0.000	0.877

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	41	44	80	271	0	0	-1
N.S.	1	1.00	0.77	0.83	1.51	5.11	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.050	1.027	0.530	0.425	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	36	106	565	0	0	-1
N.S.	1	1.00	1.02	0.55	1.61	8.56	0.00	0.00	-0.02
time (sec)	N/A	0.098	0.123	1.203	0.536	0.601	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	58	69	395	1423	0	0	-1
N.S.	1	1.00	0.55	0.65	3.73	13.42	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.073	1.266	0.496	0.445	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	46	51	341	254	0	0	-1
N.S.	1	1.00	0.72	0.80	5.33	3.97	0.00	0.00	-0.02
time (sec)	N/A	0.095	0.061	1.358	0.485	0.454	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	35	881	612	0	0	71
N.S.	1	1.00	0.76	0.92	23.18	16.11	0.00	0.00	1.87
time (sec)	N/A	0.088	0.038	1.248	0.529	0.436	0.000	0.000	0.899

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	41	67	1641	1410	0	0	305
N.S.	1	1.00	0.53	0.87	21.31	18.31	0.00	0.00	3.96
time (sec)	N/A	0.100	0.078	1.495	0.582	0.431	0.000	0.000	0.158

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	51	57	2378	2511	0	0	457
N.S.	1	1.00	0.44	0.50	20.68	21.83	0.00	0.00	3.97
time (sec)	N/A	0.105	0.100	1.779	0.658	0.450	0.000	0.000	0.950

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	151	43	0	4704	0	0	-1
N.S.	1	1.00	0.81	0.23	0.00	25.16	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.396	1.427	0.000	1.131	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	88	43	0	1670	0	0	-1
N.S.	1	1.00	0.70	0.34	0.00	13.25	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.291	1.260	0.000	0.910	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	65	41	0	624	0	0	-1
N.S.	1	1.00	1.05	0.66	0.00	10.06	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.041	0.964	0.000	0.880	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	46	0	605	0	0	-1
N.S.	1	1.00	0.98	0.85	0.00	11.20	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.034	0.983	0.000	0.643	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	69	58	0	1445	0	0	-1
N.S.	1	1.00	0.65	0.55	0.00	13.63	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.444	1.233	0.000	0.685	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	102	80	0	3880	0	0	-1
N.S.	1	1.00	0.61	0.48	0.00	23.23	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.634	1.289	0.000	0.836	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	214	366	0	25	0	0	-1
N.S.	1	1.00	0.73	1.25	0.00	0.09	0.00	0.00	-0.00
time (sec)	N/A	0.217	1.430	1.698	0.000	0.113	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	150	233	0	25	0	0	-1
N.S.	1	1.00	0.89	1.39	0.00	0.15	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.379	1.575	0.000	0.085	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	69	140	0	16	0	0	-1
N.S.	1	1.00	1.15	2.33	0.00	0.27	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.111	1.030	0.000	0.098	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	154	215	0	25	0	0	-1
N.S.	1	1.00	0.76	1.06	0.00	0.12	0.00	0.00	-0.00
time (sec)	N/A	0.137	0.423	1.464	0.000	0.113	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	210	519	0	25	0	0	-1
N.S.	1	1.00	0.78	1.92	0.00	0.09	0.00	0.00	-0.00
time (sec)	N/A	0.210	2.384	1.493	0.000	0.134	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	169	71	0	6380	0	0	-1
N.S.	1	1.00	0.73	0.31	0.00	27.50	0.00	0.00	-0.00
time (sec)	N/A	0.194	1.535	1.530	0.000	0.943	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	122	71	0	2454	0	0	-1
N.S.	1	1.00	0.78	0.46	0.00	15.73	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.439	1.266	0.000	1.123	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	86	69	0	1052	0	0	-1
N.S.	1	1.00	0.96	0.77	0.00	11.69	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.110	0.882	0.000	0.856	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	62	0	1000	0	0	-1
N.S.	1	1.00	0.88	0.79	0.00	12.82	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.114	0.977	0.000	0.636	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	90	84	0	2406	0	0	-1
N.S.	1	1.00	0.64	0.60	0.00	17.19	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.336	1.204	0.000	0.730	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	199	123	113	0	5509	0	0	-1
N.S.	1	0.98	0.61	0.56	0.00	27.14	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.942	1.280	0.000	0.822	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	224	385	0	25	0	0	-1
N.S.	1	1.00	0.73	1.26	0.00	0.08	0.00	0.00	-0.00
time (sec)	N/A	0.257	2.009	1.770	0.000	0.120	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	188	413	0	25	0	0	-1
N.S.	1	1.00	0.72	1.59	0.00	0.10	0.00	0.00	-0.00
time (sec)	N/A	0.167	2.259	1.572	0.000	0.109	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	169	428	0	16	0	0	-1
N.S.	1	1.00	0.97	2.46	0.00	0.09	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.526	1.162	0.000	0.152	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	184	327	0	46	0	0	-1
N.S.	1	1.00	0.72	1.28	0.00	0.18	0.00	0.00	-0.00
time (sec)	N/A	0.192	1.904	1.391	0.000	0.122	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	229	540	0	46	0	0	-1
N.S.	1	1.00	0.75	1.76	0.00	0.15	0.00	0.00	-0.00
time (sec)	N/A	0.252	3.691	1.522	0.000	0.155	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	116	43	0	4100	0	2265	-1
N.S.	1	1.00	0.82	0.30	0.00	28.87	0.00	15.95	-0.01
time (sec)	N/A	0.133	0.330	1.447	0.000	0.638	0.000	9.720	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	85	43	0	1320	0	702	-1
N.S.	1	1.00	0.96	0.48	0.00	14.83	0.00	7.89	-0.01
time (sec)	N/A	0.075	0.084	1.309	0.000	0.529	0.000	2.618	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	41	0	433	0	0	-1
N.S.	1	1.00	1.07	1.00	0.00	10.56	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.031	0.965	0.000	0.518	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	0	410	0	0	-1
N.S.	1	1.00	1.00	1.06	0.00	12.42	0.00	0.00	-0.03
time (sec)	N/A	0.043	0.026	0.998	0.000	0.494	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	72	44	0	1144	0	0	-1
N.S.	1	1.00	0.94	0.57	0.00	14.86	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.143	1.434	0.000	0.506	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	100	54	0	3086	0	0	-1
N.S.	1	1.00	0.79	0.43	0.00	24.49	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.262	1.611	0.000	0.520	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	206	366	0	2755	0	1227	-1
N.S.	1	1.00	0.94	1.67	0.00	12.58	0.00	5.60	-0.00
time (sec)	N/A	0.140	1.816	1.737	0.000	0.162	0.000	4.359	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	109	239	0	703	0	0	-1
N.S.	1	1.00	0.70	1.53	0.00	4.51	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.316	1.513	0.000	0.120	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	68	86	0	147	0	0	-1
N.S.	1	1.00	1.13	1.43	0.00	2.45	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.057	0.866	0.000	0.102	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	105	216	0	674	0	0	-1
N.S.	1	1.00	0.51	1.04	0.00	3.26	0.00	0.00	-0.00
time (sec)	N/A	0.129	0.326	1.634	0.000	0.116	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	208	522	0	2584	0	0	-1
N.S.	1	1.00	0.73	1.83	0.00	9.07	0.00	0.00	-0.00
time (sec)	N/A	0.207	2.876	1.918	0.000	0.131	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	113	103	0	10168	0	2452	-1
N.S.	1	1.00	0.60	0.55	0.00	54.37	0.00	13.11	-0.01
time (sec)	N/A	0.179	0.348	12.440	0.000	0.798	0.000	15.803	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	79	103	0	4050	0	0	-1
N.S.	1	1.00	0.65	0.84	0.00	33.20	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.090	2.072	0.000	0.556	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	93	0	1370	0	0	-1
N.S.	1	1.00	0.84	1.35	0.00	19.86	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.055	1.020	0.000	0.518	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	35	0	1137	0	0	-1
N.S.	1	1.00	0.81	0.61	0.00	19.95	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.060	1.079	0.000	0.501	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	69	43	0	3228	0	0	-1
N.S.	1	1.00	0.63	0.39	0.00	29.35	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.108	2.233	0.000	0.584	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	94	43	0	7562	0	0	-1
N.S.	1	1.00	0.56	0.26	0.00	45.28	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.234	8.832	0.000	0.721	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	212	352	0	7400	0	0	-1
N.S.	1	1.00	0.77	1.28	0.00	26.91	0.00	0.00	-0.00
time (sec)	N/A	0.192	1.599	6.750	0.000	0.274	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	158	257	0	2612	0	0	-1
N.S.	1	1.00	0.73	1.18	0.00	12.04	0.00	0.00	-0.00
time (sec)	N/A	0.146	0.974	2.013	0.000	0.154	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	100	253	0	1464	0	0	-1
N.S.	1	1.00	0.87	2.20	0.00	12.73	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.123	1.175	0.000	0.119	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	153	218	0	2436	0	0	-1
N.S.	1	1.00	0.65	0.92	0.00	10.28	0.00	0.00	-0.00
time (sec)	N/A	0.180	0.601	2.108	0.000	0.139	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	214	522	0	6862	0	0	-1
N.S.	1	1.00	0.63	1.53	0.00	20.12	0.00	0.00	-0.00
time (sec)	N/A	0.279	2.658	6.097	0.000	0.201	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	114	213	0	20298	0	3690	-1
N.S.	1	1.00	0.49	0.92	0.00	87.49	0.00	15.91	-0.00
time (sec)	N/A	0.214	0.420	3.133	0.000	1.379	0.000	21.379	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	82	213	0	10506	0	1981	-1
N.S.	1	1.00	0.50	1.31	0.00	64.45	0.00	12.15	-0.01
time (sec)	N/A	0.114	0.093	2.082	0.000	1.012	0.000	7.363	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	60	173	0	4200	0	0	-1
N.S.	1	1.00	0.61	1.75	0.00	42.42	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.102	1.539	0.000	0.639	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	49	65	0	3084	0	0	-1
N.S.	1	1.00	0.59	0.78	0.00	37.16	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.043	1.530	0.000	0.498	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	69	73	0	7594	0	0	-1
N.S.	1	1.00	0.48	0.51	0.00	53.10	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.208	2.135	0.000	0.656	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	117	73	0	15102	0	0	-1
N.S.	1	1.00	0.56	0.35	0.00	72.61	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.299	3.030	0.000	1.028	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	252	663	0	15718	0	0	-1
N.S.	1	1.00	0.76	1.99	0.00	47.20	0.00	0.00	-0.00
time (sec)	N/A	0.287	2.776	2.665	0.000	0.553	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	215	799	0	8226	0	0	-1
N.S.	1	1.00	0.78	2.92	0.00	30.02	0.00	0.00	-0.00
time (sec)	N/A	0.204	2.137	2.193	0.000	0.289	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	190	406	0	5442	0	0	-1
N.S.	1	1.00	0.76	1.62	0.00	21.68	0.00	0.00	-0.00
time (sec)	N/A	0.201	1.090	1.757	0.000	0.190	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	226	642	0	7847	0	0	-1
N.S.	1	1.00	0.64	1.83	0.00	22.36	0.00	0.00	-0.00
time (sec)	N/A	0.283	2.279	2.148	0.000	0.257	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	247	923	0	13823	0	0	-1
N.S.	1	1.00	0.64	2.40	0.00	35.90	0.00	0.00	-0.00
time (sec)	N/A	0.381	2.394	2.937	0.000	0.410	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	0	0	0	27	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.22	0.00	0.00	-0.01
time (sec)	N/A	0.096	7.845	2.731	0.000	0.561	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	90	0	0	25	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.23	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.195	2.099	0.000	0.436	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	23	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.37	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.055	1.018	0.000	0.407	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	23	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.43	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.068	1.205	0.000	0.374	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	71	0	0	25	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.278	1.536	0.000	0.501	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.24	0.00	0.00	-0.01
time (sec)	N/A	0.085	27.714	1.329	0.000	0.466	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.24	0.00	0.00	-0.01
time (sec)	N/A	0.070	4.025	1.285	0.000	0.441	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.070	4.015	1.326	0.000	0.413	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.24	0.00	0.00	-0.01
time (sec)	N/A	0.071	23.974	1.434	0.000	0.431	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	136	132	0	1115	0	209	1129
N.S.	1	1.00	0.89	0.87	0.00	7.34	0.00	1.38	7.43
time (sec)	N/A	0.166	0.245	1.020	0.000	1.186	0.000	0.429	0.917

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	0	0	0	0	-1
N.S.	1	1.00	1.00	0.75	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	0.017	6.380	0.000	0.000	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	0	1663	0	0	-1
N.S.	1	1.00	1.00	0.76	0.00	36.96	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.018	5.716	0.000	1.510	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	113	0	0	-1
N.S.	1	1.00	1.00	0.83	0.00	3.90	0.00	0.00	-0.03
time (sec)	N/A	0.064	0.019	7.474	0.000	0.552	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	38	0	156	0	0	-1
N.S.	1	1.00	0.96	0.81	0.00	3.32	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.017	1.447	0.000	0.426	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [66] had the largest ratio of [25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	21	0.143
2	A	3	2	1.00	21	0.095
3	A	3	3	1.00	21	0.143
4	A	2	1	1.00	19	0.053
5	A	3	2	1.00	12	0.167
6	A	2	2	1.00	19	0.105
7	A	2	2	1.00	21	0.095
8	A	2	2	1.00	21	0.095
9	A	3	3	1.00	21	0.143
10	A	6	6	1.00	23	0.261
11	A	3	2	1.00	23	0.087
12	A	2	2	1.06	23	0.087
13	A	3	2	1.00	21	0.095
14	A	1	1	1.00	14	0.071
15	A	4	3	1.00	21	0.143
16	A	4	4	1.28	23	0.174
17	A	5	4	1.00	23	0.174
18	A	4	3	1.00	23	0.130
19	A	7	6	1.00	23	0.261
20	A	3	2	1.00	23	0.087
21	A	3	2	1.00	23	0.087
22	A	3	2	1.00	21	0.095
23	A	2	2	1.00	14	0.143
24	A	4	3	1.00	21	0.143
25	A	5	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	5	4	1.00	23	0.174
27	A	5	4	1.00	23	0.174
28	A	4	3	1.00	23	0.130
29	A	6	6	1.00	23	0.261
30	A	4	3	1.00	23	0.130
31	A	5	5	1.00	23	0.217
32	A	3	3	1.00	23	0.130
33	A	3	3	1.00	23	0.130
34	A	2	2	1.00	21	0.095
35	A	2	2	1.00	14	0.143
36	A	4	4	1.00	21	0.190
37	A	3	3	1.00	23	0.130
38	A	5	5	1.00	23	0.217
39	A	4	3	1.00	23	0.130
40	A	6	6	1.00	23	0.261
41	A	4	3	1.00	23	0.130
42	A	5	5	1.00	23	0.217
43	A	3	3	1.00	23	0.130
44	A	4	4	1.00	23	0.174
45	A	3	3	1.00	21	0.143
46	A	4	4	1.00	14	0.286
47	A	5	5	1.00	21	0.238
48	A	4	4	1.00	23	0.174
49	A	6	6	1.00	23	0.261
50	A	5	4	1.00	23	0.174
51	A	4	3	1.00	23	0.130
52	A	4	4	1.00	23	0.174
53	A	5	4	1.00	23	0.174
54	A	4	3	1.00	21	0.143
55	A	5	5	1.00	14	0.357
56	A	6	6	1.00	21	0.286
57	A	5	5	1.00	23	0.217
58	A	7	6	1.00	23	0.261
59	A	6	5	1.00	23	0.217
60	A	3	3	1.00	8	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	8	0.250
62	A	3	2	1.00	8	0.250
63	A	2	2	1.00	10	0.200
64	A	4	4	1.00	10	0.400
65	A	5	5	1.00	10	0.500
66	A	5	5	1.00	25	0.200
67	A	4	4	1.00	23	0.174
68	A	6	5	1.00	23	0.217
69	A	4	4	1.00	25	0.160
70	A	5	5	1.00	25	0.200
71	A	7	7	1.00	25	0.280
72	A	6	6	1.00	25	0.240
73	A	2	2	1.00	16	0.125
74	A	7	7	1.00	25	0.280
75	A	7	7	1.00	25	0.280
76	A	6	5	1.00	25	0.200
77	A	5	4	1.00	23	0.174
78	A	7	6	1.00	23	0.261
79	A	7	6	1.00	25	0.240
80	A	5	4	1.00	25	0.160
81	A	6	5	1.00	25	0.200
82	A	8	7	1.00	25	0.280
83	A	7	6	1.00	25	0.240
84	A	6	6	1.00	16	0.375
85	A	6	6	1.00	25	0.240
86	A	7	7	1.00	25	0.280
87	A	7	7	1.00	16	0.438
88	A	3	3	1.00	10	0.300
89	A	3	3	1.00	12	0.250
90	A	1	1	1.00	12	0.083
91	A	2	2	1.00	10	0.200
92	A	2	2	1.00	12	0.167
93	A	4	4	1.00	10	0.400
94	A	4	4	1.00	12	0.333
95	A	4	4	1.00	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	6	6	1.00	10	0.600
97	A	6	6	1.00	12	0.500
98	A	4	4	1.00	25	0.160
99	A	3	3	1.00	23	0.130
100	A	3	3	1.00	23	0.130
101	A	4	4	1.00	25	0.160
102	A	6	6	1.00	25	0.240
103	A	5	5	1.00	25	0.200
104	A	2	2	1.00	16	0.125
105	A	5	5	1.00	25	0.200
106	A	7	7	1.00	25	0.280
107	A	4	4	1.00	25	0.160
108	A	2	2	1.00	23	0.087
109	A	4	4	1.00	23	0.174
110	A	6	6	1.00	25	0.240
111	A	7	7	1.00	25	0.280
112	A	6	6	1.00	25	0.240
113	A	6	6	1.00	25	0.240
114	A	4	4	1.00	16	0.250
115	A	7	7	1.00	25	0.280
116	A	5	5	1.00	25	0.200
117	A	3	3	1.00	25	0.120
118	A	3	3	1.00	23	0.130
119	A	6	6	1.00	23	0.261
120	A	7	7	1.00	25	0.280
121	A	5	5	1.00	25	0.200
122	A	7	6	1.00	25	0.240
123	A	7	7	1.00	16	0.438
124	A	8	8	1.00	25	0.320
125	A	3	3	1.00	10	0.300
126	A	1	1	1.00	12	0.083
127	A	2	2	1.00	10	0.200
128	A	3	3	1.00	12	0.250
129	A	2	2	1.00	12	0.167
130	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	5	5	1.00	23	0.217
132	A	4	4	1.00	23	0.174
133	A	3	3	1.00	21	0.143
134	A	3	3	1.00	21	0.143
135	A	3	3	1.00	23	0.130
136	A	3	3	1.00	23	0.130
137	A	3	3	1.00	23	0.130
138	A	3	3	1.00	23	0.130
139	A	3	3	1.00	23	0.130
140	A	3	3	1.00	23	0.130
141	A	7	4	1.00	21	0.190
142	A	8	4	1.00	21	0.190
143	A	6	4	1.00	21	0.190
144	A	6	4	1.00	19	0.210
145	A	3	1	1.00	12	0.083
146	A	5	4	1.00	19	0.210
147	A	5	4	1.00	21	0.190
148	A	4	3	1.00	21	0.143
149	A	5	3	1.00	21	0.143
150	A	10	4	1.00	23	0.174
151	A	11	4	1.00	23	0.174
152	A	8	5	1.00	21	0.238
153	A	8	4	1.00	14	0.286
154	A	7	5	1.00	21	0.238
155	A	8	5	1.00	23	0.217
156	A	6	4	1.00	23	0.174
157	A	7	5	1.00	23	0.217
158	A	8	6	1.00	23	0.261
159	A	6	4	1.00	23	0.174
160	A	9	4	1.00	23	0.174
161	A	13	4	1.00	23	0.174
162	A	14	5	1.00	21	0.238
163	A	10	4	1.00	14	0.286
164	A	12	5	1.00	21	0.238
165	A	10	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	10	6	1.00	23	0.261
167	A	9	6	1.00	23	0.261
168	A	11	7	1.00	23	0.304
169	A	8	5	1.00	23	0.217
170	A	11	6	1.00	23	0.261
171	A	15	6	1.00	23	0.261
172	A	15	7	1.00	23	0.304
173	A	14	5	1.00	23	0.217
174	A	13	5	1.00	23	0.217
175	A	11	5	1.00	23	0.217
176	A	11	4	1.00	21	0.190
177	A	11	4	1.00	14	0.286
178	A	14	6	1.00	21	0.286
179	A	15	6	1.00	23	0.261
180	A	15	7	1.00	23	0.304
181	A	16	7	1.00	23	0.304
182	A	12	8	1.00	8	1.000
183	A	12	8	1.00	10	0.800
184	A	6	6	1.00	21	0.286
185	A	3	2	1.00	21	0.095
186	A	5	5	1.00	21	0.238
187	A	2	1	1.00	19	0.053
188	A	4	2	1.00	12	0.167
189	A	4	3	1.00	19	0.158
190	A	4	4	1.00	21	0.190
191	A	4	4	1.00	21	0.190
192	A	4	3	1.00	21	0.143
193	A	4	4	1.00	21	0.190
194	A	3	2	1.00	21	0.095
195	A	5	5	1.00	21	0.238
196	A	3	2	1.00	23	0.087
197	A	7	6	1.00	23	0.261
198	A	3	2	1.00	21	0.095
199	A	6	5	1.00	14	0.357
200	A	4	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	6	5	1.00	23	0.217
202	A	5	4	1.00	23	0.174
203	A	6	5	1.00	23	0.217
204	A	6	5	1.00	23	0.217
205	A	5	4	1.00	23	0.174
206	A	6	5	1.00	23	0.217
207	A	3	2	1.00	23	0.087
208	A	3	2	1.00	23	0.087
209	A	3	2	1.00	21	0.095
210	A	4	3	1.00	21	0.143
211	A	5	4	1.00	23	0.174
212	A	6	5	1.00	23	0.217
213	A	7	5	1.00	23	0.217
214	A	8	5	1.00	23	0.217
215	A	8	5	1.00	23	0.217
216	A	8	5	1.00	23	0.217
217	A	9	6	1.00	23	0.261
218	A	8	5	1.00	14	0.357
219	A	8	5	1.00	23	0.217
220	A	8	5	1.00	23	0.217
221	A	7	5	1.00	23	0.217
222	A	6	5	1.00	23	0.217
223	A	5	4	1.00	23	0.174
224	A	4	3	1.00	23	0.130
225	A	3	2	1.00	23	0.087
226	A	3	2	1.00	23	0.087
227	A	3	2	1.00	23	0.087
228	A	3	2	1.00	23	0.087
229	A	6	5	1.00	24	0.208
230	A	6	5	1.00	24	0.208
231	A	4	4	1.00	24	0.167
232	A	4	4	1.00	22	0.182
233	A	7	6	1.00	22	0.273
234	A	7	6	1.00	24	0.250
235	A	7	5	1.00	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	7	5	1.00	24	0.208
237	A	4	3	1.00	24	0.125
238	A	4	3	1.00	15	0.200
239	A	6	4	1.00	24	0.167
240	A	6	4	1.00	24	0.167
241	A	7	6	1.00	24	0.250
242	A	5	5	1.00	24	0.208
243	A	5	5	1.00	24	0.208
244	A	5	5	1.00	24	0.208
245	A	5	5	1.00	22	0.227
246	A	11	7	1.00	22	0.318
247	A	14	9	1.00	24	0.375
248	A	6	5	1.00	24	0.208
249	A	7	6	1.00	24	0.250
250	A	5	4	1.00	24	0.167
251	A	5	4	1.00	15	0.267
252	A	7	5	1.00	24	0.208
253	A	6	6	1.00	24	0.250
254	A	6	6	1.00	24	0.250
255	A	6	6	1.00	24	0.250
256	A	6	5	1.00	24	0.208
257	A	6	6	1.00	22	0.273
258	A	16	7	1.00	22	0.318
259	A	9	7	1.00	24	0.292
260	A	6	5	1.00	24	0.208
261	A	6	5	1.00	24	0.208
262	A	6	5	1.00	24	0.208
263	A	6	5	1.00	15	0.333
264	A	8	6	1.00	24	0.250
265	A	3	3	1.00	10	0.300
266	A	10	6	1.00	8	0.750
267	A	17	5	1.00	10	0.500
268	A	7	3	1.00	10	0.300
269	A	9	3	1.00	10	0.300
270	A	17	6	1.00	8	0.750

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	8	6	1.00	8	0.750
272	A	9	3	1.00	8	0.375
273	A	17	6	1.00	10	0.600
274	A	7	3	1.00	10	0.300
275	A	10	6	1.00	10	0.600
276	A	3	2	1.00	15	0.133
277	A	3	3	1.00	15	0.200
278	A	2	2	1.00	15	0.133
279	A	2	2	1.00	15	0.133
280	A	2	2	1.00	13	0.154
281	A	3	3	1.00	13	0.231
282	A	4	3	1.00	15	0.200
283	A	5	4	1.00	21	0.190
284	A	3	2	1.00	21	0.095
285	A	4	4	1.00	21	0.190
286	A	2	1	1.00	19	0.053
287	A	3	3	1.00	19	0.158
288	A	3	3	1.00	21	0.143
289	A	3	3	1.00	21	0.143
290	A	2	1	1.00	21	0.048
291	A	4	4	1.00	21	0.190
292	A	3	2	1.00	21	0.095
293	A	6	5	1.00	23	0.217
294	A	3	2	1.00	23	0.087
295	A	5	5	1.00	23	0.217
296	A	3	2	1.00	21	0.095
297	A	4	3	1.00	21	0.143
298	A	5	4	1.00	23	0.174
299	A	5	4	1.00	23	0.174
300	A	4	3	1.00	23	0.130
301	A	4	4	1.00	23	0.174
302	A	3	2	1.00	23	0.087
303	A	5	5	1.00	23	0.217
304	A	7	6	1.00	23	0.261
305	A	3	2	1.00	23	0.087

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	6	6	1.00	23	0.261
307	A	3	2	1.00	21	0.095
308	A	4	3	1.00	21	0.143
309	A	6	5	1.00	23	0.217
310	A	5	4	1.00	23	0.174
311	A	5	4	1.00	23	0.174
312	A	6	5	1.00	23	0.217
313	A	4	3	1.00	23	0.130
314	A	5	5	1.00	23	0.217
315	A	3	2	1.00	23	0.087
316	A	4	3	1.00	23	0.130
317	A	6	6	1.00	23	0.261
318	A	4	3	1.00	23	0.130
319	A	5	5	1.00	23	0.217
320	A	3	3	1.00	23	0.130
321	A	4	4	1.00	23	0.174
322	A	2	2	1.00	21	0.095
323	A	4	4	1.00	21	0.190
324	A	3	3	1.00	23	0.130
325	A	5	5	1.00	23	0.217
326	A	4	3	1.00	23	0.130
327	A	6	6	1.00	23	0.261
328	A	4	3	1.00	23	0.130
329	A	6	6	1.00	23	0.261
330	A	5	4	1.00	23	0.174
331	A	5	5	1.00	23	0.217
332	A	3	3	1.00	23	0.130
333	A	3	3	1.00	23	0.130
334	A	3	3	1.00	21	0.143
335	A	5	5	1.00	21	0.238
336	A	5	4	1.00	23	0.174
337	A	6	6	1.00	23	0.261
338	A	5	4	1.00	23	0.174
339	A	6	6	1.00	23	0.261
340	A	4	4	1.00	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	4	3	1.00	23	0.130
342	A	4	4	1.00	23	0.174
343	A	4	4	1.00	23	0.174
344	A	4	3	1.00	21	0.143
345	A	6	6	1.00	21	0.286
346	A	6	5	1.00	23	0.217
347	A	7	6	1.00	23	0.261
348	A	6	5	1.00	23	0.217
349	A	4	3	1.00	15	0.200
350	A	3	3	1.00	15	0.200
351	A	5	4	1.00	15	0.267
352	A	5	5	1.00	25	0.200
353	A	4	4	1.00	23	0.174
354	A	6	6	1.00	23	0.261
355	A	4	4	1.00	25	0.160
356	A	5	5	1.00	25	0.200
357	A	7	7	1.00	25	0.280
358	A	6	6	1.00	25	0.240
359	A	2	2	1.00	16	0.125
360	A	2	2	1.00	25	0.080
361	A	5	5	1.00	25	0.200
362	A	6	5	1.00	25	0.200
363	A	5	4	1.00	23	0.174
364	A	7	7	1.00	23	0.304
365	A	7	7	1.00	25	0.280
366	A	5	4	1.00	25	0.160
367	A	6	5	1.00	25	0.200
368	A	8	7	1.00	25	0.280
369	A	7	7	1.00	25	0.280
370	A	6	6	1.00	16	0.375
371	A	6	6	1.00	25	0.240
372	A	5	5	1.00	25	0.200
373	A	4	4	1.00	25	0.160
374	A	3	3	1.00	23	0.130
375	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	4	4	1.00	25	0.160
377	A	6	6	1.00	25	0.240
378	A	5	5	1.00	25	0.200
379	A	2	2	1.00	16	0.125
380	A	7	7	1.00	25	0.280
381	A	5	5	1.00	25	0.200
382	A	4	4	1.00	25	0.160
383	A	2	2	1.00	23	0.087
384	A	4	4	1.00	23	0.174
385	A	6	6	1.00	25	0.240
386	A	7	7	1.00	25	0.280
387	A	6	6	1.00	25	0.240
388	A	2	2	1.00	25	0.080
389	A	4	4	1.00	16	0.250
390	A	5	5	1.00	25	0.200
391	A	5	5	1.00	25	0.200
392	A	3	3	1.00	25	0.120
393	A	3	3	1.00	23	0.130
394	A	6	6	1.00	23	0.261
395	A	7	7	1.00	25	0.280
396	A	5	5	1.00	25	0.200
397	A	5	5	1.00	25	0.200
398	A	7	7	1.00	16	0.438
399	A	6	6	1.00	25	0.240
400	A	3	3	1.00	25	0.120
401	A	5	5	1.00	23	0.217
402	A	4	4	0.95	23	0.174
403	A	3	3	1.00	21	0.143
404	A	3	3	1.00	21	0.143
405	A	3	3	1.00	23	0.130
406	A	3	3	1.00	23	0.130
407	A	3	3	1.00	23	0.130
408	A	3	3	1.00	14	0.214
409	A	3	3	1.00	23	0.130
410	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	4	3	1.00	25	0.120
412	A	4	3	1.00	25	0.120
413	A	4	3	1.00	23	0.130
414	A	19	13	1.00	23	0.565
415	A	4	3	1.00	25	0.120
416	A	4	3	1.00	25	0.120
417	A	4	3	1.00	23	0.130
418	A	19	13	1.00	23	0.565
419	A	6	4	1.00	23	0.174
420	A	5	4	1.00	23	0.174
421	A	2	2	1.00	21	0.095
422	A	6	4	1.00	23	0.174
423	A	5	4	1.00	23	0.174
424	A	2	2	1.00	21	0.095
425	A	4	4	1.00	13	0.308
426	A	5	4	1.00	25	0.160
427	A	5	4	1.00	25	0.160
428	A	4	4	1.00	23	0.174
429	A	5	5	1.00	23	0.217
430	A	7	7	1.00	25	0.280
431	A	7	6	1.00	25	0.240
432	A	6	6	1.00	25	0.240
433	A	5	5	1.00	25	0.200
434	A	5	4	1.00	25	0.160
435	A	5	4	1.00	25	0.160
436	A	5	4	1.00	25	0.160
437	A	5	4	1.00	25	0.160
438	A	5	4	1.00	25	0.160
439	A	4	4	1.00	23	0.174
440	A	4	4	1.00	23	0.174
441	A	6	6	1.00	25	0.240
442	A	5	4	1.00	25	0.160
443	A	4	4	1.00	25	0.160
444	A	4	4	1.00	25	0.160
445	A	4	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	5	4	1.00	25	0.160
447	A	5	4	1.00	25	0.160
448	A	5	4	1.00	25	0.160
449	A	4	4	1.00	23	0.174
450	A	5	5	1.00	23	0.217
451	A	6	6	1.00	25	0.240
452	A	5	5	1.00	25	0.200
453	A	5	5	1.00	25	0.200
454	A	4	4	1.00	25	0.160
455	A	5	4	1.00	25	0.160
456	A	5	4	1.00	25	0.160
457	A	6	6	1.00	25	0.240
458	A	5	5	1.00	25	0.200
459	A	4	4	1.00	23	0.174
460	A	4	4	1.00	23	0.174
461	A	5	5	1.00	25	0.200
462	A	6	6	1.00	25	0.240
463	A	7	7	1.00	25	0.280
464	A	6	6	1.00	25	0.240
465	A	2	2	1.00	16	0.125
466	A	6	6	1.00	25	0.240
467	A	7	7	1.00	25	0.280
468	A	7	6	1.00	25	0.240
469	A	6	5	1.00	25	0.200
470	A	5	4	1.00	23	0.174
471	A	5	4	1.00	23	0.174
472	A	6	5	1.00	25	0.200
473	A	7	6	0.98	25	0.240
474	A	8	8	1.00	25	0.320
475	A	7	7	1.00	25	0.280
476	A	6	6	1.00	16	0.375
477	A	7	7	1.00	25	0.280
478	A	8	8	1.00	25	0.320
479	A	5	5	1.00	25	0.200
480	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	3	3	1.00	23	0.130
482	A	3	3	1.00	23	0.130
483	A	4	4	1.00	25	0.160
484	A	5	5	1.00	25	0.200
485	A	5	5	1.00	25	0.200
486	A	6	6	1.00	25	0.240
487	A	2	2	1.00	16	0.125
488	A	6	6	1.00	25	0.240
489	A	7	7	1.00	25	0.280
490	A	6	6	1.00	25	0.240
491	A	5	5	1.00	25	0.200
492	A	4	4	1.00	23	0.174
493	A	4	4	1.00	23	0.174
494	A	5	5	1.00	25	0.200
495	A	6	6	1.00	25	0.240
496	A	6	6	1.00	25	0.240
497	A	5	5	1.00	25	0.200
498	A	4	4	1.00	16	0.250
499	A	7	7	1.00	25	0.280
500	A	8	7	1.00	25	0.280
501	A	7	6	1.00	25	0.240
502	A	6	5	1.00	25	0.200
503	A	5	4	1.00	23	0.174
504	A	5	4	1.00	23	0.174
505	A	6	5	1.00	25	0.200
506	A	7	6	1.00	25	0.240
507	A	7	6	1.00	25	0.240
508	A	6	6	1.00	25	0.240
509	A	7	7	1.00	16	0.438
510	A	8	8	1.00	25	0.320
511	A	9	8	1.00	25	0.320
512	A	3	3	1.00	25	0.120
513	A	3	3	1.00	23	0.130
514	A	2	2	1.00	21	0.095
515	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	3	3	1.00	23	0.130
517	A	3	3	1.00	23	0.130
518	A	3	3	1.00	23	0.130
519	A	3	3	1.00	23	0.130
520	A	3	3	1.00	23	0.130
521	A	12	11	1.00	15	0.733
522	A	4	4	1.00	15	0.267
523	A	5	5	1.00	15	0.333
524	A	4	4	1.00	15	0.267
525	A	5	5	1.00	15	0.333

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=89

$$\frac{1}{16}(6a-5b)x-\frac{(6a-5b)\cosh(c+dx)\sinh(c+dx)}{16d}+\frac{(6a-5b)\cosh(c+dx)\sinh^3(c+dx)}{24d}+\frac{b\cosh(c+dx)\,\mathrm{si}}{6d}$$

```
[Out] 1/16*(6*a-5*b)*x-1/16*(6*a-5*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/24*(6*a-5*b)*co
sh(d*x+c)*sinh(d*x+c)^3/d+1/6*b*cosh(d*x+c)*sinh(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3093, 2715, 8}

$$\frac{(6a-5b)\sinh^3(c+dx)\cosh(c+dx)}{24d}-\frac{(6a-5b)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x(6a-5b)+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((6*a - 5*b)*x)/16 - ((6*a - 5*b)*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) + ((6
*a - 5*b)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24*d) + (b*Cosh[c + d*x]*Sinh[c +
 d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{b\cosh(c+dx)\sinh^5(c+dx)}{6d}+\frac{1}{6}(6a-5b)\int \sinh^4(c+dx)\,dx \\
&= \frac{(6a-5b)\cosh(c+dx)\sinh^3(c+dx)}{24d}+\frac{b\cosh(c+dx)\sinh^5(c+}{6d} \\
&= -\frac{(6a-5b)\cosh(c+dx)\sinh(c+dx)}{16d}+\frac{(6a-5b)\cosh(c+dx)}{24d} \\
&= \frac{1}{16}(6a-5b)x-\frac{(6a-5b)\cosh(c+dx)\sinh(c+dx)}{16d}+(6a-5b
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 68, normalized size = 0.76

$$\frac{72ac-60bc+72adx-60bdx+(-48a+45b)\sinh(2(c+dx))+(6a-9b)\sinh(4(c+dx))+b\sinh(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (72*a*c - 60*b*c + 72*a*d*x - 60*b*d*x + (-48*a + 45*b)*Sinh[2*(c + d*x)] +
 (6*a - 9*b)*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 1.27, size = 67, normalized size = 0.75

method	result
default	$\frac{\left(-\frac{3b}{16}+\frac{a}{8}\right)\sinh(4dx+4c)}{4d}+\frac{\left(\frac{15b}{32}-\frac{a}{2}\right)\sinh(2dx+2c)}{2d}+\frac{3ax}{8}-\frac{5bx}{16}+\frac{b\sinh(6dx+6c)}{192d}$
risch	$-\frac{5bx}{16}+\frac{3ax}{8}+\frac{b\,\mathrm{e}^{6dx+6c}}{384d}+\frac{\mathrm{e}^{4dx+4c}a}{64d}-\frac{3\,\mathrm{e}^{4dx+4c}b}{128d}+\frac{15\,\mathrm{e}^{2dx+2c}b}{128d}-\frac{\mathrm{e}^{2dx+2c}a}{8d}-\frac{15\,\mathrm{e}^{-2dx-2c}b}{128d}+\frac{\mathrm{e}^{-2dx-2c}a}{8d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-3/16*b+1/8*a)/d*sinh(4*d*x+4*c)+1/2*(15/32*b-1/2*a)*sinh(2*d*x+2*c)/d
+3/8*a*x-5/16*b*x+1/192*b/d*sinh(6*d*x+6*c)
```

Maxima [A]
time = 0.27, size = 150, normalized size = 1.69

$$\frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{384}\,b\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/384*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*
c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-
4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Fricas [**A**]
time = 0.39, size = 122, normalized size = 1.37

$$\frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+2\left(5\,b\cosh\left(dx+c\right)^{3}+3\,(2\,a-3\,b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+6\,(6\,a-5\,b)dx+3\left(b\cosh\left(dx+c\right)^{5}+2\,(2\,a-3\,b)\cosh\left(dx+c\right)^{3}-(16\,a-15\,b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 + 3*(2*a -
 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(6*a - 5*b)*d*x + 3*(b*cosh(d*x +
c)^5 + 2*(2*a - 3*b)*cosh(d*x + c)^3 - (16*a - 15*b)*cosh(d*x + c))*sinh(d*
x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(82) = 164$.
time = 0.45, size = 258, normalized size = 2.90

$$\begin{cases} \frac{3ax\sinh^{4}\left(c+dx\right)}{8}-\frac{3ax\sinh^{2}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{4}+\frac{3ax\cosh^{4}\left(c+dx\right)}{8}+\frac{5a\sinh^{3}\left(c+dx\right)\cosh\left(c+dx\right)}{8d}-\frac{3a\sinh\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{8d}+\frac{5bx\sinh^{6}\left(c+dx\right)}{16}-\frac{15bx\sinh^{4}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{16}+\frac{15bx\sinh^{2}\left(c+dx\right)\cosh^{4}\left(c+dx\right)}{16}-\frac{5bx\cosh^{6}\left(c+dx\right)}{16}+\frac{11b\sinh^{5}\left(c+dx\right)\cosh\left(c+dx\right)}{16d}-\frac{5b\sinh^{3}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{6d}+\frac{5b\sinh\left(c+dx\right)\cosh^{5}\left(c+dx\right)}{16d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^{2}\left(c\right)\right)\sinh^{4}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
- 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*b*x*sinh(c + d*x)**6/16 - 15
*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b*x*sinh(c + d*x)**2*cosh(c
+ d*x)**4/16 - 5*b*x*cosh(c + d*x)**6/16 + 11*b*sinh(c + d*x)**5*cosh(c + d
*x)/(16*d) - 5*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*sinh(c + d*x
)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sinh(c)**4, Tru
e))
```

Giac [**A**]
time = 0.43, size = 125, normalized size = 1.40

$$\frac{1}{16}\,(6\,a-5\,b)x+\frac{be^{(6\,dx+6\,c)}}{384\,d}+\frac{(2\,a-3\,b)e^{(4\,dx+4\,c)}}{128\,d}-\frac{(16\,a-15\,b)e^{(2\,dx+2\,c)}}{128\,d}+\frac{(16\,a-15\,b)e^{(-2\,dx-2\,c)}}{128\,d}-\frac{(2\,a-3\,b)e^{(-4\,dx-4\,c)}}{128\,d}-\frac{be^{(-6\,dx-6\,c)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*(6*a - 5*b)*x + 1/384*b*e^(6*d*x + 6*c)/d + 1/128*(2*a - 3*b)*e^(4*d*x
 + 4*c)/d - 1/128*(16*a - 15*b)*e^(2*d*x + 2*c)/d + 1/128*(16*a - 15*b)*e^(
-2*d*x - 2*c)/d - 1/128*(2*a - 3*b)*e^(-4*d*x - 4*c)/d - 1/384*b*e^(-6*d*x
- 6*c)/d
```

Mupad [B]
time = 0.78, size = 76, normalized size = 0.85

$$\frac{\frac{3\,a\,\mathrm{sinh}\left(4\,c+4\,d\,x\right)}{2}-12\,a\,\mathrm{sinh}\left(2\,c+2\,d\,x\right)+\frac{45\,b\,\mathrm{sinh}\left(2\,c+2\,d\,x\right)}{4}-\frac{9\,b\,\mathrm{sinh}\left(4\,c+4\,d\,x\right)}{4}+\frac{b\,\mathrm{sinh}\left(6\,c+6\,d\,x\right)}{4}+18\,a\,d\,x-15\,b\,d\,x}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] ((3*a*sinh(4*c + 4*d*x))/2 - 12*a*sinh(2*c + 2*d*x) + (45*b*sinh(2*c + 2*d*
x))/4 - (9*b*sinh(4*c + 4*d*x))/4 + (b*sinh(6*c + 6*d*x))/4 + 18*a*d*x - 15
*b*d*x)/(48*d)
```

3.2 $\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=53

$$-\frac{(a-b)\cosh(c+dx)}{d}+\frac{(a-2b)\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}$$

```
[Out] -(a-b)*cosh(d*x+c)/d+1/3*(a-2*b)*cosh(d*x+c)^3/d+1/5*b*cosh(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3092, 380}

$$\frac{(a-2b)\cosh^3(c+dx)}{3d}-\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -(((a - b)*Cosh[c + d*x])/d) + ((a - 2*b)*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[
c + d*x]^5)/(5*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= -\frac{\operatorname{Subst}\left(\int\left(1-x^2\right)\left(a-b+bx^2\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\operatorname{Subst}\left(\int\left(a\left(1-\frac{b}{a}\right)-(a-2b)x^2-bx^4\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{(a-b)\cosh(c+dx)}{d}+\frac{(a-2b)\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+}{5d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 77, normalized size = 1.45

$$-\frac{3a\cosh(c+dx)}{4d}+\frac{5b\cosh(c+dx)}{8d}+\frac{a\cosh(3(c+dx))}{12d}-\frac{5b\cosh(3(c+dx))}{48d}+\frac{b\cosh(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) + (5*b*Cosh[c + d*x])/(8*d) + (a*Cosh[3*(c + d*x
)])/(12*d) - (5*b*Cosh[3*(c + d*x)])/(48*d) + (b*Cosh[5*(c + d*x)])/(80*d)
```

Maple [A]
time = 0.65, size = 55, normalized size = 1.04

method	result
default	$\frac{\left(-\frac{5b}{16}+\frac{a}{4}\right)\cosh(3dx+3c)}{3d}+\frac{\left(\frac{5b}{8}-\frac{3a}{4}\right)\cosh(dx+c)}{d}+\frac{b\cosh(5dx+5c)}{80d}$
risch	$\frac{b\,\mathrm{e}^{5dx+5c}}{160d}+\frac{\mathrm{e}^{3dx+3c}a}{24d}-\frac{5\,\mathrm{e}^{3dx+3c}b}{96d}-\frac{3\,\mathrm{e}^{dx+c}a}{8d}+\frac{5b\,\mathrm{e}^{dx+c}}{16d}-\frac{3\,\mathrm{e}^{-dx-c}a}{8d}+\frac{5\,\mathrm{e}^{-dx-c}b}{16d}+\frac{\mathrm{e}^{-3dx-3c}a}{24d}-\frac{5\,\mathrm{e}^{-3dx-3c}b}{96d}+b\,\mathrm{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-5/16*b+1/4*a)/d*cosh(3*d*x+3*c)+(5/8*b-3/4*a)*cosh(d*x+c)/d+1/80*b*co
sh(5*d*x+5*c)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(49) = 98.
time = 0.26, size = 141, normalized size = 2.66

$$\frac{1}{480}\,b\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 1
50*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/24*a*
(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/
d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(49) = 98.
time = 0.41, size = 102, normalized size = 1.92

$$\frac{3\,b\cosh\left(dx+c\right)^{5}+15\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+5\,(4\,a-5\,b)\cosh\left(dx+c\right)^{3}+15\,\left(2\,b\cosh\left(dx+c\right)^{3}+(4\,a-5\,b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}-30\,(6\,a-5\,b)\cosh\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 + 5*(4*a -
5*b)*cosh(d*x + c)^3 + 15*(2*b*cosh(d*x + c)^3 + (4*a - 5*b)*cosh(d*x + c))
*sinh(d*x + c)^2 - 30*(6*a - 5*b)*cosh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(42) = 84$.
time = 0.27, size = 105, normalized size = 1.98

$$\begin{cases} \frac{a\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a\cosh^3(c+dx)}{3d} + \frac{b\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4b\sinh^2(c+dx)\cosh^3(c+dx)}{3d} + \frac{8b\cosh^5(c+dx)}{15d} & \text{for } d \neq 0 \\ x\left(a + b\sinh^2(c)\right)\sinh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d)
+ b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sinh(c + d*x)**2*cosh(c + d*x)**
3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sin
h(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.
time = 0.43, size = 112, normalized size = 2.11

$$\frac{be^{(5\,dx+5\,c)}}{160\,d} + \frac{(4\,a-5\,b)e^{(3\,dx+3\,c)}}{96\,d} - \frac{(6\,a-5\,b)e^{(dx+c)}}{16\,d} - \frac{(6\,a-5\,b)e^{(-dx-c)}}{16\,d} + \frac{(4\,a-5\,b)e^{(-3\,dx-3\,c)}}{96\,d} + \frac{be^{(-5\,dx-5\,c)}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/160*b*e^(5*d*x + 5*c)/d + 1/96*(4*a - 5*b)*e^(3*d*x + 3*c)/d - 1/16*(6*a
- 5*b)*e^(d*x + c)/d - 1/16*(6*a - 5*b)*e^(-d*x - c)/d + 1/96*(4*a - 5*b)*e
^(-3*d*x - 3*c)/d + 1/160*b*e^(-5*d*x - 5*c)/d
```

Mupad [B]
time = 0.64, size = 57, normalized size = 1.08

$$\frac{15\,b\cosh(c+d\,x) - 15\,a\cosh(c+d\,x) + 5\,a\cosh(c+d\,x)^3 - 10\,b\cosh(c+d\,x)^3 + 3\,b\cosh(c+d\,x)^5}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (15*b*cosh(c + d*x) - 15*a*cosh(c + d*x) + 5*a*cosh(c + d*x)^3 - 10*b*cosh(
c + d*x)^3 + 3*b*cosh(c + d*x)^5)/(15*d)
```

3.3 $\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$-\frac{1}{8}(4a-3b)x+\frac{(4a-3b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}$$

```
[Out] -1/8*(4*a-3*b)*x+1/8*(4*a-3*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b*cosh(d*x+c)*
sinh(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3093, 2715, 8}

$$\frac{(4a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}-\frac{1}{8}x(4a-3b)+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/8*((4*a - 3*b)*x) + ((4*a - 3*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b
*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{4}(-4a+3b)\int \sinh^2(c+dx)\,d\\ &= \frac{(4a-3b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sinh^3(c+}{4d}\\ &= -\frac{1}{8}(4a-3b)x+\frac{(4a-3b)\cosh(c+dx)\sinh(c+dx)}{8d}+b\cosh(\end{aligned}$$

Mathematica [A]
time = 0.06, size = 47, normalized size = 0.77

$$\frac{-4(4a-3b)(c+dx)+8(a-b)\sinh(2(c+dx))+b\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-4*(4*a - 3*b)*(c + d*x) + 8*(a - b)*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x
)])/(32*d)
```

Maple [A]
time = 0.76, size = 46, normalized size = 0.75

method	result	size
default	$\frac{\left(-\frac{b}{2}+\frac{a}{2}\right)\sinh(2dx+2c)}{2d}-\frac{ax}{2}+\frac{3bx}{8}+\frac{b\sinh(4dx+4c)}{32d}$	46
risch	$\frac{3bx}{8}-\frac{ax}{2}+\frac{\mathrm{e}^{4dx+4c}b}{64d}+\frac{\mathrm{e}^{2dx+2c}a}{8d}-\frac{\mathrm{e}^{2dx+2c}b}{8d}-\frac{\mathrm{e}^{-2dx-2c}a}{8d}+\frac{\mathrm{e}^{-2dx-2c}b}{8d}-\frac{\mathrm{e}^{-4dx-4c}b}{64d}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1/2*b+1/2*a)*sinh(2*d*x+2*c)/d-1/2*a*x+3/8*b*x+1/32*b/d*sinh(4*d*x+4*
c)
```

Maxima [A]
time = 0.27, size = 97, normalized size = 1.59

$$\frac{1}{64}\,b\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{8}\,a\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c
)/d)
```

Fricas [A]
time = 0.53, size = 64, normalized size = 1.05

$$\frac{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}-(4\,a-3\,b)dx+\left(b\cosh\left(dx+c\right)^{3}+4\,(a-b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 - (4*a - 3*b)*d*x + (b*cosh(d*x + c)^3
 + 4*(a - b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(53) = 106.
time = 0.19, size = 158, normalized size = 2.59

$$\begin{cases} \frac{ax\sinh^{2}\left(c+dx\right)}{2}-\frac{ax\cosh^{2}\left(c+dx\right)}{2}+\frac{a\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d}+\frac{3bx\sinh^{4}\left(c+dx\right)}{8}-\frac{3bx\sinh^{2}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{4}+\frac{3bx\cosh^{4}\left(c+dx\right)}{8}+\frac{5b\sinh^{3}\left(c+dx\right)\cosh\left(c+dx\right)}{8d}-\frac{3b\sinh\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{8d} & \text{for}\ d\neq 0\\ x\left(a+b\sinh^{2}\left(c\right)\right)\sinh^{2}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x
)*cosh(c + d*x)/(2*d) + 3*b*x*sinh(c + d*x)**4/8 - 3*b*x*sinh(c + d*x)**2*c
osh(c + d*x)**2/4 + 3*b*x*cosh(c + d*x)**4/8 + 5*b*sinh(c + d*x)**3*cosh(c
+ d*x)/(8*d) - 3*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a +
 b*sinh(c)**2)*sinh(c)**2, True))
```

Giac [A]
time = 0.42, size = 79, normalized size = 1.30

$$-\frac{1}{8}\,(4\,a-3\,b)x+\frac{be^{(4\,dx+4\,c)}}{64\,d}+\frac{(a-b)e^{(2\,dx+2\,c)}}{8\,d}-\frac{(a-b)e^{(-2\,dx-2\,c)}}{8\,d}-\frac{be^{(-4\,dx-4\,c)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*a - 3*b)*x + 1/64*b*e^(4*d*x + 4*c)/d + 1/8*(a - b)*e^(2*d*x + 2*c)
/d - 1/8*(a - b)*e^(-2*d*x - 2*c)/d - 1/64*b*e^(-4*d*x - 4*c)/d
```

Mupad [B]
time = 0.11, size = 50, normalized size = 0.82

$$\frac{\frac{a\sinh(2\,c+2\,d\,x)}{4}-\frac{b\sinh(2\,c+2\,d\,x)}{4}+\frac{b\sinh(4\,c+4\,d\,x)}{32}}{d}-\frac{a\,x}{2}+\frac{3\,b\,x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] ((a*sinh(2*c + 2*d*x))/4 - (b*sinh(2*c + 2*d*x))/4 + (b*sinh(4*c + 4*d*x))/
32)/d - (a*x)/2 + (3*b*x)/8
```

3.4 $\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=32

$$\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}$$

```
[Out] (a-b)*cosh(d*x+c)/d+1/3*b*cosh(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.053$, Rules used = $\{3092\}$

$$\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - b)*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/(3*d)
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx&=\frac{\text{Subst}\left(\int\left(a-b+bx^2\right)\,dx,x,\cosh(c+dx)\right)}{d}\\&=\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 1.66

$$\frac{a\cosh(c)\cosh(dx)}{d}-\frac{3b\cosh(c+dx)}{4d}+\frac{b\cosh(3(c+dx))}{12d}+\frac{a\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d - (3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])
/(12*d) + (a*Sinh[c]*Sinh[d*x])/d
```

Maple [A]
time = 0.55, size = 32, normalized size = 1.00

method	result	size
default	$\frac{\left(-\frac{3b}{4}+a\right)\cosh(dx+c)}{d}+\frac{b\cosh(3dx+3c)}{12d}$	32
risch	$\frac{\mathrm{e}^{3dx+3c}b}{24d}+\frac{\mathrm{e}^{dx+c}a}{2d}-\frac{3b\,\mathrm{e}^{dx+c}}{8d}+\frac{\mathrm{e}^{-dx-c}a}{2d}-\frac{3\,\mathrm{e}^{-dx-c}b}{8d}+\frac{\mathrm{e}^{-3dx-3c}b}{24d}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] (-3/4*b+a)*cosh(d*x+c)/d+1/12*b/d*cosh(3*d*x+3*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.
time = 0.27, size = 67, normalized size = 2.09

$$\frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
 - 3*c)/d) + a*cosh(d*x + c)/d
```

Fricas [A]
time = 0.42, size = 48, normalized size = 1.50

$$\frac{b\cosh\left(dx+c\right)^{3}+3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+3\,(4\,a-3\,b)\cosh\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + 3*(4*a - 3*b)
*cosh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.
time = 0.12, size = 56, normalized size = 1.75

$$\begin{cases} \frac{a\cosh\left(c+dx\right)}{d}+\frac{b\sinh^{2}\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2b\cosh^{3}\left(c+dx\right)}{3d} & \text{for}\ d\neq 0 \\ x\left(a+b\sinh^{2}\left(c\right)\right)\sinh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*cosh(c + d*x)/d + b*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*b*cos
h(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sinh(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(30) = 60$.
time = 0.42, size = 70, normalized size = 2.19

$$\frac{be^{(3\,dx+3\,c)}}{24\,d} + \frac{(4\,a-3\,b)e^{(dx+c)}}{8\,d} + \frac{(4\,a-3\,b)e^{(-dx-c)}}{8\,d} + \frac{be^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*b*e^(3*d*x + 3*c)/d + 1/8*(4*a - 3*b)*e^(d*x + c)/d + 1/8*(4*a - 3*b)*
e^(-d*x - c)/d + 1/24*b*e^(-3*d*x - 3*c)/d
```

Mupad [B]
time = 0.59, size = 34, normalized size = 1.06

$$\frac{3\,a\cosh(c+d\,x) - 3\,b\cosh(c+d\,x) + b\cosh(c+d\,x)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (3*a*cosh(c + d*x) - 3*b*cosh(c + d*x) + b*cosh(c + d*x)^3)/(3*d)
```

3.5 $\int \left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=30

$$ax-\frac{bx}{2}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}$$

```
[Out] a*x-1/2*b*x+1/2*b*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {2715, 8}

$$ax+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sinh[c + d*x]^2,x]
```

```
[Out] a*x - (b*x)/2 + (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^2(c+dx)\right)\,dx &= ax+b\int \sinh^2(c+dx)\,dx\\ &= ax+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{1}{2}b\int 1\,dx\\ &= ax-\frac{bx}{2}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 1.20

$$ax+\frac{b(-c-dx)}{2d}+\frac{b\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sinh[c + d*x]^2,x]
```

```
[Out] a*x + (b*(-c - d*x))/(2*d) + (b*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.74, size = 32, normalized size = 1.07

method	result	size
default	$ax + \frac{b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right)}{d}$	32
derivativedivides	$\frac{(dx+c)a+b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right)}{d}$	37
risch	$ax - \frac{bx}{2} + \frac{e^{2dx+2c}b}{8d} - \frac{e^{-2dx-2c}b}{8d}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b/d*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)
```

Maxima [A]
time = 0.27, size = 38, normalized size = 1.27

$$-\frac{1}{8}\,b\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a*x
```

Fricas [A]
time = 0.41, size = 30, normalized size = 1.00

$$\frac{(2\,a-b)dx + b\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a - b)*d*x + b*cosh(d*x + c)*sinh(d*x + c))/d
```

Sympy [A]
time = 0.07, size = 51, normalized size = 1.70

$$ax + b\left(\begin{cases} \frac{x\sinh^2\left(c+dx\right)}{2} - \frac{x\cosh^2\left(c+dx\right)}{2} + \frac{\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d} & \text{for}\ d \neq 0 \\ x\sinh^2\left(c\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)**2,x)
```

```
[Out] a*x + b*Piecewise((x*sinh(c + d*x)**2/2 - x*cosh(c + d*x)**2/2 + sinh(c + d
*x)*cosh(c + d*x)/(2*d), Ne(d, 0)), (x*sinh(c)**2, True))
```

Giac [A]
time = 0.41, size = 38, normalized size = 1.27

$$-\frac{1}{8}\,b\left(4\,x-\frac{e^{\left(2\,dx+2\,c\right)}}{d}+\frac{e^{\left(-2\,dx-2\,c\right)}}{d}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a*x
```

Mupad [B]
time = 0.07, size = 23, normalized size = 0.77

$$a\,x-\frac{b\,x}{2}+\frac{b\sinh(2\,c+2\,d\,x)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sinh(c + d*x)^2,x)
```

```
[Out] a*x - (b*x)/2 + (b*sinh(2*c + 2*d*x))/(4*d)
```

3.6 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=25

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh(c+dx)}{d}$$

```
[Out] -a*arctanh(cosh(d*x+c))/d+b*cosh(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {3093, 3855}

$$\frac{b\cosh(c+dx)}{d}-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((a*ArcTanh[Cosh[c + d*x]])/d) + (b*Cosh[c + d*x])/d
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx&=\frac{b\cosh(c+dx)}{d}+a\int \operatorname{csch}(c+dx)\,dx\\&=-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh(c+dx)}{d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(25) = 50.
time = 0.03, size = 62, normalized size = 2.48

$$\frac{b\cosh(c)\cosh(dx)}{d}-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (b*Cosh[c]*Cosh[d*x])/d - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[Sinh[c/2
+ (d*x)/2]])/d + (b*Sinh[c]*Sinh[d*x])/d
```

Maple [A]
time = 0.77, size = 24, normalized size = 0.96

method	result	size
derivativedivides	$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\cosh(dx+c)}{d}$	24
default	$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\cosh(dx+c)}{d}$	24
risch	$\frac{b\,\mathrm{e}^{dx+c}}{2d}+\frac{\mathrm{e}^{-dx-c}b}{2d}+\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*cosh(d*x+c))
```

Maxima [A]
time = 0.28, size = 43, normalized size = 1.72

$$\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(25) = 50.
time = 0.41, size = 126, normalized size = 5.04

$$\frac{b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}-2\left(a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)+2\left(a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)+b}{2\left(d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^
2 - 2*(a*cosh(d*x + c) + a*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
 + 1) + 2*(a*cosh(d*x + c) + a*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
+ c) - 1) + b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^{2}\left(c + dx\right)\right)\operatorname{csch}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*csch(c + d*x), x)
```

Giac [**A**]
time = 0.41, size = 50, normalized size = 2.00

$$\frac{be^{(dx+c)} + be^{(-dx-c)} - 2\,a\log\left(e^{(dx+c)} + 1\right) + 2\,a\log\left(\left|e^{(dx+c)} - 1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b*e^(d*x + c) + b*e^(-d*x - c) - 2*a*log(e^(d*x + c) + 1) + 2*a*log(ab
s(e^(d*x + c) - 1)))/d
```

Mupad [**B**]
time = 0.14, size = 66, normalized size = 2.64

$$\frac{b\,\mathrm{e}^{-c-d\,x}}{2\,d} + \frac{b\,\mathrm{e}^{c+d\,x}}{2\,d} - \frac{2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/sinh(c + d*x),x)
```

```
[Out] (b*exp(- c - d*x))/(2*d) + (b*exp(c + d*x))/(2*d) - (2*atan((a*exp(d*x)*exp
(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-d^2)^(1/2)
```

3.7 $\int \text{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=16

$$bx-\frac{a\coth(c+dx)}{d}$$

```
[Out] b*x-a*coth(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3091, 8}

$$bx-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] b*x - (a*Coth[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \text{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= -\frac{a\coth(c+dx)}{d}+b\int 1\,dx\\ &= bx-\frac{a\coth(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$bx-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] b*x - (a*Coth[c + d*x])/d
```

Maple [A]
time = 1.02, size = 24, normalized size = 1.50

method	result	size
risch	$bx-\frac{2a}{d\left(\mathrm{e}^{2dx+2c}-1\right)}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] b*x-2*a/d/(exp(2*d*x+2*c)-1)
```

Maxima [A]
time = 0.26, size = 23, normalized size = 1.44

$$bx+\frac{2\,a}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*x + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.
time = 0.38, size = 36, normalized size = 2.25

$$-\frac{a\cosh\left(dx+c\right)-\left(bdx+a\right)\sinh\left(dx+c\right)}{d\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*cosh(d*x + c) - (b*d*x + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^{2}\left(c+dx\right)\right)\operatorname{csch}^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*csch(c + d*x)**2, x)
```

Giac [A]
time = 0.43, size = 28, normalized size = 1.75

$$\frac{(dx+c)b-\frac{2\,a}{e^{(2\,dx+2\,c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] ((d*x + c)*b - 2*a/(e^(2*d*x + 2*c) - 1))/d
```

Mupad [B]
time = 0.57, size = 23, normalized size = 1.44

$$b\,x-\frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/sinh(c + d*x)^2,x)
```

```
[Out] b*x - (2*a)/(d*(exp(2*c + 2*d*x) - 1))
```

3.8 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=40

$$\frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

```
[Out] 1/2*(a-2*b)*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3091, 3855}

$$\frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - 2*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/
(2*d)
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx&=-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{1}{2}(a-2b)\int \operatorname{csch}(c+dx)\,dx\\&=\frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 99 vs. 2(40) = 80.
time = 0.03, size = 99, normalized size = 2.48

$$-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/8*(a*Csch[(c + d*x)/2]^2)/d - (b*Log[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Si
nh[c/2 + (d*x)/2]])/d - (a*Log[Tanh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d*x
)/2]^2)/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(36) = 72$.

time = 1.08, size = 97, normalized size = 2.42

method	result	size
risch	$-\frac{a\,\mathrm{e}^{dx+c}\left(1+\mathrm{e}^{2dx+2c}\right)}{d\left(\mathrm{e}^{2dx+2c}-1\right)^{2}}+\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)b}{d}-\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{2d}-\frac{\ln\left(\mathrm{e}^{dx+c}+1\right)b}{d}+\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{2d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -a*exp(d*x+c)*(1+exp(2*d*x+2*c))/d/(exp(2*d*x+2*c)-1)^2+1/d*ln(exp(d*x+c)-1
)*b-1/2*a/d*ln(exp(d*x+c)-1)-1/d*ln(exp(d*x+c)+1)*b+1/2*a/d*ln(exp(d*x+c)+1
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(36) = 72$.

time = 0.26, size = 125, normalized size = 3.12

$$\frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c)
+ e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b*(l
og(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(36) = 72$.

time = 0.40, size = 484, normalized size = 12.10

$$-\frac{2\,a\cosh\left(dx+c\right)^{3}+6\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+2\,a\sinh\left(dx+c\right)^{3}+2\,a\cosh\left(dx+c\right)-\left(\left(a-2\,b\right)\cosh\left(dx+c\right)^{4}+4\,\left(a-2\,b\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\left(a-2\,b\right)\sinh\left(dx+c\right)^{4}-2\,\left(a-2\,b\right)\cosh\left(dx+c\right)^{2}+2\,\left(3\,\left(a-2\,b\right)\cosh\left(dx+c\right)^{2}-a+2\,b\right)\sinh\left(dx+c\right)^{2}+4\,\left(\left(a-2\,b\right)\cosh\left(dx+c\right)^{3}-\left(a-2\,b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+a-2\,b\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)+\left(\left(a-2\,b\right)\cosh\left(dx+c\right)^{4}+4\,\left(a-2\,b\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\left(a-2\,b\right)\sinh\left(dx+c\right)^{4}-2\,\left(a-2\,b\right)\cosh\left(dx+c\right)^{2}+2\,\left(3\,\left(a-2\,b\right)\cosh\left(dx+c\right)^{2}-a+2\,b\right)\sinh\left(dx+c\right)^{2}+4\,\left(\left(a-2\,b\right)\cosh\left(dx+c\right)^{3}-\left(a-2\,b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+a-2\,b\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)+2\,\left(3\,a\cosh\left(dx+c\right)^{2}+a\right)\sinh\left(dx+c\right)}{2\,\left(d\cosh\left(dx+c\right)^{4}+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{4}-2\,d\cosh\left(dx+c\right)^{2}+2\,\left(3\,d\cosh\left(dx+c\right)^{2}-d\right)\sinh\left(dx+c\right)^{2}+4\,\left(d\cosh\left(dx+c\right)^{3}-d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(d*
x + c)^3 + 2*a*cosh(d*x + c) - ((a - 2*b)*cosh(d*x + c)^4 + 4*(a - 2*b)*cos
```

```
h(d*x + c)*sinh(d*x + c)^3 + (a - 2*b)*sinh(d*x + c)^4 - 2*(a - 2*b)*cosh(d
*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^2 + 4*(
(a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*
b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a - 2*b)*cosh(d*x + c)^4 + 4*
(a - 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a - 2*b)*sinh(d*x + c)^4 - 2*(a
- 2*b)*cosh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x
 + c)^2 + 4*((a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x
+ c) + a - 2*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x +
c)^2 + a)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x +
c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d
)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) +
 d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^2\left(c+dx\right)\right)\operatorname{csch}^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*csch(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(36) = 72.
time = 0.44, size = 96, normalized size = 2.40

$$\frac{(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((a - 2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a - 2*b)*log(e^(d*x +
 c) + e^(-d*x - c) - 2) - 4*a*(e^(d*x + c) + e^(-d*x - c))/((e^(d*x + c) +
e^(-d*x - c))^2 - 4))/d
```

Mupad [**B**]
time = 0.13, size = 131, normalized size = 3.28

$$\frac{\operatorname{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a\,\sqrt{-d^2}-2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^2-4\,a\,b+4\,b^2}}\right)\,\sqrt{a^2-4\,a\,b+4\,b^2}}{\sqrt{-d^2}}-\frac{a\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) - 2*b*(-d^2)^(1/2)))/(d*(a^2 - 4*a*b
 + 4*b^2)^(1/2)))*(a^2 - 4*a*b + 4*b^2)^(1/2))/(-d^2)^(1/2) - (a*exp(c + d*
x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a*exp(c + d*x))/(d*(exp(4*c + 4*d*x) -
2*exp(2*c + 2*d*x) + 1))
```

3.9 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=43

$$\frac{(2a-3b)\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

```
[Out] 1/3*(2*a-3*b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)*csch(d*x+c)^2/d
```

Rubi [A]

time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3091, 3852, 8}

$$\frac{(2a-3b)\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((2*a - 3*b)*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= -\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+\frac{1}{3}(-2a+3b)\int \operatorname{csch}^2(c+dx)\,d\\ &= -\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+\frac{(i(2a-3b))\mathrm{Subst}(\int 1\,dx,x,-i}{3d}\\ &= \frac{(2a-3b)\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 49, normalized size = 1.14

$$\frac{2a\coth(c+dx)}{3d}-\frac{b\coth(c+dx)}{d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) - (b*Coth[c + d*x])/d - (a*Coth[c + d*x]*Csch[c +
 d*x]^2)/(3*d)
```

Maple [**A**]
time = 0.99, size = 62, normalized size = 1.44

method	result	size
risch	$-\frac{2\left(3b\,\mathrm{e}^{4dx+4c}+6a\,\mathrm{e}^{2dx+2c}-6b\,\mathrm{e}^{2dx+2c}-2a+3b\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(3*b*exp(4*d*x+4*c)+6*a*exp(2*d*x+2*c)-6*b*exp(2*d*x+2*c)-2*a+3*b)/d/(
exp(2*d*x+2*c)-1)^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(39) = 78$.
time = 0.27, size = 113, normalized size = 2.63

$$\frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)+\frac{2\,b}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(39) = 78$.
time = 0.42, size = 159, normalized size = 3.70

$$\frac{4\,\left((a-3\,b)\cosh\left(dx+c\right)^{2}-2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a-3\,b)\sinh\left(dx+c\right)^{2}-3\,a+3\,b\right)}{3\,\left(d\cosh\left(dx+c\right)^{4}+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{4}-4\,d\cosh\left(dx+c\right)^{2}+2\,\left(3\,d\cosh\left(dx+c\right)^{2}-2\,d\right)\sinh\left(dx+c\right)^{2}+4\,\left(d\cosh\left(dx+c\right)^{3}-d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 4/3*((a - 3*b)*cosh(d*x + c)^2 - 2*a*cosh(d*x + c)*sinh(d*x + c) + (a - 3*b
)*sinh(d*x + c)^2 - 3*a + 3*b)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(
d*x + c)^3 + d*sinh(d*x + c)^4 - 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)
^2 - 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*
x + c) + 3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.42, size = 61, normalized size = 1.42

$$-\frac{2\left(3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}-2\,a+3\,b\right)}{3\,d(e^{(2\,dx+2\,c)}-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) - 2*a
 + 3*b)/(d*(e^(2*d*x + 2*c) - 1)^3)
```

Mupad [**B**]
time = 0.60, size = 61, normalized size = 1.42

$$-\frac{2\left(3\,b-2\,a+6\,a\,\mathrm{e}^{2\,c+2\,d\,x}-6\,b\,\mathrm{e}^{2\,c+2\,d\,x}+3\,b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{3\,d\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/sinh(c + d*x)^4,x)
```

```
[Out] -(2*(3*b - 2*a + 6*a*exp(2*c + 2*d*x) - 6*b*exp(2*c + 2*d*x) + 3*b*exp(4*c
+ 4*d*x)))/(3*d*(exp(2*c + 2*d*x) - 1)^3)
```

3.10 $\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=146

$$\frac{1}{128}\left(48a^2-80ab+35b^2\right)x-\frac{(80a^2-176ab+93b^2)\cosh(c+dx)\sinh(c+dx)}{128d}+\frac{(48a^2-208ab+139b^2)\cos}{192}$$

```
[Out] 1/128*(48*a^2-80*a*b+35*b^2)*x-1/128*(80*a^2-176*a*b+93*b^2)*cosh(d*x+c)*si
nh(d*x+c)/d+1/192*(48*a^2-208*a*b+139*b^2)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/48
*(16*a-13*b)*b*cosh(d*x+c)^5*sinh(d*x+c)/d+1/8*b^2*cosh(d*x+c)^3*sinh(d*x+c
)^5/d
```

Rubi [A]

time = 0.13, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3266, 474, 466, 1171, 393, 212}

$$\frac{(48a^2-208ab+139b^2)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{(80a^2-176ab+93b^2)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x\left(48a^2-80ab+35b^2\right)+\frac{b(16a-13b)\sinh(c+dx)\cosh^5(c+dx)}{48d}+\frac{b^2\sinh^5(c+dx)\cosh^3(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((48*a^2 - 80*a*b + 35*b^2)*x)/128 - ((80*a^2 - 176*a*b + 93*b^2)*Cosh[c +
d*x]*Sinh[c + d*x])/(128*d) + ((48*a^2 - 208*a*b + 139*b^2)*Cosh[c + d*x]^3
*Sinh[c + d*x])/(192*d) + ((16*a - 13*b)*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(
48*d) + (b^2*Cosh[c + d*x]^3*Sinh[c + d*x]^5)/(8*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
```

```
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a-(a-b)x^2\right)^2}{(1-x^2)^5}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh^3(c+dx)\sinh^5(c+dx)}{8d} - \frac{\text{Subst}\Big(\int \frac{x^4(-8a^2+5b^2+8(a-b)^2x}{(1-x^2)^4}}{8d}$$

$$= \frac{(16a-13b)b\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b^2\cosh^3(c+dx)\,\mathrm{si}}{8d}$$

$$= \frac{(48a^2-208ab+139b^2)\cosh^3(c+dx)\sinh(c+dx)}{192d} + (16a -$$

$$= -\frac{(80a^2-176ab+93b^2)\cosh(c+dx)\sinh(c+dx)}{128d} + (48a^2 -$$

$$= \frac{1}{128}\left(48a^2-80ab+35b^2\right)x - \frac{(80a^2-176ab+93b^2)\cosh(c+}{128d}$$

Mathematica [A]
time = 0.14, size = 133, normalized size = 0.91

$$\frac{1152a^2c-1920abc+840b^2c+1152a^2dx-1920abdx+840b^2dx-96(8a^2-15ab+7b^2)\sinh(2(c+dx))+24(4a^2-12ab+7b^2)\sinh(4(c+dx))+32ab\sinh(6(c+dx))-32b^2\sinh(6(c+dx))+3b^2\sinh(8(c+dx))}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (1152*a^2*c - 1920*a*b*c + 840*b^2*c + 1152*a^2*d*x - 1920*a*b*d*x + 840*b^
2*d*x - 96*(8*a^2 - 15*a*b + 7*b^2)*Sinh[2*(c + d*x)] + 24*(4*a^2 - 12*a*b
+ 7*b^2)*Sinh[4*(c + d*x)] + 32*a*b*Sinh[6*(c + d*x)] - 32*b^2*Sinh[6*(c +
d*x)] + 3*b^2*Sinh[8*(c + d*x)])/(3072*d)
```

Maple [A]
time = 1.34, size = 118, normalized size = 0.81

method	result
default	$\frac{\left(-\frac{1}{16}b^2+\frac{1}{16}ab\right)\sinh(6dx+6c)}{6d}+\frac{\left(-\frac{7}{16}b^2+\frac{15}{16}ab-\frac{1}{2}a^2\right)\sinh(2dx+2c)}{2d}+\frac{\left(\frac{7}{32}b^2-\frac{3}{8}ab+\frac{1}{8}a^2\right)\sinh(4dx+4c)}{4d}+\frac{3a^2x}{8}+\frac{35b^2x}{128}-$
risch	$\frac{35b^2x}{128}-\frac{5abx}{8}+\frac{3a^2x}{8}+\frac{b^2e^{8dx+8c}}{2048d}+\frac{b\,e^{6dx+6c}a}{192d}-\frac{b^2e^{6dx+6c}}{192d}+\frac{e^{4dx+4c}a^2}{64d}-\frac{3\,e^{4dx+4c}ab}{64d}+\frac{7\,e^{4dx+4c}b^2}{256d}-\frac{e^{2dx+2c}a}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-1/16*b^2+1/16*a*b)/d*sinh(6*d*x+6*c)+1/2*(-7/16*b^2+15/16*a*b-1/2*a^2
)*sinh(2*d*x+2*c)/d+1/4*(7/32*b^2-3/8*a*b+1/8*a^2)/d*sinh(4*d*x+4*c)+3/8*a^
2*x+35/128*b^2*x-5/8*a*b*x+1/1024*b^2/d*sinh(8*d*x+8*c)
```

Maxima [A]
time = 0.27, size = 267, normalized size = 1.83

$$\frac{1}{64}\,a^{2}\left(24\,x+\frac{e^{\left(4\,dx+4\,c\right)}}{d}-\frac{8\,e^{\left(2\,dx+2\,c\right)}}{d}+\frac{8\,e^{\left(-2\,dx-2\,c\right)}}{d}-\frac{e^{\left(-4\,dx-4\,c\right)}}{d}\right)-\frac{1}{6144}\,b^{2}\left(\frac{\left(32\,e^{\left(-2\,dx-2\,c\right)}-168\,e^{\left(-4\,dx-4\,c\right)}+672\,e^{\left(-6\,dx-6\,c\right)}-3\right)e^{\left(8\,dx+8\,c\right)}}{d}-\frac{1680\,\left(dx+c\right)}{d}-\frac{672\,e^{\left(-2\,dx-2\,c\right)}-168\,e^{\left(-4\,dx-4\,c\right)}+32\,e^{\left(-6\,dx-6\,c\right)}-3\,e^{\left(-8\,dx-8\,c\right)}}{d}\right)-\frac{1}{192}\,ab\left(\frac{\left(9\,e^{\left(-2\,dx-2\,c\right)}-45\,e^{\left(-4\,dx-4\,c\right)}-1\right)e^{\left(6\,dx+6\,c\right)}}{d}+\frac{120\,\left(dx+c\right)}{d}+\frac{45\,e^{\left(-2\,dx-2\,c\right)}-9\,e^{\left(-4\,dx-4\,c\right)}+e^{\left(-6\,dx-6\,c\right)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/6144*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d
*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d
- (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^
(-8*d*x - 8*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) -
 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Fricas [A]
time = 0.39, size = 238, normalized size = 1.63

$$\frac{3\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{7}+3\,\left(7\,b^{2}\cosh\left(dx+c\right)^{3}+8\,\left(ab-b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{5}+\left(21\,b^{2}\cosh\left(dx+c\right)^{5}+80\,\left(ab-b^{2}\right)\cosh\left(dx+c\right)^{3}+12\,\left(4\,a^{2}-12\,ab+7\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+3\,\left(48\,a^{2}-80\,ab+35\,b^{2}\right)dx+3\,\left(b^{2}\cosh\left(dx+c\right)^{7}+8\,\left(ab-b^{2}\right)\cosh\left(dx+c\right)^{5}+4\,\left(4\,a^{2}-12\,ab+7\,b^{2}\right)\cosh\left(dx+c\right)^{3}-8\,\left(8\,a^{2}-15\,ab+7\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^2*cosh(d*x + c)^3 + 8*(
a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^2*cosh(d*x + c)^5 + 80*(a
*b - b^2)*cosh(d*x + c)^3 + 12*(4*a^2 - 12*a*b + 7*b^2)*cosh(d*x + c))*sinh
(d*x + c)^3 + 3*(48*a^2 - 80*a*b + 35*b^2)*d*x + 3*(b^2*cosh(d*x + c)^7 + 8
*(a*b - b^2)*cosh(d*x + c)^5 + 4*(4*a^2 - 12*a*b + 7*b^2)*cosh(d*x + c)^3 -
 8*(8*a^2 - 15*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(143) = 286$.
time = 1.04, size = 490, normalized size = 3.36

$$\begin{cases} \frac{3a^{2}x\sinh^{4}\left(c+dx\right)}{8}-\frac{3a^{2}x\sinh^{2}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{4}+\frac{3a^{2}x\cosh^{4}\left(c+dx\right)}{8}+\frac{5a^{2}\sinh^{3}\left(c+dx\right)\cosh\left(c+dx\right)}{8d}-\frac{3a^{2}\sinh\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{8d}+\frac{5abx\sinh^{6}\left(c+dx\right)}{8}-\frac{15abx\sinh^{4}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{8}+\frac{15abx\sinh^{2}\left(c+dx\right)\cosh^{4}\left(c+dx\right)}{8}-\frac{5abx\cosh^{6}\left(c+dx\right)}{8}+\frac{11ab\sinh^{5}\left(c+dx\right)\cosh\left(c+dx\right)}{8d}-\frac{5ab\sinh^{3}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{3d}+\frac{5ab\sinh\left(c+dx\right)\cosh^{5}\left(c+dx\right)}{8d}+\frac{35b^{2}x\sinh^{8}\left(c+dx\right)}{128}-\frac{35b^{2}x\sinh^{6}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{32}+\frac{105b^{2}x\sinh^{4}\left(c+dx\right)\cosh^{4}\left(c+dx\right)}{64}-\frac{35b^{2}x\sinh^{2}\left(c+dx\right)\cosh^{6}\left(c+dx\right)}{32}+\frac{35b^{2}x\cosh^{8}\left(c+dx\right)}{128}+\frac{93b^{2}\sinh^{7}\left(c+dx\right)\cosh\left(c+dx\right)}{128d}-\frac{511b^{2}\sinh^{5}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{384d}+\frac{385b^{2}\sinh^{3}\left(c+dx\right)\cosh^{5}\left(c+dx\right)}{384d}-\frac{35b^{2}\sinh\left(c+dx\right)\cosh^{7}\left(c+dx\right)}{128d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^{2}\left(c\right)\right)^{2}\sinh^{4}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((3*a**2*x*sinh(c + d*x)**4/8 - 3*a**2*x*sinh(c + d*x)**2*cosh(c +
 d*x)**2/4 + 3*a**2*x*cosh(c + d*x)**4/8 + 5*a**2*sinh(c + d*x)**3*cosh(c +
 d*x)/(8*d) - 3*a**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*a*b*x*sinh(c
+ d*x)**6/8 - 15*a*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/8 + 15*a*b*x*sinh(
c + d*x)**2*cosh(c + d*x)**4/8 - 5*a*b*x*cosh(c + d*x)**6/8 + 11*a*b*sinh(c
```

```
 + d*x)**5*cosh(c + d*x)/(8*d) - 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3
*d) + 5*a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d) + 35*b**2*x*sinh(c + d*x)*
*8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b**2*x*sinh(c
 + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**
6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**2*sinh(c + d*x)**7*cosh(c + d
*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b**2
*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**2*sinh(c + d*x)*cosh(c +
 d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**4, True))
```

Giac [A]

time = 0.47, size = 215, normalized size = 1.47

$$\frac{1}{128}\left(48\,a^{2}-80\,ab+35\,b^{2}\right)x+\frac{b^{2}e^{\left(8\,dx+8\,c\right)}}{2048\,d}-\frac{b^{2}e^{\left(-8\,dx-8\,c\right)}}{2048\,d}+\frac{\left(ab-b^{2}\right)e^{\left(6\,dx+6\,c\right)}}{192\,d}+\frac{\left(4\,a^{2}-12\,ab+7\,b^{2}\right)e^{\left(4\,dx+4\,c\right)}}{256\,d}-\frac{\left(8\,a^{2}-15\,ab+7\,b^{2}\right)e^{\left(2\,dx+2\,c\right)}}{64\,d}+\frac{\left(8\,a^{2}-15\,ab+7\,b^{2}\right)e^{\left(-2\,dx-2\,c\right)}}{64\,d}-\frac{\left(4\,a^{2}-12\,ab+7\,b^{2}\right)e^{\left(-4\,dx-4\,c\right)}}{256\,d}-\frac{\left(ab-b^{2}\right)e^{\left(-6\,dx-6\,c\right)}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/128*(48*a^2 - 80*a*b + 35*b^2)*x + 1/2048*b^2*e^(8*d*x + 8*c)/d - 1/2048*
b^2*e^(-8*d*x - 8*c)/d + 1/192*(a*b - b^2)*e^(6*d*x + 6*c)/d + 1/256*(4*a^2
 - 12*a*b + 7*b^2)*e^(4*d*x + 4*c)/d - 1/64*(8*a^2 - 15*a*b + 7*b^2)*e^(2*d
*x + 2*c)/d + 1/64*(8*a^2 - 15*a*b + 7*b^2)*e^(-2*d*x - 2*c)/d - 1/256*(4*a
^2 - 12*a*b + 7*b^2)*e^(-4*d*x - 4*c)/d - 1/192*(a*b - b^2)*e^(-6*d*x - 6*c
)/d
```

Mupad [B]

time = 0.90, size = 149, normalized size = 1.02

$$\frac{12\,a^{2}\sinh\left(4\,c+4\,d\,x\right)-96\,a^{2}\sinh\left(2\,c+2\,d\,x\right)-84\,b^{2}\sinh\left(2\,c+2\,d\,x\right)+21\,b^{2}\sinh\left(4\,c+4\,d\,x\right)-4\,b^{2}\sinh\left(6\,c+6\,d\,x\right)+\frac{3\,b^{2}\sinh\left(8\,c+8\,d\,x\right)}{8}+180\,a\,b\sinh\left(2\,c+2\,d\,x\right)-36\,a\,b\sinh\left(4\,c+4\,d\,x\right)+4\,a\,b\sinh\left(6\,c+6\,d\,x\right)+144\,a^{2}\,d\,x+105\,b^{2}\,d\,x-240\,a\,b\,d\,x}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (12*a^2*sinh(4*c + 4*d*x) - 96*a^2*sinh(2*c + 2*d*x) - 84*b^2*sinh(2*c + 2*
d*x) + 21*b^2*sinh(4*c + 4*d*x) - 4*b^2*sinh(6*c + 6*d*x) + (3*b^2*sinh(8*c
 + 8*d*x))/8 + 180*a*b*sinh(2*c + 2*d*x) - 36*a*b*sinh(4*c + 4*d*x) + 4*a*b
*sinh(6*c + 6*d*x) + 144*a^2*d*x + 105*b^2*d*x - 240*a*b*d*x)/(384*d)
```

3.11 $\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=85

$$-\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{(a-3b)(a-b)\cosh^3(c+dx)}{3d}+\frac{(2a-3b)b\cosh^5(c+dx)}{5d}+\frac{b^2\cosh^7(c+dx)}{7d}$$

```
[Out] -(a-b)^2*cosh(d*x+c)/d+1/3*(a-3*b)*(a-b)*cosh(d*x+c)^3/d+1/5*(2*a-3*b)*b*co
sh(d*x+c)^5/d+1/7*b^2*cosh(d*x+c)^7/d
```

Rubi [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3265, 380}

$$\frac{b(2a-3b)\cosh^5(c+dx)}{5d}+\frac{(a-3b)(a-b)\cosh^3(c+dx)}{3d}-\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -(((a - b)^2*Cosh[c + d*x])/d) + ((a - 3*b)*(a - b)*Cosh[c + d*x]^3)/(3*d)
+ ((2*a - 3*b)*b*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]^7)/(7*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = -\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a-b+bx^2\right)^2\,dx,x,\cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int\left((a-b)^2+(a-3b)(-a+b)x^2-(2a-3b)bx^4-b^2\right.\right.}{d}$$

$$= -\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{(a-3b)(a-b)\cosh^3(c+dx)}{3d}+\frac{(2a}{}$$

Mathematica [A]
time = 0.04, size = 154, normalized size = 1.81

$$-\frac{3a^2\cosh(c+dx)}{4d}+\frac{5ab\cosh(c+dx)}{4d}-\frac{35b^2\cosh(c+dx)}{64d}+\frac{a^2\cosh(3(c+dx))}{12d}-\frac{5ab\cosh(3(c+dx))}{24d}+\frac{7b^2\cosh(3(c+dx))}{64d}+\frac{ab\cosh(5(c+dx))}{40d}-\frac{7b^2\cosh(5(c+dx))}{320d}+\frac{b^2\cosh(7(c+dx))}{448d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-3*a^2*Cosh[c + d*x])/(4*d) + (5*a*b*Cosh[c + d*x])/(4*d) - (35*b^2*Cosh[c
 + d*x])/(64*d) + (a^2*Cosh[3*(c + d*x)])/(12*d) - (5*a*b*Cosh[3*(c + d*x)]
)/(24*d) + (7*b^2*Cosh[3*(c + d*x)])/(64*d) + (a*b*Cosh[5*(c + d*x)])/(40*d
) - (7*b^2*Cosh[5*(c + d*x)])/(320*d) + (b^2*Cosh[7*(c + d*x)])/(448*d)
```

Maple [A]
time = 0.70, size = 97, normalized size = 1.14

method	result
default	$\frac{\left(-\frac{7}{64}b^2+\frac{1}{8}ab\right)\cosh(5dx+5c)}{5d}+\frac{\left(-\frac{35}{64}b^2+\frac{5}{4}ab-\frac{3}{4}a^2\right)\cosh(dx+c)}{d}+\frac{\left(\frac{21}{64}b^2-\frac{5}{8}ab+\frac{1}{4}a^2\right)\cosh(3dx+3c)}{3d}+\frac{b^2\cosh(7dx+7c)}{448d}$
risch	$\frac{b^2e^{7dx+7c}}{896d}+\frac{b\,e^{5dx+5c}a}{80d}-\frac{7b^2e^{5dx+5c}}{640d}+\frac{e^{3dx+3c}a^2}{24d}-\frac{5\,e^{3dx+3c}ab}{48d}+\frac{7\,e^{3dx+3c}b^2}{128d}-\frac{3\,e^{dx+c}a^2}{8d}+\frac{5ab\,e^{dx+c}}{8d}-\frac{35\,e^{dx+c}}{128d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(-7/64*b^2+1/8*a*b)*cosh(5*d*x+5*c)/d+(-35/64*b^2+5/4*a*b-3/4*a^2)*cosh
(d*x+c)/d+1/3*(21/64*b^2-5/8*a*b+1/4*a^2)/d*cosh(3*d*x+3*c)+1/448*b^2*cosh(
7*d*x+7*c)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(79) = 158$.
time = 0.27, size = 247, normalized size = 2.91

$$-\frac{1}{4480}\,b^2\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d}\right)+\frac{1}{240}\,ab\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{1}{24}\,a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5*c)
/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(
-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e
^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(79) = 158$.
time = 0.42, size = 213, normalized size = 2.51

$$\frac{15\,b^{2}\cosh\left(dx+c\right)^{7}+105\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}+21\,\left(8\,ab-7\,b^{2}\right)\cosh\left(dx+c\right)^{5}+105\,\left(5\,b^{2}\cosh\left(dx+c\right)^{3}+\left(8\,ab-7\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}+35\,\left(16\,a^{2}-40\,ab+21\,b^{2}\right)\cosh\left(dx+c\right)^{3}+105\,\left(3\,b^{2}\cosh\left(dx+c\right)^{5}+2\,\left(8\,ab-7\,b^{2}\right)\cosh\left(dx+c\right)^{3}+\left(16\,a^{2}-40\,ab+21\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}-105\,\left(48\,a^{2}-80\,ab+35\,b^{2}\right)\cosh\left(dx+c\right)}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(15*b^2*cosh(d*x + c)^7 + 105*b^2*cosh(d*x + c)*sinh(d*x + c)^6 + 21
*(8*a*b - 7*b^2)*cosh(d*x + c)^5 + 105*(5*b^2*cosh(d*x + c)^3 + (8*a*b - 7*
b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 35*(16*a^2 - 40*a*b + 21*b^2)*cosh(d*
x + c)^3 + 105*(3*b^2*cosh(d*x + c)^5 + 2*(8*a*b - 7*b^2)*cosh(d*x + c)^3 +
 (16*a^2 - 40*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 105*(48*a^2 -
80*a*b + 35*b^2)*cosh(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(68) = 136$.
time = 0.68, size = 204, normalized size = 2.40

$$\begin{cases} \frac{a^{2}\sinh^{2}\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2a^{2}\cosh^{3}\left(c+dx\right)}{3d}+\frac{2ab\sinh^{4}\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{8ab\sinh^{2}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{3d}+\frac{16ab\cosh^{5}\left(c+dx\right)}{15d}+\frac{b^{2}\sinh^{6}\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2b^{2}\sinh^{4}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{d}+\frac{8b^{2}\sinh^{2}\left(c+dx\right)\cosh^{5}\left(c+dx\right)}{5d}-\frac{16b^{2}\cosh^{7}\left(c+dx\right)}{35d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^{2}\left(c\right)\right)^{2}\sinh^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3/
(3*d) + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 8*a*b*sinh(c + d*x)**2*cos
h(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15*d) + b**2*sinh(c + d*x)**
6*cosh(c + d*x)/d - 2*b**2*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 8*b**2*sin
h(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**2*cosh(c + d*x)**7/(35*d), Ne(
d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**3, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(79) = 158$.
time = 0.43, size = 196, normalized size = 2.31

$$\frac{b^{2}e^{\left(7\,dx+7\,c\right)}}{896\,d}+\frac{b^{2}e^{\left(-7\,dx-7\,c\right)}}{896\,d}+\frac{\left(8\,ab-7\,b^{2}\right)e^{\left(5\,dx+5\,c\right)}}{640\,d}+\frac{\left(16\,a^{2}-40\,ab+21\,b^{2}\right)e^{\left(3\,dx+3\,c\right)}}{384\,d}-\frac{\left(48\,a^{2}-80\,ab+35\,b^{2}\right)e^{\left(dx+c\right)}}{128\,d}-\frac{\left(48\,a^{2}-80\,ab+35\,b^{2}\right)e^{\left(-dx-c\right)}}{128\,d}+\frac{\left(16\,a^{2}-40\,ab+21\,b^{2}\right)e^{\left(-3\,dx-3\,c\right)}}{384\,d}+\frac{\left(8\,ab-7\,b^{2}\right)e^{\left(-5\,dx-5\,c\right)}}{640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/896*b^2*e^(7*d*x + 7*c)/d + 1/896*b^2*e^(-7*d*x - 7*c)/d + 1/640*(8*a*b -
 7*b^2)*e^(5*d*x + 5*c)/d + 1/384*(16*a^2 - 40*a*b + 21*b^2)*e^(3*d*x + 3*c
)/d - 1/128*(48*a^2 - 80*a*b + 35*b^2)*e^(d*x + c)/d - 1/128*(48*a^2 - 80*a
*b + 35*b^2)*e^(-d*x - c)/d + 1/384*(16*a^2 - 40*a*b + 21*b^2)*e^(-3*d*x -
3*c)/d + 1/640*(8*a*b - 7*b^2)*e^(-5*d*x - 5*c)/d
```

Mupad [B]
time = 0.23, size = 112, normalized size = 1.32

$$\frac{\frac{a^2\cosh(c+d\,x)^3}{3}-a^2\cosh(c+d\,x)+\frac{2\,a\,b\cosh(c+d\,x)^5}{5}-\frac{4\,a\,b\cosh(c+d\,x)^3}{3}+2\,a\,b\cosh(c+d\,x)+\frac{b^2\cosh(c+d\,x)^7}{7}-\frac{3\,b^2\cosh(c+d\,x)^5}{5}+b^2\cosh(c+d\,x)^3-b^2\cosh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] ((a^2*cosh(c + d*x)^3)/3 - b^2*cosh(c + d*x) - a^2*cosh(c + d*x) + b^2*cosh
(c + d*x)^3 - (3*b^2*cosh(c + d*x)^5)/5 + (b^2*cosh(c + d*x)^7)/7 + 2*a*b*c
osh(c + d*x) - (4*a*b*cosh(c + d*x)^3)/3 + (2*a*b*cosh(c + d*x)^5)/5)/d
```

3.12 $\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=110

$$-\frac{1}{16}\left(8a^2-12ab+5b^2\right)x+\frac{\left(8a^2-20ab+11b^2\right)\cosh(c+dx)\sinh(c+dx)}{16d}+\frac{(4a-3b)b\cosh^3(c+dx)\sinh(c+$$

```
[Out] -1/16*(8*a^2-12*a*b+5*b^2)*x+1/16*(8*a^2-20*a*b+11*b^2)*cosh(d*x+c)*sinh(d*
x+c)/d+1/8*(4*a-3*b)*b*cosh(d*x+c)^3*sinh(d*x+c)/d+1/6*b^2*cosh(d*x+c)^3*si
nh(d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3249, 3248}

$$\frac{\left(16a^2-36ab+15b^2\right)\sinh(c+dx)\cosh(c+dx)}{48d}-\frac{1}{16}x\left(8a^2-12ab+5b^2\right)+\frac{b(4a-5b)\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/16*((8*a^2 - 12*a*b + 5*b^2)*x) + ((16*a^2 - 36*a*b + 15*b^2)*Cosh[c + d
*x]*Sinh[c + d*x])/(48*d) + ((4*a - 5*b)*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(
24*d) + (Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(6*d)
```

Rule 3248

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rule 3249

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e
 + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p
 + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Gt
Q[p, 0]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d} - \frac{1}{6}\int (a -$$

$$= -\frac{1}{16}\left(8a^2-12ab+5b^2\right)x + \frac{(16a^2-36ab+15b^2)\cosh(c+dx}{48d}$$

Mathematica [A]
time = 0.13, size = 99, normalized size = 0.90

$$\frac{-96a^2c+144abc-60b^2c-96a^2dx+144abdx-60b^2dx+(48a^2-96ab+45b^2)\sinh(2(c+dx))+3(4a-3b)b\sinh(4(c+dx))+b^2\sinh(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-96*a^2*c + 144*a*b*c - 60*b^2*c - 96*a^2*d*x + 144*a*b*d*x - 60*b^2*d*x +
 (48*a^2 - 96*a*b + 45*b^2)*Sinh[2*(c + d*x)] + 3*(4*a - 3*b)*b*Sinh[4*(c +
 d*x)] + b^2*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 0.97, size = 89, normalized size = 0.81

method	result
default	$\frac{\left(-\frac{3}{16}b^2+\frac{1}{4}ab\right)\sinh(4dx+4c)}{4d}+\frac{\left(\frac{15}{32}b^2-ab+\frac{1}{2}a^2\right)\sinh(2dx+2c)}{2d}-\frac{a^2x}{2}-\frac{5b^2x}{16}+\frac{3abx}{4}+\frac{b^2\sinh(6dx+6c)}{192d}$
risch	$-\frac{5b^2x}{16}+\frac{3abx}{4}-\frac{a^2x}{2}+\frac{b^2\mathrm{e}^{6dx+6c}}{384d}+\frac{\mathrm{e}^{4dx+4c}ab}{32d}-\frac{3\,\mathrm{e}^{4dx+4c}b^2}{128d}+\frac{\mathrm{e}^{2dx+2c}a^2}{8d}-\frac{\mathrm{e}^{2dx+2c}ab}{4d}+\frac{15\,\mathrm{e}^{2dx+2c}b^2}{128d}-\frac{\mathrm{e}^{-2dx-2}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-3/16*b^2+1/4*a*b)/d*sinh(4*d*x+4*c)+1/2*(15/32*b^2-a*b+1/2*a^2)*sinh(
2*d*x+2*c)/d-1/2*a^2*x-5/16*b^2*x+3/4*a*b*x+1/192*b^2/d*sinh(6*d*x+6*c)
```

Maxima [A]
time = 0.27, size = 189, normalized size = 1.72

$$\frac{1}{32}\,ab\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{8}\,a^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{384}\,b^2\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x -
```

```
 2*c)/d) - 1/384*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d
*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) +
 e^(-6*d*x - 6*c))/d)
```

Fricas [A]
time = 0.40, size = 149, normalized size = 1.35

$$\frac{3\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+2\,\left(5\,b^{2}\cosh\left(dx+c\right)^{3}+3\,\left(4\,ab-3\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}-6\,\left(8\,a^{2}-12\,ab+5\,b^{2}\right)dx+3\,\left(b^{2}\cosh\left(dx+c\right)^{5}+2\,\left(4\,ab-3\,b^{2}\right)\cosh\left(dx+c\right)^{3}+\left(16\,a^{2}-32\,ab+15\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^2*cosh(d*x + c)^3 + 3*(4
*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(8*a^2 - 12*a*b + 5*b^2)*d
*x + 3*(b^2*cosh(d*x + c)^5 + 2*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + (16*a^2 -
 32*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(104) = 208$.
time = 0.49, size = 332, normalized size = 3.02

$$\begin{cases} \frac{a^{2} x \sinh^{2}{\left(c + d x \right)}}{2} - \frac{a^{2} x \cosh^{2}{\left(c + d x \right)}}{2} + \frac{a^{2} \sinh{\left(c + d x \right)} \cosh{\left(c + d x \right)}}{2 d} + \frac{3 a b x \sinh^{4}{\left(c + d x \right)}}{4} - \frac{3 a b x \sinh^{2}{\left(c + d x \right)} \cosh^{2}{\left(c + d x \right)}}{2} + \frac{3 a b x \cosh^{4}{\left(c + d x \right)}}{4} + \frac{5 a b \sinh^{3}{\left(c + d x \right)} \cosh{\left(c + d x \right)}}{4 d} - \frac{3 a b \sinh{\left(c + d x \right)} \cosh^{3}{\left(c + d x \right)}}{4 d} + \frac{5 b^{2} x \sinh^{6}{\left(c + d x \right)}}{16} - \frac{15 b^{2} x \sinh^{4}{\left(c + d x \right)} \cosh^{2}{\left(c + d x \right)}}{16} + \frac{15 b^{2} x \sinh^{2}{\left(c + d x \right)} \cosh^{4}{\left(c + d x \right)}}{16} - \frac{5 b^{2} x \cosh^{6}{\left(c + d x \right)}}{16} + \frac{11 b^{2} \sinh^{5}{\left(c + d x \right)} \cosh{\left(c + d x \right)}}{16 d} - \frac{5 b^{2} \sinh^{3}{\left(c + d x \right)} \cosh^{3}{\left(c + d x \right)}}{6 d} + \frac{5 b^{2} \sinh{\left(c + d x \right)} \cosh^{5}{\left(c + d x \right)}}{16 d} & \text{for}\: d \neq 0 \\x \left(a + b \sinh^{2}{\left(c \right)}\right)^{2} \sinh^{2}{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh(
c + d*x)**2*cosh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c
+ d*x)**3*cosh(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d)
+ 5*b**2*x*sinh(c + d*x)**6/16 - 15*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**
2/16 + 15*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b**2*x*cosh(c + d
*x)**6/16 + 11*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b**2*sinh(c +
 d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16
*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**2, True))
```

Giac [A]
time = 0.44, size = 159, normalized size = 1.45

$$-\frac{1}{16}\,{\left(8\,a^{2}-12\,ab+5\,b^{2}\right)}x+\frac{b^{2}e^{\left(6\,dx+6\,c\right)}}{384\,d}-\frac{b^{2}e^{\left(-6\,dx-6\,c\right)}}{384\,d}+\frac{{\left(4\,ab-3\,b^{2}\right)}e^{\left(4\,dx+4\,c\right)}}{128\,d}+\frac{{\left(16\,a^{2}-32\,ab+15\,b^{2}\right)}e^{\left(2\,dx+2\,c\right)}}{128\,d}-\frac{{\left(16\,a^{2}-32\,ab+15\,b^{2}\right)}e^{\left(-2\,dx-2\,c\right)}}{128\,d}-\frac{{\left(4\,ab-3\,b^{2}\right)}e^{\left(-4\,dx-4\,c\right)}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/16*(8*a^2 - 12*a*b + 5*b^2)*x + 1/384*b^2*e^(6*d*x + 6*c)/d - 1/384*b^2*
e^(-6*d*x - 6*c)/d + 1/128*(4*a*b - 3*b^2)*e^(4*d*x + 4*c)/d + 1/128*(16*a^
```

```
2 - 32*a*b + 15*b^2)*e^(2*d*x + 2*c)/d - 1/128*(16*a^2 - 32*a*b + 15*b^2)*e
^(-2*d*x - 2*c)/d - 1/128*(4*a*b - 3*b^2)*e^(-4*d*x - 4*c)/d
```

Mupad [B]
time = 0.23, size = 108, normalized size = 0.98

$$\frac{12\,a^2\sinh(2\,c+2\,d\,x)+\frac{45\,b^2\sinh(2\,c+2\,d\,x)}{4}-\frac{9\,b^2\sinh(4\,c+4\,d\,x)}{4}+\frac{b^2\sinh(6\,c+6\,d\,x)}{4}-24\,a\,b\sinh(2\,c+2\,d\,x)+3\,a\,b\sinh(4\,c+4\,d\,x)-24\,a^2\,d\,x-15\,b^2\,d\,x+36\,a\,b\,d\,x}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (12*a^2*sinh(2*c + 2*d*x) + (45*b^2*sinh(2*c + 2*d*x))/4 - (9*b^2*sinh(4*c
+ 4*d*x))/4 + (b^2*sinh(6*c + 6*d*x))/4 - 24*a*b*sinh(2*c + 2*d*x) + 3*a*b*
sinh(4*c + 4*d*x) - 24*a^2*d*x - 15*b^2*d*x + 36*a*b*d*x)/(48*d)
```

3.13 $\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=57

$$\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{2(a-b)b\cosh^3(c+dx)}{3d}+\frac{b^2\cosh^5(c+dx)}{5d}$$

```
[Out] (a-b)^2*cosh(d*x+c)/d+2/3*(a-b)*b*cosh(d*x+c)^3/d+1/5*b^2*cosh(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3265, 200}

$$\frac{2b(a-b)\cosh^3(c+dx)}{3d}+\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)^2*Cosh[c + d*x])/d + (2*(a - b)*b*Cosh[c + d*x]^3)/(3*d) + (b^2*Co
sh[c + d*x]^5)/(5*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int\left(a-b+bx^2\right)^2\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left(a^2\left(1+\frac{b(-2a+b)}{a^2}\right)+2ab\left(1-\frac{b}{a}\right)x^2+b^2x^4\right)\,dx,x,\cosh\right.}{d}\\ &= \frac{(a-b)^2\cosh(c+dx)}{d}+\frac{2(a-b)b\cosh^3(c+dx)}{3d}+\frac{b^2\cosh^5(c+}{5d}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 111, normalized size = 1.95

$$\frac{a^2\cosh(c)\cosh(dx)}{d}-\frac{3ab\cosh(c+dx)}{2d}+\frac{5b^2\cosh(c+dx)}{8d}+\frac{ab\cosh(3(c+dx))}{6d}-\frac{5b^2\cosh(3(c+dx))}{48d}+\frac{b^2\cosh(5(c+dx))}{80d}+\frac{a^2\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Cosh[c]*Cosh[d*x])/d - (3*a*b*Cosh[c + d*x])/(2*d) + (5*b^2*Cosh[c + d
*x])/(8*d) + (a*b*Cosh[3*(c + d*x)])/(6*d) - (5*b^2*Cosh[3*(c + d*x)])/(48*
d) + (b^2*Cosh[5*(c + d*x)])/(80*d) + (a^2*Sinh[c]*Sinh[d*x])/d
```

Maple [**A**]
time = 0.65, size = 66, normalized size = 1.16

method	result
default	$\frac{\left(-\frac{5}{16}b^2+\frac{1}{2}ab\right)\cosh(3dx+3c)}{3d}+\frac{\left(\frac{5}{8}b^2-\frac{3}{2}ab+a^2\right)\cosh(dx+c)}{d}+\frac{b^2\cosh(5dx+5c)}{80d}$
risch	$\frac{b^2 \mathrm{e}^{5dx+5c}}{160d}+\frac{\mathrm{e}^{3dx+3c}ab}{12d}-\frac{5\,\mathrm{e}^{3dx+3c}b^2}{96d}+\frac{\mathrm{e}^{dx+c}a^2}{2d}-\frac{3ab\,\mathrm{e}^{dx+c}}{4d}+\frac{5\,\mathrm{e}^{dx+c}b^2}{16d}+\frac{\mathrm{e}^{-dx-c}a^2}{2d}-\frac{3\,\mathrm{e}^{-dx-c}ab}{4d}+\frac{5\,\mathrm{e}^{-dx-c}b^2}{16d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-5/16*b^2+1/2*a*b)/d*cosh(3*d*x+3*c)+(5/8*b^2-3/2*a*b+a^2)*cosh(d*x+c)
/d+1/80*b^2*cosh(5*d*x+5*c)/d
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(53) = 106$.
time = 0.27, size = 157, normalized size = 2.75

$$\frac{1}{480}\,b^2\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{1}{12}\,ab\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a^2\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/480*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/12*
a*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3
*c)/d) + a^2*cosh(d*x + c)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(53) = 106$.
time = 0.39, size = 122, normalized size = 2.14

$$\frac{3\,b^2\cosh\left(dx+c\right)^5+15\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+5\,\left(8\,ab-5\,b^2\right)\cosh\left(dx+c\right)^3+15\,\left(2\,b^2\cosh\left(dx+c\right)^3+\left(8\,ab-5\,b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2+30\,\left(8\,a^2-12\,ab+5\,b^2\right)\cosh\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^2*cosh(d*x + c)^5 + 15*b^2*cosh(d*x + c)*sinh(d*x + c)^4 + 5*(8*
a*b - 5*b^2)*cosh(d*x + c)^3 + 15*(2*b^2*cosh(d*x + c)^3 + (8*a*b - 5*b^2)*
cosh(d*x + c))*sinh(d*x + c)^2 + 30*(8*a^2 - 12*a*b + 5*b^2)*cosh(d*x + c))
/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(49) = 98$.
time = 0.28, size = 128, normalized size = 2.25

$$\begin{cases} \frac{a^2 \cosh{(c+dx)}}{d} + \frac{2ab \sinh^2{(c+dx)} \cosh{(c+dx)}}{d} - \frac{4ab \cosh^3{(c+dx)}}{3d} + \frac{b^2 \sinh^4{(c+dx)} \cosh{(c+dx)}}{d} - \frac{4b^2 \sinh^2{(c+dx)} \cosh^3{(c+dx)}}{3d} + \frac{8b^2 \cosh^5{(c+dx)}}{15d} & \text{for } d \neq 0 \\ x\left(a + b \sinh^2{(c)}\right)^2 \sinh{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*cosh(c + d*x)/d + 2*a*b*sinh(c + d*x)**2*cosh(c + d*x)/d -
4*a*b*cosh(c + d*x)**3/(3*d) + b**2*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*
*2*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 8*b**2*cosh(c + d*x)**5/(15*d)
, Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(53) = 106$.
time = 0.43, size = 138, normalized size = 2.42

$$\frac{b^2 e^{(5\,dx+5\,c)}}{160\,d} + \frac{b^2 e^{(-5\,dx-5\,c)}}{160\,d} + \frac{(8\,ab - 5\,b^2)e^{(3\,dx+3\,c)}}{96\,d} + \frac{(8\,a^2 - 12\,ab + 5\,b^2)e^{(dx+c)}}{16\,d} + \frac{(8\,a^2 - 12\,ab + 5\,b^2)e^{(-dx-c)}}{16\,d} + \frac{(8\,ab - 5\,b^2)e^{(-3\,dx-3\,c)}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/160*b^2*e^(5*d*x + 5*c)/d + 1/160*b^2*e^(-5*d*x - 5*c)/d + 1/96*(8*a*b -
5*b^2)*e^(3*d*x + 3*c)/d + 1/16*(8*a^2 - 12*a*b + 5*b^2)*e^(d*x + c)/d + 1/
16*(8*a^2 - 12*a*b + 5*b^2)*e^(-d*x - c)/d + 1/96*(8*a*b - 5*b^2)*e^(-3*d*x
 - 3*c)/d
```

Mupad [B]
time = 0.65, size = 76, normalized size = 1.33

$$\frac{15\,a^2\cosh(c+d\,x) + 10\,a\,b\cosh(c+d\,x)^3 - 30\,a\,b\cosh(c+d\,x) + 3\,b^2\cosh(c+d\,x)^5 - 10\,b^2\cosh(c+d\,x)^3 + 15\,b^2\cosh(c+d\,x)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (15*a^2*cosh(c + d*x) + 15*b^2*cosh(c + d*x) - 10*b^2*cosh(c + d*x)^3 + 3*b
^2*cosh(c + d*x)^5 - 30*a*b*cosh(c + d*x) + 10*a*b*cosh(c + d*x)^3)/(15*d)
```

3.14 $\int \left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=72

$$\frac{1}{8}\left(8a^2-8ab+3b^2\right)x+\frac{(8a-3b)b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^2\cosh(c+dx)\sinh^3(c+dx)}{4d}$$

```
[Out] 1/8*(8*a^2-8*a*b+3*b^2)*x+1/8*(8*a-3*b)*b*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b^2
*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3258}

$$\frac{1}{8}x\left(8a^2-8ab+3b^2\right)+\frac{b(8a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 8*a*b + 3*b^2)*x)/8 + ((8*a - 3*b)*b*Cosh[c + d*x]*Sinh[c + d*x])
/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 3258

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[(8*a^2 +
8*a*b + 3*b^2)*(x/8), x] + (-Simp[b^2*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)),
x] - Simp[b*(8*a + 3*b)*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{1}{8}\left(8a^2-8ab+3b^2\right)x+\frac{(8a-3b)b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^2\cosh(c+}{}$$

Mathematica [A]
time = 0.09, size = 60, normalized size = 0.83

$$\frac{4\left(8a^2-8ab+3b^2\right)(c+dx)+8(2a-b)b\sinh(2(c+dx))+b^2\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (4*(8*a^2 - 8*a*b + 3*b^2)*(c + d*x) + 8*(2*a - b)*b*Sinh[2*(c + d*x)] + b^
2*Sinh[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 0.81, size = 58, normalized size = 0.81

method	result	size
default	$a^{2} x+\frac{\left(a b -\frac{1}{2} b^{2}\right) \sinh \left(2 d x +2 c \right)}{2 d}+\frac{3 b^{2} x}{8}-a b x +\frac{b^{2} \sinh \left(4 d x +4 c \right)}{32 d}$	58
risch	$\frac{3 b^{2} x}{8}+a^{2} x -a b x +\frac{{\mathrm e}^{4 d x +4 c} b^{2}}{64 d}-\frac{{\mathrm e}^{2 d x +2 c} b^{2}}{8 d}+\frac{{\mathrm e}^{2 d x +2 c} a b}{4 d}+\frac{{\mathrm e}^{-2 d x -2 c} b^{2}}{8 d}-\frac{{\mathrm e}^{-2 d x -2 c} a b}{4 d}-\frac{{\mathrm e}^{-4 d x -4 c} b^{2}}{64 d}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+1/2*(a*b-1/2*b^2)*sinh(2*d*x+2*c)/d+3/8*b^2*x-a*b*x+1/32*b^2/d*sinh(4
*d*x+4*c)
```

Maxima [A]
time = 0.28, size = 105, normalized size = 1.46

$$\frac{1}{64} \, b^{2} {\left(24 \, x + \frac{e^{\left(4 \, d x + 4 \, c\right)}}{d} - \frac{8 \, e^{\left(2 \, d x + 2 \, c\right)}}{d} + \frac{8 \, e^{\left(-2 \, d x - 2 \, c\right)}}{d} - \frac{e^{\left(-4 \, d x - 4 \, c\right)}}{d}\right)} - \frac{1}{4} \, a b {\left(4 \, x - \frac{e^{\left(2 \, d x + 2 \, c\right)}}{d} + \frac{e^{\left(-2 \, d x - 2 \, c\right)}}{d}\right)} + a^{2} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x -
 2*c)/d) + a^2*x
```

Fricas [A]
time = 0.38, size = 80, normalized size = 1.11

$$\frac{b^{2} \cosh \left(d x + c\right) \sinh \left(d x + c\right)^{3} + {\left(8 \, a^{2} - 8 \, a b + 3 \, b^{2}\right)} d x + {\left(b^{2} \cosh \left(d x + c\right)^{3} + 4 \, {\left(2 \, a b - b^{2}\right)} \cosh \left(d x + c\right)\right)} \sinh \left(d x + c\right)}{8 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)*sinh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*d*x + (b^2
*cosh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(60) = 120$.
time = 0.21, size = 168, normalized size = 2.33

$$\begin{cases} a^{2} x + a b x \sinh^{2}{\left(c + d x \right)} - a b x \cosh^{2}{\left(c + d x \right)} + \frac{a b \sinh{\left(c + d x \right)} \cosh{\left(c + d x \right)}}{d} + \frac{3 b^{2} x \sinh^{4}{\left(c + d x \right)}}{8} - \frac{3 b^{2} x \sinh^{2}{\left(c + d x \right)} \cosh^{2}{\left(c + d x \right)}}{4} + \frac{3 b^{2} x \cosh^{4}{\left(c + d x \right)}}{8} + \frac{5 b^{2} \sinh^{3}{\left(c + d x \right)} \cosh{\left(c + d x \right)}}{8 d} - \frac{3 b^{2} \sinh{\left(c + d x \right)} \cosh^{3}{\left(c + d x \right)}}{8 d} & \text{for}\: d \neq 0 \\x \left(a + b \sinh^{2}{\left(c \right)}\right)^{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x + a*b*x*sinh(c + d*x)**2 - a*b*x*cosh(c + d*x)**2 + a*b*s
inh(c + d*x)*cosh(c + d*x)/d + 3*b**2*x*sinh(c + d*x)**4/8 - 3*b**2*x*sinh(
c + d*x)**2*cosh(c + d*x)**2/4 + 3*b**2*x*cosh(c + d*x)**4/8 + 5*b**2*sinh(
c + d*x)**3*cosh(c + d*x)/(8*d) - 3*b**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*
d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2, True))
```

Giac [**A**]
time = 0.44, size = 101, normalized size = 1.40

$$\frac{1}{8}\left(8\,a^{2}-8\,ab+3\,b^{2}\right)x+\frac{b^{2}e^{(4\,dx+4\,c)}}{64\,d}-\frac{b^{2}e^{(-4\,dx-4\,c)}}{64\,d}+\frac{(2\,ab-b^{2})e^{(2\,dx+2\,c)}}{8\,d}-\frac{(2\,ab-b^{2})e^{(-2\,dx-2\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 - 8*a*b + 3*b^2)*x + 1/64*b^2*e^(4*d*x + 4*c)/d - 1/64*b^2*e^(-4
*d*x - 4*c)/d + 1/8*(2*a*b - b^2)*e^(2*d*x + 2*c)/d - 1/8*(2*a*b - b^2)*e^(
-2*d*x - 2*c)/d
```

Mupad [**B**]
time = 0.10, size = 67, normalized size = 0.93

$$a^{2}\,x+\frac{3\,b^{2}\,x}{8}-a\,b\,x-\frac{b^{2}\,\sinh(2\,c+2\,d\,x)}{4\,d}+\frac{b^{2}\,\sinh(4\,c+4\,d\,x)}{32\,d}+\frac{a\,b\,\sinh(2\,c+2\,d\,x)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] a^2*x + (3*b^2*x)/8 - a*b*x - (b^2*sinh(2*c + 2*d*x))/(4*d) + (b^2*sinh(4*c
 + 4*d*x))/(32*d) + (a*b*sinh(2*c + 2*d*x))/(2*d)
```

3.15 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=52

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{(2a-b)b\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}$$

```
[Out] -a^2*arctanh(cosh(d*x+c))/d+(2*a-b)*b*cosh(d*x+c)/d+1/3*b^2*cosh(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3265, 398, 212}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a-b)\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTanh[Cosh[c + d*x]])/d) + ((2*a - b)*b*Cosh[c + d*x])/d + (b^2*Co
sh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^2}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \left(-(2a-b)b-b^2x^2+\frac{a^2}{1-x^2}\right)\,dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{(2a-b)b\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{a^2\text{Subst}\left(\int \frac{1}{1-x^2}\,d\right.}{} \\
&= -\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{(2a-b)b\cosh(c+dx)}{d}+\frac{b^2\cosh^3}{3}
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 104, normalized size = 2.00

$$\frac{2ab\cosh(c)\cosh(dx)}{d}-\frac{3b^2\cosh(c+dx)}{4d}+\frac{b^2\cosh(3(c+dx))}{12d}-\frac{a^2\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a^2\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{2ab\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*a*b*Cosh[c]*Cosh[d*x])/d - (3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cosh[3*(c
+ d*x)])/(12*d) - (a^2*Log[Cosh[c/2 + (d*x)/2]])/d + (a^2*Log[Sinh[c/2 + (d
*x)/2]])/d + (2*a*b*Sinh[c]*Sinh[d*x])/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(50) = 100.
time = 1.16, size = 112, normalized size = 2.15

method	result
default	$\frac{b^2\left(\frac{\left(\cosh^3(dx+c)\right)}{3}+\cosh(dx+c)-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab\left(\cosh(dx+c)-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)-2b^2\left(\cosh(dx+c)-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)-}{d}$
risch	$\frac{e^{3dx+3c}b^2}{24d}+\frac{ab\,e^{dx+c}}{d}-\frac{3\,e^{dx+c}b^2}{8d}+\frac{e^{-dx-c}ab}{d}-\frac{3\,e^{-dx-c}b^2}{8d}+\frac{e^{-3dx-3c}b^2}{24d}+\frac{a^2\ln\left(e^{dx+c}-1\right)}{d}-\frac{a^2\ln\left(e^{dx+c}+1\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x+c)))+2*a*b*(cosh(
d*x+c)-2*arctanh(exp(d*x+c)))-2*b^2*(cosh(d*x+c)-2*arctanh(exp(d*x+c)))-2*a
^2*arctanh(exp(d*x+c))+4*a*b*arctanh(exp(d*x+c))-2*b^2*arctanh(exp(d*x+c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(50) = 100.

time = 0.27, size = 102, normalized size = 1.96

$$\frac{1}{24}\,b^{2}\left(\frac{e^{\left(3\,dx+3\,c\right)}}{d}-\frac{9\,e^{\left(dx+c\right)}}{d}-\frac{9\,e^{\left(-dx-c\right)}}{d}+\frac{e^{\left(-3\,dx-3\,c\right)}}{d}\right)+ab\left(\frac{e^{\left(dx+c\right)}}{d}+\frac{e^{\left(-dx-c\right)}}{d}\right)+\frac{a^{2}\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) + a*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a^2*log(tanh(1/2*d*x +
 1/2*c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(50) = 100$.
time = 0.46, size = 492, normalized size = 9.46

$$\frac{b^{2}\cosh\left(dx+c\right)^{6}+6\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+b^{2}\sinh\left(dx+c\right)^{6}+3\,\left(8\,ab-3\,b^{2}\right)\cosh\left(dx+c\right)^{4}+3\,\left(5\,b^{2}\cosh\left(dx+c\right)^{2}+8\,ab-3\,b^{2}\right)\sinh\left(dx+c\right)^{4}+4\,\left(5\,b^{2}\cosh\left(dx+c\right)^{3}+3\,\left(8\,ab-3\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+3\,\left(8\,ab-3\,b^{2}\right)\cosh\left(dx+c\right)^{2}+3\,\left(5\,b^{2}\cosh\left(dx+c\right)^{4}+6\,\left(8\,ab-3\,b^{2}\right)\cosh\left(dx+c\right)^{2}+8\,ab-3\,b^{2}\right)\sinh\left(dx+c\right)^{2}+b^{2}-24\,\left(a^{2}\cosh\left(dx+c\right)^{3}+3\,a^{2}\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)+3\,a^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+a^{2}\sinh\left(dx+c\right)^{3}\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)+24\,\left(a^{2}\cosh\left(dx+c\right)^{3}+3\,a^{2}\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)+3\,a^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+a^{2}\sinh\left(dx+c\right)^{3}\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)+6\,\left(b^{2}\cosh\left(dx+c\right)^{5}+2\,\left(8\,ab-3\,b^{2}\right)\cosh\left(dx+c\right)^{3}+\left(8\,ab-3\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{24\,\left(d\cosh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+d\sinh\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(
d*x + c)^6 + 3*(8*a*b - 3*b^2)*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 +
 8*a*b - 3*b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 + 3*(8*a*b - 3*b
^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(8*a*b - 3*b^2)*cosh(d*x + c)^2 + 3*
(5*b^2*cosh(d*x + c)^4 + 6*(8*a*b - 3*b^2)*cosh(d*x + c)^2 + 8*a*b - 3*b^2)
*sinh(d*x + c)^2 + b^2 - 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c)^2*si
nh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(d*x + c)^3)*lo
g(cosh(d*x + c) + sinh(d*x + c) + 1) + 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh
(d*x + c)^2*sinh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(
d*x + c)^3)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(b^2*cosh(d*x + c)^5
 + 2*(8*a*b - 3*b^2)*cosh(d*x + c)^3 + (8*a*b - 3*b^2)*cosh(d*x + c))*sinh(
d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh
(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^{2}\left(c+dx\right)\right)^{2}\operatorname{csch}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)**2*csch(c + d*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(50) = 100.
time = 0.43, size = 110, normalized size = 2.12

$$\frac{b^{2} e^{\left(3 \, dx + 3 \, c\right)} + 24 \, a b e^{\left(dx + c\right)} - 9 \, b^{2} e^{\left(dx + c\right)} - 24 \, a^{2} \log\left(e^{\left(dx + c\right)} + 1\right) + 24 \, a^{2} \log\left(\left| e^{\left(dx + c\right)} - 1 \right|\right) + \left(24 \, a b e^{\left(2 \, dx + 2 \, c\right)} - 9 \, b^{2} e^{\left(2 \, dx + 2 \, c\right)} + b^{2}\right) e^{\left(-3 \, dx - 3 \, c\right)}}{24 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(b^2*e^(3*d*x + 3*c) + 24*a*b*e^(d*x + c) - 9*b^2*e^(d*x + c) - 24*a^2
*log(e^(d*x + c) + 1) + 24*a^2*log(abs(e^(d*x + c) - 1)) + (24*a*b*e^(2*d*x
 + 2*c) - 9*b^2*e^(2*d*x + 2*c) + b^2)*e^(-3*d*x - 3*c))/d
```

Mupad [**B**]
time = 0.16, size = 116, normalized size = 2.23

$$\frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}+\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}+\frac{b\,\mathrm{e}^{-c-d\,x}\,\left(8\,a-3\,b\right)}{8\,d}+\frac{b\,\mathrm{e}^{c+d\,x}\,\left(8\,a-3\,b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/sinh(c + d*x),x)
```

```
[Out] (b^2*exp(- 3*c - 3*d*x))/(24*d) - (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2)
)/(d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^2)^(1/2) + (b^2*exp(3*c + 3*d*x))/(24*d
) + (b*exp(- c - d*x)*(8*a - 3*b))/(8*d) + (b*exp(c + d*x)*(8*a - 3*b))/(8*
d)
```

3.16 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}(4a-b)bx-\frac{a^2\coth(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

```
[Out] 1/2*(4*a-b)*b*x-a^2*coth(d*x+c)/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3266, 473, 393, 212}

$$\frac{(2a^2+b^2)\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^2\cosh^2(c+dx)\coth(c+dx)}{d}+\frac{1}{2}bx(4a-b)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((4*a - b)*b*x)/2 - (a^2*Cosh[c + d*x]^2*Coth[c + d*x])/d + ((2*a^2 + b^2)*
Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{x^2\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{a^2\cosh^2(c+dx)\coth(c+dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{a(a+2b)+(a-b)^2x^2}{\left(1-x^2\right)^2}\,dx\right.}{d}\\
&= -\frac{a^2\cosh^2(c+dx)\coth(c+dx)}{d} + \frac{\left(2a^2+b^2\right)\cosh(c+dx)\,\mathrm{sin}}{2d}\\
&= \frac{1}{2}(4a-b)bx - \frac{a^2\cosh^2(c+dx)\coth(c+dx)}{d} + \frac{\left(2a^2+b^2\right)\mathrm{cos}}{}
\end{aligned}
$$

Mathematica [**A**]
time = 0.11, size = 56, normalized size = 1.12

$$2abx + \frac{b^2(-c-dx)}{2d} - \frac{a^2\coth(c+dx)}{d} + \frac{b^2\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] 2*a*b*x + (b^2*(-c - d*x))/(2*d) - (a^2*Coth[c + d*x])/d + (b^2*Sinh[2*(c +
 d*x)])/(4*d)
```

Maple [**A**]
time = 1.29, size = 68, normalized size = 1.36

method	result	size
risch	$2abx - \frac{b^2x}{2} + \frac{\mathrm{e}^{2dx+2c}b^2}{8d} - \frac{\mathrm{e}^{-2dx-2c}b^2}{8d} - \frac{2a^2}{d\left(\mathrm{e}^{2dx+2c}-1\right)}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b*x-1/2*b^2*x+1/8/d*exp(2*d*x+2*c)*b^2-1/8/d*exp(-2*d*x-2*c)*b^2-2*a^2/
d/(exp(2*d*x+2*c)-1)
```

Maxima [A]
time = 0.27, size = 63, normalized size = 1.26

$$-\frac{1}{8}\,b^{2}\left(4\,x-\frac{e^{\left(2\,dx+2\,c\right)}}{d}+\frac{e^{\left(-2\,dx-2\,c\right)}}{d}\right)+2\,abx+\frac{2\,a^{2}}{d\left(e^{\left(-2\,dx-2\,c\right)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a*b*x + 2*a^2/(
d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [A]
time = 0.49, size = 89, normalized size = 1.78

$$\frac{b^{2}\cosh\left(dx+c\right)^{3}+3\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}-\left(8\,a^{2}+b^{2}\right)\cosh\left(dx+c\right)+4\,\left(\left(4\,ab-b^{2}\right)dx+2\,a^{2}\right)\sinh\left(dx+c\right)}{8\,d\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 - (8*a^2 + b
^2)*cosh(d*x + c) + 4*((4*a*b - b^2)*d*x + 2*a^2)*sinh(d*x + c))/(d*sinh(d*
x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(46) = 92.
time = 0.43, size = 135, normalized size = 2.70

$$\frac{b^{2}e^{\left(2\,dx+2\,c\right)}+4\,\left(4\,ab-b^{2}\right)\left(dx+c\right)-\frac{4\,abe^{\left(4\,dx+4\,c\right)}-b^{2}e^{\left(4\,dx+4\,c\right)}+16\,a^{2}e^{\left(2\,dx+2\,c\right)}-4\,abe^{\left(2\,dx+2\,c\right)}+2\,b^{2}e^{\left(2\,dx+2\,c\right)}-b^{2}}{e^{\left(4\,dx+4\,c\right)}-e^{\left(2\,dx+2\,c\right)}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(b^2*e^(2*d*x + 2*c) + 4*(4*a*b - b^2)*(d*x + c) - (4*a*b*e^(4*d*x + 4*
c) - b^2*e^(4*d*x + 4*c) + 16*a^2*e^(2*d*x + 2*c) - 4*a*b*e^(2*d*x + 2*c) +
 2*b^2*e^(2*d*x + 2*c) - b^2)/(e^(4*d*x + 4*c) - e^(2*d*x + 2*c)))/d
```

Mupad [B]
time = 0.66, size = 67, normalized size = 1.34

$$\frac{b\,x\,\left(4\,a-b\right)}{2}-\frac{2\,a^2}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{b^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}+\frac{b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/sinh(c + d*x)^2,x)
```

```
[Out] (b*x*(4*a - b))/2 - (2*a^2)/(d*(exp(2*c + 2*d*x) - 1)) - (b^2*exp(- 2*c - 2
*d*x))/(8*d) + (b^2*exp(2*c + 2*d*x))/(8*d)
```

3.17 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=56

$$\frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b^2\cosh(c+dx)}{d}-\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d}$$

```
[Out] 1/2*a*(a-4*b)*arctanh(cosh(d*x+c))/d+b^2*cosh(d*x+c)/d-1/2*a^2*coth(d*x+c)*
csch(d*x+c)/d
```

Rubi [A]

time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$, Rules used = {3265, 398, 393, 212}

$$-\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a*(a - 4*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + (b^2*Cosh[c + d*x])/d - (a^2*C
oth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a-b+bx^2\right)^2}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(b^2+\frac{a(a-2b)+2abx^2}{\left(1-x^2\right)^2}\right)\,dx, x, \cosh(c+dx)\right)}{d}\\
&= \frac{b^2\cosh(c+dx)}{d}+\frac{\operatorname{Subst}\left(\int \frac{a(a-2b)+2abx^2}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= \frac{b^2\cosh(c+dx)}{d}-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+(a(a-4b))\\
&= \frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b^2\cosh(c+dx)}{d}-a^2\coth(
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 134 vs. 2(56) = 112.
time = 0.05, size = 134, normalized size = 2.39

$$\frac{b^2\cosh(c)\cosh(dx)}{d}-\frac{a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{2ab\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{2ab\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{b^2\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (b^2*Cosh[c]*Cosh[d*x])/d - (a^2*Csch[(c + d*x)/2]^2)/(8*d) - (2*a*b*Log[Co
sh[c/2 + (d*x)/2]])/d + (2*a*b*Log[Sinh[c/2 + (d*x)/2]])/d - (a^2*Log[Tanh[
(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/(8*d) + (b^2*Sinh[c]*Sinh[
d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(52) = 104.
time = 1.41, size = 137, normalized size = 2.45

method	result
risch	$\frac{e^{dx+c}b^2}{2d}+\frac{e^{-dx-c}b^2}{2d}-\frac{a^2e^{dx+c}\left(1+e^{2dx+2c}\right)}{d\left(e^{2dx+2c}-1\right)^2}-\frac{a^2\ln\left(e^{dx+c}-1\right)}{2d}+\frac{2a\ln\left(e^{dx+c}-1\right)b}{d}+\frac{a^2\ln\left(e^{dx+c}+1\right)}{2d}-\frac{2a\ln\left(e^{dx+c}+1\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*exp(d*x+c)*b^2+1/2/d*exp(-d*x-c)*b^2-a^2*exp(d*x+c)*(1+exp(2*d*x+2*c)
)/d/(exp(2*d*x+2*c)-1)^2-1/2*a^2/d*ln(exp(d*x+c)-1)+2*a/d*ln(exp(d*x+c)-1)*
b+1/2*a^2/d*ln(exp(d*x+c)+1)-2*a/d*ln(exp(d*x+c)+1)*b
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(52) = 104$.
time = 0.27, size = 157, normalized size = 2.80

$$\frac{1}{2}\,b^{2}\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}\,a^{2}\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)-2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d
 - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-
2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*a*b*(log(e^(-d*x - c) + 1)/d - l
og(e^(-d*x - c) - 1)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(52) = 104$.
time = 0.40, size = 902, normalized size = 16.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d
*x + c)^6 - (2*a^2 + b^2)*cosh(d*x + c)^4 + (15*b^2*cosh(d*x + c)^2 - 2*a^2
 - b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 - (2*a^2 + b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 - (2*a^2 + b^2)*cosh(d*x + c)^2 + (15*b^2*cosh(d*x +
 c)^4 - 6*(2*a^2 + b^2)*cosh(d*x + c)^2 - 2*a^2 - b^2)*sinh(d*x + c)^2 + b^
2 + ((a^2 - 4*a*b)*cosh(d*x + c)^5 + 5*(a^2 - 4*a*b)*cosh(d*x + c)*sinh(d*x
 + c)^4 + (a^2 - 4*a*b)*sinh(d*x + c)^5 - 2*(a^2 - 4*a*b)*cosh(d*x + c)^3 +
 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c)^3 + 2*(5*(
a^2 - 4*a*b)*cosh(d*x + c)^3 - 3*(a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)
^2 + (a^2 - 4*a*b)*cosh(d*x + c) + (5*(a^2 - 4*a*b)*cosh(d*x + c)^4 - 6*(a^
2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c))*log(cosh(d*x + c)
+ sinh(d*x + c) + 1) - ((a^2 - 4*a*b)*cosh(d*x + c)^5 + 5*(a^2 - 4*a*b)*cos
h(d*x + c)*sinh(d*x + c)^4 + (a^2 - 4*a*b)*sinh(d*x + c)^5 - 2*(a^2 - 4*a*b
```

```
)*cosh(d*x + c)^3 + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(
d*x + c)^3 + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^3 - 3*(a^2 - 4*a*b)*cosh(d*x
+ c))*sinh(d*x + c)^2 + (a^2 - 4*a*b)*cosh(d*x + c) + (5*(a^2 - 4*a*b)*cosh
(d*x + c)^4 - 6*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c))
*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*b^2*cosh(d*x + c)^5 - 2*(2*a
^2 + b^2)*cosh(d*x + c)^3 - (2*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*
cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 - 2
*d*cosh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 2*(5*d*c
osh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*
d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(52) = 104.
time = 0.43, size = 125, normalized size = 2.23

$$\frac{2\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{4\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}+\left(a^2-4\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-\left(a^2-4\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^2*(e^(d*x + c) + e^(-d*x - c)) - 4*a^2*(e^(d*x + c) + e^(-d*x - c)
)/((e^(d*x + c) + e^(-d*x - c))^2 - 4) + (a^2 - 4*a*b)*log(e^(d*x + c) + e^
(-d*x - c) + 2) - (a^2 - 4*a*b)*log(e^(d*x + c) + e^(-d*x - c) - 2))/d
```

Mupad [**B**]
time = 0.67, size = 179, normalized size = 3.20

$$\frac{\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a^2\,\sqrt{-d^2}-4\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4-8\,a^3\,b+16\,a^2\,b^2}}\right)\,\sqrt{a^4-8\,a^3\,b+16\,a^2\,b^2}}{\sqrt{-d^2}}+\frac{b^2\,e^{c+d\,x}}{2\,d}+\frac{b^2\,e^{-c-d\,x}}{2\,d}-\frac{a^2\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^2\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) - 4*a*b*(-d^2)^(1/2)))/(d*(a^4 - 8
*a^3*b + 16*a^2*b^2)^(1/2)))*(a^4 - 8*a^3*b + 16*a^2*b^2)^(1/2))/(-d^2)^(1/
2) + (b^2*exp(c + d*x))/(2*d) + (b^2*exp(- c - d*x))/(2*d) - (a^2*exp(c + d
*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^2*exp(c + d*x))/(d*(exp(4*c + 4*d*x)
 - 2*exp(2*c + 2*d*x) + 1))
```

3.18 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=40

$$b^2x+\frac{a(a-2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}$$

```
[Out] b^2*x+a*(a-2*b)*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {3266, 472, 213}

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a(a-2b)\coth(c+dx)}{d}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] b^2*x + (a*(a - 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{x^4(1-x^2)}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^4}-\frac{a(a-2b)}{x^2}-\frac{b^2}{-1+x^2}\right)\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{a(a-2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}-\frac{b^2\text{Subst}\left(\int \frac{1}{-1+x}\right.}{}\\
&= b^2x+\frac{a(a-2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 85 vs. 2(40) = 80.
time = 0.49, size = 85, normalized size = 2.12

$$
\frac{4\left(b+a\operatorname{csch}^2(c+dx)\right)^2\left(3b^2(c+dx)-a\coth(c+dx)\left(-2a+6b+a\operatorname{csch}^2(c+dx)\right)\right)\sinh^4(c+dx)}{3d(2a-b+b\cosh(2(c+dx)))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (4*(b + a*Csch[c + d*x]^2)^2*(3*b^2*(c + d*x) - a*Coth[c + d*x]*(-2*a + 6*b
 + a*Csch[c + d*x]^2))*Sinh[c + d*x]^4)/(3*d*(2*a - b + b*Cosh[2*(c + d*x)]
)^2)
```

Maple [**A**]
time = 1.36, size = 69, normalized size = 1.72

method	result	size
risch	$b^2x-\frac{4a\left(3b\,e^{4dx+4c}+3a\,e^{2dx+2c}-6b\,e^{2dx+2c}-a+3b\right)}{3d\left(e^{2dx+2c}-1\right)^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x-4/3*a*(3*b*exp(4*d*x+4*c)+3*a*exp(2*d*x+2*c)-6*b*exp(2*d*x+2*c)-a+3*b
)/d/(exp(2*d*x+2*c)-1)^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(38) = 76.
time = 0.28, size = 121, normalized size = 3.02

$$
b^2x+\frac{4}{3}\,a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)+\frac{4\,ab}{d(e^{(-2\,dx-2\,c)}-1)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*x + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x -
4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(38) = 76$.
time = 0.42, size = 174, normalized size = 4.35

$$\frac{2\left(a^2-3\,ab\right)\cosh\left(dx+c\right)^3+6\left(a^2-3\,ab\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+\left(3\,b^2dx-2\,a^2+6\,ab\right)\sinh\left(dx+c\right)^3-6\left(a^2-ab\right)\cosh\left(dx+c\right)-3\left(3\,b^2dx-\left(3\,b^2dx-2\,a^2+6\,ab\right)\cosh\left(dx+c\right)^2-2\,a^2+6\,ab\right)\sinh\left(dx+c\right)}{3\left(d\sinh\left(dx+c\right)^3+3\left(d\cosh\left(dx+c\right)^2-d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(a^2 - 3*a*b)*cosh(d*x + c)^3 + 6*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d
*x + c)^2 + (3*b^2*d*x - 2*a^2 + 6*a*b)*sinh(d*x + c)^3 - 6*(a^2 - a*b)*cos
h(d*x + c) - 3*(3*b^2*d*x - (3*b^2*d*x - 2*a^2 + 6*a*b)*cosh(d*x + c)^2 - 2
*a^2 + 6*a*b)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)
*sinh(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(38) = 76$.
time = 0.43, size = 81, normalized size = 2.02

$$\frac{3\left(dx+c\right)b^2-\frac{4\left(3\,abe^{\left(4\,dx+4\,c\right)}+3\,a^2e^{\left(2\,dx+2\,c\right)}-6\,abe^{\left(2\,dx+2\,c\right)}-a^2+3\,ab\right)}{\left(e^{\left(2\,dx+2\,c\right)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*b^2 - 4*(3*a*b*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*c) - 6
*a*b*e^(2*d*x + 2*c) - a^2 + 3*a*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B]
time = 0.62, size = 166, normalized size = 4.15

$$b^2\,x-\frac{\frac{4\,a\,b}{3\,d}-\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b-a^2\right)}{3\,d}+\frac{4\,a\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}+\frac{\frac{4\,\left(a\,b-a^2\right)}{3\,d}-\frac{4\,a\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{4\,a\,b}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/sinh(c + d*x)^4,x)
```

```
[Out] b^2*x - ((4*a*b)/(3*d) - (8*exp(2*c + 2*d*x)*(a*b - a^2))/(3*d) + (4*a*b*ex
p(4*c + 4*d*x))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c +
 6*d*x) - 1) + ((4*(a*b - a^2))/(3*d) - (4*a*b*exp(2*c + 2*d*x))/(3*d))/(ex
p(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (4*a*b)/(3*d*(exp(2*c + 2*d*x) -
 1))
```

3.19 $\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=261

$$\frac{3}{256}(4a-3b)\left(8a^2-14ab+7b^2\right)x-\frac{(576a^3-1744a^2b+1678ab^2-525b^3)\cosh(c+dx)\sinh(c+dx)}{1280d}+\frac{(48a^3-}{}$$

```
[Out] 3/256*(4*a-3*b)*(8*a^2-14*a*b+7*b^2)*x-1/1280*(576*a^3-1744*a^2*b+1678*a*b^
2-525*b^3)*cosh(d*x+c)*sinh(d*x+c)/d+1/640*(48*a^3-272*a^2*b+314*a*b^2-105*
b^3)*cosh(d*x+c)^3*sinh(d*x+c)/d+3/80*(2*a-3*b)*cosh(d*x+c)^5*sinh(d*x+c)^3
*(a-(a-b)*tanh(d*x+c)^2)^2/d+1/10*cosh(d*x+c)^7*sinh(d*x+c)^3*(a-(a-b)*tanh
(d*x+c)^2)^3/d-1/160*b*cosh(d*x+c)^3*sinh(d*x+c)^3*(a*(14*a-9*b)-(22*a-21*b
)*(a-b)*tanh(d*x+c)^2)/d
```

Rubi [A]
time = 0.31, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3266, 478, 591, 466, 393, 212}

$$\frac{3}{256}x(4a-3b)\left(8a^2-14ab+7b^2\right)+\frac{(48a^3-272a^2b+314ab^2-105b^3)\sinh(c+dx)\cosh^3(c+dx)}{640d}-\frac{(576a^3-1744a^2b+1678ab^2-525b^3)\sinh(c+dx)\cosh(c+dx)}{1280d}+\frac{\sinh^3(c+dx)\cosh^7(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{10d}+\frac{3(2a-3b)\sinh^3(c+dx)\cosh^5(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{80d}-\frac{b\sinh^3(c+dx)\cosh^3(c+dx)\left(a(14a-9b)-(22a-21b)(a-b)\tanh^2(c+dx)\right)}{160d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*x)/256 - ((576*a^3 - 1744*a^2*b + 1
678*a*b^2 - 525*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(1280*d) + ((48*a^3 - 272
*a^2*b + 314*a*b^2 - 105*b^3)*Cosh[c + d*x]^3*Sinh[c + d*x])/(640*d) + (3*(
2*a - 3*b)*Cosh[c + d*x]^5*Sinh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^2)
/(80*d) + (Cosh[c + d*x]^7*Sinh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^3)
/(10*d) - (b*Cosh[c + d*x]^3*Sinh[c + d*x]^3*(a*(14*a - 9*b) - (22*a - 21*b
)*(a - b)*Tanh[c + d*x]^2))/(160*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{x^4\left(a-(a-b)x^2\right)^3}{(1-x^2)^6}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\cosh^7(c+dx)\sinh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{10d} - \\
&= \frac{3(2a-3b)\cosh^5(c+dx)\sinh^3(c+dx)\left(a-(a-b)\tanh^2(c+\right.}{80d} \\
&= \frac{3(2a-3b)\cosh^5(c+dx)\sinh^3(c+dx)\left(a-(a-b)\tanh^2(c+\right.}{80d} \\
&= \frac{(48a^3-272a^2b+314ab^2-105b^3)\cosh^3(c+dx)\sinh(c+dx)}{640d} + \\
&= -\frac{(576a^3-1744a^2b+1678ab^2-525b^3)\cosh(c+dx)\sinh(c+}{1280d} \\
&= \frac{3}{256}(4a-3b)\left(8a^2-14ab+7b^2\right)x - \frac{(576a^3-1744a^2b+1678}{}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 162, normalized size = 0.62

$$\frac{120(4a-3b)\left(8a^2-14ab+7b^2\right)(c+dx)-20(128a^3-360a^2b+336ab^2-105b^3)\sinh(2(c+dx))+40(8a^3-36a^2b+42ab^2-15b^3)\sinh(4(c+dx))+10b(16a^2-32ab+15b^2)\sinh(6(c+dx))+5(6a-5b)b^2\sinh(8(c+dx))+2b^3\sinh(10(c+dx))}{10240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (120*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*(c + d*x) - 20*(128*a^3 - 360*a^2
*b + 336*a*b^2 - 105*b^3)*Sinh[2*(c + d*x)] + 40*(8*a^3 - 36*a^2*b + 42*a*b
^2 - 15*b^3)*Sinh[4*(c + d*x)] + 10*b*(16*a^2 - 32*a*b + 15*b^2)*Sinh[6*(c
+ d*x)] + 5*(6*a - 5*b)*b^2*Sinh[8*(c + d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(
10240*d)
```

Maple [A]
time = 1.51, size = 177, normalized size = 0.68

method	result
default	$\frac{\left(-\frac{5}{256}b^3+\frac{3}{128}a\,b^2\right)\sinh(8dx+8c)}{8d}+\frac{\left(\frac{45}{512}b^3-\frac{3}{16}a\,b^2+\frac{3}{32}a^2b\right)\sinh(6dx+6c)}{6d}+\frac{\left(-\frac{15}{64}b^3+\frac{21}{32}a\,b^2-\frac{9}{16}a^2b+\frac{1}{8}a^3\right)\sinh(4dx+4c)}{4d}+$
risch	$-\frac{15a^2bx}{16}-\frac{63b^3x}{256}+\frac{9\,\mathrm{e}^{-4dx-4c}a^2b}{128d}-\frac{21\,\mathrm{e}^{-4dx-4c}a\,b^2}{256d}+\frac{3a^3x}{8}+\frac{105a\,b^2x}{128}-\frac{45\,\mathrm{e}^{-2dx-2c}a^2b}{128d}+\frac{21\,\mathrm{e}^{-2dx-2c}a\,b^2}{64d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-5/256*b^3+3/128*a*b^2)/d*sinh(8*d*x+8*c)+1/6*(45/512*b^3-3/16*a*b^2+3
/32*a^2*b)/d*sinh(6*d*x+6*c)+1/4*(-15/64*b^3+21/32*a*b^2-9/16*a^2*b+1/8*a^3
)/d*sinh(4*d*x+4*c)+1/2*(105/256*b^3-21/16*a*b^2+45/32*a^2*b-1/2*a^3)*sinh(
2*d*x+2*c)/d+3/8*a^3*x-63/256*b^3*x+105/128*a*b^2*x-15/16*a^2*b*x+1/5120*b^
3/d*sinh(10*d*x+10*c)
```

Maxima [A]
time = 0.28, size = 405, normalized size = 1.55

$$\frac{1}{64}\,a^{3}\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{20480}\,b^{3}\left(\frac{\left(25\,e^{(-2\,dx-2\,c)}-150\,e^{(-4\,dx-4\,c)}+600\,e^{(-6\,dx-6\,c)}-2100\,e^{(-8\,dx-8\,c)}-2\right)e^{(10\,dx+10\,c)}}{d}+\frac{5040\,(dx+c)}{d}+\frac{2100\,e^{(-2\,dx-2\,c)}-600\,e^{(-4\,dx-4\,c)}+150\,e^{(-6\,dx-6\,c)}-25\,e^{(-8\,dx-8\,c)}+2\,e^{(-10\,dx-10\,c)}}{d}\right)-\frac{1}{2048}\,ab^{2}\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+672\,e^{(-6\,dx-6\,c)}-3\right)e^{(8\,dx+8\,c)}}{d}-\frac{1680\,(dx+c)}{d}-\frac{672\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+32\,e^{(-6\,dx-6\,c)}-3\,e^{(-8\,dx-8\,c)}}{d}\right)-\frac{1}{128}\,a^{2}b\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((25*e^(-2*d*x - 2*c) - 150*e^(-4*
d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x +
10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c)
+ 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 1
/2048*a*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x -
6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 16
8*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) - 1/128*a
^2*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 12
0*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
))/d)
```

Fricas [A]
time = 0.41, size = 406, normalized size = 1.56

$$\frac{5\,b^{3}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{9}+10\,\left(6\,b^{3}\cosh\left(dx+c\right)^{3}+\left(6\,ab^{2}-5\,b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{7}+\left(126\,b^{3}\cosh\left(dx+c\right)^{5}+70\,\left(6\,ab^{2}-5\,b^{3}\right)\cosh\left(dx+c\right)^{3}+15\,\left(16\,a^{2}b-32\,ab^{2}+15\,b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{5}+10\,\left(6\,b^{3}\cosh\left(dx+c\right)^{7}+7\,\left(6\,ab^{2}-5\,b^{3}\right)\cosh\left(dx+c\right)^{5}+5\,\left(16\,a^{2}b-32\,ab^{2}+15\,b^{3}\right)\cosh\left(dx+c\right)^{3}+4\,\left(8\,a^{3}-36\,a^{2}b+42\,ab^{2}-15\,b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+30\,\left(32\,a^{3}-80\,a^{2}b+70\,ab^{2}-21\,b^{3}\right)dx+5\,\left(b^{3}\cosh\left(dx+c\right)^{9}+2\,\left(6\,ab^{2}-5\,b^{3}\right)\cosh\left(dx+c\right)^{7}+3\,\left(16\,a^{2}b-32\,ab^{2}+15\,b^{3}\right)\cosh\left(dx+c\right)^{5}+8\,\left(8\,a^{3}-36\,a^{2}b+42\,ab^{2}-15\,b^{3}\right)\cosh\left(dx+c\right)^{3}-2\,\left(128\,a^{3}-360\,a^{2}b+336\,ab^{2}-105\,b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2560*(5*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 10*(6*b^3*cosh(d*x + c)^3 + (
6*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (126*b^3*cosh(d*x + c)^5
+ 70*(6*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + 15*(16*a^2*b - 32*a*b^2 + 15*b^3)*
cosh(d*x + c))*sinh(d*x + c)^5 + 10*(6*b^3*cosh(d*x + c)^7 + 7*(6*a*b^2 - 5
*b^3)*cosh(d*x + c)^5 + 5*(16*a^2*b - 32*a*b^2 + 15*b^3)*cosh(d*x + c)^3 +
4*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3
0*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*d*x + 5*(b^3*cosh(d*x + c)^9 + 2*
(6*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 3*(16*a^2*b - 32*a*b^2 + 15*b^3)*cosh(d
*x + c)^5 + 8*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*cosh(d*x + c)^3 - 2*(1
28*a^3 - 360*a^2*b + 336*a*b^2 - 105*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(248) = 496$.
time = 2.04, size = 777, normalized size = 2.98

$$\begin{cases} \frac{3a^{3}x\sinh^{4}(c+dx)}{8} - \frac{3a^{3}x\sinh^{2}(c+dx)\cosh^{2}(c+dx)}{4} + \frac{3a^{3}x\cosh^{4}(c+dx)}{8} + \frac{5a^{3}\sinh^{3}(c+dx)\cosh(c+dx)}{8d} - \frac{3a^{3}\sinh(c+dx)\cosh^{3}(c+dx)}{8d} + \frac{15a^{2}bx\sinh^{6}(c+dx)}{16} - \frac{45a^{2}bx\sinh^{4}(c+dx)\cosh^{2}(c+dx)}{16} + \frac{45a^{2}bx\sinh^{2}(c+dx)\cosh^{4}(c+dx)}{16} - \frac{15a^{2}bx\cosh^{6}(c+dx)}{16} + \frac{33a^{2}b\sinh^{5}(c+dx)\cosh(c+dx)}{16d} - \frac{5a^{2}b\sinh^{3}(c+dx)\cosh^{3}(c+dx)}{2d} + \frac{15a^{2}b\sinh(c+dx)\cosh^{5}(c+dx)}{16d} + \frac{105ab^{2}x\sinh^{8}(c+dx)}{128} - \frac{105ab^{2}x\sinh^{6}(c+dx)\cosh^{2}(c+dx)}{32} + \frac{315ab^{2}x\sinh^{4}(c+dx)\cosh^{4}(c+dx)}{64} - \frac{105ab^{2}x\sinh^{2}(c+dx)\cosh^{6}(c+dx)}{32} + \frac{105ab^{2}x\cosh^{8}(c+dx)}{128} + \frac{279ab^{2}\sinh^{7}(c+dx)\cosh(c+dx)}{128d} - \frac{511ab^{2}\sinh^{5}(c+dx)\cosh^{3}(c+dx)}{128d} + \frac{385ab^{2}\sinh^{3}(c+dx)\cosh^{5}(c+dx)}{128d} - \frac{105ab^{2}\sinh(c+dx)\cosh^{7}(c+dx)}{128d} + \frac{63b^{3}x\sinh^{10}(c+dx)}{256} - \frac{315b^{3}x\sinh^{8}(c+dx)\cosh^{2}(c+dx)}{256} + \frac{315b^{3}x\sinh^{6}(c+dx)\cosh^{4}(c+dx)}{128} - \frac{315b^{3}x\sinh^{4}(c+dx)\cosh^{6}(c+dx)}{128} + \frac{315b^{3}x\sinh^{2}(c+dx)\cosh^{8}(c+dx)}{256} - \frac{63b^{3}x\cosh^{10}(c+dx)}{256} + \frac{193b^{3}\sinh^{9}(c+dx)\cosh(c+dx)}{256d} - \frac{237b^{3}\sinh^{7}(c+dx)\cosh^{3}(c+dx)}{128d} + \frac{21b^{3}\sinh^{5}(c+dx)\cosh^{5}(c+dx)}{10d} - \frac{147b^{3}\sinh^{3}(c+dx)\cosh^{7}(c+dx)}{128d} + \frac{63b^{3}\sinh(c+dx)\cosh^{9}(c+dx)}{256d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^{2}(c)\right)^{3}\sinh^{4}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((3*a**3*x*sinh(c + d*x)**4/8 - 3*a**3*x*sinh(c + d*x)**2*cosh(c +
 d*x)**2/4 + 3*a**3*x*cosh(c + d*x)**4/8 + 5*a**3*sinh(c + d*x)**3*cosh(c +
 d*x)/(8*d) - 3*a**3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 15*a**2*b*x*sin
h(c + d*x)**6/16 - 45*a**2*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 45*a*
*2*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 15*a**2*b*x*cosh(c + d*x)**6/
16 + 33*a**2*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*a**2*b*sinh(c + d*
x)**3*cosh(c + d*x)**3/(2*d) + 15*a**2*b*sinh(c + d*x)*cosh(c + d*x)**5/(16
*d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)**6*cos
h(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 105*
a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c + d*x)*
*8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a*b**2*sin
h(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)**3*cosh(c
 + d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + 63
*b**3*x*sinh(c + d*x)**10/256 - 315*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**
2/256 + 315*b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 315*b**3*x*sinh(
c + d*x)**4*cosh(c + d*x)**6/128 + 315*b**3*x*sinh(c + d*x)**2*cosh(c + d*x
)**8/256 - 63*b**3*x*cosh(c + d*x)**10/256 + 193*b**3*sinh(c + d*x)**9*cosh
(c + d*x)/(256*d) - 237*b**3*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + 21
*b**3*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) - 147*b**3*sinh(c + d*x)**3*
cosh(c + d*x)**7/(128*d) + 63*b**3*sinh(c + d*x)*cosh(c + d*x)**9/(256*d),
Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**4, True))
```

Giac [**A**]
time = 0.46, size = 325, normalized size = 1.25

$$\frac{b^{3}e^{(10\,dx+10\,c)}}{10240\,d} - \frac{b^{3}e^{(-10\,dx-10\,c)}}{10240\,d} + \frac{3}{256}\,\left(32\,a^{3} - 80\,a^{2}b + 70\,ab^{2} - 21\,b^{3}\right)x + \frac{\left(6\,ab^{2} - 5\,b^{3}\right)e^{(8\,dx+8\,c)}}{4096\,d} + \frac{\left(16\,a^{2}b - 32\,ab^{2} + 15\,b^{3}\right)e^{(6\,dx+6\,c)}}{2048\,d} + \frac{\left(8\,a^{3} - 36\,a^{2}b + 42\,ab^{2} - 15\,b^{3}\right)e^{(4\,dx+4\,c)}}{512\,d} - \frac{\left(128\,a^{3} - 360\,a^{2}b + 336\,ab^{2} - 105\,b^{3}\right)e^{(2\,dx+2\,c)}}{1024\,d} + \frac{\left(128\,a^{3} - 360\,a^{2}b + 336\,ab^{2} - 105\,b^{3}\right)e^{(-2\,dx-2\,c)}}{1024\,d} - \frac{\left(8\,a^{3} - 36\,a^{2}b + 42\,ab^{2} - 15\,b^{3}\right)e^{(-4\,dx-4\,c)}}{512\,d} - \frac{\left(16\,a^{2}b - 32\,ab^{2} + 15\,b^{3}\right)e^{(-6\,dx-6\,c)}}{2048\,d} - \frac{\left(6\,ab^{2} - 5\,b^{3}\right)e^{(-8\,dx-8\,c)}}{4096\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10240*b^3*e^(10*d*x + 10*c)/d - 1/10240*b^3*e^(-10*d*x - 10*c)/d + 3/256*
(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*x + 1/4096*(6*a*b^2 - 5*b^3)*e^(8*d
*x + 8*c)/d + 1/2048*(16*a^2*b - 32*a*b^2 + 15*b^3)*e^(6*d*x + 6*c)/d + 1/5
12*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*e^(4*d*x + 4*c)/d - 1/1024*(128*a
^3 - 360*a^2*b + 336*a*b^2 - 105*b^3)*e^(2*d*x + 2*c)/d + 1/1024*(128*a^3 -
 360*a^2*b + 336*a*b^2 - 105*b^3)*e^(-2*d*x - 2*c)/d - 1/512*(8*a^3 - 36*a^
```

```
2*b + 42*a*b^2 - 15*b^3)*e^(-4*d*x - 4*c)/d - 1/2048*(16*a^2*b - 32*a*b^2 +
 15*b^3)*e^(-6*d*x - 6*c)/d - 1/4096*(6*a*b^2 - 5*b^3)*e^(-8*d*x - 8*c)/d
```

Mupad [**B**]
time = 1.20, size = 239, normalized size = 0.92

$$\frac{40\,a^3\,\sinh(4\,c+4\,d\,x)-320\,a^3\,\sinh(2\,c+2\,d\,x)+\frac{525\,b^3\,\sinh(2\,c+2\,d\,x)}{2}-75\,b^3\,\sinh(4\,c+4\,d\,x)+\frac{75\,b^3\,\sinh(6\,c+6\,d\,x)}{4}-\frac{25\,b^3\,\sinh(8\,c+8\,d\,x)}{8}+\frac{b^3\,\sinh(10\,c+10\,d\,x)}{4}-840\,a\,b^2\,\sinh(2\,c+2\,d\,x)+900\,a^2\,b\,\sinh(2\,c+2\,d\,x)+210\,a\,b^2\,\sinh(4\,c+4\,d\,x)-180\,a^2\,b\,\sinh(4\,c+4\,d\,x)-40\,a\,b^2\,\sinh(6\,c+6\,d\,x)+20\,a^2\,b\,\sinh(6\,c+6\,d\,x)+\frac{15\,a\,b^2\,\sinh(8\,c+8\,d\,x)}{4}+480\,a^3\,d\,x-315\,b^3\,d\,x+1050\,a\,b^2\,d\,x-1200\,a^2\,b\,d\,x}{1280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (40*a^3*sinh(4*c + 4*d*x) - 320*a^3*sinh(2*c + 2*d*x) + (525*b^3*sinh(2*c +
 2*d*x))/2 - 75*b^3*sinh(4*c + 4*d*x) + (75*b^3*sinh(6*c + 6*d*x))/4 - (25*
b^3*sinh(8*c + 8*d*x))/8 + (b^3*sinh(10*c + 10*d*x))/4 - 840*a*b^2*sinh(2*c
 + 2*d*x) + 900*a^2*b*sinh(2*c + 2*d*x) + 210*a*b^2*sinh(4*c + 4*d*x) - 180
*a^2*b*sinh(4*c + 4*d*x) - 40*a*b^2*sinh(6*c + 6*d*x) + 20*a^2*b*sinh(6*c +
 6*d*x) + (15*a*b^2*sinh(8*c + 8*d*x))/4 + 480*a^3*d*x - 315*b^3*d*x + 1050
*a*b^2*d*x - 1200*a^2*b*d*x)/(1280*d)
```

3.20 $\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=115

$$-\frac{(a-b)^3\cosh(c+dx)}{d}+\frac{(a-4b)(a-b)^2\cosh^3(c+dx)}{3d}+\frac{3(a-2b)(a-b)b\cosh^5(c+dx)}{5d}+\frac{(3a-4b)b^2\cosh^7}{7d}$$

```
[Out] -(a-b)^3*cosh(d*x+c)/d+1/3*(a-4*b)*(a-b)^2*cosh(d*x+c)^3/d+3/5*(a-2*b)*(a-b
)*b*cosh(d*x+c)^5/d+1/7*(3*a-4*b)*b^2*cosh(d*x+c)^7/d+1/9*b^3*cosh(d*x+c)^9
/d
```

Rubi [A]

time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3265, 380}

$$\frac{b^2(3a-4b)\cosh^7(c+dx)}{7d}+\frac{3b(a-2b)(a-b)\cosh^5(c+dx)}{5d}+\frac{(a-4b)(a-b)^2\cosh^3(c+dx)}{3d}-\frac{(a-b)^3\cosh(c+dx)}{d}+\frac{b^3\cosh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -(((a - b)^3*Cosh[c + d*x])/d) + ((a - 4*b)*(a - b)^2*Cosh[c + d*x]^3)/(3*d
) + (3*(a - 2*b)*(a - b)*b*Cosh[c + d*x]^5)/(5*d) + ((3*a - 4*b)*b^2*Cosh[c
 + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(9*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= -\frac{\operatorname{Subst}\left(\int \left(1-x^2\right)\left(a-b+bx^2\right)^3\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\operatorname{Subst}\left(\int \left((a-b)^3-(a-4b)(a-b)^2x^2+3(a-2b)b(-a+b)\right.\right.}{d}\\ &= -\frac{(a-b)^3\cosh(c+dx)}{d}+\frac{(a-4b)(a-b)^2\cosh^3(c+dx)}{3d}+3(a\end{aligned}$$

Mathematica [A]
time = 0.53, size = 127, normalized size = 1.10

$$\frac{-1890(4a-3b)\left(8a^2-14ab+7b^2\right)\cosh(c+dx)+420\left(16a^3-60a^2b+63ab^2-21b^3\right)\cosh(3(c+dx))+756(4a-3b)(a-b)b\cosh(5(c+dx))+135(4a-3b)b^2\cosh(7(c+dx))+35b^3\cosh(9(c+dx))}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-1890*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*Cosh[c + d*x] + 420*(16*a^3 - 6
0*a^2*b + 63*a*b^2 - 21*b^3)*Cosh[3*(c + d*x)] + 756*(4*a - 3*b)*(a - b)*b*
Cosh[5*(c + d*x)] + 135*(4*a - 3*b)*b^2*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(
c + d*x)])/(80640*d)
```

Maple [A]
time = 0.80, size = 147, normalized size = 1.28

method	result
default	$\frac{\left(-\frac{9}{256}b^3+\frac{3}{64}a\,b^2\right)\cosh(7dx+7c)}{7d}+\frac{\left(\frac{9}{64}b^3-\frac{21}{64}a\,b^2+\frac{3}{16}a^2b\right)\cosh(5dx+5c)}{5d}+\frac{\left(-\frac{21}{64}b^3+\frac{63}{64}a\,b^2-\frac{15}{16}a^2b+\frac{1}{4}a^3\right)\cosh(3dx+3c)}{3d}+$
risch	$\frac{b^3\mathrm{e}^{9dx+9c}}{4608d}+\frac{b^3\mathrm{e}^{-9dx-9c}}{4608d}-\frac{9b^3\mathrm{e}^{7dx+7c}}{3584d}+\frac{9b^3\mathrm{e}^{5dx+5c}}{640d}+\frac{\mathrm{e}^{3dx+3c}a^3}{24d}-\frac{7\,\mathrm{e}^{3dx+3c}b^3}{128d}-\frac{3\,\mathrm{e}^{dx+c}a^3}{8d}+\frac{63\,\mathrm{e}^{dx+c}b^3}{256d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(-9/256*b^3+3/64*a*b^2)*cosh(7*d*x+7*c)/d+1/5*(9/64*b^3-21/64*a*b^2+3/1
6*a^2*b)*cosh(5*d*x+5*c)/d+1/3*(-21/64*b^3+63/64*a*b^2-15/16*a^2*b+1/4*a^3)
/d*cosh(3*d*x+3*c)+(63/128*b^3-105/64*a*b^2+15/8*a^2*b-3/4*a^3)*cosh(d*x+c)
/d+1/2304*b^3*cosh(9*d*x+9*c)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(107) = 214.
time = 0.27, size = 376, normalized size = 3.27

$$-\frac{1}{161280}b^3\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\frac{39690\,e^{(-dx-c)}-8820\,e^{(-3\,dx-3\,c)}+2268\,e^{(-5\,dx-5\,c)}-405\,e^{(-7\,dx-7\,c)}+35\,e^{(-9\,dx-9\,c)}}{d}\right)-\frac{3}{4480}ab^2\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d}\right)+\frac{1}{160}a^2b\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d
*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x
 - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c
) + 35*e^(-9*d*x - 9*c))/d) - 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-
4*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x
 - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d)
 + 1/160*a^2*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c
```

```
)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) +
1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(107) = 214$.
time = 0.41, size = 373, normalized size = 3.24

$$\frac{35\,b^3\cosh\left(dx+c\right)^9+315\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8+135\,\left(4\,ab^2-3\,b^3\right)\cosh\left(dx+c\right)^7+105\,\left(28\,b^3\cosh\left(dx+c\right)^3+9\,\left(4\,ab^2-3\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^6+756\,\left(4\,a^2b-7\,ab^2+3\,b^3\right)\cosh\left(dx+c\right)^5+315\,\left(14\,b^3\cosh\left(dx+c\right)^5+15\,\left(4\,ab^2-3\,b^3\right)\cosh\left(dx+c\right)^3+12\,\left(4\,a^2b-7\,ab^2+3\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^4+420\,\left(16\,a^3-60\,a^2b+63\,ab^2-21\,b^3\right)\cosh\left(dx+c\right)^3+315\,\left(4\,b^3\cosh\left(dx+c\right)^7+9\,\left(4\,ab^2-3\,b^3\right)\cosh\left(dx+c\right)^5+24\,\left(4\,a^2b-7\,ab^2+3\,b^3\right)\cosh\left(dx+c\right)^3+4\,\left(16\,a^3-60\,a^2b+63\,ab^2-21\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2-1890\,\left(32\,a^3-80\,a^2b+70\,ab^2-21\,b^3\right)\cosh\left(dx+c\right)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^3*cosh(d*x + c)^9 + 315*b^3*cosh(d*x + c)*sinh(d*x + c)^8 + 1
35*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^7 + 105*(28*b^3*cosh(d*x + c)^3 + 9*(4*a
*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 756*(4*a^2*b - 7*a*b^2 + 3*b
^3)*cosh(d*x + c)^5 + 315*(14*b^3*cosh(d*x + c)^5 + 15*(4*a*b^2 - 3*b^3)*co
sh(d*x + c)^3 + 12*(4*a^2*b - 7*a*b^2 + 3*b^3)*cosh(d*x + c))*sinh(d*x + c)
^4 + 420*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*cosh(d*x + c)^3 + 315*(4*b
^3*cosh(d*x + c)^7 + 9*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + 24*(4*a^2*b - 7*
a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*
cosh(d*x + c))*sinh(d*x + c)^2 - 1890*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^
3)*cosh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(100) = 200$.
time = 1.43, size = 330, normalized size = 2.87

$$\begin{cases}\frac{a^3\sinh^2\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2a^3\cosh^3\left(c+dx\right)}{3d}+\frac{3a^2b\sinh^4\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{4a^2b\sinh^2\left(c+dx\right)\cosh^3\left(c+dx\right)}{d}+\frac{8a^2b\cosh^5\left(c+dx\right)}{5d}+\frac{3ab^2\sinh^6\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{6ab^2\sinh^4\left(c+dx\right)\cosh^3\left(c+dx\right)}{d}+\frac{24ab^2\sinh^2\left(c+dx\right)\cosh^5\left(c+dx\right)}{5d}-\frac{48ab^2\cosh^7\left(c+dx\right)}{35d}+\frac{b^3\sinh^8\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{8b^3\sinh^6\left(c+dx\right)\cosh^3\left(c+dx\right)}{3d}+\frac{16b^3\sinh^4\left(c+dx\right)\cosh^5\left(c+dx\right)}{5d}-\frac{64b^3\sinh^2\left(c+dx\right)\cosh^7\left(c+dx\right)}{35d}+\frac{128b^3\cosh^9\left(c+dx\right)}{315d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^2\left(c\right)\right)^3\sinh^3\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**3*cosh(c + d*x)**3/
(3*d) + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*a**2*b*sinh(c + d*x)*
*2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5/(5*d) + 3*a*b**2*sinh(c +
 d*x)**6*cosh(c + d*x)/d - 6*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 2
4*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 48*a*b**2*cosh(c + d*x)*
*7/(35*d) + b**3*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**3*sinh(c + d*x)**6
*cosh(c + d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) -
 64*b**3*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**3*cosh(c + d*x)*
*9/(315*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(107) = 214$.
time = 0.45, size = 296, normalized size = 2.57

$$\frac{b^3e^{\left(9\,dx+9\,c\right)}}{4608\,d}+\frac{b^3e^{\left(-9\,dx-9\,c\right)}}{4608\,d}+\frac{3\,\left(4\,ab^2-3\,b^3\right)e^{\left(7\,dx+7\,c\right)}}{3584\,d}+\frac{3\,\left(4\,a^2b-7\,ab^2+3\,b^3\right)e^{\left(5\,dx+5\,c\right)}}{640\,d}+\frac{\left(16\,a^3-60\,a^2b+63\,ab^2-21\,b^3\right)e^{\left(3\,dx+3\,c\right)}}{384\,d}-\frac{3\,\left(32\,a^3-80\,a^2b+70\,ab^2-21\,b^3\right)e^{\left(dx+c\right)}}{256\,d}-\frac{3\,\left(32\,a^3-80\,a^2b+70\,ab^2-21\,b^3\right)e^{\left(-dx-c\right)}}{256\,d}+\frac{\left(16\,a^3-60\,a^2b+63\,ab^2-21\,b^3\right)e^{\left(-3\,dx-3\,c\right)}}{384\,d}+\frac{3\,\left(4\,a^2b-7\,ab^2+3\,b^3\right)e^{\left(-5\,dx-5\,c\right)}}{640\,d}+\frac{3\,\left(4\,ab^2-3\,b^3\right)e^{\left(-7\,dx-7\,c\right)}}{3584\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4608*b^3*e^(9*d*x + 9*c)/d + 1/4608*b^3*e^(-9*d*x - 9*c)/d + 3/3584*(4*a*
b^2 - 3*b^3)*e^(7*d*x + 7*c)/d + 3/640*(4*a^2*b - 7*a*b^2 + 3*b^3)*e^(5*d*x
 + 5*c)/d + 1/384*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*e^(3*d*x + 3*c)/d
 - 3/256*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*e^(d*x + c)/d - 3/256*(32*
a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*e^(-d*x - c)/d + 1/384*(16*a^3 - 60*a^2
*b + 63*a*b^2 - 21*b^3)*e^(-3*d*x - 3*c)/d + 3/640*(4*a^2*b - 7*a*b^2 + 3*b
^3)*e^(-5*d*x - 5*c)/d + 3/3584*(4*a*b^2 - 3*b^3)*e^(-7*d*x - 7*c)/d
```

Mupad [B]
time = 0.43, size = 185, normalized size = 1.61

$$\frac{\frac{a^3\cosh(c+d\,x)^3}{3}-a^3\cosh(c+d\,x)+\frac{3\,a^2\,b\cosh(c+d\,x)^5}{5}-2\,a^2\,b\cosh(c+d\,x)^3+3\,a^2\,b\cosh(c+d\,x)+\frac{3\,a\,b^2\cosh(c+d\,x)^7}{7}-\frac{9\,a\,b^2\cosh(c+d\,x)^5}{5}+3\,a\,b^2\cosh(c+d\,x)^3-3\,a\,b^2\cosh(c+d\,x)+\frac{b^3\cosh(c+d\,x)^9}{9}-\frac{4\,b^3\cosh(c+d\,x)^7}{7}+\frac{6\,b^3\cosh(c+d\,x)^5}{5}-\frac{4\,b^3\cosh(c+d\,x)^3}{3}+b^3\cosh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (b^3*cosh(c + d*x) - a^3*cosh(c + d*x) + (a^3*cosh(c + d*x)^3)/3 - (4*b^3*c
osh(c + d*x)^3)/3 + (6*b^3*cosh(c + d*x)^5)/5 - (4*b^3*cosh(c + d*x)^7)/7 +
 (b^3*cosh(c + d*x)^9)/9 + 3*a*b^2*cosh(c + d*x)^3 - 2*a^2*b*cosh(c + d*x)^
3 - (9*a*b^2*cosh(c + d*x)^5)/5 + (3*a^2*b*cosh(c + d*x)^5)/5 + (3*a*b^2*co
sh(c + d*x)^7)/7 - 3*a*b^2*cosh(c + d*x) + 3*a^2*b*cosh(c + d*x))/d
```

3.21 $\int \sinh^2(c+dx) \left(a+b\sinh^2(c+dx)\right)^3 \, dx$

Optimal. Leaf size=181

$$-\frac{1}{128}\left(64a^3-144a^2b+120ab^2-35b^3\right)x+\frac{\left(96a^3-376a^2b+360ab^2-105b^3\right)\cosh(c+dx)\sinh(c+dx)}{384d}+b(2$$

```
[Out] -1/128*(64*a^3-144*a^2*b+120*a*b^2-35*b^3)*x+1/384*(96*a^3-376*a^2*b+360*a*
b^2-105*b^3)*cosh(d*x+c)*sinh(d*x+c)/d+1/192*b*(24*a^2-64*a*b+35*b^2)*cosh(
d*x+c)*sinh(d*x+c)^3/d+1/48*(6*a-7*b)*cosh(d*x+c)*sinh(d*x+c)*(a+b*sinh(d*x
+c)^2)^2/d+1/8*cosh(d*x+c)*sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$,
Rules used = {3249, 3248}

$$\frac{b\left(24a^2-64ab+35b^2\right)\sinh^3(c+dx)\cosh(c+dx)}{192d}+\frac{\left(96a^3-376a^2b+360ab^2-105b^3\right)\sinh(c+dx)\cosh(c+dx)}{384d}-\frac{1}{128}x\left(64a^3-144a^2b+120ab^2-35b^3\right)+\frac{\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3}{8d}+\frac{(6a-7b)\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -1/128*((64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*x) + ((96*a^3 - 376*a^2*b
 + 360*a*b^2 - 105*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(384*d) + (b*(24*a^2 -
 64*a*b + 35*b^2)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(192*d) + ((6*a - 7*b)*Cos
h[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(48*d) + (Cosh[c + d*x]
*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3)/(8*d)
```

Rule 3248

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rule 3249

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e
 + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p
 + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Gt
Q[p, 0]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3}{8d} - \frac{1}{8}\int (a -$$

$$= \frac{(6a-7b)\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{48d} +$$

$$= -\frac{1}{128}\left(64a^3-144a^2b+120ab^2-35b^3\right)x + \frac{(96a^3-376a^2b+}{}$$

Mathematica [A]
time = 0.21, size = 130, normalized size = 0.72

$$\frac{-24(64a^3-144a^2b+120ab^2-35b^3)(c+dx)+48(16a^3-48a^2b+45ab^2-14b^3)\sinh(2(c+dx))+24b(12a^2-18ab+7b^2)\sinh(4(c+dx))+16(3a-2b)b^2\sinh(6(c+dx))+3b^3\sinh(8(c+dx))}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-24*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*(c + d*x) + 48*(16*a^3 - 48*
a^2*b + 45*a*b^2 - 14*b^3)*Sinh[2*(c + d*x)] + 24*b*(12*a^2 - 18*a*b + 7*b^
2)*Sinh[4*(c + d*x)] + 16*(3*a - 2*b)*b^2*Sinh[6*(c + d*x)] + 3*b^3*Sinh[8*
(c + d*x)])/(3072*d)
```

Maple [A]
time = 1.19, size = 140, normalized size = 0.77

method	result
default	$\frac{\left(-\frac{1}{16}b^3+\frac{3}{32}a\,b^2\right)\sinh(6dx+6c)}{6d}+\frac{\left(\frac{7}{32}b^3-\frac{9}{16}a\,b^2+\frac{3}{8}a^2b\right)\sinh(4dx+4c)}{4d}+\frac{\left(-\frac{7}{16}b^3+\frac{45}{32}a\,b^2-\frac{3}{2}a^2b+\frac{1}{2}a^3\right)\sinh(2dx+2c)}{2d}-\frac{a^3x}{2}-$
risch	$\frac{35b^3x}{128}-\frac{15a\,b^2x}{16}+\frac{9a^2bx}{8}-\frac{a^3x}{2}+\frac{b^3\mathrm{e}^{8dx+8c}}{2048d}+\frac{b^2\mathrm{e}^{6dx+6c}a}{128d}-\frac{b^3\mathrm{e}^{6dx+6c}}{192d}+\frac{3\,\mathrm{e}^{4dx+4c}a^2b}{64d}-\frac{9\,\mathrm{e}^{4dx+4c}a\,b^2}{128d}+\frac{7\,\mathrm{e}^{4dx+4c}}{256d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-1/16*b^3+3/32*a*b^2)/d*sinh(6*d*x+6*c)+1/4*(7/32*b^3-9/16*a*b^2+3/8*a
^2*b)/d*sinh(4*d*x+4*c)+1/2*(-7/16*b^3+45/32*a*b^2-3/2*a^2*b+1/2*a^3)*sinh(
2*d*x+2*c)/d-1/2*a^3*x+35/128*b^3*x-15/16*a*b^2*x+9/8*a^2*b*x+1/1024*b^3/d*
sinh(8*d*x+8*c)
```

Maxima [A]
time = 0.28, size = 306, normalized size = 1.69

$$\frac{3}{64}a^2b\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{6144}b^3\left(\frac{(32e^{(-2dx-2c)}-168e^{(-4dx-4c)}+672e^{(-6dx-6c)}-3)e^{(8dx+8c)}}{d}-\frac{1680(dx+c)}{d}-\frac{672e^{(-2dx-2c)}-168e^{(-4dx-4c)}+32e^{(-6dx-6c)}-3e^{(-8dx-8c)}}{d}\right)-\frac{1}{128}ab^2\left(\frac{(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1)e^{(6dx+6c)}}{d}+\frac{120(dx+c)}{d}+\frac{45e^{(-2dx-2c)}-9e^{(-4dx-4c)}+e^{(-6dx-6c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x
 - 2*c)/d) - 1/6144*b^3*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*
e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x
 - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/
d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x +
 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c))/d)
```

Fricas [**A**]
time = 0.40, size = 269, normalized size = 1.49

$$\frac{3\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+3\left(7\,b^3\cosh\left(dx+c\right)^3+4\left(3\,ab^2-2\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^5+\left(21\,b^3\cosh\left(dx+c\right)^5+40\left(3\,ab^2-2\,b^3\right)\cosh\left(dx+c\right)^3+12\left(12\,a^2b-18\,ab^2+7\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3-3\left(64\,a^3-144\,a^2b+120\,ab^2-35\,b^3\right)dx+3\left(b^3\cosh\left(dx+c\right)^7+4\left(3\,ab^2-2\,b^3\right)\cosh\left(dx+c\right)^5+4\left(12\,a^2b-18\,ab^2+7\,b^3\right)\cosh\left(dx+c\right)^3+4\left(16\,a^3-48\,a^2b+45\,ab^2-14\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^3*cosh(d*x + c)^3 + 4*(
3*a*b^2 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^3*cosh(d*x + c)^5 +
 40*(3*a*b^2 - 2*b^3)*cosh(d*x + c)^3 + 12*(12*a^2*b - 18*a*b^2 + 7*b^3)*co
sh(d*x + c))*sinh(d*x + c)^3 - 3*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*
d*x + 3*(b^3*cosh(d*x + c)^7 + 4*(3*a*b^2 - 2*b^3)*cosh(d*x + c)^5 + 4*(12*
a^2*b - 18*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 48*a^2*b + 45*a*b^2
 - 14*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(177) = 354$.
time = 1.05, size = 561, normalized size = 3.10

$$\begin{cases} \frac{a^3x\sinh^2{(c+dx)}}{2}-\frac{a^3x\cosh^2{(c+dx)}}{2}+\frac{a^3\sinh{(c+dx)}\cosh{(c+dx)}}{2d}+\frac{9a^2bx\sinh^4{(c+dx)}}{8}-\frac{9a^2bx\sinh^2{(c+dx)}\cosh^2{(c+dx)}}{4}+\frac{9a^2bx\cosh^4{(c+dx)}}{8}+\frac{15a^2b\sinh^3{(c+dx)}\cosh{(c+dx)}}{8d}-\frac{9a^2b\sinh{(c+dx)}\cosh^3{(c+dx)}}{8d}+\frac{15ab^2x\sinh^6{(c+dx)}}{16}-\frac{45ab^2x\sinh^4{(c+dx)}\cosh^2{(c+dx)}}{16}+\frac{45ab^2x\sinh^2{(c+dx)}\cosh^4{(c+dx)}}{16}-\frac{15ab^2x\cosh^6{(c+dx)}}{16}+\frac{33ab^2\sinh^5{(c+dx)}\cosh{(c+dx)}}{16d}-\frac{5ab^2\sinh^3{(c+dx)}\cosh^3{(c+dx)}}{2d}+\frac{15ab^2\sinh{(c+dx)}\cosh^5{(c+dx)}}{16d}+\frac{35b^3x\sinh^8{(c+dx)}}{128}-\frac{35b^3x\sinh^6{(c+dx)}\cosh^2{(c+dx)}}{32}+\frac{105b^3x\sinh^4{(c+dx)}\cosh^4{(c+dx)}}{64}-\frac{35b^3x\sinh^2{(c+dx)}\cosh^6{(c+dx)}}{32}+\frac{35b^3x\cosh^8{(c+dx)}}{128}+\frac{93b^3\sinh^7{(c+dx)}\cosh{(c+dx)}}{128d}-\frac{511b^3\sinh^5{(c+dx)}\cosh^3{(c+dx)}}{384d}+\frac{385b^3\sinh^3{(c+dx)}\cosh^5{(c+dx)}}{384d}-\frac{35b^3\sinh{(c+dx)}\cosh^7{(c+dx)}}{128d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^2{(c)}\right)^3\sinh^2{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b*x
*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 15*a
**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cosh(c
+ d*x)**3/(8*d) + 15*a*b**2*x*sinh(c + d*x)**6/16 - 45*a*b**2*x*sinh(c + d*
x)**4*cosh(c + d*x)**2/16 + 45*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/1
6 - 15*a*b**2*x*cosh(c + d*x)**6/16 + 33*a*b**2*sinh(c + d*x)**5*cosh(c + d
*x)/(16*d) - 5*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) + 15*a*b**2*s
inh(c + d*x)*cosh(c + d*x)**5/(16*d) + 35*b**3*x*sinh(c + d*x)**8/128 - 35*
b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b**3*x*sinh(c + d*x)**4*c
```

```
osh(c + d*x)**4/64 - 35*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b*
*3*x*cosh(c + d*x)**8/128 + 93*b**3*sinh(c + d*x)**7*cosh(c + d*x)/(128*d)
- 511*b**3*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b**3*sinh(c + d*
x)**3*cosh(c + d*x)**5/(384*d) - 35*b**3*sinh(c + d*x)*cosh(c + d*x)**7/(12
8*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**2, True))
```

Giac [A]
time = 0.45, size = 251, normalized size = 1.39

$$\frac{b^3e^{(8\,dx+8\,c)}}{2048\,d}-\frac{b^3e^{(-8\,dx-8\,c)}}{2048\,d}-\frac{1}{128}\,(64\,a^3-144\,a^2b+120\,ab^2-35\,b^3)x+\frac{(3\,ab^2-2\,b^3)e^{(6\,dx+6\,c)}}{384\,d}+\frac{(12\,a^2b-18\,ab^2+7\,b^3)e^{(4\,dx+4\,c)}}{256\,d}+\frac{(16\,a^3-48\,a^2b+45\,ab^2-14\,b^3)e^{(2\,dx+2\,c)}}{128\,d}-\frac{(16\,a^3-48\,a^2b+45\,ab^2-14\,b^3)e^{(-2\,dx-2\,c)}}{128\,d}-\frac{(12\,a^2b-18\,ab^2+7\,b^3)e^{(-4\,dx-4\,c)}}{256\,d}-\frac{(3\,ab^2-2\,b^3)e^{(-6\,dx-6\,c)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2048*b^3*e^(8*d*x + 8*c)/d - 1/2048*b^3*e^(-8*d*x - 8*c)/d - 1/128*(64*a^
3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*x + 1/384*(3*a*b^2 - 2*b^3)*e^(6*d*x +
6*c)/d + 1/256*(12*a^2*b - 18*a*b^2 + 7*b^3)*e^(4*d*x + 4*c)/d + 1/128*(16*
a^3 - 48*a^2*b + 45*a*b^2 - 14*b^3)*e^(2*d*x + 2*c)/d - 1/128*(16*a^3 - 48*
a^2*b + 45*a*b^2 - 14*b^3)*e^(-2*d*x - 2*c)/d - 1/256*(12*a^2*b - 18*a*b^2
+ 7*b^3)*e^(-4*d*x - 4*c)/d - 1/384*(3*a*b^2 - 2*b^3)*e^(-6*d*x - 6*c)/d
```

Mupad [B]
time = 0.97, size = 181, normalized size = 1.00

$$\frac{96\,a^3\,\sinh(2\,c+2\,d\,x)-84\,b^3\,\sinh(2\,c+2\,d\,x)+21\,b^3\,\sinh(4\,c+4\,d\,x)-4\,b^3\,\sinh(6\,c+6\,d\,x)+\frac{3\,b^3\,\sinh(8\,c+8\,d\,x)}{8}+270\,a\,b^2\,\sinh(2\,c+2\,d\,x)-288\,a^2\,b\,\sinh(2\,c+2\,d\,x)-54\,a\,b^2\,\sinh(4\,c+4\,d\,x)+36\,a^2\,b\,\sinh(4\,c+4\,d\,x)+6\,a\,b^2\,\sinh(6\,c+6\,d\,x)-192\,a^3\,d\,x+105\,b^3\,d\,x-360\,a\,b^2\,d\,x+432\,a^2\,b\,d\,x}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (96*a^3*sinh(2*c + 2*d*x) - 84*b^3*sinh(2*c + 2*d*x) + 21*b^3*sinh(4*c + 4*
d*x) - 4*b^3*sinh(6*c + 6*d*x) + (3*b^3*sinh(8*c + 8*d*x))/8 + 270*a*b^2*si
nh(2*c + 2*d*x) - 288*a^2*b*sinh(2*c + 2*d*x) - 54*a*b^2*sinh(4*c + 4*d*x)
+ 36*a^2*b*sinh(4*c + 4*d*x) + 6*a*b^2*sinh(6*c + 6*d*x) - 192*a^3*d*x + 10
5*b^3*d*x - 360*a*b^2*d*x + 432*a^2*b*d*x)/(384*d)
```

3.22 $\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=79

$$\frac{(a-b)^3\cosh(c+dx)}{d}+\frac{(a-b)^2b\cosh^3(c+dx)}{d}+\frac{3(a-b)b^2\cosh^5(c+dx)}{5d}+\frac{b^3\cosh^7(c+dx)}{7d}$$

```
[Out] (a-b)^3*cosh(d*x+c)/d+(a-b)^2*b*cosh(d*x+c)^3/d+3/5*(a-b)*b^2*cosh(d*x+c)^5
/d+1/7*b^3*cosh(d*x+c)^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$,
Rules used = {3265, 200}

$$\frac{3b^2(a-b)\cosh^5(c+dx)}{5d}+\frac{b(a-b)^2\cosh^3(c+dx)}{d}+\frac{(a-b)^3\cosh(c+dx)}{d}+\frac{b^3\cosh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - b)^3*Cosh[c + d*x])/d + ((a - b)^2*b*Cosh[c + d*x]^3)/d + (3*(a - b)*
b^2*Cosh[c + d*x]^5)/(5*d) + (b^3*Cosh[c + d*x]^7)/(7*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int\left(a-b+bx^2\right)^3\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left(a^3\left(1-\frac{b(3a^2-3ab+b^2)}{a^3}\right)+3a^2b\left(1+\frac{b(-2a+b)}{a^2}\right)x^2+3ab^2\right.\right.}{d}\\ &= \frac{(a-b)^3\cosh(c+dx)}{d}+\frac{(a-b)^2b\cosh^3(c+dx)}{d}+\frac{3(a-b)b^2\text{co}}{5a}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 94, normalized size = 1.19

$$\frac{\cosh(c+dx)\left(1120a^3-2800a^2b+2492ab^2-762b^3+b(560a^2-784ab+299b^2)\cosh(2(c+dx))+6(14a-9b)b^2\cosh(4(c+dx))+5b^3\cosh(6(c+dx))\right)}{1120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Cosh[c + d*x]*(1120*a^3 - 2800*a^2*b + 2492*a*b^2 - 762*b^3 + b*(560*a^2 -
 784*a*b + 299*b^2)*Cosh[2*(c + d*x)] + 6*(14*a - 9*b)*b^2*Cosh[4*(c + d*x)
] + 5*b^3*Cosh[6*(c + d*x)]))/(1120*d)
```

Maple [A]
time = 0.68, size = 108, normalized size = 1.37

method	result
default	$\frac{\left(-\frac{7}{64}b^3+\frac{3}{16}a\,b^2\right)\cosh(5dx+5c)}{5d}+\frac{\left(\frac{21}{64}b^3-\frac{15}{16}a\,b^2+\frac{3}{4}a^2b\right)\cosh(3dx+3c)}{3d}+\frac{\left(-\frac{35}{64}b^3+\frac{15}{8}a\,b^2-\frac{9}{4}a^2b+a^3\right)\cosh(dx+c)}{d}+\frac{b^3\cosh(7}{448}$
risch	$\frac{b^3\mathrm{e}^{7dx+7c}}{896d}-\frac{7b^3\mathrm{e}^{5dx+5c}}{640d}+\frac{3b^2\mathrm{e}^{5dx+5c}a}{160d}+\frac{\mathrm{e}^{3dx+3c}a^2b}{8d}-\frac{5\,\mathrm{e}^{3dx+3c}a\,b^2}{32d}+\frac{7\,\mathrm{e}^{3dx+3c}b^3}{128d}+\frac{\mathrm{e}^{dx+c}a^3}{2d}-\frac{9\,\mathrm{e}^{dx+c}a^2b}{8d}+15\,\mathrm{e}^{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(-7/64*b^3+3/16*a*b^2)*cosh(5*d*x+5*c)/d+1/3*(21/64*b^3-15/16*a*b^2+3/4
*a^2*b)/d*cosh(3*d*x+3*c)+(-35/64*b^3+15/8*a*b^2-9/4*a^2*b+a^3)*cosh(d*x+c)
/d+1/448*b^3*cosh(7*d*x+7*c)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(75) = 150$.
time = 0.30, size = 263, normalized size = 3.33

$$-\frac{1}{4480}\,b^3\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d}\right)+\frac{1}{160}\,ab^2\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{1}{8}\,a^2b\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a^3\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/160*a*b^2*(3*e^(5*d*x + 5*
c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e
^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/8*a^2*b*(e^(3*d*x + 3*c)/d -
9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + a^3*cosh(d*x + c
)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(75) = 150$.
time = 0.43, size = 234, normalized size = 2.96

$$\frac{5\,b^3\cosh\left(dx+c\right)^7+35\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6+7\left(12\,ab^2-7\,b^3\right)\cosh\left(dx+c\right)^5+35\left(5\,b^3\cosh\left(dx+c\right)^3+\left(12\,ab^2-7\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^4+35\left(16\,a^2b-20\,ab^2+7\,b^3\right)\cosh\left(dx+c\right)^3+35\left(3\,b^3\cosh\left(dx+c\right)^5+2\left(12\,ab^2-7\,b^3\right)\cosh\left(dx+c\right)^3+3\left(16\,a^2b-20\,ab^2+7\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2+35\left(64\,a^3-144\,a^2b+120\,ab^2-35\,b^3\right)\cosh\left(dx+c\right)}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b^3*cosh(d*x + c)^7 + 35*b^3*cosh(d*x + c)*sinh(d*x + c)^6 + 7*(1
2*a*b^2 - 7*b^3)*cosh(d*x + c)^5 + 35*(5*b^3*cosh(d*x + c)^3 + (12*a*b^2 -
7*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 35*(16*a^2*b - 20*a*b^2 + 7*b^3)*co
sh(d*x + c)^3 + 35*(3*b^3*cosh(d*x + c)^5 + 2*(12*a*b^2 - 7*b^3)*cosh(d*x +
 c)^3 + 3*(16*a^2*b - 20*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 35
*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*cosh(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(68) = 136$.
time = 0.68, size = 221, normalized size = 2.80

$$\begin{cases} \frac{a^3\cosh\left(c+dx\right)}{d}+\frac{3a^2b\sinh^2\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2a^2b\cosh^3\left(c+dx\right)}{d}+\frac{3ab^2\sinh^4\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{4ab^2\sinh^2\left(c+dx\right)\cosh^3\left(c+dx\right)}{d}+\frac{8ab^2\cosh^5\left(c+dx\right)}{5d}+\frac{b^3\sinh^6\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2b^3\sinh^4\left(c+dx\right)\cosh^3\left(c+dx\right)}{d}+\frac{8b^3\sinh^2\left(c+dx\right)\cosh^5\left(c+dx\right)}{5d}-\frac{16b^3\cosh^7\left(c+dx\right)}{35d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^2\left(c\right)\right)^3\sinh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*cosh(c + d*x)/d + 3*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)/d
 - 2*a**2*b*cosh(c + d*x)**3/d + 3*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)/d
- 4*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a*b**2*cosh(c + d*x)**5/
(5*d) + b**3*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b**3*sinh(c + d*x)**4*cos
h(c + d*x)**3/d + 8*b**3*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**3*
cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c), True)
)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(75) = 150$.
time = 0.46, size = 222, normalized size = 2.81

$$\frac{b^3e^{\left(7\,dx+7\,c\right)}}{896\,d}+\frac{b^3e^{\left(-7\,dx-7\,c\right)}}{896\,d}+\frac{\left(12\,ab^2-7\,b^3\right)e^{\left(5\,dx+5\,c\right)}}{640\,d}+\frac{\left(16\,a^2b-20\,ab^2+7\,b^3\right)e^{\left(3\,dx+3\,c\right)}}{128\,d}+\frac{\left(64\,a^3-144\,a^2b+120\,ab^2-35\,b^3\right)e^{\left(dx+c\right)}}{128\,d}+\frac{\left(64\,a^3-144\,a^2b+120\,ab^2-35\,b^3\right)e^{\left(-dx-c\right)}}{128\,d}+\frac{\left(16\,a^2b-20\,ab^2+7\,b^3\right)e^{\left(-3\,dx-3\,c\right)}}{128\,d}+\frac{\left(12\,ab^2-7\,b^3\right)e^{\left(-5\,dx-5\,c\right)}}{640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/896*b^3*e^(7*d*x + 7*c)/d + 1/896*b^3*e^(-7*d*x - 7*c)/d + 1/640*(12*a*b^
2 - 7*b^3)*e^(5*d*x + 5*c)/d + 1/128*(16*a^2*b - 20*a*b^2 + 7*b^3)*e^(3*d*x
 + 3*c)/d + 1/128*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*e^(d*x + c)/d +
```

```
1/128*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*e^(-d*x - c)/d + 1/128*(16
*a^2*b - 20*a*b^2 + 7*b^3)*e^(-3*d*x - 3*c)/d + 1/640*(12*a*b^2 - 7*b^3)*e^
(-5*d*x - 5*c)/d
```

Mupad [B]
time = 0.26, size = 129, normalized size = 1.63

$$\frac{a^3\cosh(c+d\,x)+a^2\,b\cosh(c+d\,x)^3-3\,a^2\,b\cosh(c+d\,x)+\frac{3\,a\,b^2\cosh(c+d\,x)^5}{5}-2\,a\,b^2\cosh(c+d\,x)^3+3\,a\,b^2\cosh(c+d\,x)+\frac{b^3\cosh(c+d\,x)^7}{7}-\frac{3\,b^3\cosh(c+d\,x)^5}{5}+b^3\cosh(c+d\,x)^3-b^3\cosh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (a^3*cosh(c + d*x) - b^3*cosh(c + d*x) + b^3*cosh(c + d*x)^3 - (3*b^3*cosh(
c + d*x)^5)/5 + (b^3*cosh(c + d*x)^7)/7 - 2*a*b^2*cosh(c + d*x)^3 + a^2*b*c
osh(c + d*x)^3 + (3*a*b^2*cosh(c + d*x)^5)/5 + 3*a*b^2*cosh(c + d*x) - 3*a^
2*b*cosh(c + d*x))/d
```

3.23 $\int \left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=128

$$\frac{1}{16}(2a-b)\left(8a^2-8ab+5b^2\right)x+\frac{b(64a^2-54ab+15b^2)\cosh(c+dx)\sinh(c+dx)}{48d}+\frac{5(2a-b)b^2\cosh(c+dx)\,\mathrm{s}}{24d}$$

```
[Out] 1/16*(2*a-b)*(8*a^2-8*a*b+5*b^2)*x+1/48*b*(64*a^2-54*a*b+15*b^2)*cosh(d*x+c
)*sinh(d*x+c)/d+5/24*(2*a-b)*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d+1/6*b*cosh(d*x
+c)*sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3259, 3248}

$$\frac{b(64a^2-54ab+15b^2)\sinh(c+dx)\cosh(c+dx)}{48d}+\frac{1}{16}x(2a-b)\left(8a^2-8ab+5b^2\right)+\frac{5b^2(2a-b)\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{b\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((2*a - b)*(8*a^2 - 8*a*b + 5*b^2)*x)/16 + (b*(64*a^2 - 54*a*b + 15*b^2)*Co
sh[c + d*x]*Sinh[c + d*x])/(48*d) + (5*(2*a - b)*b^2*Cosh[c + d*x]*Sinh[c +
 d*x]^3)/(24*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)
/(6*d)
```

Rule 3248

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^3\,dx=\frac{b\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}+\frac{1}{6}\int\left(a+b\sinh^2(c+d\right.$$

$$=\frac{1}{16}(2a-b)\left(8a^2-8ab+5b^2\right)x+\frac{b(64a^2-54ab+15b^2)\cosh(c+dx)\sinh(}{48d}$$

Mathematica [A]
time = 0.17, size = 95, normalized size = 0.74

$$\frac{12(2a-b)\left(8a^2-8ab+5b^2\right)(c+dx)+9b\left(16a^2-16ab+5b^2\right)\sinh(2(c+dx))+9(2a-b)b^2\sinh(4(c+dx))+b^3\sinh(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (12*(2*a - b)*(8*a^2 - 8*a*b + 5*b^2)*(c + d*x) + 9*b*(16*a^2 - 16*a*b + 5*
b^2)*Sinh[2*(c + d*x)] + 9*(2*a - b)*b^2*Sinh[4*(c + d*x)] + b^3*Sinh[6*(c
+ d*x)])/(192*d)
```

Maple [A]
time = 0.91, size = 102, normalized size = 0.80

method	result
default	$a^3x+\frac{\left(-\frac{3}{16}b^3+\frac{3}{8}a\,b^2\right)\sinh(4dx+4c)}{4d}+\frac{\left(\frac{15}{32}b^3-\frac{3}{2}a\,b^2+\frac{3}{2}a^2b\right)\sinh(2dx+2c)}{2d}-\frac{5b^3x}{16}+\frac{9a\,b^2x}{8}-\frac{3a^2bx}{2}+\frac{b^3\sinh(6dx+6c)}{192d}$
risch	$a^3x-\frac{5b^3x}{16}+\frac{9a\,b^2x}{8}-\frac{3a^2bx}{2}+\frac{b^3\mathrm{e}^{6dx+6c}}{384d}-\frac{3\,\mathrm{e}^{4dx+4c}b^3}{128d}+\frac{3\,\mathrm{e}^{4dx+4c}a\,b^2}{64d}+\frac{3\,\mathrm{e}^{2dx+2c}a^2b}{8d}-\frac{3\,\mathrm{e}^{2dx+2c}a\,b^2}{8d}+\frac{15\,\mathrm{e}^{2dx}}{128}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+1/4*(-3/16*b^3+3/8*a*b^2)/d*sinh(4*d*x+4*c)+1/2*(15/32*b^3-3/2*a*b^2+
3/2*a^2*b)*sinh(2*d*x+2*c)/d-5/16*b^3*x+9/8*a*b^2*x-3/2*a^2*b*x+1/192*b^3/d
*sinh(6*d*x+6*c)
```

Maxima [A]
time = 0.28, size = 197, normalized size = 1.54

$$\frac{3}{64}\,ab^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{3}{8}\,a^2b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^3x-\frac{1}{384}\,b^3\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d
```

```
*x - 2*c)/d) + a^3*x - 1/384*b^3*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c)
 - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Fricas [A]
time = 0.41, size = 165, normalized size = 1.29

$$\frac{3\,b^{3}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+2\,\left(5\,b^{3}\cosh\left(dx+c\right)^{3}+9\,\left(2\,ab^{2}-b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+6\,\left(16\,a^{3}-24\,a^{2}b+18\,ab^{2}-5\,b^{3}\right)dx+3\,\left(b^{3}\cosh\left(dx+c\right)^{5}+6\,\left(2\,ab^{2}-b^{3}\right)\cosh\left(dx+c\right)^{3}+3\,\left(16\,a^{2}b-16\,ab^{2}+5\,b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^3*cosh(d*x + c)^3 + 9*(2
*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(16*a^3 - 24*a^2*b + 18*a*
b^2 - 5*b^3)*d*x + 3*(b^3*cosh(d*x + c)^5 + 6*(2*a*b^2 - b^3)*cosh(d*x + c)
^3 + 3*(16*a^2*b - 16*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(119) = 238.
time = 0.49, size = 350, normalized size = 2.73

$$\begin{cases} a^{3}x+\frac{3a^{2}bx\sinh^{2}\left(c+dx\right)}{2}-\frac{3a^{2}bx\cosh^{2}\left(c+dx\right)}{2}+\frac{3a^{2}b\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d}+\frac{9ab^{2}x\sinh^{4}\left(c+dx\right)}{8}-\frac{9ab^{2}x\sinh^{2}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{4}+\frac{9ab^{2}x\cosh^{4}\left(c+dx\right)}{8}+\frac{15ab^{2}\sinh^{3}\left(c+dx\right)\cosh\left(c+dx\right)}{8d}-\frac{9ab^{2}\sinh\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{8d}+\frac{5b^{3}x\sinh^{6}\left(c+dx\right)}{16}-\frac{15b^{3}x\sinh^{4}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{16}+\frac{15b^{3}x\sinh^{2}\left(c+dx\right)\cosh^{4}\left(c+dx\right)}{16}-\frac{5b^{3}x\cosh^{6}\left(c+dx\right)}{16}+\frac{11b^{3}\sinh^{5}\left(c+dx\right)\cosh\left(c+dx\right)}{16d}-\frac{5b^{3}\sinh^{3}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{6d}+\frac{5b^{3}\sinh\left(c+dx\right)\cosh^{5}\left(c+dx\right)}{16d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^{2}\left(c\right)\right)^{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x*sinh(c + d*x)**2/2 - 3*a**2*b*x*cosh(c + d*x
)**2/2 + 3*a**2*b*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 9*a*b**2*x*sinh(c + d
*x)**4/8 - 9*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a*b**2*x*cosh
(c + d*x)**4/8 + 15*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a*b**2*
sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*b**3*x*sinh(c + d*x)**6/16 - 15*b*
*3*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b**3*x*sinh(c + d*x)**2*cosh
(c + d*x)**4/16 - 5*b**3*x*cosh(c + d*x)**6/16 + 11*b**3*sinh(c + d*x)**5*c
osh(c + d*x)/(16*d) - 5*b**3*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*
*3*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*
*3, True))
```

Giac [A]
time = 0.42, size = 177, normalized size = 1.38

$$\frac{b^{3}e^{\left(6\,dx+6\,c\right)}}{384\,d}-\frac{b^{3}e^{\left(-6\,dx-6\,c\right)}}{384\,d}+\frac{1}{16}\,\left(16\,a^{3}-24\,a^{2}b+18\,ab^{2}-5\,b^{3}\right)x+\frac{3\,\left(2\,ab^{2}-b^{3}\right)e^{\left(4\,dx+4\,c\right)}}{128\,d}+\frac{3\,\left(16\,a^{2}b-16\,ab^{2}+5\,b^{3}\right)e^{\left(2\,dx+2\,c\right)}}{128\,d}-\frac{3\,\left(16\,a^{2}b-16\,ab^{2}+5\,b^{3}\right)e^{\left(-2\,dx-2\,c\right)}}{128\,d}-\frac{3\,\left(2\,ab^{2}-b^{3}\right)e^{\left(-4\,dx-4\,c\right)}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/384*b^3*e^(6*d*x + 6*c)/d - 1/384*b^3*e^(-6*d*x - 6*c)/d + 1/16*(16*a^3 -
 24*a^2*b + 18*a*b^2 - 5*b^3)*x + 3/128*(2*a*b^2 - b^3)*e^(4*d*x + 4*c)/d +
```

```
 3/128*(16*a^2*b - 16*a*b^2 + 5*b^3)*e^(2*d*x + 2*c)/d - 3/128*(16*a^2*b -
16*a*b^2 + 5*b^3)*e^(-2*d*x - 2*c)/d - 3/128*(2*a*b^2 - b^3)*e^(-4*d*x - 4*
c)/d
```

Mupad [B]

time = 0.75, size = 123, normalized size = 0.96

$$\frac{\frac{45\,b^3\sinh(2\,c+2\,d\,x)}{4}-\frac{9\,b^3\sinh(4\,c+4\,d\,x)}{4}+\frac{b^3\sinh(6\,c+6\,d\,x)}{4}-36\,a\,b^2\sinh(2\,c+2\,d\,x)+36\,a^2\,b\sinh(2\,c+2\,d\,x)+\frac{9\,a\,b^2\sinh(4\,c+4\,d\,x)}{2}+48\,a^3\,d\,x-15\,b^3\,d\,x+54\,a\,b^2\,d\,x-72\,a^2\,b\,d\,x}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] ((45*b^3*sinh(2*c + 2*d*x))/4 - (9*b^3*sinh(4*c + 4*d*x))/4 + (b^3*sinh(6*c
 + 6*d*x))/4 - 36*a*b^2*sinh(2*c + 2*d*x) + 36*a^2*b*sinh(2*c + 2*d*x) + (9
*a*b^2*sinh(4*c + 4*d*x))/2 + 48*a^3*d*x - 15*b^3*d*x + 54*a*b^2*d*x - 72*a
^2*b*d*x)/(48*d)
```

3.24 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=83

$$-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(3a^2-3ab+b^2)\cosh(c+dx)}{d}+\frac{(3a-2b)b^2\cosh^3(c+dx)}{3d}+\frac{b^3\cosh^5(c+dx)}{5d}$$

```
[Out] -a^3*arctanh(cosh(d*x+c))/d+b*(3*a^2-3*a*b+b^2)*cosh(d*x+c)/d+1/3*(3*a-2*b)
*b^2*cosh(d*x+c)^3/d+1/5*b^3*cosh(d*x+c)^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3265, 398, 212}

$$-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(3a^2-3ab+b^2)\cosh(c+dx)}{d}+\frac{b^2(3a-2b)\cosh^3(c+dx)}{3d}+\frac{b^3\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -((a^3*ArcTanh[Cosh[c + d*x]])/d) + (b*(3*a^2 - 3*a*b + b^2)*Cosh[c + d*x])
/d + ((3*a - 2*b)*b^2*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^3}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\Bigl(\int \Bigl(-b(3a^2-3ab+b^2)-(3a-2b)b^2x^2-b^3x^4+\frac{a^3}{1-x^2}}{d}$$

$$= \frac{b(3a^2-3ab+b^2)\cosh(c+dx)}{d} + \frac{(3a-2b)b^2\cosh^3(c+dx)}{3d} +$$

$$= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d} + \frac{b(3a^2-3ab+b^2)\cosh(c+dx)}{d} +$$

Mathematica [A]
time = 0.15, size = 83, normalized size = 1.00

$$\frac{30b(24a^2-18ab+5b^2)\cosh(c+dx)+5(12a-5b)b^2\cosh(3(c+dx))+3\left(b^3\cosh(5(c+dx))+80a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (30*b*(24*a^2 - 18*a*b + 5*b^2)*Cosh[c + d*x] + 5*(12*a - 5*b)*b^2*Cosh[3*(
c + d*x)] + 3*(b^3*Cosh[5*(c + d*x)] + 80*a^3*Log[Tanh[(c + d*x)/2]]))/(240
*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(79) = 158.
time = 1.11, size = 224, normalized size = 2.70

method	result
default	$\frac{b^3\left(\frac{\left(\cosh^5(dx+c)\right)}{5}+\frac{\left(\cosh^3(dx+c)\right)}{3}+\cosh(dx+c)-2\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+3a\,b^2\left(\frac{\left(\cosh^3(dx+c)\right)}{3}+\cosh(dx+c)-2\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)-3b^3}{}$
risch	$\frac{b^3\mathrm{e}^{5dx+5c}}{160d}+\frac{\mathrm{e}^{3dx+3c}a\,b^2}{8d}-\frac{5\,\mathrm{e}^{3dx+3c}b^3}{96d}+\frac{3\,\mathrm{e}^{dx+c}a^2b}{2d}-\frac{9\,\mathrm{e}^{dx+c}a\,b^2}{8d}+\frac{5\,\mathrm{e}^{dx+c}b^3}{16d}+\frac{3\,\mathrm{e}^{-dx-c}a^2b}{2d}-\frac{9\,\mathrm{e}^{-dx-c}a\,b^2}{8d}+5\,\mathrm{e}^{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/5*cosh(d*x+c)^5+1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x
+c)))+3*a*b^2*(1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x+c)))-3*b^3*(
1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x+c)))+3*a^2*b*(cosh(d*x+c)-2
*arctanh(exp(d*x+c)))-6*a*b^2*(cosh(d*x+c)-2*arctanh(exp(d*x+c)))+3*b^3*(co
sh(d*x+c)-2*arctanh(exp(d*x+c)))-2*a^3*arctanh(exp(d*x+c))+6*a^2*b*arctanh(
exp(d*x+c))-6*a*b^2*arctanh(exp(d*x+c))+2*b^3*arctanh(exp(d*x+c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(79) = 158.
time = 0.28, size = 193, normalized size = 2.33

$$\frac{1}{480}\,b^{3}\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{1}{8}\,ab^{2}\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3}{2}\,a^{2}b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{a^{3}\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/480*b^3*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/8*a
*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x -
3*c)/d) + 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a^3*log(tanh(1/2*d*x
 + 1/2*c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. 2(79) = 158.
time = 0.42, size = 1128, normalized size = 13.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^3*cosh(d*x + c)^10 + 30*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^
3*sinh(d*x + c)^10 + 5*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^8 + 5*(27*b^3*cosh(
d*x + c)^2 + 12*a*b^2 - 5*b^3)*sinh(d*x + c)^8 + 40*(9*b^3*cosh(d*x + c)^3
+ (12*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(24*a^2*b - 18*a*b
^2 + 5*b^3)*cosh(d*x + c)^6 + 10*(63*b^3*cosh(d*x + c)^4 + 72*a^2*b - 54*a*
b^2 + 15*b^3 + 14*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(
189*b^3*cosh(d*x + c)^5 + 70*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + 45*(24*a^
2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(24*a^2*b - 18*
a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 10*(63*b^3*cosh(d*x + c)^6 + 35*(12*a*b^2
- 5*b^3)*cosh(d*x + c)^4 + 72*a^2*b - 54*a*b^2 + 15*b^3 + 45*(24*a^2*b - 18
*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(9*b^3*cosh(d*x + c)^
7 + 7*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 15*(24*a^2*b - 18*a*b^2 + 5*b^3)
*cosh(d*x + c)^3 + 3*(24*a^2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*b^3 + 5*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2 + 5*(27*b^3*cosh(d*x
+ c)^8 + 28*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^6 + 90*(24*a^2*b - 18*a*b^2 +
5*b^3)*cosh(d*x + c)^4 + 12*a*b^2 - 5*b^3 + 36*(24*a^2*b - 18*a*b^2 + 5*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 480*(a^3*cosh(d*x + c)^5 + 5*a^3*cosh(
d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^3*
cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)*sinh(d*x + c)^4 + a^3
*sinh(d*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 480*(a^3*cosh(d*
x + c)^5 + 5*a^3*cosh(d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sin
```

```
h(d*x + c)^2 + 10*a^3*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)
*sinh(d*x + c)^4 + a^3*sinh(d*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) -
 1) + 10*(3*b^3*cosh(d*x + c)^9 + 4*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 18
*(24*a^2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 12*(24*a^2*b - 18*a*b^2 +
5*b^3)*cosh(d*x + c)^3 + (12*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(
d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*d*cosh(d*x + c)^
3*sinh(d*x + c)^2 + 10*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*d*cosh(d*x + c
)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(79) = 158.
time = 0.44, size = 202, normalized size = 2.43

$$\frac{3\,b^3e^{(5\,dx+5\,c)}+60\,ab^2e^{(3\,dx+3\,c)}-25\,b^3e^{(3\,dx+3\,c)}+720\,a^2be^{(dx+c)}-540\,ab^2e^{(dx+c)}+150\,b^3e^{(dx+c)}-480\,a^3\log\left(e^{(dx+c)}+1\right)+480\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right)+\left(720\,a^2be^{(4\,dx+4\,c)}-540\,ab^2e^{(4\,dx+4\,c)}+150\,b^3e^{(4\,dx+4\,c)}+60\,ab^2e^{(2\,dx+2\,c)}-25\,b^3e^{(2\,dx+2\,c)}+3\,b^3\right)e^{(-5\,dx-5\,c)}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/480*(3*b^3*e^(5*d*x + 5*c) + 60*a*b^2*e^(3*d*x + 3*c) - 25*b^3*e^(3*d*x +
 3*c) + 720*a^2*b*e^(d*x + c) - 540*a*b^2*e^(d*x + c) + 150*b^3*e^(d*x + c)
 - 480*a^3*log(e^(d*x + c) + 1) + 480*a^3*log(abs(e^(d*x + c) - 1)) + (720*
a^2*b*e^(4*d*x + 4*c) - 540*a*b^2*e^(4*d*x + 4*c) + 150*b^3*e^(4*d*x + 4*c)
 + 60*a*b^2*e^(2*d*x + 2*c) - 25*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-5*d*x - 5
*c))/d
```

Mupad [**B**]
time = 0.30, size = 184, normalized size = 2.22

$$\frac{\mathrm{e}^{c+d\,x}\,\left(24\,a^2\,b-18\,a\,b^2+5\,b^3\right)}{16\,d}-\frac{2\,\mathrm{atan}\left(\frac{a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}}+\frac{\mathrm{e}^{-c-d\,x}\,\left(24\,a^2\,b-18\,a\,b^2+5\,b^3\right)}{16\,d}+\frac{b^3\,\mathrm{e}^{-5\,c-5\,d\,x}}{160\,d}+\frac{b^3\,\mathrm{e}^{5\,c+5\,d\,x}}{160\,d}+\frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}\,\left(12\,a-5\,b\right)}{96\,d}+\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}\,\left(12\,a-5\,b\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/sinh(c + d*x),x)
```

```
[Out] (exp(c + d*x)*(24*a^2*b - 18*a*b^2 + 5*b^3))/(16*d) - (2*atan((a^3*exp(d*x)
*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/2) + (exp(- c
 - d*x)*(24*a^2*b - 18*a*b^2 + 5*b^3))/(16*d) + (b^3*exp(- 5*c - 5*d*x))/(1
60*d) + (b^3*exp(5*c + 5*d*x))/(160*d) + (b^2*exp(- 3*c - 3*d*x)*(12*a - 5*
b))/(96*d) + (b^2*exp(3*c + 3*d*x)*(12*a - 5*b))/(96*d)
```

3.25 $\int \mathbf{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=137

$$\frac{3}{8}b\left(8a^2-4ab+b^2\right)x-\frac{a(2a+b)(4a+b)\coth(c+dx)}{8d}+\frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b)\tanh^2(c+\right.}{4d}$$

```
[Out] 3/8*b*(8*a^2-4*a*b+b^2)*x-1/8*a*(2*a+b)*(4*a+b)*coth(d*x+c)/d+1/4*b*cosh(d*
x+c)^4*coth(d*x+c)*(a-(a-b)*tanh(d*x+c)^2)^2/d+1/8*b*cosh(d*x+c)^2*coth(d*x
+c)*(a*(4*a+b)-(4*a-3*b)*(a-b)*tanh(d*x+c)^2)/d
```

Rubi [A]

time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3266, 479, 591, 464, 212}

$$\frac{3}{8}bx\left(8a^2-4ab+b^2\right)-\frac{a(2a+b)(4a+b)\coth(c+dx)}{8d}+\frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{4d}+\frac{b\cosh^2(c+dx)\coth(c+dx)\left(a(4a+b)-(4a-3b)(a-b)\tanh^2(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*b*(8*a^2 - 4*a*b + b^2)*x)/8 - (a*(2*a + b)*(4*a + b)*Coth[c + d*x])/(8*
d) + (b*Cosh[c + d*x]^4*Coth[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(4*d
) + (b*Cosh[c + d*x]^2*Coth[c + d*x]*(a*(4*a + b) - (4*a - 3*b)*(a - b)*Tan
h[c + d*x]^2))/(8*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
```

```
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{x^2\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{4d} +\\
&= \frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{4d} +\\
&= -\frac{a(2a+b)(4a+b)\coth(c+dx)}{8d} + b\cosh^4(c+dx)\coth(c+\\
&= \frac{3}{8}b\left(8a^2-4ab+b^2\right)x - \frac{a(2a+b)(4a+b)\coth(c+dx)}{8d} +
\end{aligned}$$

Mathematica [A]

time = 1.24, size = 113, normalized size = 0.82

$$\frac{\left(b+a\operatorname{csch}^2(c+dx)\right)^3\sinh^6(c+dx)\left(12b(8a^2-4ab+b^2)(c+dx)-32a^3\coth(c+dx)+8(3a-b)b^2\sinh(2(c+dx))+b^3\sinh(4(c+dx))\right)}{4d(2a-b+b\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((b + a*Csch[c + d*x]^2)^3*Sinh[c + d*x]^6*(12*b*(8*a^2 - 4*a*b + b^2)*(c +
 d*x) - 32*a^3*Coth[c + d*x] + 8*(3*a - b)*b^2*Sinh[2*(c + d*x)] + b^3*Sinh
[4*(c + d*x)]))/(4*d*(2*a - b + b*Cosh[2*(c + d*x)])^3)
```

Maple [A]
time = 1.14, size = 147, normalized size = 1.07

method	result
risch	$3a^2bx-\frac{3a\,b^2x}{2}+\frac{3b^3x}{8}+\frac{\mathrm{e}^{4dx+4c}b^3}{64d}+\frac{3\,\mathrm{e}^{2dx+2c}a\,b^2}{8d}-\frac{\mathrm{e}^{2dx+2c}b^3}{8d}-\frac{3\,\mathrm{e}^{-2dx-2c}a\,b^2}{8d}+\frac{\mathrm{e}^{-2dx-2c}b^3}{8d}-\frac{\mathrm{e}^{-4dx-4c}b^3}{64d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a^2*b*x-3/2*a*b^2*x+3/8*b^3*x+1/64/d*exp(4*d*x+4*c)*b^3+3/8/d*exp(2*d*x+2
*c)*a*b^2-1/8/d*exp(2*d*x+2*c)*b^3-3/8/d*exp(-2*d*x-2*c)*a*b^2+1/8/d*exp(-2
*d*x-2*c)*b^3-1/64/d*exp(-4*d*x-4*c)*b^3-2*a^3/d/(exp(2*d*x+2*c)-1)
```

Maxima [A]
time = 0.28, size = 130, normalized size = 0.95

$$\frac{1}{64}\,b^3\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{3}{8}\,ab^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+3\,a^2bx+\frac{2\,a^3}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*b^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x
 - 2*c)/d) + 3*a^2*b*x + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [A]
time = 0.40, size = 169, normalized size = 1.23

$$\frac{b^3\cosh\left(dx+c\right)^5+5\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+3\left(8\,ab^2-3\,b^3\right)\cosh\left(dx+c\right)^3+\left(10\,b^3\cosh\left(dx+c\right)^3+9\left(8\,ab^2-3\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2-8\left(8\,a^3+3\,ab^2-b^3\right)\cosh\left(dx+c\right)+8\left(8\,a^3+3\left(8\,a^2b-4\,ab^2+b^3\right)dx\right)\sinh\left(dx+c\right)}{64\,d\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(b^3*cosh(d*x + c)^5 + 5*b^3*cosh(d*x + c)*sinh(d*x + c)^4 + 3*(8*a*b^
2 - 3*b^3)*cosh(d*x + c)^3 + (10*b^3*cosh(d*x + c)^3 + 9*(8*a*b^2 - 3*b^3)*
cosh(d*x + c))*sinh(d*x + c)^2 - 8*(8*a^3 + 3*a*b^2 - b^3)*cosh(d*x + c) +
8*(8*a^3 + 3*(8*a^2*b - 4*a*b^2 + b^3)*d*x)*sinh(d*x + c))/(d*sinh(d*x + c)
)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**A**]
time = 0.46, size = 177, normalized size = 1.29

$$\frac{b^{3}e^{(4\,dx+4\,c)} + 24\,ab^{2}e^{(2\,dx+2\,c)} - 8\,b^{3}e^{(2\,dx+2\,c)} + 24\,(8\,a^{2}b - 4\,ab^{2} + b^{3})(dx + c) - \frac{128\,a^{3}}{e^{(2\,dx+2\,c)}-1} - \left(144\,a^{2}be^{(4\,dx+4\,c)} - 72\,ab^{2}e^{(4\,dx+4\,c)} + 18\,b^{3}e^{(4\,dx+4\,c)} + 24\,ab^{2}e^{(2\,dx+2\,c)} - 8\,b^{3}e^{(2\,dx+2\,c)} + b^{3}\right)e^{(-4\,dx-4\,c)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(b^3*e^(4*d*x + 4*c) + 24*a*b^2*e^(2*d*x + 2*c) - 8*b^3*e^(2*d*x + 2*c
) + 24*(8*a^2*b - 4*a*b^2 + b^3)*(d*x + c) - 128*a^3/(e^(2*d*x + 2*c) - 1)
- (144*a^2*b*e^(4*d*x + 4*c) - 72*a*b^2*e^(4*d*x + 4*c) + 18*b^3*e^(4*d*x +
 4*c) + 24*a*b^2*e^(2*d*x + 2*c) - 8*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x -
 4*c))/d
```

Mupad [**B**]
time = 0.76, size = 121, normalized size = 0.88

$$\frac{3\,b\,x\,\left(8\,a^2-4\,a\,b+b^2\right)}{8}-\frac{2\,a^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{b^3\,{\mathrm{e}}^{-4\,c-4\,d\,x}}{64\,d}+\frac{b^3\,{\mathrm{e}}^{4\,c+4\,d\,x}}{64\,d}-\frac{b^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}\,\left(3\,a-b\right)}{8\,d}+\frac{b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a-b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/sinh(c + d*x)^2,x)
```

```
[Out] (3*b*x*(8*a^2 - 4*a*b + b^2))/8 - (2*a^3)/(d*(exp(2*c + 2*d*x) - 1)) - (b^3
*exp(- 4*c - 4*d*x))/(64*d) + (b^3*exp(4*c + 4*d*x))/(64*d) - (b^2*exp(- 2*
c - 2*d*x)*(3*a - b))/(8*d) + (b^2*exp(2*c + 2*d*x)*(3*a - b))/(8*d)
```

3.26 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=83

$$\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{(3a-b)b^2\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}-\frac{a^3\coth(c+dx)\text{csch}(c+dx)}{2d}$$

```
[Out] 1/2*a^2*(a-6*b)*arctanh(cosh(d*x+c))/d+(3*a-b)*b^2*cosh(d*x+c)/d+1/3*b^3*co
sh(d*x+c)^3/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3265, 398, 393, 212}

$$-\frac{a^3\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b^2(3a-b)\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^2*(a - 6*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + ((3*a - b)*b^2*Cosh[c + d*x]
)/d + (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

<u>Rubi steps</u>

$$\begin{aligned}
\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^3}{(1-x^2)^2}\,dx,x,\cosh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left((3a-b)b^2+b^3x^2+\frac{a^2(a-3b)+3a^2bx^2}{(1-x^2)^2}\right)dx,x,\cosh(c+\right.}{d}\\
&= \frac{(3a-b)b^2\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}+\frac{\operatorname{Subst}\left(\int \frac{a^2(a-3}{(1-}\right.}{}\\
&= \frac{(3a-b)b^2\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}-\frac{a^3\coth(c+dx)}{2d}\\
&= \frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{(3a-b)b^2\cosh(c+dx)}{d}+
\end{aligned}$$

<u>Mathematica</u> [**B**] Leaf count is larger than twice the leaf count of optimal. 210 vs. 2(83) = 166.
time = 3.04, size = 210, normalized size = 2.53

$$-\frac{\left(-18(4a-b)b^2\cosh(c)\cosh(dx)-2b^3\cosh(3c)\cosh(3dx)+3a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)-12a^3\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+72a^2b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+12a^3\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)-72a^2b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+3a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)-72ab^2\sinh(c)\sinh(dx)+18b^3\sinh(c)\sinh(dx)-2b^3\sinh(3c)\sinh(3dx)\right)\left(a+b\sinh^2(c+dx)\right)^3}{3d(2a-b+b\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -1/3*((-18*(4*a - b)*b^2*Cosh[c]*Cosh[d*x] - 2*b^3*Cosh[3*c]*Cosh[3*d*x] +
3*a^3*Csch[(c + d*x)/2]^2 - 12*a^3*Log[Cosh[(c + d*x)/2]] + 72*a^2*b*Log[Co
sh[(c + d*x)/2]] + 12*a^3*Log[Sinh[(c + d*x)/2]] - 72*a^2*b*Log[Sinh[(c + d
*x)/2]] + 3*a^3*Sech[(c + d*x)/2]^2 - 72*a*b^2*Sinh[c]*Sinh[d*x] + 18*b^3*S
inh[c]*Sinh[d*x] - 2*b^3*Sinh[3*c]*Sinh[3*d*x])*(a + b*Sinh[c + d*x]^2)^3)/
(d*(2*a - b + b*Cosh[2*(c + d*x)])^3)
```

<u>Maple</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(77) = 154.
time = 1.30, size = 208, normalized size = 2.51

method	result

risch	$\frac{\mathrm{e}^{3dx+3c}b^{3}}{24d}+\frac{3\,\mathrm{e}^{dx+c}a\,b^{2}}{2d}-\frac{3\,\mathrm{e}^{dx+c}b^{3}}{8d}+\frac{3\,\mathrm{e}^{-dx-c}a\,b^{2}}{2d}-\frac{3\,\mathrm{e}^{-dx-c}b^{3}}{8d}+\frac{\mathrm{e}^{-3dx-3c}b^{3}}{24d}-\frac{a^{3}\mathrm{e}^{dx+c}\left(1+\mathrm{e}^{2dx+2c}\right)}{d\left(\mathrm{e}^{2dx+2c}-1\right)^{2}}-\frac{a^{3}\ln\left(\mathrm{e}^{dx+c}-\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*b^3+3/2/d*exp(d*x+c)*a*b^2-3/8/d*exp(d*x+c)*b^3+3/2/d
*exp(-d*x-c)*a*b^2-3/8/d*exp(-d*x-c)*b^3+1/24/d*exp(-3*d*x-3*c)*b^3-a^3*exp
(d*x+c)*(1+exp(2*d*x+2*c))/d/(exp(2*d*x+2*c)-1)^2-1/2*a^3/d*ln(exp(d*x+c)-1
)+3*a^2/d*ln(exp(d*x+c)-1)*b+1/2*a^3/d*ln(exp(d*x+c)+1)-3*a^2/d*ln(exp(d*x+
c)+1)*b
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(77) = 154$.
time = 0.28, size = 217, normalized size = 2.61

$$\frac{1}{24}\,b^{3}\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3}{2}\,ab^{2}\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}\,a^{3}\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)-3\,a^{2}b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) + 3/2*a*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^3*(log(e^(
-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x -
3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 3*a^2*b*(log(e^(-d
*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1814 vs. $2(77) = 154$.
time = 0.46, size = 1814, normalized size = 21.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*sin
h(d*x + c)^10 + (36*a*b^2 - 11*b^3)*cosh(d*x + c)^8 + (45*b^3*cosh(d*x + c)
^2 + 36*a*b^2 - 11*b^3)*sinh(d*x + c)^8 + 8*(15*b^3*cosh(d*x + c)^3 + (36*a
*b^2 - 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(12*a^3 + 18*a*b^2 - 5*b^
3)*cosh(d*x + c)^6 + 2*(105*b^3*cosh(d*x + c)^4 - 12*a^3 - 18*a*b^2 + 5*b^3
 + 14*(36*a*b^2 - 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*b^3*cosh
(d*x + c)^5 + 14*(36*a*b^2 - 11*b^3)*cosh(d*x + c)^3 - 3*(12*a^3 + 18*a*b^2
 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(12*a^3 + 18*a*b^2 - 5*b^3)*co
```

```
sh(d*x + c)^4 + 2*(105*b^3*cosh(d*x + c)^6 + 35*(36*a*b^2 - 11*b^3)*cosh(d*
x + c)^4 - 12*a^3 - 18*a*b^2 + 5*b^3 - 15*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*b^3*cosh(d*x + c)^7 + 7*(36*a*b^2 - 11*
b^3)*cosh(d*x + c)^5 - 5*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^3 - (12*
a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + (36*a*b^2 -
11*b^3)*cosh(d*x + c)^2 + (45*b^3*cosh(d*x + c)^8 + 28*(36*a*b^2 - 11*b^3)*
cosh(d*x + c)^6 - 30*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^4 + 36*a*b^2
 - 11*b^3 - 12*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 12*((a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 7*(a^3 - 6*a^2*b)*cosh(d*x + c)*si
nh(d*x + c)^6 + (a^3 - 6*a^2*b)*sinh(d*x + c)^7 - 2*(a^3 - 6*a^2*b)*cosh(d*
x + c)^5 - (2*a^3 - 12*a^2*b - 21*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 + 5*(7*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 2*(a^3 - 6*a^2*b)*cosh(d*x
 + c))*sinh(d*x + c)^4 + (a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (35*(a^3 - 6*a^2
*b)*cosh(d*x + c)^4 + a^3 - 6*a^2*b - 20*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + (21*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 - 20*(a^3 - 6*a^2*b)*c
osh(d*x + c)^3 + 3*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(a^3
 - 6*a^2*b)*cosh(d*x + c)^6 - 10*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + 3*(a^3 -
 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
 + 1) - 12*((a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 7*(a^3 - 6*a^2*b)*cosh(d*x +
c)*sinh(d*x + c)^6 + (a^3 - 6*a^2*b)*sinh(d*x + c)^7 - 2*(a^3 - 6*a^2*b)*co
sh(d*x + c)^5 - (2*a^3 - 12*a^2*b - 21*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sin
h(d*x + c)^5 + 5*(7*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 2*(a^3 - 6*a^2*b)*cos
h(d*x + c))*sinh(d*x + c)^4 + (a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (35*(a^3 -
6*a^2*b)*cosh(d*x + c)^4 + a^3 - 6*a^2*b - 20*(a^3 - 6*a^2*b)*cosh(d*x + c)
^2)*sinh(d*x + c)^3 + (21*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 - 20*(a^3 - 6*a^2
*b)*cosh(d*x + c)^3 + 3*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7
*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 - 10*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + 3*(
a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
 + c) - 1) + 2*(5*b^3*cosh(d*x + c)^9 + 4*(36*a*b^2 - 11*b^3)*cosh(d*x + c)
^7 - 6*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^5 - 4*(12*a^3 + 18*a*b^2 -
 5*b^3)*cosh(d*x + c)^3 + (36*a*b^2 - 11*b^3)*cosh(d*x + c))*sinh(d*x + c))
/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7
 - 2*d*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^5 + 5*(
7*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^
3 + (35*d*cosh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21
*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c
)^2 + (7*d*cosh(d*x + c)^6 - 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*si
nh(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(77) = 154$.
time = 0.45, size = 174, normalized size = 2.10

$$\frac{b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+36\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-12\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{24\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}+6\left(a^3-6\,a^2b\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-6\left(a^3-6\,a^2b\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(b^3*(e^(d*x + c) + e^(-d*x - c))^3 + 36*a*b^2*(e^(d*x + c) + e^(-d*x
- c)) - 12*b^3*(e^(d*x + c) + e^(-d*x - c)) - 24*a^3*(e^(d*x + c) + e^(-d*x
 - c))/((e^(d*x + c) + e^(-d*x - c))^2 - 4) + 6*(a^3 - 6*a^2*b)*log(e^(d*x
+ c) + e^(-d*x - c) + 2) - 6*(a^3 - 6*a^2*b)*log(e^(d*x + c) + e^(-d*x - c)
 - 2))/d
```

Mupad [**B**]
time = 0.22, size = 229, normalized size = 2.76

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\left(a^3\,\sqrt{-d^2}-6\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6-12\,a^5\,b+36\,a^4\,b^2}}\right)\,\sqrt{a^6-12\,a^5\,b+36\,a^4\,b^2}}{\sqrt{-d^2}}+\frac{b^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}+\frac{b^3\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}+\frac{3\,b^2\,\mathrm{e}^{c+d\,x}\,\left(4\,a-b\right)}{8\,d}+\frac{3\,b^2\,\mathrm{e}^{-c-d\,x}\,\left(4\,a-b\right)}{8\,d}-\frac{a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) - 6*a^2*b*(-d^2)^(1/2)))/(d*(a^6 -
 12*a^5*b + 36*a^4*b^2)^(1/2)))*(a^6 - 12*a^5*b + 36*a^4*b^2)^(1/2))/(-d^2)
^(1/2) + (b^3*exp(- 3*c - 3*d*x))/(24*d) + (b^3*exp(3*c + 3*d*x))/(24*d) +
(3*b^2*exp(c + d*x)*(4*a - b))/(8*d) + (3*b^2*exp(- c - d*x)*(4*a - b))/(8*
d) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(c + d*x))/(
d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.27 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=113

$$\frac{1}{2}(6a-b)b^2x+\frac{a(2a^2-5ab-2b^2)\coth(c+dx)}{2d}-\frac{a^2(2a+3b)\coth^3(c+dx)}{6d}+\frac{b\cosh^2(c+dx)\coth^3(c+dx)\,(}{2d}$$

```
[Out] 1/2*(6*a-b)*b^2*x+1/2*a*(2*a^2-5*a*b-2*b^2)*coth(d*x+c)/d-1/6*a^2*(2*a+3*b)
*coth(d*x+c)^3/d+1/2*b*cosh(d*x+c)^2*coth(d*x+c)^3*(a-(a-b)*tanh(d*x+c)^2)^
2/d
```

Rubi [A]
time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3266, 479, 584, 213}

$$\frac{a(2a^2-5ab-2b^2)\coth(c+dx)}{2d}-\frac{a^2(2a+3b)\coth^3(c+dx)}{6d}+\frac{1}{2}b^2x(6a-b)+\frac{b\cosh^2(c+dx)\coth^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((6*a - b)*b^2*x)/2 + (a*(2*a^2 - 5*a*b - 2*b^2)*Coth[c + d*x])/(2*d) - (a^
2*(2*a + 3*b)*Coth[c + d*x]^3)/(6*d) + (b*Cosh[c + d*x]^2*Coth[c + d*x]^3*(
a - (a - b)*Tanh[c + d*x]^2)^2)/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
```

```
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{x^4\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{b\cosh^2(c+dx)\coth^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{2d} +\\
&= \frac{b\cosh^2(c+dx)\coth^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{2d} +\\
&= \frac{a(2a^2-5ab-2b^2)\coth(c+dx)}{2d} - \frac{a^2(2a+3b)\coth^3(c+dx)}{6d}\\
&= \frac{1}{2}(6a-b)b^2x + \frac{a(2a^2-5ab-2b^2)\coth(c+dx)}{2d} - a^2(2a+3b)
\end{aligned}
$$

Mathematica [A]
time = 1.62, size = 107, normalized size = 0.95

$$\frac{2\left(b+a\operatorname{csch}^2(c+dx)\right)^3\sinh^6(c+dx)\left(-4a^2\coth(c+dx)\left(-2a+9b+a\operatorname{csch}^2(c+dx)\right)+3b^2(2(6a-b)(c+dx)+b\sinh(2(c+dx)))\right)}{3d(2a-b+b\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (2*(b + a*Csch[c + d*x]^2)^3*Sinh[c + d*x]^6*(-4*a^2*Coth[c + d*x]*(-2*a +
9*b + a*Csch[c + d*x]^2) + 3*b^2*(2*(6*a - b)*(c + d*x) + b*Sinh[2*(c + d*x
)])))/(3*d*(2*a - b + b*Cosh[2*(c + d*x)])^3)
```

Maple [A]
time = 1.36, size = 113, normalized size = 1.00

method	result	size
risch	$3a\,b^{2}x-\frac{b^{3}x}{2}+\frac{{\mathrm e}^{2dx+2c}b^{3}}{8d}-\frac{{\mathrm e}^{-2dx-2c}b^{3}}{8d}-\frac{2a^{2}\left(9b\,{\mathrm e}^{4dx+4c}+6a\,{\mathrm e}^{2dx+2c}-18b\,{\mathrm e}^{2dx+2c}-2a+9b\right)}{3d\left({\mathrm e}^{2dx+2c}-1\right)^{3}}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a*b^2*x-1/2*b^3*x+1/8/d*exp(2*d*x+2*c)*b^3-1/8/d*exp(-2*d*x-2*c)*b^3-2/3*
a^2*(9*b*exp(4*d*x+4*c)+6*a*exp(2*d*x+2*c)-18*b*exp(2*d*x+2*c)-2*a+9*b)/d/(
exp(2*d*x+2*c)-1)^3
```

Maxima [**A**]
time = 0.28, size = 161, normalized size = 1.42

$$-\frac{1}{8}\,b^{3}\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+3\,ab^{2}x+\frac{4}{3}\,a^{3}\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)+\frac{6\,a^{2}b}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 3*a*b^2*x + 4/3*a
^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) - 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(106) = 212$.
time = 0.39, size = 281, normalized size = 2.49

$$\frac{3\,b^{3}\cosh\left(dx+c\right)^{5}+15\,b^{3}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+\left(16\,a^{3}-72\,a^{2}b-9\,b^{3}\right)\cosh\left(dx+c\right)^{3}-4\left(4\,a^{3}-18\,a^{2}b-3\left(6\,ab^{2}-b^{3}\right)dx\right)\sinh\left(dx+c\right)^{3}+3\left(10\,b^{3}\cosh\left(dx+c\right)^{3}+\left(16\,a^{3}-72\,a^{2}b-9\,b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}-6\left(8\,a^{3}-12\,a^{2}b-b^{3}\right)\cosh\left(dx+c\right)+12\left(4\,a^{3}-18\,a^{2}b-3\left(6\,ab^{2}-b^{3}\right)dx-\left(4\,a^{3}-18\,a^{2}b-3\left(6\,ab^{2}-b^{3}\right)dx\right)\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)}{24\left(d\sinh\left(dx+c\right)^{3}+3\left(d\cosh\left(dx+c\right)^{2}-d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^3*cosh(d*x + c)^5 + 15*b^3*cosh(d*x + c)*sinh(d*x + c)^4 + (16*a^
3 - 72*a^2*b - 9*b^3)*cosh(d*x + c)^3 - 4*(4*a^3 - 18*a^2*b - 3*(6*a*b^2 -
b^3)*d*x)*sinh(d*x + c)^3 + 3*(10*b^3*cosh(d*x + c)^3 + (16*a^3 - 72*a^2*b
- 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 6*(8*a^3 - 12*a^2*b - b^3)*cosh(d
*x + c) + 12*(4*a^3 - 18*a^2*b - 3*(6*a*b^2 - b^3)*d*x - (4*a^3 - 18*a^2*b
- 3*(6*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^3
 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 154, normalized size = 1.36

$$\frac{3\,b^3e^{(2\,dx+2\,c)}+12\,(6\,ab^2-b^3)(dx+c)-3\left(12\,ab^2e^{(2\,dx+2\,c)}-2\,b^3e^{(2\,dx+2\,c)}+b^3\right)e^{(-2\,dx-2\,c)}-\frac{16\,(9\,a^2be^{(4\,dx+4\,c)}+6\,a^3e^{(2\,dx+2\,c)}-18\,a^2be^{(2\,dx+2\,c)}-2\,a^3+9\,a^2b)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^3*e^(2*d*x + 2*c) + 12*(6*a*b^2 - b^3)*(d*x + c) - 3*(12*a*b^2*e^
(2*d*x + 2*c) - 2*b^3*e^(2*d*x + 2*c) + b^3)*e^(-2*d*x - 2*c) - 16*(9*a^2*b
*e^(4*d*x + 4*c) + 6*a^3*e^(2*d*x + 2*c) - 18*a^2*b*e^(2*d*x + 2*c) - 2*a^3
 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B]
time = 0.14, size = 222, normalized size = 1.96

$$\frac{\frac{2\,(3\,a^2\,b-2\,a^3)}{3\,d}-\frac{2\,a^2\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,a^2\,b}{d}-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(3\,a^2\,b-2\,a^3)}{3\,d}+\frac{2\,a^2\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}+\frac{b^2\,x\,(6\,a-b)}{2}-\frac{b^3\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}+\frac{b^3\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}-\frac{2\,a^2\,b}{d\,(\mathrm{e}^{2\,c+2\,d\,x}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/sinh(c + d*x)^4,x)
```

```
[Out] ((2*(3*a^2*b - 2*a^3))/(3*d) - (2*a^2*b*exp(2*c + 2*d*x))/d)/(exp(4*c + 4*d
*x) - 2*exp(2*c + 2*d*x) + 1) - ((2*a^2*b)/d - (4*exp(2*c + 2*d*x)*(3*a^2*b
 - 2*a^3))/(3*d) + (2*a^2*b*exp(4*c + 4*d*x))/d)/(3*exp(2*c + 2*d*x) - 3*ex
p(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + (b^2*x*(6*a - b))/2 - (b^3*exp(- 2
*c - 2*d*x))/(8*d) + (b^3*exp(2*c + 2*d*x))/(8*d) - (2*a^2*b)/(d*(exp(2*c +
 2*d*x) - 1))
```

3.28 $\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{a^3\text{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,b^{7/2}d}+\frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d}-\frac{(a+2b)\cosh^3(c+dx)}{3b^2d}+\frac{\cosh^5(c+dx)}{5bd}$$

```
[Out] (a^2+a*b+b^2)*cosh(d*x+c)/b^3/d-1/3*(a+2*b)*cosh(d*x+c)^3/b^2/d+1/5*cosh(d*
x+c)^5/b/d-a^3*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/b^(7/2)/d/(a-b)^(1/2
)
```

Rubi [A]
time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 398, 211}

$$-\frac{a^3\text{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{7/2}d\sqrt{a-b}}+\frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d}-\frac{(a+2b)\cosh^3(c+dx)}{3b^2d}+\frac{\cosh^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((a^3*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(7/2)*d)
) + ((a^2 + a*b + b^2)*Cosh[c + d*x])/(b^3*d) - ((a + 2*b)*Cosh[c + d*x]^3)
/(3*b^2*d) + Cosh[c + d*x]^5/(5*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{a^2+ab+b^2}{b^3}+\frac{(a+2b)x^2}{b^2}-\frac{x^4}{b}+\frac{a^3}{b^3(a-b+bx^2)}\right)\,dx, x, \cosh(c+dx)\right)}{d}\\
&= \frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d}-\frac{(a+2b)\cosh^3(c+dx)}{3b^2d}+\frac{\cosh^5(c+dx)}{5bd}-a^3\text{Subs}\\
&= -\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,b^{7/2}d}+\frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d}-\frac{(a+2b)\cosh^3(c+}{3b^2d}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.61, size = 165, normalized size = 1.51

$$
\frac{-\frac{240a^3\left(\text{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\text{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}}+30\sqrt{b}\,(8a^2+6ab+5b^2)\cosh(c+dx)-5b^{3/2}(4a+5b)\cosh(3(c+dx))+3b^{5/2}\cosh(5(c+dx))}{240b^{7/2}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((-240*a^3*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + A
rcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b] +
30*Sqrt[b]*(8*a^2 + 6*a*b + 5*b^2)*Cosh[c + d*x] - 5*b^(3/2)*(4*a + 5*b)*Co
sh[3*(c + d*x)] + 3*b^(5/2)*Cosh[5*(c + d*x)])/(240*b^(7/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(97) = 194$.
time = 1.18, size = 296, normalized size = 2.72

method	result
derivativedivides	$-\frac{a^3\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{b^3\sqrt{ab-b^2}}+\frac{1}{5b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{-4a-3b}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-$
default	$-\frac{a^3\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{b^3\sqrt{ab-b^2}}+\frac{1}{5b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{-4a-3b}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-$

risch	$\frac{e^{5dx+5c}}{160bd}-\frac{5\,e^{3dx+3c}}{96bd}-\frac{e^{3dx+3c}a}{24b^2d}+\frac{e^{dx+c}a^2}{2b^3d}+\frac{3a\,e^{dx+c}}{8b^2d}+\frac{5\,e^{dx+c}}{16bd}+\frac{e^{-dx-c}a^2}{2b^3d}+\frac{3\,e^{-dx-c}a}{8b^2d}+\frac{5\,e^{-dx-c}}{16bd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3/b^3/(a*b-b^2)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2-2*a+4*b
)/(a*b-b^2)^(1/2))+1/5/b/(tanh(1/2*d*x+1/2*c)+1)^5-1/2/b/(tanh(1/2*d*x+1/2*
c)+1)^4-1/8*(-4*a-3*b)/b^2/(tanh(1/2*d*x+1/2*c)+1)^2-1/12*(4*a-b)/b^2/(tanh
(1/2*d*x+1/2*c)+1)^3-1/8/b^3*(-8*a^2-4*a*b-3*b^2)/(tanh(1/2*d*x+1/2*c)+1)-1
/5/b/(tanh(1/2*d*x+1/2*c)-1)^5-1/2/b/(tanh(1/2*d*x+1/2*c)-1)^4-1/8*(-4*a-3*
b)/b^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/12*(-4*a+b)/b^2/(tanh(1/2*d*x+1/2*c)-1)^
3-1/8*(8*a^2+4*a*b+3*b^2)/b^3/(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(3*b^2*e^(10*d*x + 10*c) + 3*b^2 - 5*(4*a*b*e^(8*c) + 5*b^2*e^(8*c))*
e^(8*d*x) + 30*(8*a^2*e^(6*c) + 6*a*b*e^(6*c) + 5*b^2*e^(6*c))*e^(6*d*x) +
30*(8*a^2*e^(4*c) + 6*a*b*e^(4*c) + 5*b^2*e^(4*c))*e^(4*d*x) - 5*(4*a*b*e^(
2*c) + 5*b^2*e^(2*c))*e^(2*d*x))*e^(-5*d*x - 5*c)/(b^3*d) - 1/128*integrate
(256*(a^3*e^(3*d*x + 3*c) - a^3*e^(d*x + c))/(b^4*e^(4*d*x + 4*c) + b^4 + 2
*(2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1588 vs. $2(97) = 194$.
time = 0.53, size = 3242, normalized size = 29.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/480*(3*(a*b^3 - b^4)*cosh(d*x + c)^10 + 30*(a*b^3 - b^4)*cosh(d*x + c)*s
inh(d*x + c)^9 + 3*(a*b^3 - b^4)*sinh(d*x + c)^10 - 5*(4*a^2*b^2 + a*b^3 -
5*b^4)*cosh(d*x + c)^8 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4 - 27*(a*b^3 - b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^8 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^3 - (4*a
^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(8*a^3*b - 2*a^
2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)
```

```
^4 + 24*a^3*b - 6*a^2*b^2 - 3*a*b^3 - 15*b^4 - 14*(4*a^2*b^2 + a*b^3 - 5*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(189*(a*b^3 - b^4)*cosh(d*x + c)^5
- 70*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^3 + 45*(8*a^3*b - 2*a^2*b^2
- a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(8*a^3*b - 2*a^2*b^2 -
 a*b^3 - 5*b^4)*cosh(d*x + c)^4 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)^6 - 35
*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^4 + 24*a^3*b - 6*a^2*b^2 - 3*a*b
^3 - 15*b^4 + 45*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + 3*a*b^3 - 3*b^4 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^7 - 7*(4
*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^5 + 15*(8*a^3*b - 2*a^2*b^2 - a*b^3
 - 5*b^4)*cosh(d*x + c)^3 + 3*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*
x + c))*sinh(d*x + c)^3 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^2 + 5
*(27*(a*b^3 - b^4)*cosh(d*x + c)^8 - 28*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*
x + c)^6 + 90*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^4 - 4*a^2
*b^2 - a*b^3 + 5*b^4 + 36*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 - 240*(a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^4*si
nh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^3*cosh(d*x + c)
^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)*sinh(d*x + c)^4 + a^3*sinh(d*x + c
)^5)*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
 c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x
 + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*c
osh(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x
 + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*
x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d
*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d
*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + b)) + 10*(3*(a*b^3 - b^4)*cosh(d*x + c)^9 - 4*(
4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^7 + 18*(8*a^3*b - 2*a^2*b^2 - a*b^
3 - 5*b^4)*cosh(d*x + c)^5 + 12*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(
d*x + c)^3 - (4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a*
b^4 - b^5)*d*cosh(d*x + c)^5 + 5*(a*b^4 - b^5)*d*cosh(d*x + c)^4*sinh(d*x +
 c) + 10*(a*b^4 - b^5)*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*(a*b^4 - b^5)
*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*(a*b^4 - b^5)*d*cosh(d*x + c)*sinh(d
*x + c)^4 + (a*b^4 - b^5)*d*sinh(d*x + c)^5), 1/480*(3*(a*b^3 - b^4)*cosh(d
*x + c)^10 + 30*(a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + 3*(a*b^3 - b^
4)*sinh(d*x + c)^10 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^8 - 5*(4*
a^2*b^2 + a*b^3 - 5*b^4 - 27*(a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8
 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^3 - (4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d
*x + c))*sinh(d*x + c)^7 + 30*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*
x + c)^6 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)^4 + 24*a^3*b - 6*a^2*b^2 - 3*
a*b^3 - 15*b^4 - 14*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x +
 c)^6 + 4*(189*(a*b^3 - b^4)*cosh(d*x + c)^5 - 70*(4*a^2*b^2 + a*b^3 - 5*b^
4)*cosh(d*x + c)^3 + 45*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 30*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^
4 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)^6 - 35*(4*a^2*b^2 + a*b^3 - 5*b^4)*c
osh(d*x + c)^4 + 24*a^3*b - 6*a^2*b^2 - 3*a*b^3 - 15*b^4 + 45*(8*a^3*b - 2*
```

```
a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a*b^3 - 3*b^4
 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^7 - 7*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh
(d*x + c)^5 + 15*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^3 + 3*
(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*(4
*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^2 + 5*(27*(a*b^3 - b^4)*cosh(d*x +
c)^8 - 28*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 90*(8*a^3*b - 2*a^2
*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^4 - 4*a^2*b^2 - a*b^3 + 5*b^4 + 36*(8*a
^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 480*(a
^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x
+ c)^3*sinh(d*x + c)^2 + 10*a^3*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cos
h(d*x + c)*sinh(d*x + c)^4 + a^3*sinh(d*x + c)^5)*sqrt(a*b - b^2)*arctan(-1
/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)
^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x
 + c))/sqrt(a*b - b^2)) + 480*(a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^4*
sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^3*cosh(d*x +
c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)*sinh...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 1.64, size = 415, normalized size = 3.81

$$\frac{e^{-5c-5dx}}{160\,b\,d}-\frac{\sqrt{a^6}\left(2\,\mathrm{atan}\left(\frac{a^3\,e^{d\,x}\,e^c\,\sqrt{b^7\,d^2\,\left(a-b\right)}}{2\,b^3\,d\,\left(a-b\right)\,\sqrt{a^6}}\right)+2\,\mathrm{atan}\left(\frac{\left(e^{d\,x}\,e^c\left(\frac{2\,a^7}{b^{11}\,d\,{\left(a-b\right)}^2\,\sqrt{a^6}}-\frac{4\left(2\,a^4\,b^4\,d\,\sqrt{a^6}-2\,a^5\,b^3\,d\,\sqrt{a^6}\right)}{a^3\,b^8\,\left(a-b\right)\,\sqrt{a\,b^7\,d^2-b^8\,d^2}\,\sqrt{b^7\,d^2\,\left(a-b\right)}}\right)+\frac{2\,a^7\,e^{3\,c}\,e^{3\,d\,x}}{b^{11}\,d\,{\left(a-b\right)}^2\,\sqrt{a^6}}\right)\left(b^9\,\sqrt{a\,b^7\,d^2-b^8\,d^2}-a\,b^8\,\sqrt{a\,b^7\,d^2-b^8\,d^2}\right)}{4\,a^4}\right)\right)}{2\,\sqrt{a\,b^7\,d^2-b^8\,d^2}}+\frac{e^{5\,c+5\,d\,x}}{160\,b\,d}+\frac{e^{c+d\,x}\,\left(8\,a^2+6\,a\,b+5\,b^2\right)}{16\,b^3\,d}+\frac{e^{-c-d\,x}\,\left(8\,a^2+6\,a\,b+5\,b^2\right)}{16\,b^3\,d}-\frac{e^{-3\,c-3\,d\,x}\,\left(4\,a+5\,b\right)}{96\,b^2\,d}-\frac{e^{3\,c+3\,d\,x}\,\left(4\,a+5\,b\right)}{96\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^7/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] exp(- 5*c - 5*d*x)/(160*b*d) - ((a^6)^(1/2)*(2*atan((a^3*exp(d*x)*exp(c)*(b
^7*d^2*(a - b))^(1/2))/(2*b^3*d*(a - b)*(a^6)^(1/2))) + 2*atan(((exp(d*x)*e
xp(c)*((2*a^7)/(b^11*d*(a - b)^2*(a^6)^(1/2)) - (4*(2*a^4*b^4*d*(a^6)^(1/2)
 - 2*a^5*b^3*d*(a^6)^(1/2)))/(a^3*b^8*(a - b)*(a*b^7*d^2 - b^8*d^2)^(1/2)*(
b^7*d^2*(a - b))^(1/2))) + (2*a^7*exp(3*c)*exp(3*d*x))/(b^11*d*(a - b)^2*(a
^6)^(1/2)))*(b^9*(a*b^7*d^2 - b^8*d^2)^(1/2) - a*b^8*(a*b^7*d^2 - b^8*d^2)^
(1/2)))/(4*a^4))))/(2*(a*b^7*d^2 - b^8*d^2)^(1/2)) + exp(5*c + 5*d*x)/(160*
b*d) + (exp(c + d*x)*(6*a*b + 8*a^2 + 5*b^2))/(16*b^3*d) + (exp(- c - d*x)*
(6*a*b + 8*a^2 + 5*b^2))/(16*b^3*d) - (exp(- 3*c - 3*d*x)*(4*a + 5*b))/(96*
b^2*d) - (exp(3*c + 3*d*x)*(4*a + 5*b))/(96*b^2*d)
```

3.29 $\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{(8a^2+4ab+3b^2)\,x}{8b^3}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}\,b^3d}-\frac{(4a+3b)\cosh(c+dx)\sinh(c+dx)}{8b^2d}+\frac{\cosh(c+dx)}{4}$$

```
[Out] 1/8*(8*a^2+4*a*b+3*b^2)*x/b^3-1/8*(4*a+3*b)*cosh(d*x+c)*sinh(d*x+c)/b^2/d+1
/4*cosh(d*x+c)*sinh(d*x+c)^3/b/d-a^(5/2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^
(1/2))/b^3/d/(a-b)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3266, 481, 592, 536, 212, 214}

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{b^3d\sqrt{a-b}}+\frac{x(8a^2+4ab+3b^2)}{8b^3}-\frac{(4a+3b)\sinh(c+dx)\cosh(c+dx)}{8b^2d}+\frac{\sinh^3(c+dx)\cosh(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((8*a^2 + 4*a*b + 3*b^2)*x)/(8*b^3) - (a^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c
+ d*x])/Sqrt[a]])/(Sqrt[a - b]*b^3*d) - ((4*a + 3*b)*Cosh[c + d*x]*Sinh[c +
 d*x])/(8*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\Big(\int \frac{x^6}{(1-x^2)^3(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\Big)}{d}$$

$$= \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} - \frac{\text{Subst}\Big(\int \frac{x^2(3a+(a+3b)x^2)}{(1-x^2)^2(a+(-a+b)x^2)}\,dx, x, \tanh(c+dx)\Big)}{4bd}$$

$$= -\frac{(4a+3b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} - \frac{\text{Subst}\Big(}{}$$

$$= -\frac{(4a+3b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} - \frac{a^3\text{Subst}}{}$$

$$= \frac{(8a^2+4ab+3b^2)\,x}{8b^3} - \frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}\,b^3d} - \frac{(4a+3b)\cosh(c+}{8b^2d}$$

Mathematica [A]
time = 0.32, size = 97, normalized size = 0.80

$$\frac{4(8a^2+4ab+3b^2)(c+dx) - \frac{32a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} - 8b(a+b)\sinh(2(c+dx)) + b^2\sinh(4(c+dx))}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (4*(8*a^2 + 4*a*b + 3*b^2)*(c + d*x) - (32*a^(5/2)*ArcTanh[(Sqrt[a - b]*Tan
h[c + d*x])/Sqrt[a]])/Sqrt[a - b] - 8*b*(a + b)*Sinh[2*(c + d*x)] + b^2*Sin
h[4*(c + d*x)])/(32*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(107) = 214$.
time = 1.26, size = 417, normalized size = 3.45

method	result
risch	$\frac{x\,a^2}{b^3} + \frac{ax}{2b^2} + \frac{3x}{8b} + \frac{e^{4dx+4c}}{64bd} - \frac{a\,e^{2dx+2c}}{8b^2d} - \frac{e^{2dx+2c}}{8bd} + \frac{a\,e^{-2dx-2c}}{8b^2d} + \frac{e^{-2dx-2c}}{8bd} - \frac{e^{-4dx-4c}}{64bd} + \frac{\sqrt{a\,(a-}}{}$

derivativedivides	$\frac{1}{4b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{4a+b}{8b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{4a+3b}{8b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-8a^{2}-4ab-3b^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}\right.\right.}{8b^{3}}$
default	$\frac{1}{4b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{4a+b}{8b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{4a+3b}{8b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-8a^{2}-4ab-3b^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}\right.\right.}{8b^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/b/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/8*(4
*a+b)/b^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/8*(4*a+3*b)/b^2/(tanh(1/2*d*x+1/2*c)-
1)+1/8/b^3*(-8*a^2-4*a*b-3*b^2)*ln(tanh(1/2*d*x+1/2*c)-1)-1/4/b/(tanh(1/2*d
*x+1/2*c)+1)^4+1/2/b/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(-4*a-b)/b^2/(tanh(1/2*d
*x+1/2*c)+1)^2-1/8*(4*a+3*b)/b^2/(tanh(1/2*d*x+1/2*c)+1)+1/8*(8*a^2+4*a*b+3
*b^2)/b^3*ln(tanh(1/2*d*x+1/2*c)+1)+2*a^4/b^3*(1/2*((-b*(a-b))^(1/2)+b)/a/(
-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b
*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1
/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))))
```

<u>Maxima</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(107) = 214$.
time = 0.49, size = 1725, normalized size = 14.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh
(d*x + c)^8 + 8*(8*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b + b^2)
*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b - 2*b^2)*sinh(d*x + c)^
6 + 8*(7*b^2*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(35*b^2*cosh(d*x + c)^4 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*x - 60*(a*b + b^
2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8*a^2 +
 4*a*b + 3*b^2)*d*x*cosh(d*x + c) - 20*(a*b + b^2)*cosh(d*x + c)^3)*sinh(d*
x + c)^3 + 8*(a*b + b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 12*(8
*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b + b^2)*cosh(d*x + c)^4
+ 2*a*b + 2*b^2)*sinh(d*x + c)^2 + 32*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x
 + c)^3*sinh(d*x + c) + 6*a^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^2*cosh(
d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4)*sqrt(a/(a - b))*log((b^2*co
sh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 +
 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*
sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b
^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b
- b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 -
3*a*b + b^2)*sqrt(a/(a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d
*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d
*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + b)) - b^2 + 8*(b^2*cosh(d*x + c)^7 + 4*(8*a^2 +
 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c)^5 + 2*(a*
b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cos
h(d*x + c)^3*sinh(d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^
3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4), 1/64*(b^2*cosh(
d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 8*
(8*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b + b^2)*cosh(d*x + c)^6
 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b - 2*b^2)*sinh(d*x + c)^6 + 8*(7*b^2*cos
h(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cos
h(d*x + c)^4 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*x - 60*(a*b + b^2)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8*a^2 + 4*a*b + 3*b^2)*
d*x*cosh(d*x + c) - 20*(a*b + b^2)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a*
b + b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 12*(8*a^2 + 4*a*b + 3
```

```
*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b + b^2)*cosh(d*x + c)^4 + 2*a*b + 2*b^2)
*sinh(d*x + c)^2 - 64*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)^3*sinh(d*x
 + c) + 6*a^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^2*cosh(d*x + c)*sinh(d*
x + c)^3 + a^2*sinh(d*x + c)^4)*sqrt(-a/(a - b))*arctan(1/2*(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-
a/(a - b))/a) - b^2 + 8*(b^2*cosh(d*x + c)^7 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*
x*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c)^5 + 2*(a*b + b^2)*cosh(d*x
+ c))*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^3*d*cosh(d*x + c)*
sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.68, size = 208, normalized size = 1.72

$$-\frac{\frac{64\,a^{3}\arctan\left(\frac{b e^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{\sqrt{-a^{2}+ab}\,b^{3}}-\frac{8\,(8\,a^{2}+4\,ab+3\,b^{2})(dx+c)}{b^{3}}-\frac{b e^{(4\,dx+4\,c)}-8\,a e^{(2\,dx+2\,c)}-8\,b e^{(2\,dx+2\,c)}}{b^{2}}+\frac{(48\,a^{2}e^{(4\,dx+4\,c)}+24\,ab e^{(4\,dx+4\,c)}+18\,b^{2}e^{(4\,dx+4\,c)}-8\,ab e^{(2\,dx+2\,c)}-8\,b^{2}e^{(2\,dx+2\,c)}+b^{2})e^{(-4\,dx-4\,c)}}{b^{3}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/64*(64*a^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(s
qrt(-a^2 + a*b)*b^3) - 8*(8*a^2 + 4*a*b + 3*b^2)*(d*x + c)/b^3 - (b*e^(4*d*
x + 4*c) - 8*a*e^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c))/b^2 + (48*a^2*e^(4*d*
x + 4*c) + 24*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) - 8*a*b*e^(2*d*x
 + 2*c) - 8*b^2*e^(2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c)/b^3)/d
```

Mupad [**B**]
time = 1.07, size = 266, normalized size = 2.20

$$\frac{x\,(8\,a^{2}+4\,a\,b+3\,b^{2})}{8\,b^{3}}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}}{64\,b\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}}{64\,b\,d}+\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a+b)}{8\,b^{2}\,d}-\frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)}{8\,b^{2}\,d}+\frac{a^{5/2}\,\ln\left(\frac{4\,a^{3}\,\mathrm{e}^{2\,c+2\,d\,x}}{b^{4}}-\frac{2\,a^{5/2}\,(b\,d+2\,a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x})}{b^{4}\,d\,\sqrt{a-b}}\right)}{2\,b^{3}\,d\,\sqrt{a-b}}-\frac{a^{5/2}\,\ln\left(\frac{4\,a^{3}\,\mathrm{e}^{2\,c+2\,d\,x}}{b^{4}}+\frac{2\,a^{5/2}\,(b\,d+2\,a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x})}{b^{4}\,d\,\sqrt{a-b}}\right)}{2\,b^{3}\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^6/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (x*(4*a*b + 8*a^2 + 3*b^2))/(8*b^3) - exp(- 4*c - 4*d*x)/(64*b*d) + exp(4*c
 + 4*d*x)/(64*b*d) + (exp(- 2*c - 2*d*x)*(a + b))/(8*b^2*d) - (exp(2*c + 2*
```

```
d*x)*(a + b))/(8*b^2*d) + (a^(5/2)*log((4*a^3*exp(2*c + 2*d*x))/b^4 - (2*a^
(5/2)*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(b^4*d*(a - b)
^(1/2))))/(2*b^3*d*(a - b)^(1/2)) - (a^(5/2)*log((4*a^3*exp(2*c + 2*d*x))/b
^4 + (2*a^(5/2)*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(b^4
*d*(a - b)^(1/2))))/(2*b^3*d*(a - b)^(1/2))
```

3.30 $\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=79

$$\frac{a^2\operatorname{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,b^{5/2}d}-\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}$$

```
[Out] -(a+b)*cosh(d*x+c)/b^2/d+1/3*cosh(d*x+c)^3/b/d+a^2*arctan(cosh(d*x+c)*b^(1/
2)/(a-b)^(1/2))/b^(5/2)/d/(a-b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 398, 211}

$$\frac{a^2\operatorname{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{5/2}d\sqrt{a-b}}-\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a^2*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(5/2)*d) -
 ((a + b)*Cosh[c + d*x])/(b^2*d) + Cosh[c + d*x]^3/(3*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a+b}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a-b+bx^2)}\right)\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}+\frac{a^2\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+d\right.}{b^2d}$$

$$= \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,b^{5/2}d}-\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 134, normalized size = 1.70

$$\frac{\frac{12a^2\left(\text{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\text{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}}-3\sqrt{b}\,(4a+3b)\cosh(c+dx)+b^{3/2}\cosh(3(c+dx))}{12b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((12*a^2*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + Arc
Tan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b] - 3*
Sqrt[b]*(4*a + 3*b)*Cosh[c + d*x] + b^(3/2)*Cosh[3*(c + d*x)])/(12*b^(5/2)*
d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(69) = 138.
time = 1.09, size = 179, normalized size = 2.27

method	result
derivativedivides	$\frac{\frac{a^2\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{b^2\sqrt{ab-b^2}}+\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2a+b}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-}{d}$
default	$\frac{\frac{a^2\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{b^2\sqrt{ab-b^2}}+\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2a+b}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-}{d}$

risch	$\frac{e^{3dx+3c}}{24bd}-\frac{a\,e^{dx+c}}{2b^2d}-\frac{3\,e^{dx+c}}{8bd}-\frac{e^{-dx-c}a}{2b^2d}-\frac{3\,e^{-dx-c}}{8bd}+\frac{e^{-3dx-3c}}{24bd}-\frac{a^2\ln\left(e^{2dx+2c}-\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^2}}+1\right)}{2\sqrt{-ab+b^2}\,db^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2/b^2/(a*b-b^2)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2-2*a+4*b)
/(a*b-b^2)^(1/2))+1/3/b/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/b/(tanh(1/2*d*x+1/2*c
)+1)^2-1/2*(2*a+b)/b^2/(tanh(1/2*d*x+1/2*c)+1)-1/3/b/(tanh(1/2*d*x+1/2*c)-1
)^3-1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/b^2*(-b-2*a)/(tanh(1/2*d*x+1/2*c)-1
))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(4*a*e^(4*c) + 3*b*e^(4*c))*e^(4*d*x) + 3*(4*a*e^(2*c) + 3*b*e^(2*
c))*e^(2*d*x) - b*e^(6*d*x + 6*c) - b)*e^(-3*d*x - 3*c)/(b^2*d) + 1/32*inte
grate(64*(a^2*e^(3*d*x + 3*c) - a^2*e^(d*x + c))/(b^3*e^(4*d*x + 4*c) + b^3
 + 2*(2*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(69) = 138$.
time = 0.46, size = 1668, normalized size = 21.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(a*b^2 - b^3)*cosh(d*x + c)*sinh(d
*x + c)^5 + (a*b^2 - b^3)*sinh(d*x + c)^6 - 3*(4*a^2*b - a*b^2 - 3*b^3)*cos
h(d*x + c)^4 - 3*(4*a^2*b - a*b^2 - 3*b^3 - 5*(a*b^2 - b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + 4*(5*(a*b^2 - b^3)*cosh(d*x + c)^3 - 3*(4*a^2*b - a*b^2
 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 - b^3 - 3*(4*a^2*b - a*b^2
 - 3*b^3)*cosh(d*x + c)^2 + 3*(5*(a*b^2 - b^3)*cosh(d*x + c)^4 - 4*a^2*b +
a*b^2 + 3*b^3 - 6*(4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 - 12*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^2*c
osh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(d*x + c)^3)*sqrt(-a*b + b^2)*log((b
*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 -
```

```
2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*
x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)
- 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 +
(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b
)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^
4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*
x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) +
b)) + 6*((a*b^2 - b^3)*cosh(d*x + c)^5 - 2*(4*a^2*b - a*b^2 - 3*b^3)*cosh(d
*x + c)^3 - (4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3
 - b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 - b^4)*d*cosh(d*x + c)^2*sinh(d*x + c)
 + 3*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a*b^3 - b^4)*d*sinh(d
*x + c)^3), 1/24*((a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(a*b^2 - b^3)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a*b^2 - b^3)*sinh(d*x + c)^6 - 3*(4*a^2*b - a*b^2 -
 3*b^3)*cosh(d*x + c)^4 - 3*(4*a^2*b - a*b^2 - 3*b^3 - 5*(a*b^2 - b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a*b^2 - b^3)*cosh(d*x + c)^3 - 3*(4*a^
2*b - a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 - b^3 - 3*(4*a^
2*b - a*b^2 - 3*b^3)*cosh(d*x + c)^2 + 3*(5*(a*b^2 - b^3)*cosh(d*x + c)^4 -
 4*a^2*b + a*b^2 + 3*b^3 - 6*(4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c)^2*sinh(d*x + c
) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(d*x + c)^3)*sqrt(a*b - b
^2)*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*
sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a -
3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) - 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh
(d*x + c)^2*sinh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(
d*x + c)^3)*sqrt(a*b - b^2)*arctan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + si
nh(d*x + c))/(a - b)) + 6*((a*b^2 - b^3)*cosh(d*x + c)^5 - 2*(4*a^2*b - a*b
^2 - 3*b^3)*cosh(d*x + c)^3 - (4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c))*sinh
(d*x + c))/((a*b^3 - b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 - b^4)*d*cosh(d*x +
c)^2*sinh(d*x + c) + 3*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a*b
^3 - b^4)*d*sinh(d*x + c)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.38, size = 348, normalized size = 4.41

$$\frac{\left(2\,\mathrm{atan}\left(\frac{a^2\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\sqrt{b^5\,d^2\,\left(a-b\right)}}{2\,b^2\,d\,\left(a-b\right)\,\sqrt{a^4}}\right)+2\,\mathrm{atan}\left(\left({\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\left(\frac{2\,a^2}{b^8\,d\,{\left(a-b\right)}^2\,\sqrt{a^4}}-\frac{4\,\left(2\,a^3\,b^3\,d\,\sqrt{a^4}-2\,a^4\,b^2\,d\,\sqrt{a^4}\right)}{a^5\,b^6\,\left(a-b\right)\,\sqrt{a\,b^5\,d^2-b^6\,d^2}\,\sqrt{b^5\,d^2\,\left(a-b\right)}}\right)+\frac{2\,a^2\,{\mathrm{e}}^{3\,c}\,{\mathrm{e}}^{3\,d\,x}}{b^8\,d\,{\left(a-b\right)}^2\,\sqrt{a^4}}\right)\,\left(\frac{b^7\,\sqrt{a\,b^5\,d^2-b^6\,d^2}}{4}-\frac{a\,b^6\,\sqrt{a\,b^5\,d^2-b^6\,d^2}}{4}\right)\right)\right)\,\sqrt{a^4}}{2\,\sqrt{a\,b^5\,d^2-b^6\,d^2}}+\frac{{\mathrm{e}}^{-3\,c-3\,d\,x}}{24\,b\,d}+\frac{{\mathrm{e}}^{3\,c+3\,d\,x}}{24\,b\,d}-\frac{{\mathrm{e}}^{c+d\,x}\,\left(4\,a+3\,b\right)}{8\,b^2\,d}-\frac{{\mathrm{e}}^{-c-d\,x}\,\left(4\,a+3\,b\right)}{8\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^5/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] ((2*atan((a^2*exp(d*x)*exp(c)*(b^5*d^2*(a - b))^(1/2))/(2*b^2*d*(a - b)*(a^
4)^(1/2))) + 2*atan((exp(d*x)*exp(c)*((2*a^2)/(b^8*d*(a - b)^2*(a^4)^(1/2))
 - (4*(2*a^3*b^3*d*(a^4)^(1/2) - 2*a^4*b^2*d*(a^4)^(1/2)))/(a^5*b^6*(a - b)
*(a*b^5*d^2 - b^6*d^2)^(1/2)*(b^5*d^2*(a - b))^(1/2))) + (2*a^2*exp(3*c)*ex
p(3*d*x))/(b^8*d*(a - b)^2*(a^4)^(1/2)))*((b^7*(a*b^5*d^2 - b^6*d^2)^(1/2))
/4 - (a*b^6*(a*b^5*d^2 - b^6*d^2)^(1/2))/4)))*(a^4)^(1/2))/(2*(a*b^5*d^2 -
b^6*d^2)^(1/2)) + exp(- 3*c - 3*d*x)/(24*b*d) + exp(3*c + 3*d*x)/(24*b*d) -
 (exp(c + d*x)*(4*a + 3*b))/(8*b^2*d) - (exp(- c - d*x)*(4*a + 3*b))/(8*b^2
*d)
```

3.31 $\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=79

$$-\frac{(2a+b)x}{2b^2}+\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}\,b^2d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

```
[Out] -1/2*(2*a+b)*x/b^2+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d+a^(3/2)*arctanh((a-b)^(1
/2)*tanh(d*x+c)/a^(1/2))/b^2/d/(a-b)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3266, 481, 536, 212, 214}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{b^2d\sqrt{a-b}}-\frac{x(2a+b)}{2b^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/2*((2*a + b)*x)/b^2 + (a^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[
a]])/(Sqrt[a - b]*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
```

```
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1-x^2)(a+(-a+b)x^2)}\,dx, x, \tanh(c+dx)\right)}{2bd}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \tanh(c+dx)\right)}{b^2d} - (2a +\\
&= -\frac{(2a+b)x}{2b^2} + \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}\,b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 71, normalized size = 0.90

$$
\frac{-2(2a+b)(c+dx) + \frac{4a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} + b\sinh(2(c+dx))}{4b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-2*(2*a + b)*(c + d*x) + (4*a^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sq
rt[a]])/Sqrt[a - b] + b*Sinh[2*(c + d*x)])/(4*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(67) = 134$.
time = 1.04, size = 301, normalized size = 3.81

method	result
risch	$-\frac{x}{2b}-\frac{ax}{b^{2}}+\frac{e^{2dx+2c}}{8bd}-\frac{e^{-2dx-2c}}{8bd}+\frac{\sqrt{a(a-b)}\,a\ln\left(e^{2dx+2c}-\frac{2\sqrt{a(a-b)}-2a+b}{b}\right)}{2(a-b)d\,b^{2}}-\frac{\sqrt{a(a-$
derivativedivides	$\frac{-\frac{2a^{3}\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}}{2a\sqrt{-b(a-b)}\sqrt{}}\right)}{b^{2}}}{}$
default	$\frac{-\frac{2a^{3}\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}}{2a\sqrt{-b(a-b)}\sqrt{}}\right)}{b^{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^3/b^2*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(
1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2)))-1/2/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/b/(tanh(1/2*d*x+1/2*c)+1
)+1/2/b^2*(-b-2*a)*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/b/(tanh(1/2*d*x+1/2*c)-1)^
2+1/2/b/(tanh(1/2*d*x+1/2*c)-1)+1/2*(2*a+b)/b^2*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(67) = 134$.
time = 0.46, size = 859, normalized size = 10.87

$$\left[-\frac{4\,(2\,a+b)d x\cosh\left(dx+c\right)^{2}-b\cosh\left(dx+c\right)^{4}-4\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}-b\sinh\left(dx+c\right)^{4}+2\,\left(2\,(2\,a+b)dx-3\,b\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)^{2}-4\,\left(a\cosh\left(dx+c\right)^{2}+2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^{2}\right)\sqrt{\frac{a}{a-b}}\log\left(\frac{b^{2}\cosh\left(dx+c\right)^{4}+4\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+b^{2}\sinh\left(dx+c\right)^{4}+2\,\left(2\,ab-b^{2}\right)\cosh\left(dx+c\right)^{2}+2\,\left(3\,b^{2}\cosh\left(dx+c\right)^{2}+2\,ab-b^{2}\right)\sinh\left(dx+c\right)^{2}+8\,a^{2}-8\,ab+b^{2}+4\,\left(b^{2}\cosh\left(dx+c\right)^{3}+\left(2\,ab-b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)-4\,\left(\left(ab-b^{2}\right)\cosh\left(dx+c\right)^{2}+2\,\left(ab-b^{2}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(ab-b^{2}\right)\sinh\left(dx+c\right)^{2}+2\,a^{2}-3\,ab+b^{2}\right)\sqrt{\frac{a}{a-b}}}{b\cosh\left(dx+c\right)^{4}+4\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+b\sinh\left(dx+c\right)^{4}+2\,\left(2\,a-b\right)\cosh\left(dx+c\right)^{2}+2\,\left(3\,b\cosh\left(dx+c\right)^{2}+2\,a-b\right)\sinh\left(dx+c\right)^{2}+4\,\left(b\cosh\left(dx+c\right)^{3}+\left(2\,a-b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+b}\right)+4\,\left(2\,(2\,a+b)dx\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)^{3}\right)\sinh\left(dx+c\right)+b}{8\,\left(b^{2}d\cosh\left(dx+c\right)^{2}+2\,b^{2}d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^{2}d\sinh\left(dx+c\right)^{2}\right)},\ -\frac{4\,(2\,a+b)dx\cosh\left(dx+c\right)^{2}-b\cosh\left(dx+c\right)^{4}-4\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}-b\sinh\left(dx+c\right)^{4}+2\,\left(2\,(2\,a+b)dx-3\,b\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)^{2}-8\,\left(a\cosh\left(dx+c\right)^{2}+2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^{2}\right)\sqrt{-\frac{a}{a-b}}\arctan\left(\frac{\left(b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}+2\,a-b\right)\sqrt{-\frac{a}{a-b}}}{2\,a}\right)+4\,\left(2\,(2\,a+b)dx\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)^{3}\right)\sinh\left(dx+c\right)+b}{8\,\left(b^{2}d\cosh\left(dx+c\right)^{2}+2\,b^{2}d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^{2}d\sinh\left(dx+c\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a + b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a + b)*d*x - 3*b*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*
x + c) + a*sinh(d*x + c)^2)*sqrt(a/(a - b))*log((b^2*cosh(d*x + c)^4 + 4*b^
2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cos
h(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*
a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c) - 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)
*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/
(a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d
*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x
 + c) + b)) + 4*(2*(2*a + b)*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*
x + c) + b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) +
b^2*d*sinh(d*x + c)^2), -1/8*(4*(2*a + b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x
+ c)^4 - 4*b*cosh(d*x + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a
+ b)*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 8*(a*cosh(d*x + c)^2 + 2*
a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*sqrt(-a/(a - b))*arctan(
1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^
2 + 2*a - b)*sqrt(-a/(a - b))/a) + 4*(2*(2*a + b)*d*x*cosh(d*x + c) - b*cos
h(d*x + c)^3)*sinh(d*x + c) + b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x
+ c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.68, size = 126, normalized size = 1.59

$$\frac{\frac{8\,a^2 \arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b^2} - \frac{4\,(dx+c)(2\,a+b)}{b^2} + \frac{e^{(2\,dx+2\,c)}}{b} + \frac{\left(4\,ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}-b\right)e^{(-2\,dx-2\,c)}}{b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt
(-a^2 + a*b)*b^2) - 4*(d*x + c)*(2*a + b)/b^2 + e^(2*d*x + 2*c)/b + (4*a*e^
(2*d*x + 2*c) + 2*b*e^(2*d*x + 2*c) - b)*e^(-2*d*x - 2*c)/b^2)/d
```

Mupad [**B**]
time = 0.94, size = 216, normalized size = 2.73

$$\frac{\mathrm{e}^{2\,c+2\,d\,x}}{8\,b\,d}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b\,d}-\frac{x\,\left(2\,a+b\right)}{2\,b^2}+\frac{a^{3/2}\,\ln\left(-\frac{4\,a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{b^3}-\frac{2\,a^{3/2}\,\left(b\,d+2\,a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{b^3\,d\,\sqrt{a-b}}\right)}{2\,b^2\,d\,\sqrt{a-b}}-\frac{a^{3/2}\,\ln\left(\frac{2\,a^{3/2}\,\left(b\,d+2\,a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{b^3\,d\,\sqrt{a-b}}-\frac{4\,a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{b^3}\right)}{2\,b^2\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] exp(2*c + 2*d*x)/(8*b*d) - exp(- 2*c - 2*d*x)/(8*b*d) - (x*(2*a + b))/(2*b^
2) + (a^(3/2)*log(- (4*a^2*exp(2*c + 2*d*x))/b^3 - (2*a^(3/2)*(b*d + 2*a*d*
exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(b^3*d*(a - b)^(1/2))))/(2*b^2*d*
(a - b)^(1/2)) - (a^(3/2)*log((2*a^(3/2)*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*
d*exp(2*c + 2*d*x)))/(b^3*d*(a - b)^(1/2)) - (4*a^2*exp(2*c + 2*d*x))/b^3))
/(2*b^2*d*(a - b)^(1/2))
```

3.32 $\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=56

$$-\frac{a\text{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,b^{3/2}d}+\frac{\cosh(c+dx)}{bd}$$

```
[Out] cosh(d*x+c)/b/d-a*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/b^(3/2)/d/(a-b)^(
1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 396, 211}

$$\frac{\cosh(c+dx)}{bd}-\frac{a\text{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{3/2}d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((a*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(3/2)*d))
+ Cosh[c + d*x]/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)}{bd} - \frac{a\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{bd} \\
&= -\frac{a\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,b^{3/2}d} + \frac{\cosh(c+dx)}{bd}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 107, normalized size = 1.91

$$
\frac{-\frac{a\left(\text{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\text{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}}+\sqrt{b}\cosh(c+dx)}{b^{3/2}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-((a*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan
[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b]) + Sqrt
[b]*Cosh[c + d*x])/(b^(3/2)*d)
```

Maple [A]
time = 0.95, size = 93, normalized size = 1.66

method	result	size
derivativedivides	$\frac{\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{a\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{b\sqrt{ab-b^2}}}{d}$	93
default	$\frac{\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{a\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{b\sqrt{ab-b^2}}}{d}$	93
risch	$\frac{e^{dx+c}}{2bd}+\frac{e^{-dx-c}}{2bd}-\frac{a\ln\left(e^{2dx+2c}+\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^2}}+1\right)}{2\sqrt{-ab+b^2}\,db}+\frac{a\ln\left(e^{2dx+2c}-\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^2}}+1\right)}{2\sqrt{-ab+b^2}\,db}$	141

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b/(tanh(1/2*d*x+1/2*c)+1)-1/b/(tanh(1/2*d*x+1/2*c)-1)-a/b/(a*b-b^2)^
(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2-2*a+4*b)/(a*b-b^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/8*integrate(16*(a*e^(3*d*x
 + 3*c) - a*e^(d*x + c))/(b^2*e^(4*d*x + 4*c) + b^2 + 2*(2*a*b*e^(2*c) - b^
2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(48) = 96.
time = 0.45, size = 746, normalized size = 13.32

$$\left[\frac{(ab-b^2)\cosh(dx+c)^2+2(ab-b^2)\cosh(dx+c)\sinh(dx+c)+(ab-b^2)\sinh(dx+c)^2-\sqrt{-ab+b^2}\,(a\cosh(dx+c)+a\sinh(dx+c))\log\left(\frac{b\cosh(dx+c)^4+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2(2a-3b)\cosh(dx+c)^2+2\left(3b\cosh(dx+c)^2-2a+3b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3-(2a-3b)\cosh(dx+c)\right)\sinh(dx+c)+4\left(\cosh(dx+c)^3+3\cosh(dx+c)\sinh(dx+c)^2+\sinh(dx+c)^3+\left(3\cosh(dx+c)^2+1\right)\sinh(dx+c)+\cosh(dx+c)\right)\sqrt{-ab+b^2}+b}{b\cosh(dx+c)^4+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2(2a-b)\cosh(dx+c)^2+2\left(3b\cosh(dx+c)^2+2a-b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3+(2a-b)\cosh(dx+c)\right)\sinh(dx+c)+b}\right)+ab-b^2}{2\left((ab^2-b^3)d\cosh(dx+c)+(ab^2-b^3)d\sinh(dx+c)\right)}, \frac{(ab-b^2)\cosh(dx+c)^2+2(ab-b^2)\cosh(dx+c)\sinh(dx+c)+(ab-b^2)\sinh(dx+c)^2-2\sqrt{ab-b^2}\,(a\cosh(dx+c)+a\sinh(dx+c))\arctan\left(-\frac{b\cosh(dx+c)^3+3b\cosh(dx+c)\sinh(dx+c)^2+b\sinh(dx+c)^3+(4a-3b)\cosh(dx+c)+\left(3b\cosh(dx+c)^2+4a-3b\right)\sinh(dx+c)}{2\sqrt{ab-b^2}}\right)+2\sqrt{ab-b^2}\,(a\cosh(dx+c)+a\sinh(dx+c))\arctan\left(-\frac{\sqrt{ab-b^2}\,(\cosh(dx+c)+\sinh(dx+c))}{2(a-b)}\right)+ab-b^2}{2\left((ab^2-b^3)d\cosh(dx+c)+(ab^2-b^3)d\sinh(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x +
c) + (a*b - b^2)*sinh(d*x + c)^2 - sqrt(-a*b + b^2)*(a*cosh(d*x + c) + a*si
nh(d*x + c))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 -
 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 +
 sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*s
qrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3
 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c
))*sinh(d*x + c) + b)) + a*b - b^2)/((a*b^2 - b^3)*d*cosh(d*x + c) + (a*b^2
 - b^3)*d*sinh(d*x + c)), 1/2*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*
cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 - 2*sqrt(a*b - b^
2)*(a*cosh(d*x + c) + a*sinh(d*x + c))*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b
*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x +
 c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) + 2
*sqrt(a*b - b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c))*arctan(-1/2*sqrt(a*b -
 b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + a*b - b^2)/((a*b^2 - b^3)*
d*cosh(d*x + c) + (a*b^2 - b^3)*d*sinh(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 1.17, size = 293, normalized size = 5.23

$$\frac{\mathrm{e}^{c+d\,x}}{2\,b\,d}-\frac{\left(2\,\mathrm{atan}\left(\frac{a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{b^3\,d^2\,\left(a-b\right)}}{2\,b\,d\,\left(a-b\right)\,{\left(a^2\right)}^{3/2}}\right)+2\,\mathrm{atan}\left(\left(\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\left(\frac{2\,a^3}{b^5\,d\,{\left(a-b\right)}^2\,{\left(a^2\right)}^{3/2}}-\frac{4\,\left(2\,b^2\,d\,{\left(a^2\right)}^{3/2}-2\,a\,b\,d\,{\left(a^2\right)}^{3/2}\right)}{a^3\,b^4\,\left(a-b\right)\,\sqrt{a\,b^3\,d^2-b^4\,d^2}\,\sqrt{b^3\,d^2\,\left(a-b\right)}}\right)+\frac{2\,a^3\,\mathrm{e}^{3\,c}\,\mathrm{e}^{3\,d\,x}}{b^5\,d\,{\left(a-b\right)}^2\,{\left(a^2\right)}^{3/2}}\right)\,\left(\frac{b^5\,\sqrt{a\,b^3\,d^2-b^4\,d^2}}{4}-\frac{a\,b^4\,\sqrt{a\,b^3\,d^2-b^4\,d^2}}{4}\right)\right)\right)\,\sqrt{a^2}}{2\,\sqrt{a\,b^3\,d^2-b^4\,d^2}}+\frac{\mathrm{e}^{-c-d\,x}}{2\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] exp(c + d*x)/(2*b*d) - ((2*atan((a^3*exp(d*x)*exp(c)*(b^3*d^2*(a - b))^(1/2
))/(2*b*d*(a - b)*(a^2)^(3/2))) + 2*atan((exp(d*x)*exp(c)*((2*a^3)/(b^5*d*(
a - b)^2*(a^2)^(3/2)) - (4*(2*b^2*d*(a^2)^(3/2) - 2*a*b*d*(a^2)^(3/2)))/(a^
3*b^4*(a - b)*(a*b^3*d^2 - b^4*d^2)^(1/2)*(b^3*d^2*(a - b))^(1/2))) + (2*a^
3*exp(3*c)*exp(3*d*x))/(b^5*d*(a - b)^2*(a^2)^(3/2)))*((b^5*(a*b^3*d^2 - b^
4*d^2)^(1/2))/4 - (a*b^4*(a*b^3*d^2 - b^4*d^2)^(1/2))/4)))*(a^2)^(1/2))/(2*
(a*b^3*d^2 - b^4*d^2)^(1/2)) + exp(- c - d*x)/(2*b*d)
```

3.33 $\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{b}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}\,bd}$$

```
[Out] x/b-arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/b/d/(a-b)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3250, 3260, 214}

$$\frac{x}{b}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a - b]*b
*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sinh^2(c+dx)}\,dx}{b}\\
&= \frac{x}{b}-\frac{a\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{bd}\\
&= \frac{x}{b}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}\,bd}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 50, normalized size = 1.00

$$\frac{c+dx-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (c + d*x - (Sqrt[a]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a -
b])/(b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(42) = 84$.
time = 1.01, size = 216, normalized size = 4.32

method	result
risch	$\frac{x}{b}+\frac{\sqrt{a(a-b)}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{a(a-b)}+2a-b}{b}\right)}{2(a-b)db}-\frac{\sqrt{a(a-b)}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{a(a-b)}-}{b}\right.}{2(a-b)db}$
derivativedivides	$\frac{\frac{2a^2\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(\right.}{2a\sqrt{-b(a-b)}\sqrt{(}}\right.}{b}}{d}$

default	$\frac{2a^2\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{-b(a-b)}-b\right)\operatorname{arctanh}\left(-\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\right.}}\right.}{b}$ over d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2/b*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2
)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)
*a)^(1/2)))+1/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/b*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(42) = 84$.
time = 0.44, size = 464, normalized size = 9.28

$$\left[\frac{2\,dx+\sqrt{\frac{a}{a-b}}\log\left(\frac{b^2\cosh(dx+c)^4+4\,b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4+2\,(2\,ab-b^2)\cosh(dx+c)^2+2\left(3\,b^2\cosh(dx+c)^2+2\,ab-b^2\right)\sinh(dx+c)^2+8\,a^2-8\,ab+b^2+4\left(b^2\cosh(dx+c)^3+(2\,ab-b^2)\cosh(dx+c)\right)\sinh(dx+c)+4\left((ab-b^2)\cosh(dx+c)^2+2\,(ab-b^2)\cosh(dx+c)\sinh(dx+c)+(ab-b^2)\sinh(dx+c)^2+2\,a^2-3\,ab+b^2\right)\sqrt{\frac{a}{a-b}}}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\,(2\,a-b)\cosh(dx+c)^2+2\left(3\,b\cosh(dx+c)^2+2\,a-b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3+(2\,a-b)\cosh(dx+c)\right)\sinh(dx+c)+b}\right)}{2\,bd},\ \frac{dx-\sqrt{-\frac{a}{a-b}}\arctan\left(\frac{\left(b\cosh(dx+c)^2+2\,b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+2\,a-b\right)\sqrt{-\frac{a}{a-b}}}{2\,a}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt(a/(a - b))*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c
)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 +
 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b +
```

```
b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) +
 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)
 + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/(a - b)))/(b*c
osh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*
(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)
^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/(
b*d), (d*x - sqrt(-a/(a - b))*arctan(1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a/(a - b))/a))/(b*d
)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.27, size = 64, normalized size = 1.28

$$-\frac{\frac{a\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b}-\frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(a*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
 a*b)*b) - (d*x + c)/b)/d
```

Mupad [**B**]
time = 1.22, size = 473, normalized size = 9.46

$$\frac{x}{b}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\left(b^5\,\sqrt{b^3\,d^2-a\,b^2\,d^2}-a\,b^4\,\sqrt{b^3\,d^2-a\,b^2\,d^2}\right)\,\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\left(\frac{2\,\left(8\,a^2-8\,a\,b+b^2\right)\,\left(8\,a^{5/2}\,\sqrt{b^3\,d^2-a\,b^2\,d^2}-8\,a^{3/2}\,b\,\sqrt{b^3\,d^2-a\,b^2\,d^2}+\sqrt{a}\,b^2\,\sqrt{b^3\,d^2-a\,b^2\,d^2}\right)}{b^8\,d\,{\left(a-b\right)}^2\,\sqrt{b^3\,d^2-a\,b^2\,d^2}}+\frac{4\,\sqrt{a}\,\left(4\,a-2\,b\right)\,\left(8\,d\,a^3\,b-12\,d\,a^2\,b^2+4\,d\,a\,b^3\right)}{b^7\,\left(a-b\right)\,\sqrt{b^3\,d^2-a\,b^2\,d^2}\,\sqrt{-b^2\,d^2\,\left(a-b\right)}}\right)+\frac{2\,\left(2\,a^{3/2}\,b\,\sqrt{b^3\,d^2-a\,b^2\,d^2}-\sqrt{a}\,b^2\,\sqrt{b^3\,d^2-a\,b^2\,d^2}\right)\,\left(8\,a^2-8\,a\,b+b^2\right)}{b^8\,d\,{\left(a-b\right)}^2\,\sqrt{b^3\,d^2-a\,b^2\,d^2}}+\frac{4\,\sqrt{a}\,\left(2\,a^2\,b^2\,d-2\,a\,b^3\,d\right)\,\left(4\,a-2\,b\right)}{b^7\,\left(a-b\right)\,\sqrt{b^3\,d^2-a\,b^2\,d^2}\,\sqrt{-b^2\,d^2\,\left(a-b\right)}}\right)}{4\,a}\right)}{\sqrt{b^3\,d^2-a\,b^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] x/b - (a^(1/2)*atan(((b^5*(b^3*d^2 - a*b^2*d^2)^(1/2) - a*b^4*(b^3*d^2 - a*
b^2*d^2)^(1/2))*(exp(2*c)*exp(2*d*x)*((2*(8*a^2 - 8*a*b + b^2)*(8*a^(5/2)*(
b^3*d^2 - a*b^2*d^2)^(1/2) - 8*a^(3/2)*b*(b^3*d^2 - a*b^2*d^2)^(1/2) + a^(1
```

```
/2)*b^2*(b^3*d^2 - a*b^2*d^2)^(1/2)))/(b^8*d*(a - b)^2*(b^3*d^2 - a*b^2*d^2
)^(1/2)) + (4*a^(1/2)*(4*a - 2*b)*(4*a*b^3*d - 12*a^2*b^2*d + 8*a^3*b*d))/(
b^7*(a - b)*(b^3*d^2 - a*b^2*d^2)^(1/2)*(-b^2*d^2*(a - b))^(1/2))) + (2*(2*
a^(3/2)*b*(b^3*d^2 - a*b^2*d^2)^(1/2) - a^(1/2)*b^2*(b^3*d^2 - a*b^2*d^2)^(
1/2))*(8*a^2 - 8*a*b + b^2))/(b^8*d*(a - b)^2*(b^3*d^2 - a*b^2*d^2)^(1/2))
+ (4*a^(1/2)*(2*a^2*b^2*d - 2*a*b^3*d)*(4*a - 2*b))/(b^7*(a - b)*(b^3*d^2 -
 a*b^2*d^2)^(1/2)*(-b^2*d^2*(a - b))^(1/2))))/(4*a))/(b^3*d^2 - a*b^2*d^2)
^(1/2)
```

3.34 $\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,\sqrt{b}\,d}$$

```
[Out] arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3265, 211}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{b}\,d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(Sqrt[a - b]*Sqrt[b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,\sqrt{b}\,d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 91, normalized size = 2.28

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\operatorname{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqrt
[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(Sqrt[a - b]*Sqrt[b]*d)
```

Maple [A]
time = 0.82, size = 51, normalized size = 1.28

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{d\sqrt{ab-b^2}}$	51
default	$\frac{\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{d\sqrt{ab-b^2}}$	51
risch	$-\frac{\ln\left(e^{2dx+2c}-\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^2}}+1\right)}{2\sqrt{-ab+b^2}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^2}}+1\right)}{2\sqrt{-ab+b^2}\,d}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(a*b-b^2)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2-2*a+4*b)/(a*b-b^2
)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(d*x + c)/(b*sinh(d*x + c)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(32) = 64.
time = 0.44, size = 502, normalized size = 12.55

$$\left[-\frac{\sqrt{-ab+b^2}\log\left(\frac{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2\,(2\,a-3\,b)\cosh(dx+c)^2+2\left(3\,b\cosh(dx+c)^2-2\,a+3\,b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3-(2\,a-3\,b)\cosh(dx+c)\right)\sinh(dx+c)-4\left(\cosh(dx+c)^3+3\cosh(dx+c)\sinh(dx+c)^2+\sinh(dx+c)^3+\left(3\cosh(dx+c)^2+1\right)\sinh(dx+c)+\cosh(dx+c)\right)\sqrt{-ab+b^2}+b}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\,(2\,a-b)\cosh(dx+c)^2+2\left(3\,b\cosh(dx+c)^2+2\,a-b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3+(2\,a-b)\cosh(dx+c)\right)\sinh(dx+c)+b}\right)}{2\,(ab-b^2)d},\ \frac{\sqrt{ab-b^2}\arctan\left(-\frac{b\cosh(dx+c)^3+3\,b\cosh(dx+c)\sinh(dx+c)^2+b\sinh(dx+c)^3+(4\,a-3\,b)\cosh(dx+c)+\left(3\,b\cosh(dx+c)^2+4\,a-3\,b\right)\sinh(dx+c)}{2\sqrt{ab-b^2}}\right)-\sqrt{ab-b^2}\arctan\left(-\frac{\sqrt{ab-b^2}\left(\cosh(dx+c)+\sinh(dx+c)\right)}{2\,(a-b)}\right)}{(ab-b^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*
x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*
x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d
*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(
d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*co
sh(d*x + c))*sinh(d*x + c) + b))/((a*b - b^2)*d), (sqrt(a*b - b^2)*arctan(-
1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c
)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*
x + c))/sqrt(a*b - b^2)) - sqrt(a*b - b^2)*arctan(-1/2*sqrt(a*b - b^2)*(cos
h(d*x + c) + sinh(d*x + c))/(a - b)))/((a*b - b^2)*d)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 367433 vs. $2(32) = 64$.
time = 117.79, size = 367433, normalized size = 9185.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/sinh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tanh(c/2 +
d*x/2))/(b*d), Eq(a, 0)), (-2/(b*d*tanh(c/2 + d*x/2)**2 + b*d), Eq(a, b)),
(cosh(c + d*x)/(a*d), Eq(b, 0)), (x*sinh(c)/(a + b*sinh(c)**2), Eq(d, 0)),
(-74*a**37*b*log(-sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/
2))/(2*a**38*b*d - 5478*a**37*b**2*d + 148*a**37*b*d*sqrt(-a*b + b**2) + 25
02532*a**36*b**3*d - 135124*a**36*b**2*d*sqrt(-a*b + b**2) - 456961248*a**3
5*b**4*d + 36983424*a**35*b**3*d*sqrt(-a*b + b**2) + 44602414272*a**34*b**5
*d - 4809599808*a**34*b**4*d*sqrt(-a*b + b**2) - 2698911348224*a**33*b**6*d
 + 363524561920*a**33*b**5*d*sqrt(-a*b + b**2) + 110776036340736*a**32*b**7
*d - 17891931206656*a**32*b**6*d*sqrt(-a*b + b**2) - 3275718126403584*a**31
*b**8*d + 616808259780608*a**31*b**7*d*sqrt(-a*b + b**2) + 728547276296028l
6*a**30*b**9*d - 15666762815766528*a**30*b**8*d*sqrt(-a*b + b**2) - 1258467
596957384704*a**29*b**10*d + 30423030383522176*a**29*b**9*d*sqrt(-a*b + b*
*2) + 17306140891880620032*a**28*b**11*d - 464520617439526912 0*a**28*b**10*
d*sqrt(-a*b + b**2) - 193199008739227598848*a**27*b**12*d + 570019388028595
73248*a**27*b**11*d*sqrt(-a*b + b**2) + 1778515685235870400512*a**26*b**13*
d - 5720299074193761239 04*a**26*b**12*d*sqrt(-a*b + b**2) - 13673782930644 6
13988352*a**25*b**14*d + 476102010976912539648 0*a**25*b**13*d*sqrt(-a*b + b
**2) + 88722183139577965838336*a**24*b**15*d - 3324427608271268243046 4*a**2
```

```
4*b**14*d*sqrt(-a*b + b**2) - 490030319626953299066880*a**23*b**16*d + 1965
893232475255078912OO*a**23*b**15*d*sqrt(-a*b + b**2) + 23202646598809994605
36320*a**22*b**17*d - 992185245208510642257920*a**22*b**16*d*sqrt(-a*b + b*
*2) - 947323742305031456555008O*a**21*b**18*d + 430103113560420123672576O*a
**21*b**17*d*sqrt(-a*b + b**2) + 33508135815008970573086720*a**20*b**19*d -
 16096759227611109665013760*a**20*b**18*d*sqrt(-a*b + b**2) - 1030660010072
81297455841280*a**19*b**20*d + 52224655042483407940485120*a**19*b**19*d*sqr
t(-a*b + b**2) + 276460659949410743463444480*a**18*b**21*d - 14735375623101
0598099353600*a**18*b**20*d*sqrt(-a*b + b**2) - 648017072918162395858206720
*a**17*b**22*d + 362406002426292494841937920*a**17*b**21*d*sqrt(-a*b + b**2
) + 1328967367029204488741191680*a**16*b**23*d - 77807030927656552325185536
0*a**16*b**22*d*sqrt(-a*b + b**2) - 2385704631316968523085905920*a**15*b**2
4*d + 1459214048234341782006005760*a**15*b**23*d*sqrt(-a*b + b**2) + 374752
9655246565869805895680*a**14*b**25*d - 2390143076492438985108357120*a**14*b
**24*d*sqrt(-a*b + b**2) - 5144960127422757831513735168*a**13*b**26*d + 341
5726795297830225523507200*a**13*b**25*d*sqrt(-a*b + b**2) + 616034336892617
9873261617152*a**12*b**27*d - 4250434573627220170723295232*a**12*b**26*d*sq
rt(-a*b + b**2) - 6412553052386662194178162688*a**11*b**28*d + 459139020036
1817020475375616*a**11*b**27*d*sqrt(-a*b + b**2) + 577744396413111418133623
6032*a**10*b**29*d - 4286847917414518496444809216*a**10*b**28*d*sqrt(-a*b +
 b**2) - 4478521132381256779508482048*a**9*b**30*d + 3439316943952955273874
767872*a**9*b**29*d*sqrt(-a*b + b**2) + 2963524367539447964941418496*a**8*b
**31*d - 2352691778556737395705774080*a**8*b**30*d*sqrt(-a*b + b**2) - 1656
691644402709111026745344*a**7*b**32*d + 1358109508879153777946394624*a**7*b
**31*d*sqrt(-a*b + b**2) + 771639221199942160467623936*a**6*b**33*d - 65251
7637350019802209452032*a**6*b**32*d*sqrt(-a*b + b**2) - 2938576292183731375
58667264*a**5*b**34*d + 256080136201453725194125312*a**5*b**33*d*sqrt(-a*b
+ b**2) + 89102865177381478141526016*a**4*b**35*d - 79945311123381698013691
904*a**4*b**34*d*sqrt(-a*b + b**2) - 20683374899158687330336768*a**3*b**36*
d + 19090166507000540776366080*a**3*b**35*d*sqrt(-a*b + b**2) + 34508693073
56993239908352*a**2*b**37*d - 3273780564249381544394752*a**2*b**36*d*sqrt(-
a*b + b**2) - 368344585663832326668288*a*b**38*d + 358899852698093036240896
*a*b**37*d*sqrt(-a*b + b**2) + 18889465931478580854784*b**39*d - 1888946593
1478580854784*b**38*d*sqrt(-a*b + b**2)) - 74*a**37*b*log(sqrt(1 - 2*b/a -
2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*a**38*b*d - 5478*a**37*b**2*
d + 148*a**37*b*d*sqrt(-a*b + b**2) + 2502532*a**36*b**3*d - 135124*a**36*b
**2*d*sqrt(-a*b + b**2) - 456961248*a**35*b**4*d + 36983424*a**35*b**3*d*sq
rt(-a*b + b**2) + 44602414272*a**34*b**5*d - 4809599808*a**34*b**4*d*sqrt(-
a*b + b**2) - 2698911348224*a**33*b**6*d + 363524561920*a**33*b**5*d*sqrt(-
a*b + b**2) + 110776036340736*a**32*b**7*d - 17891931206656*a**32*b**6*d*sq
rt(-a*b + b**2) - 3275718126403584*a**31*b**8*d + 616808259780608*a**31*b**
7*d*sqrt(-a*b + b**2) + 72854727629602816*a**30*b**9*d - 15666762815766528*
a**30*b**8*d*sqrt(-a*b + b**2) - 1258467596957384704*a**29*b**10*d + 304230
30383522176*a**29*b**9*d*sqrt(-a*b + b**2) + 17306140891880620032*a**28*b*
*11*d - 464520617439526912O*a**28*b**10*d*sqrt(-a*b + b**2) - 1931990087392
```

```
27598848*a**27*b**12*d + 57001938802859573248*a**27*b**11*d*sqrt(-a*b + b**
2) + 177851568523587040051 2*a**26*b**13*d - 57202990741937612390 4*a**26*b**
12*d*sqrt(-a*b + b**2) - 136737829306446139883 5...
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 0.99, size = 116, normalized size = 2.90

$$\frac{\ln\left(-\frac{4\,\left(a+a\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{b^2\,\left(a-b\right)}-\frac{8\,a\,\mathrm{e}^{c+d\,x}}{{\left(-b\right)}^{5/2}\,\sqrt{a-b}}\right)-\ln\left(\frac{8\,a\,\mathrm{e}^{c+d\,x}}{{\left(-b\right)}^{5/2}\,\sqrt{a-b}}-\frac{4\,\left(a+a\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{b^2\,\left(a-b\right)}\right)}{2\,\sqrt{-b}\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (log(- (4*(a + a*exp(2*c + 2*d*x)))/(b^2*(a - b)) - (8*a*exp(c + d*x))/((-b
)^(5/2)*(a - b)^(1/2))) - log((8*a*exp(c + d*x))/((-b)^(5/2)*(a - b)^(1/2))
 - (4*(a + a*exp(2*c + 2*d*x)))/(b^2*(a - b))))/(2*(-b)^(1/2)*d*(a - b)^(1/
2))
```

3.35 $\int \frac{1}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a-b}\,d}$$

```
[Out] arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/d/a^(1/2)/(a-b)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3260, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a-b}\,d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 40, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(32) = 64$.
time = 1.14, size = 180, normalized size = 4.50

method	result
risch	$\frac{\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}-2a^2+2ab}{b\sqrt{a^2-ab}}\right)}{2\sqrt{a^2-ab}\,d}-\frac{\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}+2a^2-2a}{b\sqrt{a^2-ab}}\right.}{2\sqrt{a^2-ab}\,d}$
derivativedivides	$\frac{2a\left(\frac{\left(-\sqrt{-b(a-b)}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2\sqrt{-b(a-b)}\,a\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(-\sqrt{-b(a-b)}+b\right)\arctan}{2a\sqrt{-b(a-b)}\,\sqrt{}}\right.}{d}$
default	$\frac{2a\left(\frac{\left(-\sqrt{-b(a-b)}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2\sqrt{-b(a-b)}\,a\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(-\sqrt{-b(a-b)}+b\right)\arctan}{2a\sqrt{-b(a-b)}\,\sqrt{}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*a*(1/2*(-(-b*(a-b))^(1/2)-b)/(-b*(a-b))^(1/2)/a/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(
1/2))-1/2*(-(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2
```

```
*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(
1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(32) = 64$.
time = 0.43, size = 430, normalized size = 10.75

$$\left[\frac{\log\left(\frac{b^2\cosh(dx+c)^4+4b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4+2\left(2ab-b^2\right)\cosh(dx+c)^2+2\left(3b^2\cosh(dx+c)^2+2ab-b^2\right)\sinh(dx+c)^2+8a^2-8ab+b^2+4\left(b^2\cosh(dx+c)^3+\left(2ab-b^2\right)\cosh(dx+c)\right)\sinh(dx+c)-4\left(b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+2a-b\right)\sqrt{a^2-ab}}{b\cosh(dx+c)^4+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2(2a-b)\cosh(dx+c)^2+2\left(3b\cosh(dx+c)^2+2a-b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3+(2a-b)\cosh(dx+c)\right)\sinh(dx+c)+b}\right)}{2\sqrt{a^2-ab}\,d}, -\frac{\sqrt{-a^2+ab}\arctan\left(-\frac{\left(b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+2a-b\right)\sqrt{-a^2+ab}}{2\left(a^2-ab\right)}\right)}{(a^2-ab)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*s
inh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2
 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c
)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2
*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*
b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c
)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(
d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c)
+ b))/(sqrt(a^2 - a*b)*d), -sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2
 + a*b)/(a^2 - a*b))/((a^2 - a*b)*d)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 15870 vs. $2(32) = 64$.
time = 17.10, size = 15870, normalized size = 396.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/sinh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-tanh(c/2 +
 d*x/2)/(2*d) - 1/(2*d*tanh(c/2 + d*x/2)))/b, Eq(a, 0)), (2*tanh(c/2 + d*x/
2)/(b*d*tanh(c/2 + d*x/2)**2 + b*d), Eq(a, b)), (x/a, Eq(b, 0)), (x/(a + b*
sinh(c)**2), Eq(d, 0)), (-6*a**3*b*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*
log(-sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(10*a**4*
b*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b +
b**2)/a) - 2*a**4*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/
a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) - 50*a**3*b**2*d*sqrt(1 - 2*b/a
- 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + 26*a**3*
b*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/
a + 2*sqrt(-a*b + b**2)/a) + 72*a**2*b**3*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b +
b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) - 56*a**2*b**2*d*sqrt(-a*b
 + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a
*b + b**2)/a) - 32*a*b**4*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1
- 2*b/a + 2*sqrt(-a*b + b**2)/a) + 32*a*b**3*d*sqrt(-a*b + b**2)*sqrt(1 - 2
*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a)) + 6*
a**3*b*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*log(sqrt(1 - 2*b/a + 2*sqrt(
-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(10*a**4*b*d*sqrt(1 - 2*b/a - 2*sqrt(-
a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) - 2*a**4*d*sqrt(-a*b
 + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a
*b + b**2)/a) - 50*a**3*b**2*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt
(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + 26*a**3*b*d*sqrt(-a*b + b**2)*sqrt(1
- 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) +
72*a**2*b**3*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*s
qrt(-a*b + b**2)/a) - 56*a**2*b**2*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*s
qrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) - 32*a*b**4*d*s
qrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)
/a) + 32*a*b**3*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)
*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a)) - 4*a**3*b*sqrt(1 - 2*b/a + 2*sqr
t(-a*b + b**2)/a)*log(-sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh(c/2 +
 d*x/2))/(10*a**4*b*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/
a + 2*sqrt(-a*b + b**2)/a) - 2*a**4*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*
sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) - 50*a**3*b**2
*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b
**2)/a) + 26*a**3*b*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2
)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + 72*a**2*b**3*d*sqrt(1 - 2*b/
a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) - 56*a**
2*b**2*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 -
 2*b/a + 2*sqrt(-a*b + b**2)/a) - 32*a*b**4*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b
+ b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + 32*a*b**3*d*sqrt(-a*b
+ b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*
b + b**2)/a)) + 4*a**3*b*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a)*log(sqrt(1
 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(10*a**4*b*d*sqrt(1
- 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) -
2*a**4*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 -
```

```
 2*b/a + 2*sqrt(-a*b + b**2)/a) - 50*a**3*b**2*d*sqrt(1 - 2*b/a - 2*sqrt(-a
*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + 26*a**3*b*d*sqrt(-a
*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(
-a*b + b**2)/a) + 72*a**2*b**3*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sq
rt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) - 56*a**2*b**2*d*sqrt(-a*b + b**2)*sq
rt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/
a) - 32*a*b**4*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2
*sqrt(-a*b + b**2)/a) + 32*a*b**3*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sq
rt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a)) + a**3*sqrt(-a*
b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*log(-sqrt(1 - 2*b/a + 2*s
qrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(10*a**4*b*d*sqrt(1 - 2*b/a - 2*sq
rt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) - 2*a**4*d*sqrt(
-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqr
t(-a*b + b**2)/a) - 50*a**3*b**2*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*
sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + 26*a**3*b*d*sqrt(-a*b + b**2)*sqr
t(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a
) + 72*a**2*b**3*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a +
 2*sqrt(-a*b + b**2)/a) - 56*a**2*b**2*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a -
 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) - 32*a*b**4
*d*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b
**2)/a) + 32*a*b**3*d*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2
)/a)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a)) -...
```

Giac [A]
time = 0.56, size = 47, normalized size = 1.18

$$\frac{\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b
)*d)
```

Mupad [B]
time = 0.45, size = 146, normalized size = 3.65

$$\frac{\ln\left(-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{b}-\frac{2\,\left(b\,d+2\,a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b\,d\,\sqrt{a-b}}\right)-\ln\left(\frac{2\,\left(b\,d+2\,a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b\,d\,\sqrt{a-b}}-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{b}\right)}{2\,\sqrt{a}\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (log(- (4*exp(2*c + 2*d*x))/b - (2*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*d*exp(
2*c + 2*d*x)))/(a^(1/2)*b*d*(a - b)^(1/2))) - log((2*(b*d + 2*a*d*exp(2*c +
 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(1/2)*b*d*(a - b)^(1/2)) - (4*exp(2*c +
 2*d*x))/b))/(2*a^(1/2)*d*(a - b)^(1/2))
```

3.36 $\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}\,d}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

```
[Out] -arctanh(cosh(d*x+c))/a/d-arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a
/d/(a-b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3265, 400, 212, 211}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{ad\sqrt{a-b}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a*Sqrt[a - b]*d))
- ArcTanh[Cosh[c + d*x]]/(a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cosh(c+dx)\right)}{ad}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx,x,\cosh(c+dx)\right)}{ad}\\ &= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}\,d}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 124, normalized size = 2.07

$$\frac{-\sqrt{b}\,\operatorname{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)-\sqrt{b}\,\operatorname{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\sqrt{a-b}\,\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a\sqrt{a-b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
[Out] (-(Sqrt[b]*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]) - S
qrt[b]*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + Sqrt[a
 - b]*Log[Tanh[(c + d*x)/2]])/(a*Sqrt[a - b]*d)
```

Maple [A]
time = 1.39, size = 72, normalized size = 1.20

method	result
derivativedivides	$\frac{-\frac{b\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{a\sqrt{ab-b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{-\frac{b\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{a\sqrt{ab-b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$

risch	$-\frac{\ln\left(e^{dx+c}+1\right)}{da}+\frac{\ln\left(e^{dx+c}-1\right)}{da}+\frac{\sqrt{-b\,(a-b)}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-b\,(a-b)}\,e^{dx+c}}{b}+1\right)}{2(a-b)da}-$ $\sqrt{-b\,(a\,-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a*b/(a*b-b^2)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2-2*a+4*b)/
(a*b-b^2)^(1/2))+1/a*ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a*b*e^(4*d*x + 4*c) + a*
b + 2*(2*a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(52) = 104$.
time = 0.42, size = 586, normalized size = 9.77

$$\left[\frac{\sqrt{-\frac{b}{a-b}}\log\left(\frac{b\cosh(dx+c)^4+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2(2a-3b)\cosh(dx+c)^2+2\left(3b\cosh(dx+c)^2-2a+3b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3-(2a-3b)\cosh(dx+c)\right)\sinh(dx+c)-4\left((a-b)\cosh(dx+c)^3+3(a-b)\cosh(dx+c)\sinh(dx+c)^2+(a-b)\sinh(dx+c)^3+(a-b)\cosh(dx+c)+\left(3(a-b)\cosh(dx+c)^2+a-b\right)\sinh(dx+c)\right)\sqrt{-\frac{b}{a-b}}+b}{b\cosh(dx+c)^4+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2(2a-b)\cosh(dx+c)^2+2\left(3b\cosh(dx+c)^2+2a-b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3+(2a-b)\cosh(dx+c)\right)\sinh(dx+c)+b}\right)-2\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)+2\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)}{2ad},\ -\frac{\sqrt{\frac{b}{a-b}}\arctan\left(\frac{1}{2}\sqrt{\frac{b}{a-b}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)\right)-\sqrt{\frac{b}{a-b}}\arctan\left(\frac{\left(b\cosh(dx+c)^3+3b\cosh(dx+c)\sinh(dx+c)^2+b\sinh(dx+c)^3+(4a-3b)\cosh(dx+c)+\left(3b\cosh(dx+c)^2+4a-3b\right)\sinh(dx+c)\right)\sqrt{\frac{b}{a-b}}}{2b}\right)+\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)-\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/(a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*
x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(
d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c)
+ (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
+ 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)
) - 2*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*log(cosh(d*x + c) + sinh(d
*x + c) - 1))/(a*d), -(sqrt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x
 + c) + sinh(d*x + c))) - sqrt(b/(a - b))*arctan(1/2*(b*cosh(d*x + c)^3 + 3
*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x
 + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))*sqrt(b/(a - b))/b)
```

```
 + log(cosh(d*x + c) + sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh(d*x +
c) - 1))/(a*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(c+dx\right)}{a+b\sinh^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sinh(c + d*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 1.06, size = 323, normalized size = 5.38

$$-\frac{2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\left(16\,a^2\,\sqrt{-a^2\,d^2}+9\,b^2\,\sqrt{-a^2\,d^2}-24\,a\,b\,\sqrt{-a^2\,d^2}\right)}{16\,d\,a^3-24\,d\,a^2\,b+9\,d\,a\,b^2}\right)}{\sqrt{-a^2\,d^2}}-\frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{b}\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\sqrt{a^2\,d^2\,\left(a-b\right)}}{2\,a\,d\,\left(a-b\right)}\right)+2\,\mathrm{atan}\left(\frac{4\,a^4\,d^2\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c+4\,a^2\,b^2\,d^2\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c+b\,{\mathrm{e}}^{3\,c}\,{\mathrm{e}}^{3\,d\,x}\,\sqrt{a^3\,d^2-a^2\,b\,d^2}\,\sqrt{a^2\,d^2\,\left(a-b\right)}-8\,a^3\,b\,d^2\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c+b\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\sqrt{a^3\,d^2-a^2\,b\,d^2}\,\sqrt{a^2\,d^2\,\left(a-b\right)}}{\sqrt{b}\,d\,\left(2\,a\,b-2\,a^2\right)\,\sqrt{a^2\,d^2\,\left(a-b\right)}}\right)\right)}{2\,\sqrt{a^3\,d^2-a^2\,b\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] - (2*atan((exp(d*x)*exp(c)*(16*a^2*(-a^2*d^2)^(1/2) + 9*b^2*(-a^2*d^2)^(1/2
) - 24*a*b*(-a^2*d^2)^(1/2)))/(16*a^3*d + 9*a*b^2*d - 24*a^2*b*d)))/(-a^2*d
^2)^(1/2) - (b^(1/2)*(2*atan((b^(1/2)*exp(d*x)*exp(c)*(a^2*d^2*(a - b))^(1/
2))/(2*a*d*(a - b))) + 2*atan((4*a^4*d^2*exp(d*x)*exp(c) + 4*a^2*b^2*d^2*ex
p(d*x)*exp(c) + b*exp(3*c)*exp(3*d*x)*(a^3*d^2 - a^2*b*d^2)^(1/2)*(a^2*d^2*
(a - b))^(1/2) - 8*a^3*b*d^2*exp(d*x)*exp(c) + b*exp(d*x)*exp(c)*(a^3*d^2 -
 a^2*b*d^2)^(1/2)*(a^2*d^2*(a - b))^(1/2))/(b^(1/2)*d*(2*a*b - 2*a^2)*(a^2*
d^2*(a - b))^(1/2)))))/(2*(a^3*d^2 - a^2*b*d^2)^(1/2))
```

3.37 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}\,d}-\frac{\coth(c+dx)}{ad}$$

```
[Out] -coth(d*x+c)/a/d-b*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/d/(a-b)
^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3266, 464, 214}

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d\sqrt{a-b}}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a - b]*d))
 - Coth[c + d*x]/(a*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
```

```
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^2(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{\coth(c+dx)}{ad} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \tanh(c+dx)\right)}{ad}\\
&= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}\,d} - \frac{\coth(c+dx)}{ad}
\end{aligned}
$$

Mathematica [**A**]
time = 0.19, size = 57, normalized size = 1.00

$$
\frac{-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} - \sqrt{a}\,\coth(c+dx)}{a^{3/2}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a - b]) - Sqrt[a]*
Coth[c + d*x])/(a^(3/2)*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(49) = 98$.
time = 1.40, size = 208, normalized size = 3.65

method	result
risch	$-\frac{2}{da\left(e^{2dx+2c}-1\right)}+\frac{b\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}+2a^2-2ab}{b\sqrt{a^2-ab}}\right)}{2\sqrt{a^2-ab}\,da}-\frac{b\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}}{b}\right.}{2\sqrt{a^2-}}$

derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+2b\left(\frac{\left(\sqrt{-b\left(a-b\right)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}\right)}{2a\sqrt{-b\left(a-b\right)}\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}-\frac{\left(\sqrt{-b\left(a-b\right)}\right.}{2a\sqrt{-b\left(\right.}}\right.}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+2b\left(\frac{\left(\sqrt{-b\left(a-b\right)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}\right)}{2a\sqrt{-b\left(a-b\right)}\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}-\frac{\left(\sqrt{-b\left(a-b\right)}\right.}{2a\sqrt{-b\left(\right.}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)+2*b*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^
(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)))-1/2/a/tanh(1/2*d*x+1/2*c))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(49) = 98$.
time = 0.47, size = 675, normalized size = 11.84

$$\left[\frac{\left(b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}-b\right)\sqrt{a^{2}-ab}\,\log\left(\frac{b^{2}\cosh(dx+c)^{4}+4\,b^{2}\cosh(dx+c)\sinh(dx+c)^{3}+b^{2}\sinh(dx+c)^{4}+2\,(2\,ab-b^{2})\cosh(dx+c)^{2}+2\,\left(3\,b^{2}\cosh(dx+c)^{2}+2\,ab-b^{2}\right)\sinh(dx+c)^{2}+8\,a^{2}-8\,ab+b^{2}+4\,\left(b^{2}\cosh(dx+c)^{3}+(2\,ab-b^{2})\cosh(dx+c)\right)\sinh(dx+c)+4\,\left(b\cosh(dx+c)^{2}+2\,b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^{2}+2\,a-b\right)\sqrt{a^{2}-ab}}{b\cosh(dx+c)^{4}+4\,b\cosh(dx+c)\sinh(dx+c)^{3}+b\sinh(dx+c)^{4}+2\,(2\,a-b)\cosh(dx+c)^{2}+2\,\left(3\,b\cosh(dx+c)^{2}+2\,a-b\right)\sinh(dx+c)^{2}+4\,\left(b\cosh(dx+c)^{3}+(2\,a-b)\cosh(dx+c)\right)\sinh(dx+c)+b}\right)-4\,a^{2}+4\,ab}{2\,\left((a^{3}-a^{2}b)d\cosh\left(dx+c\right)^{2}+2\,(a^{3}-a^{2}b)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a^{3}-a^{2}b)d\sinh\left(dx+c\right)^{2}-(a^{3}-a^{2}b)d\right)},\ \frac{\left(b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}-b\right)\sqrt{-a^{2}+ab}\,\arctan\left(-\frac{\left(b\cosh(dx+c)^{2}+2\,b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^{2}+2\,a-b\right)\sqrt{-a^{2}+ab}}{2\,(a^{2}-ab)}\right)-2\,a^{2}+2\,ab}{(a^{3}-a^{2}b)d\cosh\left(dx+c\right)^{2}+2\,(a^{3}-a^{2}b)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a^{3}-a^{2}b)d\sinh\left(dx+c\right)^{2}-(a^{3}-a^{2}b)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c
)^2 - b)*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sin
h(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3
*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 +
 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(b
*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*
a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c
)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
+ c))*sinh(d*x + c) + b)) - 4*a^2 + 4*a*b)/((a^3 - a^2*b)*d*cosh(d*x + c)^2
 + 2*(a^3 - a^2*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - a^2*b)*d*sinh(d*x
 + c)^2 - (a^3 - a^2*b)*d), ((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*
x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(
-a^2 + a*b)/(a^2 - a*b)) - 2*a^2 + 2*a*b)/((a^3 - a^2*b)*d*cosh(d*x + c)^2
+ 2*(a^3 - a^2*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - a^2*b)*d*sinh(d*x
+ c)^2 - (a^3 - a^2*b)*d)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sinh(c + d*x)**2), x)
```

Giac [A]

time = 0.63, size = 72, normalized size = 1.26

$$-\frac{\frac{b\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,a}+\frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
 a*b)*a) + 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [B]
time = 0.47, size = 176, normalized size = 3.09

$$\frac{b\,\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{a}-\frac{2\,\left(b\,d+2\,a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^{3/2}\,d\,\sqrt{a-b}}\right)}{2\,a^{3/2}\,d\,\sqrt{a-b}}-\frac{2}{a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{b\,\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{a}+\frac{2\,\left(b\,d+2\,a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^{3/2}\,d\,\sqrt{a-b}}\right)}{2\,a^{3/2}\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (b*log((4*exp(2*c + 2*d*x))/a - (2*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*d*exp(
2*c + 2*d*x)))/(a^(3/2)*d*(a - b)^(1/2))))/(2*a^(3/2)*d*(a - b)^(1/2)) - 2/
(a*d*(exp(2*c + 2*d*x) - 1)) - (b*log((4*exp(2*c + 2*d*x))/a + (2*(b*d + 2*
a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(3/2)*d*(a - b)^(1/2))))/(
2*a^(3/2)*d*(a - b)^(1/2))
```

3.38 $\int \frac{\mathbf{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2\sqrt{a-b}\,d}+\frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d}-\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

```
[Out] 1/2*(a+2*b)*arctanh(cosh(d*x+c))/a^2/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d+b^(3
/2)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/a^2/d/(a-b)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3265, 425, 536, 212, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2d\sqrt{a-b}}+\frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d}-\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a^2*Sqrt[a - b]*d) +
 ((a + 2*b)*ArcTanh[Cosh[c + d*x]])/(2*a^2*d) - (Coth[c + d*x]*Csch[c + d*x
])/(2*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+bx^2)}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}+\frac{\operatorname{Subst}\left(\int \frac{a+b+bx^2}{(1-x^2)(a-b+bx^2)}\,dx, x, \cosh(c+dx)\right)}{2ad}\\ &= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{a^2d}+(a+2b\\ &= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2\sqrt{a-b}\,d}+\frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d}-\frac{\coth(c+dx)\operatorname{csc}}{2ad}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.47, size = 201, normalized size = 2.28

$$-\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^4(c+dx)\left(2a\sqrt{a-b}\cosh(c+dx)-2\left(2b^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+2b^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)-\sqrt{a-b}\,(a+2b)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)\sinh^2(c+dx)\right)}{8a^2\sqrt{a-b}\,d\left(b+a\operatorname{csch}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/8*((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^4*(2*a*Sqrt[a - b]*Cosh
[c + d*x] - 2*(2*b^(3/2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqr
t[a - b]] + 2*b^(3/2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a
```

```
 - b]] - Sqrt[a - b]*(a + 2*b)*Log[Tanh[(c + d*x)/2]])*Sinh[c + d*x]^2))/(a
^2*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))
```

Maple [A]
time = 1.65, size = 113, normalized size = 1.28

method	result
derivativedivides	$\frac{\frac{\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{\left(-2a-4b\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{2}}+\frac{b^{2}\arctan\left(\frac{2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{a^{2}\sqrt{ab-b^{2}}}}{d}$
default	$\frac{\frac{\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{\left(-2a-4b\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{2}}+\frac{b^{2}\arctan\left(\frac{2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{a^{2}\sqrt{ab-b^{2}}}}{d}$
risch	$-\frac{e^{dx+c}\left(1+e^{2dx+2c}\right)}{da\left(e^{2dx+2c}-1\right)^{2}}+\frac{b\ln\left(e^{dx+c}+1\right)}{a^{2}d}+\frac{\ln\left(e^{dx+c}+1\right)}{2da}-\frac{b\ln\left(e^{dx+c}-1\right)}{a^{2}d}-\frac{\ln\left(e^{dx+c}-1\right)}{2da}+\frac{\sqrt{-b\left(a-b\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a-1/8/a/tanh(1/2*d*x+1/2*c)^2+1/4/a^2*(-2*a-
4*b)*ln(tanh(1/2*d*x+1/2*c))+b^2/a^2/(a*b-b^2)^(1/2)*arctan(1/4*(2*a*tanh(1
/2*d*x+1/2*c)^2-2*a+4*b)/(a*b-b^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*
c) + a*d) + 1/2*(a + 2*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - 1/2*(a +
2*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 8*integrate(1/4*(b^2*e^(3*d*x
+ 3*c) - b^2*e^(d*x + c))/(a^2*b*e^(4*d*x + 4*c) + a^2*b + 2*(2*a^3*e^(2*c)
 - a^2*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(76) = 152$.
time = 0.45, size = 1837, normalized size = 20.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] [-1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(d
*x + c)^3 - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh
(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x +
c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-b/(
a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh
(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a
+ 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*
sinh(d*x + c) + 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d
*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*co
sh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c
)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*c
osh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*c
osh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 2*a*cosh(d*
x + c) - ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x +
c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a +
2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c
)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) +
 sinh(d*x + c) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2
 + 2*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)
*cosh(d*x + c)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(co
sh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c
))/(a^2*d*cosh(d*x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*s
inh(d*x + c)^4 - 2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)
^2 - a^2*d)*sinh(d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c
))*sinh(d*x + c)), -1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x +
 c)^2 + 2*a*sinh(d*x + c)^3 - 2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh
(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c
)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x
 + c) + b)*sqrt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x + c) + sinh
(d*x + c))) + 2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*
sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*
x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(
b/(a - b))*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^
2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 +
4*a - 3*b)*sinh(d*x + c))*sqrt(b/(a - b))/b) + 2*a*cosh(d*x + c) - ((a + 2*
b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2*b)*
sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x + c
)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3 - (a + 2*b)*c
osh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sinh(d*x + c) +
1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)
*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3
```

```
- (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sin
h(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(a^2*d*cosh(d*
x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(d*x + c)^4 -
2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(
d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*sinh(d*x + c))
]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*sinh(c + d*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 1.40, size = 571, normalized size = 6.49

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a^7\,\sqrt{-a^4\,d^2}+18\,b^7\,\sqrt{-a^4\,d^2}-36\,a^2\,b^5\,\sqrt{-a^4\,d^2}-30\,a^3\,b^4\,\sqrt{-a^4\,d^2}+12\,a^4\,b^3\,\sqrt{-a^4\,d^2}+21\,a^5\,b^2\,\sqrt{-a^4\,d^2}+9\,a\,b^6\,\sqrt{-a^4\,d^2}+8\,a^6\,b\,\sqrt{-a^4\,d^2}\right)}{a^8\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+9\,a^2\,b^6\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}-18\,a^4\,b^4\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}-6\,a^5\,b^3\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+9\,a^6\,b^2\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+6\,a^7\,b\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}}\right)\,\sqrt{a^2+4\,a\,b+4\,b^2}}{\sqrt{-a^4\,d^2}}-\frac{\mathrm{e}^{c+d\,x}}{a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,\mathrm{e}^{c+d\,x}}{a\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{{\left(-b\right)}^{3/2}\,\ln\left(\frac{64\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)\,\left(a^3+3\,a^2\,b-3\,b^3\right)}{a^5\,{\left(a-b\right)}^2}-\frac{128\,\mathrm{e}^{c+d\,x}\,\left(a^3+3\,a^2\,b-3\,b^3\right)}{a^5\,\sqrt{-b}\,{\left(a-b\right)}^{3/2}}\right)}{2\,a^2\,d\,\sqrt{a-b}}+\frac{{\left(-b\right)}^{3/2}\,\ln\left(\frac{64\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)\,\left(a^3+3\,a^2\,b-3\,b^3\right)}{a^5\,{\left(a-b\right)}^2}+\frac{128\,\mathrm{e}^{c+d\,x}\,\left(a^3+3\,a^2\,b-3\,b^3\right)}{a^5\,\sqrt{-b}\,{\left(a-b\right)}^{3/2}}\right)}{2\,a^2\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^7*(-a^4*d^2)^(1/2) + 18*b^7*(-a^4*d^2)^(1/2) - 36
*a^2*b^5*(-a^4*d^2)^(1/2) - 30*a^3*b^4*(-a^4*d^2)^(1/2) + 12*a^4*b^3*(-a^4*
d^2)^(1/2) + 21*a^5*b^2*(-a^4*d^2)^(1/2) + 9*a*b^6*(-a^4*d^2)^(1/2) + 8*a^6
*b*(-a^4*d^2)^(1/2)))/(a^8*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 9*a^2*b^6*d*(4*a
*b + a^2 + 4*b^2)^(1/2) - 18*a^4*b^4*d*(4*a*b + a^2 + 4*b^2)^(1/2) - 6*a^5*
b^3*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 9*a^6*b^2*d*(4*a*b + a^2 + 4*b^2)^(1/2)
 + 6*a^7*b*d*(4*a*b + a^2 + 4*b^2)^(1/2)))*(4*a*b + a^2 + 4*b^2)^(1/2))/(-a
```

```
^4*d^2)^(1/2) - exp(c + d*x)/(a*d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*x)
)/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - ((-b)^(3/2)*log((64*(
exp(2*c + 2*d*x) + 1)*(3*a^2*b + a^3 - 3*b^3))/(a^5*(a - b)^2) - (128*exp(c
 + d*x)*(3*a^2*b + a^3 - 3*b^3))/(a^5*(-b)^(1/2)*(a - b)^(3/2))))/(2*a^2*d*
(a - b)^(1/2)) + ((-b)^(3/2)*log((64*(exp(2*c + 2*d*x) + 1)*(3*a^2*b + a^3
- 3*b^3))/(a^5*(a - b)^2) + (128*exp(c + d*x)*(3*a^2*b + a^3 - 3*b^3))/(a^5
*(-b)^(1/2)*(a - b)^(3/2))))/(2*a^2*d*(a - b)^(1/2))
```

3.39 $\int \frac{\mathbf{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}\,d}+\frac{(a+b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}$$

```
[Out] (a+b)*coth(d*x+c)/a^2/d-1/3*coth(d*x+c)^3/a/d+b^2*arctanh((a-b)^(1/2)*tanh(
d*x+c)/a^(1/2))/a^(5/2)/d/(a-b)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3266, 472, 214}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d\sqrt{a-b}}+\frac{(a+b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b]*d)
+ ((a + b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\Big(\int \frac{(1-x^2)^2}{x^4(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\Big)}{d} \\
&= \frac{\operatorname{Subst}\Big(\int \Big(\frac{1}{ax^4}+\frac{-a-b}{a^2x^2}+\frac{b^2}{a^2(a-(a-b)x^2)}\Big)\,dx, x, \tanh(c+dx)\Big)}{d} \\
&= \frac{(a+b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}+\frac{b^2\operatorname{Subst}\Big(\int \frac{1}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)}{a^2d} \\
&= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}\,d}+\frac{(a+b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}
\end{aligned}
$$

Mathematica [A]
time = 0.44, size = 126, normalized size = 1.62

$$
-\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2(c+dx)\left(-3b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\sqrt{a-b}\coth(c+dx)\left(-2a-3b+a\operatorname{csch}^2(c+dx)\right)\right)}{6a^{5/2}\sqrt{a-b}\,d\left(b+a\operatorname{csch}^2(c+dx)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/6*((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*(-3*b^2*ArcTanh[(Sqrt
[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a - b]*Coth[c + d*x]*(-2*a -
 3*b + a*Csch[c + d*x]^2)))/(a^(5/2)*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(68) = 136.
time = 1.56, size = 266, normalized size = 3.41

method	result
risch	$-\frac{2\left(-3b\,e^{4dx+4c}+6a\,e^{2dx+2c}+6b\,e^{2dx+2c}-2a-3b\right)}{3d\,a^2\left(e^{2dx+2c}-1\right)^3}+\frac{b^2\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}-2a^2+2ab}{b\sqrt{a^2-ab}}\right)}{2\sqrt{a^2-ab}\,d\,a^2}-$

derivativedivides	$\frac{-\frac{\frac{a\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{2}}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{-3a-4b}{8a^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^{2}\left(\frac{\left(\sqrt{-b\,(a-b)}\right.}{2a\sqrt{-b\,(a}}\right.}{}}{}$
default	$\frac{-\frac{\frac{a\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{2}}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{-3a-4b}{8a^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^{2}\left(\frac{\left(\sqrt{-b\,(a-b)}\right.}{2a\sqrt{-b\,(a}}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/a^2*(1/3*a*tanh(1/2*d*x+1/2*c)^3-3*a*tanh(1/2*d*x+1/2*c)-4*b*tanh
(1/2*d*x+1/2*c))-1/24/a/tanh(1/2*d*x+1/2*c)^3-1/8/a^2*(-3*a-4*b)/tanh(1/2*d
*x+1/2*c)-2*b^2/a*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-
a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(68) = 136$.
time = 0.48, size = 1972, normalized size = 25.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b - a*b^2)*sinh(d*x + c)^4 + 8*a^3 + 4*a^2*b - 1
2*a*b^2 - 24*(a^3 - a*b^2)*cosh(d*x + c)^2 - 24*(a^3 - a*b^2 - 3*(a^2*b - a
*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cos
h(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 +
3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4
*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*c
osh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^
2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c
))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*c
osh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^
2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(
d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)
*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 +
b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 +
2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*
sinh(d*x + c) + b)) + 48*((a^2*b - a*b^2)*cosh(d*x + c)^3 - (a^3 - a*b^2)*c
osh(d*x + c))*sinh(d*x + c))/((a^4 - a^3*b)*d*cosh(d*x + c)^6 + 6*(a^4 - a^
3*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - a^3*b)*d*sinh(d*x + c)^6 - 3*
(a^4 - a^3*b)*d*cosh(d*x + c)^4 + 3*(5*(a^4 - a^3*b)*d*cosh(d*x + c)^2 - (a
^4 - a^3*b)*d)*sinh(d*x + c)^4 + 3*(a^4 - a^3*b)*d*cosh(d*x + c)^2 + 4*(5*(
a^4 - a^3*b)*d*cosh(d*x + c)^3 - 3*(a^4 - a^3*b)*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(5*(a^4 - a^3*b)*d*cosh(d*x + c)^4 - 6*(a^4 - a^3*b)*d*cosh(d*x
+ c)^2 + (a^4 - a^3*b)*d)*sinh(d*x + c)^2 - (a^4 - a^3*b)*d + 6*((a^4 - a^3
*b)*d*cosh(d*x + c)^5 - 2*(a^4 - a^3*b)*d*cosh(d*x + c)^3 + (a^4 - a^3*b)*d
*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 24
*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b - a*b^2)*sinh(d*x
 + c)^4 + 4*a^3 + 2*a^2*b - 6*a*b^2 - 12*(a^3 - a*b^2)*cosh(d*x + c)^2 - 12
*(a^3 - a*b^2 - 3*(a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(b^2
*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^
6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4
 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*s
inh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2
*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c
```

```
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-
a^2 + a*b)/(a^2 - a*b)) + 24*((a^2*b - a*b^2)*cosh(d*x + c)^3 - (a^3 - a*b^
2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 - a^3*b)*d*cosh(d*x + c)^6 + 6*(a^4
- a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - a^3*b)*d*sinh(d*x + c)^6
- 3*(a^4 - a^3*b)*d*cosh(d*x + c)^4 + 3*(5*(a^4 - a^3*b)*d*cosh(d*x + c)^2
- (a^4 - a^3*b)*d)*sinh(d*x + c)^4 + 3*(a^4 - a^3*b)*d*cosh(d*x + c)^2 + 4*
(5*(a^4 - a^3*b)*d*cosh(d*x + c)^3 - 3*(a^4 - a^3*b)*d*cosh(d*x + c))*sinh(
d*x + c)^3 + 3*(5*(a^4 - a^3*b)*d*cosh(d*x + c)^4 - 6*(a^4 - a^3*b)*d*cosh(
d*x + c)^2 + (a^4 - a^3*b)*d)*sinh(d*x + c)^2 - (a^4 - a^3*b)*d + 6*((a^4 -
 a^3*b)*d*cosh(d*x + c)^5 - 2*(a^4 - a^3*b)*d*cosh(d*x + c)^3 + (a^4 - a^3*
b)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*sinh(c + d*x)**2), x)
```

Giac [**A**]
time = 0.70, size = 118, normalized size = 1.51

$$\frac{\frac{3\,b^2\arctan\left(\frac{b e^{(2\,dx+2\,c)}+2\,a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,a^2}+\frac{2\left(3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+2\,a+3\,b\right)}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt
(-a^2 + a*b)*a^2) + 2*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 6*b*e^(2
*d*x + 2*c) + 2*a + 3*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [**B**]
time = 1.18, size = 350, normalized size = 4.49

$$\frac{2\,b}{a^2\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{8}{3\,a\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}-\frac{4}{a\,d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{b^2\,\ln\left(\frac{4\,b^2\,\left(2\,a\,b-b^2+8\,a^2\,{\mathrm{e}}^{2\,c+2\,d\,x}+b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}-8\,a\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{a^5\,\left(a-b\right)}-\frac{8\,b^2\,\left(b+4\,a\,{\mathrm{e}}^{2\,c+2\,d\,x}-2\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{a^{9/2}\,\sqrt{a-b}}\right)}{2\,a^{5/2}\,d\,\sqrt{a-b}}+\frac{b^2\,\ln\left(\frac{4\,b^2\,\left(2\,a\,b-b^2+8\,a^2\,{\mathrm{e}}^{2\,c+2\,d\,x}+b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}-8\,a\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{a^5\,\left(a-b\right)}+\frac{8\,b^2\,\left(b+4\,a\,{\mathrm{e}}^{2\,c+2\,d\,x}-2\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{a^{9/2}\,\sqrt{a-b}}\right)}{2\,a^{5/2}\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (2*b)/(a^2*d*(exp(2*c + 2*d*x) - 1)) - 8/(3*a*d*(3*exp(2*c + 2*d*x) - 3*exp
(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - 4/(a*d*(exp(4*c + 4*d*x) - 2*exp(2
*c + 2*d*x) + 1)) - (b^2*log((4*b^2*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) +
 b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a^5*(a - b)) - (8*b^2*(b
+ 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(9/2)*(a - b)^(1/2))))/(
2*a^(5/2)*d*(a - b)^(1/2)) + (b^2*log((4*b^2*(2*a*b - b^2 + 8*a^2*exp(2*c +
 2*d*x) + b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a^5*(a - b)) + (
8*b^2*(b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(9/2)*(a - b)^(
1/2))))/(2*a^(5/2)*d*(a - b)^(1/2))
```

3.40 $\int \frac{\mathbf{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=130

$$-\frac{b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3\sqrt{a-b}\,d}-\frac{(3a^2+4ab+8b^2)\tanh^{-1}(\cosh(c+dx))}{8a^3d}+\frac{(3a+4b)\coth(c+dx)\mathrm{csch}(c+d}{8a^2d}$$

```
[Out] -1/8*(3*a^2+4*a*b+8*b^2)*arctanh(cosh(d*x+c))/a^3/d+1/8*(3*a+4*b)*coth(d*x+
c)*csch(d*x+c)/a^2/d-1/4*coth(d*x+c)*csch(d*x+c)^3/a/d-b^(5/2)*arctan(cosh(
d*x+c)*b^(1/2)/(a-b)^(1/2))/a^3/d/(a-b)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3265, 425, 541, 536, 212, 211}

$$-\frac{b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3d\sqrt{a-b}}+\frac{(3a+4b)\coth(c+dx)\mathrm{csch}(c+dx)}{8a^2d}-\frac{(3a^2+4ab+8b^2)\tanh^{-1}(\cosh(c+dx))}{8a^3d}-\frac{\coth(c+dx)\mathrm{csch}^3(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a^3*Sqrt[a - b]*d)
) - ((3*a^2 + 4*a*b + 8*b^2)*ArcTanh[Cosh[c + d*x]])/(8*a^3*d) + ((3*a + 4*
b)*Coth[c + d*x]*Csch[c + d*x])/(8*a^2*d) - (Coth[c + d*x]*Csch[c + d*x]^3)
/(4*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a-b+bx^2)}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= -\frac{\coth(c+dx)\operatorname{csch}^3(c+dx)}{4ad} - \frac{\text{Subst}\left(\int \frac{3a+b+3bx^2}{(1-x^2)^2(a-b+bx^2)}\,dx, x, \cosh(c+dx)\right)}{4ad}\\
&= \frac{(3a+4b)\coth(c+dx)\operatorname{csch}(c+dx)}{8a^2d} - \frac{\coth(c+dx)\operatorname{csch}^3(c+dx)}{4ad} -\\
&= \frac{(3a+4b)\coth(c+dx)\operatorname{csch}(c+dx)}{8a^2d} - \frac{\coth(c+dx)\operatorname{csch}^3(c+dx)}{4ad} -\\
&= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3\sqrt{a-b}\,d} - \frac{(3a^2+4ab+8b^2)\tanh^{-1}(\cosh(c+dx))}{8a^3d} +
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.25, size = 295, normalized size = 2.27

$$-\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2(c+dx)\left(64b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+64b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)-2a\sqrt{a-b}\,(3a+4b)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+a^2\sqrt{a-b}\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)-8\sqrt{a-b}\,(3a^2+4ab+8b^2)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-2a\sqrt{a-b}\,(3a+4b)\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)-a^2\sqrt{a-b}\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)\right)}{128a^3\sqrt{a-b}\,d\left(b+a\operatorname{csch}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/128*((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*(64*b^(5/2)*ArcTan[
(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + 64*b^(5/2)*ArcTan[(S
qrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] - 2*a*Sqrt[a - b]*(3*a +
 4*b)*Csch[(c + d*x)/2]^2 + a^2*Sqrt[a - b]*Csch[(c + d*x)/2]^4 - 8*Sqrt[a
- b]*(3*a^2 + 4*a*b + 8*b^2)*Log[Tanh[(c + d*x)/2]] - 2*a*Sqrt[a - b]*(3*a
+ 4*b)*Sech[(c + d*x)/2]^2 - a^2*Sqrt[a - b]*Sech[(c + d*x)/2]^4))/(a^3*Sqr
t[a - b]*d*(b + a*Csch[c + d*x]^2))
```

Maple [A]
time = 5.52, size = 156, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a-4b\right)^2}{64a^3}-\frac{-4a-4b}{32a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{1}{64a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{(6a^2+8ab+16b^2)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a^3}-\frac{b^3\arctan\left(2a\right.}{a^3}}{d}$
default	$\frac{\frac{\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a-4b\right)^2}{64a^3}-\frac{-4a-4b}{32a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{1}{64a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{(6a^2+8ab+16b^2)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a^3}-\frac{b^3\arctan\left(2a\right.}{a^3}}{d}$
risch	$\frac{e^{dx+c}\left(3a\,e^{6dx+6c}+4b\,e^{6dx+6c}-11a\,e^{4dx+4c}-4b\,e^{4dx+4c}-11a\,e^{2dx+2c}-4b\,e^{2dx+2c}+3a+4b\right)}{4d\,a^2\left(e^{2dx+2c}-1\right)^4}+\frac{3\ln\left(e^{dx+c}-1\right)}{8da}+b\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/64*(a*tanh(1/2*d*x+1/2*c)^2-4*a-4*b)^2/a^3-1/32*(-4*a-4*b)/a^2/tanh(
1/2*d*x+1/2*c)^2-1/64/a/tanh(1/2*d*x+1/2*c)^4+1/16/a^3*(6*a^2+8*a*b+16*b^2)
*ln(tanh(1/2*d*x+1/2*c))-b^3/a^3/(a*b-b^2)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d
*x+1/2*c)^2-2*a+4*b)/(a*b-b^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*((3*a*e^(7*c) + 4*b*e^(7*c))*e^(7*d*x) - (11*a*e^(5*c) + 4*b*e^(5*c))*e
^(5*d*x) - (11*a*e^(3*c) + 4*b*e^(3*c))*e^(3*d*x) + (3*a*e^c + 4*b*e^c)*e^(
d*x))/(a^2*d*e^(8*d*x + 8*c) - 4*a^2*d*e^(6*d*x + 6*c) + 6*a^2*d*e^(4*d*x +
 4*c) - 4*a^2*d*e^(2*d*x + 2*c) + a^2*d) - 1/8*(3*a^2 + 4*a*b + 8*b^2)*log(
(e^(d*x + c) + 1)*e^(-c))/(a^3*d) + 1/8*(3*a^2 + 4*a*b + 8*b^2)*log((e^(d*x
 + c) - 1)*e^(-c))/(a^3*d) - 32*integrate(1/16*(b^3*e^(3*d*x + 3*c) - b^3*e
^(d*x + c))/(a^3*b*e^(4*d*x + 4*c) + a^3*b + 2*(2*a^4*e^(2*c) - a^3*b*e^(2*
c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2976 vs. $2(116) = 232$.
time = 0.55, size = 5809, normalized size = 44.68

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(3*a^2 + 4*a*b)*cosh(d*x + c)^7 + 14*(3*a^2 + 4*a*b)*cosh(d*x + c)*
sinh(d*x + c)^6 + 2*(3*a^2 + 4*a*b)*sinh(d*x + c)^7 - 2*(11*a^2 + 4*a*b)*co
sh(d*x + c)^5 + 2*(21*(3*a^2 + 4*a*b)*cosh(d*x + c)^2 - 11*a^2 - 4*a*b)*sin
h(d*x + c)^5 + 10*(7*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 - (11*a^2 + 4*a*b)*cos
h(d*x + c))*sinh(d*x + c)^4 - 2*(11*a^2 + 4*a*b)*cosh(d*x + c)^3 + 2*(35*(3
*a^2 + 4*a*b)*cosh(d*x + c)^4 - 10*(11*a^2 + 4*a*b)*cosh(d*x + c)^2 - 11*a^
2 - 4*a*b)*sinh(d*x + c)^3 + 2*(21*(3*a^2 + 4*a*b)*cosh(d*x + c)^5 - 10*(11
*a^2 + 4*a*b)*cosh(d*x + c)^3 - 3*(11*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x
+ c)^2 + 4*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2
*sinh(d*x + c)^8 - 4*b^2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - b^2)*
sinh(d*x + c)^6 + 6*b^2*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^3 - 3*b^2*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 - 30*b^2*cosh(d*
x + c)^2 + 3*b^2)*sinh(d*x + c)^4 - 4*b^2*cosh(d*x + c)^2 + 8*(7*b^2*cosh(d
*x + c)^5 - 10*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*(7*b^2*cosh(d*x + c)^6 - 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d*x + c)^2
- b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 - 3*b^2*cosh(d*x + c)
^5 + 3*b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a -
 b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*
x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3
*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sin
h(d*x + c) - 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x
+ c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(
d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c)^4
 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh
(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh
(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 2*(3*a^2 + 4*a
```

```
*b)*cosh(d*x + c) - ((3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 + 4
*a*b + 8*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 + 4*a*b + 8*b^2)*sinh(
d*x + c)^8 - 4*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 + 4*a*
b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^6 + 8*(7*
(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*
x + c))*sinh(d*x + c)^5 + 6*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35
*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 - 30*(3*a^2 + 4*a*b + 8*b^2)*cosh(
d*x + c)^2 + 9*a^2 + 12*a*b + 24*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 + 4*a*b
 + 8*b^2)*cosh(d*x + c)^5 - 10*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 + 3*
(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a^2 + 4*a*b +
 8*b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 - 15
*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 9*(3*a^2 + 4*a*b + 8*b^2)*cosh(d
*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + 8*b^2
+ 8*((3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^7 - 3*(3*a^2 + 4*a*b + 8*b^2)*co
sh(d*x + c)^5 + 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 - (3*a^2 + 4*a*b
+ 8*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) +
1) + ((3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 + 4*a*b + 8*b^2)*c
osh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 + 4*a*b + 8*b^2)*sinh(d*x + c)^8 - 4*
(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh
(d*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 4*a*b
+ 8*b^2)*cosh(d*x + c)^3 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*
x + c)^5 + 6*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 + 4*a*b
 + 8*b^2)*cosh(d*x + c)^4 - 30*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 + 9*
a^2 + 12*a*b + 24*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(
d*x + c)^5 - 10*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 4*a*b
+ 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*
x + c)^2 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 - 15*(3*a^2 + 4*a*b
 + 8*b^2)*cosh(d*x + c)^4 + 9*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a
^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + 8*b^2 + 8*((3*a^2 + 4
*a*b + 8*b^2)*cosh(d*x + c)^7 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^5 +
 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 - (3*a^2 + 4*a*b + 8*b^2)*cosh(d
*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(3*a^
2 + 4*a*b)*cosh(d*x + c)^6 - 5*(11*a^2 + 4*a*b)*cosh(d*x + c)^4 - 3*(11*a^2
 + 4*a*b)*cosh(d*x + c)^2 + 3*a^2 + 4*a*b)*sinh(d*x + c))/(a^3*d*cosh(d*x +
 c)^8 + 8*a^3*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d*sinh(d*x + c)^8 - 4*a
^3*d*cosh(d*x + c)^6 + 6*a^3*d*cosh(d*x + c)^4 + 4*(7*a^3*d*cosh(d*x + c)^2
 - a^3*d)*sinh(d*x + c)^6 - 4*a^3*d*cosh(d*x + c)^2 + 8*(7*a^3*d*cosh(d*x +
 c)^3 - 3*a^3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^3*d*cosh(d*x + c)^
4 - 30*a^3*d*cosh(d*x + c)^2 + 3*a^3*d)*sinh(d*x + c)^4 + a^3*d + 8*(7*a^3*
d*cosh(d*x + c)^5 - 10*a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(7*a^3*d*cosh(d*x + c)^6 - 15*a^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 5.66, size = 1639, normalized size = 12.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^5*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (exp(c + d*x)*(4*a*b + 3*a^2))/(4*a^3*d*(exp(2*c + 2*d*x) - 1)) - (atan((ex
p(d*x)*exp(c)*(243*a^12*(-a^6*d^2)^(1/2) + 18432*b^12*(-a^6*d^2)^(1/2) + 69
12*a^2*b^10*(-a^6*d^2)^(1/2) - 30720*a^3*b^9*(-a^6*d^2)^(1/2) - 26880*a^4*b
^8*(-a^6*d^2)^(1/2) - 24192*a^5*b^7*(-a^6*d^2)^(1/2) + 5024*a^6*b^6*(-a^6*d
^2)^(1/2) + 13408*a^7*b^5*(-a^6*d^2)^(1/2) + 17160*a^8*b^4*(-a^6*d^2)^(1/2)
 + 9540*a^9*b^3*(-a^6*d^2)^(1/2) + 4563*a^10*b^2*(-a^6*d^2)^(1/2) + 9216*a*
b^11*(-a^6*d^2)^(1/2) + 1134*a^11*b*(-a^6*d^2)^(1/2)))/(81*a^13*d*(64*a*b^3
 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 2304*a^3*b^10*d*(64*a*b^
3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) - 3840*a^6*b^7*d*(64*a*b^
3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) - 1440*a^7*b^6*d*(64*a*b^
3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) - 864*a^8*b^5*d*(64*a*b^3
 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 1600*a^9*b^4*d*(64*a*b^3
 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 1200*a^10*b^3*d*(64*a*b^
3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 945*a^11*b^2*d*(64*a*b^
3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 270*a^12*b*d*(64*a*b^3
+ 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2)))*(64*a*b^3 + 24*a^3*b + 9*
a^4 + 64*b^4 + 64*a^2*b^2)^(1/2))/(4*(-a^6*d^2)^(1/2)) - (6*exp(c + d*x))/(
a*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*
exp(c + d*x))/(a*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6
```

```
*d*x) + exp(8*c + 8*d*x) + 1)) - ((2*atan((b^3*exp(d*x)*exp(c)*(a^6*d^2*(a
- b))^(1/2)*(15*a^4*b + 9*a^5 - 48*b^5 + 40*a^3*b^2))/(2*a^3*d*(b^5)^(1/2)*
(6*a^5*b - 48*a*b^5 + 9*a^6 + 48*b^6 - 40*a^3*b^3 + 25*a^4*b^2))) + 2*atan(
(exp(d*x)*exp(c)*((4*(18*a^9*d*(b^5)^(1/2) - 96*a^4*d*(b^5)^(3/2) + 96*a^3*
b*d*(b^5)^(3/2) + 12*a^8*b*d*(b^5)^(1/2) - 80*a^6*b^3*d*(b^5)^(1/2) + 50*a^
7*b^2*d*(b^5)^(1/2)))/(a^8*b^4*(a - b)*(a^7*d^2 - a^6*b*d^2)^(1/2)*(a*b - a
^2)*(a^6*d^2*(a - b))^(1/2)*(15*a^4*b + 9*a^5 - 48*b^5 + 40*a^3*b^2)) + (2*
(40*a^3*b^5*(a^7*d^2 - a^6*b*d^2)^(1/2) - 48*b^8*(a^7*d^2 - a^6*b*d^2)^(1/2
) + 15*a^4*b^4*(a^7*d^2 - a^6*b*d^2)^(1/2) + 9*a^5*b^3*(a^7*d^2 - a^6*b*d^2
)^(1/2)))/(a^11*b*d*(a - b)*(a^7*d^2 - a^6*b*d^2)^(1/2)*(a*b - a^2)*(b^5)^(
1/2)*(6*a^5*b - 48*a*b^5 + 9*a^6 + 48*b^6 - 40*a^3*b^3 + 25*a^4*b^2))) + (2
*exp(3*c)*exp(3*d*x)*(40*a^3*b^5*(a^7*d^2 - a^6*b*d^2)^(1/2) - 48*b^8*(a^7*
d^2 - a^6*b*d^2)^(1/2) + 15*a^4*b^4*(a^7*d^2 - a^6*b*d^2)^(1/2) + 9*a^5*b^3
*(a^7*d^2 - a^6*b*d^2)^(1/2)))/(a^11*b*d*(a - b)*(a^7*d^2 - a^6*b*d^2)^(1/2
)*(a*b - a^2)*(b^5)^(1/2)*(6*a^5*b - 48*a*b^5 + 9*a^6 + 48*b^6 - 40*a^3*b^3
 + 25*a^4*b^2)))*((a^11*b*(a^7*d^2 - a^6*b*d^2)^(1/2))/4 + (a^9*b^3*(a^7*d^
2 - a^6*b*d^2)^(1/2))/4 - (a^10*b^2*(a^7*d^2 - a^6*b*d^2)^(1/2))/2)))*(b^5)
^(1/2))/(2*(a^7*d^2 - a^6*b*d^2)^(1/2)) - (exp(c + d*x)*(a - 4*b))/(2*a^2*d
*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.41 $\int \frac{\operatorname{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a-b}\,d}-\frac{(a^2+ab+b^2)\coth(c+dx)}{a^3d}+\frac{(2a+b)\coth^3(c+dx)}{3a^2d}-\frac{\coth^5(c+dx)}{5ad}$$

```
[Out] -(a^2+a*b+b^2)*coth(d*x+c)/a^3/d+1/3*(2*a+b)*coth(d*x+c)^3/a^2/d-1/5*coth(d
*x+c)^5/a/d-b^3*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2)/d/(a-b)^(1
/2)
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3266, 472, 214}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d\sqrt{a-b}}+\frac{(2a+b)\coth^3(c+dx)}{3a^2d}-\frac{(a^2+ab+b^2)\coth(c+dx)}{a^3d}-\frac{\coth^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*Sqrt[a - b]*d
)) - ((a^2 + a*b + b^2)*Coth[c + d*x])/(a^3*d) + ((2*a + b)*Coth[c + d*x]^3
)/(3*a^2*d) - Coth[c + d*x]^5/(5*a*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
```

```
IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^6\left(a-(a-b)x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^6}+\frac{-2a-b}{a^2x^4}+\frac{a^2+ab+b^2}{a^3x^2}+\frac{b^3}{a^3\left(-a+(a-b)x^2\right)}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{\left(a^2+ab+b^2\right)\coth(c+dx)}{a^3d}+\frac{(2a+b)\coth^3(c+dx)}{3a^2d}-\frac{\coth^5(c+dx)}{5ad}+\frac{b^3\mathrm{S}}{}\\
&= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a-b}\,d}-\frac{\left(a^2+ab+b^2\right)\coth(c+dx)}{a^3d}+\frac{(2a+b)\,\mathrm{cot}}{3a^2}
\end{aligned}
$$

Mathematica [**A**]
time = 0.95, size = 155, normalized size = 1.41

$$
-\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2(c+dx)\left(15b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\,\sqrt{a-b}\,\coth(c+dx)\left(8a^2+10ab+15b^2-a(4a+5b)\operatorname{csch}^2(c+dx)+3a^2\operatorname{csch}^4(c+dx)\right)\right)}{30a^{7/2}\sqrt{a-b}\,d\left(b+a\operatorname{csch}^2(c+dx)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/30*((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*(15*b^3*ArcTanh[(Sqr
t[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a - b]*Coth[c + d*x]*(8*a^2
 + 10*a*b + 15*b^2 - a*(4*a + 5*b)*Csch[c + d*x]^2 + 3*a^2*Csch[c + d*x]^4)
))/(a^(7/2)*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(98) = 196.
time = 1.58, size = 347, normalized size = 3.15

method	result
risch	$-\frac{2\left(15b^2\mathrm{e}^{8dx+8c}-30ab\,\mathrm{e}^{6dx+6c}-60b^2\mathrm{e}^{6dx+6c}+80a^2\mathrm{e}^{4dx+4c}+70ab\,\mathrm{e}^{4dx+4c}+90b^2\mathrm{e}^{4dx+4c}-40a^2\mathrm{e}^{2dx+2c}-50ab\,\mathrm{e}^{2dx+2c}\right.}{15d\,a^3\left(\mathrm{e}^{2dx+2c}-1\right)^5}$

derivativedivides	$-\frac{\frac{a^{2}\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{5a^{2}\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{4a\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{3}+10a^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+12a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b+16b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32a^{3}}+$
default	$-\frac{\frac{a^{2}\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{5a^{2}\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{4a\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{3}+10a^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+12a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b+16b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32a^{3}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/32/a^3*(1/5*a^2*tanh(1/2*d*x+1/2*c)^5-5/3*a^2*tanh(1/2*d*x+1/2*c)^3
-4/3*a*tanh(1/2*d*x+1/2*c)^3*b+10*a^2*tanh(1/2*d*x+1/2*c)+12*a*tanh(1/2*d*x
+1/2*c)*b+16*b^2*tanh(1/2*d*x+1/2*c))+2*b^3/a^2*(1/2*((-b*(a-b))^(1/2)+b)/a
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(
-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)))-1/160/a/tanh(1/2*d*x+1/2*c)^
5-1/96*(-5*a-4*b)/a^2/tanh(1/2*d*x+1/2*c)^3-1/32/a^3*(10*a^2+12*a*b+16*b^2)
/tanh(1/2*d*x+1/2*c))
```

<u>Maxima</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2142 vs. $2(98) = 196$.
time = 0.48, size = 4540, normalized size = 41.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(60*(a^2*b^2 - a*b^3)*cosh(d*x + c)^8 + 480*(a^2*b^2 - a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^7 + 60*(a^2*b^2 - a*b^3)*sinh(d*x + c)^8 - 120*(a^3*b
 + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^6 - 120*(a^3*b + a^2*b^2 - 2*a*b^3 - 14
*(a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 240*(14*(a^2*b^2 - a*
b^3)*cosh(d*x + c)^3 - 3*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c))*sinh(d*
x + c)^5 + 40*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^4 + 40*(1
05*(a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3
- 45*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*a^4
+ 8*a^3*b + 20*a^2*b^2 - 60*a*b^3 + 160*(21*(a^2*b^2 - a*b^3)*cosh(d*x + c)
^5 - 15*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^3 + (8*a^4 - a^3*b + 2*a^
2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 40*(4*a^4 + a^3*b + a^2*b
^2 - 6*a*b^3)*cosh(d*x + c)^2 + 40*(42*(a^2*b^2 - a*b^3)*cosh(d*x + c)^6 -
45*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^4 - 4*a^4 - a^3*b - a^2*b^2 +
6*a*b^3 + 6*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 - 15*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 +
 b^3*sinh(d*x + c)^10 - 5*b^3*cosh(d*x + c)^8 + 10*b^3*cosh(d*x + c)^6 + 5*
(9*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^8 + 40*(3*b^3*cosh(d*x + c)^3 -
 b^3*cosh(d*x + c))*sinh(d*x + c)^7 - 10*b^3*cosh(d*x + c)^4 + 10*(21*b^3*c
osh(d*x + c)^4 - 14*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^6 + 4*(63*b^3*
cosh(d*x + c)^5 - 70*b^3*cosh(d*x + c)^3 + 15*b^3*cosh(d*x + c))*sinh(d*x +
 c)^5 + 5*b^3*cosh(d*x + c)^2 + 10*(21*b^3*cosh(d*x + c)^6 - 35*b^3*cosh(d*
x + c)^4 + 15*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^4 + 40*(3*b^3*cosh(d
*x + c)^7 - 7*b^3*cosh(d*x + c)^5 + 5*b^3*cosh(d*x + c)^3 - b^3*cosh(d*x +
c))*sinh(d*x + c)^3 - b^3 + 5*(9*b^3*cosh(d*x + c)^8 - 28*b^3*cosh(d*x + c)
^6 + 30*b^3*cosh(d*x + c)^4 - 12*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2
 + 10*(b^3*cosh(d*x + c)^9 - 4*b^3*cosh(d*x + c)^7 + 6*b^3*cosh(d*x + c)^5
- 4*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)
*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(
d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2
*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3
+ (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
```

```
+ 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)
) + 80*(6*(a^2*b^2 - a*b^3)*cosh(d*x + c)^7 - 9*(a^3*b + a^2*b^2 - 2*a*b^3)
*cosh(d*x + c)^5 + 2*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^3
- (4*a^4 + a^3*b + a^2*b^2 - 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 -
 a^4*b)*d*cosh(d*x + c)^10 + 10*(a^5 - a^4*b)*d*cosh(d*x + c)*sinh(d*x + c)
^9 + (a^5 - a^4*b)*d*sinh(d*x + c)^10 - 5*(a^5 - a^4*b)*d*cosh(d*x + c)^8 +
 5*(9*(a^5 - a^4*b)*d*cosh(d*x + c)^2 - (a^5 - a^4*b)*d)*sinh(d*x + c)^8 +
10*(a^5 - a^4*b)*d*cosh(d*x + c)^6 + 40*(3*(a^5 - a^4*b)*d*cosh(d*x + c)^3
- (a^5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*(a^5 - a^4*b)*d*c
osh(d*x + c)^4 - 14*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + (a^5 - a^4*b)*d)*sinh
(d*x + c)^6 - 10*(a^5 - a^4*b)*d*cosh(d*x + c)^4 + 4*(63*(a^5 - a^4*b)*d*co
sh(d*x + c)^5 - 70*(a^5 - a^4*b)*d*cosh(d*x + c)^3 + 15*(a^5 - a^4*b)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 10*(21*(a^5 - a^4*b)*d*cosh(d*x + c)^6 - 35*(
a^5 - a^4*b)*d*cosh(d*x + c)^4 + 15*(a^5 - a^4*b)*d*cosh(d*x + c)^2 - (a^5
- a^4*b)*d)*sinh(d*x + c)^4 + 5*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + 40*(3*(a^
5 - a^4*b)*d*cosh(d*x + c)^7 - 7*(a^5 - a^4*b)*d*cosh(d*x + c)^5 + 5*(a^5 -
 a^4*b)*d*cosh(d*x + c)^3 - (a^5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^3
+ 5*(9*(a^5 - a^4*b)*d*cosh(d*x + c)^8 - 28*(a^5 - a^4*b)*d*cosh(d*x + c)^6
 + 30*(a^5 - a^4*b)*d*cosh(d*x + c)^4 - 12*(a^5 - a^4*b)*d*cosh(d*x + c)^2
+ (a^5 - a^4*b)*d)*sinh(d*x + c)^2 - (a^5 - a^4*b)*d + 10*((a^5 - a^4*b)*d*
cosh(d*x + c)^9 - 4*(a^5 - a^4*b)*d*cosh(d*x + c)^7 + 6*(a^5 - a^4*b)*d*cos
h(d*x + c)^5 - 4*(a^5 - a^4*b)*d*cosh(d*x + c)^3 + (a^5 - a^4*b)*d*cosh(d*x
 + c))*sinh(d*x + c)), -1/15*(30*(a^2*b^2 - a*b^3)*cosh(d*x + c)^8 + 240*(a
^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + 30*(a^2*b^2 - a*b^3)*sinh(d
*x + c)^8 - 60*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^6 - 60*(a^3*b + a^
2*b^2 - 2*a*b^3 - 14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 1
20*(14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^3 - 3*(a^3*b + a^2*b^2 - 2*a*b^3)*co
sh(d*x + c))*sinh(d*x + c)^5 + 20*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cos
h(d*x + c)^4 + 20*(105*(a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 8*a^4 - a^3*b +
2*a^2*b^2 - 9*a*b^3 - 45*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 16*a^4 + 4*a^3*b + 10*a^2*b^2 - 30*a*b^3 + 80*(21*(a^2*b^2 - a
*b^3)*cosh(d*x + c)^5 - 15*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^3 + (8
*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(98) = 196.

time = 0.69, size = 213, normalized size = 1.94

$$-\frac{\frac{15\,b^{3}\arctan\left(\frac{b e^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{\sqrt{-a^{2}+ab}\,a^{3}}+\frac{2\,\left(15\,b^{2}e^{(8\,dx+8\,c)}-30\,abe^{(6\,dx+6\,c)}-60\,b^{2}e^{(6\,dx+6\,c)}+80\,a^{2}e^{(4\,dx+4\,c)}+70\,abe^{(4\,dx+4\,c)}+90\,b^{2}e^{(4\,dx+4\,c)}-40\,a^{2}e^{(2\,dx+2\,c)}-50\,abe^{(2\,dx+2\,c)}-60\,b^{2}e^{(2\,dx+2\,c)}+8\,a^{2}+10\,ab+15\,b^{2}\right)}{a^{3}\left(e^{(2\,dx+2\,c)}-1\right)^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*b^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(s
qrt(-a^2 + a*b)*a^3) + 2*(15*b^2*e^(8*d*x + 8*c) - 30*a*b*e^(6*d*x + 6*c) -
 60*b^2*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x + 4*c) + 70*a*b*e^(4*d*x + 4*c) +
 90*b^2*e^(4*d*x + 4*c) - 40*a^2*e^(2*d*x + 2*c) - 50*a*b*e^(2*d*x + 2*c) -
 60*b^2*e^(2*d*x + 2*c) + 8*a^2 + 10*a*b + 15*b^2)/(a^3*(e^(2*d*x + 2*c) -
1)^5))/d
```

Mupad [B]
time = 1.21, size = 479, normalized size = 4.35

$$\frac{4\,b}{a^{2}\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{32}{5\,a\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1\right)}-\frac{2\,b^{2}}{a^{3}\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{8\,\left(4\,a-b\right)}{3\,a^{2}\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{16}{a\,d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}+\frac{b^{3}\,\ln\left(\frac{4\,b^{4}\,\left(2\,a\,b-b^{2}+8\,a^{2}\,\mathrm{e}^{2\,c+2\,d\,x}+b^{2}\,\mathrm{e}^{2\,c+2\,d\,x}-8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^{7}\,\left(a-b\right)}-\frac{8\,b^{4}\,\left(b+4\,a\,\mathrm{e}^{2\,c+2\,d\,x}-2\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^{13/2}\,\sqrt{a-b}}\right)}{2\,a^{7/2}\,d\,\sqrt{a-b}}-\frac{b^{3}\,\ln\left(\frac{4\,b^{4}\,\left(2\,a\,b-b^{2}+8\,a^{2}\,\mathrm{e}^{2\,c+2\,d\,x}+b^{2}\,\mathrm{e}^{2\,c+2\,d\,x}-8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^{7}\,\left(a-b\right)}+\frac{8\,b^{4}\,\left(b+4\,a\,\mathrm{e}^{2\,c+2\,d\,x}-2\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^{13/2}\,\sqrt{a-b}}\right)}{2\,a^{7/2}\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^6*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (4*b)/(a^2*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - 32/(5*a*d*(5*ex
p(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*
d*x) + exp(10*c + 10*d*x) - 1)) - (2*b^2)/(a^3*d*(exp(2*c + 2*d*x) - 1)) -
(8*(4*a - b))/(3*a^2*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c +
 6*d*x) - 1)) - 16/(a*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*
c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (b^3*log((4*b^4*(2*a*b - b^2 + 8*a^2*
exp(2*c + 2*d*x) + b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a^7*(a
- b)) - (8*b^4*(b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(13/2)
*(a - b)^(1/2))))/(2*a^(7/2)*d*(a - b)^(1/2)) - (b^3*log((4*b^4*(2*a*b - b^
2 + 8*a^2*exp(2*c + 2*d*x) + b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x))
)/(a^7*(a - b)) + (8*b^4*(b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))
/(a^(13/2)*(a - b)^(1/2))))/(2*a^(7/2)*d*(a - b)^(1/2))
```

3.42 $\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=102

$$\frac{x}{b^2}-\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2(a-b)^{3/2}b^2d}-\frac{a\tanh(c+dx)}{2(a-b)bd\left(a-(a-b)\tanh^2(c+dx)\right)}$$

```
[Out] x/b^2-1/2*(2*a-3*b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/(a-b)^
(3/2)/b^2/d-1/2*a*tanh(d*x+c)/(a-b)/b/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {3266, 481, 536, 212, 214}

$$-\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2b^2d(a-b)^{3/2}}-\frac{a\tanh(c+dx)}{2bd(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] x/b^2 - (Sqrt[a]*(2*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/
(2*(a - b)^(3/2)*b^2*d) - (a*Tanh[c + d*x])/(2*(a - b)*b*d*(a - (a - b)*Tan
h[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a-(a-b)x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a\tanh(c+dx)}{2(a-b)bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\Big(\int \frac{a+(a-2b)x^2}{(1-x^2)(a+(-a+b)x^2)}\,dx, x}{2(a-b)bd}$$

$$= -\frac{a\tanh(c+dx)}{2(a-b)bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\big(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx}{b^2d}$$

$$= \frac{x}{b^2} - \frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2(a-b)^{3/2}b^2d} - \frac{a\tanh(c+d}{2(a-b)bd\,\big(a-(a-b)\,\text{t}}$$

Mathematica [A]
time = 0.58, size = 99, normalized size = 0.97

$$-\frac{-2(c+dx)+\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}}+\frac{ab\sinh(2(c+dx))}{(a-b)(2a-b+b\cosh(2(c+dx)))}}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*(-2*(c + d*x) + (Sqrt[a]*(2*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x
])/Sqrt[a]])/(a - b)^(3/2) + (a*b*Sinh[2*(c + d*x)])/((a - b)*(2*a - b + b*
Cosh[2*(c + d*x)])))/(b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(90) = 180$.

time = 1.09, size = 319, normalized size = 3.13

method	result
risch	$\frac{x}{b^2}+\frac{a\left(2a\,e^{2dx+2c}-b\,e^{2dx+2c}+b\right)}{b^2(a-b)d\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)}+\frac{\sqrt{a\left(a-b\right)}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{a\left(a-b\right)}+2a-b}{b}\right)a}{2(a-b)^2 d\,b^2}-$
derivativedivides	$\frac{2a\left(\frac{-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a-b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+(2a-3b)a\left(\frac{\left(\sqrt{-b\left(a-b\right)}+b\right)\arctan\left(\frac{}{\sqrt{\left(2\sqrt{-}\right.}}\right.}{2a\sqrt{-b\left(a-b\right)}\sqrt{\left(2\sqrt{-b\left(a\right.}\right.}}\right.\right.}{b^2}$

default	$2a\left(\frac{-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a-b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(2a-3b)a\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{}{\sqrt{\left(2\sqrt{}\right)}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b}\right)}}\right)}{}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a/b^2*((-1/2*b/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/2*b/(a-b)*tanh(1/2*d*x+
1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x
+1/2*c)^2+a)+1/2*(2*a-3*b)/(a-b)*a*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(
1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/
2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))))+1/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/b^2*ln
(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(91) = 182$.

time = 0.49, size = 1772, normalized size = 17.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b - b^2)*d*x*cosh(d*x + c)^4 + 16*(a*b - b^2)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a*b - b^2)*d*x*sinh(d*x + c)^4 + 4*(a*b - b^2)*d*x + 4
*(2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*cosh(d*x + c)^2 + 4*(6*(a*b -
b^2)*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*sinh(
d*x + c)^2 + ((2*a*b - 3*b^2)*cosh(d*x + c)^4 + 4*(2*a*b - 3*b^2)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (2*a*b - 3*b^2)*sinh(d*x + c)^4 + 2*(4*a^2 - 8*a*b +
 3*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a^2 - 8*
a*b + 3*b^2)*sinh(d*x + c)^2 + 2*a*b - 3*b^2 + 4*((2*a*b - 3*b^2)*cosh(d*x
+ c)^3 + (4*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a -
b))*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*si
nh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2
+ 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)
^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a*b - b^2)*cosh(d*x +
 c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x +
c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/(a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(
d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2
+ 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3
+ (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*a*b + 8*(2*(a*b - b^2)*d
*x*cosh(d*x + c)^3 + (2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*cosh(d*x +
 c))*sinh(d*x + c))/((a*b^3 - b^4)*d*cosh(d*x + c)^4 + 4*(a*b^3 - b^4)*d*co
sh(d*x + c)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*sinh(d*x + c)^4 + 2*(2*a^2*b^
2 - 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a*b^3 - b^4)*d*cosh(d*x + c)^2
 + (2*a^2*b^2 - 3*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 - b^4)*d + 4*((a
*b^3 - b^4)*d*cosh(d*x + c)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*d*cosh(d*x + c)
)*sinh(d*x + c)), 1/2*(2*(a*b - b^2)*d*x*cosh(d*x + c)^4 + 8*(a*b - b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a*b - b^2)*d*x*sinh(d*x + c)^4 + 2*(a*
b - b^2)*d*x + 2*(2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*cosh(d*x + c)^
2 + 2*(6*(a*b - b^2)*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - 3*a*b + b^2)*d*x + 2*
a^2 - a*b)*sinh(d*x + c)^2 - ((2*a*b - 3*b^2)*cosh(d*x + c)^4 + 4*(2*a*b -
3*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b - 3*b^2)*sinh(d*x + c)^4 + 2*
(4*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - 3*b^2)*cosh(d*x + c
)^2 + 4*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^2 + 2*a*b - 3*b^2 + 4*((2*a*b -
3*b^2)*cosh(d*x + c)^3 + (4*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x +
c))*sqrt(-a/(a - b))*arctan(1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh
(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a/(a - b))/a) + 2*a*b + 4*(2
*(a*b - b^2)*d*x*cosh(d*x + c)^3 + (2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a
*b)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 - b^4)*d*cosh(d*x + c)^4 + 4*(a*b
^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*sinh(d*x + c)^4
```

```
 + 2*(2*a^2*b^2 - 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a*b^3 - b^4)*d*c
osh(d*x + c)^2 + (2*a^2*b^2 - 3*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 -
b^4)*d + 4*((a*b^3 - b^4)*d*cosh(d*x + c)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.36, size = 168, normalized size = 1.65

$$-\frac{\frac{\left(2\,a^{2}-3\,ab\right)\arctan\left(\frac{be^{\left(2\,dx+2\,c\right)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{\left(ab^{2}-b^{3}\right)\sqrt{-a^{2}+ab}}-\frac{2\,\left(2\,a^{2}e^{\left(2\,dx+2\,c\right)}-abe^{\left(2\,dx+2\,c\right)}+ab\right)}{\left(ab^{2}-b^{3}\right)\left(be^{\left(4\,dx+4\,c\right)}+4\,ae^{\left(2\,dx+2\,c\right)}-2\,be^{\left(2\,dx+2\,c\right)}+b\right)}-\frac{2\,\left(dx+c\right)}{b^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 - 3*a*b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 +
a*b))/((a*b^2 - b^3)*sqrt(-a^2 + a*b)) - 2*(2*a^2*e^(2*d*x + 2*c) - a*b*e^(
2*d*x + 2*c) + a*b)/((a*b^2 - b^3)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c)
 - 2*b*e^(2*d*x + 2*c) + b)) - 2*(d*x + c)/b^2)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)^4}{{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^2, x)
```

3.43 $\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{(a-2b)\text{ArcTan}\left(\frac{\sqrt{b}\,\cosh(c+dx)}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}b^{3/2}d}-\frac{a\cosh(c+dx)}{2(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)}$$

```
[Out] 1/2*(a-2*b)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/b^(3/2)/d-1
/2*a*cosh(d*x+c)/(a-b)/b/d/(a-b+b*cosh(d*x+c)^2)
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 393, 211}

$$\frac{(a-2b)\text{ArcTan}\left(\frac{\sqrt{b}\,\cosh(c+dx)}{\sqrt{a-b}}\right)}{2b^{3/2}d(a-b)^{3/2}}-\frac{a\cosh(c+dx)}{2bd(a-b)\left(a+b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - 2*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*(a - b)^(3/2)*b^
(3/2)*d) - (a*Cosh[c + d*x])/(2*(a - b)*b*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1-x^2}{(a-b+bx^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= -\frac{a\cosh(c+dx)}{2(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)}+\frac{(a-2b)\operatorname{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cos\right.}{2(a-b)bd}\\ &= \frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}b^{3/2}d}-\frac{a\cosh(c+dx)}{2(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.43, size = 141, normalized size = 1.57

$$\frac{\frac{(a-2b)\left(\operatorname{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\operatorname{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{(a-b)^{3/2}}-\frac{2a\sqrt{b}\cosh(c+dx)}{(a-b)(2a-b+b\cosh(2(c+dx)))}}{2b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (((a - 2*b)*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] +
ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/(a - b)^(3/2)
 - (2*a*Sqrt[b]*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(
2*b^(3/2)*d)
```

Maple [A]
time = 1.04, size = 155, normalized size = 1.72

method	result
derivativedivides	$\frac{\frac{8(2a-4b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16a}{\left(16ab-16b^2\right)\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)}+\frac{4(2a-4b)\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{\left(16ab-16b^2\right)\sqrt{ab-b^2}}}{d}$
default	$\frac{\frac{8(2a-4b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16a}{\left(16ab-16b^2\right)\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)}+\frac{4(2a-4b)\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{\left(16ab-16b^2\right)\sqrt{ab-b^2}}}{d}$
risch	$-\frac{a\,\mathrm{e}^{dx+c}\left(1+\mathrm{e}^{2dx+2c}\right)}{bd(a-b)\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)}+\frac{\ln\left(\mathrm{e}^{2dx+2c}-\frac{2(a-b)\mathrm{e}^{dx+c}}{\sqrt{-ab+b^2}}+1\right)}{2\sqrt{-ab+b^2}\,(a-b)d}-\frac{\ln\left(\mathrm{e}^{2dx+2c}-\frac{2(a-b)\mathrm{e}^{d}}{\sqrt{-ab}}\right.}{4\sqrt{-ab+b^2}\,(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(8*((2*a-4*b)*tanh(1/2*d*x+1/2*c)^2-2*a)/(16*a*b-16*b^2)/(a*tanh(1/2*d*
x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+4*(2*a-4*
b)/(16*a*b-16*b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2-2*
a+4*b)/(a*b-b^2)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(a*e^(3*d*x + 3*c) + a*e^(d*x + c))/(a*b^2*d - b^3*d + (a*b^2*d*e^(4*c) -
b^3*d*e^(4*c))*e^(4*d*x) + 2*(2*a^2*b*d*e^(2*c) - 3*a*b^2*d*e^(2*c) + b^3*d
*e^(2*c))*e^(2*d*x)) + 1/8*integrate(8*((a*e^(3*c) - 2*b*e^(3*c))*e^(3*d*x)
 - (a*e^c - 2*b*e^c)*e^(d*x))/(a*b^2 - b^3 + (a*b^2*e^(4*c) - b^3*e^(4*c))*
e^(4*d*x) + 2*(2*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) + b^3*e^(2*c))*e^(2*d*x)),
 x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(78) = 156$.
time = 0.44, size = 1889, normalized size = 20.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b - a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^2 + 4*(a^2*b - a*b^2)*sinh(d*x + c)^3 + ((a*b - 2*b^2)*cosh(
d*x + c)^4 + 4*(a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b - 2*b^2)*
sinh(d*x + c)^4 + 2*(2*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a*b - 2
*b^2)*cosh(d*x + c)^2 + 2*a^2 - 5*a*b + 2*b^2)*sinh(d*x + c)^2 + a*b - 2*b^
2 + 4*((a*b - 2*b^2)*cosh(d*x + c)^3 + (2*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*
(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 -
(2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x
+ c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x +
```

```
 c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*
x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 +
2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 +
(2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(a^2*b - a*b^2)*cosh(d*x +
 c) + 4*(a^2*b - a*b^2 + 3*(a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/
((a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 4*(a^2*b^3 - 2*a*b^4 + b^5)*
d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^3 - 2*a*b^4 + b^5)*d*sinh(d*x + c)
^4 + 2*(2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 2*(3*(a^
2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4
 - b^5)*d)*sinh(d*x + c)^2 + (a^2*b^3 - 2*a*b^4 + b^5)*d + 4*((a^2*b^3 - 2*
a*b^4 + b^5)*d*cosh(d*x + c)^3 + (2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d*
cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a^2*b - a*b^2)*cosh(d*x + c)^3 + 6*
(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2*b - a*b^2)*sinh(d*x
+ c)^3 - ((a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(a*b - 2*b^2)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a*b - 2*b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 5*a*b + 2*b^2)*co
sh(d*x + c)^2 + 2*(3*(a*b - 2*b^2)*cosh(d*x + c)^2 + 2*a^2 - 5*a*b + 2*b^2)
*sinh(d*x + c)^2 + a*b - 2*b^2 + 4*((a*b - 2*b^2)*cosh(d*x + c)^3 + (2*a^2
 - 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b - b^2)*arctan(-1/2*
(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
+ (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x +
c))/sqrt(a*b - b^2)) + ((a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(a*b - 2*b^2)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a*b - 2*b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 5*a
*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a*b - 2*b^2)*cosh(d*x + c)^2 + 2*a^2 -
5*a*b + 2*b^2)*sinh(d*x + c)^2 + a*b - 2*b^2 + 4*((a*b - 2*b^2)*cosh(d*x +
c)^3 + (2*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b - b^2
)*arctan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + 2*
(a^2*b - a*b^2)*cosh(d*x + c) + 2*(a^2*b - a*b^2 + 3*(a^2*b - a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c))/((a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 4*
(a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^3 - 2*a*
b^4 + b^5)*d*sinh(d*x + c)^4 + 2*(2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d*
cosh(d*x + c)^2 + 2*(3*(a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (2*a^3
*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d)*sinh(d*x + c)^2 + (a^2*b^3 - 2*a*b^4 +
 b^5)*d + 4*((a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^3 + (2*a^3*b^2 - 5*a
^2*b^3 + 4*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)^3}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^2, x)
```

3.44 $\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a-b)^{3/2}d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*cosh(d*x+c)*sinh(d*x+c)/(a-b)/d/(a+b*sinh(d*x+c)^2)-1/2*arctanh((a-b)^(
1/2)*tanh(d*x+c)/a^(1/2))/(a-b)^(3/2)/d/a^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3252, 12, 3260, 214}

$$\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a - b)^(3/2)*d)
 + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)}-\frac{\int \frac{a}{a+b\sinh^2(c+dx)}\,dx}{2a(a-b)}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)}-\frac{\int \frac{1}{a+b\sinh^2(c+dx)}\,dx}{2(a-b)}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)}-\frac{\operatorname{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx,x,\tanh(c+dx)\right)}{2(a-b)d}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a-b)^{3/2}d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 81, normalized size = 0.96

$$
\frac{-\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{3/2}}+\frac{\sinh(2(c+dx))}{(a-b)(2a-b+b\cosh(2(c+dx)))}}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-(ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a - b)^(3/2))) +
Sinh[2*(c + d*x)]/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(72) = 144$.
time = 0.99, size = 275, normalized size = 3.27

method	result
risch	$-\frac{2a\,\mathrm{e}^{2dx+2c}-b\,\mathrm{e}^{2dx+2c}+b}{bd(a-b)\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}+2a^2-2ab}{b\sqrt{a^2-ab}}\right)}{4\sqrt{a^2-ab}\,(a-b)d}-$ ln(

derivativedivides	$\frac{-\frac{8\left(-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a-b)}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a-b)}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{a\left(\frac{\left(-\sqrt{-b(a-b)}-b\right)\arctan\left(\frac{a\,\mathrm{ta}}{\sqrt{\left(2\sqrt{-b(a}\right.}}\right.}{2\sqrt{-b(a-b)}\,a\sqrt{\left(2\sqrt{-b(a-b}\right.}}\right.}{}}{d}$
default	$\frac{-\frac{8\left(-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a-b)}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a-b)}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{a\left(\frac{\left(-\sqrt{-b(a-b)}-b\right)\arctan\left(\frac{a\,\mathrm{ta}}{\sqrt{\left(2\sqrt{-b(a}\right.}}\right.}{2\sqrt{-b(a-b)}\,a\sqrt{\left(2\sqrt{-b(a-b}\right.}}\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-8*(-1/8/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/8/(a-b)*tanh(1/2*d*x+1/2*c))/(a
*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+
a)-1/(a-b)*a*(1/2*(-(-b*(a-b))^(1/2)-b)/(-b*(a-b))^(1/2)/a/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2))-1/2*(-(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2))))
```

Maxima **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(72) = 144$.
time = 0.44, size = 1523, normalized size = 18.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*
(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b
+ a*b^2)*sinh(d*x + c)^2 + (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(
d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b
^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x +
 c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^
2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)
^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b
^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b
 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(
d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a
 - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 +
 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/((a^3
*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - 2*a^2*b^3 + a*b^
4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*sinh(d
*x + c)^4 + 2*(2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 +
 2*(3*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (2*a^4*b - 5*a^3*b^
2 + 4*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*d
 + 4*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (2*a^4*b - 5*a^3*b^
2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b - 2*a
*b^2 + 2*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 4*(2*a^3 - 3*a^2*b + a
*b^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x +
c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sin
h(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 +
 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(
-a^2 + a*b)/(a^2 - a*b)))/((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4
+ 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^
2 - 2*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(2*a^4*b - 5*a^3*b^2 + 4*a^2*b
^3 - a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d
*x + c)^2 + (2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 +
(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d + 4*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d
*x + c)^3 + (2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh
(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.20, size = 135, normalized size = 1.61

$$-\frac{\frac{\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,(a-b)}+\frac{2\,\left(2\,ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+b\right)}{(ab-b^2)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2
 + a*b)*(a - b)) + 2*(2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)/((a*b -
b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)))/
d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)^2}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^2, x)
```

3.45 $\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=81

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}\sqrt{b}\,d}+\frac{\cosh(c+dx)}{2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)}$$

```
[Out] 1/2*cosh(d*x+c)/(a-b)/d/(a-b+b*cosh(d*x+c)^2)+1/2*arctan(cosh(d*x+c)*b^(1/2
)/(a-b)^(1/2))/(a-b)^(3/2)/d/b^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3265, 205, 211}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2\sqrt{b}\,d(a-b)^{3/2}}+\frac{\cosh(c+dx)}{2d(a-b)\left(a+b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(2*(a - b)^(3/2)*Sqrt[b]*d) + C
osh[c + d*x]/(2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a-b+bx^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= \frac{\cosh(c+dx)}{2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{2(a-b)d}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}\sqrt{b}\,d} + \frac{\cosh(c+dx)}{2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 130, normalized size = 1.60

$$
\frac{\frac{\text{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\text{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}\sqrt{b}} + \frac{2\cosh(c+dx)}{(a-b)(2a-b+b\cosh(2(c+dx)))}}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqr
t[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/((a - b)^(3/2)*Sqrt[b]) +
 (2*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(69) = 138$.
time = 0.90, size = 146, normalized size = 1.80

method	result
derivativedivides	$\frac{\frac{-\frac{(-2b+a)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)a}+\frac{2}{2a-2b}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{2(a-b)\sqrt{ab-b^2}}}{d}$
default	$\frac{\frac{-\frac{(-2b+a)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)a}+\frac{2}{2a-2b}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a+4b}{4\sqrt{ab-b^2}}\right)}{2(a-b)\sqrt{ab-b^2}}}{d}$
risch	$\frac{\mathrm{e}^{dx+c}\left(1+\mathrm{e}^{2dx+2c}\right)}{d(a-b)\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)}-\frac{\ln\left(\mathrm{e}^{2dx+2c}-\frac{2(a-b)\mathrm{e}^{dx+c}}{\sqrt{-ab+b^2}}+1\right)}{4\sqrt{-ab+b^2}\,(a-b)d}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{2(a-b)\mathrm{e}^{dx+c}}{\sqrt{-ab+b}}\right.}{4\sqrt{-ab+b^2}\,(a-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/2*(-2*b+a)/(a-b)/a*tanh(1/2*d*x+1/2*c)^2+1/2/(a-b))/(a*tanh(1/2*
d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2/(a-
b)/(a*b-b^2)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2-2*a+4*b)/(a*b-b^2)
^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (e^(3*d*x + 3*c) + e^(d*x + c))/(a*b*d - b^2*d + (a*b*d*e^(4*c) - b^2*d*e^(
4*c))*e^(4*d*x) + 2*(2*a^2*d*e^(2*c) - 3*a*b*d*e^(2*c) + b^2*d*e^(2*c))*e^(
2*d*x)) + 1/2*integrate(2*(e^(3*d*x + 3*c) - e^(d*x + c))/(a*b - b^2 + (a*b
*e^(4*c) - b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^2*e^(2*c) - 3*a*b*e^(2*c) + b^2*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(69) = 138$.
time = 0.44, size = 1628, normalized size = 20.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b - b^2)*cosh(d*x + c)^3 + 12*(a*b - b^2)*cosh(d*x + c)*sinh(d*x
 + c)^2 + 4*(a*b - b^2)*sinh(d*x + c)^3 + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x
 + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2
*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (
2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-a*b + b^2)*log((b*cosh(d*x
 + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a -
3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(cosh
(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d
*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh
(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*
a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2
```

```
+ 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(
a*b - b^2)*cosh(d*x + c) + 4*(3*(a*b - b^2)*cosh(d*x + c)^2 + a*b - b^2)*si
nh(d*x + c))/((a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 4*(a^2*b^2 - 2*
a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 - 2*a*b^3 + b^4)*d*
sinh(d*x + c)^4 + 2*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*d*cosh(d*x + c)^2
 + 2*(3*(a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^2 + (2*a^3*b - 5*a^2*b^2
+ 4*a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*d + 4*((a^2
*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a*b - b^2)*cosh(d*x + c)^3 +
6*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a*b - b^2)*sinh(d*x + c)^3
 + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)
^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d
*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) +
 b)*sqrt(a*b - b^2)*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh
(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x
 + c)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) - (b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d
*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d
*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(a*b - b^2)*arc
tan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + 2*(a*b
- b^2)*cosh(d*x + c) + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + a*b - b^2)*sinh(d
*x + c))/((a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 4*(a^2*b^2 - 2*a*b^
3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 - 2*a*b^3 + b^4)*d*sinh
(d*x + c)^4 + 2*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*d*cosh(d*x + c)^2 + 2
*(3*(a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^2 + (2*a^3*b - 5*a^2*b^2 + 4*
a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*d + 4*((a^2*b^2
 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)
*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)}{{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2)^2, x)
```

3.46 $\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=95

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}d}-\frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*(2*a-b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(3/2)/d-
1/2*b*cosh(d*x+c)*sinh(d*x+c)/a/(a-b)/d/(a+b*sinh(d*x+c)^2)
```

Rubi [A]

time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3263, 12, 3260, 214}

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}}-\frac{b\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^(-2),x]
```

```
[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)
^(3/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*a*(a - b)*d*(a + b*Sinh[c +
d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= -\frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)}-\frac{\int \frac{-2a+b}{a+b\sinh^2(c+dx)}\,dx}{2a(a-b)}\\
&= -\frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)}+\frac{(2a-b)\int \frac{1}{a+b\sinh^2(c+dx)}\,dx}{2a(a-b)}\\
&= -\frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)}+\frac{(2a-b)\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx,x,\tanh(c\right.}{2a(a-b)d}\\
&= \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}d}-\frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [**A**]
time = 0.20, size = 96, normalized size = 1.01

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}d}-\frac{b\sinh(2(c+dx))}{2a(a-b)d(2a-b+b\cosh(2(c+dx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^(-2),x]
```

```
[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)
^(3/2)*d) - (b*Sinh[2*(c + d*x)])/(2*a*(a - b)*d*(2*a - b + b*Cosh[2*(c + d
*x)]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(83) = 166.
time = 1.20, size = 285, normalized size = 3.00

method	result

derivativedivides	$\dfrac{-\dfrac{2\left(\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a-b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a(a-b)}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\dfrac{(-b+2a)\left(\dfrac{\left(\sqrt{-b\,(a-b)}+b\right)\arctan\left(\dfrac{}{\sqrt{\left(2\sqrt{-}\right.}}\right.}{2a\sqrt{-b\,(a-b)}\sqrt{\left(2\sqrt{-b\,(a}\right.}}\right.}{}}{d}$
default	$\dfrac{-\dfrac{2\left(\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a-b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a(a-b)}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\dfrac{(-b+2a)\left(\dfrac{\left(\sqrt{-b\,(a-b)}+b\right)\arctan\left(\dfrac{}{\sqrt{\left(2\sqrt{-}\right.}}\right.}{2a\sqrt{-b\,(a-b)}\sqrt{\left(2\sqrt{-b\,(a}\right.}}\right.}{}}{d}$
risch	$\frac{2a\,e^{2dx+2c}-b\,e^{2dx+2c}+b}{da(a-b)\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}-2a^2+2ab}{b\sqrt{a^2-ab}}\right)b}{4\sqrt{a^2-ab}\,(a-b)da}+\frac{\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(1/2*b/a/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/2*b/a/(a-b)*tanh(1/2*d*x+1/2
*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/
2*c)^2+a)-(-b+2*a)/(a-b)*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(83) = 166$.
time = 0.43, size = 1617, normalized size = 17.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*(
2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b +
 a*b^2)*sinh(d*x + c)^2 + ((2*a*b - b^2)*cosh(d*x + c)^4 + 4*(2*a*b - b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b - b^2)*sinh(d*x + c)^4 + 2*(4*a^2 -
4*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*a^2 -
 4*a*b + b^2)*sinh(d*x + c)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(d*x + c
)^3 + (4*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*l
og((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*
x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a
*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 +
(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cos
h(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b
*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 +
2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x +
c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))
/((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b - 2*a^3*b^2 +
a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*d*
sinh(d*x + c)^4 + 2*(2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)
^2 + 2*(3*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (2*a^5 - 5*a^4*
b + 4*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)
*d + 4*((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (2*a^5 - 5*a^4*b
+ 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a^2*b - 2*a*
b^2 + 2*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 4*(2*a^3 - 3*a^2*b + a*
b^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x + c
)^2 - ((2*a*b - b^2)*cosh(d*x + c)^4 + 4*(2*a*b - b^2)*cosh(d*x + c)*sinh(d
*x + c)^3 + (2*a*b - b^2)*sinh(d*x + c)^4 + 2*(4*a^2 - 4*a*b + b^2)*cosh(d*
x + c)^2 + 2*(3*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*a*b + b^2)*sinh(d
*x + c)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(d*x + c)^3 + (4*a^2 - 4*a*b
 + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(
d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)
*sqrt(-a^2 + a*b)/(a^2 - a*b)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x +
 c)^4 + 4*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^4*b - 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^5 - 5*a^4*b + 4*a^3
```

```
*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*co
sh(d*x + c)^2 + (2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2
+ (a^4*b - 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh
(d*x + c)^3 + (2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh
(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.57, size = 144, normalized size = 1.52

$$\frac{\frac{(2\,a-b)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^2-ab)\sqrt{-a^2+ab}}+\frac{2\,\left(2\,ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+b\right)}{(a^2-ab)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(
(a^2 - a*b)*sqrt(-a^2 + a*b)) + 2*(2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c)
+ b)/((a^2 - a*b)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x +
 2*c) + b)))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(1/(a + b*sinh(c + d*x)^2)^2, x)
```

3.47 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

<u>**Optimal**</u>. Leaf size=110

$$-\frac{(3a-2b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2(a-b)^{3/2}d}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{b\cosh(c+dx)}{2a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)}$$

```
[Out] -arctanh(cosh(d*x+c))/a^2/d-1/2*b*cosh(d*x+c)/a/(a-b)/d/(a-b+b*cosh(d*x+c)^
2)-1/2*(3*a-2*b)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^2/(a-b)^
(3/2)/d
```

<u>**Rubi**</u> [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$,
Rules used = {3265, 425, 536, 212, 211}

$$-\frac{\sqrt{b}\,(3a-2b)\text{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2d(a-b)^{3/2}}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{b\cosh(c+dx)}{2ad(a-b)\left(a+b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a^2
*(a - b)^(3/2)*d) - ArcTanh[Cosh[c + d*x]]/(a^2*d) - (b*Cosh[c + d*x])/(2*a
*(a - b)*d*(a - b + b*Cosh[c + d*x]^2))
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 425</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh(c+dx)}{2a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-2a+b+bx^2}{(1-x^2)(a-b+bx^2)}\,dx, x, \cosh}{2a(a-b)d}$$

$$= -\frac{b\cosh(c+dx)}{2a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{a^2d}$$

$$= -\frac{(3a-2b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2(a-b)^{3/2}d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{}{2a(a-b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 176, normalized size = 1.60

$$\frac{\frac{\sqrt{b}(-3a+2b)\text{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}+\frac{\sqrt{b}(-3a+2b)\text{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{2ab\cosh(c+dx)}{(a-b)(2a-b+b\cosh(2(c+dx)))}+2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((Sqrt[b]*(-3*a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[
a - b]])/(a - b)^(3/2) + (Sqrt[b]*(-3*a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*
Tanh[(c + d*x)/2])/Sqrt[a - b]])/(a - b)^(3/2) - (2*a*b*Cosh[c + d*x])/((a
- b)*(2*a - b + b*Cosh[2*(c + d*x)])) + 2*Log[Tanh[(c + d*x)/2]])/(2*a^2*d)
```

Maple [A]

time = 1.50, size = 171, normalized size = 1.55

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}}-\frac{4b\left(\frac{-\frac{(-2b+a)\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a-b)}+\frac{a}{4a-4b}}{a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(3a-2b)\arctan\left(\frac{2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2}{4\sqrt{ab-b^{2}}}\right.}{8(a-b)\sqrt{ab-b^{2}}}\right.}{a^{2}}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}}-\frac{4b\left(\frac{-\frac{(-2b+a)\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a-b)}+\frac{a}{4a-4b}}{a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(3a-2b)\arctan\left(\frac{2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2}{4\sqrt{ab-b^{2}}}\right.}{8(a-b)\sqrt{ab-b^{2}}}\right.}{a^{2}}}{d}$
risch	$-\frac{b\,\mathrm{e}^{dx+c}\left(1+\mathrm{e}^{2dx+2c}\right)}{a(a-b)d\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)}+\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)}{a^{2}d}-\frac{\ln\left(\mathrm{e}^{dx+c}+1\right)}{a^{2}d}+\frac{3\sqrt{-b\,(a-b)}\,\ln\left(\mathrm{e}^{2dx+}\right.}{4(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^2*ln(tanh(1/2*d*x+1/2*c))-4*b/a^2*((-1/4*(-2*b+a)/(a-b)*tanh(1/2*d
*x+1/2*c)^2+1/4*a/(a-b))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2
+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/8*(3*a-2*b)/(a-b)/(a*b-b^2)^(1/2)*arctan(1/
4*(2*a*tanh(1/2*d*x+1/2*c)^2-2*a+4*b)/(a*b-b^2)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(4*c)
- a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(2*a^3*d*e^(2*c) - 3*a^2*b*d*e^(2*c) + a*b
^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log((e^(
d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/2*((3*a*b*e^(3*c) - 2*b^2*e^(
3*c))*e^(3*d*x) - (3*a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^3*b - a^2*b^2 + (a^3*
b*e^(4*c) - a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 3*a^3*b*e^(2*c)
 + a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1256 vs. $2(98) = 196$.
time = 0.51, size = 2529, normalized size = 22.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b
*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) - ((3*a*b - 2*b^2)*cosh(d*x + c)^4 +
 4*(3*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - 2*b^2)*sinh(d*x
 + c)^4 + 2*(6*a^2 - 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - 2*b^2)*
cosh(d*x + c)^2 + 6*a^2 - 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a*b - 2*b^2 +
4*((3*a*b - 2*b^2)*cosh(d*x + c)^3 + (6*a^2 - 7*a*b + 2*b^2)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-b/(a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)
*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3
*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2
*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a - b)*cosh(d*x + c)^3 + 3*(a
- b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh
(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a -
 b)) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d
*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x
 + c) + b)) + 4*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cos
h(d*x + c)^2 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh
(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b +
 b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1)
- 4*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x + c)^2
 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2
+ a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b + b^2)*cosh(d
*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*a*b*c
osh(d*x + c)^2 + a*b)*sinh(d*x + c))/((a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 +
 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b - a^2*b^2)*d*
sinh(d*x + c)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a
^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + (2*a^4 - 3*a^3*b + a^2*b^2)*d)*sinh(d*x
 + c)^2 + (a^3*b - a^2*b^2)*d + 4*((a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 + (2
*a^4 - 3*a^3*b + a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a*b*cosh
(d*x + c)^3 + 6*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d*x + c)^3 +
 2*a*b*cosh(d*x + c) + ((3*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(3*a*b - 2*b^2)
*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - 2*b^2)*sinh(d*x + c)^4 + 2*(6*a^2
 - 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - 2*b^2)*cosh(d*x + c)^2 +
6*a^2 - 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a*b - 2*b^2 + 4*((3*a*b - 2*b^2)
```

```
*cosh(d*x + c)^3 + (6*a^2 - 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x + c) + sinh(d*x + c))) -
 ((3*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(3*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (3*a*b - 2*b^2)*sinh(d*x + c)^4 + 2*(6*a^2 - 7*a*b + 2*b^2)*cosh
(d*x + c)^2 + 2*(3*(3*a*b - 2*b^2)*cosh(d*x + c)^2 + 6*a^2 - 7*a*b + 2*b^2)
*sinh(d*x + c)^2 + 3*a*b - 2*b^2 + 4*((3*a*b - 2*b^2)*cosh(d*x + c)^3 + (6*
a^2 - 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a - b))*arctan(1
/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)
^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x
 + c))*sqrt(b/(a - b))/b) + 2*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*
a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a
*b + b^2)*sinh(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2
*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(
d*x + c) + 1) - 2*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*c
osh(d*x + c)^2 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*si
nh(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b
 + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1
) + 2*(3*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/((a^3*b - a^2*b^2)*d*cos
h(d*x + c)^4 + 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b
 - a^2*b^2)*d*sinh(d*x + c)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*d*cosh(d*x +
c)^2 + 2*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + (2*a^4 - 3*a^3*b + a^2*b^
2)*d)*sinh(d*x + c)^2 + (a^3*b - a^2*b^2)*d + 4*((a^3*b - a^2*b^2)*d*cosh(d
*x + c)^3 + (2*a^4 - 3*a^3*b + a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
```

```
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(c+d\,x\right)\,\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^2), x)
```

3.48 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=142

$$-\frac{(4a-3b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a-b)^{3/2}d}-\frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{(2a^2-4ab+3b^2)\tanh(c+}{2a^2(a-b)d\left(a-(a-b)\tanh^2(c\right.}$$

```
[Out] -1/2*(4*a-3*b)*b*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a-b)^(3/
2)/d-coth(d*x+c)/a/d/(a-(a-b)*tanh(d*x+c)^2)+1/2*(2*a^2-4*a*b+3*b^2)*tanh(d
*x+c)/a^2/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3266, 473, 393, 214}

$$-\frac{b(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a-b)^{3/2}}+\frac{(2a^2-4ab+3b^2)\tanh(c+dx)}{2a^2d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((4*a - 3*b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*
(a - b)^(3/2)*d) - Coth[c + d*x]/(a*d*(a - (a - b)*Tanh[c + d*x]^2)) + ((2*
a^2 - 4*a*b + 3*b^2)*Tanh[c + d*x])/(2*a^2*(a - b)*d*(a - (a - b)*Tanh[c +
d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\Big(\int \frac{a-3b+ax^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+d}{ad}\\
&= -\frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\left(2a^2-4ab+3b^2\right)\tanh(c+dx)}{2a^2(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)}\\
&= -\frac{(4a-3b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a-b)^{3/2}d} - \frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A]
time = 0.52, size = 170, normalized size = 1.20

$$-\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^5(c+dx)\left(2\sqrt{a}\sqrt{a-b}\cosh(c+dx)\left(4a^2-6ab+3b^2+(2a-3b)b\cosh(2(c+dx))\right)-2b(-4a+3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)(2a-b+b\cosh(2(c+dx)))\sinh(c+dx)\right)}{16a^{5/2}(a-b)^{3/2}d\left(b+a\operatorname{csch}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/16*((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^5*(2*Sqrt[a]*Sqrt[a -
b]*Cosh[c + d*x]*(4*a^2 - 6*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*(c + d*x)])
- 2*b*(-4*a + 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b +
b*Cosh[2*(c + d*x)])*Sinh[c + d*x]))/(a^(5/2)*(a - b)^(3/2)*d*(b + a*Csch[c
 + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(130) = 260$.

time = 1.51, size = 316, normalized size = 2.23

method	result
derivativedivides	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^{2}}+\frac{4b\Bigg(\frac{\frac{b\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a-4b}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a-4b}}{a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(4a-3b)a\Bigg(\frac{\left(\sqrt{-b\left(a-b\right)}+b\right)\text{arcta}}{2a\sqrt{-b\left(a-b\right)}}}{}}{}$
default	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^{2}}+\frac{4b\Bigg(\frac{\frac{b\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a-4b}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a-4b}}{a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(4a-3b)a\Bigg(\frac{\left(\sqrt{-b\left(a-b\right)}+b\right)\text{arcta}}{2a\sqrt{-b\left(a-b\right)}}}{}}{}$
risch	$-\frac{4ab\,\mathrm{e}^{4dx+4c}-3b^{2}\mathrm{e}^{4dx+4c}+8a^{2}\mathrm{e}^{2dx+2c}-14ab\,\mathrm{e}^{2dx+2c}+6b^{2}\mathrm{e}^{2dx+2c}+2ab-3b^{2}}{a^{2}(a-b)d\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)\left(\mathrm{e}^{2dx+2c}-1\right)}+\frac{\ln\Bigg(\mathrm{e}^{2dx+2c}+\frac{2a\sqrt{a^{2}-ab}-b\sqrt{a}}{b\sqrt{a^{2}-}}}{\sqrt{a^{2}-ab}\,(a-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^2*tanh(1/2*d*x+1/2*c)+4*b/a^2*((1/4*b/(a-b)*tanh(1/2*d*x+1/2*c)
^3+1/4*b/(a-b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d
*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/4*(4*a-3*b)/(a-b)*a*(1/2*((-b*(a
-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arcta
n(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b
))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))))-1/2/a^2/tanh
(1/2*d*x+1/2*c))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1366 vs. $2(131) = 262$.
time = 0.43, size = 2988, normalized size = 21.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 16*(4*a^3*b - 7*
a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(4*a^3*b - 7*a^2*b^2 +
 3*a*b^3)*sinh(d*x + c)^4 + 8*a^3*b - 20*a^2*b^2 + 12*a*b^3 + 8*(4*a^4 - 11
*a^3*b + 10*a^2*b^2 - 3*a*b^3)*cosh(d*x + c)^2 + 8*(4*a^4 - 11*a^3*b + 10*a
^2*b^2 - 3*a*b^3 + 3*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 - ((4*a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 6*(4*a*b^2 - 3*b^3)*cosh(
d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 - 3*b^3)*sinh(d*x + c)^6 + (16*a^2*b -
24*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + (16*a^2*b - 24*a*b^2 + 9*b^3 + 15*(4*a*
b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(4*a*b^2 - 3*b^3)*cosh
```

```
(d*x + c)^3 + (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
- 4*a*b^2 + 3*b^3 - (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^2 + (15*(4*
a*b^2 - 3*b^3)*cosh(d*x + c)^4 - 16*a^2*b + 24*a*b^2 - 9*b^3 + 6*(16*a^2*b
- 24*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - 3*b^
3)*cosh(d*x + c)^5 + 2*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^3 - (16*
a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log
((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x
+ c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b
 - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2
*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(
d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*c
osh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*
(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)
^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) +
16*((4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 11*a^3*b + 1
0*a^2*b^2 - 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 2*a^4*b^2 + a^
3*b^3)*d*cosh(d*x + c)^6 + 6*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*
sinh(d*x + c)^5 + (a^5*b - 2*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^6 + (4*a^6
- 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^4 + (15*(a^5*b - 2*a^4
*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*
b^3)*d)*sinh(d*x + c)^4 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cos
h(d*x + c)^2 + 4*(5*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 + (4*a^
6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (
15*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^4 + 6*(4*a^6 - 11*a^5*b +
10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^2 - (4*a^6 - 11*a^5*b + 10*a^4*b^2
- 3*a^3*b^3)*d)*sinh(d*x + c)^2 - (a^5*b - 2*a^4*b^2 + a^3*b^3)*d + 2*(3*(a
^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 + 2*(4*a^6 - 11*a^5*b + 10*a^
4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^3 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a
^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(4*a^3*b - 7*a^2*b^2 + 3*a
*b^3)*cosh(d*x + c)^4 + 8*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sin
h(d*x + c)^3 + 2*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*sinh(d*x + c)^4 + 4*a^3*b
- 10*a^2*b^2 + 6*a*b^3 + 4*(4*a^4 - 11*a^3*b + 10*a^2*b^2 - 3*a*b^3)*cosh(d
*x + c)^2 + 4*(4*a^4 - 11*a^3*b + 10*a^2*b^2 - 3*a*b^3 + 3*(4*a^3*b - 7*a^2
*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((4*a*b^2 - 3*b^3)*cosh(
d*x + c)^6 + 6*(4*a*b^2 - 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 -
 3*b^3)*sinh(d*x + c)^6 + (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + (
16*a^2*b - 24*a*b^2 + 9*b^3 + 15*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 4*(5*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^3 + (16*a^2*b - 24*a*b^2 +
9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*a*b^2 + 3*b^3 - (16*a^2*b - 24*a*
b^2 + 9*b^3)*cosh(d*x + c)^2 + (15*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^4 - 16*a
^2*b + 24*a*b^2 - 9*b^3 + 6*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + 2*(16*a^2*b - 24
*a*b^2 + 9*b^3)*cosh(d*x + c)^3 - (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x +
c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*co
sh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(
```

```
a^2 - a*b)) + 8*((4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 + (4*a^4 -
 11*a^3*b + 10*a^2*b^2 - 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 2
*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 6*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*c
osh(d*x + c)*sinh(d*x + c)^5 + (a^5*b - 2*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c
)^6 + (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^4 + (15*(
a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + (4*a^6 - 11*a^5*b + 10*a^4
*b^2 - 3*a^3*b^3)*d)*sinh(d*x + c)^4 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a
^3*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x +
 c)^3 + (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d
*x + c)^3 + (15*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^4 + 6*(4*a^6
- 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^2 - (4*a^6 - 11*a^5*b
+ 10*a^4*b^2 - 3*a^3*b^3)*d)*sinh(d*x + c)^2 - (a^5*b - 2*a^4*b^2 + a^3*b^3
)*d + 2*(3*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.68, size = 229, normalized size = 1.61

$$-\frac{\frac{\left(4\,ab-3\,b^2\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\left(a^3-a^2b\right)\sqrt{-a^2+ab}}+\frac{2\,\left(4\,abe^{(4\,dx+4\,c)}-3\,b^2e^{(4\,dx+4\,c)}+8\,a^2e^{(2\,dx+2\,c)}-14\,abe^{(2\,dx+2\,c)}+6\,b^2e^{(2\,dx+2\,c)}+2\,ab-3\,b^2\right)}{\left(a^3-a^2b\right)\left(be^{(6\,dx+6\,c)}+4\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}-4\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}-b\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((4*a*b - 3*b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 +
a*b))/((a^3 - a^2*b)*sqrt(-a^2 + a*b)) + 2*(4*a*b*e^(4*d*x + 4*c) - 3*b^2*e
^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 14*a*b*e^(2*d*x + 2*c) + 6*b^2*e^(
2*d*x + 2*c) + 2*a*b - 3*b^2)/((a^3 - a^2*b)*(b*e^(6*d*x + 6*c) + 4*a*e^(4*
d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - 4*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c
) - b)))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\mathrm{sinh}\left(c+d\,x\right)}^2\,{\left(b\,{\mathrm{sinh}\left(c+d\,x\right)}^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2),x)
[Out] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2), x)
```

3.49 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=161

$$\frac{(5a-4b)b^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3(a-b)^{3/2}d}+\frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d}-\frac{(a-2b)b\cosh(c+dx)}{2a^2(a-b)d\left(a-b+b\cosh^2(c+d\right.}$$

```
[Out] 1/2*(5*a-4*b)*b^(3/2)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/a^3/(a-b)^(3/
2)/d+1/2*(a+4*b)*arctanh(cosh(d*x+c))/a^3/d-1/2*(a-2*b)*b*cosh(d*x+c)/a^2/(
a-b)/d/(a-b+b*cosh(d*x+c)^2)-1/2*coth(d*x+c)*csch(d*x+c)/a/d/(a-b+b*cosh(d*
x+c)^2)
```

Rubi [A]

time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3265, 425, 541, 536, 212, 211}

$$\frac{b^{3/2}(5a-4b)\operatorname{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3d(a-b)^{3/2}}+\frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d}-\frac{b(a-2b)\cosh(c+dx)}{2a^2d(a-b)\left(a+b\cosh^2(c+dx)-b\right)}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*a^3*(a
 - b)^(3/2)*d) + ((a + 4*b)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) - ((a - 2*b)*
b*Cosh[c + d*x])/(2*a^2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)) - (Coth[c +
d*x]*Csch[c + d*x])/(2*a*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\Big(\int \frac{1}{(1-x^2)^2(a-b+bx^2)^2}\,dx, x, \cosh(c+dx)\Big)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}\Big(\int \frac{a+b+3bx^2}{(1-x^2)(a-b+bx^2)^2}\,dx, x, \cosh(c+d}{2ad}$$

$$= -\frac{(a-2b)b\cosh(c+dx)}{2a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)} -$$

$$= -\frac{(a-2b)b\cosh(c+dx)}{2a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)} +$$

$$= \frac{(5a-4b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3(a-b)^{3/2}d} + \frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d} - \frac{}{2a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.88, size = 350, normalized size = 2.17

$$\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^3(c+dx)\left(\frac{8ab^2\coth(c+dx)}{a-b} + \frac{4(5a-4b)b^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)(2a-b+b\cosh(2(c+dx)))\operatorname{csch}(c+dx)}{(a-b)^{3/2}} + \frac{4(5a-4b)b^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)(2a-b+b\cosh(2(c+dx)))\operatorname{csch}(c+dx)}{(a-b)^{3/2}} - a(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}(c+dx) - 4(a+4b)(2a-b+b\cosh(2(c+dx)))\operatorname{csch}(c+dx)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - a(2a-b+b\cosh(2(c+dx)))\operatorname{csch}(c+dx)\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\right)}{32a^3d\left(b+a\operatorname{csch}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^3*((8*a*b^2*Coth[c + d*x])/(
a - b) + (4*(5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/
2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x])/(a - b)^(3/
2) + (4*(5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/
Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x])/(a - b)^(3/2) -
 a*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[(c + d*x)/2]^2*Csch[c + d*x] - 4*(a
 + 4*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]*Log[Tanh[(c + d*x)/2]
] - a*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]*Sech[(c + d*x)/2]^2))/(
32*a^3*d*(b + a*Csch[c + d*x]^2)^2)
```

Maple [A]
time = 1.78, size = 213, normalized size = 1.32

method	result

derivativedivides	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{1}{8a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-8b-2a)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{2b^2\left(\frac{-\frac{(-2b+a)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)}+\frac{a}{2a-2b}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}\right.\right.}\right.}{a^3}}{d}$
default	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{1}{8a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-8b-2a)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{2b^2\left(\frac{-\frac{(-2b+a)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)}+\frac{a}{2a-2b}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}\right.\right.}\right.}{a^3}}{d}$
risch	$-\frac{e^{dx+c}\left(ab\,e^{6dx+6c}-2b^2e^{6dx+6c}+4a^2e^{4dx+4c}-5ab\,e^{4dx+4c}+2b^2e^{4dx+4c}+4a^2e^{2dx+2c}-5ab\,e^{2dx+2c}+2b^2e^{2dx+2c}+ab-2b\right.}{d\,a^2\left(e^{2dx+2c}-1\right)^2(a-b)\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a^2-1/8/a^2/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-
8*b-2*a)*ln(tanh(1/2*d*x+1/2*c))+2*b^2/a^3*((-1/2*(-2*b+a)/(a-b)*tanh(1/2*d
*x+1/2*c)^2+1/2*a/(a-b))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2
+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/4*(5*a-4*b)/(a-b)/(a*b-b^2)^(1/2)*arctan(1/
4*(2*a*tanh(1/2*d*x+1/2*c)^2-2*a+4*b)/(a*b-b^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -((a*b*e^(7*c) - 2*b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) - 5*a*b*e^(5*c)
+ 2*b^2*e^(5*c))*e^(5*d*x) + (4*a^2*e^(3*c) - 5*a*b*e^(3*c) + 2*b^2*e^(3*c)
)*e^(3*d*x) + (a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^3*b*d - a^2*b^2*d + (a^3*b*
d*e^(8*c) - a^2*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^4*d*e^(6*c) - 2*a^3*b*d*e^(
6*c) + a^2*b^2*d*e^(6*c))*e^(6*d*x) - 2*(4*a^4*d*e^(4*c) - 7*a^3*b*d*e^(4*c
) + 3*a^2*b^2*d*e^(4*c))*e^(4*d*x) + 4*(a^4*d*e^(2*c) - 2*a^3*b*d*e^(2*c) +
 a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*(a + 4*b)*log((e^(d*x + c) + 1)*e^(-c)
)/(a^3*d) - 1/2*(a + 4*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) + 8*integra
te(1/8*((5*a*b^2*e^(3*c) - 4*b^3*e^(3*c))*e^(3*d*x) - (5*a*b^2*e^c - 4*b^3*
e^c)*e^(d*x))/(a^4*b - a^3*b^2 + (a^4*b*e^(4*c) - a^3*b^2*e^(4*c))*e^(4*d*x
) + 2*(2*a^5*e^(2*c) - 3*a^4*b*e^(2*c) + a^3*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4252 vs. $2(145) = 290$.
time = 0.53, size = 8059, normalized size = 50.06

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b - 2*a*b^2)*cosh(d*x + c)^7 + 28*(a^2*b - 2*a*b^2)*cosh(d*x
+ c)*sinh(d*x + c)^6 + 4*(a^2*b - 2*a*b^2)*sinh(d*x + c)^7 + 4*(4*a^3 - 5*a
^2*b + 2*a*b^2)*cosh(d*x + c)^5 + 4*(4*a^3 - 5*a^2*b + 2*a*b^2 + 21*(a^2*b
- 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*b - 2*a*b^2)*cosh(
d*x + c)^3 + (4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 4
*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 4*(35*(a^2*b - 2*a*b^2)*cosh
(d*x + c)^4 + 4*a^3 - 5*a^2*b + 2*a*b^2 + 10*(4*a^3 - 5*a^2*b + 2*a*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(a^2*b - 2*a*b^2)*cosh(d*x + c)^5 +
10*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 - 5*a^2*b + 2*a*b
^2)*cosh(d*x + c))*sinh(d*x + c)^2 - ((5*a*b^2 - 4*b^3)*cosh(d*x + c)^8 + 8
*(5*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2 - 4*b^3)*sinh(d
*x + c)^8 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 4*(5*a^2*b - 9*
a*b^2 + 4*b^3 + 7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7
*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^5 - 2*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^4 +
 2*(35*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 20*a^2*b + 31*a*b^2 - 12*b^3 + 3
0*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*a*
b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)
^3 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*a*b^
2 - 4*b^3 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(5*a*b^2 -
 4*b^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 +
5*a^2*b - 9*a*b^2 + 4*b^3 - 3*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 8*((5*a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(5*a^2*b - 9*
a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x +
c)^3 + (5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a
 - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(
d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a +
 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*s
inh(d*x + c) + 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*
x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*cos
h(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c)
^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*co
sh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*co
sh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(a^2*b - 2
*a*b^2)*cosh(d*x + c) - 2*((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^8 + 8*(a
^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b + 3*a*b^2 -
```

```
4*b^3)*sinh(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^
6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^3
 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(
4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^4 + 2*(35*(a^2*b + 3*a*b
^2 - 4*b^3)*cosh(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a*b^2 - 12*b^3 + 30*(a^3
 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2*
b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)
*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c))*sin
h(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3
)*cosh(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15*(a^
3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + a^3 + 2*a^2*b - 7*a*b^2 +
4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(a^3 + 2*a^2*b - 7
*a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cos
h(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x +
c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*((a^2*b + 3*a*b^2 - 4*b^3)*c
osh(d*x + c)^8 + 8*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7
+ (a^2*b + 3*a*b^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2 +
4*b^3)*cosh(d*x + c)^6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b + 3*
a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 - 4
*b^3)*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*
sinh(d*x + c)^5 - 2*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^4 +
 2*(35*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a*b
^2 - 12*b^3 + 30*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(a^3 + 2*a^2
*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^
3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*a^
2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cos
h(d*x + c)^6 + 15*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + a^3 +
 2*a^2*b - 7*a*b^2 + 4*b^3 - 3*(4*a^3 + 9*a^2*b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(c+d\,x\right)^3\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2), x)
```

3.50 $\int \frac{\mathbf{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=174

$$\frac{(6a-5b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a-b)^{3/2}d}+\frac{(2a^2+ab-5b^2)\coth(c+dx)}{2a^3(a-b)d}-\frac{(2a-5b)\coth^3(c+dx)}{6a^2(a-b)d}-\frac{bcs}{2a(a-b}$$

```
[Out] 1/2*(6*a-5*b)*b^2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2)/(a-b)^(3
/2)/d+1/2*(2*a^2+a*b-5*b^2)*coth(d*x+c)/a^3/(a-b)/d-1/6*(2*a-5*b)*coth(d*x+
c)^3/a^2/(a-b)/d-1/2*b*csch(d*x+c)^3*sech(d*x+c)/a/(a-b)/d/(a-(a-b)*tanh(d*
x+c)^2)
```

Rubi [A]
time = 0.17, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3266, 479, 584, 214}

$$\frac{b^2(6a-5b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a-b)^{3/2}}-\frac{(2a-5b)\coth^3(c+dx)}{6a^2d(a-b)}+\frac{(2a^2+ab-5b^2)\coth(c+dx)}{2a^3d(a-b)}-\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{2ad(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((6*a - 5*b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(7/2)*(
a - b)^(3/2)*d) + ((2*a^2 + a*b - 5*b^2)*Coth[c + d*x])/(2*a^3*(a - b)*d) -
 ((2*a - 5*b)*Coth[c + d*x]^3)/(6*a^2*(a - b)*d) - (b*Csch[c + d*x]^3*Sech[
c + d*x])/(2*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^4\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{2a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)\left(2a-5b+(-2a+b)x^2\right)}{x^4\left(a+(-a+b)x^2\right)}\right.}{2a(a-b)d} \\
&= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{2a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \left(\frac{2a-5b}{ax^4} + \frac{-2a^2-ab+5b^2}{a^2x^2} +\right.\right.}{2a(} \\
&= \frac{\left(2a^2+ab-5b^2\right)\coth(c+dx)}{2a^3(a-b)d} - \frac{(2a-5b)\coth^3(c+dx)}{6a^2(a-b)d} - \frac{b\operatorname{csch}^3(c+}{2a(a-b)d\left(a-\right.} \\
&= \frac{(6a-5b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a-b)^{3/2}d} + \frac{\left(2a^2+ab-5b^2\right)\coth(c+dx)}{2a^3(a-b)d} -
\end{aligned}$$

Mathematica [A]
time = 0.83, size = 210, normalized size = 1.21

$$\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^4(c+dx)\left(\frac{3(6a-5b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)(2a-b+b\cosh(2(c+dx)))}{(a-b)^{3/2}} + 4\sqrt{a}\,(a+3b)(2a-b+b\cosh(2(c+dx)))\coth(c+dx) - 2a^{3/2}(2a-b+b\cosh(2(c+dx)))\coth(c+dx)\operatorname{csch}^2(c+dx) - \frac{3\sqrt{a}\,b^3\sinh(2(c+dx))}{a-b}\right)}{24a^{7/2}d\left(b+a\operatorname{csch}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^4*((3*(6*a - 5*b)*b^2*ArcTan
h[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b + b*Cosh[2*(c + d*x)]))/(a
- b)^(3/2) + 4*Sqrt[a]*(a + 3*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d
*x] - 2*a^(3/2)*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d*x]*Csch[c + d*x]
^2 - (3*Sqrt[a]*b^3*Sinh[2*(c + d*x)])/(a - b)))/(24*a^(7/2)*d*(b + a*Csch[
c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(158) = 316$.

time = 1.61, size = 371, normalized size = 2.13

method	result
derivativedivides	$-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{1}{24a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-8b-3a}{8a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-$ $2b^2\left(\frac{\frac{b\left(\tanh^3\right.}{2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a}\right.$

default	$-\frac{\frac{a\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{3}}-\frac{1}{24a^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{-8b-3a}{8a^{3}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-$ $2b^{2}\Bigg($ $\frac{b\left(\tanh\right.}{2}$ $a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2$
risch	$-\frac{-18a\,b^{2}\mathrm{e}^{8dx+8c}+15b^{3}\mathrm{e}^{8dx+8c}-36a^{2}b\,\mathrm{e}^{6dx+6c}+102a\,b^{2}\mathrm{e}^{6dx+6c}-60b^{3}\mathrm{e}^{6dx+6c}+48a^{3}\mathrm{e}^{4dx+4c}+20a^{2}b\,\mathrm{e}^{4dx+4c}-158a\,b^{2}}{3a^{3}d\left(\mathrm{e}^{2dx+2c}-1\right)^{3}(a-b)\left(b\,\mathrm{e}^{4dx+4c}+4\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/a^3*(1/3*a*tanh(1/2*d*x+1/2*c)^3-3*a*tanh(1/2*d*x+1/2*c)-8*b*tanh
(1/2*d*x+1/2*c))-1/24/a^2/tanh(1/2*d*x+1/2*c)^3-1/8/a^3*(-8*b-3*a)/tanh(1/2
*d*x+1/2*c)-2*b^2/a^3*((1/2*b/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/2*b/(a-b)*tanh(
1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh
(1/2*d*x+1/2*c)^2+a)+1/2*(6*a-5*b)/(a-b)*a*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*
(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a
-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is ’assume(b-a>0)’, see ‘assume?‘ for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3427 vs. $2(159) = 318$.
time = 0.47, size = 7110, normalized size = 40.86

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^8 + 96*(6*a^3*b^
2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + 12*(6*a^3*b^2 - 1
1*a^2*b^3 + 5*a*b^4)*sinh(d*x + c)^8 + 24*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^
3 - 10*a*b^4)*cosh(d*x + c)^6 + 24*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*
a*b^4 + 14*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 48*(14*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^3 + 3*(6*a^4*
b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 16
*a^4*b + 16*a^3*b^2 - 92*a^2*b^3 + 60*a*b^4 - 8*(24*a^5 - 14*a^4*b - 89*a^3
*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^4 - 8*(24*a^5 - 14*a^4*b - 89*
a^3*b^2 + 124*a^2*b^3 - 45*a*b^4 - 105*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*c
osh(d*x + c)^4 - 45*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 32*(21*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d
*x + c)^5 + 15*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)
^3 - (24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c
))*sinh(d*x + c)^3 + 8*(8*a^5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 - 30*a*b^
4)*cosh(d*x + c)^2 + 8*(42*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)
^6 + 8*a^5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 - 30*a*b^4 + 45*(6*a^4*b - 2
3*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^4 - 6*(24*a^5 - 14*a^4*b -
 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*
((6*a*b^3 - 5*b^4)*cosh(d*x + c)^10 + 10*(6*a*b^3 - 5*b^4)*cosh(d*x + c)*si
nh(d*x + c)^9 + (6*a*b^3 - 5*b^4)*sinh(d*x + c)^10 + (24*a^2*b^2 - 50*a*b^3
 + 25*b^4)*cosh(d*x + c)^8 + (24*a^2*b^2 - 50*a*b^3 + 25*b^4 + 45*(6*a*b^3
- 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(6*a*b^3 - 5*b^4)*cosh(d*
x + c)^3 + (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^7
- 2*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^6 + 2*(105*(6*a*b^3 - 5*
b^4)*cosh(d*x + c)^4 - 36*a^2*b^2 + 60*a*b^3 - 25*b^4 + 14*(24*a^2*b^2 - 50
*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(6*a*b^3 - 5*b^4)
*cosh(d*x + c)^5 + 14*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^3 - 3*
(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(36*a^2
*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^4 + 2*(105*(6*a*b^3 - 5*b^4)*cosh(d
*x + c)^6 + 35*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^4 + 36*a^2*b^
2 - 60*a*b^3 + 25*b^4 - 15*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 - 6*a*b^3 + 5*b^4 + 8*(15*(6*a*b^3 - 5*b^4)*cosh(d*x + c)
```

```
^7 + 7*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^5 - 5*(36*a^2*b^2 - 6
0*a*b^3 + 25*b^4)*cosh(d*x + c)^3 + (36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d
*x + c))*sinh(d*x + c)^3 - (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^2
 + (45*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^8 + 28*(24*a^2*b^2 - 50*a*b^3 + 25*b
^4)*cosh(d*x + c)^6 - 30*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^4 -
 24*a^2*b^2 + 50*a*b^3 - 25*b^4 + 12*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^9 + 4*(2
4*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^7 - 6*(36*a^2*b^2 - 60*a*b^3 +
 25*b^4)*cosh(d*x + c)^5 + 4*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)
^3 - (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^
2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b
^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x +
c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x
 + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2
- a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
 + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*s
inh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x +
 c) + b)) + 16*(6*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^7 + 9*(6
*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^5 - 2*(24*a^5 -
14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^3 + (8*a^5 -
2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 - 30*a*b^4)*cosh(d*x + c))*sinh(d*x + c))
/((a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^10 + 10*(a^6*b - 2*a^5*b^2
+ a^4*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^6*b - 2*a^5*b^2 + a^4*b^3)*
d*sinh(d*x + c)^10 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x
 + c)^8 + (45*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (4*a^7 - 13
*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^8 - 2*(6*a^7 - 17*a^6*b +
 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^6 + 8*(15*(a^6*b - 2*a^5*b^2 + a^4
*b^3)*d*cosh(d*x + c)^3 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cos
h(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d
*x + c)^4 + 14*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^
2 - (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(6*a
^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^4 + 4*(63*(a^6*b -
2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 14*(4*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 0.72, size = 220, normalized size = 1.26

$$\frac{\frac{3\left(6\,ab^{2}-5\,b^{3}\right)\arctan\left(\frac{be^{\left(2\,dx+2\,c\right)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{\left(a^{4}-a^{3}b\right)\sqrt{-a^{2}+ab}}+\frac{6\left(2\,ab^{2}e^{\left(2\,dx+2\,c\right)}-b^{3}e^{\left(2\,dx+2\,c\right)}+b^{3}\right)}{\left(a^{4}-a^{3}b\right)\left(be^{\left(4\,dx+4\,c\right)}+4\,ae^{\left(2\,dx+2\,c\right)}-2\,be^{\left(2\,dx+2\,c\right)}+b\right)}+\frac{8\left(3\,be^{\left(4\,dx+4\,c\right)}-3\,ae^{\left(2\,dx+2\,c\right)}-6\,be^{\left(2\,dx+2\,c\right)}+a+3\,b\right)}{a^{3}\left(e^{\left(2\,dx+2\,c\right)}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(6*a*b^2 - 5*b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2
 + a*b))/((a^4 - a^3*b)*sqrt(-a^2 + a*b)) + 6*(2*a*b^2*e^(2*d*x + 2*c) - b^
3*e^(2*d*x + 2*c) + b^3)/((a^4 - a^3*b)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x +
 2*c) - 2*b*e^(2*d*x + 2*c) + b)) + 8*(3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x +
 2*c) - 6*b*e^(2*d*x + 2*c) + a + 3*b)/(a^3*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\mathrm{sinh}\left(c+d\,x\right)}^4\,{\left(b\,{\mathrm{sinh}\left(c+d\,x\right)}^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2), x)
```

3.51 $\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=124

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,(a-b)^{5/2}d}+\frac{\tanh^3(c+dx)}{4(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2}-\frac{3\tanh(c+dx)}{8(a-b)^2d\left(a-(a-b)\tanh^2(c+\right.}$$

```
[Out] 3/8*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/(a-b)^(5/2)/d/a^(1/2)+1/4*tanh
(d*x+c)^3/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)^2-3/8*tanh(d*x+c)/(a-b)^2/d/(a-(a
-b)*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3266, 294, 214}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,d(a-b)^{5/2}}-\frac{3\tanh(c+dx)}{8d(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh^3(c+dx)}{4d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a - b)^(5/2)*d
) + Tanh[c + d*x]^3/(4*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2)^2) - (3*Tanh
[c + d*x])/(8*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
```

```
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(a-(a-b)x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh^3(c+dx)}{4(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{3\text{Subst}\Big(\int \frac{x^2}{(a+(-a+b)x^2)^2}\,dx, x, \tanh(}{4(a-b)d}$$

$$= \frac{\tanh^3(c+dx)}{4(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{3\tanh(c+dx)}{8(a-b)^2d\,\big(a-(a-b)\tanh^2(c+}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,(a-b)^{5/2}d} + \frac{\tanh^3(c+dx)}{4(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{}{8}$$

Mathematica [A]
time = 0.93, size = 104, normalized size = 0.84

$$\frac{\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{5/2}} + \frac{(-8a+5b+(2a-5b)\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b)^2(2a-b+b\cosh(2(c+dx)))^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)) +
 ((-8*a + 5*b + (2*a - 5*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a - b)^
2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(110) = 220.
time = 1.02, size = 357, normalized size = 2.88

method	result

derivativedivides	$\frac{-\frac{32\left(\frac{3a\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128\left(a^{2}-2ab+b^{2}\right)}-\frac{(11a-20b)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128\left(a^{2}-2ab+b^{2}\right)}-\frac{(11a-20b)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128\left(a^{2}-2ab+b^{2}\right)}+\frac{3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{128\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^{2}}-\frac{3a\left(\frac{\left(\sqrt{-b\,(a}\right.}{2a\sqrt{-}}\right.}{}}{}$
default	$\frac{-\frac{32\left(\frac{3a\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128\left(a^{2}-2ab+b^{2}\right)}-\frac{(11a-20b)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128\left(a^{2}-2ab+b^{2}\right)}-\frac{(11a-20b)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128\left(a^{2}-2ab+b^{2}\right)}+\frac{3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{128\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^{2}}-\frac{3a\left(\frac{\left(\sqrt{-b\,(a}\right.}{2a\sqrt{-}}\right.}{}}{}$
risch	$-\frac{8a^{2}b\,e^{6dx+6c}-16a\,b^{2}e^{6dx+6c}+5b^{3}e^{6dx+6c}+16a^{3}e^{4dx+4c}-56a^{2}b\,e^{4dx+4c}+46a\,b^{2}e^{4dx+4c}-15b^{3}e^{4dx+4c}+8a^{2}b\,e^{2dx+2}}{4b^{2}d(a-b)^{2}\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-32*(3/128*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/128*(11*a-20*b)/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-1/128*(11*a-20*b)/(a^2-2*a*b+b^2)*tanh
(1/2*d*x+1/2*c)^3+3/128*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*
d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-3/4/(
a^2-2*a*b+b^2)*a*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-
2*b)*a)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2465 vs. $2(112) = 224$.
time = 0.65, size = 5186, normalized size = 41.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)^6 + 2
4*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)*sinh(d*x + c)
^5 + 4*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*sinh(d*x + c)^6 + 8*a^
3*b^2 - 28*a^2*b^3 + 20*a*b^4 + 4*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2
*b^3 + 15*a*b^4)*cosh(d*x + c)^4 + 4*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*
a^2*b^3 + 15*a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*
a*b^4)*cosh(d*x + c)^3 + (16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 15
*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^4*b - 40*a^3*b^2 + 47*a^2*b
^3 - 15*a*b^4)*cosh(d*x + c)^2 + 4*(8*a^4*b - 40*a^3*b^2 + 47*a^2*b^3 - 15*
a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)^4 +
6*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 - 3*(b^4*cosh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x +
 c)^7 + b^4*sinh(d*x + c)^8 + 4*(2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(7*b^4*
cosh(d*x + c)^2 + 2*a*b^3 - b^4)*sinh(d*x + c)^6 + 8*(7*b^4*cosh(d*x + c)^3
 + 3*(2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2*b^2 - 8*a*b^
3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*b^4*cosh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b^
3 + 3*b^4 + 30*(2*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + b^4 + 8*(
7*b^4*cosh(d*x + c)^5 + 10*(2*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^2*b^2 - 8
*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b^3 - b^4)*cosh(d*x
 + c)^2 + 4*(7*b^4*cosh(d*x + c)^6 + 15*(2*a*b^3 - b^4)*cosh(d*x + c)^4 + 2
*a*b^3 - b^4 + 3*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 8*(b^4*cosh(d*x + c)^7 + 3*(2*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^2*
b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a*b^3 - b^4)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*
sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2
*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^
2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4
*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 +
 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x
```

```
+ c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d
*x + c))*sinh(d*x + c) + b)) + 8*(3*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*
a*b^4)*cosh(d*x + c)^5 + 2*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 +
15*a*b^4)*cosh(d*x + c)^3 + (8*a^4*b - 40*a^3*b^2 + 47*a^2*b^3 - 15*a*b^4)*
cosh(d*x + c))*sinh(d*x + c))/((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*
cosh(d*x + c)^8 + 8*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x +
c)*sinh(d*x + c)^7 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*sinh(d*x +
 c)^8 + 4*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*
x + c)^6 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^2
 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d)*sinh(d*x + c)
^6 + 2*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a
*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*
cosh(d*x + c)^3 + 3*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6
- a*b^7)*d*cosh(d*x + c)^4 + 30*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*
b^6 + a*b^7)*d*cosh(d*x + c)^2 + (8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*
a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d)*sinh(d*x + c)^4 + 4*(2*a^5*b^3 - 7*a^4*b
^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^4 - 3*a
^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^5 + 10*(2*a^5*b^3 - 7*a^4*b^4 +
 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^3 + (8*a^6*b^2 - 32*a^5*b^3
 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*cosh(d*x + c))*sinh(d*
x + c)^3 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^6
 + 15*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x +
c)^4 + 3*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3
*a*b^7)*d*cosh(d*x + c)^2 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6
+ a*b^7)*d)*sinh(d*x + c)^2 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d +
 8*((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^7 + 3*(2*a^5*
b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^5 + (8*a^6
*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*cosh(
d*x + c)^3 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh
(d*x + c))*sinh(d*x + c)), -1/8*(2*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a
*b^4)*cosh(d*x + c)^6 + 12*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*co
sh(d*x + c)*sinh(d*x + c)^5 + 2*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^
4)*sinh(d*x + c)^6 + 4*a^3*b^2 - 14*a^2*b^3 + 10*a*b^4 + 2*(16*a^5 - 72*a^4
*b + 102*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4)*cosh(d*x + c)^4 + 2*(16*a^5 - 72*
a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4 + 1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(112) = 224$.
time = 3.80, size = 282, normalized size = 2.27

$$\frac{\frac{3\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{(a^2-2ab+b^2)\sqrt{-a^2+ab}} - \frac{2\left(8a^2be^{(6dx+6c)} - 16ab^2e^{(6dx+6c)} + 5b^3e^{(6dx+6c)} + 16a^3e^{(4dx+4c)} - 56a^2be^{(4dx+4c)} + 46ab^2e^{(4dx+4c)} - 15b^3e^{(4dx+4c)} + 8a^2be^{(2dx+2c)} - 32ab^2e^{(2dx+2c)} + 15b^3e^{(2dx+2c)} + 2ab^2 - 5b^3\right)}{(a^2b^2-2ab^3+b^4)\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-2be^{(2dx+2c)}+b\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^2 - 2
*a*b + b^2)*sqrt(-a^2 + a*b)) - 2*(8*a^2*b*e^(6*d*x + 6*c) - 16*a*b^2*e^(6*
d*x + 6*c) + 5*b^3*e^(6*d*x + 6*c) + 16*a^3*e^(4*d*x + 4*c) - 56*a^2*b*e^(4
*d*x + 4*c) + 46*a*b^2*e^(4*d*x + 4*c) - 15*b^3*e^(4*d*x + 4*c) + 8*a^2*b*e
^(2*d*x + 2*c) - 32*a*b^2*e^(2*d*x + 2*c) + 15*b^3*e^(2*d*x + 2*c) + 2*a*b^
2 - 5*b^3)/((a^2*b^2 - 2*a*b^3 + b^4)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2
*c) - 2*b*e^(2*d*x + 2*c) + b)^2))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)^4}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^3, x)
```

3.52 $\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=135

$$\frac{(a-4b)\text{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}b^{3/2}d}-\frac{a\cosh(c+dx)}{4(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)^2}+\frac{(a-4b)\cosh(c+dx)}{8(a-b)^2bd\left(a-b+b\cosh^2(c+\right.}$$

```
[Out] 1/8*(a-4*b)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/b^(3/2)/d-1
/4*a*cosh(d*x+c)/(a-b)/b/d/(a-b+b*cosh(d*x+c)^2)^2+1/8*(a-4*b)*cosh(d*x+c)/
(a-b)^2/b/d/(a-b+b*cosh(d*x+c)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3265, 393, 205, 211}

$$\frac{(a-4b)\text{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8b^{3/2}d(a-b)^{5/2}}+\frac{(a-4b)\cosh(c+dx)}{8bd(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{a\cosh(c+dx)}{4bd(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - 4*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*(a - b)^(5/2)*b^
(3/2)*d) - (a*Cosh[c + d*x])/(4*(a - b)*b*d*(a - b + b*Cosh[c + d*x]^2)^2)
+ ((a - 4*b)*Cosh[c + d*x])/(8*(a - b)^2*b*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{(a-b+bx^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\cosh(c+dx)}{4(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{(a-4b)\text{Subst}\left(\int \frac{1}{(a-b+bx^2)^2}\,dx, x, c\right.}{4(a-b)bd}$$

$$= -\frac{a\cosh(c+dx)}{4(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{(a-4b)\cosh(c+dx)}{8(a-b)^2bd\left(a-b+b\cosh^2(c+d\right.}$$

$$= \frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}b^{3/2}d} - \frac{a\cosh(c+dx)}{4(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{}{8(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.90, size = 170, normalized size = 1.26

$$\frac{\frac{(a-4b)\left(\text{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\text{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{(a-b)^{5/2}}+\frac{2\sqrt{b}\cosh(c+dx)\left(-2a^2-5ab+4b^2+(a-4b)b\cosh(2(c+dx))\right)}{(a-b)^2(2a-b+b\cosh(2(c+dx)))^2}}{8b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (((a - 4*b)*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] +
ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/(a - b)^(5/2)
 + (2*Sqrt[b]*Cosh[c + d*x]*(-2*a^2 - 5*a*b + 4*b^2 + (a - 4*b)*b*Cosh[2*(c
 + d*x)]))/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*b^(3/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(121) = 242$.
time = 1.41, size = 281, normalized size = 2.08

method	result
derivativedivides	$\frac{\frac{\frac{a(a-4b)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4b\left(a^2-2ab+b^2\right)}-\frac{\left(3a^3-2a^2b-8a\,b^2+16b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ab\left(a^2-2ab+b^2\right)}+\frac{\left(3a^2+4ab-16b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4b\left(a^2-2ab+b^2\right)}-\frac{(2b+a)a}{4b\left(a^2-2ab+b^2\right)}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}+}{d}$
default	$\frac{\frac{\frac{a(a-4b)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4b\left(a^2-2ab+b^2\right)}-\frac{\left(3a^3-2a^2b-8a\,b^2+16b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ab\left(a^2-2ab+b^2\right)}+\frac{\left(3a^2+4ab-16b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4b\left(a^2-2ab+b^2\right)}-\frac{(2b+a)a}{4b\left(a^2-2ab+b^2\right)}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}+}{d}$
risch	$-\frac{\mathrm{e}^{dx+c}(-ab\,\mathrm{e}^{6dx+6c}+4b^2\mathrm{e}^{6dx+6c}+4a^2\mathrm{e}^{4dx+4c}+9ab\,\mathrm{e}^{4dx+4c}-4b^2\mathrm{e}^{4dx+4c}+4a^2\mathrm{e}^{2dx+2c}+9ab\,\mathrm{e}^{2dx+2c}-4b^2\mathrm{e}^{2dx+2c}-ab}{4b(a-b)^2d\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(8*(1/32*a*(a-4*b)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/32*(3*a^3-
2*a^2*b-8*a*b^2+16*b^3)/a/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/32*(3*a
^2+4*a*b-16*b^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/32*(2*b+a)*a/b/(
a^2-2*a*b+b^2))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh
(1/2*d*x+1/2*c)^2+a)^2+1/8*(a-4*b)/b/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan
(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2-2*a+4*b)/(a*b-b^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((a*b*e^(7*c) - 4*b^2*e^(7*c))*e^(7*d*x) - (4*a^2*e^(5*c) + 9*a*b*e^(5*
c) - 4*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) + 9*a*b*e^(3*c) - 4*b^2*e^(3
*c))*e^(3*d*x) + (a*b*e^c - 4*b^2*e^c)*e^(d*x))/(a^2*b^3*d - 2*a*b^4*d + b^
5*d + (a^2*b^3*d*e^(8*c) - 2*a*b^4*d*e^(8*c) + b^5*d*e^(8*c))*e^(8*d*x) + 4
*(2*a^3*b^2*d*e^(6*c) - 5*a^2*b^3*d*e^(6*c) + 4*a*b^4*d*e^(6*c) - b^5*d*e^(
6*c))*e^(6*d*x) + 2*(8*a^4*b*d*e^(4*c) - 24*a^3*b^2*d*e^(4*c) + 27*a^2*b^3*
d*e^(4*c) - 14*a*b^4*d*e^(4*c) + 3*b^5*d*e^(4*c))*e^(4*d*x) + 4*(2*a^3*b^2*
d*e^(2*c) - 5*a^2*b^3*d*e^(2*c) + 4*a*b^4*d*e^(2*c) - b^5*d*e^(2*c))*e^(2*d
*x)) + 1/8*integrate(2*((a*e^(3*c) - 4*b*e^(3*c))*e^(3*d*x) - (a*e^c - 4*b*
e^c)*e^(d*x))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*b^2*e^(4*c) - 2*a*b^3*e^(4*c)
 + b^4*e^(4*c))*e^(4*d*x) + 2*(2*a^3*b*e^(2*c) - 5*a^2*b^2*e^(2*c) + 4*a*b^
3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3293 vs. $2(121) = 242$.
time = 0.46, size = 6087, normalized size = 45.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^7 + 28*(a^2*b^2 - 5*a*b^
3 + 4*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(a^2*b^2 - 5*a*b^3 + 4*b^4)*si
nh(d*x + c)^7 - 4*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^5
- 4*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4 - 21*(a^2*b^2 - 5*a*b^3 + 4*b^4
)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh
(d*x + c)^3 - (4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(
d*x + c)^4 - 4*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 + 4
*(35*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^4 - 4*a^3*b - 5*a^2*b^2 + 13
*a*b^3 - 4*b^4 - 10*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 4*(21*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^5 - 10
*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 - 3*(4*a^3*b + 5*
a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((a*b^2 - 4*b^
3)*cosh(d*x + c)^8 + 8*(a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b
^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^6
 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3 + 7*(a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(2*a^2*b - 9*a*b^2 + 4
*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^3 - 40*a^2*b + 35*a*b^2 - 12*
b^3)*cosh(d*x + c)^4 + 2*(35*(a*b^2 - 4*b^3)*cosh(d*x + c)^4 + 8*a^3 - 40*a
^2*b + 35*a*b^2 - 12*b^3 + 30*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b - 9*a*
b^2 + 4*b^3)*cosh(d*x + c)^3 + (8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(
d*x + c))*sinh(d*x + c)^3 + a*b^2 - 4*b^3 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3)*c
osh(d*x + c)^2 + 4*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15*(2*a^2*b - 9*a*b
^2 + 4*b^3)*cosh(d*x + c)^4 + 2*a^2*b - 9*a*b^2 + 4*b^3 + 3*(8*a^3 - 40*a^2
*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a*b^2 - 4*b^
3)*cosh(d*x + c)^7 + 3*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + (8*a^3
 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^3 + (2*a^2*b - 9*a*b^2 + 4*b
^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh
(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*co
sh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c)
^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2
 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)
^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*co
sh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*co
sh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(a^2*b^2 -
```

```
 5*a*b^3 + 4*b^4)*cosh(d*x + c) + 4*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x
 + c)^6 - 5*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^4 + a^2*
b^2 - 5*a*b^3 + 4*b^4 - 3*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x +
 c)^8 + 8*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)*sinh(d*x +
c)^7 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*sinh(d*x + c)^8 + 4*(2*a^4*b
^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b
^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4
+ 9*a^2*b^5 - 5*a*b^6 + b^7)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b^2 - 32*a^4*b^3
 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^
3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^3 + 3*(2*a^4*b^3 - 7*a^3
*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*
(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^4 + 30*(2*a^4*b^3 - 7
*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + (8*a^5*b^2 - 32*a
^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d)*sinh(d*x + c)^4 + 4
*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + 8*
(7*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^5 + 10*(2*a^4*b^3
- 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^3 + (8*a^5*b^2 - 3
2*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^
6 + 15*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^
4 + 3*(8*a^5*b^2 - 32*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)
*d*cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d)
*sinh(d*x + c)^2 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d + 8*((a^3*b^4 -
3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^7 + 3*(2*a^4*b^3 - 7*a^3*b^4 + 9
*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^5 + (8*a^5*b^2 - 32*a^4*b^3 + 51*
a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c)^3 + (2*a^4*b^3 - 7
*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*
(2*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^7 + 14*(a^2*b^2 - 5*a*b^3 + 4*
b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(a^2*b^2 - 5*a*b^3 + 4*b^4)*sinh(d*x
 + c)^7 - 2*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)^3}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^3, x)
```

3.53 $\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=139

$$-\frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^{5/2}d}+\frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2}+\frac{(2a+b)\cosh(c+dx)\sinh(c+}{8a(a-b)^2d\left(a+b\sinh^2(c+dx\right.}$$

```
[Out] -1/8*(4*a-b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(5/2)/d
+1/4*cosh(d*x+c)*sinh(d*x+c)/(a-b)/d/(a+b*sinh(d*x+c)^2)^2+1/8*(2*a+b)*cosh
(d*x+c)*sinh(d*x+c)/a/(a-b)^2/d/(a+b*sinh(d*x+c)^2)
```

Rubi [A]

time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3252, 12, 3260, 214}

$$-\frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a-b)^{5/2}}+\frac{(2a+b)\sinh(c+dx)\cosh(c+dx)}{8ad(a-b)^2\left(a+b\sinh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{4d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -1/8*((4*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a -
 b)^(5/2)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(4*(a - b)*d*(a + b*Sinh[c + d
*x]^2)^2) + ((2*a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*a*(a - b)^2*d*(a + b
*Sinh[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
```

```
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx &= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2}-\frac{\int \frac{a-2a\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx}{4a(a-b)}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2}+\frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)}-\frac{\int \frac{}{a+}}{8}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2}+\frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)}-\frac{(4a}{}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2}+\frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)}-\frac{(4a}{}\\
&= -\frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^{5/2}d}+\frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2}+
\end{aligned}$$

Mathematica [A]
time = 0.90, size = 121, normalized size = 0.87

$$\frac{-\frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{5/2}}+\frac{(8a^2-4ab-b^2+b(2a+b)\cosh(2(c+dx)))\sinh(2(c+dx))}{a(a-b)^2(2a-b+b\cosh(2(c+dx)))^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-(((4*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a - b
)^(5/2))) + ((8*a^2 - 4*a*b - b^2 + b*(2*a + b)*Cosh[2*(c + d*x)])*Sinh[2*(
c + d*x)])/(a*(a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(125) = 250$.

time = 1.32, size = 401, normalized size = 2.88

method	result
derivativedivides	$-\frac{\frac{8\left(-\frac{(4a-b)\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(4a^{2}-9ab-4b^{2}\right)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(4a^{2}-9ab-4b^{2}\right)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a\left(a^{2}-2ab+b^{2}\right)}-\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^{2}}}{}$
default	$-\frac{\frac{8\left(-\frac{(4a-b)\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(4a^{2}-9ab-4b^{2}\right)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(4a^{2}-9ab-4b^{2}\right)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a\left(a^{2}-2ab+b^{2}\right)}-\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^{2}}}{}$
risch	$-\frac{4a\,b^{2}e^{6dx+6c}-b^{3}e^{6dx+6c}+16a^{3}e^{4dx+4c}-8a^{2}b\,e^{4dx+4c}-2a\,b^{2}e^{4dx+4c}+3b^{3}e^{4dx+4c}+16a^{2}b\,e^{2dx+2c}-4a\,b^{2}e^{2dx+2c}-3}{4b(a-b)^{2}d\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)^{2}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-8*(-1/32*(4*a-b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+1/32*(4*a^2-9*
a*b-4*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/32*(4*a^2-9*a*b-4*b^2)
/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-1/32*(4*a-b)/(a^2-2*a*b+b^2)*tanh(
1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh
(1/2*d*x+1/2*c)^2+a)^2-1/4*(4*a-b)/(a^2-2*a*b+b^2)*(1/2*(-(-b*(a-b))^(1/2)-
b)/(-b*(a-b))^(1/2)/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*(-(-b*(a-b))^(1/2)+b
)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2632 vs. $2(125) = 250$.
time = 0.46, size = 5519, normalized size = 39.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 + 24*(4*a^3*b^2 -
 5*a^2*b^3 + a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(4*a^3*b^2 - 5*a^2*b^
3 + a*b^4)*sinh(d*x + c)^6 + 8*a^3*b^2 - 4*a^2*b^3 - 4*a*b^4 + 4*(16*a^5 -
24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 4*(16*a^5 - 2
4*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4 + 15*(4*a^3*b^2 - 5*a^2*b^3 + a*b
^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4
)*cosh(d*x + c)^3 + (16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^4*b - 20*a^3*b^2 + a^2*b^3 + 3*a*b^
4)*cosh(d*x + c)^2 + 4*(16*a^4*b - 20*a^3*b^2 + a^2*b^3 + 3*a*b^4 + 15*(4*a
^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 24*a^4*b + 6*a^3*
b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a*b^3 - b
^4)*cosh(d*x + c)^8 + 8*(4*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*
a*b^3 - b^4)*sinh(d*x + c)^8 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^
6 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4 + 7*(4*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(
d*x + c)^6 + 8*(7*(4*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(8*a^2*b^2 - 6*a*b^3
+ b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(32*a^3*b - 40*a^2*b^2 + 20*a*b^3
 - 3*b^4)*cosh(d*x + c)^4 + 2*(35*(4*a*b^3 - b^4)*cosh(d*x + c)^4 + 32*a^3*
b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4 + 30*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 4*a*b^3 - b^4 + 8*(7*(4*a*b^3 - b^4)*cosh(d*x +
c)^5 + 10*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^3 + (32*a^3*b - 40*a^2*
b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2*b^2 - 6*a
*b^3 + b^4)*cosh(d*x + c)^2 + 4*(7*(4*a*b^3 - b^4)*cosh(d*x + c)^6 + 15*(8*
a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^2*b^2 - 6*a*b^3 + b^4 + 3*(3
2*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 8*((4*a*b^3 - b^4)*cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*
x + c)^5 + (32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (8*
a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log(
(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x +
```

```
 c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b
- b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*
a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*co
sh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(
2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^
2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 8
*(3*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^5 + 2*(16*a^5 - 24*a^4*b
+ 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (16*a^4*b - 20*a^3*b^2
 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^3 - 3*a^4*b^4 +
 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^
5 - a^2*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3
*b^5 - a^2*b^6)*d*sinh(d*x + c)^8 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 -
5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*
b^5 - a^2*b^6)*d*cosh(d*x + c)^2 + (2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a
^3*b^5 + a^2*b^6)*d)*sinh(d*x + c)^6 + 2*(8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3
 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 -
 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 7*a^5*
b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(35*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^4 + 30*(2*
a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^2 +
(8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d
)*sinh(d*x + c)^4 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*
b^6)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d
*cosh(d*x + c)^5 + 10*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*
b^6)*d*cosh(d*x + c)^3 + (8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 +
17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 -
3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^6 + 15*(2*a^6*b^2 - 7*a^5*
b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4 + 3*(8*a^7*b - 32*
a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)
^2 + (2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d)*sinh(d*x
+ c)^2 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d + 8*((a^5*b^3 - 3*a^
4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^7 + 3*(2*a^6*b^2 - 7*a^5*b^3 +
 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^5 + (8*a^7*b - 32*a^6*b^2
 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)^3 + (2
*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*si
nh(d*x + c)), -1/8*(2*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 + 12*
(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(4*a^3*b^
2 - 5*a^2*b^3 + a*b^4)*sinh(d*x + c)^6 + 4*a^3*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(125) = 250.
time = 1.85, size = 277, normalized size = 1.99

$$-\frac{\frac{(4\,a-b)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{-a^2+ab}}+\frac{2\,\left(4\,ab^2e^{(6\,dx+6\,c)}-b^3e^{(6\,dx+6\,c)}+16\,a^3e^{(4\,dx+4\,c)}-8\,a^2be^{(4\,dx+4\,c)}-2\,ab^2e^{(4\,dx+4\,c)}+3\,b^3e^{(4\,dx+4\,c)}+16\,a^2be^{(2\,dx+2\,c)}-4\,ab^2e^{(2\,dx+2\,c)}-3\,b^3e^{(2\,dx+2\,c)}+2\,ab^2+b^3\right)}{(a^3b-2\,a^2b^2+ab^3)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((4*a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/
((a^3 - 2*a^2*b + a*b^2)*sqrt(-a^2 + a*b)) + 2*(4*a*b^2*e^(6*d*x + 6*c) - b
^3*e^(6*d*x + 6*c) + 16*a^3*e^(4*d*x + 4*c) - 8*a^2*b*e^(4*d*x + 4*c) - 2*a
*b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 16*a^2*b*e^(2*d*x + 2*c) - 4
*a*b^2*e^(2*d*x + 2*c) - 3*b^3*e^(2*d*x + 2*c) + 2*a*b^2 + b^3)/((a^3*b - 2
*a^2*b^2 + a*b^3)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x +
 2*c) + b)^2))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)^2}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^3, x)
```

3.54 $\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=118

$$\frac{3\mathrm{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}\sqrt{b}\,d}+\frac{\cosh(c+dx)}{4(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2}+\frac{3\cosh(c+dx)}{8(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)}$$

```
[Out] 1/4*cosh(d*x+c)/(a-b)/d/(a-b+b*cosh(d*x+c)^2)^2+3/8*cosh(d*x+c)/(a-b)^2/d/(
a-b+b*cosh(d*x+c)^2)+3/8*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/(a-b)^(5/2
)/d/b^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3265, 205, 211}

$$\frac{3\mathrm{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8\sqrt{b}\,d(a-b)^{5/2}}+\frac{3\cosh(c+dx)}{8d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}+\frac{\cosh(c+dx)}{4d(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*(a - b)^(5/2)*Sqrt[b]*d)
 + Cosh[c + d*x]/(4*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)^2) + (3*Cosh[c +
d*x])/(8*(a - b)^2*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a-b+bx^2)^3}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)}{4(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{(a-b+bx^2)^2}\,dx, x, \cosh(c+dx\right.}{4(a-b)d} \\
&= \frac{\cosh(c+dx)}{4(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{3\cosh(c+dx)}{8(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)} \\
&= \frac{3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}\sqrt{b}\,d} + \frac{\cosh(c+dx)}{4(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{}{8(a-b)^2}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.53, size = 149, normalized size = 1.26

$$
\frac{\frac{3\left(\operatorname{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\operatorname{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{(a-b)^{5/2}\sqrt{b}} + \frac{2\cosh(c+dx)(10a-7b+3b\cosh(2(c+dx)))}{(a-b)^2(2a-b+b\cosh(2(c+dx)))^2}}{8d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(
Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/((a - b)^(5/2)*Sqrt[b
]) + (2*Cosh[c + d*x]*(10*a - 7*b + 3*b*Cosh[2*(c + d*x)]))/((a - b)^2*(2*a
 - b + b*Cosh[2*(c + d*x)])^2))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(104) = 208$.
time = 0.95, size = 277, normalized size = 2.35

method	result
risch	$\frac{e^{dx+c}\left(3b\,e^{6dx+6c}+20a\,e^{4dx+4c}-11b\,e^{4dx+4c}+20a\,e^{2dx+2c}-11b\,e^{2dx+2c}+3b\right)}{4(a-b)^2d\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)^2} - \frac{3\ln\left(e^{2dx+2c}-\frac{2(a-b)e^{dx+c}}{\sqrt{-ab+b^2}}+1\right)}{16\sqrt{-ab+b^2}\,(a-b)^2d}$

derivativedivides	$\frac{\frac{-\frac{\left(5a^2-16ab+8b^2\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a\left(a^2-2ab+b^2\right)}+\frac{\left(15a^3-46a^2b+56a\,b^2-16b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2\left(a^2-2ab+b^2\right)}-\frac{\left(15a^2-32ab+8b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a\left(a^2-2ab+b^2\right)}+}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}}{d}$
default	$\frac{\frac{-\frac{\left(5a^2-16ab+8b^2\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a\left(a^2-2ab+b^2\right)}+\frac{\left(15a^3-46a^2b+56a\,b^2-16b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2\left(a^2-2ab+b^2\right)}-\frac{\left(15a^2-32ab+8b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a\left(a^2-2ab+b^2\right)}+}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/8*(5*a^2-16*a*b+8*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+1
/8/a^2*(15*a^3-46*a^2*b+56*a*b^2-16*b^3)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c
)^4-1/8*(15*a^2-32*a*b+8*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+1/8*(
5*a-2*b)/(a^2-2*a*b+b^2))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^
2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+3/8/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan
(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2-2*a+4*b)/(a*b-b^2)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((20*a*e^(5*c) - 11*b*e^(5*c))*e^(5*d*x) + (20*a*e^(3*c) - 11*b*e^(3*c)
)*e^(3*d*x) + 3*b*e^(7*d*x + 7*c) + 3*b*e^(d*x + c))/(a^2*b^2*d - 2*a*b^3*d
 + b^4*d + (a^2*b^2*d*e^(8*c) - 2*a*b^3*d*e^(8*c) + b^4*d*e^(8*c))*e^(8*d*x
) + 4*(2*a^3*b*d*e^(6*c) - 5*a^2*b^2*d*e^(6*c) + 4*a*b^3*d*e^(6*c) - b^4*d*
e^(6*c))*e^(6*d*x) + 2*(8*a^4*d*e^(4*c) - 24*a^3*b*d*e^(4*c) + 27*a^2*b^2*d
*e^(4*c) - 14*a*b^3*d*e^(4*c) + 3*b^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^3*b*d*e
^(2*c) - 5*a^2*b^2*d*e^(2*c) + 4*a*b^3*d*e^(2*c) - b^4*d*e^(2*c))*e^(2*d*x)
) + 1/2*integrate(3/2*(e^(3*d*x + 3*c) - e^(d*x + c))/(a^2*b - 2*a*b^2 + b^
3 + (a^2*b*e^(4*c) - 2*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^3*e^
(2*c) - 5*a^2*b*e^(2*c) + 4*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2726 vs. $2(104) = 208$.
time = 0.47, size = 5152, normalized size = 43.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*(a*b^2 - b^3)*cosh(d*x + c)^7 + 84*(a*b^2 - b^3)*cosh(d*x + c)*si
nh(d*x + c)^6 + 12*(a*b^2 - b^3)*sinh(d*x + c)^7 + 4*(20*a^2*b - 31*a*b^2 +
 11*b^3)*cosh(d*x + c)^5 + 4*(20*a^2*b - 31*a*b^2 + 11*b^3 + 63*(a*b^2 - b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(21*(a*b^2 - b^3)*cosh(d*x + c)^3
+ (20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(20*a^2
*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 4*(105*(a*b^2 - b^3)*cosh(d*x + c
)^4 + 20*a^2*b - 31*a*b^2 + 11*b^3 + 10*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^3 + 4*(63*(a*b^2 - b^3)*cosh(d*x + c)^5 + 10*(20
*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 3*(20*a^2*b - 31*a*b^2 + 11*b
^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*
x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x + c
)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*co
sh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2 -
 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + c)^4 + 30*(2*a*b - b
^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*b^2*cos
h(d*x + c)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*b^2*
cosh(d*x + c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3*(8*a^2 - 8*a*b + 3*b
^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x
+ c)^7 + 3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x
 + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b + b^2)*log(
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
- 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(
d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c
) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3
+ (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) +
 b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c
)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(
d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c)
+ b)) + 12*(a*b^2 - b^3)*cosh(d*x + c) + 4*(21*(a*b^2 - b^3)*cosh(d*x + c)^
6 + 5*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 3*a*b^2 - 3*b^3 + 3*
(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^3*b^3 -
3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^8 + 8*(a^3*b^3 - 3*a^2*b^4 + 3*a
*b^5 - b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3*b^3 - 3*a^2*b^4 + 3*a*b^
5 - b^6)*d*sinh(d*x + c)^8 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5
 + b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*co
sh(d*x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh
(d*x + c)^6 + 2*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5
- 3*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*c
osh(d*x + c)^3 + 3*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*
d*cosh(d*x + c)^4 + 30*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*
d*cosh(d*x + c)^2 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*
```

```
b^5 - 3*b^6)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*
a*b^5 + b^6)*d*cosh(d*x + c)^2 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)
*d*cosh(d*x + c)^5 + 10*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)
*d*cosh(d*x + c)^3 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a
*b^5 - 3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3*b^3 - 3*a^2*b^4
+ 3*a*b^5 - b^6)*d*cosh(d*x + c)^6 + 15*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4
- 5*a*b^5 + b^6)*d*cosh(d*x + c)^4 + 3*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 -
 41*a^2*b^4 + 17*a*b^5 - 3*b^6)*d*cosh(d*x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3
+ 9*a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh(d*x + c)^2 + (a^3*b^3 - 3*a^2*b^4 + 3*
a*b^5 - b^6)*d + 8*((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^7
 + 3*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d*x + c)^5
+ (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5 - 3*b^6)*d*cos
h(d*x + c)^3 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d
*x + c))*sinh(d*x + c)), 1/8*(6*(a*b^2 - b^3)*cosh(d*x + c)^7 + 42*(a*b^2 -
 b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 6*(a*b^2 - b^3)*sinh(d*x + c)^7 + 2*(
20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^5 + 2*(20*a^2*b - 31*a*b^2 + 11
*b^3 + 63*(a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(21*(a*b^2 -
b^3)*cosh(d*x + c)^3 + (20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d
*x + c)^4 + 2*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 2*(105*(a*b^
2 - b^3)*cosh(d*x + c)^4 + 20*a^2*b - 31*a*b^2 + 11*b^3 + 10*(20*a^2*b - 31
*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(63*(a*b^2 - b^3)*cos
h(d*x + c)^5 + 10*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 3*(20*a^
2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x +
 c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
```

```
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)}{{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2)^3, x)
```

3.55 $\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

<u>**Optimal**</u>. Leaf size=154

$$\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{5/2}d}-\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2}-\frac{3(2a-b)b\cosh(c+dx}{8a^2(a-b)^2d\left(a+b\,\mathrm{si}\right.}$$

```
[Out] 1/8*(8*a^2-8*a*b+3*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a
-b)^(5/2)/d-1/4*b*cosh(d*x+c)*sinh(d*x+c)/a/(a-b)/d/(a+b*sinh(d*x+c)^2)^2-3
/8*(2*a-b)*b*cosh(d*x+c)*sinh(d*x+c)/a^2/(a-b)^2/d/(a+b*sinh(d*x+c)^2)
```

<u>**Rubi**</u> [A]
time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3263, 3252, 12, 3260, 214}

$$-\frac{3b(2a-b)\sinh(c+dx)\cosh(c+dx)}{8a^2d(a-b)^2\left(a+b\sinh^2(c+dx)\right)}+\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{5/2}}-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^(-3),x]
```

```
[Out] ((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a
^(5/2)*(a - b)^(5/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(4*a*(a - b)*d*(a
 + b*Sinh[c + d*x]^2)^2) - (3*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*a
^2*(a - b)^2*d*(a + b*Sinh[c + d*x]^2))
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 214</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

<u>Rule 3252</u>

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{\int \frac{-4a+3b+2b\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx}{4a(a-b)}$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3(2a-b)b\cosh(c+dx)\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} -$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3(2a-b)b\cosh(c+dx)\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} +$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3(2a-b)b\cosh(c+dx)\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} +$$

$$= \frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{5/2}d} - \frac{b\cosh(c+dx)\sinh(c+}{4a(a-b)d\left(a+b\sinh^2(c+\right.}$$

Mathematica [A]
time = 0.81, size = 132, normalized size = 0.86

$$\frac{\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}} + \frac{\sqrt{a}\,b(-16a^2+16ab-3b^2+3b(-2a+b)\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b)^2(2a-b+b\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^(-3),x]
```

```
[Out] (((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a
- b)^(5/2) + (Sqrt[a]*b*(-16*a^2 + 16*a*b - 3*b^2 + 3*b*(-2*a + b)*Cosh[2*(
c + d*x)])*Sinh[2*(c + d*x)])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2)
)/(8*a^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(140) = 280$.

time = 1.43, size = 418, normalized size = 2.71

method	result
derivativedivides	$-\frac{2\left(\frac{b(8a-5b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a\left(a^2-2ab+b^2\right)}-\frac{\left(8a^2-29ab+12b^2\right)b\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2\left(a^2-2ab+b^2\right)}-\frac{\left(8a^2-29ab+12b^2\right)b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2\left(a^2-2ab+b^2\right)}+\frac{b(8a-5b)\tanh}{8a\left(a^2-2ab+\right.}\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}$
default	$-\frac{2\left(\frac{b(8a-5b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a\left(a^2-2ab+b^2\right)}-\frac{\left(8a^2-29ab+12b^2\right)b\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2\left(a^2-2ab+b^2\right)}-\frac{\left(8a^2-29ab+12b^2\right)b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2\left(a^2-2ab+b^2\right)}+\frac{b(8a-5b)\tanh}{8a\left(a^2-2ab+\right.}\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}$
risch	$\frac{8a^2b\,e^{6dx+6c}-8a\,b^2e^{6dx+6c}+3b^3e^{6dx+6c}+48a^3e^{4dx+4c}-72a^2b\,e^{4dx+4c}+42a\,b^2e^{4dx+4c}-9b^3e^{4dx+4c}+40a^2b\,e^{2dx+2c}-4}{4d\,a^2(a-b)^2\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(1/8*b*(8*a-5*b)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(8*a^2
-29*a*b+12*b^2)/a^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-1/8*(8*a^2-29*a
*b+12*b^2)/a^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/8*b*(8*a-5*b)/a/(a
^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*
x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-1/4/a*(8*a^2-8*a*b+3*b^2)/(a^2-2*
a*b+b^2)*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-
a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
```

```
^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-
2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the ’assume’ command before evaluation *may* h
elp (example of legal syntax is ’assume(b-a>0)’, see ‘assume?‘ for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2835 vs. 2(140) = 280.
time = 0.52, size = 5925, normalized size = 38.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 24
*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^
5 + 4*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 24*a^
3*b^2 - 36*a^2*b^3 + 12*a*b^4 + 12*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2
*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 12*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a
^2*b^3 + 3*a*b^4 + 5*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 16*(5*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b
^4)*cosh(d*x + c)^3 + 3*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*
b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3
 - 9*a*b^4)*cosh(d*x + c)^2 + 4*(40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b
^4 + 15*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 18*
(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + ((8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(8*a^2*b
^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a^2*b^2 - 8*a*b^3
+ 3*b^4)*sinh(d*x + c)^8 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cos
h(d*x + c)^6 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4 + 7*(8*a^2*b^2 -
 8*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2*b^2 - 8*a*
b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*
a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d
```

```
*x + c)^4 + 64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4 + 30*(16*a^
3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a
^2*b^2 - 8*a*b^3 + 3*b^4 + 8*(7*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)
^5 + 10*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (64*a^
4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(
8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(16*a^3*b - 24*a^2*b^2 +
14*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^
4 + 3*(64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(1
6*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (64*a^4 - 128*a^
3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (16*a^3*b - 24*a^2*
b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((
b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x +
c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b -
 b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a
*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*
x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cos
h(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2
*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2
 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 8*
(3*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^5 + 6*(16*a^
5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + (40*a^4
*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6
*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 - 3*
a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 -
 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^7*b - 7*a^6*b^
2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 - 3*
a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^7*b - 7*a^6*b^2 + 9
*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^8 - 32*a^7*b +
51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*
(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^7*b
- 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c))*sinh(d*x +
c)^5 + 2*(35*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4
+ 30*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c
)^2 + (8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)
*d)*sinh(d*x + c)^4 + 4*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*
b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d
*cosh(d*x + c)^5 + 10*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^
5)*d*cosh(d*x + c)^3 + (8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4
*b^4 - 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 - 3*a^5*
b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 15*(2*a^7*b - 7*a^6*b^2 + 9*
a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^8 - 32*a^7*b + 51
*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^7*
b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^2 + (a^6*
```

```
b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d + 8*((a^6*b^2 - 3*a^5*b^3 + 3*a^4*
b^4 - a^3*b^5)*d*cosh(d*x + c)^7 + 3*(2*a^7*b -...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(140) = 280.
time = 0.81, size = 302, normalized size = 1.96

$$\frac{\frac{\left(8\,a^{2}-8\,ab+3\,b^{2}\right)\arctan\left(\frac{be^{\left(2\,dx+2\,c\right)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{\left(a^{4}-2\,a^{3}b+a^{2}b^{2}\right)\sqrt{-a^{2}+ab}}+\frac{2\,\left(8\,a^{2}be^{\left(6\,dx+6\,c\right)}-8\,ab^{2}e^{\left(6\,dx+6\,c\right)}+3\,b^{3}e^{\left(6\,dx+6\,c\right)}+48\,a^{3}e^{\left(4\,dx+4\,c\right)}-72\,a^{2}be^{\left(4\,dx+4\,c\right)}+42\,ab^{2}e^{\left(4\,dx+4\,c\right)}-9\,b^{3}e^{\left(4\,dx+4\,c\right)}+40\,a^{2}be^{\left(2\,dx+2\,c\right)}-40\,ab^{2}e^{\left(2\,dx+2\,c\right)}+9\,b^{3}e^{\left(2\,dx+2\,c\right)}+6\,ab^{2}-3\,b^{3}\right)}{\left(a^{4}-2\,a^{3}b+a^{2}b^{2}\right)\left(be^{\left(4\,dx+4\,c\right)}+4\,ae^{\left(2\,dx+2\,c\right)}-2\,be^{\left(2\,dx+2\,c\right)}+b\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((8*a^2 - 8*a*b + 3*b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(
-a^2 + a*b))/((a^4 - 2*a^3*b + a^2*b^2)*sqrt(-a^2 + a*b)) + 2*(8*a^2*b*e^(6
*d*x + 6*c) - 8*a*b^2*e^(6*d*x + 6*c) + 3*b^3*e^(6*d*x + 6*c) + 48*a^3*e^(4
*d*x + 4*c) - 72*a^2*b*e^(4*d*x + 4*c) + 42*a*b^2*e^(4*d*x + 4*c) - 9*b^3*e
^(4*d*x + 4*c) + 40*a^2*b*e^(2*d*x + 2*c) - 40*a*b^2*e^(2*d*x + 2*c) + 9*b^
3*e^(2*d*x + 2*c) + 6*a*b^2 - 3*b^3)/((a^4 - 2*a^3*b + a^2*b^2)*(b*e^(4*d*x
 + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{sinh}\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(1/(a + b*sinh(c + d*x)^2)^3, x)
```

3.56 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=166

$$-\frac{\sqrt{b}\,(15a^2-20ab+8b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3(a-b)^{5/2}d}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d}-\frac{b\cosh(c+dx)}{4a(a-b)d\left(a-b+b\cosh^2(c\right.}$$

```
[Out] -arctanh(cosh(d*x+c))/a^3/d-1/4*b*cosh(d*x+c)/a/(a-b)/d/(a-b+b*cosh(d*x+c)^
2)^2-1/8*(7*a-4*b)*b*cosh(d*x+c)/a^2/(a-b)^2/d/(a-b+b*cosh(d*x+c)^2)-1/8*(1
5*a^2-20*a*b+8*b^2)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^3/(a-
b)^(5/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3265, 425, 541, 536, 212, 211}

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d}-\frac{b(7a-4b)\cosh(c+dx)}{8a^2d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{\sqrt{b}\,(15a^2-20ab+8b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3d(a-b)^{5/2}}-\frac{b\cosh(c+dx)}{4ad(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -1/8*(Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt
[a - b]])/(a^3*(a - b)^(5/2)*d) - ArcTanh[Cosh[c + d*x]]/(a^3*d) - (b*Cosh[
c + d*x])/(4*a*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)^2) - ((7*a - 4*b)*b*Co
sh[c + d*x])/(8*a^2*(a - b)^2*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh(c+dx)}{4a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-4a+b+3bx^2}{(1-x^2)(a-b+bx^2)^2}\,dx, x, \text{co}\right.}{4a(a-b)d}$$

$$= -\frac{b\cosh(c+dx)}{4a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(7a-4b)b\cosh(c+dx)}{8a^2(a-b)^2d\left(a-b+b\cosh^2(c+\right.}$$

$$= -\frac{b\cosh(c+dx)}{4a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(7a-4b)b\cosh(c+dx)}{8a^2(a-b)^2d\left(a-b+b\cosh^2(c+\right.}$$

$$= -\frac{\sqrt{b}\,(15a^2-20ab+8b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3(a-b)^{5/2}d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.23, size = 237, normalized size = 1.43

$$-\frac{\frac{\sqrt{b}\,(15a^2-20ab+8b^2)\text{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{\sqrt{b}\,(15a^2-20ab+8b^2)\text{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{8a^2b\cosh(c+dx)}{(a-b)(2a-b+b\cosh(2(c+dx)))^2} + \frac{2a(7a-4b)b\cosh(c+dx)}{(a-b)^2(2a-b+b\cosh(2(c+dx)))} - 8\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -1/8*((Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(
c + d*x)/2])/Sqrt[a - b]])/(a - b)^(5/2) + (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^
2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(a - b)^(5/
2) + (8*a^2*b*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])^2) +
(2*a*(7*a - 4*b)*b*Cosh[c + d*x])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)]
)) - 8*Log[Tanh[(c + d*x)/2]])/(a^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $\overline{2(152)} = 304$.
time = 1.74, size = 308, normalized size = 1.86

method	result
derivativedivides	$\dfrac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3} - \frac{2b\left(\frac{-\frac{(9a^2-28ab+16b^2)a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a^2-2ab+b^2)} + \frac{3(9a^3-30a^2b+40a\,b^2-16b^3)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a^2-2ab+b^2)} - \frac{a(27a^2-68ab+}{8(}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right.}\right.}{a^3}}{d}$

default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2b\left(\frac{-\frac{\left(9a^2-28ab+16b^2\right)a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8\left(a^2-2ab+b^2\right)}+\frac{3\left(9a^3-30a^2b+40a\,b^2-16b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8\left(a^2-2ab+b^2\right)}-\frac{a\left(27a^2-68ab+3\right.}{8\left(a^2\right.}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}\right.}{a^3}}{d}$
risch	$-\frac{e^{dx+c}b\left(7ab\,e^{6dx+6c}-4b^2e^{6dx+6c}+36a^2e^{4dx+4c}-31ab\,e^{4dx+4c}+4b^2e^{4dx+4c}+36a^2e^{2dx+2c}-31ab\,e^{2dx+2c}+4b^2e^{2dx+2c}-\right.}{4d\,a^2(a-b)^2\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^3*ln(tanh(1/2*d*x+1/2*c))-2/a^3*b*((-1/8*(9*a^2-28*a*b+16*b^2)*a/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+3/8*(9*a^3-30*a^2*b+40*a*b^2-16*b^3)/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-1/8*a*(27*a^2-68*a*b+32*b^2)/(a^2-2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^2+3/8*a^2*(3*a-2*b)/(a^2-2*a*b+b^2))/(a*tanh(1/2
*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/16
*(15*a^2-20*a*b+8*b^2)/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*a*tanh
(1/2*d*x+1/2*c)^2-2*a+4*b)/(a*b-b^2)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((7*a*b^2*e^(7*c) - 4*b^3*e^(7*c))*e^(7*d*x) + (36*a^2*b*e^(5*c) - 31*
a*b^2*e^(5*c) + 4*b^3*e^(5*c))*e^(5*d*x) + (36*a^2*b*e^(3*c) - 31*a*b^2*e^(
3*c) + 4*b^3*e^(3*c))*e^(3*d*x) + (7*a*b^2*e^c - 4*b^3*e^c)*e^(d*x))/(a^4*b
^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) - 2*a^3*b^3*d*e^(8*c) +
 a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(2*a^5*b*d*e^(6*c) - 5*a^4*b^2*d*e^(6*c)
+ 4*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(8*a^6*d*e^(4*c) -
 24*a^5*b*d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) - 14*a^3*b^3*d*e^(4*c) + 3*a^2*b
^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^5*b*d*e^(2*c) - 5*a^4*b^2*d*e^(2*c) + 4*a^
3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(
-c))/(a^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) - 2*integrate(1/8*((15
*a^2*b*e^(3*c) - 20*a*b^2*e^(3*c) + 8*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^
c - 20*a*b^2*e^c + 8*b^3*e^c)*e^(d*x))/(a^5*b - 2*a^4*b^2 + a^3*b^3 + (a^5*
b*e^(4*c) - 2*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^6*e^(2*
c) - 5*a^5*b*e^(2*c) + 4*a^4*b^2*e^(2*c) - a^3*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5242 vs. $2(152) = 304$.

time = 0.53, size = 9815, normalized size = 59.13

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^7 + 28*(7*a^2*b^2 - 4*a*b^3)*
cosh(d*x + c)*sinh(d*x + c)^6 + 4*(7*a^2*b^2 - 4*a*b^3)*sinh(d*x + c)^7 + 4
*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^5 + 4*(36*a^3*b - 31*a^2*b
^2 + 4*a*b^3 + 21*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 +
20*(7*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + (36*a^3*b - 31*a^2*b^2 + 4*a*
b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*c
osh(d*x + c)^3 + 4*(35*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^4 + 36*a^3*b - 3
1*a^2*b^2 + 4*a*b^3 + 10*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 + 4*(21*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5 + 10*(36*a^3
*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + 3*(36*a^3*b - 31*a^2*b^2 + 4*a
*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cos
h(d*x + c)^8 + 8*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)*sinh(d*x + c
)^7 + (15*a^2*b^2 - 20*a*b^3 + 8*b^4)*sinh(d*x + c)^8 + 4*(30*a^3*b - 55*a^
2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^6 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a
*b^3 - 8*b^4 + 7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 8*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + 3*(30*a^3*b
 - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(120*a
^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*
(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 120*a^4 - 280*a^3*b + 269
*a^2*b^2 - 124*a*b^3 + 24*b^4 + 30*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^2*b^2 - 20*a*b^3 + 8*b^4 + 8*(7*
(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^5 + 10*(30*a^3*b - 55*a^2*b^2
 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^3 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 -
 124*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(30*a^3*b - 55*a^2*
b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b
^4)*cosh(d*x + c)^6 + 15*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*
x + c)^4 + 30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 3*(120*a^4 - 280*a^3*
b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*
((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^7 + 3*(30*a^3*b - 55*a^2*b^2
 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^5 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 -
 124*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (30*a^3*b - 55*a^2*b^2 + 36*a*b^3 -
8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a - b))*log((b*cosh(d*x + c)^
4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*c
osh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b
*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a - b)*co
sh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x
+ c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d
*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(
```

```
d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(
d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*co
sh(d*x + c))*sinh(d*x + c) + b)) + 4*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c) +
16*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)
*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(d*x + c)^8
+ 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(2*a^3*b - 5*
a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b -
 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^4 - 24*
a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*(a^2*b^2 - 2
*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 +
3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x
+ c)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x
+ c)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^4
- 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*(a^2*b^2 -
 2*a*b^3 + b^4)*cosh(d*x + c)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*
cosh(d*x + c)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b
 + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^
2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
 b^4)*cosh(d*x + c)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*
cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh
(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 16*((a^2*b^2 - 2*a*b^3
+ b^4)*cosh(d*x + c)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x
 + c)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(d*x + c)^8 + 4*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4
 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2
*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^4 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(c+d\,x\right)\,{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^3), x)
```

3.57 $\int \frac{\text{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=215

$$-\frac{3b(8a^2-12ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a-b)^{5/2}d}-\frac{(4a-5b)(2a-3b)\coth(c+dx)}{8a^3(a-b)^2d}-\frac{b\,\text{csch}(c+dx)\text{s}}{4a(a-b)d\,(a-(a-}$$

```
[Out] -3/8*b*(8*a^2-12*a*b+5*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2
)/(a-b)^(5/2)/d-1/8*(4*a-5*b)*(2*a-3*b)*coth(d*x+c)/a^3/(a-b)^2/d-1/4*b*csc
h(d*x+c)*sech(d*x+c)^3/a/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)^2-1/8*b*coth(d*x+c
)*(4*a-5*b-(4*a-b)*tanh(d*x+c)^2)/a^2/(a-b)^2/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A]

time = 0.22, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3266, 479, 591, 464, 214}

$$-\frac{(4a-5b)(2a-3b)\coth(c+dx)}{8a^3d(a-b)^2}-\frac{b\coth(c+dx)\left(-\left((4a-b)\tanh^2(c+dx)\right)+4a-5b\right)}{8a^2d(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{3b(8a^2-12ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a-b)^{5/2}}-\frac{b\,\text{csch}(c+dx)\text{sech}^3(c+dx)}{4ad(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(8*a^2 - 12*a*b + 5*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]
)/(8*a^(7/2)*(a - b)^(5/2)*d) - ((4*a - 5*b)*(2*a - 3*b)*Coth[c + d*x])/(8*
a^3*(a - b)^2*d) - (b*Csch[c + d*x]*Sech[c + d*x]^3)/(4*a*(a - b)*d*(a - (a
 - b)*Tanh[c + d*x]^2)^2) - (b*Coth[c + d*x]*(4*a - 5*b - (4*a - b)*Tanh[c
+ d*x]^2))/(8*a^2*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^2(a-(a-b)x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{(1-x^2)(4a-5b+(-4a+b)x^2)}{x^2(a+(-a+b)x^2)^2}\ d}{4a(a-b)d}$$

$$= -\frac{b\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{b\coth(c+dx)\left(4a-5b-(4a-\right.}{8a^2(a-b)^2d\left(a-(a-b)\,\text{ta}\right.}$$

$$= -\frac{(4a-5b)(2a-3b)\coth(c+dx)}{8a^3(a-b)^2d} - \frac{b\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

$$= -\frac{3b(8a^2-12ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a-b)^{5/2}d} - \frac{(4a-5b)(2a-3b)\,\text{cot}}{8a^3(a-b)^2d}$$

Mathematica [A]
time = 1.17, size = 225, normalized size = 1.05

$$\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^6(c+dx)\left(-\frac{3b(8a^2-12ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)(2a-b+b\cosh(2(c+dx)))^2}{(a-b)^{5/2}} - 8\sqrt{a}\,(2a-b+b\cosh(2(c+dx)))^2\coth(c+dx) + \frac{4a^{3/2}b^2\sinh(2(c+dx))}{a-b} + \frac{\sqrt{a}\,(10a-7b)b^2(2a-b+b\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b)^2}\right)}{64a^{7/2}d\left(b+a\operatorname{csch}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^6*((-3*b*(8*a^2 - 12*a*b + 5
*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b + b*Cosh[2*(c +
 d*x)])^2)/(a - b)^(5/2) - 8*Sqrt[a]*(2*a - b + b*Cosh[2*(c + d*x)])^2*Coth
[c + d*x] + (4*a^(3/2)*b^2*Sinh[2*(c + d*x)])/(a - b) + (Sqrt[a]*(10*a - 7*
b)*b^2*(2*a - b + b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a - b)^2))/(64*a
^(7/2)*d*(b + a*Csch[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(199) = 398.
time = 1.67, size = 442, normalized size = 2.06 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^3*tanh(1/2*d*x+1/2*c)-1/2/a^3/tanh(1/2*d*x+1/2*c)+2/a^3*b*((3/8
*a*b*(4*a-3*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(12*a^2-49*a*b+28*
```

```
b^2)*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-1/8*(12*a^2-49*a*b+28*b^2)*b/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+3/8*a*b*(4*a-3*b)/(a^2-2*a*b+b^2)*tanh
(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tan
h(1/2*d*x+1/2*c)^2+a)^2+3/8*(8*a^2-12*a*b+5*b^2)/(a^2-2*a*b+b^2)*a*(1/2*((-
b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*((-b*
(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4423 vs. 2(200) = 400.
time = 0.48, size = 9102, normalized size = 42.33

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(12*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^8
+ 96*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)*sinh(d*x
 + c)^7 + 12*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*sinh(d*x + c)^
8 + 24*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*
x + c)^6 + 24*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5 +
 14*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 32*a^4*b^2 - 136*a^3*b^3 + 164*a^2*b^4 - 60*a*b^5 + 48*(14*(8*a
^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^3 + 3*(24*a^5*b -
 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 8*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 4
5*a*b^5)*cosh(d*x + c)^4 + 8*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^
3 + 238*a^2*b^4 - 45*a*b^5 + 105*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a
*b^5)*cosh(d*x + c)^4 + 45*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4
 + 10*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(21*(8*a^4*b^2 - 20*a^3*
b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^5 + 15*(24*a^5*b - 76*a^4*b^2 + 9
```

```
1*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c)^3 + (64*a^6 - 296*a^5*b +
548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5)*cosh(d*x + c))*sinh(d*x
 + c)^3 + 8*(32*a^5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b^5)
*cosh(d*x + c)^2 + 8*(42*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*co
sh(d*x + c)^6 + 32*a^5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b
^5 + 45*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d
*x + c)^4 + 6*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4
 - 45*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^2*b^3 - 12*a*b^4 +
5*b^5)*cosh(d*x + c)^10 + 10*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)*s
inh(d*x + c)^9 + (8*a^2*b^3 - 12*a*b^4 + 5*b^5)*sinh(d*x + c)^10 + (64*a^3*
b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^8 + (64*a^3*b^2 - 136
*a^2*b^3 + 100*a*b^4 - 25*b^5 + 45*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x
+ c)^2)*sinh(d*x + c)^8 + 8*(15*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c
)^3 + (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c))*sinh(d
*x + c)^7 + 2*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*c
osh(d*x + c)^6 + 2*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b
^5 + 105*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^4 + 14*(64*a^3*b^2 -
136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*
(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^5 + 14*(64*a^3*b^2 - 136*a^2*b
^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^3 + 3*(64*a^4*b - 192*a^3*b^2 + 224*
a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*a^2*b^3 +
12*a*b^4 - 5*b^5 - 2*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25
*b^5)*cosh(d*x + c)^4 + 2*(105*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)
^6 - 64*a^4*b + 192*a^3*b^2 - 224*a^2*b^3 + 120*a*b^4 - 25*b^5 + 35*(64*a^3
*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^4 + 15*(64*a^4*b - 1
92*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 8*(15*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^7 + 7*(64*a^3*b^2
 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^5 + 5*(64*a^4*b - 192*a^
3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^3 - (64*a^4*b - 192
*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c))*sinh(d*x + c)^3
 - (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^2 + (45*(8
*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 136*a^2*b^3
 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^6 - 64*a^3*b^2 + 136*a^2*b^3 - 100*a*b
^4 + 25*b^5 + 30*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5
)*cosh(d*x + c)^4 - 12*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 +
25*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(8*a^2*b^3 - 12*a*b^4 + 5*b
^5)*cosh(d*x + c)^9 + 4*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cos
h(d*x + c)^7 + 6*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5
)*cosh(d*x + c)^5 - 4*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 2
5*b^5)*cosh(d*x + c)^3 - (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^
2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cos
h(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*
a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c) + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
```

```
inh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*
x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 +
2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 +
(2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 16*(6*(8*a^4*b^2 - 20*a^3*b^
3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^7 + 9*(24*a^5*b - 76*a^4*b^2 + 91*a
^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c)^5 + 2*(64*a^6 - 296*a^5*b + 5
48*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.81, size = 331, normalized size = 1.54

$$-\frac{\frac{3\left(8a^{2}b-12ab^{2}+5b^{3}\right)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^{2}+ab}}\right)}{\left(a^{5}-2a^{4}b+a^{3}b^{2}\right)\sqrt{-a^{2}+ab}}+\frac{2\left(16a^{2}b^{2}e^{(6dx+6c)}-20ab^{3}e^{(6dx+6c)}+7b^{4}e^{(6dx+6c)}+80a^{3}be^{(4dx+4c)}-136a^{2}b^{2}e^{(4dx+4c)}+86ab^{3}e^{(4dx+4c)}-21b^{4}e^{(4dx+4c)}+64a^{2}b^{2}e^{(2dx+2c)}-76ab^{3}e^{(2dx+2c)}+21b^{4}e^{(2dx+2c)}+10ab^{3}-7b^{4}\right)}{\left(a^{5}-2a^{4}b+a^{3}b^{2}\right)\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-2be^{(2dx+2c)}+b\right)^{2}}+\frac{16}{a^{3}\left(e^{(2dx+2c)}-1\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(8*a^2*b - 12*a*b^2 + 5*b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a -
b)/sqrt(-a^2 + a*b))/((a^5 - 2*a^4*b + a^3*b^2)*sqrt(-a^2 + a*b)) + 2*(16*a
^2*b^2*e^(6*d*x + 6*c) - 20*a*b^3*e^(6*d*x + 6*c) + 7*b^4*e^(6*d*x + 6*c) +
 80*a^3*b*e^(4*d*x + 4*c) - 136*a^2*b^2*e^(4*d*x + 4*c) + 86*a*b^3*e^(4*d*x
 + 4*c) - 21*b^4*e^(4*d*x + 4*c) + 64*a^2*b^2*e^(2*d*x + 2*c) - 76*a*b^3*e^
(2*d*x + 2*c) + 21*b^4*e^(2*d*x + 2*c) + 10*a*b^3 - 7*b^4)/((a^5 - 2*a^4*b
+ a^3*b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) +
 b)^2) + 16/(a^3*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(c+d\,x\right)^2\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3), x)
```

3.58 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=224

$$\frac{b^{3/2}(35a^2-56ab+24b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^4(a-b)^{5/2}d}+\frac{(a+6b)\tanh^{-1}(\cosh(c+dx))}{2a^4d}-\frac{(2a-3b)b\cosh(}{4a^2(a-b)d\,(a-b+b\,\mathrm{co}}$$

```
[Out] 1/8*b^(3/2)*(35*a^2-56*a*b+24*b^2)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/
a^4/(a-b)^(5/2)/d+1/2*(a+6*b)*arctanh(cosh(d*x+c))/a^4/d-1/4*(2*a-3*b)*b*co
sh(d*x+c)/a^2/(a-b)/d/(a-b+b*cosh(d*x+c)^2)^2-1/8*(a-4*b)*(4*a-3*b)*b*cosh(
d*x+c)/a^3/(a-b)^2/d/(a-b+b*cosh(d*x+c)^2)-1/2*coth(d*x+c)*csch(d*x+c)/a/d/
(a-b+b*cosh(d*x+c)^2)^2
```

Rubi [A]

time = 0.29, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3265, 425, 541, 536, 212, 211}

$$\frac{(a+6b)\tanh^{-1}(\cosh(c+dx))}{2a^4d}-\frac{b(a-4b)(4a-3b)\cosh(c+dx)}{8a^3d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{b(2a-3b)\cosh(c+dx)}{4a^2d(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}+\frac{b^{3/2}(35a^2-56ab+24b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^4d(a-b)^{5/2}}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b\cosh^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a -
 b]])/(8*a^4*(a - b)^(5/2)*d) + ((a + 6*b)*ArcTanh[Cosh[c + d*x]])/(2*a^4*d
) - ((2*a - 3*b)*b*Cosh[c + d*x])/(4*a^2*(a - b)*d*(a - b + b*Cosh[c + d*x]
^2)^2) - ((a - 4*b)*(4*a - 3*b)*b*Cosh[c + d*x])/(8*a^3*(a - b)^2*d*(a - b
+ b*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a - b + b*Cos
h[c + d*x]^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+bx^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+b+5bx^2}{(1-x^2)(a-b+bx^2)^3}\,dx, x, \cosh(c+d\right.}{2ad}$$

$$= -\frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)^2} -$$

$$= -\frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(a-4b)(4a-3b)b\cosh(c+d}{8a^3(a-b)^2d\left(a-b+b\cosh^2(c+\right.}$$

$$= -\frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(a-4b)(4a-3b)b\cosh(c+d}{8a^3(a-b)^2d\left(a-b+b\cosh^2(c+\right.}$$

$$= \frac{b^{3/2}(35a^2-56ab+24b^2)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^4(a-b)^{5/2}d} + \frac{(a+6b)\tanh^{-1}(\cosh(c+}{2a^4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.81, size = 419, normalized size = 1.87

$$\frac{(2a-b+b\cosh(2(c+dx)))\operatorname{csch}^5(c+dx)\left(\frac{8a^2b^2\coth(c+dx)}{a-b}+\frac{2a(11a-8b)b^2(2a-b+b\cosh(2(c+dx)))\coth(c+dx)}{(a-b)^2}+\frac{b^{3/2}(35a^2-56ab+24b^2)\text{ArcTan}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)(2a-b+b\cosh(2(c+dx)))^2\operatorname{csch}(c+dx)}{(a-b)^{5/2}}+\frac{b^{3/2}(35a^2-56ab+24b^2)\text{ArcTan}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)(2a-b+b\cosh(2(c+dx)))^2\operatorname{csch}(c+dx)}{(a-b)^{5/2}}-a(2a-b+b\cosh(2(c+dx)))^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}(c+dx)-4(a+6b)(2a-b+b\cosh(2(c+dx)))^2\operatorname{csch}(c+dx)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-a(2a-b+b\cosh(2(c+dx)))^2\operatorname{csch}(c+dx)\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\right)}{64a^4d\left(b+a\operatorname{csch}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^5*((8*a^2*b^2*Coth[c + d*x])
/(a - b) + (2*a*(11*a - 8*b)*b^2*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d
*x])/(a - b)^2 + (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqrt[b] - I*Sq
rt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])^2*Csc
h[c + d*x])/(a - b)^(5/2) + (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqr
t[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d
*x)])^2*Csch[c + d*x])/(a - b)^(5/2) - a*(2*a - b + b*Cosh[2*(c + d*x)])^2*
Csch[(c + d*x)/2]^2*Csch[c + d*x] - 4*(a + 6*b)*(2*a - b + b*Cosh[2*(c + d*
x)])^2*Csch[c + d*x]*Log[Tanh[(c + d*x)/2]] - a*(2*a - b + b*Cosh[2*(c + d*
x)])^2*Csch[c + d*x]*Sech[(c + d*x)/2]^2))/(64*a^4*d*(b + a*Csch[c + d*x]^2
)^3)
```

Maple [A]
time = 1.94, size = 350, normalized size = 1.56 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a^3-1/8/a^3/tanh(1/2*d*x+1/2*c)^2+1/4/a^4*(-
12*b-2*a)*ln(tanh(1/2*d*x+1/2*c))+4*b^2/a^4*((-1/16*(13*a^2-40*a*b+24*b^2)*
a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+1/16*(39*a^3-134*a^2*b+184*a*b^2-80
*b^3)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-1/16*a*(39*a^2-104*a*b+56*b^2)/
(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+1/16*a^2*(13*a-10*b)/(a^2-2*a*b+b^2))
/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)
^2+a)^2+1/32*(35*a^2-56*a*b+24*b^2)/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(
1/4*(2*a*tanh(1/2*d*x+1/2*c)^2-2*a+4*b)/(a*b-b^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((4*a^2*b^2*e^(11*c) - 19*a*b^3*e^(11*c) + 12*b^4*e^(11*c))*e^(11*d*x)
 + (32*a^3*b*e^(9*c) - 128*a^2*b^2*e^(9*c) + 129*a*b^3*e^(9*c) - 36*b^4*e^(
9*c))*e^(9*d*x) + 2*(32*a^4*e^(7*c) - 80*a^3*b*e^(7*c) + 94*a^2*b^2*e^(7*c)
 - 55*a*b^3*e^(7*c) + 12*b^4*e^(7*c))*e^(7*d*x) + 2*(32*a^4*e^(5*c) - 80*a^
3*b*e^(5*c) + 94*a^2*b^2*e^(5*c) - 55*a*b^3*e^(5*c) + 12*b^4*e^(5*c))*e^(5*
d*x) + (32*a^3*b*e^(3*c) - 128*a^2*b^2*e^(3*c) + 129*a*b^3*e^(3*c) - 36*b^4
*e^(3*c))*e^(3*d*x) + (4*a^2*b^2*e^c - 19*a*b^3*e^c + 12*b^4*e^c)*e^(d*x))/
(a^5*b^2*d - 2*a^4*b^3*d + a^3*b^4*d + (a^5*b^2*d*e^(12*c) - 2*a^4*b^3*d*e^
(12*c) + a^3*b^4*d*e^(12*c))*e^(12*d*x) + 2*(4*a^6*b*d*e^(10*c) - 11*a^5*b^
2*d*e^(10*c) + 10*a^4*b^3*d*e^(10*c) - 3*a^3*b^4*d*e^(10*c))*e^(10*d*x) + (
16*a^7*d*e^(8*c) - 64*a^6*b*d*e^(8*c) + 95*a^5*b^2*d*e^(8*c) - 62*a^4*b^3*d
*e^(8*c) + 15*a^3*b^4*d*e^(8*c))*e^(8*d*x) - 4*(8*a^7*d*e^(6*c) - 28*a^6*b*
d*e^(6*c) + 37*a^5*b^2*d*e^(6*c) - 22*a^4*b^3*d*e^(6*c) + 5*a^3*b^4*d*e^(6*
c))*e^(6*d*x) + (16*a^7*d*e^(4*c) - 64*a^6*b*d*e^(4*c) + 95*a^5*b^2*d*e^(4*
c) - 62*a^4*b^3*d*e^(4*c) + 15*a^3*b^4*d*e^(4*c))*e^(4*d*x) + 2*(4*a^6*b*d*
e^(2*c) - 11*a^5*b^2*d*e^(2*c) + 10*a^4*b^3*d*e^(2*c) - 3*a^3*b^4*d*e^(2*c)
)*e^(2*d*x)) + 1/2*(a + 6*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^4*d) - 1/2*(a
 + 6*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^4*d) + 8*integrate(1/32*((35*a^2*b
^2*e^(3*c) - 56*a*b^3*e^(3*c) + 24*b^4*e^(3*c))*e^(3*d*x) - (35*a^2*b^2*e^c
 - 56*a*b^3*e^c + 24*b^4*e^c)*e^(d*x))/(a^6*b - 2*a^5*b^2 + a^4*b^3 + (a^6*
b*e^(4*c) - 2*a^5*b^2*e^(4*c) + a^4*b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^7*e^(2*
c) - 5*a^6*b*e^(2*c) + 4*a^5*b^2*e^(2*c) - a^4*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 12128 vs. $2(206) = 412$.

time = 0.69, size = 22563, normalized size = 100.73

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(4*a^3*b^2 - 19*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^11 + 44*(4*a^3*
b^2 - 19*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^10 + 4*(4*a^3*b^2
- 19*a^2*b^3 + 12*a*b^4)*sinh(d*x + c)^11 + 4*(32*a^4*b - 128*a^3*b^2 + 129
*a^2*b^3 - 36*a*b^4)*cosh(d*x + c)^9 + 4*(32*a^4*b - 128*a^3*b^2 + 129*a^2*
b^3 - 36*a*b^4 + 55*(4*a^3*b^2 - 19*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*si
nh(d*x + c)^9 + 12*(55*(4*a^3*b^2 - 19*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^3
+ 3*(32*a^4*b - 128*a^3*b^2 + 129*a^2*b^3 - 36*a*b^4)*cosh(d*x + c))*sinh(d
*x + c)^8 + 8*(32*a^5 - 80*a^4*b + 94*a^3*b^2 - 55*a^2*b^3 + 12*a*b^4)*cosh
(d*x + c)^7 + 8*(32*a^5 - 80*a^4*b + 94*a^3*b^2 - 55*a^2*b^3 + 12*a*b^4 + 1
65*(4*a^3*b^2 - 19*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^4 + 18*(32*a^4*b - 128
*a^3*b^2 + 129*a^2*b^3 - 36*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 56*(3
3*(4*a^3*b^2 - 19*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^5 + 6*(32*a^4*b - 128*a
^3*b^2 + 129*a^2*b^3 - 36*a*b^4)*cosh(d*x + c)^3 + (32*a^5 - 80*a^4*b + 94*
a^3*b^2 - 55*a^2*b^3 + 12 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(c+d\,x\right)^3\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3), x)
```

3.59 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=259

$$\frac{b^2(48a^2-80ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a-b)^{5/2}d}+\frac{(8a^3-4a^2b-45ab^2+35b^3)\coth(c+dx)}{8a^4(a-b)^2d}-(8a^2-52a$$

```
[Out] 1/8*b^2*(48*a^2-80*a*b+35*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(
9/2)/(a-b)^(5/2)/d+1/8*(8*a^3-4*a^2*b-45*a*b^2+35*b^3)*coth(d*x+c)/a^4/(a-b
)^2/d-1/24*(8*a^2-52*a*b+35*b^2)*coth(d*x+c)^3/a^3/(a-b)^2/d-1/4*b*csch(d*x
+c)^3*sech(d*x+c)^3/a/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)^2-1/8*(10*a-7*b)*b*cs
ch(d*x+c)^3*sech(d*x+c)/a^2/(a-b)^2/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A]

time = 0.26, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3266, 479, 591, 584, 214}

$$-\frac{b(10a-7b)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{8a^2d(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{b^2(48a^2-80ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d(a-b)^{5/2}}-\frac{(8a^2-52ab+35b^2)\coth^3(c+dx)}{24a^3d(a-b)^2}+\frac{(8a^3-4a^2b-45ab^2+35b^3)\coth(c+dx)}{8a^4d(a-b)^2}-\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4ad(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (b^2*(48*a^2 - 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]
])/(8*a^(9/2)*(a - b)^(5/2)*d) + ((8*a^3 - 4*a^2*b - 45*a*b^2 + 35*b^3)*Cot
h[c + d*x])/(8*a^4*(a - b)^2*d) - ((8*a^2 - 52*a*b + 35*b^2)*Coth[c + d*x]^
3)/(24*a^3*(a - b)^2*d) - (b*Csch[c + d*x]^3*Sech[c + d*x]^3)/(4*a*(a - b)*
d*(a - (a - b)*Tanh[c + d*x]^2)^2) - ((10*a - 7*b)*b*Csch[c + d*x]^3*Sech[c
 + d*x])/(8*a^2*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^4\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(4a-7b+(-4a+b)x^2\right)}{x^4\left(a+(-a+b)x^2\right)^2}}{4a(a-b)d}$$

$$= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(10a-7b)b\operatorname{csch}^3(c+dx)\operatorname{sech}(}{8a^2(a-b)^2d\,(a-(a-b)\tanh^2(}$$

$$= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(10a-7b)b\operatorname{csch}^3(c+dx)\operatorname{sech}(}{8a^2(a-b)^2d\,(a-(a-b)\tanh^2(}$$

$$= \frac{\left(8a^3-4a^2b-45ab^2+35b^3\right)\coth(c+dx)}{8a^4(a-b)^2d} - \frac{\left(8a^2-52ab+35b^2\right)\coth^3(c+d}{24a^3(a-b)^2d}$$

$$= \frac{b^2\left(48a^2-80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a-b)^{5/2}d} + \frac{(8a^3-4a^2b-45ab^2+}{8a^4(a-}$$

Mathematica [A]
time = 1.87, size = 167, normalized size = 0.64

$$\frac{\frac{3b^2\left(48a^2-80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}} + \sqrt{a}\left(-8\coth(c+dx)\left(-2a-9b+a\operatorname{csch}^2(c+dx)\right) + \frac{3b^3\left(-32a^2+40ab-11b^2+b(-14a+11b)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{(a-b)^2(2a-b+b\cosh(2(c+dx)))^2}\right)}{24a^{9/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*b^2*(48*a^2 - 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt
[a]])/(a - b)^(5/2) + Sqrt[a]*(-8*Coth[c + d*x]*(-2*a - 9*b + a*Csch[c + d*
x]^2) + (3*b^3*(-32*a^2 + 40*a*b - 11*b^2 + b*(-14*a + 11*b)*Cosh[2*(c + d*
x)])*Sinh[2*(c + d*x)])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2)))/(24
*a^(9/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(241) = 482$.
time = 1.87, size = 497, normalized size = 1.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/a^4*(1/3*a*tanh(1/2*d*x+1/2*c)^3-3*a*tanh(1/2*d*x+1/2*c)-12*b*tan
h(1/2*d*x+1/2*c))-1/24/a^3/tanh(1/2*d*x+1/2*c)^3-1/8/a^4*(-12*b-3*a)/tanh(1
/2*d*x+1/2*c)-4*b^2/a^4*((1/16*a*b*(16*a-13*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x
+1/2*c)^7-1/16*(16*a^2-69*a*b+44*b^2)*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)
^5-1/16*(16*a^2-69*a*b+44*b^2)*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/16
*a*b*(16*a-13*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c
)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/16*(48*a^2-8
0*a*b+35*b^2)/(a^2-2*a*b+b^2)*a*(1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2
)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b
*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8519 vs. $2(243) = 486$.
time = 0.54, size = 17294, normalized size = 66.77

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(12*(48*a^4*b^3 - 128*a^3*b^4 + 115*a^2*b^5 - 35*a*b^6)*cosh(d*x + c)
^12 + 144*(48*a^4*b^3 - 128*a^3*b^4 + 115*a^2*b^5 - 35*a*b^6)*cosh(d*x + c)
*sinh(d*x + c)^11 + 12*(48*a^4*b^3 - 128*a^3*b^4 + 115*a^2*b^5 - 35*a*b^6)*
sinh(d*x + c)^12 + 72*(48*a^5*b^2 - 176*a^4*b^3 + 243*a^3*b^4 - 150*a^2*b^5
 + 35*a*b^6)*cosh(d*x + c)^10 + 72*(48*a^5*b^2 - 176*a^4*b^3 + 243*a^3*b^4
- 150*a^2*b^5 + 35*a*b^6 + 11*(48*a^4*b^3 - 128*a^3*b^4 + 115*a^2*b^5 - 35*
a*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 240*(11*(48*a^4*b^3 - 128*a^3*b^
4 + 115*a^2*b^5 - 35*a*b^6)*cosh(d*x + c)^3 + 3*(48*a^5*b^2 - 176*a^4*b^3 +
 243*a^3*b^4 - 150*a^2*b^5 + 35*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^9 + 4*(
768*a^6*b - 5408*a^5*b^2 + 12960*a^4*b^3 - 14370*a^3*b^4 + 7625*a^2*b^5 - 1
575*a*b^6)*cosh(d*x + c)^8 + 4*(768*a^6*b - 5408*a^5*b^2 + 12960*a^4*b^3 -
```

```
14370*a^3*b^4 + 7625*a^2*b^5 - 1575*a*b^6 + 1485*(48*a^4*b^3 - 128*a^3*b^4
+ 115*a^2*b^5 - 35*a*b^6)*cosh(d*x + c)^4 + 810*(48*a^5*b^2 - 176*a^4*b^3 +
 243*a^3*b^4 - 150*a^2*b^ ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.82, size = 378, normalized size = 1.46

$$\frac{\frac{3\,(48\,a^2b^2-80\,ab^3+35\,b^4)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^6-2\,a^5b+a^4b^2)\sqrt{-a^2+ab}}+\frac{6\,(24\,a^2b^3e^{(6\,dx+6\,c)}-32\,ab^4e^{(6\,dx+6\,c)}+11\,b^5e^{(6\,dx+6\,c)}+112\,a^3b^2e^{(4\,dx+4\,c)}-200\,a^2b^3e^{(4\,dx+4\,c)}+130\,ab^4e^{(4\,dx+4\,c)}-33\,b^5e^{(4\,dx+4\,c)}+88\,a^2b^3e^{(2\,dx+2\,c)}-112\,ab^4e^{(2\,dx+2\,c)}+33\,b^5e^{(2\,dx+2\,c)}+14\,ab^4-11\,b^5)}{(a^6-2\,a^5b+a^4b^2)(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b)^2}+\frac{16\,(9\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-18\,be^{(2\,dx+2\,c)}+2\,a+9\,b)}{a^4(e^{(2\,dx+2\,c)}-1)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*(48*a^2*b^2 - 80*a*b^3 + 35*b^4)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*
a - b)/sqrt(-a^2 + a*b))/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(-a^2 + a*b)) + 6*(
24*a^2*b^3*e^(6*d*x + 6*c) - 32*a*b^4*e^(6*d*x + 6*c) + 11*b^5*e^(6*d*x + 6
*c) + 112*a^3*b^2*e^(4*d*x + 4*c) - 200*a^2*b^3*e^(4*d*x + 4*c) + 130*a*b^4
*e^(4*d*x + 4*c) - 33*b^5*e^(4*d*x + 4*c) + 88*a^2*b^3*e^(2*d*x + 2*c) - 11
2*a*b^4*e^(2*d*x + 2*c) + 33*b^5*e^(2*d*x + 2*c) + 14*a*b^4 - 11*b^5)/((a^6
 - 2*a^5*b + a^4*b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d
*x + 2*c) + b)^2) + 16*(9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 18*b*e^
(2*d*x + 2*c) + 2*a + 9*b)/(a^4*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\mathrm{sinh}\left(c+d\,x\right)}^4\,{\left(b\,{\mathrm{sinh}\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3), x)
```

3.60 $\int \frac{1}{1+\sinh^2(x)}\,dx$

Optimal. Leaf size=2

$$\tanh(x)$$

```
[Out] tanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3254, 3852, 8}

$$\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)^(-1),x]
```

```
[Out] Tanh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sinh^2(x)}\,dx &= \int \operatorname{sech}^2(x)\,dx\\ &= i\text{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right)\\ &= \tanh(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 2, normalized size = 1.00

$$\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)^(-1),x]
```

```
[Out] Tanh[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(2) = 4.
time = 0.38, size = 17, normalized size = 8.50

method	result	size
risch	$-\frac{2}{1+\mathrm{e}^{2x}}$	11
default	$\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^{2}\left(\frac{x}{2}\right)+1}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 10 vs. 2(2) = 4.
time = 0.27, size = 10, normalized size = 5.00

$$\frac{2}{e^{(-2\,x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 2/(e^(-2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(2) = 4.
time = 0.42, size = 20, normalized size = 10.00

$$-\frac{2}{\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.
time = 0.28, size = 14, normalized size = 7.00

$$\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^{2}\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**2),x)
```

```
[Out] 2*tanh(x/2)/(tanh(x/2)**2 + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(2) = 4$.
time = 0.42, size = 10, normalized size = 5.00

$$-\frac{2}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2),x, algorithm="giac")
```

```
[Out] -2/(e^(2*x) + 1)
```

Mupad [B]
time = 0.04, size = 10, normalized size = 5.00

$$-\frac{2}{\mathrm{e}^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2 + 1),x)
```

```
[Out] -2/(exp(2*x) + 1)
```

3.61 $\int \frac{1}{\left(1+\sinh^2(x)\right)^2}\,dx$

Optimal. Leaf size=11

$$\tanh(x)-\frac{\tanh^3(x)}{3}$$

```
[Out] tanh(x)-1/3*tanh(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {3254, 3852}

$$\tanh(x)-\frac{\tanh^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)^(-2),x]
```

```
[Out] Tanh[x] - Tanh[x]^3/3
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+\sinh^2(x)\right)^2}\,dx &= \int \operatorname{sech}^4(x)\,dx\\ &= i\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\tanh(x)\right)\\ &= \tanh(x)-\frac{\tanh^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.55

$$\frac{2\tanh(x)}{3}+\frac{1}{3}\operatorname{sech}^2(x)\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)^(-2),x]
```

```
[Out] (2*Tanh[x])/3 + (Sech[x]^2*Tanh[x])/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(9) = 18$.
time = 0.39, size = 36, normalized size = 3.27

method	result	size
risch	$-\frac{4(3\,\mathrm{e}^{2x}+1)}{3(1+\mathrm{e}^{2x})^3}$	19
default	$-\frac{2\left(-\left(\tanh^5\left(\frac{x}{2}\right)\right)-\frac{2\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-tanh(1/2*x)^5-2/3*tanh(1/2*x)^3-tanh(1/2*x))/(tanh(1/2*x)^2+1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(9) = 18$.
time = 0.27, size = 49, normalized size = 4.45

$$\frac{4\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}+\frac{4}{3\,\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 4*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 4/3/(3*e^(-2*x) + 3*e
^(-4*x) + e^(-6*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(9) = 18$.
time = 0.48, size = 84, normalized size = 7.64

$$-\frac{8\,\left(2\,\cosh\left(x\right)+\sinh\left(x\right)\right)}{3\,\left(\cosh\left(x\right)^{5}+5\,\cosh\left(x\right)\sinh\left(x\right)^{4}+\sinh\left(x\right)^{5}+\left(10\,\cosh\left(x\right)^{2}+3\right)\sinh\left(x\right)^{3}+3\,\cosh\left(x\right)^{3}+\left(10\,\cosh\left(x\right)^{3}+9\,\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+\left(5\,\cosh\left(x\right)^{4}+9\,\cosh\left(x\right)^{2}+2\right)\sinh\left(x\right)+4\,\cosh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/3*(2*cosh(x) + sinh(x))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + (
10*cosh(x)^2 + 3)*sinh(x)^3 + 3*cosh(x)^3 + (10*cosh(x)^3 + 9*cosh(x))*sinh
(x)^2 + (5*cosh(x)^4 + 9*cosh(x)^2 + 2)*sinh(x) + 4*cosh(x))
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(8) = 16$.
time = 0.71, size = 104, normalized size = 9.45

$$\frac{6\tanh^{5}\left(\frac{x}{2}\right)}{3\tanh^{6}\left(\frac{x}{2}\right)+9\tanh^{4}\left(\frac{x}{2}\right)+9\tanh^{2}\left(\frac{x}{2}\right)+3}+\frac{4\tanh^{3}\left(\frac{x}{2}\right)}{3\tanh^{6}\left(\frac{x}{2}\right)+9\tanh^{4}\left(\frac{x}{2}\right)+9\tanh^{2}\left(\frac{x}{2}\right)+3}+\frac{6\tanh\left(\frac{x}{2}\right)}{3\tanh^{6}\left(\frac{x}{2}\right)+9\tanh^{4}\left(\frac{x}{2}\right)+9\tanh^{2}\left(\frac{x}{2}\right)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**2)**2,x)
```

```
[Out] 6*tanh(x/2)**5/(3*tanh(x/2)**6 + 9*tanh(x/2)**4 + 9*tanh(x/2)**2 + 3) + 4*t
anh(x/2)**3/(3*tanh(x/2)**6 + 9*tanh(x/2)**4 + 9*tanh(x/2)**2 + 3) + 6*tanh
(x/2)/(3*tanh(x/2)**6 + 9*tanh(x/2)**4 + 9*tanh(x/2)**2 + 3)
```

Giac [**A**]
time = 0.42, size = 18, normalized size = 1.64

$$-\frac{4\left(3\,e^{(2\,x)}+1\right)}{3\left(e^{(2\,x)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^2,x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(2*x) + 1)/(e^(2*x) + 1)^3
```

Mupad [**B**]
time = 0.59, size = 18, normalized size = 1.64

$$-\frac{4\left(3\,\mathrm{e}^{2\,x}+1\right)}{3\left(\mathrm{e}^{2\,x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2 + 1)^2,x)
```

```
[Out] -(4*(3*exp(2*x) + 1))/(3*(exp(2*x) + 1)^3)
```

3.62 $\int \frac{1}{\left(1+\sinh^2(x)\right)^3}\,dx$

Optimal. Leaf size=19

$$\tanh(x)-\frac{2\tanh^3(x)}{3}+\frac{\tanh^5(x)}{5}$$

```
[Out] tanh(x)-2/3*tanh(x)^3+1/5*tanh(x)^5
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3254, 3852}

$$\frac{\tanh^5(x)}{5}-\frac{2\tanh^3(x)}{3}+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)^(-3),x]
```

```
[Out] Tanh[x] - (2*Tanh[x]^3)/3 + Tanh[x]^5/5
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+\sinh^2(x)\right)^3}\,dx &= \int \operatorname{sech}^6(x)\,dx\\ &= i\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\tanh(x)\right)\\ &= \tanh(x)-\frac{2\tanh^3(x)}{3}+\frac{\tanh^5(x)}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.42

$$\frac{8\tanh(x)}{15}+\frac{4}{15}\operatorname{sech}^2(x)\tanh(x)+\frac{1}{5}\operatorname{sech}^4(x)\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)^(-3),x]
```

```
[Out] (8*Tanh[x])/15 + (4*Sech[x]^2*Tanh[x])/15 + (Sech[x]^4*Tanh[x])/5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(15) = 30.
time = 0.40, size = 52, normalized size = 2.74

method	result	size
risch	$-\frac{16(10\,\mathrm{e}^{4x}+5\,\mathrm{e}^{2x}+1)}{15(1+\mathrm{e}^{2x})^5}$	25
default	$-\frac{2\left(-\left(\tanh^9\left(\frac{x}{2}\right)\right)-\frac{4\left(\tanh^7\left(\frac{x}{2}\right)\right)}{3}-\frac{58\left(\tanh^5\left(\frac{x}{2}\right)\right)}{15}-\frac{4\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^5}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-tanh(1/2*x)^9-4/3*tanh(1/2*x)^7-58/15*tanh(1/2*x)^5-4/3*tanh(1/2*x)^3-
tanh(1/2*x))/(tanh(1/2*x)^2+1)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(15) = 30.
time = 0.26, size = 111, normalized size = 5.84

$$\frac{16\,e^{(-2\,x)}}{3\,(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1)}+\frac{32\,e^{(-4\,x)}}{3\,(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1)}+\frac{16}{15\,(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10
*x) + 1) + 32/3*e^(-4*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*
x) + e^(-10*x) + 1) + 16/15/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(
-8*x) + e^(-10*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(15) = 30.
time = 0.42, size = 185, normalized size = 9.74

$$-\frac{16\,(11\,\cosh\left(x\right)^{2}+18\,\cosh\left(x\right)\sinh\left(x\right)+11\,\sinh\left(x\right)^{2}+5)}{15\,(\cosh\left(x\right)^{8}+8\,\cosh\left(x\right)\sinh\left(x\right)^{7}+\sinh\left(x\right)^{8}+(28\,\cosh\left(x\right)^{2}+5)\sinh\left(x\right)^{6}+5\,\cosh\left(x\right)^{6}+2\,(28\,\cosh\left(x\right)^{3}+15\,\cosh\left(x\right))\sinh\left(x\right)^{5}+5\,(14\,\cosh\left(x\right)^{4}+15\,\cosh\left(x\right)^{2}+2)\sinh\left(x\right)^{4}+10\,\cosh\left(x\right)^{4}+4\,(14\,\cosh\left(x\right)^{5}+25\,\cosh\left(x\right)^{3}+10\,\cosh\left(x\right))\sinh\left(x\right)^{3}+(28\,\cosh\left(x\right)^{6}+75\,\cosh\left(x\right)^{4}+60\,\cosh\left(x\right)^{2}+11)\sinh\left(x\right)^{2}+11\,\cosh\left(x\right)^{2}+2\,(4\,\cosh\left(x\right)^{7}+15\,\cosh\left(x\right)^{5}+20\,\cosh\left(x\right)^{3}+9\,\cosh\left(x\right))\sinh\left(x\right)+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(x)^2 + 18*cosh(x)*sinh(x) + 11*sinh(x)^2 + 5)/(cosh(x)^8 +
8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 + 5)*sinh(x)^6 + 5*cosh(x)^
6 + 2*(28*cosh(x)^3 + 15*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 + 15*cosh(x)^
2 + 2)*sinh(x)^4 + 10*cosh(x)^4 + 4*(14*cosh(x)^5 + 25*cosh(x)^3 + 10*cosh(
x))*sinh(x)^3 + (28*cosh(x)^6 + 75*cosh(x)^4 + 60*cosh(x)^2 + 11)*sinh(x)^2
 + 11*cosh(x)^2 + 2*(4*cosh(x)^7 + 15*cosh(x)^5 + 20*cosh(x)^3 + 9*cosh(x))
*sinh(x) + 5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(17) = 34$.
time = 1.75, size = 260, normalized size = 13.68

$$\frac{30\tanh^{9}\left(\frac{x}{2}\right)}{15\tanh^{10}\left(\frac{x}{2}\right)+75\tanh^{8}\left(\frac{x}{2}\right)+150\tanh^{6}\left(\frac{x}{2}\right)+150\tanh^{4}\left(\frac{x}{2}\right)+75\tanh^{2}\left(\frac{x}{2}\right)+15}+\frac{40\tanh^{7}\left(\frac{x}{2}\right)}{15\tanh^{10}\left(\frac{x}{2}\right)+75\tanh^{8}\left(\frac{x}{2}\right)+150\tanh^{6}\left(\frac{x}{2}\right)+150\tanh^{4}\left(\frac{x}{2}\right)+75\tanh^{2}\left(\frac{x}{2}\right)+15}+\frac{116\tanh^{5}\left(\frac{x}{2}\right)}{15\tanh^{10}\left(\frac{x}{2}\right)+75\tanh^{8}\left(\frac{x}{2}\right)+150\tanh^{6}\left(\frac{x}{2}\right)+150\tanh^{4}\left(\frac{x}{2}\right)+75\tanh^{2}\left(\frac{x}{2}\right)+15}+\frac{40\tanh^{3}\left(\frac{x}{2}\right)}{15\tanh^{10}\left(\frac{x}{2}\right)+75\tanh^{8}\left(\frac{x}{2}\right)+150\tanh^{6}\left(\frac{x}{2}\right)+150\tanh^{4}\left(\frac{x}{2}\right)+75\tanh^{2}\left(\frac{x}{2}\right)+15}+\frac{30\tanh\left(\frac{x}{2}\right)}{15\tanh^{10}\left(\frac{x}{2}\right)+75\tanh^{8}\left(\frac{x}{2}\right)+150\tanh^{6}\left(\frac{x}{2}\right)+150\tanh^{4}\left(\frac{x}{2}\right)+75\tanh^{2}\left(\frac{x}{2}\right)+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**2)**3,x)
```

```
[Out] 30*tanh(x/2)**9/(15*tanh(x/2)**10 + 75*tanh(x/2)**8 + 150*tanh(x/2)**6 + 15
0*tanh(x/2)**4 + 75*tanh(x/2)**2 + 15) + 40*tanh(x/2)**7/(15*tanh(x/2)**10
+ 75*tanh(x/2)**8 + 150*tanh(x/2)**6 + 150*tanh(x/2)**4 + 75*tanh(x/2)**2 +
 15) + 116*tanh(x/2)**5/(15*tanh(x/2)**10 + 75*tanh(x/2)**8 + 150*tanh(x/2)
**6 + 150*tanh(x/2)**4 + 75*tanh(x/2)**2 + 15) + 40*tanh(x/2)**3/(15*tanh(x
/2)**10 + 75*tanh(x/2)**8 + 150*tanh(x/2)**6 + 150*tanh(x/2)**4 + 75*tanh(x
/2)**2 + 15) + 30*tanh(x/2)/(15*tanh(x/2)**10 + 75*tanh(x/2)**8 + 150*tanh(
x/2)**6 + 150*tanh(x/2)**4 + 75*tanh(x/2)**2 + 15)
```

Giac [A]
time = 0.40, size = 24, normalized size = 1.26

$$-\frac{16\left(10\,e^{(4\,x)}+5\,e^{(2\,x)}+1\right)}{15\left(e^{(2\,x)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*x) + 5*e^(2*x) + 1)/(e^(2*x) + 1)^5
```

Mupad [B]
time = 0.61, size = 24, normalized size = 1.26

$$-\frac{16\left(5\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{4\,x}+1\right)}{15\left(\mathrm{e}^{2\,x}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2 + 1)^3,x)
```

```
[Out] -(16*(5*exp(2*x) + 10*exp(4*x) + 1))/(15*(exp(2*x) + 1)^5)
```

3.63 $\int \frac{1}{1-\sinh^2(x)} \, dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3260, 212}

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sinh^2(x)} \, dx &= \text{Subst}\left(\int \frac{1}{1-2x^2} \, dx, x, \tanh(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{1-\sinh^2(x)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - Sinh[x]^2)^(-1),x]
```

```
[Out] Integrate[(1 - Sinh[x]^2)^(-1), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.
time = 0.40, size = 40, normalized size = 2.67

method	result	size
risch	$\frac{\sqrt{2}\,\ln\left(e^{2x}-3+2\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2x}-3-2\sqrt{2}\right)}{4}$	36
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{2}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+1/2*2^(1/2)*arctanh(1/4*
(2*tanh(1/2*x)+2)*2^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(12) = 24$.
time = 0.47, size = 61, normalized size = 4.07

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/4*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(12) = 24$.
time = 0.45, size = 66, normalized size = 4.40

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^2-2\sqrt{2}+3}{\cosh(x)^2+\sinh(x)^2-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^
2 - 3))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(15) = 30$.
time = 0.83, size = 209, normalized size = 13.93

$$\frac{816\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{1632\sqrt{2}+2308}+\frac{577\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{1632\sqrt{2}+2308}+\frac{816\log\left(\tanh\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{1632\sqrt{2}+2308}+\frac{577\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{1632\sqrt{2}+2308}-\frac{577\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{1632\sqrt{2}+2308}-\frac{816\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{1632\sqrt{2}+2308}-\frac{577\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{1632\sqrt{2}+2308}-\frac{816\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{1632\sqrt{2}+2308}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**2),x)
```

```
[Out] 816*log(tanh(x/2) - 1 + sqrt(2))/(1632*sqrt(2) + 2308) + 577*sqrt(2)*log(ta
nh(x/2) - 1 + sqrt(2))/(1632*sqrt(2) + 2308) + 816*log(tanh(x/2) + 1 + sqrt
(2))/(1632*sqrt(2) + 2308) + 577*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))/(1632
*sqrt(2) + 2308) - 577*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(1632*sqrt(2) +
 2308) - 816*log(tanh(x/2) - sqrt(2) - 1)/(1632*sqrt(2) + 2308) - 577*sqrt(
2)*log(tanh(x/2) - sqrt(2) + 1)/(1632*sqrt(2) + 2308) - 816*log(tanh(x/2) -
 sqrt(2) + 1)/(1632*sqrt(2) + 2308)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.
time = 0.42, size = 37, normalized size = 2.47

$$-\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6))
```

Mupad [B]
time = 0.15, size = 50, normalized size = 3.33

$$-\frac{\sqrt{2}\left(\ln\left(4\,\mathrm{e}^{2x}-\frac{\sqrt{2}\,\left(12\,\mathrm{e}^{2x}-4\right)}{4}\right)-\ln\left(4\,\mathrm{e}^{2x}+\frac{\sqrt{2}\,\left(12\,\mathrm{e}^{2x}-4\right)}{4}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^2 - 1),x)
```

```
[Out] -(2^(1/2)*(log(4*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/4) - log(4*exp(2*x)
 + (2^(1/2)*(12*exp(2*x) - 4))/4)))/4
```

3.64 $\int \frac{1}{\left(1-\sinh^2(x)\right)^2} \, dx$

Optimal. Leaf size=37

$$\frac{3\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{4\sqrt{2}}+\frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)}$$

```
[Out] 1/4*cosh(x)*sinh(x)/(1-sinh(x)^2)+3/8*arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3263, 12, 3260, 212}

$$\frac{3\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{4\sqrt{2}}+\frac{\sinh(x)\cosh(x)}{4\left(1-\sinh^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)^(-2),x]
```

```
[Out] (3*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) + (Cosh[x]*Sinh[x])/(4*(1 - Sinh[x
]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
```

```
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-\sinh^2(x)\right)^2}\,dx &= \frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)}-\frac{1}{4}\int -\frac{3}{1-\sinh^2(x)}\,dx\\
&= \frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)}+\frac{3}{4}\int \frac{1}{1-\sinh^2(x)}\,dx\\
&= \frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{3\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)}
\end{aligned}$$

Mathematica [**A**]
time = 0.09, size = 35, normalized size = 0.95

$$\frac{3\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}-\frac{\sinh(2x)}{4(-3+\cosh(2x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)^(-2),x]
```

```
[Out] (3*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) - Sinh[2*x]/(4*(-3 + Cosh[2*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(29) = 58.
time = 0.41, size = 92, normalized size = 2.49

method	result
risch	$-\frac{3\,\mathrm{e}^{2x}-1}{2(\mathrm{e}^{4x}-6\,\mathrm{e}^{2x}+1)}+\frac{3\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-3+2\sqrt{2}\right)}{16}-\frac{3\sqrt{2}\,\ln\left(\mathrm{e}^{2x}-3-2\sqrt{2}\right)}{16}$
default	$-\frac{-\frac{\tanh\left(\frac{x}{2}\right)}{4}-\frac{1}{4}}{\tanh^2\left(\frac{x}{2}\right)-2\tanh\left(\frac{x}{2}\right)-1}+\frac{3\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{8}-\frac{-\frac{\tanh\left(\frac{x}{2}\right)}{4}+\frac{1}{4}}{\tanh^2\left(\frac{x}{2}\right)+2\tanh\left(\frac{x}{2}\right)-1}+\frac{3\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right.\right.}{}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/4*tanh(1/2*x)-1/4)/(tanh(1/2*x)^2-2*tanh(1/2*x)-1)+3/8*2^(1/2)*arctanh
(1/4*(2*tanh(1/2*x)-2)*2^(1/2))-(-1/4*tanh(1/2*x)+1/4)/(tanh(1/2*x)^2+2*tan
h(1/2*x)-1)+3/8*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(27) = 54$.
time = 0.48, size = 87, normalized size = 2.35

$$\frac{3}{16}\sqrt{2}\log\left(-\frac{\sqrt{2} - e^{(-x)} + 1}{\sqrt{2} + e^{(-x)} - 1}\right) - \frac{3}{16}\sqrt{2}\log\left(-\frac{\sqrt{2} - e^{(-x)} - 1}{\sqrt{2} + e^{(-x)} + 1}\right) - \frac{3e^{(-2x)} - 1}{2\left(6e^{(-2x)} - e^{(-4x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 3/16*sqr
t(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - 1/2*(3*e^(-2*x)
 - 1)/(6*e^(-2*x) - e^(-4*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(27) = 54$.
time = 0.40, size = 216, normalized size = 5.84

$$-\frac{24\cosh(x)^2 - 3\left(\sqrt{2}\cosh(x)^4 + 4\sqrt{2}\cosh(x)\sinh(x)^3 + \sqrt{2}\sinh(x)^4 + 6\left(\sqrt{2}\cosh(x)^2 - \sqrt{2}\right)\sinh(x)^2 - 6\sqrt{2}\cosh(x)^2 + 4\left(\sqrt{2}\cosh(x)^3 - 3\sqrt{2}\cosh(x)\right)\sinh(x) + \sqrt{2}\right)\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2 - 4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x) + 3\left(2\sqrt{2}-3\right)\sinh(x)^2 - 2\sqrt{2} + 3}{\cosh(x)^2 + \sinh(x)^2 - 3}\right) + 48\cosh(x)\sinh(x) + 24\sinh(x)^2 - 8}{16\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 6\left(\cosh(x)^2 - 1\right)\sinh(x)^2 - 6\cosh(x)^2 + 4\left(\cosh(x)^3 - 3\cosh(x)\right)\sinh(x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(24*cosh(x)^2 - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 +
sqrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^2 - 6*sqrt(2)*c
osh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log
(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2*s
qrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 - 3)) + 48*co
sh(x)*sinh(x) + 24*sinh(x)^2 - 8)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x
)^4 + 6*(cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x))
*sinh(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2052 vs. $2(32) = 64$.
time = 4.60, size = 2052, normalized size = 55.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**2)**2,x)
```

```
[Out] 525888*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**4/(1402368*sqrt(2)*tanh(x/2)
**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x
/2)**2 + 1402368*sqrt(2) + 1983248) + 371859*sqrt(2)*log(tanh(x/2) - 1 + sq
rt(2))*tanh(x/2)**4/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 -
11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 19
83248) - 2231154*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(1402368
*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414
208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) - 3155328*log(tanh(x/
2) - 1 + sqrt(2))*tanh(x/2)**2/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh
(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*s
qrt(2) + 1983248) + 525888*log(tanh(x/2) - 1 + sqrt(2))/(1402368*sqrt(2)*ta
nh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)
*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) + 371859*sqrt(2)*log(tanh(x/2) -
 1 + sqrt(2))/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 118994
88*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248)
 + 525888*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**4/(1402368*sqrt(2)*tanh(x
/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tan
h(x/2)**2 + 1402368*sqrt(2) + 1983248) + 371859*sqrt(2)*log(tanh(x/2) + 1 +
 sqrt(2))*tanh(x/2)**4/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4
 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) +
 1983248) - 2231154*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1402
368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8
414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) - 3155328*log(tanh
(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*t
anh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 140236
8*sqrt(2) + 1983248) + 525888*log(tanh(x/2) + 1 + sqrt(2))/(1402368*sqrt(2)
*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt
(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) + 371859*sqrt(2)*log(tanh(x/2
) + 1 + sqrt(2))/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 118
99488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 19832
48) - 371859*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**4/(1402368*sqr
t(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*
sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) - 525888*log(tanh(x/2) -
sqrt(2) - 1)*tanh(x/2)**4/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)
**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2
) + 1983248) + 3155328*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**2/(1402368*s
qrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 841420
8*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) + 2231154*sqrt(2)*log(t
anh(x/2) - sqrt(2) - 1)*tanh(x/2)**2/(1402368*sqrt(2)*tanh(x/2)**4 + 198324
8*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 140
2368*sqrt(2) + 1983248) - 371859*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(1402
368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8
414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) - 525888*log(tanh(
x/2) - sqrt(2) - 1)/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 -
11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 19
```

```
83248) - 371859*sqrt(2)*log(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**4/(1402368*
sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 84142
08*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) - 525888*log(tanh(x/2)
 - sqrt(2) + 1)*tanh(x/2)**4/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x
/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqr
t(2) + 1983248) + 3155328*log(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**2/(140236
8*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 841
4208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) + 2231154*sqrt(2)*lo
g(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**2/(1402368*sqrt(2)*tanh(x/2)**4 + 198
3248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 +
1402368*sqrt(2) + 1983248) - 371859*sqrt(2)*log(tanh(x/2) - sqrt(2) + 1)/(1
402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2
- 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) - 525888*log(ta
nh(x/2) - sqrt(2) + 1)/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4
 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) +
 1983248) + 701184*sqrt(2)*tanh(x/2)**3/(1402368*sqrt(2)*tanh(x/2)**4 + 198
3248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 +
1402368*sqrt(2) + 1983248) + 991624*tanh(x/2)**3/(1402368*sqrt(2)*tanh(x/2)
**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x
/2)**2 + 1402368*sqrt(2) + 1983248) + 701184*sqrt(2)*tanh(x/2)/(1402368*sqr
t(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*
sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 198324...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(27) = 54$.
time = 0.41, size = 62, normalized size = 1.68

$$-\frac{3}{16}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-\frac{3e^{(2x)}-1}{2\left(e^{(4x)}-6e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^2,x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - 1/2*(3*e^(2*x) - 1)/(e^(4*x) - 6*e^(2*x) + 1)
```

Mupad [B]
time = 0.69, size = 77, normalized size = 2.08

$$\frac{3\sqrt{2}\ln\left(3e^{2x}+\frac{3\sqrt{2}\left(12e^{2x}-4\right)}{16}\right)}{16}-\frac{3\sqrt{2}\ln\left(3e^{2x}-\frac{3\sqrt{2}\left(12e^{2x}-4\right)}{16}\right)}{16}-\frac{\frac{3e^{2x}}{2}-\frac{1}{2}}{e^{4x}-6e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2 - 1)^2,x)
```

```
[Out] (3*2^(1/2)*log(3*exp(2*x) + (3*2^(1/2)*(12*exp(2*x) - 4))/16))/16 - (3*2^(1
/2)*log(3*exp(2*x) - (3*2^(1/2)*(12*exp(2*x) - 4))/16))/16 - ((3*exp(2*x))/
2 - 1/2)/(exp(4*x) - 6*exp(2*x) + 1)
```

3.65 $\int \frac{1}{\left(1-\sinh^2(x)\right)^3} \, dx$

Optimal. Leaf size=55

$$\frac{19\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{32\sqrt{2}}+\frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2}+\frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)}$$

```
[Out] 1/8*cosh(x)*sinh(x)/(1-sinh(x)^2)^2+9/32*cosh(x)*sinh(x)/(1-sinh(x)^2)+19/6
4*arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3263, 3252, 12, 3260, 212}

$$\frac{19\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{32\sqrt{2}}+\frac{9\sinh(x)\cosh(x)}{32\left(1-\sinh^2(x)\right)}+\frac{\sinh(x)\cosh(x)}{8\left(1-\sinh^2(x)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)^(-3),x]
```

```
[Out] (19*ArcTanh[Sqrt[2]*Tanh[x]])/(32*Sqrt[2]) + (Cosh[x]*Sinh[x])/(8*(1 - Sinh
[x]^2)^2) + (9*Cosh[x]*Sinh[x])/(32*(1 - Sinh[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-\sinh^2(x)\right)^3}\,dx &= \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2}-\frac{1}{8}\int \frac{-7-2\sinh^2(x)}{\left(1-\sinh^2(x)\right)^2}\,dx\\
&= \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2}+\frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)}-\frac{1}{32}\int -\frac{19}{1-\sinh^2(x)}\,dx\\
&= \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2}+\frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)}+\frac{19}{32}\int \frac{1}{1-\sinh^2(x)}\,dx\\
&= \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2}+\frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)}+\frac{19}{32}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\tanh(x)\right)\\
&= \frac{19\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{32\sqrt{2}}+\frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2}+\frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 51, normalized size = 0.93

$$\frac{19\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{32\sqrt{2}}+\frac{\sinh(2x)}{4(-3+\cosh(2x))^2}-\frac{9\sinh(2x)}{32(-3+\cosh(2x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)^(-3),x]
```

```
[Out] (19*ArcTanh[Sqrt[2]*Tanh[x]])/(32*Sqrt[2]) + Sinh[2*x]/(4*(-3 + Cosh[2*x])^
2) - (9*Sinh[2*x])/(32*(-3 + Cosh[2*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(45) = 90$.
time = 0.41, size = 124, normalized size = 2.25

method	result
risch	$-\frac{19\,e^{6x}-171\,e^{4x}+89\,e^{2x}-9}{16(e^{4x}-6\,e^{2x}+1)^{2}}+\frac{19\sqrt{2}\,\ln\left(e^{2x}-3+2\sqrt{2}\right)}{128}-\frac{19\sqrt{2}\,\ln\left(e^{2x}-3-2\sqrt{2}\right)}{128}$
default	$-\frac{-\frac{13\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{8}-\frac{11\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8}+\frac{31\tanh\left(\frac{x}{2}\right)}{8}-\frac{11}{8}}{4\left(\tanh^{2}\left(\frac{x}{2}\right)+2\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{19\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{64}-$ $\frac{-\frac{13\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{8}+\frac{11\left(\tanh^{2}\left(\frac{x}{2}\right.\right.}{8}}{4\left(\tanh^{2}\left(\frac{x}{2}\right)-2\tanh\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-13/8*tanh(1/2*x)^3-11/8*tanh(1/2*x)^2+31/8*tanh(1/2*x)-11/8)/(tanh(1
/2*x)^2+2*tanh(1/2*x)-1)^2+19/64*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1
/2))-1/4*(-13/8*tanh(1/2*x)^3+11/8*tanh(1/2*x)^2+31/8*tanh(1/2*x)+11/8)/(ta
nh(1/2*x)^2-2*tanh(1/2*x)-1)^2+19/64*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*
2^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(41) = 82$.
time = 0.48, size = 111, normalized size = 2.02

$$\frac{19}{128}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{19}{128}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-\frac{89\,e^{(-2x)}-171\,e^{(-4x)}+19\,e^{(-6x)}-9}{16\left(12\,e^{(-2x)}-38\,e^{(-4x)}+12\,e^{(-6x)}-e^{(-8x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 19/128*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 19/128
*sqrt(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - 1/16*(89*e^(
-2*x) - 171*e^(-4*x) + 19*e^(-6*x) - 9)/(12*e^(-2*x) - 38*e^(-4*x) + 12*e^(
-6*x) - e^(-8*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(41) = 82$.
time = 0.43, size = 575, normalized size = 10.45

$$-\frac{152\cosh(x)^{6}+912\cosh(x)\sinh(x)^{5}+152\sinh(x)^{6}+456\left(5\cosh(x)^{2}-3\right)\sinh(x)^{4}-1368\cosh(x)^{4}+608\left(5\cosh(x)^{3}-9\cosh(x)\right)\sinh(x)^{3}+8\left(285\cosh(x)^{4}-1026\cosh(x)^{2}+89\right)\sinh(x)^{2}+712\cosh(x)^{2}-19\left(\sqrt{2}\cosh(x)^{8}+8\sqrt{2}\cosh(x)\sinh(x)^{7}+\sqrt{2}\sinh(x)^{8}+4\left(7\sqrt{2}\cosh(x)^{2}-3\sqrt{2}\right)\sinh(x)^{6}-12\sqrt{2}\cosh(x)^{6}+8\left(7\sqrt{2}\cosh(x)^{3}-9\sqrt{2}\cosh(x)\right)\sinh(x)^{5}+2\left(35\sqrt{2}\cosh(x)^{4}-90\sqrt{2}\cosh(x)^{2}+19\sqrt{2}\right)\sinh(x)^{4}+38\sqrt{2}\cosh(x)^{4}+8\left(7\sqrt{2}\cosh(x)^{5}-30\sqrt{2}\cosh(x)^{3}+19\sqrt{2}\cosh(x)\right)\sinh(x)^{3}+4\left(7\sqrt{2}\cosh(x)^{6}-45\sqrt{2}\cosh(x)^{4}+57\sqrt{2}\cosh(x)^{2}-3\sqrt{2}\right)\sinh(x)^{2}-12\sqrt{2}\cosh(x)^{2}+8\left(\sqrt{2}\cosh(x)^{7}-9\sqrt{2}\cosh(x)^{5}+19\sqrt{2}\cosh(x)^{3}-3\sqrt{2}\cosh(x)\right)\sinh(x)+\sqrt{2}\right)\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^{2}-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^{2}-2\sqrt{2}+3}{\cosh(x)^{2}+\sinh(x)^{2}-3}\right)+16\left(57\cosh(x)^{5}-342\cosh(x)^{3}+89\cosh(x)\right)\sinh(x)-72}{128\left(\cosh(x)^{8}+8\cosh(x)\sinh(x)^{7}+\sinh(x)^{8}+4\left(7\cosh(x)^{2}-3\right)\sinh(x)^{6}-12\cosh(x)^{6}+8\left(7\cosh(x)^{3}-9\cosh(x)\right)\sinh(x)^{5}+2\left(35\cosh(x)^{4}-90\cosh(x)^{2}+19\right)\sinh(x)^{4}+38\cosh(x)^{4}+8\left(7\cosh(x)^{5}-30\cosh(x)^{3}+19\cosh(x)\right)\sinh(x)^{3}+4\left(7\cosh(x)^{6}-45\cosh(x)^{4}+57\cosh(x)^{2}-3\right)\sinh(x)^{2}-12\cosh(x)^{2}+8\left(\cosh(x)^{7}-9\cosh(x)^{5}+19\cosh(x)^{3}-3\cosh(x)\right)\sinh(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(152*cosh(x)^6 + 912*cosh(x)*sinh(x)^5 + 152*sinh(x)^6 + 456*(5*cosh
(x)^2 - 3)*sinh(x)^4 - 1368*cosh(x)^4 + 608*(5*cosh(x)^3 - 9*cosh(x))*sinh(
x)^3 + 8*(285*cosh(x)^4 - 1026*cosh(x)^2 + 89)*sinh(x)^2 + 712*cosh(x)^2 -
19*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 4
*(7*sqrt(2)*cosh(x)^2 - 3*sqrt(2))*sinh(x)^6 - 12*sqrt(2)*cosh(x)^6 + 8*(7*
sqrt(2)*cosh(x)^3 - 9*sqrt(2)*cosh(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^4
- 90*sqrt(2)*cosh(x)^2 + 19*sqrt(2))*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4 + 8*(
7*sqrt(2)*cosh(x)^5 - 30*sqrt(2)*cosh(x)^3 + 19*sqrt(2)*cosh(x))*sinh(x)^3
+ 4*(7*sqrt(2)*cosh(x)^6 - 45*sqrt(2)*cosh(x)^4 + 57*sqrt(2)*cosh(x)^2 - 3*
sqrt(2))*sinh(x)^2 - 12*sqrt(2)*cosh(x)^2 + 8*(sqrt(2)*cosh(x)^7 - 9*sqrt(2
)*cosh(x)^5 + 19*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*
log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(
2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 - 3)) + 16
*(57*cosh(x)^5 - 342*cosh(x)^3 + 89*cosh(x))*sinh(x) - 72)/(cosh(x)^8 + 8*c
osh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 3)*sinh(x)^6 - 12*cosh(x)^6
 + 8*(7*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 90*cosh(x)^2 +
 19)*sinh(x)^4 + 38*cosh(x)^4 + 8*(7*cosh(x)^5 - 30*cosh(x)^3 + 19*cosh(x))
*sinh(x)^3 + 4*(7*cosh(x)^6 - 45*cosh(x)^4 + 57*cosh(x)^2 - 3)*sinh(x)^2 -
12*cosh(x)^2 + 8*(cosh(x)^7 - 9*cosh(x)^5 + 19*cosh(x)^3 - 3*cosh(x))*sinh(
x) + 1)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5666 vs. $2(51) = 102$.
time = 11.81, size = 5666, normalized size = 103.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**2)**3,x)
```

```
[Out] 10001001174720*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**8/(33687582904320*sq
rt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2
)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh
(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 4
04250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 476414366270
72) + 7071775749331*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**8/(3368
7582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 57169723952
4864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160
*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tan
h(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) +
 47641436627072) - 84861308991972*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh
(x/2)**6/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8
 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 12
80128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 57169
7239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 3368758290
```

```
4320*sqrt(2) + 47641436627072) - 120012014096640*log(tanh(x/2) - 1 + sqrt(2
))*tanh(x/2)**6/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(
x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)*
*6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4
- 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 336
87582904320*sqrt(2) + 47641436627072) + 380038044639360*log(tanh(x/2) - 1 +
 sqrt(2))*tanh(x/2)**4/(33687582904320*sqrt(2)*tanh(x/2)**8 + 4764143662707
2*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tan
h(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x
/2)**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**
2 + 33687582904320*sqrt(2) + 47641436627072) + 268727478474578*sqrt(2)*log(
tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**4/(33687582904320*sqrt(2)*tanh(x/2)**8
+ 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851
840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 18103745
91828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt
(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) - 8486130899197
2*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(33687582904320*sqrt(2)
*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6
- 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)
**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250
994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) -
 120012014096640*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(33687582904320*
sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x
/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*ta
nh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 -
 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 4764143662
7072) + 10001001174720*log(tanh(x/2) - 1 + sqrt(2))/(33687582904320*sqrt(2)
*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6
- 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)
**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250
994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) +
 7071775749331*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))/(33687582904320*sqrt(2)
*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6
- 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)
**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250
994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) +
 10001001174720*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**8/(33687582904320*s
qrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/
2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tan
h(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 -
404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627
072) + 7071775749331*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**8/(336
87582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 5716972395
24864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 128012815036416
0*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*ta
```

```
nh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2)
+ 47641436627072) - 84861308991972*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tan
h(x/2)**6/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**
8 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1
280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 5716
97239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 336875829
04320*sqrt(2) + 47641436627072) - 1200120140966...
```

Giac [A]
time = 0.41, size = 74, normalized size = 1.35

$$-\frac{19}{128}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-\frac{19e^{(6x)}-171e^{(4x)}+89e^{(2x)}-9}{16\left(e^{(4x)}-6e^{(2x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] -19/128*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*
x) - 6)) - 1/16*(19*e^(6*x) - 171*e^(4*x) + 89*e^(2*x) - 9)/(e^(4*x) - 6*e^
(2*x) + 1)^2
```

Mupad [B]
time = 0.61, size = 112, normalized size = 2.04

$$\frac{17e^{2x}-3}{38e^{4x}-12e^{2x}-12e^{6x}+e^{8x}+1}-\frac{19\sqrt{2}\,\ln\left(\frac{19e^{2x}}{8}-\frac{19\sqrt{2}\,(12e^{2x}-4)}{128}\right)}{128}+\frac{19\sqrt{2}\,\ln\left(\frac{19e^{2x}}{8}+\frac{19\sqrt{2}\,(12e^{2x}-4)}{128}\right)}{128}-\frac{\frac{19e^{2x}}{16}-\frac{57}{16}}{e^{4x}-6e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^2 - 1)^3,x)
```

```
[Out] (17*exp(2*x) - 3)/(38*exp(4*x) - 12*exp(2*x) - 12*exp(6*x) + exp(8*x) + 1)
- (19*2^(1/2)*log((19*exp(2*x))/8 - (19*2^(1/2)*(12*exp(2*x) - 4))/128))/12
8 + (19*2^(1/2)*log((19*exp(2*x))/8 + (19*2^(1/2)*(12*exp(2*x) - 4))/128))/
128 - ((19*exp(2*x))/16 - 57/16)/(exp(4*x) - 6*exp(2*x) + 1)
```

3.66 $\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=130

$$-\frac{(a-b)(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8b^{3/2}f}-\frac{(a+3b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8bf}$$

```
[Out] -1/8*(a-b)*(a+3*b)*arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))
/b^(3/2)/f+1/4*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(3/2)/b/f-1/8*(a+3*b)*cosh
(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/b/f
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3265, 396, 201, 223, 212}

$$-\frac{(a-b)(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8b^{3/2}f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4bf}-\frac{(a+3b)\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/8*((a - b)*(a + 3*b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh
[e + f*x]^2]])/(b^(3/2)*f) - ((a + 3*b)*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e
 + f*x]^2])/(8*b*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*
b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx &= -\frac{\mathrm{Subst}\left(\int (1-x^2)\sqrt{a-b+bx^2}\,dx, x, \cosh(e+fx)\right)}{f}\\
&= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}{4bf} - \frac{(a+3b)\mathrm{Subst}\Big(\int}{}\\
&= -\frac{(a+3b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8bf} + \frac{\cosh(e+}{}\\
&= -\frac{(a+3b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8bf} + \frac{\cosh(e+}{}\\
&= -\frac{(a-b)(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a-}{}
\end{aligned}$$

Mathematica [A]

time = 0.46, size = 114, normalized size = 0.88

$$\frac{\frac{\cosh(e+fx)(a-4b+b\cosh(2(e+fx)))\sqrt{4a-2b+2b\cosh(2(e+fx))}}{2b}+\frac{(-a+b)(a+3b)\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{b^{3/2}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((Cosh[e + f*x]*(a - 4*b + b*Cosh[2*(e + f*x)])*Sqrt[4*a - 2*b + 2*b*Cosh[2
*(e + f*x)]])/(2*b) + ((-a + b)*(a + 3*b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x]
 + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]])/b^(3/2))/(8*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(114) = 228$.
time = 1.83, size = 339, normalized size = 2.61

method	result
default	$\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\,\left(4b^{\frac{5}{2}}\sqrt{b\left(\cosh^{4}\left(fx+e\right)\right)+\left(a-b\right)\left(\cosh^{2}\left(fx+e\right)\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(4*b^(5/2)*(b*cosh(f*x+e)^4+
(a-b)*cosh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2-10*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x
+e)^2)^(1/2)*b^(5/2)+2*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(3/2
)-ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b
^(1/2)+a-b)/b^(1/2))*a^2*b-2*a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4
+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*b^2+3*b^3*ln(1/2*(2*b*cos
h(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/
2)))/b^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1180 vs. $2(114) = 228$.
time = 0.52, size = 3037, normalized size = 23.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(2*((a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*
cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sin
h(f*x + e)^2 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2
 + 2*a*b - 3*b^2)*sinh(f*x + e)^4)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a
^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b
)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e
)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x
 + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x +
e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x
 + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 +
(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2
 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*co
sh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2
)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh
(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f
*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*
x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x
+ e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3
*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))
*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
 b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) +
4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4
*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e
))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*
sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*s
inh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 2*((
a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e
)^3*sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2
 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 + 2*a*b - 3
*b^2)*sinh(f*x + e)^4)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*
sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*co
sh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(
f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)
^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f
*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))
*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2)) - sqrt(2)*(b^2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x
 + e)^5 + b^2*sinh(f*x + e)^6 + (2*a*b - 7*b^2)*cosh(f*x + e)^4 + (15*b^2*c
osh(f*x + e)^2 + 2*a*b - 7*b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3
+ (2*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + (2*a*b - 7*b^2)*cosh(f*x
 + e)^2 + (15*b^2*cosh(f*x + e)^4 + 6*(2*a*b - 7*b^2)*cosh(f*x + e)^2 + 2*a
```

```
*b - 7*b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(2*a*b - 7
*b^2)*cosh(f*x + e)^3 + (2*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)^4 + 4*b^2
*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b^2*f*cosh(f*x + e)^2*sinh(f*x + e)^2
+ 4*b^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + b^2*f*sinh(f*x + e)^4), 1/64*(4*(
(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x +
e)^3*sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sinh(f*x + e)^
2 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 + 2*a*b -
3*b^2)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*co
sh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e
)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (
6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(
f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 4*((a^2 + 2*a*b
 - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^3*sinh(f*
x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 +
 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 + 2*a*b - 3*b^2)*sinh(
f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*
sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*
a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*si
nh(f*x + e) + b)) + sqrt(2)*(b^2*cosh(f*x + e)^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 890 vs. 2(114) = 228.
time = 0.70, size = 890, normalized size = 6.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b)*((2*a*e^(6*e) - 7*b*e^(6*e))*e^(-2*e)/b + e^(2*f*x + 6*e)) + 8*(a^2*e^(4
*e) + 2*a*b*e^(4*e) - 3*b^2*e^(4*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqr
t(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(
-b))/(sqrt(-b)*b) + 4*(a^2*sqrt(b)*e^(4*e) + 2*a*b^(3/2)*e^(4*e) - 3*b^(5/2
)*e^(4*e))*log(abs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/2)))/b^
2 - 4*(2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*e^(4*e) - 8*(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)^3*a*b*e^(4*e) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^2*e^(4*e) + 4*(sqrt(b)*e^(
2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
+ 2*e) + b))^2*a*b^(3/2)*e^(4*e) - 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4
*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(5/2)*e^(
4*e) + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b*e^(4*e) + 4*(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)*a*b^2*e^(4*e) - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^3*e^(4*e) + 3*b^(7/2)*e^(4*e
))/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^2 - b)^2*b))*e^(-4*e)/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(e+f\,x)^3\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.67 $\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2\sqrt{b}f}+\frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f}$$

```
[Out] 1/2*(a-b)*arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f/b^(1/2
)+1/2*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3265, 201, 223, 212}

$$\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}+\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/
(2*Sqrt[b]*f) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a-b+bx^2}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} + \frac{(a-b)\text{Subst}\Big(\int \frac{}{\sqrt{a}}$$

$$= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} + \frac{(a-b)\text{Subst}\Bigg(\int \frac{1}{1-bx}$$

$$= \frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2\sqrt{b}\,f} + \frac{\cosh(e+fx)\sqrt{}}{}$$

Mathematica [A]
time = 0.10, size = 97, normalized size = 1.18

$$\frac{\cosh(e+fx)\sqrt{2a-b+b\cosh(2(e+fx))}}{2\sqrt{2}\,f} + \frac{(a-b)\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{2\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])/(2*Sqrt[2]*f) + ((a - b
)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]])
/(2*Sqrt[b]*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(70) = 140.
time = 0.94, size = 200, normalized size = 2.44

---

method	result
default	$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(2\sqrt{b\left(\cosh^{4}\left(fx+e\right)\right)+\left(a-b\right)\left(\cosh^{2}\left(fx+e\right)\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(2*(b*cosh(f*x+e)^4+(a-b)*cos
h(f*x+e)^2)^(1/2)*b^(1/2)+a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a
-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-b*ln(1/2*(2*b*cosh(f*x+e)^2+
2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2)))/b^(1/2
)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(70) = 140$.
time = 0.55, size = 2130, normalized size = 25.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (
a - b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f
*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x
+ e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14
*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (
9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^
2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4
*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b -
4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cos
```

```
h(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)
^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x
+ e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5
 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 +
 a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sin
h(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18
*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f
*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*
cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^
3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*
x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x +
e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e
)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a - b)*cosh(f
*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2
)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*s
inh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b
)*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(
cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(
2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2
 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^
2 + 2*b*f*cosh(f*x + e)*sinh(f*x + e) + b*f*sinh(f*x + e)^2), -1/8*(2*((a -
 b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(
f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*
sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x +
 e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f
*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 +
 (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((a - b)*cosh(f*x + e)^2
+ 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2)*sqrt(-b)
*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*c
osh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*
(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)
^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) -
sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x +
 e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x
```

```
+ e)^2 + 2*b*f*cosh(f*x + e)*sinh(f*x + e) + b*f*sinh(f*x + e)^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sinh^{2}{\left(e+fx\right)}}\sinh{\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sinh(e + f*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{16,[4,2,4]%%%}+%%%{%%%{-32,[1]%%%},[4,2,3]%%%}+%%%{%%%{16,[2]
%%%},[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{sinh}(e+f\,x)\,\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}\,\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.68 $\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}$$

```
[Out] -arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh
(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A]

time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3265, 399, 223, 212, 385}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x
]^2]])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{1-x^2}\,dx, x, \cosh(e+fx)\right)}{f}\\ &= -\frac{a\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f} + b\text{Subs}\\ &= -\frac{a\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} + b\text{Sub}\\ &= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} + \sqrt{b}\tanh^{-1}\Bigg(\end{aligned}$$

Mathematica [A]
time = 0.09, size = 97, normalized size = 1.15

$$\frac{-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)+\sqrt{b}\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]]]) + Sqrt[b]*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b +
b*Cosh[2*(e + f*x)]]])/f
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(72) = 144$.
time = 1.15, size = 174, normalized size = 2.07

method	result
default	$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\,\left(\sqrt{b}\,\ln\left(\frac{2b\left(\cosh^{2}\left(fx+e\right)\right)+2\sqrt{b\left(\cosh^{4}\left(fx+e\right)\right)+(a-b}}{2\sqrt{b}}\right.\right.}{2\cosh(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(b^(1/2)*ln(1/2*(2*b*cosh(f*x
+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-a
^(1/2)*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)
^2)^(1/2)+a-b)/sinh(f*x+e)^2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(72) = 144$.
time = 0.51, size = 4423, normalized size = 52.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x +
 e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2
*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x +
e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2
+ b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^
3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*
x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cos
```

```
h(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(
14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3
 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(
a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x +
e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e
)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*
a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)
^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*
cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 +
3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3
 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(
f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x
 + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x +
 e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 2*sqrt(a)*log(-((a + b)*cosh(f*x
+ e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4
+ 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sin
h(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) +
 a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^
4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh
(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(b)*log(-(b*cosh(f*x
 + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)
*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqrt(2
)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*s
qrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e
)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/f, 1/4*(4*sqrt(-a)*arctan(sqrt(2
)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*s
qrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4
 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh
(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh
(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(b)*log((a
^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f
*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 +
 a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*
b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)
^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)
*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 +
 a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x
```

```
 + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x +
e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*
b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6
+ 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*
x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(
f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x
+ e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*si
nh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4
*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 +
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh
(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*c
osh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x
+ e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e
)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*co...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^2{(e + fx)}}\,\operatorname{csch}{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*csch(e + f*x), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}}{\sinh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x),x)
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x), x)
```

3.69 $\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=88

$$\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2\sqrt{a}\,f}-\frac{\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{2f}$$

```
[Out] 1/2*(a-b)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f/a^(1/2
)-1/2*coth(f*x+e)*csch(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3265, 386, 385, 212}

$$\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{a}\,f}-\frac{\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/
(2*Sqrt[a]*f) - (Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x
])/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{(1-x^2)^2}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{2f} + \frac{(a-}{}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{2f} + \frac{(a-}{}$$

$$= \frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2\sqrt{a}\,f} - \frac{\sqrt{a-b+b\,\mathrm{c}}}{}$$

Mathematica [**A**]
time = 0.21, size = 104, normalized size = 1.18

$$\frac{2(a-b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right) - \sqrt{2}\sqrt{a}\sqrt{2a-b+b\cosh(2(e+fx))}\,\coth(e+fx)\operatorname{csch}(e+fx)}{4\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
[Out] (2*(a - b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[e +
 f*x]*Csch[e + f*x])/(4*Sqrt[a]*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(76) = 152.
time = 1.11, size = 230, normalized size = 2.61

method	result
default	$-\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(-a\ln\left(\frac{(a+b)\left(\cosh^2(fx+e)\right)+2\sqrt{a}\,\sqrt{b\left(\cosh^4\left(fx+e\right)\right)-}}{\sinh(fx+e)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-a*ln(((a+b)*cosh(f*x+e)^2+
2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*s
inh(f*x+e)^2+b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cos
h(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2+2*a^(1/2)*((a+b*sinh(f*
x+e)^2)*cosh(f*x+e)^2)^(1/2))/a^(1/2)/sinh(f*x+e)^2/cosh(f*x+e)/(a+b*sinh(f
*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

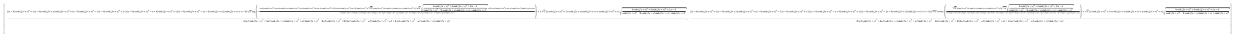
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(76) = 152$.
time = 0.45, size = 1277, normalized size = 14.51



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a - b)*cosh(f*x + e)^4 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a - b)*sinh(f*x + e)^4 - 2*(a - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f*
x + e)^2 - a + b)*sinh(f*x + e)^2 + 4*((a - b)*cosh(f*x + e)^3 - (a - b)*co
sh(f*x + e))*sinh(f*x + e) + a - b)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 +
 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a
 - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x +
e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
```

```
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*
((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/
(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3
*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e
)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*(a*cosh(f*x + e)^2 + 2
*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^4 + 4*a*f*cosh(f*x + e)*s
inh(f*x + e)^3 + a*f*sinh(f*x + e)^4 - 2*a*f*cosh(f*x + e)^2 + 2*(3*a*f*cos
h(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 4*(a*f*cosh(f*x + e)^3 - a*f*co
sh(f*x + e))*sinh(f*x + e)), -1/2*(((a - b)*cosh(f*x + e)^4 + 4*(a - b)*cos
h(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f*x + e)^4 - 2*(a - b)*cosh(f*x +
 e)^2 + 2*(3*(a - b)*cosh(f*x + e)^2 - a + b)*sinh(f*x + e)^2 + 4*((a - b)*
cosh(f*x + e)^3 - (a - b)*cosh(f*x + e))*sinh(f*x + e) + a - b)*sqrt(-a)*ar
ctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh
(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*
a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2
+ 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqr
t(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)
^2 + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e
)^4 + 4*a*f*cosh(f*x + e)*sinh(f*x + e)^3 + a*f*sinh(f*x + e)^4 - 2*a*f*cos
h(f*x + e)^2 + 2*(3*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 4*(a
*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^{2}\left(e + fx\right)}\,\operatorname{csch}^{3}\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*csch(e + f*x)**3, x)
```

Giac **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\sinh\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^3,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^3, x)
```

3.70 $\int \mathbf{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=144

$$-\frac{(a-b)(3a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8a^{3/2}f}+\frac{(3a+b)\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\mathrm{cs}}{8af}$$

```
[Out] -1/8*(a-b)*(3*a+b)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))
/a^(3/2)/f-1/4*(a-b+b*cosh(f*x+e)^2)^(3/2)*coth(f*x+e)*csch(f*x+e)^3/a/f+1/
8*(3*a+b)*coth(f*x+e)*csch(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3265, 390, 386, 385, 212}

$$-\frac{(a-b)(3a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}^3(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4af}+\frac{(3a+b)\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{8af}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/8*((a - b)*(3*a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh
[e + f*x]^2]])/(a^(3/2)*f) + ((3*a + b)*Sqrt[a - b + b*Cosh[e + f*x]^2]*Cot
h[e + f*x]*Csch[e + f*x])/(8*a*f) - ((a - b + b*Cosh[e + f*x]^2)^(3/2)*Coth
[e + f*x]*Csch[e + f*x]^3)/(4*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{(1-x^2)^3}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\left(a-b+b\cosh^2(e+fx)\right)^{3/2}\coth(e+fx)\operatorname{csch}^3(e+fx)}{4af} -$$

$$= \frac{(3a+b)\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{8af}$$

$$= \frac{(3a+b)\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{8af}$$

$$= -\frac{(a-b)(3a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8a^{3/2}f} +$$

Mathematica [A]

time = 0.38, size = 129, normalized size = 0.90

$$\frac{(-6a^2+4ab+2b^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)-\sqrt{2}\sqrt{a}\sqrt{2a-b+b\cosh(2(e+fx))}\coth(e+fx)\operatorname{csch}(e+fx)\left(-3a+b+2a\operatorname{csch}^2(e+fx)\right)}{16a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-6*a^2 + 4*a*b + 2*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a
- b + b*Cosh[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e +
f*x)]]*Coth[e + f*x]*Csch[e + f*x]*(-3*a + b + 2*a*Csch[e + f*x]^2))/(16*a^
(3/2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(128) = 256$.
time = 103.82, size = 381, normalized size = 2.65

method	result
default	$\frac{\sqrt{\left(a+b\left(\sinh^2(fx+e)\right)\right)\left(\cosh^2(fx+e)\right)}\left(6\sqrt{\left(a+b\left(\sinh^2(fx+e)\right)\right)\left(\cosh^2(fx+e)\right)}\,(\sinh^2(\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(6*((a+b*sinh(f*x+e)^2)*cosh
(f*x+e)^2)^(1/2)*sinh(f*x+e)^2*a^(5/2)-3*a^3*ln(((a+b)*cosh(f*x+e)^2+2*a^(1
/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*
x+e)^4+2*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*
x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^4*a^2+ln(((a+b)*cosh(f*x+e)^2
+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*
b^2*sinh(f*x+e)^4*a-2*b*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+
e)^2*a^(3/2)-4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*a^(5/2))/sinh(f*x+
e)^4/a^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1646 vs. $2(128) = 256$.
time = 0.61, size = 3395, normalized size = 23.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^8 + 8*(3*a^2 - 2*a*b - b^2)*co
sh(f*x + e)*sinh(f*x + e)^7 + (3*a^2 - 2*a*b - b^2)*sinh(f*x + e)^8 - 4*(3*
a^2 - 2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x +
e)^2 - 3*a^2 + 2*a*b + b^2)*sinh(f*x + e)^6 + 8*(7*(3*a^2 - 2*a*b - b^2)*co
sh(f*x + e)^3 - 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*
(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 + 2*(35*(3*a^2 - 2*a*b - b^2)*cosh(f*
x + e)^4 - 30*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 + 9*a^2 - 6*a*b - 3*b^2
)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^5 - 10*(3*a^2
- 2*a*b - b^2)*cosh(f*x + e)^3 + 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sin
h(f*x + e)^3 - 4*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 + 4*(7*(3*a^2 - 2*a*
b - b^2)*cosh(f*x + e)^6 - 15*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 + 9*(3*
a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 - 3*a^2 + 2*a*b + b^2)*sinh(f*x + e)^2 +
 3*a^2 - 2*a*b - b^2 + 8*((3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^7 - 3*(3*a^2
- 2*a*b - b^2)*cosh(f*x + e)^5 + 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^3 -
(3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-((a + b)*c
osh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x
 + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a
- b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*
x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f
*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 +
 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((3*a^
2 - a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - a*b)*cosh(f*x + e)*sinh(f*x + e)^5 +
(3*a^2 - a*b)*sinh(f*x + e)^6 - (11*a^2 - a*b)*cosh(f*x + e)^4 + (15*(3*a^2
 - a*b)*cosh(f*x + e)^2 - 11*a^2 + a*b)*sinh(f*x + e)^4 + 4*(5*(3*a^2 - a*b
)*cosh(f*x + e)^3 - (11*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e)^3 - (11*a^2
 - a*b)*cosh(f*x + e)^2 + (15*(3*a^2 - a*b)*cosh(f*x + e)^4 - 6*(11*a^2 - a
*b)*cosh(f*x + e)^2 - 11*a^2 + a*b)*sinh(f*x + e)^2 + 3*a^2 - a*b + 2*(3*(3
*a^2 - a*b)*cosh(f*x + e)^5 - 2*(11*a^2 - a*b)*cosh(f*x + e)^3 - (11*a^2 -
a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e
)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2)))/(a^2*f*cosh(f*x + e)^8 + 8*a^2*f*cosh(f*x + e)*sinh(f*x + e)^7 +
```

```
a^2*f*sinh(f*x + e)^8 - 4*a^2*f*cosh(f*x + e)^6 + 6*a^2*f*cosh(f*x + e)^4 +
 4*(7*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^6 + 8*(7*a^2*f*cosh(f*x
+ e)^3 - 3*a^2*f*cosh(f*x + e))*sinh(f*x + e)^5 - 4*a^2*f*cosh(f*x + e)^2 +
 2*(35*a^2*f*cosh(f*x + e)^4 - 30*a^2*f*cosh(f*x + e)^2 + 3*a^2*f)*sinh(f*x
 + e)^4 + 8*(7*a^2*f*cosh(f*x + e)^5 - 10*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*c
osh(f*x + e))*sinh(f*x + e)^3 + a^2*f + 4*(7*a^2*f*cosh(f*x + e)^6 - 15*a^2
*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^2 + 8*(
a^2*f*cosh(f*x + e)^7 - 3*a^2*f*cosh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^3 -
 a^2*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(((3*a^2 - 2*a*b - b^2)*cosh(f*x
+ e)^8 + 8*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (3*a^2 - 2
*a*b - b^2)*sinh(f*x + e)^8 - 4*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^6 + 4*(
7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 - 3*a^2 + 2*a*b + b^2)*sinh(f*x + e
)^6 + 8*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^3 - 3*(3*a^2 - 2*a*b - b^2)*
cosh(f*x + e))*sinh(f*x + e)^5 + 6*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 +
2*(35*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 - 30*(3*a^2 - 2*a*b - b^2)*cosh
(f*x + e)^2 + 9*a^2 - 6*a*b - 3*b^2)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 2*a*b
 - b^2)*cosh(f*x + e)^5 - 10*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^3 + 3*(3*a^
2 - 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(3*a^2 - 2*a*b - b^2)*c
osh(f*x + e)^2 + 4*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^6 - 15*(3*a^2 - 2
*a*b - b^2)*cosh(f*x + e)^4 + 9*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 - 3*a
^2 + 2*a*b + b^2)*sinh(f*x + e)^2 + 3*a^2 - 2*a*b - b^2 + 8*((3*a^2 - 2*a*b
 - b^2)*cosh(f*x + e)^7 - 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^5 + 3*(3*a^
2 - 2*a*b - b^2)*cosh(f*x + e)^3 - (3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sin
h(f*x + e))*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*
sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*
a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*si
nh(f*x + e) + b)) + sqrt(2)*((3*a^2 - a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - a*b
)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - a*b)*sinh(f*x + e)^6 - (11*a^2 -
 a*b)*cosh(f*x + e)^4 + (15*(3*a^2 - a*b)*cosh(f*x + e)^2 - 11*a^2 + a*b)*s
inh(f*x + e)^4 + 4*(5*(3*a^2 - a*b)*cosh(f*x + ...
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\sinh\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^5,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^5, x)
```

3.71 $\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=300

$$\frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf}+\frac{\cosh(e+fx)\sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx}}{5f}$$

```
[Out] 1/15*(a-4*b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+1/5*cosh
(f*x+e)*sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/15*(2*a^2+3*a*b-8*b^2)*
(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/15*(a-4*b)*(1/(1+sinh(
f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+
e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sech(
f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/15*(2*a^2+3*a*b-8*b^2)*(a+b*sinh(f*
x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Rubi [A]
time = 0.23, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3267, 489, 596, 545, 429, 506, 422}

$$\frac{(2a^2+3ab-8b^2)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{15b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(2a^2+3ab-8b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15b^2f}-\frac{(a-4b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{15bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(a-4b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf}+\frac{\sinh^3(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f
) + (Cosh[e + f*x]*Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(5*f) + ((2
*a^2 + 3*a*b - 8*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*
x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) - ((a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Se
ch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2*(a +
 b*Sinh[e + f*x]^2))/a]) - ((2*a^2 + 3*a*b - 8*b^2)*Sqrt[a + b*Sinh[e + f*x
]^2]*Tanh[e + f*x])/(15*b^2*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
```

```
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{1+x^2}}\,dx, x, \text{s}\right.}{f}$$

$$= \frac{\cosh(e+fx)\sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{5f} - \frac{\left(\sqrt{\cosh^2\right.}{}$$

$$= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\text{co}}{}$$

$$= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\text{co}}{}$$

$$= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\text{co}}{}$$

$$= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\text{co}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.96, size = 210, normalized size = 0.70

$$\frac{16ia(2a^2+3ab-8b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) - 32ia(a^2+ab-2b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) + \sqrt{2}\,b(8a^2-48ab+25b^2+4(4a-7b)b\cosh(2(e+fx))+3b^2\cosh(4(e+fx)))\sinh(2(e+fx))}{240b^2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((16*I)*a*(2*a^2 + 3*a*b - 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticE[I*(e + f*x), b/a] - (32*I)*a*(a^2 + a*b - 2*b^2)*Sqrt[(2*a - b + b
*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2 - 48*
a*b + 25*b^2 + 4*(4*a - 7*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*x)])
*Sinh[2*(e + f*x)])/(240*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [**A**]
time = 2.56, size = 512, normalized size = 1.71

method	result
default	$\frac{3\sqrt{-\frac{b}{a}}\,b^2(\sinh^7(fx+e))+4\sqrt{-\frac{b}{a}}\,ab(\sinh^5(fx+e))-\sqrt{-\frac{b}{a}}\,b^2(\sinh^5(fx+e))+\sqrt{-\frac{b}{a}}\,a^2(\sinh^3(fx+e))-4\sqrt{-\frac{b}{a}}\,b^2(\mathrm{si}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^7+4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5
-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-4*(-1/a*
b)^(1/2)*b^2*sinh(f*x+e)^3+a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+7*a*((a+b*sinh(f*
x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*b-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-2*((a+b*sinh(f*x+e)^2)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))*a^2-3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh
(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+(-
1/a*b)^(1/2)*a^2*sinh(f*x+e)-4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/b/(-1/a*b)^(
1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.09, size = 25, normalized size = 0.08

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\sinh\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{32,[4,2,4]%%%}+%%%{%%%{-64,[1]%%%},[4,2,3]%%%}+%%%{%%%{32,[2]
%%%},[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{sinh}(e+f\,x)^4\,\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}\,\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.72 $\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=177

$$\frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{i(a-2b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{3bf\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}+\frac{ia(a-}{}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*I*(a-2*b)*(cos(
I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a
+b*sinh(f*x+e)^2)^(1/2)/b/f/(1+b*sinh(f*x+e)^2/a)^(1/2)+1/3*I*a*(a-b)*(cos(
I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))*(1
+b*sinh(f*x+e)^2/a)^(1/2)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3249, 3251, 3257, 3256, 3262, 3261}

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{i(a-2b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((I/3)*(a
 - 2*b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[
1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*
Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3249

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e
 + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p
 + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Gt
Q[p, 0]
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
```

```
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{1}{3}\int \frac{a-}{\sqrt{\ }}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{(a-2b)}{\ }$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\Big((a-2b}{\ }$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{i(a-2b)}{\ }$$

Mathematica [A]
time = 0.59, size = 170, normalized size = 0.96

$$\frac{-2i\sqrt{2}\,a(a-2b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+2i\sqrt{2}\,a(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+b(2a-b+b\cosh(2(e+fx)))\sinh(2(e+fx))}{6bf\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])
*Sinh[2*(e + f*x)])/(6*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.16, size = 353, normalized size = 1.99

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, b(\cosh^4(fx+e))\sinh(fx+e)+\sqrt{-\frac{b}{a}}\, a(\cosh^2(fx+e))\sinh(fx+e)-\sqrt{-\frac{b}{a}}\, b(\cosh^2(fx+e))\sinh(fx+e)-2a\sqrt{\frac{b(\cosh}{\ }}}{\ }$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b*cosh(f*x+e)^4*sinh(f*x+e)+(-1/a*b)^(1/2)*a*cosh(f*x+e
)^2*sinh(f*x+e)-(-1/a*b)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)-2*a*(b/a*cosh(f*
x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^
(1/2),(a/b)^(1/2))+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2
)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a
-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))*a-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/(-1/a*b)^(1/2)/cosh(f*x+e)/
(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)
```

Fricas [F]
time = 0.12, size = 25, normalized size = 0.14

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\sinh\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sinh^{2}\left(e+fx\right)}\,\sinh^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sinh(e + f*x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(e+f\,x)^2\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.73 $\int \sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(
1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3257, 3256}

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
(b*Sinh[e + f*x]^2)/a])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$
$$= -\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

Mathematica [A]
time = 0.07, size = 69, normalized size = 1.15

$$-\frac{ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a]
)/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 0.95, size = 140, normalized size = 2.33

method	result
default	$\frac{\sqrt{\frac{a+b\left(\sinh^2(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\left(a\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\right.\right.}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/
cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F]
time = 0.11, size = 16, normalized size = 0.27

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sinh^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{b\,\sinh\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.74 $\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=199

$$-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+f}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] -coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sin
h(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/
2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)
^2)/a)^(1/2)+b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*Elliptic
F(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(
f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(a+b*sinh(f
*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]

time = 0.12, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3267, 486, 21, 433, 429, 506, 422}

$$\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - (EllipticE[ArcTan[Sinh[e
 + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (b*EllipticF[ArcTan[Sinh[e + f*x
]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e +
 f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e
+ f*x])/f
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\Big(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\Big)\text{Subst}\Big(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{1+x^2}}\,dx, x,}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\Big(\sqrt{\cosh^2(e+fx)}\ \text{se}}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\Big(b\sqrt{\cosh^2(e+fx)}\ \text{s}}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\Big(b\sqrt{\cosh^2(e+fx)}\ \text{s}}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{bF\big(\tan^{-1}(\sinh(e+f}{af\sqrt{\frac{\text{sec}}{}}}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{E\big(\tan^{-1}(\sinh(e+f}{f\sqrt{\frac{\text{sech}}{}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.43, size = 151, normalized size = 0.76

$$\frac{\sqrt{2}\,(-2a+b-b\cosh(2(e+fx)))\coth(e+fx)-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\big(i(e+fx)\,\big|\,\frac{b}{a}\big)+2i(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\big(i(e+fx)\,\big|\,\frac{b}{a}\big)}{2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (2*I)*(a - b)*
Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(2*f*S
qrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 0.99, size = 166, normalized size = 0.83

method	result

default	$\frac{-\sqrt{-\frac{b}{a}}\, b\left(\cosh^{4}(fx+e)\right)+b\sqrt{\frac{b\left(\cosh^{2}(fx+e)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\sinh(fx+e)\,\mathrm{EllipticE}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}}, \sqrt{}\right.}{\sinh(fx+e)\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\left(\sinh^{2}(fx+e)\right)}\, f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+b*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*sinh(f*x+e)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))-(-1/a*b)^(1/2)*a*cosh(f*x+e)^2+(-1/a*b)^(1/2)*b*cosh(f*x+e)^2)/sinh(f*x+
e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2, x)
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(215) = 430$.
time = 0.10, size = 542, normalized size = 2.72

$$\frac{\left((2a-b)\cosh(fx+e)^{2}+2(2a-b)\cosh(fx+e)\sinh(fx+e)+(2a-b)\sinh(fx+e)^{2}-2\left(b\cosh(fx+e)^{2}+2b\cosh(fx+e)\sinh(fx+e)+b\sinh(fx+e)^{2}-b\right)\sqrt{\frac{a^{2}-ab}{b^{2}}}-2a+b\right)\sqrt{b}\sqrt{\frac{2b\sqrt{\frac{a^{2}-ab}{b^{2}}}-2a+b}{b}}\,E(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^{2}-ab}{b^{2}}}-2a+b}{b}}\left(\cosh(fx+e)+\sinh(fx+e)\right)\right)\mid\frac{8a^{2}-8ab+b^{2}+4(2ab-b^{2})\sqrt{\frac{a^{2}-ab}{b^{2}}}}{b^{2}})-2\left((2a-b)\cosh(fx+e)^{2}+2(2a-b)\cosh(fx+e)\sinh(fx+e)+(2a-b)\sinh(fx+e)^{2}-2a+b\right)\sqrt{b}\sqrt{\frac{2b\sqrt{\frac{a^{2}-ab}{b^{2}}}-2a+b}{b}}\,F(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^{2}-ab}{b^{2}}}-2a+b}{b}}\left(\cosh(fx+e)+\sinh(fx+e)\right)\right)\mid\frac{8a^{2}-8ab+b^{2}+4(2ab-b^{2})\sqrt{\frac{a^{2}-ab}{b^{2}}}}{b^{2}})-\sqrt{2}\left(b\cosh(fx+e)+b\sinh(fx+e)\right)\sqrt{\frac{b\cosh(fx+e)^{2}+b\sinh(fx+e)^{2}+2a-b}{\cosh(fx+e)^{2}-2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^{2}}}}{bf\cosh(fx+e)^{2}+2bf\cosh(fx+e)\sinh(fx+e)+bf\sinh(fx+e)^{2}-bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (((2*a - b)*cosh(f*x + e)^2 + 2*(2*a - b)*cosh(f*x + e)*sinh(f*x + e) + (2*
a - b)*sinh(f*x + e)^2 - 2*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x
 + e) + b*sinh(f*x + e)^2 - b)*sqrt((a^2 - a*b)/b^2) - 2*a + b)*sqrt(b)*sqrt
((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt(
(a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8
*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((2*a - b)*cos
h(f*x + e)^2 + 2*(2*a - b)*cosh(f*x + e)*sinh(f*x + e) + (2*a - b)*sinh(f*x
 + e)^2 - 2*a + b)*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*el
liptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e
) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b
)/b^2))/b^2) - sqrt(2)*(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((b*cosh(f*x
 + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
```

```
inh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^2 + 2*b*f*cosh(f*x + e
)*sinh(f*x + e) + b*f*sinh(f*x + e)^2 - b*f)
```

Sympy **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^2{(e+fx)}}\,\operatorname{csch}^2{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*csch(e + f*x)**2, x)
```

Giac **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad **[F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}}{{\mathrm{sinh}\left(e+f\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^2,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^2, x)
```

3.75 $\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=276

$$\frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{(2a-b)E}{}$$

```
[Out] 1/3*(2*a-b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-1/3*coth(f*x+e)*csch(
f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/3*(2*a-b)*(1/(1+sinh(f*x+e)^2))^(1/2
)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-
b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*s
inh(f*x+e)^2)/a)^(1/2)-1/3*b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^
(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x
+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/
2)-1/3*(2*a-b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a/f
```

Rubi [A]

time = 0.20, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3267, 486, 597, 545, 429, 506, 422}

$$-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\operatorname{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(2a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}+\frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (Coth[e + f
*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + ((2*a - b)*Ellipti
cE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2
])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*EllipticF
[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((2*a - b)*Sqr
t[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
```

```
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{\sqrt{a+bx^2}}{x^4\sqrt{1+x^2}}\,dx,x,\mathrm{si}\right.}{f}$$

$$= -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(\sqrt{\cosh}\right.}{}$$

$$= \frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth(e+fx)\mathrm{cs}}{}$$

$$= \frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth(e+fx)\mathrm{cs}}{}$$

$$= \frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth(e+fx)\mathrm{cs}}{}$$

$$= \frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth(e+fx)\mathrm{cs}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.01, size = 208, normalized size = 0.75

$$\frac{\frac{(4(2a^2-4ab+b^2)\cosh(2(e+fx))-(2a-b)(8a-3b-b\cosh(4(e+fx))))\coth(e+fx)\operatorname{csch}^2(e+fx)}{2\sqrt{2}}+2ia(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-4ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{6af\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (((4*(2*a^2 - 4*a*b + b^2)*Cosh[2*(e + f*x)] - (2*a - b)*(8*a - 3*b - b*Cos
h[4*(e + f*x)]))*Coth[e + f*x]*Csch[e + f*x]^2)/(2*Sqrt[2]) + (2*I)*a*(2*a
- b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] -
(4*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*
x), b/a])/(6*a*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [**A**]
time = 5.33, size = 436, normalized size = 1.58

method	result
default	$2\sqrt{-\frac{b}{a}}\, ab(\sinh^6(fx+e))-\sqrt{-\frac{b}{a}}\, b^2(\sinh^6(fx+e))+b\sqrt{\frac{a+b(\sinh^2(fx+e))}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\Big(\sinh(fx+e$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+b*
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(c
osh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*s
inh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elliptic
E(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3+((a+b*sinh(f*x+
e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(
a/b)^(1/2))*b^2*sinh(f*x+e)^3+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4-(-1/a*b)^(
1/2)*b^2*sinh(f*x+e)^4+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2-2*(-1/a*b)^(1/2)*a*
b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2)/a/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x
+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^4, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2144 vs. $2(280) = 560$.
time = 0.13, size = 2144, normalized size = 7.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(((4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^6 + 6*(4*a^2 - 4*a*b + b^2)*cosh
(f*x + e)*sinh(f*x + e)^5 + (4*a^2 - 4*a*b + b^2)*sinh(f*x + e)^6 - 3*(4*a^
```

```
2 - 4*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)
^2 - 4*a^2 + 4*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(4*a^2 - 4*a*b + b^2)*cosh
(f*x + e)^3 - 3*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(4
*a^2 - 4*a*b + b^2)*cosh(f*x + e)^2 + 3*(5*(4*a^2 - 4*a*b + b^2)*cosh(f*x +
 e)^4 - 6*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^2 + 4*a^2 - 4*a*b + b^2)*sinh
(f*x + e)^2 - 4*a^2 + 4*a*b - b^2 + 6*((4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^
5 - 2*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^3 + (4*a^2 - 4*a*b + b^2)*cosh(f*
x + e))*sinh(f*x + e) - 2*((2*a*b - b^2)*cosh(f*x + e)^6 + 6*(2*a*b - b^2)*
cosh(f*x + e)*sinh(f*x + e)^5 + (2*a*b - b^2)*sinh(f*x + e)^6 - 3*(2*a*b -
b^2)*cosh(f*x + e)^4 + 3*(5*(2*a*b - b^2)*cosh(f*x + e)^2 - 2*a*b + b^2)*si
nh(f*x + e)^4 + 4*(5*(2*a*b - b^2)*cosh(f*x + e)^3 - 3*(2*a*b - b^2)*cosh(f
*x + e))*sinh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f*x + e)^2 + 3*(5*(2*a*b -
b^2)*cosh(f*x + e)^4 - 6*(2*a*b - b^2)*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(
f*x + e)^2 - 2*a*b + b^2 + 6*((2*a*b - b^2)*cosh(f*x + e)^5 - 2*(2*a*b - b^
2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2
- a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic
_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + si
nh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2)
)/b^2) - 2*((2*a^2 - a*b)*cosh(f*x + e)^6 + 6*(2*a^2 - a*b)*cosh(f*x + e)*s
inh(f*x + e)^5 + (2*a^2 - a*b)*sinh(f*x + e)^6 - 3*(2*a^2 - a*b)*cosh(f*x +
 e)^4 + 3*(5*(2*a^2 - a*b)*cosh(f*x + e)^2 - 2*a^2 + a*b)*sinh(f*x + e)^4 +
 4*(5*(2*a^2 - a*b)*cosh(f*x + e)^3 - 3*(2*a^2 - a*b)*cosh(f*x + e))*sinh(f
*x + e)^3 + 3*(2*a^2 - a*b)*cosh(f*x + e)^2 + 3*(5*(2*a^2 - a*b)*cosh(f*x +
 e)^4 - 6*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f*x + e)^2 - 2*
a^2 + a*b + 6*((2*a^2 - a*b)*cosh(f*x + e)^5 - 2*(2*a^2 - a*b)*cosh(f*x + e
)^3 + (2*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b - b^2)*cosh(f*x
+ e)^6 + 6*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b - b^2)*sinh(f*x
 + e)^6 - 3*(a*b - b^2)*cosh(f*x + e)^4 + 3*(5*(a*b - b^2)*cosh(f*x + e)^2
- a*b + b^2)*sinh(f*x + e)^4 + 4*(5*(a*b - b^2)*cosh(f*x + e)^3 - 3*(a*b -
b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(a*b - b^2)*cosh(f*x + e)^2 + 3*(5*
(a*b - b^2)*cosh(f*x + e)^4 - 6*(a*b - b^2)*cosh(f*x + e)^2 + a*b - b^2)*si
nh(f*x + e)^2 - a*b + b^2 + 6*((a*b - b^2)*cosh(f*x + e)^5 - 2*(a*b - b^2)*
cosh(f*x + e)^3 + (a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b
)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(ar
csin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*
x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2
) - sqrt(2)*((2*a*b - b^2)*cosh(f*x + e)^5 + 5*(2*a*b - b^2)*cosh(f*x + e)*
sinh(f*x + e)^4 + (2*a*b - b^2)*sinh(f*x + e)^5 - 2*(3*a*b - b^2)*cosh(f*x
+ e)^3 + 2*(5*(2*a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + b^2)*sinh(f*x + e)^3
- b^2*cosh(f*x + e) + 2*(5*(2*a*b - b^2)*cosh(f*x + e)^3 - 3*(3*a*b - b^2)*
cosh(f*x + e))*sinh(f*x + e)^2 + (5*(2*a*b - b^2)*cosh(f*x + e)^4 - 6*(3*a*
b - b^2)*cosh(f*x + e)^2 - b^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2)))/(a*b*f*cosh(f*x + e)^6 + 6*a*b*f*cosh(f*x + e)*sinh(f
*x + e)^5 + a*b*f*sinh(f*x + e)^6 - 3*a*b*f*cosh(f*x + e)^4 + 3*a*b*f*cosh(
```

```
f*x + e)^2 + 3*(5*a*b*f*cosh(f*x + e)^2 - a*b*f)*sinh(f*x + e)^4 + 4*(5*a*b
*f*cosh(f*x + e)^3 - 3*a*b*f*cosh(f*x + e))*sinh(f*x + e)^3 - a*b*f + 3*(5*
a*b*f*cosh(f*x + e)^4 - 6*a*b*f*cosh(f*x + e)^2 + a*b*f)*sinh(f*x + e)^2 +
6*(a*b*f*cosh(f*x + e)^5 - 2*a*b*f*cosh(f*x + e)^3 + a*b*f*cosh(f*x + e))*s
inh(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\sinh\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^4,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^4, x)
```

3.76 $\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=177

$$-\frac{(a-b)^2(a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{16b^{3/2}f}-\frac{(a-b)(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e}{16bf}$$

```
[Out] -1/16*(a-b)^2*(a+5*b)*arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/
2))/b^(3/2)/f-1/24*(a+5*b)*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(3/2)/b/f+1/6*
cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(5/2)/b/f-1/16*(a-b)*(a+5*b)*cosh(f*x+e)*
(a-b+b*cosh(f*x+e)^2)^(1/2)/b/f
```

Rubi [A]
time = 0.13, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3265, 396, 201, 223, 212}

$$-\frac{(a-b)^2(a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{16b^{3/2}f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6bf}-\frac{(a+5b)\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{24bf}-\frac{(a-b)(a+5b)\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{16bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/16*((a - b)^2*(a + 5*b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*C
osh[e + f*x]^2]])/(b^(3/2)*f) - ((a - b)*(a + 5*b)*Cosh[e + f*x]*Sqrt[a - b
 + b*Cosh[e + f*x]^2])/(16*b*f) - ((a + 5*b)*Cosh[e + f*x]*(a - b + b*Cosh[
e + f*x]^2)^(3/2))/(24*b*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(5
/2))/(6*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx &= -\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a-b+bx^2\right)^{3/2}\,dx,x,\cosh(e+fx)\right)}{f}\\
&= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{5/2}}{6bf}-\frac{(a+5b)\text{Subst}}{}\\
&= -\frac{(a+5b)\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}{24bf}+\frac{\text{cos}}{}\\
&= -\frac{(a-b)(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{16bf}-\\
\\
&= -\frac{(a-b)(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{16bf}-\\
&= -\frac{(a-b)^2(a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{16b^{3/2}f}-
\end{aligned}$$

Mathematica [A]

time = 0.41, size = 151, normalized size = 0.85

$$\frac{\sqrt{2}\sqrt{b}\sqrt{2a-b+b\cosh(2(e+fx))}\left((6a^2-51ab+37b^2)\cosh(e+fx)+b((7a-8b)\cosh(3(e+fx))+b\cosh(5(e+fx)))\right)-12(a-b)^2(a+5b)\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{192b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*Sqrt[b]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*((6*a^2 - 51*a*b + 37*
b^2)*Cosh[e + f*x] + b*((7*a - 8*b)*Cosh[3*(e + f*x)] + b*Cosh[5*(e + f*x)]
)) - 12*(a - b)^2*(a + 5*b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a -
b + b*Cosh[2*(e + f*x)]]])/(192*b^(3/2)*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(157) = 314$.
time = 2.42, size = 483, normalized size = 2.73

method	result
default	$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(16\sqrt{b\left(\cosh^{4}\left(fx+e\right)\right)+\left(a-b\right)\left(\cosh^{2}\left(fx+e\right)\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(16*(b*cosh(f*x+e)^4+(a-b)*c
osh(f*x+e)^2)^(1/2)*b^(7/2)*cosh(f*x+e)^4+4*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x
+e)^2)^(1/2)*b^(5/2)*(-13*b+7*a)*cosh(f*x+e)^2+66*b^(7/2)*(b*cosh(f*x+e)^4+
(a-b)*cosh(f*x+e)^2)^(1/2)-72*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)
*b^(5/2)+6*a^2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(3/2)-3*ln(1/2
*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a
-b)/b^(1/2))*a^3*b-9*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cos
h(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*a^2*b^2+27*b^3*a*ln(1/2*(2*b*cosh(f
*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))
-15*b^4*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(
1/2)*b^(1/2)+a-b)/b^(1/2)))/b^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1965 vs. $2(157) = 314$.

time = 0.55, size = 4608, normalized size = 26.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(6*((a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^6 + 6*(a^3 + 3*a
^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 + 3*a^2*b -
 9*a*b^2 + 5*b^3)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 + 3*a^2*b - 9*a
*b^2 + 5*b^3)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 + 3*a^2*b - 9*a*b^2
 + 5*b^3)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*
b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*sinh
(f*x + e)^6)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sin
h(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*
(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh
(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4
*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b
^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*
cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b
^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^
2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^
2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 - sq
rt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh
(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(
f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^
3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f
*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 +
 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x +
e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x
 + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 +
6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*c
osh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cos
h(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 6*((a^3 + 3*a^2*b - 9*a*b^
2 + 5*b^3)*cosh(f*x + e)^6 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x +
 e)^5*sinh(f*x + e) + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^4*
sinh(f*x + e)^2 + 20*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^3*sinh
(f*x + e)^3 + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^2*sinh(f*x
 + e)^4 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)*sinh(f*x + e)^5
 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*sinh(f*x + e)^6)*sqrt(b)*log(-(b*cosh(
f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a -
 b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 - sqr
t(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1
```

```
)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x
+ e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(2)*(b^3*cosh(f*x + e)^1
0 + 10*b^3*cosh(f*x + e)*sinh(f*x + e)^9 + b^3*sinh(f*x + e)^10 + (7*a*b^2
- 8*b^3)*cosh(f*x + e)^8 + (45*b^3*cosh(f*x + e)^2 + 7*a*b^2 - 8*b^3)*sinh(
f*x + e)^8 + 8*(15*b^3*cosh(f*x + e)^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e))*s
inh(f*x + e)^7 + (6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^6 + (210*b^3*c
osh(f*x + e)^4 + 6*a^2*b - 51*a*b^2 + 37*b^3 + 28*(7*a*b^2 - 8*b^3)*cosh(f*
x + e)^2)*sinh(f*x + e)^6 + 2*(126*b^3*cosh(f*x + e)^5 + 28*(7*a*b^2 - 8*b^
3)*cosh(f*x + e)^3 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e))*sinh(f*
x + e)^5 + (6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^4 + (210*b^3*cosh(f*
x + e)^6 + 70*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^4 + 6*a^2*b - 51*a*b^2 + 37*b
^3 + 15*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*
(30*b^3*cosh(f*x + e)^7 + 14*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^5 + 5*(6*a^2*b
 - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^3 + (6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(
f*x + e))*sinh(f*x + e)^3 + b^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e)^2 + (45*b
^3*cosh(f*x + e)^8 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^6 + 15*(6*a^2*b - 5
1*a*b^2 + 37*b^3)*cosh(f*x + e)^4 + 7*a*b^2 - 8*b^3 + 6*(6*a^2*b - 51*a*b^2
 + 37*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*b^3*cosh(f*x + e)^9 + 4*
(7*a*b^2 - 8*b^3)*cosh(f*x + e)^7 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*
x + e)^5 + 2*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^3 + (7*a*b^2 - 8*b
^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2)))/(b^2*f*cosh(f*x + e)^6 + 6*b^2*f*cosh(f*x + e)^5*sinh(f*x + e) + 1
5*b^2*f*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*b^2*f*cosh(f*x + e)^3*sinh(f*x
 + e)^3 + 15*b^2*f*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*b^2*f*cosh(f*x + e)*
sinh(f*x + e)^5 + b^2*f*sinh(f*x + e)^6), 1/384*(12*((a^3 + 3*a^2*b - 9*a*b
^2 + 5*b^3)*cosh(f*x + e)^6 + 6*(a^3 + 3*a^2*b ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1585 vs. 2(157) = 314.
time = 1.07, size = 1585, normalized size = 8.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] 1/384*(((b*e^(2*f*x + 10*e) + (7*a*b^2*e^(14*e) - 8*b^3*e^(14*e))*e^(-6*e)/
b^2)*e^(2*f*x) + (6*a^2*b*e^(12*e) - 51*a*b^2*e^(12*e) + 37*b^3*e^(12*e))*e
^(-6*e)/b^2)*sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
2*e) + b) + 24*(a^3*e^(6*e) + 3*a^2*b*e^(6*e) - 9*a*b^2*e^(6*e) + 5*b^3*e^(
6*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))/(sqrt(-b)*b) + 12*(a^3*sqr
t(b)*e^(6*e) + 3*a^2*b^(3/2)*e^(6*e) - 9*a*b^(5/2)*e^(6*e) + 5*b^(7/2)*e^(6
*e))*log(abs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/2)))/b^2 - 2*
(12*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
 - 2*b*e^(2*f*x + 2*e) + b))^5*a^3*e^(6*e) - 108*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5
*a^2*b*e^(6*e) + 132*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^2*e^(6*e) - 45*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))^5*b^3*e^(6*e) + 48*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2)*
e^(6*e) - 120*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a*b^(5/2)*e^(6*e) + 63*(sqrt(b)*e^
(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b))^4*b^(7/2)*e^(6*e) + 32*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4
*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^3*b*e^(6*
e) + 96*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*b^2*e^(6*e) - 156*(sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))^3*a*b^3*e^(6*e) + 50*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*
e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^4*e^(6*e) + 108*(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b
*e^(2*f*x + 2*e) + b))^2*a*b^(7/2)*e^(6*e) - 78*(sqrt(b)*e^(2*f*x + 2*e) -
sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*
b^(9/2)*e^(6*e) - 12*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^3*b^2*e^(6*e) - 36*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))*a^2*b^3*e^(6*e) + 72*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b^4*e^(6*e
) - 21*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
*e) - 2*b*e^(2*f*x + 2*e) + b))*b^5*e^(6*e) - 36*a*b^(9/2)*e^(6*e) + 31*b^(
11/2)*e^(6*e))/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - b)^3*b))*e^(-6*e)/f
```

Mupad **[F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(e+f\,x)^3\,{\left(b\,{\sinh(e+f\,x)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.77 $\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=121

$$\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8\sqrt{b}\,f}+\frac{3(a-b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8f}+\cosh($$

```
[Out] 1/4*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(3/2)/f+3/8*(a-b)^2*arctanh(cosh(f*x+
e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f/b^(1/2)+3/8*(a-b)*cosh(f*x+e)*(a-
b+b*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3265, 201, 223, 212}

$$\frac{3(a-b)\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{8f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4f}+\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a - b)^2*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2
]])/(8*Sqrt[b]*f) + (3*(a - b)*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2
])/(8*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int\left(a-b+bx^2\right)^{3/2}\,dx,x,\cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a-b))\text{Subst}(}{}$$

$$= \frac{3(a-b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8f} + \frac{\cosh(e+}{}$$

$$= \frac{3(a-b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8f} + \frac{\cosh(e+}{}$$

$$= \frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8\sqrt{b}\,f} + \frac{3(a-b)\,\mathrm{c}}{}$$

Mathematica [**A**]
time = 0.21, size = 111, normalized size = 0.92

$$\frac{\frac{1}{2}\cosh(e+fx)(5a-4b+b\cosh(2(e+fx)))\sqrt{4a-2b+2b\cosh(2(e+fx))}+\frac{3(a-b)^2\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{\sqrt{b}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((Cosh[e + f*x]*(5*a - 4*b + b*Cosh[2*(e + f*x)])*Sqrt[4*a - 2*b + 2*b*Cosh
[2*(e + f*x)]])/2 + (3*(a - b)^2*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2
*a - b + b*Cosh[2*(e + f*x)]]])/Sqrt[b])/(8*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(105) = 210$.
time = 0.93, size = 336, normalized size = 2.78

method	result
default	$\frac{\sqrt{(a+b(\sinh^2(fx+e)))(\cosh^2(fx+e))}\left(4b^{\frac{3}{2}}\sqrt{b(\cosh^4(fx+e))+(a-b)(\cosh^2(fx+e)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(4*b^(3/2)*(b*cosh(f*x+e)^4+
(a-b)*cosh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2-10*b^(3/2)*(b*cosh(f*x+e)^4+(a-b)*
cosh(f*x+e)^2)^(1/2)+10*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/
2)+3*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2
)*b^(1/2)+a-b)/b^(1/2))*a^2-6*b*a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e
)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))+3*b^2*ln(1/2*(2*b*cosh
(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2
)))/b^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1151 vs. $2(105) = 210$.
time = 0.52, size = 2977, normalized size = 24.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/64*(6*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(
f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2*sinh(f*x +
 e)^2 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b
+ b^2)*sinh(f*x + e)^4)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f
*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x
+ e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14
```

```
*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (
9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^
2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4
*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b -
4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cos
h(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)
^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x
+ e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5
 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 +
 a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sin
h(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18
*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f
*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*
cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^
3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*
x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x +
e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e
)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 6*((a^2 - 2*a*b
 + b^2)*cosh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3*sinh(f*x +
e) + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 2*a*b
 + b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^4
)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*s
inh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b
)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(
cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(
2)*(b^2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*
x + e)^6 + (10*a*b - 7*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 10*
a*b - 7*b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (10*a*b - 7*b^2)*
cosh(f*x + e))*sinh(f*x + e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e)^2 + (15*b^2
*cosh(f*x + e)^4 + 6*(10*a*b - 7*b^2)*cosh(f*x + e)^2 + 10*a*b - 7*b^2)*sin
h(f*x + e)^2 + b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(10*a*b - 7*b^2)*cosh(f*x
 + e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*x + e)^
3*sinh(f*x + e) + 6*b*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh(f*x +
e)*sinh(f*x + e)^3 + b*f*sinh(f*x + e)^4), -1/64*(12*((a^2 - 2*a*b + b^2)*c
osh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a
^2 - 2*a*b + b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 2*a*b + b^2)*c
osh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^4)*sqrt(-b
)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*s
```

```
inh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*
x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b -
 b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh
(f*x + e))*sinh(f*x + e))) + 12*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 4*(a
^2 - 2*a*b + b^2)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 2*a*b + b^2)*cos
h(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x
 + e)^3 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(cos
h(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b
*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x +
 e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +
 e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(2)*(b^2*cosh(f*
x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 897 vs. 2(105) = 210.
time = 0.76, size = 897, normalized size = 7.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b)*(b*e^(2*f*x + 6*e) + (10*a*b*e^(6*e) - 7*b^2*e^(6*e))*e^(-2*e)/b) - 24*(
a^2*e^(4*e) - 2*a*b*e^(4*e) + b^2*e^(4*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))
/sqrt(-b))/sqrt(-b) - 12*(a^2*sqrt(b)*e^(4*e) - 2*a*b^(3/2)*e^(4*e) + b^(5/
2)*e^(4*e))*log(abs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/2)))/b
 - 4*(10*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*e^(4*e) - 12*(sqrt(b)*e^(2*f*x + 2*
e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
```

```
))^3*a*b*e^(4*e) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^2*e^(4*e) + 8*(sqrt(b)*e^
(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b))^2*a*b^(3/2)*e^(4*e) - 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(5/2)*e^
(4*e) - 6*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
+ 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b*e^(4*e) + 8*(sqrt(b)*e^(2*f*x + 2*
e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
))*a*b^2*e^(4*e) - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^3*e^(4*e) - 4*a*b^(5/2)*e^(
4*e) + 3*b^(7/2)*e^(4*e))/((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - b)^2)*e^(-4*e)/f
```

Mupad **[F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(e+f\,x)\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.78 $\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}+\frac{(3a-b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2f}+b\cosh$$

```
[Out] -a^(3/2)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f+1/2*(3*
a-b)*arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2
*b*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3265, 427, 537, 223, 212, 385}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}+\frac{b\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}+\frac{\sqrt{b}\,(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Co
sh[e + f*x]^2]])/(2*f) + (b*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/
(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{1-x^2}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{b\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{-(a-b)(2}{(1-x^2)\sqrt{}}\right.}{}$$

$$= \frac{b\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} - \frac{a^2\text{Subst}\left(\int \frac{}{(1-x^2}\right.}{}$$

$$= \frac{b\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} - \frac{a^2\text{Subst}\left(\int \frac{1}{1-ax^2}\right.}{}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} + \frac{(3a-b)\sqrt{b}\ \text{t}}{}$$

Mathematica [A]
time = 0.38, size = 136, normalized size = 1.07

$$\frac{-4a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)+b\cosh(e+fx)\sqrt{4a-2b+2b\cosh(2(e+fx))}-2\sqrt{b}\,(-3a+b)\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-4*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2
*(e + f*x)]]] + b*Cosh[e + f*x]*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]] - 2
*Sqrt[b]*(-3*a + b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Co
sh[2*(e + f*x)]]])/(4*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(109) = 218$.
time = 1.25, size = 268, normalized size = 2.11

method	result
default	$-\frac{\sqrt{(a+b(\sinh^2(fx+e)))(\cosh^2(fx+e))}\left(b^{\frac{3}{2}}\ln\left(\frac{2b(\cosh^2(fx+e))+2\sqrt{b(\cosh^4(fx+e))+(a-}}{2\sqrt{b}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(b^(3/2)*ln(1/2*(2*b*cosh(f*
x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))+
2*a^(3/2)*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x
+e)^2)^(1/2)+a-b)/(cosh(f*x+e)^2-1))-3*b^(1/2)*a*ln(1/2*(2*b*cosh(f*x+e)^2+
2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-2*b*(b*
cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(
1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(109) = 218$.
time = 0.57, size = 5565, normalized size = 43.82

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x + e)
 + (3*a - b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*
cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cos
h(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 +
 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)
^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4
+ 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)
^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^
2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^
3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*
x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x
+ e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x +
e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e
))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^
4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*
cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh
(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*
a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^
2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(c
osh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(
f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f
*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) - 4*(a*cosh
(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2)*sqrt(a)*
log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (
a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*
x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*
x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x
+ e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*co
sh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + (
(3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x + e) + (3*a
- b)*sinh(f*x + e)^2)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
```

```
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cos
h(f*x + e)^2 + a - b)*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*
sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)) - sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e)
 + b*sinh(f*x + e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))
/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*sinh(f*x + e) + f*sinh(f*x + e)^2),
 1/8*(8*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x +
 e)^2)*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
 e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(
f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b
)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*
x + e) + b)) - ((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(
f*x + e) + (3*a - b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 +
8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^
2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x
+ e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(
f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x
 + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(
f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3
 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*
b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)
*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e
)^2)*sinh(f*x + e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*s
inh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cos
h(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*...
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x), x)
```

3.79 $\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{\sqrt{a}\,(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}-a\sqrt{a-}$$

```
[Out] b^(3/2)*arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f+1/2*(a-3
*b)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))*a^(1/2)/f-1/2*
a*coth(f*x+e)*csch(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3265, 424, 537, 223, 212, 385}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}+\frac{\sqrt{a}\,(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2f}-\frac{a\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(a - 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e +
f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*C
osh[e + f*x]^2]])/f - (a*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch
[e + f*x])/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a-b+bx^2\right)^{3/2}}{\left(1-x^2\right)^2}dx,x,\cosh(e+fx)\right)}{f}$$

$$=-\frac{a\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{2f}-$$

$$=-\frac{a\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{2f}+$$

$$=-\frac{a\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{2f}+$$

$$=\frac{\sqrt{a}\,(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2f}+$$

Mathematica [A]
time = 0.46, size = 143, normalized size = 1.10

$$\frac{2\sqrt{a}\,(a-3b)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)-a\sqrt{4a-2b+2b\cosh(2(e+fx))}\,\coth(e+fx)\mathrm{csch}(e+fx)+4b^{3/2}\log\left(\sqrt{2}\,\sqrt{b}\,\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*Sqrt[a]*(a - 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b +
 b*Cosh[2*(e + f*x)]]] - a*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]]*Coth[e +
 f*x]*Csch[e + f*x] + 4*b^(3/2)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*
a - b + b*Cosh[2*(e + f*x)]]])/(4*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(112) = 224.
time = 1.26, size = 297, normalized size = 2.28

method	result
default	$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\,\left(a^{\frac{3}{2}}\ln\left(\frac{(a+b)\left(\cosh^{2}(fx+e)\right)+2\sqrt{a}\,\sqrt{b\left(\cosh^{4}\left(fx+e\right)\right)+}}{\sinh(fx+e)^{2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(a^(3/2)*ln(((a+b)*cosh(f*x+e
)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^
2)*sinh(f*x+e)^2+2*b^(3/2)*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-
b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*sinh(f*x+e)^2-3*a^(1/2)*b*ln(
((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+
a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2-2*a*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(
1/2))/sinh(f*x+e)^2/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(112) = 224$.
time = 0.63, size = 6622, normalized size = 50.94

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
 e)^4 - 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 +
 4*(b*cosh(f*x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log((a^
2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*
x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 +
a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b
)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^
4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*
cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 +
a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x
+ e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e
)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b
^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 +
 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x
 + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f
*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x +
 e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sin
h(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*
a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(
f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*co
sh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x +
 e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)
^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 +
 sinh(f*x + e)^6)) - ((a - 3*b)*cosh(f*x + e)^4 + 4*(a - 3*b)*cosh(f*x + e)
*sinh(f*x + e)^3 + (a - 3*b)*sinh(f*x + e)^4 - 2*(a - 3*b)*cosh(f*x + e)^2
+ 2*(3*(a - 3*b)*cosh(f*x + e)^2 - a + 3*b)*sinh(f*x + e)^2 + 4*((a - 3*b)*
cosh(f*x + e)^3 - (a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt(a)
*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 +
(a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f
*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(
f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)
^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f
*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f
*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x
```

```
 + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*c
osh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) +
(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
- 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*
cosh(f*x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log(-(b*cosh(
f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a -
 b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqr
t(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1
)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x
+ e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - 2*sqrt(2)*(a*cosh(f*x + e)^2
 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x + e)*s
inh(f*x + e)^3 + f*sinh(f*x + e)^4 - 2*f*cosh(f*x + e)^2 + 2*(3*f*cosh(f*x
+ e)^2 - f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)^3 - f*cosh(f*x + e))*sinh(
f*x + e) + f), -1/4*(2*((a - 3*b)*cosh(f*x + e)^4 + 4*(a - 3*b)*cosh(f*x +
e)*sinh(f*x + e)^3 + (a - 3*b)*sinh(f*x + e)^4 - 2*(a - 3*b)*cosh(f*x + e)^
2 + 2*(3*(a - 3*b)*cosh(f*x + e)^2 - a + 3*b)*sinh(f*x + e)^2 + 4*((a - 3*b
)*cosh(f*x + e)^3 - (a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt(
-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(
b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x +
 e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))
 - (b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)
^4 - 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*
(b*cosh(f*x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log((a^2*b
*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^3,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^3, x)
```

3.80 $\int \textbf{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=135

$$-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8\sqrt{a}\,f}+\frac{3(a-b)\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\text{csch}(e}{8f}$$

```
[Out] -1/4*(a-b+b*cosh(f*x+e)^2)^(3/2)*coth(f*x+e)*csch(f*x+e)^3/f-3/8*(a-b)^2*ar
ctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f/a^(1/2)+3/8*(a-b)*
coth(f*x+e)*csch(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3265, 386, 385, 212}

$$-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8\sqrt{a}\,f}-\frac{\coth(e+fx)\text{csch}^3(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4f}+\frac{3(a-b)\coth(e+fx)\text{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a - b)^2*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^
2]])/(8*Sqrt[a]*f) + (3*(a - b)*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*
x]*Csch[e + f*x])/(8*f) - ((a - b + b*Cosh[e + f*x]^2)^(3/2)*Coth[e + f*x]*
Csch[e + f*x]^3)/(4*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{\left(1-x^2\right)^3}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\left(a-b+b\cosh^2(e+fx)\right)^{3/2}\coth(e+fx)\operatorname{csch}^3(e+fx)}{4f} -$$

$$= \frac{3(a-b)\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx}{8f}$$

$$= \frac{3(a-b)\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx}{8f}$$

$$= -\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8\sqrt{a}\,f} + \frac{3(a-b}{}$$

Mathematica [A]
time = 0.39, size = 123, normalized size = 0.91

$$\frac{-6(a-b)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)+\sqrt{2}\sqrt{a}\sqrt{2a-b+b\cosh(2(e+fx))}\,\coth(e+fx)\operatorname{csch}(e+fx)\left(3a-5b-2a\operatorname{csch}^2(e+fx)\right)}{16\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-6*(a - b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh
[2*(e + f*x)]]] + Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[
e + f*x]*Csch[e + f*x]*(3*a - 5*b - 2*a*Csch[e + f*x]^2))/(16*Sqrt[a]*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(119) = 238$.
time = 1.39, size = 379, normalized size = 2.81

method	result
default	$\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(-3a^2\ln\left(\frac{(a+b)\left(\cosh^2(fx+e)\right)+2\sqrt{a}\sqrt{b\left(\cosh^4\left(fx+e\right)\right)+}}{\sinh(fx+e)^2}\right)\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-3*a^2*ln(((a+b)*cosh(f*x+e
)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^
2)*sinh(f*x+e)^4+6*a*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(
a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^4-3*b^2*ln(((a+b)
*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/s
inh(f*x+e)^2)*sinh(f*x+e)^4+6*a^(3/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(
1/2)*sinh(f*x+e)^2-10*b*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*a^(1/2)*s
inh(f*x+e)^2-4*a^(3/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/a^(1/2)/s
inh(f*x+e)^4/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1515 vs. $2(119) = 238$.
time = 0.61, size = 3133, normalized size = 23.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*cosh(
f*x + e)*sinh(f*x + e)^7 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^8 - 4*(a^2 - 2
```

```
*a*b + b^2)*cosh(f*x + e)^6 + 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^
2 + 2*a*b - b^2)*sinh(f*x + e)^6 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3
 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(a^2 - 2*a*b +
b^2)*cosh(f*x + e)^4 + 2*(35*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 30*(a^2
- 2*a*b + b^2)*cosh(f*x + e)^2 + 3*a^2 - 6*a*b + 3*b^2)*sinh(f*x + e)^4 + 8
*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 10*(a^2 - 2*a*b + b^2)*cosh(f*x +
 e)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(a^2 - 2*a
*b + b^2)*cosh(f*x + e)^2 + 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 - 15*(
a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 9*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2
- a^2 + 2*a*b - b^2)*sinh(f*x + e)^2 + a^2 - 2*a*b + b^2 + 8*((a^2 - 2*a*b
+ b^2)*cosh(f*x + e)^7 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 3*(a^2 - 2
*a*b + b^2)*cosh(f*x + e)^3 - (a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f
*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a
 + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)
^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a
 - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x +
 e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x
+ e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x +
 e) + 1)) + 2*sqrt(2)*((3*a^2 - 5*a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - 5*a*b)*
cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - 5*a*b)*sinh(f*x + e)^6 - (11*a^2 -
 5*a*b)*cosh(f*x + e)^4 + (15*(3*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*a^2 + 5*
a*b)*sinh(f*x + e)^4 + 4*(5*(3*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a
*b)*cosh(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e)^2 + (15
*(3*a^2 - 5*a*b)*cosh(f*x + e)^4 - 6*(11*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*
a^2 + 5*a*b)*sinh(f*x + e)^2 + 3*a^2 - 5*a*b + 2*(3*(3*a^2 - 5*a*b)*cosh(f*
x + e)^5 - 2*(11*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x +
 e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*
cosh(f*x + e)^8 + 8*a*f*cosh(f*x + e)*sinh(f*x + e)^7 + a*f*sinh(f*x + e)^8
 - 4*a*f*cosh(f*x + e)^6 + 4*(7*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^6
+ 6*a*f*cosh(f*x + e)^4 + 8*(7*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*s
inh(f*x + e)^5 + 2*(35*a*f*cosh(f*x + e)^4 - 30*a*f*cosh(f*x + e)^2 + 3*a*f
)*sinh(f*x + e)^4 - 4*a*f*cosh(f*x + e)^2 + 8*(7*a*f*cosh(f*x + e)^5 - 10*a
*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a*f*cosh(f
*x + e)^6 - 15*a*f*cosh(f*x + e)^4 + 9*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x
+ e)^2 + a*f + 8*(a*f*cosh(f*x + e)^7 - 3*a*f*cosh(f*x + e)^5 + 3*a*f*cosh(
f*x + e)^3 - a*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(3*((a^2 - 2*a*b + b^2)
*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (a
^2 - 2*a*b + b^2)*sinh(f*x + e)^8 - 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 +
 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e
)^6 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e)^5 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 2*(35*
```

```
(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 30*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^
2 + 3*a^2 - 6*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(
f*x + e)^5 - 10*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)
*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 + 4
*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 - 15*(a^2 - 2*a*b + b^2)*cosh(f*x +
 e)^4 + 9*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x
 + e)^2 + a^2 - 2*a*b + b^2 + 8*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^7 - 3*(a
^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 -
 (a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a)*arctan(sqrt(2)*
(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqr
t(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 +
 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f
*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f
*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*((3*a^2
- 5*a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - 5*a*b)*cosh(f*x + e)*sinh(f*x + e)^5
+ (3*a^2 - 5*a*b)*sinh(f*x + e)^6 - (11*a^2 - 5*a*b)*cosh(f*x + e)^4 + (15*
(3*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*a^2 + 5*a*b)*sinh(f*x + e)^4 + 4*(5*(3
*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e))*sinh(f*x +
e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e)^2 + (15*(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2145 vs. $2(119) = 238$.
time = 0.90, size = 2145, normalized size = 15.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*(a^2 - 2*a*b + b^2)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^
(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/s
qrt(-a))/sqrt(-a) - 2*(3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^2 - 6*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^7*a*b - 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
```

```
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*b^2 - 21*(sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
+ b))^6*a^2*sqrt(b) - 22*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a*b^(3/2) + 35*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))^6*b^(5/2) - 44*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^3 - 105*(sqrt(
b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(
2*f*x + 2*e) + b))^5*a^2*b + 246*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^2 - 105*(sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))^5*b^3 - 292*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^3*sqrt(b) +
735*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
 - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2) - 690*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4
*a*b^(5/2) + 175*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*b^(7/2) - 176*(sqrt(b)*e^(2*f*x
 + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e
) + b))^3*a^4 + 936*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^3*b - 1623*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^3*a^2*b^2 + 950*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*b^3 - 175*(sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))^3*b^4 + 528*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^4*sqrt(b) - 1640*(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b
*e^(2*f*x + 2*e) + b))^2*a^3*b^(3/2) + 1761*(sqrt(b)*e^(2*f*x + 2*e) - sqrt
(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^2*
b^(5/2) - 714*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(7/2) + 105*(sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))^2*b^(9/2) + 192*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^5 - 912*(sqrt(b)*e^(2*f*x
 + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e
) + b))*a^4*b + 1540*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^3*b^2 - 963*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))*a^2*b^3 + 282*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b^4 - 35*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))*b^5 - 192*a^5*sqrt(b) + 560*a^4*b^(3/2) - 500*a^3*b^(5/2) + 213*a
^2*b^(7/2) - 46*a*b^(9/2) + 5*b^(11/2))/((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - 2*(sq
```

```
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))*sqrt(b) - 4*a + b)^4)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^5,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^5, x)
```

3.81 $\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$

Optimal. Leaf size=199

$$\frac{(a-b)^2(5a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{16a^{3/2}f}-\frac{(a-b)(5a+b)\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+}{16af}$$

```
[Out] 1/16*(a-b)^2*(5*a+b)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2
))/a^(3/2)/f+1/24*(5*a+b)*(a-b+b*cosh(f*x+e)^2)^(3/2)*coth(f*x+e)*csch(f*x+
e)^3/a/f-1/6*(a-b+b*cosh(f*x+e)^2)^(5/2)*coth(f*x+e)*csch(f*x+e)^5/a/f-1/16
*(a-b)*(5*a+b)*coth(f*x+e)*csch(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.13, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3265, 390, 386, 385, 212}

$$\frac{(a-b)^2(5a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{16a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}^5(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6af}+\frac{(5a+b)\coth(e+fx)\operatorname{csch}^3(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{24af}-\frac{(a-b)(5a+b)\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{16af}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^2*(5*a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e
+ f*x]^2]])/(16*a^(3/2)*f) - ((a - b)*(5*a + b)*Sqrt[a - b + b*Cosh[e + f*x
]^2]*Coth[e + f*x]*Csch[e + f*x])/(16*a*f) + ((5*a + b)*(a - b + b*Cosh[e +
 f*x]^2)^(3/2)*Coth[e + f*x]*Csch[e + f*x]^3)/(24*a*f) - ((a - b + b*Cosh[e
 + f*x]^2)^(5/2)*Coth[e + f*x]*Csch[e + f*x]^5)/(6*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
```

```
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \text{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{\left(1-x^2\right)^4}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\left(a-b+b\cosh^2(e+fx)\right)^{5/2}\coth(e+fx)\text{csch}^5(e+fx)}{6af} +$$

$$= \frac{(5a+b)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}\coth(e+fx)\text{csch}^3(e+}{24af}$$

$$= -\frac{(a-b)(5a+b)\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\text{cs}}{16af}$$

$$= -\frac{(a-b)(5a+b)\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\text{cs}}{16af}$$

$$= \frac{(a-b)^2(5a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{16a^{3/2}f} -$$

Mathematica [A]
time = 0.70, size = 174, normalized size = 0.87

$$\frac{\frac{(a-b)^2(5a+b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)}{a^{3/2}}-\frac{\sqrt{a-\frac{b}{2}+\frac{1}{2}b\cosh(2(e+fx))}\,(149a^2-122ab+9b^2-4(25a^2-36ab+3b^2)\cosh(2(e+fx))+(15a^2-22ab+3b^2)\cosh(4(e+fx)))\coth(e+fx)\operatorname{csch}^5(e+fx)}{24a}}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (((a - b)^2*(5*a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b
+ b*Cosh[2*(e + f*x)]]])/a^(3/2) - (Sqrt[a - b/2 + (b*Cosh[2*(e + f*x)])/2]
*(149*a^2 - 122*a*b + 9*b^2 - 4*(25*a^2 - 36*a*b + 3*b^2)*Cosh[2*(e + f*x)]
 + (15*a^2 - 22*a*b + 3*b^2)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]
^5)/(24*a))/(16*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 568 vs. 2(179) = 358.
time = 1.48, size = 569, normalized size = 2.86

$$-\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(30a^{\frac{7}{2}}\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(\sinh^4\left(f\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/96*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(30*a^(7/2)*((a+b*sinh(f*x+
e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^4-44*b*((a+b*sinh(f*x+e)^2)*cosh(f*x
+e)^2)^(1/2)*sinh(f*x+e)^4*a^(5/2)-15*a^4*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)
*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e
)^6+27*a^3*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(
f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^6-9*b^2*ln(((a+b)*cosh(f*x+
e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)
^2)*sinh(f*x+e)^6*a^2-3*b^3*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e
)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^6*a-20*a^(7/
2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^2+6*b^2*((a+b*sinh
(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^4*a^(3/2)+28*b*((a+b*sinh(f*x+e
)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^2*a^(5/2)+16*a^(7/2)*((a+b*sinh(f*x+e
)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^6/a^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+
e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^7, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3633 vs. $2(179) = 358$.
time = 0.97, size = 7369, normalized size = 37.03

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*((5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^12 + 12*(5*a^3 -
9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (5*a^3 - 9*a^2*b
+ 3*a*b^2 + b^3)*sinh(f*x + e)^12 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cos
h(f*x + e)^10 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 11*(5*a^3 - 9*a^2*b +
3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(5*a^3 - 9*a^2*b
+ 3*a*b^2 + b^3)*cosh(f*x + e)^3 - 3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh
(f*x + e))*sinh(f*x + e)^9 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x
+ e)^8 + 15*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 5*a^3 -
 9*a^2*b + 3*a*b^2 + b^3 - 18*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x +
e)^2)*sinh(f*x + e)^8 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x +
 e)^5 - 30*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 + 5*(5*a^3 - 9
*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^7 - 20*(5*a^3 - 9*a^2*
b + 3*a*b^2 + b^3)*cosh(f*x + e)^6 + 4*(231*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^
3)*cosh(f*x + e)^6 - 315*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4
- 25*a^3 + 45*a^2*b - 15*a*b^2 - 5*b^3 + 105*(5*a^3 - 9*a^2*b + 3*a*b^2 + b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 +
b^3)*cosh(f*x + e)^7 - 63*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5
 + 35*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 - 5*(5*a^3 - 9*a^2*
b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(5*a^3 - 9*a^2*b + 3
*a*b^2 + b^3)*cosh(f*x + e)^4 + 15*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*co
sh(f*x + e)^8 - 84*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^6 + 70*(
5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 5*a^3 - 9*a^2*b + 3*a*b^
2 + b^3 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x +
e)^4 + 20*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^9 - 36*(5*a^3
 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^7 + 42*(5*a^3 - 9*a^2*b + 3*a*b^2
 + b^3)*cosh(f*x + e)^5 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e
)^3 + 3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 +
5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(
f*x + e)^2 + 6*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^10 - 45*
(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^8 + 70*(5*a^3 - 9*a^2*b + 3
*a*b^2 + b^3)*cosh(f*x + e)^6 - 50*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f
```

```
*x + e)^4 - 5*a^3 + 9*a^2*b - 3*a*b^2 - b^3 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2
 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 12*((5*a^3 - 9*a^2*b + 3*a*b^2 +
 b^3)*cosh(f*x + e)^11 - 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^
9 + 10*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^7 - 10*(5*a^3 - 9*a^
2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5 + 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*
cosh(f*x + e)^3 - (5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x
 + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh
(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*
(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x +
e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3
*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x
 + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*
x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x
 + e) + 1)) - 2*sqrt(2)*((15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^10 + 1
0*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^9 + (15*a^3 - 2
2*a^2*b + 3*a*b^2)*sinh(f*x + e)^10 - (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f
*x + e)^8 - (85*a^3 - 122*a^2*b + 9*a*b^2 - 45*(15*a^3 - 22*a^2*b + 3*a*b^2
)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(15*a^3 - 22*a^2*b + 3*a*b^2)*co
sh(f*x + e)^3 - (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e))*sinh(f*x + e)
^7 + 2*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^6 + 2*(105*(15*a^3 - 22*
a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + 99*a^3 - 50*a^2*b + 3*a*b^2 - 14*(85*a^3
 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(15*a^3 -
22*a^2*b + 3*a*b^2)*cosh(f*x + e)^5 - 14*(85*a^3 - 122*a^2*b + 9*a*b^2)*cos
h(f*x + e)^3 + 3*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e)
^5 + 2*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + 2*(105*(15*a^3 - 22*
a^2*b + 3*a*b^2)*cosh(f*x + e)^6 - 35*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f
*x + e)^4 + 99*a^3 - 50*a^2*b + 3*a*b^2 + 15*(99*a^3 - 50*a^2*b + 3*a*b^2)*
cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(15*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh
(f*x + e)^7 - 7*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^5 + 5*(99*a^3
- 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^3 + (99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(
f*x + e))*sinh(f*x + e)^3 + 15*a^3 - 22*a^2*b + 3*a*b^2 - (85*a^3 - 122*a^2
*b + 9*a*b^2)*cosh(f*x + e)^2 + (45*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x
+ e)^8 - 28*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^6 + 30*(99*a^3 - 5
0*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 - 85*a^3 + 1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**7*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4684 vs. $2(179) = 358$.
time = 1.69, size = 4684, normalized size = 23.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*arctan(-1/2*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b) - sqrt(b))/sqrt(-a))/(sqrt(-a)*a) - 2*(15*(sqrt(b)*e^(2*f*x + 2*e) - sq
rt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^11*a
^3 - 27*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b))^11*a^2*b + 9*(sqrt(b)*e^(2*f*x + 2*e) - sq
rt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^11*a
*b^2 + 3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
 2*e) - 2*b*e^(2*f*x + 2*e) + b))^11*b^3 - 165*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^10*
a^3*sqrt(b) + 297*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^10*a^2*b^(3/2) + 93*(sqrt(b)*e^(
2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
+ 2*e) + b))^10*a*b^(5/2) - 33*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^10*b^(7/2) - 340*(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b
*e^(2*f*x + 2*e) + b))^9*a^4 + 1437*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^9*a^3*b + 615*
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b))^9*a^2*b^2 - 1237*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b
*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^9*a*b^3
+ 165*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*
e) - 2*b*e^(2*f*x + 2*e) + b))^9*b^4 + 3060*(sqrt(b)*e^(2*f*x + 2*e) - sqrt
(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^8*a^4*
sqrt(b) + 4305*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^8*a^3*b^(3/2) - 11373*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))^8*a^2*b^(5/2) + 5463*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^8*a*b^(7/2) - 495
*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
2*b*e^(2*f*x + 2*e) + b))^8*b^(9/2) + 3168*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(
b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^5 +
 16464*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^4*b - 49146*(sqrt(b)*e^(2*f*x + 2*e) -
sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*
a^3*b^2 + 43362*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
```

```
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^2*b^3 - 13302*(sqrt(b)*e^(2*f*
x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*
e) + b))^7*a*b^4 + 990*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*b^5 + 26976*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))^6*a^5*sqrt(b) - 106832*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f
*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a^4*b^(3/2) +
 148926*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a^3*b^(5/2) - 86646*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))^6*a^2*b^(7/2) + 20370*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a*b^(9/2) - 1386*(sqr
t(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e
^(2*f*x + 2*e) + b))^6*b^(11/2) + 12672*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^6 - 11
3568*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^5*a^5*b + 255432*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a
^4*b^2 - 243990*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^3*b^3 + 106302*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^5*a^2*b^4 - 20538*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*
e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^5 + 1386*(sqrt(b
)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b))^5*b^6 - 63360*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^6*sqrt(b) + 24
8928*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^4*a^5*b^(3/2) - 373128*(sqrt(b)*e^(2*f*x + 2*
e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
))^4*a^4*b^(5/2) + 253458*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + ...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^7,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^7, x)
```

3.82 $\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=367

$$\frac{(a^2-11ab+8b^2)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf}+\frac{2(4a-3b)\cosh(e+fx)\sinh^3(e+fx}{35f}$$

```
[Out] 1/35*(a^2-11*a*b+8*b^2)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b
/f+2/35*(4*a-3*b)*cosh(f*x+e)*sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/7
*b*cosh(f*x+e)*sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2)/f+2/35*(a-2*b)*(a^2+
4*a*b-4*b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(
sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/35*(a^2-1
1*a*b+8*b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(
sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/35*(a-2*b)*
(a^2+4*a*b-4*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Rubi [A]
time = 0.32, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3267, 488, 596, 545, 429, 506, 422}

$$-\frac{(a^2-11ab+8b^2)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\operatorname{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{35bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2(a-2b)\left(a^2+4ab-4b^2\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{35b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a-2b)\left(a^2+4ab-4b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35b^2f}+\frac{(a^2-11ab+8b^2)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf}+\frac{b\sinh^5(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{7f}+\frac{2(4a-3b)\sinh^3(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a^2 - 11*a*b + 8*b^2)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x
]^2])/(35*b*f) + (2*(4*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]^3*Sqrt[a + b*Si
nh[e + f*x]^2])/(35*f) + (b*Cosh[e + f*x]*Sinh[e + f*x]^5*Sqrt[a + b*Sinh[e
 + f*x]^2])/(7*f) + (2*(a - 2*b)*(a^2 + 4*a*b - 4*b^2)*EllipticE[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b^2*f*
Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a^2 - 11*a*b + 8*b^2
)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(35*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2
*(a - 2*b)*(a^2 + 4*a*b - 4*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])
/(35*b^2*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{x^4\left(a+bx^2\right)^{3/2}}{\sqrt{1+x^2}}\,dx,x\right.}{f}$$

$$= \frac{b\cosh(e+fx)\sinh^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{7f} + \left(\sqrt{\right.$$

$$= \frac{2(4a-3b)\cosh(e+fx)\sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx}}{35f}$$

$$= \frac{\left(a^2-11ab+8b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e}}{35bf}$$

$$= \frac{\left(a^2-11ab+8b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e}}{35bf}$$

$$= \frac{\left(a^2-11ab+8b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e}}{35bf}$$

$$= \frac{\left(a^2-11ab+8b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e}}{35bf}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.91, size = 262, normalized size = 0.71

$$\frac{128ia\left(a^3+2a^2b-12ab^2+8b^3\right)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-64ia\left(2a^3+3a^2b-13ab^2+8b^3\right)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,b\left(32a^3-496a^2b+684ab^2-250b^3+b\left(144a^2-480ab+299b^2\right)\cosh(2(e+fx))+2(26a-27b)b^2\cosh(4(e+fx))+5b^3\cosh(6(e+fx))\right)\sinh(2(e+fx))}{2240b^2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((128*I)*a*(a^3 + 2*a^2*b - 12*a*b^2 + 8*b^3)*Sqrt[(2*a - b + b*Cosh[2*(e +
 f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (64*I)*a*(2*a^3 + 3*a^2*b - 13*a*b
^2 + 8*b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x),
b/a] + Sqrt[2]*b*(32*a^3 - 496*a^2*b + 684*a*b^2 - 250*b^3 + b*(144*a^2 - 4
80*a*b + 299*b^2)*Cosh[2*(e + f*x)] + 2*(26*a - 27*b)*b^2*Cosh[4*(e + f*x)]
 + 5*b^3*Cosh[6*(e + f*x)])*Sinh[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a - b + b
*Cosh[2*(e + f*x)]])
```

Maple [**A**]
time = 1.40, size = 743, normalized size = 2.02

method	result
default	$5\sqrt{-\frac{b}{a}}\, b^{3}\left(\cosh^{8}\left(f x+e\right)\right) \sinh\left(f x+e\right)+\left(13\sqrt{-\frac{b}{a}}\, a\, b^{2}-21\sqrt{-\frac{b}{a}}\, b^{3}\right)\left(\cosh^{6}\left(f x+e\right)\right) \sinh\left(f x+e\right)+\left(9\sqrt{-\frac{b}{a}}\, a^{2} b-43\sqrt{-\frac{b}{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(5*(-1/a*b)^(1/2)*b^3*cosh(f*x+e)^8*sinh(f*x+e)+(13*(-1/a*b)^(1/2)*a*b
^2-21*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^6*sinh(f*x+e)+(9*(-1/a*b)^(1/2)*a^2*b
-43*(-1/a*b)^(1/2)*a*b^2+35*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^4*sinh(f*x+e)+(
(-1/a*b)^(1/2)*a^3-20*(-1/a*b)^(1/2)*a^2*b+38*(-1/a*b)^(1/2)*a*b^2-19*(-1/a
*b)^(1/2)*b^3)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3
+15*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-32*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a*b^2+16*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(
sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)
)*a^3-4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+24*(b/a*cosh(f*x+e)^2+(a-b)/a)
^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*b^2-16*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)/b/(-1/a*b)^(1/2)/cosh(f*x+
e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.11, size = 38, normalized size = 0.10

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^{6}+a\sinh\left(fx+e\right)^{4}\right)\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^6 + a*sinh(f*x + e)^4)*sqrt(b*sinh(f*x + e)^2 + a
), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{32,[4,2,4]%%%}+%%%{%%%{-64,[1]%%%},[4,2,3]%%%}+%%%{%%%{32,[2]
%%%},[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh(e+f\,x)^4\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.83 $\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=236

$$\frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}+\frac{\cosh(e+fx)\sinh(e+fx)\,(a+b\sinh^2(e+fx)}{5f}$$

```
[Out] 1/5*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f+1/15*(3*a-4*b)*cosh
(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/15*I*(3*a^2-13*a*b+8*b^2)
*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/
2))*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(1+b*sinh(f*x+e)^2/a)^(1/2)+1/15*I*a*(3*a
-4*b)*(a-b)*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x
),(b/a)^(1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$,
Rules used = {3249, 3251, 3257, 3256, 3262, 3261}

$$-\frac{i(3a^2-13ab+8b^2)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}+\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}+\frac{(3a-4b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}+\frac{ia(3a-4b)(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{15bf\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f
) + (Cosh[e + f*x]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(5*f) - ((I
/15)*(3*a^2 - 13*a*b + 8*b^2)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/15)*a*(3*a - 4*b)*(
a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sq
rt[a + b*Sinh[e + f*x]^2])
```

Rule 3249

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e
 + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p
 + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Gt
Q[p, 0]
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx &= \frac{\cosh(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}-\frac{1}{5}\int (\\
&= \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}+\\
&= \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}+\\
&= \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}+\\
&= \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}+
\end{aligned}
$$

Mathematica [A]
time = 0.94, size = 213, normalized size = 0.90

$$
\frac{-16ia(3a^2-13ab+8b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+16ia(3a^2-7ab+4b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,b(48a^2-68ab+25b^2+4(9a-7b)b\cosh(2(e+fx))+3b^2\cosh(4(e+fx)))\sinh(2(e+fx))}{240bf\sqrt{2a-b+b\cosh(2(e+fx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-16*I)*a*(3*a^2 - 13*a*b + 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]
*EllipticE[I*(e + f*x), b/a] + (16*I)*a*(3*a^2 - 7*a*b + 4*b^2)*Sqrt[(2*a -
 b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(48*a^
2 - 68*a*b + 25*b^2 + 4*(9*a - 7*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e +
 f*x)])*Sinh[2*(e + f*x)])/(240*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.18, size = 535, normalized size = 2.27

method	result

default	$3\sqrt{-\frac{b}{a}}\, b^2(\cosh^6(fx+e))\sinh(fx+e)+\bigg(9\sqrt{-\frac{b}{a}}\, ab-10\sqrt{-\frac{b}{a}}\, b^2\bigg)(\cosh^4(fx+e))\sinh(fx+e)+\bigg(6\sqrt{-\frac{b}{a}}\, a^2-13\sqrt{-\frac{b}{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(3*(-1/a*b)^(1/2)*b^2*cosh(f*x+e)^6*sinh(f*x+e)+(9*(-1/a*b)^(1/2)*a*b-
10*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(6*(-1/a*b)^(1/2)*a^2-13*(
-1/a*b)^(1/2)*a*b+7*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)-9*a^2*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))+17*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*
x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-8*(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*b^2+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-13*(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*a*b+8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/(-1/a*b)^
(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.12, size = 38, normalized size = 0.16

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^4+a\sinh\left(fx+e\right)^2\right)\sqrt{b\sinh\left(fx+e\right)^2+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^4 + a*sinh(f*x + e)^2)*sqrt(b*sinh(f*x + e)^2 + a
), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh(e+f\,x)^2\,{\left(b\,\sinh(e+f\,x)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.84 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=174

$$\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}+ ia(a$$

```
[Out] 1/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-2/3*I*(2*a-b)*(co
s(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*
(a+b*sinh(f*x+e)^2)^(1/2)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)+1/3*I*a*(a-b)*(cos(
I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))*(1
+b*sinh(f*x+e)^2/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3259, 3251, 3257, 3256, 3262, 3261}

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)
/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b
/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a-b)+2(2}{\sqrt{a+b}}$$

$$=\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{1}{3}(a(a-b))\int \frac{}{\sqrt{a}}$$

$$=\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{\Big(2(2a-b)\sqrt{a+b\mathrm{s}}}{}$$

$$=\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)E\big(ie+if}{3f\sqrt{1+}}$$

Mathematica [A]
time = 0.54, size = 169, normalized size = 0.97

$$\frac{-4i\sqrt{2}\,a(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+2i\sqrt{2}\,a(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+b(2a-b+b\cosh(2(e+fx)))\sinh(2(e+fx))}{6f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])
*Sinh[2*(e + f*x)])/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.10, size = 428, normalized size = 2.46

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\,b^2\left(\cosh^4(fx+e)\right)\sinh(fx+e)+\sqrt{-\frac{b}{a}}\,ab\left(\cosh^2(fx+e)\right)\sinh(fx+e)-\sqrt{-\frac{b}{a}}\,b^2\left(\cosh^2(fx+e)\right)\sinh(fx+e)+3a^2\sqrt{\frac{b(}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*cosh(f*x+e)^4*sinh(f*x+e)+(-1/a*b)^(1/2)*a*b*cosh(f
*x+e)^2*sinh(f*x+e)-(-1/a*b)^(1/2)*b^2*cosh(f*x+e)^2*sinh(f*x+e)+3*a^2*(b/a
*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))-5*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+2*(b/a*cosh
(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*b^2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b^2)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1
/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.10, size = 16, normalized size = 0.09

$$\text{integral}\Big(\big(b\sinh\left(fx+e\right)^2+a\big)^{\frac{3}{2}},x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.85 $\int \mathbf{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=204

$$-\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{(a+b)E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2}}{f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] -a*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-(a+b)*(1/(1+sinh(f*x+e)^2))^(1/2
)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-
b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sin
h(f*x+e)^2)/a)^(1/2)+2*b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2
)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*
(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(a+
b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.15, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3267, 485, 545, 429, 506, 422}

$$\frac{2b\,\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - ((a + b)*EllipticE[Arc
Tan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*
Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*b*EllipticF[ArcTan[
Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*Sqrt[a + b*Sinh[e
 + f*x]^2]*Tanh[e + f*x])/f
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\Bigg(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\Bigg)\operatorname{Subst}\Bigg(\int\frac{(a+bx^2)^{3/2}}{x^2\sqrt{1+x^2}}\,dx, x}{f}$$

$$= -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\Bigg(\sqrt{\cosh^2(e+fx)}}{}$$

$$= -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\Bigg(2ab\sqrt{\cosh^2(e+}}{}$$

$$= -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{2bF\big(\tan^{-1}(\sinh(e}{f\sqrt{}}$$

$$= -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{(a+b)E\big(\tan^{-1}(\mathrm{si}}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.74, size = 155, normalized size = 0.76

$$-\frac{a\left(\sqrt{2}\,(2a-b+b\cosh(2(e+fx)))\coth(e+fx)+2i(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2i(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)\right)}{2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(a*(Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Coth[e + f*x] + (2*I)*(a +
 b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (
2*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x),
 b/a]))/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.14, size = 243, normalized size = 1.19

method	result

default	$\frac{-\sqrt{-\frac{b}{a}}\, ab(\cosh^4(fx+e))+\left(-\sqrt{-\frac{b}{a}}\, a^2+\sqrt{-\frac{b}{a}}\, ab\right)(\cosh^2(fx+e))+\sinh(fx+e)\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\, \sqrt{\frac{b(\cosh^2(fx-}{a}}}{\sinh(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-1/a*b)^(1/2)*a*b*cosh(f*x+e)^4+(-(-1/a*b)^(1/2)*a^2+(-1/a*b)^(1/2)*a*b)
*cosh(f*x+e)^2+sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a
)^(1/2)*b*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a-EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2
),(a/b)^(1/2))*a+EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b))/sinh
(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^2, x)
```

Fricas [F]
time = 0.10, size = 46, normalized size = 0.23

$$\operatorname{integral}\left(\left(b\operatorname{csch}\left(fx+e\right)^{2}\sinh\left(fx+e\right)^{2}+a\operatorname{csch}\left(fx+e\right)^{2}\right)\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*csch(f*x + e)^2*sinh(f*x + e)^2 + a*csch(f*x + e)^2)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{64,[4,6,4]%%%}+%%%{%%%{-128,[1]%%%},[4,6,3]%%%}+%%%{%%%{64,[2
]%%%},
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^2,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^2, x)
```

3.86 $\int \mathbf{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=267

$$\frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{a\coth(e+fx)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+2(a-$$

```
[Out] 2/3*(a-2*b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*a*coth(f*x+e)*csch(
f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/f+2/3*(a-2*b)*(1/(1+sinh(f*x+e)^2))^(1/2
)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-
b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sin
h(f*x+e)^2)/a)^(1/2)-1/3*(a-3*b)*b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*se
ch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/
a)^(1/2)-2/3*(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.21, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3267, 485, 597, 545, 429, 506, 422}

$$-\frac{b(a-3b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2(a-2b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{a\coth(e+fx)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(a - 2*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (a*Coth[e +
 f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + (2*(a - 2*b)*Ell
ipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a - 3*b)
*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2
*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2))))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
```

```
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{x^4\sqrt{1+x^2}}\,dx,\right.}{f}$$

$$= -\frac{a\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} +$$

$$= \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - a\coth(e+$$

$$= \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - a\coth(e+$$

$$= \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - a\coth(e+$$

$$= \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - a\coth(e+$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.66, size = 213, normalized size = 0.80

$$\frac{\frac{(-8a^2+13ab-6b^2+2(2a^2-7ab+4b^2)\cosh(2(e+fx))+(a-2b)b\cosh(4(e+fx)))\coth(e+fx)\operatorname{csch}^2(e+fx)}{\sqrt{2}}+4ia(a-2b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2i(2a^2-5ab+3b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{6f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (((-8*a^2 + 13*a*b - 6*b^2 + 2*(2*a^2 - 7*a*b + 4*b^2)*Cosh[2*(e + f*x)] +
(a - 2*b)*b*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] + (4*
I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x
), b/a] - (2*I)*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)]
)/a]*EllipticF[I*(e + f*x), b/a])/(6*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [**A**]
time = 1.34, size = 454, normalized size = 1.70

method	result
default	$\dfrac{2\sqrt{-\frac{b}{a}}\,ab(\sinh^6(fx+e))-4\sqrt{-\frac{b}{a}}\,b^2(\sinh^6(fx+e))+b\sqrt{\frac{a+b(\sinh^2(fx+e))}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\Big(\sinh(fx+e)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+
b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))*a*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3+4*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4-3*(-1/
a*b)^(1/2)*a*b*sinh(f*x+e)^4-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+(-1/a*b)^(1
/2)*a^2*sinh(f*x+e)^2-5*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2
)/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^4, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2222 vs. 2(271) = 542.
time = 0.14, size = 2222, normalized size = 8.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(((2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^6 + 6*(2*a^2 - 5*a*b + 2*b^2)*
cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2 - 5*a*b + 2*b^2)*sinh(f*x + e)^6 - 3
```

```
*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^4 + 3*(5*(2*a^2 - 5*a*b + 2*b^2)*cos
h(f*x + e)^2 - 2*a^2 + 5*a*b - 2*b^2)*sinh(f*x + e)^4 + 4*(5*(2*a^2 - 5*a*b
 + 2*b^2)*cosh(f*x + e)^3 - 3*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e))*sinh(f
*x + e)^3 + 3*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^2 + 3*(5*(2*a^2 - 5*a*b
 + 2*b^2)*cosh(f*x + e)^4 - 6*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^2 + 2*a
^2 - 5*a*b + 2*b^2)*sinh(f*x + e)^2 - 2*a^2 + 5*a*b - 2*b^2 + 6*((2*a^2 - 5
*a*b + 2*b^2)*cosh(f*x + e)^5 - 2*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^3 +
 (2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b - 2*b^2)*co
sh(f*x + e)^6 + 6*(a*b - 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b - 2*b^
2)*sinh(f*x + e)^6 - 3*(a*b - 2*b^2)*cosh(f*x + e)^4 + 3*(5*(a*b - 2*b^2)*c
osh(f*x + e)^2 - a*b + 2*b^2)*sinh(f*x + e)^4 + 4*(5*(a*b - 2*b^2)*cosh(f*x
 + e)^3 - 3*(a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(a*b - 2*b^2)*
cosh(f*x + e)^2 + 3*(5*(a*b - 2*b^2)*cosh(f*x + e)^4 - 6*(a*b - 2*b^2)*cosh
(f*x + e)^2 + a*b - 2*b^2)*sinh(f*x + e)^2 - a*b + 2*b^2 + 6*((a*b - 2*b^2)
*cosh(f*x + e)^5 - 2*(a*b - 2*b^2)*cosh(f*x + e)^3 + (a*b - 2*b^2)*cosh(f*x
 + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 -
a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*
a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - ((2*a^2 - 7*a*b + 3*b^2)*cosh(f*x
+ e)^6 + 6*(2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2 -
 7*a*b + 3*b^2)*sinh(f*x + e)^6 - 3*(2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e)^4
 + 3*(5*(2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e)^2 - 2*a^2 + 7*a*b - 3*b^2)*si
nh(f*x + e)^4 + 4*(5*(2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e)^3 - 3*(2*a^2 - 7
*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(2*a^2 - 7*a*b + 3*b^2)*co
sh(f*x + e)^2 + 3*(5*(2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e)^4 - 6*(2*a^2 - 7
*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a^2 - 7*a*b + 3*b^2)*sinh(f*x + e)^2 - 2*
a^2 + 7*a*b - 3*b^2 + 6*((2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e)^5 - 2*(2*a^2
 - 7*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e))*
sinh(f*x + e) - 2*((a*b - b^2)*cosh(f*x + e)^6 + 6*(a*b - b^2)*cosh(f*x + e
)*sinh(f*x + e)^5 + (a*b - b^2)*sinh(f*x + e)^6 - 3*(a*b - b^2)*cosh(f*x +
e)^4 + 3*(5*(a*b - b^2)*cosh(f*x + e)^2 - a*b + b^2)*sinh(f*x + e)^4 + 4*(5
*(a*b - b^2)*cosh(f*x + e)^3 - 3*(a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3
 + 3*(a*b - b^2)*cosh(f*x + e)^2 + 3*(5*(a*b - b^2)*cosh(f*x + e)^4 - 6*(a*
b - b^2)*cosh(f*x + e)^2 + a*b - b^2)*sinh(f*x + e)^2 - a*b + b^2 + 6*((a*b
 - b^2)*cosh(f*x + e)^5 - 2*(a*b - b^2)*cosh(f*x + e)^3 + (a*b - b^2)*cosh(
f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2
 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2
) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*
(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*((a*b - 2*b^2)*cosh(f*x
 + e)^5 + 5*(a*b - 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^4 + (a*b - 2*b^2)*sin
h(f*x + e)^5 - (3*a*b - 4*b^2)*cosh(f*x + e)^3 + (10*(a*b - 2*b^2)*cosh(f*x
 + e)^2 - 3*a*b + 4*b^2)*sinh(f*x + e)^3 - 2*b^2*cosh(f*x + e) + (10*(a*b -
 2*b^2)*cosh(f*x + e)^3 - 3*(3*a*b - 4*b^2)*cosh(f*x + e))*sinh(f*x + e)^2
+ (5*(a*b - 2*b^2)*cosh(f*x + e)^4 - 3*(3*a*b - 4*b^2)*cosh(f*x + e)^2 - 2*
b^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
```

```
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*
cosh(f*x + e)^6 + 6*b*f*cosh(f*x + e)*sinh(f*x + e)^5 + b*f*sinh(f*x + e)^6
 - 3*b*f*cosh(f*x + e)^4 + 3*(5*b*f*cosh(f*x + e)^2 - b*f)*sinh(f*x + e)^4
+ 3*b*f*cosh(f*x + e)^2 + 4*(5*b*f*cosh(f*x + e)^3 - 3*b*f*cosh(f*x + e))*s
inh(f*x + e)^3 + 3*(5*b*f*cosh(f*x + e)^4 - 6*b*f*cosh(f*x + e)^2 + b*f)*si
nh(f*x + e)^2 - b*f + 6*(b*f*cosh(f*x + e)^5 - 2*b*f*cosh(f*x + e)^3 + b*f*
cosh(f*x + e))*sinh(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.54Unable to divide
, perhaps due to rounding error%%%{1024,[8,10,8]%%%}+%%%{%%%{-4096,[1]%%%},
[8,10,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\sinh(e+f\,x)^2+a\right)}^{3/2}}{{\sinh\left(e+f\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^4,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^4, x)
```

3.87 $\int \left(a+b\sinh^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=232

$$\frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d}+\frac{b\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+d\right.}{5d}$$

```
[Out] 1/5*b*cosh(d*x+c)*sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^(3/2)/d+4/15*(2*a-b)*b*co
sh(d*x+c)*sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^(1/2)/d-1/15*I*(23*a^2-23*a*b+8*b
^2)*(cos(I*c+I*d*x)^2)^(1/2)/cos(I*c+I*d*x)*EllipticE(sin(I*c+I*d*x),(b/a)^
(1/2))*(a+b*sinh(d*x+c)^2)^(1/2)/d/(1+b*sinh(d*x+c)^2/a)^(1/2)+4/15*I*a*(a-
b)*(2*a-b)*(cos(I*c+I*d*x)^2)^(1/2)/cos(I*c+I*d*x)*EllipticF(sin(I*c+I*d*x)
,(b/a)^(1/2))*(1+b*sinh(d*x+c)^2/a)^(1/2)/d/(a+b*sinh(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3259, 3249, 3251, 3257, 3256, 3262, 3261}

$$-\frac{i(23a^2-23ab+8b^2)\sqrt{a+b\sinh^2(c+dx)}\,E\left(ic+idx\middle|\frac{b}{a}\right)}{15d\sqrt{\frac{b\sinh^2(c+dx)}{a}+1}}+\frac{b\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^{3/2}}{5d}+\frac{4b(2a-b)\sinh(c+dx)\cosh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d}+\frac{4ia(a-b)(2a-b)\sqrt{\frac{b\sinh^2(c+dx)}{a}+1}\,F\left(ic+idx\middle|\frac{b}{a}\right)}{15d\sqrt{a+b\sinh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^(5/2),x]
```

```
[Out] (4*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x]*Sqrt[a + b*Sinh[c + d*x]^2])/(15
*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^(3/2))/(5*d) -
 ((I/15)*(23*a^2 - 23*a*b + 8*b^2)*EllipticE[I*c + I*d*x, b/a]*Sqrt[a + b*S
inh[c + d*x]^2])/(d*Sqrt[1 + (b*Sinh[c + d*x]^2)/a]) + (((4*I)/15)*a*(a - b
)*(2*a - b)*EllipticF[I*c + I*d*x, b/a]*Sqrt[1 + (b*Sinh[c + d*x]^2)/a])/(d
*Sqrt[a + b*Sinh[c + d*x]^2])
```

Rule 3249

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e
 + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p
 + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Gt
Q[p, 0]
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^{5/2}\,dx=\frac{b\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^{3/2}}{5d}+\frac{1}{5}\int\sqrt{a+b\sinh^2(}$$

$$=\frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d}+\frac{b\cosh(c+}{}$$

$$=\frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d}+\frac{b\cosh(c+}{}$$

$$=\frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d}+\frac{b\cosh(c+}{}$$

$$=\frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d}+\frac{b\cosh(c+}{}$$

Mathematica [A]
time = 0.95, size = 208, normalized size = 0.90

$$\frac{-16ia(23a^2-23ab+8b^2)\sqrt{\frac{2a-b+b\cosh(2(c+dx))}{a}}E\left(i(c+dx)\,\middle|\,\frac{b}{a}\right)+64ia(2a^2-3ab+b^2)\sqrt{\frac{2a-b+b\cosh(2(c+dx))}{a}}F\left(i(c+dx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,b(88a^2-88ab+25b^2+28(2a-b)b\cosh(2(c+dx))+3b^2\cosh(4(c+dx)))\sinh(2(c+dx))}{240d\sqrt{2a-b+b\cosh(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^(5/2),x]
```

```
[Out] ((-16*I)*a*(23*a^2 - 23*a*b + 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(c + d*x)])/a
]*EllipticE[I*(c + d*x), b/a] + (64*I)*a*(2*a^2 - 3*a*b + b^2)*Sqrt[(2*a -
b + b*Cosh[2*(c + d*x)])/a]*EllipticF[I*(c + d*x), b/a] + Sqrt[2]*b*(88*a^2
 - 88*a*b + 25*b^2 + 28*(2*a - b)*b*Cosh[2*(c + d*x)] + 3*b^2*Cosh[4*(c + d
*x)])*Sinh[2*(c + d*x)])/(240*d*Sqrt[2*a - b + b*Cosh[2*(c + d*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(268) = 536$.
time = 1.40, size = 609, normalized size = 2.62

---

method	result
default	$3\sqrt{-\frac{b}{a}}\, b^3 (\cosh^6(dx+c)) \sinh(dx+c)+\left(14\sqrt{-\frac{b}{a}}\, a\, b^2-10\sqrt{-\frac{b}{a}}\, b^3\right)(\cosh^4(dx+c)) \sinh(dx+c)+\left(11\sqrt{-\frac{b}{a}}\, a^2 b-18\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(3*(-1/a*b)^(1/2)*b^3*cosh(d*x+c)^6*sinh(d*x+c)+(14*(-1/a*b)^(1/2)*a*b
^2-10*(-1/a*b)^(1/2)*b^3)*cosh(d*x+c)^4*sinh(d*x+c)+(11*(-1/a*b)^(1/2)*a^2*
b-18*(-1/a*b)^(1/2)*a*b^2+7*(-1/a*b)^(1/2)*b^3)*cosh(d*x+c)^2*sinh(d*x+c)+1
5*a^3*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/2)*(cosh(d*x+c)^2)^(1/2)*EllipticF(sin
h(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))-34*a^2*b*(b/a*cosh(d*x+c)^2+(a-b)/a)^(
1/2)*(cosh(d*x+c)^2)^(1/2)*EllipticF(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2)
)+27*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/2)*(cosh(d*x+c)^2)^(1/2)*EllipticF(sinh
(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2-8*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/
2)*(cosh(d*x+c)^2)^(1/2)*EllipticF(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))*
b^3+23*a^2*b*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/2)*(cosh(d*x+c)^2)^(1/2)*Ellipt
icE(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))-23*(b/a*cosh(d*x+c)^2+(a-b)/a)^
(1/2)*(cosh(d*x+c)^2)^(1/2)*EllipticE(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2
))*a*b^2+8*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/2)*(cosh(d*x+c)^2)^(1/2)*Elliptic
E(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)/(-1/a*b)^(1/2)/cosh(d*x+c)/(
a+b*sinh(d*x+c)^2)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^(5/2), x)
```

Fricas [F]
time = 0.15, size = 45, normalized size = 0.19

$$\operatorname{integral}\left(\left(b^2\sinh\left(dx+c\right)^4+2\,ab\sinh\left(dx+c\right)^2+a^2\right)\sqrt{b\sinh\left(dx+c\right)^2+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sinh(d*x + c)^4 + 2*a*b*sinh(d*x + c)^2 + a^2)*sqrt(b*sinh(d*
x + c)^2 + a), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{sinh}\left(c+d\,x\right)}^2+a\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^(5/2),x)
```

```
[Out] int((a + b*sinh(c + d*x)^2)^(5/2), x)
```

3.88 $\int \sqrt{1+\sinh^2(x)}\,dx$

Optimal. Leaf size=11

$$\sqrt{\cosh^2(x)}\,\tanh(x)$$

```
[Out] (cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3255, 3286, 2717}

$$\sqrt{\cosh^2(x)}\,\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sinh[x]^2],x]
```

```
[Out] Sqrt[Cosh[x]^2]*Tanh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+\sinh^2(x)}\,dx &= \int \sqrt{\cosh^2(x)}\,dx\\ &= \left(\sqrt{\cosh^2(x)}\,\mathrm{sech}(x)\right)\int \cosh(x)\,dx\\ &= \sqrt{\cosh^2(x)}\,\tanh(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\sqrt{\cosh^2(x)}\,\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sinh[x]^2],x]
```

```
[Out] Sqrt[Cosh[x]^2]*Tanh[x]
```

Maple [A]
time = 0.81, size = 14, normalized size = 1.27

method	result	size
default	$\frac{\sinh(x)\sqrt{\frac{1}{2}+\frac{\cosh(2x)}{2}}}{\cosh(x)}$	14
risch	$\frac{\sqrt{\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}\,\mathrm{e}^{2x}}{2+2\,\mathrm{e}^{2x}}-\frac{\sqrt{\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}{2(1+\mathrm{e}^{2x})}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] sinh(x)*(cosh(x)^2)^(1/2)/cosh(x)
```

Maxima [A]
time = 0.48, size = 11, normalized size = 1.00

$$-\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

Fricas [A]
time = 0.41, size = 2, normalized size = 0.18

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sinh(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sinh^2(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(x)**2 + 1), x)
```

Giac [A]
time = 0.41, size = 11, normalized size = 1.00

$$-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

Mupad [B]
time = 0.07, size = 2, normalized size = 0.18

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)^2 + 1)^(1/2),x)
```

```
[Out] sinh(x)
```

3.89 $\int \sqrt{-1-\sinh^2(x)}\,dx$

Optimal. Leaf size=13

$$\sqrt{-\cosh^2(x)}\,\tanh(x)$$

```
[Out] (-cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{3255, 3286, 2717\}$

$$\sqrt{-\cosh^2(x)}\,\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Sinh[x]^2],x]
```

```
[Out] Sqrt[-Cosh[x]^2]*Tanh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1-\sinh^2(x)}\,dx &= \int \sqrt{-\cosh^2(x)}\,dx\\ &= \left(\sqrt{-\cosh^2(x)}\,\operatorname{sech}(x)\right)\int \cosh(x)\,dx\\ &= \sqrt{-\cosh^2(x)}\,\tanh(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\sqrt{-\cosh^2(x)}\,\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Sinh[x]^2],x]
```

```
[Out] Sqrt[-Cosh[x]^2]*Tanh[x]
```

Maple [A]
time = 0.76, size = 15, normalized size = 1.15

method	result	size
default	$-\frac{\sinh(x)\cosh(x)}{\sqrt{-\left(\cosh^2(x)\right)}}$	15
risch	$\frac{\sqrt{-\left(1+e^{2x}\right)^2 e^{-2x}}\, e^{2x}}{2+2\,e^{2x}}-\frac{\sqrt{-\left(1+e^{2x}\right)^2 e^{-2x}}}{2(1+e^{2x})}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -sinh(x)*cosh(x)/(-cosh(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.
time = 0.48, size = 25, normalized size = 1.92

$$-\frac{e^{(-2\,x)}}{2\,\sqrt{-e^{(-2\,x)}}}+\frac{1}{2\,\sqrt{-e^{(-2\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-2*x)/sqrt(-e^(-2*x)) + 1/2/sqrt(-e^(-2*x))
```

Fricas [C] Result contains complex when optimal does not.
time = 0.41, size = 14, normalized size = 1.08

$$\frac{1}{2}\left(i\,e^{(2\,x)}-i\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(2*x) - I)*e^(-x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\sinh^{2}\left(x\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-sinh(x)**2 - 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 11, normalized size = 0.85

$$-\frac{1}{2}i\,e^{(-x)}+\frac{1}{2}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*e^(-x) + 1/2*I*e^x
```

Mupad [B]
time = 0.17, size = 5, normalized size = 0.38

$$\sinh(x)\ 1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- sinh(x)^2 - 1)^(1/2),x)
```

```
[Out] sinh(x)*1i
```

3.90 $\int \sqrt{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=11

$$-iE(ix|-1)$$

```
[Out] -I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticE(I*sinh(x),I)
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{3256\}$

$$-iE(ix|-1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sinh[x]^2],x]
```

```
[Out] (-I)*EllipticE[I*x, -1]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{1-\sinh^2(x)}\,dx = -iE(ix|-1)$$

Mathematica [A]
time = 0.02, size = 11, normalized size = 1.00

$$-iE(ix|-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sinh[x]^2],x]
```

```
[Out] (-I)*EllipticE[I*x, -1]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(21) = 42.
time = 1.00, size = 51, normalized size = 4.64

method	result	size
default	$\frac{\sqrt{-\left(-1+\sinh^{2}\left(x\right)\right)\left(\cosh^{2}\left(x\right)\right)}\,\sqrt{\frac{1}{2}+\frac{\cosh\left(2x\right)}{2}}\,\left(2\,\mathrm{EllipticF}\left(\sinh\left(x\right),i\right)-\mathrm{EllipticE}\left(\sinh\left(x\right),i\right)\right)}{\sqrt{1-\left(\sinh^{4}\left(x\right)\right)}\,\cosh\left(x\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)*(2*EllipticF(sinh(x),I)
-EllipticE(sinh(x),I))/(1-sinh(x)^4)^(1/2)/cosh(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-sinh(x)^2 + 1), x)
```

Fricas [F]
time = 0.09, size = 12, normalized size = 1.09

$$\text{integral}\left(\sqrt{-\sinh\left(x\right)^{2}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-sinh(x)^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{1-\sinh^{2}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - sinh(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-sinh(x)^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.09

$$\int \sqrt{1-\sinh \left (x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - sinh(x)^2)^(1/2),x)
```

```
[Out] int((1 - sinh(x)^2)^(1/2), x)
```

3.91 $\int \sqrt{-1+\sinh^2(x)}\, dx$

Optimal. Leaf size=33

$$-\frac{iE(ix|-1)\sqrt{-1+\sinh^2(x)}}{\sqrt{1-\sinh^2(x)}}$$

```
[Out] -I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticE(I*sinh(x),I)*(-1+sinh(x)^2)^(1/2)/(1
-sinh(x)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3257, 3256}

$$-\frac{i\sqrt{\sinh^2(x)-1}\,E(ix|-1)}{\sqrt{1-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sinh[x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*x, -1]*Sqrt[-1 + Sinh[x]^2])/Sqrt[1 - Sinh[x]^2]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1+\sinh^2(x)}\,dx &= \frac{\sqrt{-1+\sinh^2(x)}\int \sqrt{1-\sinh^2(x)}\,dx}{\sqrt{1-\sinh^2(x)}}\\ &= -\frac{iE(ix|-1)\sqrt{-1+\sinh^2(x)}}{\sqrt{1-\sinh^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$\frac{i\sqrt{3-\cosh(2x)}\,E(ix|-1)}{\sqrt{-3+\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sinh[x]^2],x]
```

```
[Out] (I*Sqrt[3 - Cosh[2*x]]*EllipticE[I*x, -1])/Sqrt[-3 + Cosh[2*x]]
```

Maple [A]
time = 0.77, size = 61, normalized size = 1.85

method	result	size
default	$\frac{i\sqrt{\left(-1+\sinh^{2}\left(x\right)\right)\left(\cosh^{2}\left(x\right)\right)}\,\sqrt{\frac{1}{2}+\frac{\cosh\left(2x\right)}{2}}\,\sqrt{1-\left(\sinh^{2}\left(x\right)\right)}\,\mathrm{EllipticE}\left(i\sinh\left(x\right),i\right)}{\sqrt{\sinh^{4}\left(x\right)-1}\,\cosh\left(x\right)\sqrt{-1+\sinh^{2}\left(x\right)}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*((-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)*(1-sinh(x)^2)^(1/2)*El
lipticE(I*sinh(x),I)/(sinh(x)^4-1)^(1/2)/cosh(x)/(-1+sinh(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(x)^2 - 1), x)
```

Fricas [F]
time = 0.09, size = 10, normalized size = 0.30

$$\mathrm{integral}\left(\sqrt{\sinh\left(x\right)^{2}-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sinh(x)^2 - 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sinh^2(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)**2)**(1/2),x)
[Out] Integral(sqrt(sinh(x)**2 - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sinh(x)^2 - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{\sinh(x)^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)^2 - 1)^(1/2),x)
[Out] int((sinh(x)^2 - 1)^(1/2), x)
```

3.92 $\int \sqrt{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=42

$$-\frac{iE\left(ix\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(x)}}{\sqrt{1+\frac{b\sinh^2(x)}{a}}}$$

```
[Out] -I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticE(I*sinh(x),(b/a)^(1/2))*(a+b*sinh(x)^
2)^(1/2)/(1+b*sinh(x)^2/a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{3257, 3256\}$

$$-\frac{i\sqrt{a+b\sinh^2(x)}\,E\left(ix\middle|\frac{b}{a}\right)}{\sqrt{\frac{b\sinh^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*x, b/a]*Sqrt[a + b*Sinh[x]^2])/Sqrt[1 + (b*Sinh[x]^2)/a]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(x)}\,dx=\frac{\sqrt{a+b\sinh^2(x)}\int\sqrt{1+\frac{b\sinh^2(x)}{a}}\,dx}{\sqrt{1+\frac{b\sinh^2(x)}{a}}}$$

$$=-\frac{iE\left(ix\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(x)}}{\sqrt{1+\frac{b\sinh^2(x)}{a}}}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 1.29

$$-\frac{ia\sqrt{\frac{2a-b+b\cosh(2x)}{a}}\,E\left(ix\middle|\frac{b}{a}\right)}{\sqrt{2a-b+b\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticE[I*x, b/a])/Sqrt[2*a - b +
 b*Cosh[2*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(49) = 98$.
time = 1.18, size = 109, normalized size = 2.60

method	result
default	$\frac{\sqrt{\frac{a+b(\sinh^2(x))}{a}}\,\sqrt{\frac{1}{2}+\frac{\cosh(2x)}{2}}\,\bigg(a\,\mathrm{EllipticF}\bigg(\sinh(x)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\bigg)-b\,\mathrm{EllipticF}\bigg(\sinh(x)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\bigg)+b\,\mathrm{EllipticE}}{\sqrt{-\frac{b}{a}}\,\cosh(x)\sqrt{a+b\left(\sinh^2(x)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(x)^2)/a)^(1/2)*(cosh(x)^2)^(1/2)*(a*EllipticF(sinh(x)*(-1/a*b)^(
1/2),(a/b)^(1/2))-b*EllipticF(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))+b*Ellipti
cE(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/cosh(x)/(a+b*sinh(x)
^2)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x)^2 + a), x)
```

Fricas [**F**]
time = 0.10, size = 12, normalized size = 0.29

$$\text{integral}\left(\sqrt{b\sinh\left(x\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(x)^2 + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sinh^{2}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{b\sinh\left(x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(x)^2)^(1/2),x)
```

```
[Out] int((a + b*sinh(x)^2)^(1/2), x)
```

3.93 $\int \left(1+\sinh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=29

$$\frac{2}{3}\sqrt{\cosh^2(x)}\,\tanh(x)+\frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)$$

```
[Out] 1/3*(cosh(x)^2)^(3/2)*tanh(x)+2/3*(cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3255, 3282, 3286, 2717}

$$\frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)+\frac{2}{3}\sqrt{\cosh^2(x)}\,\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)^(3/2),x]
```

```
[Out] (2*Sqrt[Cosh[x]^2]*Tanh[x])/3 + ((Cosh[x]^2)^(3/2)*Tanh[x])/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(1+\sinh^2(x)\right)^{3/2}\,dx &= \int \cosh^2(x)^{3/2}\,dx\\ &= \frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)+\frac{2}{3}\int \sqrt{\cosh^2(x)}\,dx\\ &= \frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)+\frac{1}{3}\left(2\sqrt{\cosh^2(x)}\,\mathrm{sech}(x)\right)\int \cosh(x)\,dx\\ &= \frac{2}{3}\sqrt{\cosh^2(x)}\,\tanh(x)+\frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.79

$$\frac{1}{12}\sqrt{\cosh^2(x)}\,\mathrm{sech}(x)(9\sinh(x)+\sinh(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[Cosh[x]^2]*Sech[x]*(9*Sinh[x] + Sinh[3*x]))/12
```

Maple [A]
time = 0.76, size = 21, normalized size = 0.72

method	result	siz
default	$\frac{\sqrt{\frac{1}{2}+\frac{\cosh(2x)}{2}}\;\sinh(x)\left(\sinh^2(x)+3\right)}{3\cosh(x)}$	21
risch	$\frac{\mathrm{e}^{4x}\sqrt{\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}{24+24\,\mathrm{e}^{2x}}+\frac{3\sqrt{\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}\;\mathrm{e}^{2x}}{8(1+\mathrm{e}^{2x})}-\frac{3\sqrt{\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}{8(1+\mathrm{e}^{2x})}-\frac{\mathrm{e}^{-2x}\sqrt{\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}{24(1+\mathrm{e}^{2x})}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(cosh(x)^2)^(1/2)*sinh(x)*(sinh(x)^2+3)/cosh(x)
```

Maxima [A]
time = 0.48, size = 23, normalized size = 0.79

$$\frac{1}{24}\,e^{(3\,x)}-\frac{3}{8}\,e^{(-x)}-\frac{1}{24}\,e^{(-3\,x)}+\frac{3}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*x) - 3/8*e^(-x) - 1/24*e^(-3*x) + 3/8*e^x
```

Fricas [A]
time = 0.43, size = 17, normalized size = 0.59

$$\frac{1}{12}\sinh\left(x\right)^{3}+\frac{1}{4}\left(\cosh\left(x\right)^{2}+3\right)\sinh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*sinh(x)^3 + 1/4*(cosh(x)^2 + 3)*sinh(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\sinh^{2}\left(x\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)**2)**(3/2),x)
```

```
[Out] Integral((sinh(x)**2 + 1)**(3/2), x)
```

Giac [A]
time = 0.42, size = 25, normalized size = 0.86

$$-\frac{1}{24}\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)}+\frac{1}{24}\,e^{(3\,x)}+\frac{3}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(9*e^(2*x) + 1)*e^(-3*x) + 1/24*e^(3*x) + 3/8*e^x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int{\left({\mathrm{sinh}\left(x\right)}^2+1\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)^2 + 1)^(3/2),x)
```

```
[Out] int((sinh(x)^2 + 1)^(3/2), x)
```

3.94 $\int \left(-1-\sinh^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=33

$$-\frac{2}{3}\sqrt{-\cosh^2(x)}\,\tanh(x)+\frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)$$

```
[Out] 1/3*(-cosh(x)^2)^(3/2)*tanh(x)-2/3*(-cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3255, 3282, 3286, 2717}

$$\frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)-\frac{2}{3}\sqrt{-\cosh^2(x)}\,\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[-Cosh[x]^2]*Tanh[x])/3 + ((-Cosh[x]^2)^(3/2)*Tanh[x])/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(-1-\sinh^2(x)\right)^{3/2}\,dx &= \int \left(-\cosh^2(x)\right)^{3/2}\,dx\\ &= \frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)-\frac{2}{3}\int \sqrt{-\cosh^2(x)}\,dx\\ &= \frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)-\frac{1}{3}\left(2\sqrt{-\cosh^2(x)}\,\operatorname{sech}(x)\right)\int \cosh(x)\,dx\\ &= -\frac{2}{3}\sqrt{-\cosh^2(x)}\,\tanh(x)+\frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 25, normalized size = 0.76

$$-\frac{1}{12}\sqrt{-\cosh^2(x)}\,\operatorname{sech}(x)(9\sinh(x)+\sinh(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Sinh[x]^2)^(3/2),x]
[Out] -1/12*(Sqrt[-Cosh[x]^2]*Sech[x]*(9*Sinh[x] + Sinh[3*x]))
```

Maple [**A**]
time = 0.71, size = 21, normalized size = 0.64

method	result
default	$\frac{\cosh(x)\sinh(x)\left(\cosh^2(x)+2\right)}{3\sqrt{-\left(\cosh^2(x)\right)}}$
risch	$-\frac{e^{4x}\sqrt{-\left(1+e^{2x}\right)^2 e^{-2x}}}{24\left(1+e^{2x}\right)}-\frac{3\sqrt{-\left(1+e^{2x}\right)^2 e^{-2x}}\,e^{2x}}{8\left(1+e^{2x}\right)}+\frac{3\sqrt{-\left(1+e^{2x}\right)^2 e^{-2x}}}{8\left(1+e^{2x}\right)}+\frac{e^{-2x}\sqrt{-(1+e^{2x}}}{24+24\,e^{2x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*cosh(x)*sinh(x)*(cosh(x)^2+2)/(-cosh(x)^2)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(25) = 50$.
time = 0.50, size = 53, normalized size = 1.61

$$\frac{3\,e^{(-2\,x)}}{8\,\left(-e^{(-2\,x)}\right)^{\frac{3}{2}}}-\frac{3\,e^{(-4\,x)}}{8\,\left(-e^{(-2\,x)}\right)^{\frac{3}{2}}}-\frac{e^{(-6\,x)}}{24\,\left(-e^{(-2\,x)}\right)^{\frac{3}{2}}}+\frac{1}{24\,\left(-e^{(-2\,x)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*e^(-2*x)/(-e^(-2*x))^(3/2) - 3/8*e^(-4*x)/(-e^(-2*x))^(3/2) - 1/24*e^(-
6*x)/(-e^(-2*x))^(3/2) + 1/24/(-e^(-2*x))^(3/2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.55, size = 26, normalized size = 0.79

$$\frac{1}{24}\left(-i\,e^{(6\,x)}-9i\,e^{(4\,x)}+9i\,e^{(2\,x)}+i\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(-I*e^(6*x) - 9*I*e^(4*x) + 9*I*e^(2*x) + I)*e^(-3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\sinh^{2}\left(x\right)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)**2)**(3/2),x)
```

```
[Out] Integral((-sinh(x)**2 - 1)**(3/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 25, normalized size = 0.76

$$\frac{1}{24}i\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)}-\frac{1}{24}i\,e^{(3\,x)}-\frac{3}{8}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*I*(9*e^(2*x) + 1)*e^(-3*x) - 1/24*I*e^(3*x) - 3/8*I*e^x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int{\left(-{\mathrm{sinh}\left(x\right)}^2-1\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- sinh(x)^2 - 1)^(3/2),x)
```

```
[Out] int((- sinh(x)^2 - 1)^(3/2), x)
```

3.95 $\int \left(1-\sinh^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=45

$$-2iE(ix|-1)+\frac{2}{3}iF(ix|-1)-\frac{1}{3}\cosh(x)\sinh(x)\sqrt{1-\sinh^2(x)}$$

```
[Out] -2*I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticE(I*sinh(x),I)+2/3*I*(cosh(x)^2)^(1/
2)/cosh(x)*EllipticF(I*sinh(x),I)-1/3*cosh(x)*sinh(x)*(1-sinh(x)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3259, 3251, 3256, 3261}

$$\frac{2}{3}iF(ix|-1)-2iE(ix|-1)-\frac{1}{3}\sinh(x)\sqrt{1-\sinh^2(x)}\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] (-2*I)*EllipticE[I*x, -1] + ((2*I)/3)*EllipticF[I*x, -1] - (Cosh[x]*Sinh[x]
*Sqrt[1 - Sinh[x]^2])/3
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
```

```
0]
```

Rubi steps

$$
\begin{aligned}
\int \left(1-\sinh^2(x)\right)^{3/2}\,dx &= -\frac{1}{3}\cosh(x)\sinh(x)\sqrt{1-\sinh^2(x)}+\frac{1}{3}\int \frac{4-6\sinh^2(x)}{\sqrt{1-\sinh^2(x)}}\,dx\\
&= -\frac{1}{3}\cosh(x)\sinh(x)\sqrt{1-\sinh^2(x)}-\frac{2}{3}\int \frac{1}{\sqrt{1-\sinh^2(x)}}\,dx+2\int \sqrt{1-\sinh^2(}\\
&= -2iE(ix|-1)+\frac{2}{3}iF(ix|-1)-\frac{1}{3}\cosh(x)\sinh(x)\sqrt{1-\sinh^2(x)}
\end{aligned}
$$

Mathematica [**A**]
time = 0.05, size = 45, normalized size = 1.00

$$\frac{1}{12}\left(-24iE(ix|-1)+8iF(ix|-1)-\sqrt{6-2\cosh(2x)}\,\sinh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] ((-24*I)*EllipticE[I*x, -1] + (8*I)*EllipticF[I*x, -1] - Sqrt[6 - 2*Cosh[2*
x]]*Sinh[2*x])/12
```

Maple [**A**]
time = 0.98, size = 103, normalized size = 2.29

method	result
default	$\frac{\sqrt{-\left(-1+\sinh^2(x)\right)\left(\cosh^2(x)\right)}\left(\left(\cosh^4(x)\right)\sinh(x)+10\sqrt{-\left(\cosh^2(x)\right)+2}\,\sqrt{\frac{1}{2}+\frac{\cosh(2x)}{2}}\ \mathrm{EllipticF(si}\right.}{3\sqrt{1-\left(\sinh^4(x)\right)}\,\cosh(x)\sqrt{1-(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-(-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^4*sinh(x)+10*(-cosh(x)^2+2)^
(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh(x),I)-6*(-cosh(x)^2+2)^(1/2)*(cosh(x
)^2)^(1/2)*EllipticE(sinh(x),I)-2*cosh(x)^2*sinh(x))/(1-sinh(x)^4)^(1/2)/co
sh(x)/(1-sinh(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((-sinh(x)^2 + 1)^(3/2), x)
```

Fricas [F]
time = 0.08, size = 12, normalized size = 0.27

$$\mathrm{integral}\left(\left(-\sinh\left(x\right)^{2}+1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(3/2),x, algorithm="fricas")
[Out] integral((-sinh(x)^2 + 1)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(1-\sinh^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)**2)**(3/2),x)
[Out] Integral((1 - sinh(x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(3/2),x, algorithm="giac")
[Out] integrate((-sinh(x)^2 + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int{\left(1-{\mathrm{sinh}\left(x\right)}^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - sinh(x)^2)^(3/2),x)
[Out] int((1 - sinh(x)^2)^(3/2), x)
```

3.96 $\int \left(-1+\sinh^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=87

$$\frac{2iF(ix|-1)\sqrt{1-\sinh^2(x)}}{3\sqrt{-1+\sinh^2(x)}}+\frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}+\frac{2iE(ix|-1)\sqrt{-1+\sinh^2(x)}}{\sqrt{1-\sinh^2(x)}}$$

```
[Out] 2/3*I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticF(I*sinh(x),I)*(1-sinh(x)^2)^(1/2)/
(-1+sinh(x)^2)^(1/2)+1/3*cosh(x)*sinh(x)*(-1+sinh(x)^2)^(1/2)+2*I*(cosh(x)^
2)^(1/2)/cosh(x)*EllipticE(I*sinh(x),I)*(-1+sinh(x)^2)^(1/2)/(1-sinh(x)^2)^
(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3259, 3251, 3257, 3256, 3262, 3261}

$$\frac{1}{3}\sinh(x)\sqrt{\sinh^2(x)-1}\,\cosh(x)+\frac{2i\sqrt{1-\sinh^2(x)}\,F(ix|-1)}{3\sqrt{\sinh^2(x)-1}}+\frac{2i\sqrt{\sinh^2(x)-1}\,E(ix|-1)}{\sqrt{1-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sinh[x]^2)^(3/2),x]
```

```
[Out] (((2*I)/3)*EllipticF[I*x, -1]*Sqrt[1 - Sinh[x]^2])/Sqrt[-1 + Sinh[x]^2] + (
Cosh[x]*Sinh[x]*Sqrt[-1 + Sinh[x]^2])/3 + ((2*I)*EllipticE[I*x, -1]*Sqrt[-1
 + Sinh[x]^2])/Sqrt[1 - Sinh[x]^2]
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
```

```
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(-1+\sinh^2(x)\right)^{3/2}\,dx &= \frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}+\frac{1}{3}\int\frac{4-6\sinh^2(x)}{\sqrt{-1+\sinh^2(x)}}\,dx\\ &= \frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}-\frac{2}{3}\int\frac{1}{\sqrt{-1+\sinh^2(x)}}\,dx-2\int\sqrt{-1+\\ &= \frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}-\frac{\left(2\sqrt{1-\sinh^2(x)}\right)\int\frac{1}{\sqrt{1-\sinh^2(x)}}\,d}{3\sqrt{-1+\sinh^2(x)}}\\ &= \frac{2iF(ix|-1)\sqrt{1-\sinh^2(x)}}{3\sqrt{-1+\sinh^2(x)}}+\frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}+\frac{2iE(ix|-}{\sqrt{}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 78, normalized size = 0.90

$$\frac{-24i\sqrt{3-\cosh(2x)}\,E(ix|-1)+8i\sqrt{3-\cosh(2x)}\,F(ix|-1)+\frac{-6\sinh(2x)+\sinh(4x)}{\sqrt{2}}}{12\sqrt{-3+\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sinh[x]^2)^(3/2),x]
```

```
[Out] ((-24*I)*Sqrt[3 - Cosh[2*x]]*EllipticE[I*x, -1] + (8*I)*Sqrt[3 - Cosh[2*x]]
*EllipticF[I*x, -1] + (-6*Sinh[2*x] + Sinh[4*x])/Sqrt[2])/(12*Sqrt[-3 + Cos
h[2*x]])
```

Maple [A]
time = 0.97, size = 106, normalized size = 1.22

method	result
default	$\frac{\sqrt{\left(-1+\sinh^{2}\left(x\right)\right)\left(\cosh^{2}\left(x\right)\right)}\,\Bigg(\left(\cosh^{4}\left(x\right)\right)\sinh\left(x\right)+2 i\sqrt{\frac{1}{2}+\frac{\cosh\left(2 x\right)}{2}}\,\sqrt{-\left(\cosh^{2}\left(x\right)\right)+2}\,\mathrm{EllipticF}(i \sinh}{3\sqrt{\sinh^{4}\left(x\right)-1}\,\cosh\left(x\right)\sqrt{-1+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^4*sinh(x)+2*I*(cosh(x)^2)^(1/
2)*(-cosh(x)^2+2)^(1/2)*EllipticF(I*sinh(x),I)-6*I*(cosh(x)^2)^(1/2)*(-cosh
(x)^2+2)^(1/2)*EllipticE(I*sinh(x),I)-2*cosh(x)^2*sinh(x))/(sinh(x)^4-1)^(1
/2)/cosh(x)/(-1+sinh(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((sinh(x)^2 - 1)^(3/2), x)
```

Fricas [F]
time = 0.09, size = 10, normalized size = 0.11

$$\mathrm{integral}\left(\left(\sinh\left(x\right)^{2}-1\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((sinh(x)^2 - 1)^(3/2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sinh^{2}\left(x\right)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)**2)**(3/2),x)
```

```
[Out] Integral((sinh(x)**2 - 1)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sinh(x)^2 - 1)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left({\mathrm{sinh}\left(x\right)}^2-1\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)^2 - 1)^(3/2),x)
```

```
[Out] int((sinh(x)^2 - 1)^(3/2), x)
```

3.97 $\int \left(a+b\sinh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=123

$$\frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}-\frac{2i(2a-b)E\left(ix\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(x)}}{3\sqrt{1+\frac{b\sinh^2(x)}{a}}}+\frac{ia(a-b)F\left(ix\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(}{a}}}{3\sqrt{a+b\sinh^2(x)}}$$

```
[Out] 1/3*b*cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2)-2/3*I*(2*a-b)*(cosh(x)^2)^(1/2)
/cosh(x)*EllipticE(I*sinh(x),(b/a)^(1/2))*(a+b*sinh(x)^2)^(1/2)/(1+b*sinh(x
)^2/a)^(1/2)+1/3*I*a*(a-b)*(cosh(x)^2)^(1/2)/cosh(x)*EllipticF(I*sinh(x),(b
/a)^(1/2))*(1+b*sinh(x)^2/a)^(1/2)/(a+b*sinh(x)^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3259, 3251, 3257, 3256, 3262, 3261}

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\sinh^2(x)}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(x)}{a}+1}\;F\left(ix\middle|\frac{b}{a}\right)}{3\sqrt{a+b\sinh^2(x)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(x)}\;E\left(ix\middle|\frac{b}{a}\right)}{3\sqrt{\frac{b\sinh^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x]^2)^(3/2),x]
```

```
[Out] (b*Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2])/3 - (((2*I)/3)*(2*a - b)*Elliptic
E[I*x, b/a]*Sqrt[a + b*Sinh[x]^2])/Sqrt[1 + (b*Sinh[x]^2)/a] + ((I/3)*a*(a
- b)*EllipticF[I*x, b/a]*Sqrt[1 + (b*Sinh[x]^2)/a])/Sqrt[a + b*Sinh[x]^2]
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(x)\right)^{3/2}\,dx = \frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}+\frac{1}{3}\int \frac{a(3a-b)+2(2a-b)b\sinh^2(x)}{\sqrt{a+b\sinh^2(x)}}\,dx$$

$$= \frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}-\frac{1}{3}(a(a-b))\int \frac{1}{\sqrt{a+b\sinh^2(x)}}\,dx+\frac{1}{3}$$

$$= \frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}+\frac{\left(2(2a-b)\sqrt{a+b\sinh^2(x)}\right)\int\sqrt{1+\frac{b}{\,}}}{3\sqrt{1+\frac{b\sinh^2(x)}{a}}}$$

$$= \frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}-\frac{2i(2a-b)E\left(ix\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(x)}}{3\sqrt{1+\frac{b\sinh^2(x)}{a}}}+$$

Mathematica [A]
time = 0.28, size = 132, normalized size = 1.07

$$\frac{-8ia(2a-b)\sqrt{\frac{2a-b+b\cosh(2x)}{a}}\,E\left(ix\middle|\frac{b}{a}\right)+4ia(a-b)\sqrt{\frac{2a-b+b\cosh(2x)}{a}}\,F\left(ix\middle|\frac{b}{a}\right)+\sqrt{2}\,b(2a-b+b\cosh(2x))\sinh(2x)}{12\sqrt{2a-b+b\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^2)^(3/2),x]
```

```
[Out] ((-8*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticE[I*x, b/a] + (
4*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticF[I*x, b/a] + Sqrt[2
]*b*(2*a - b + b*Cosh[2*x])*Sinh[2*x])/(12*Sqrt[2*a - b + b*Cosh[2*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(129) = 258.
time = 1.14, size = 333, normalized size = 2.71

method	result
default	$\sqrt{-\frac{b}{a}}\, b^2 (\cosh^4(x)) \sinh(x)+\sqrt{-\frac{b}{a}}\, ab(\cosh^2(x)) \sinh(x)-\sqrt{-\frac{b}{a}}\, b^2 (\cosh^2(x)) \sinh(x)+3a^2 \sqrt{\frac{b(\cosh^2(x))}{a}+\frac{a-b}{a}}\, \sqrt{\frac{1}{2}+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*cosh(x)^4*sinh(x)+(-1/a*b)^(1/2)*a*b*cosh(x)^2*sinh
(x)-(-1/a*b)^(1/2)*b^2*cosh(x)^2*sinh(x)+3*a^2*(b/a*cosh(x)^2+(a-b)/a)^(1/2
)*(cosh(x)^2)^(1/2)*EllipticF(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))-5*a*b*(b/
a*cosh(x)^2+(a-b)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh(x)*(-1/a*b)^(1/
2),(a/b)^(1/2))+2*(b/a*cosh(x)^2+(a-b)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticF
(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+4*a*b*(b/a*cosh(x)^2+(a-b)/a)^(1/2
)*(cosh(x)^2)^(1/2)*EllipticE(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))-2*(b/a*co
sh(x)^2+(a-b)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticE(sinh(x)*(-1/a*b)^(1/2),(
a/b)^(1/2))*b^2)/(-1/a*b)^(1/2)/cosh(x)/(a+b*sinh(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x)^2 + a)^(3/2), x)
```

Fricas [**F**]
time = 0.10, size = 12, normalized size = 0.10

$$\mathrm{integral}\left(\left(b\sinh\left(x\right)^{2}+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(x)^2 + a)^(3/2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sinh(x)**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x)^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(b\,{\mathrm{sinh}\left(x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(x)^2)^(3/2),x)
```

```
[Out] int((a + b*sinh(x)^2)^(3/2), x)
```

3.98 $\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=83

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2b^{3/2}f}+\frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf}$$

```
[Out] -1/2*(a+b)*arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/b^(3/2)
/f+1/2*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/b/f
```

Rubi [A]

time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3265, 396, 223, 212}

$$\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2bf}-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/2*((a + b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^
2]])/(b^(3/2)*f) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*b*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf} - \frac{(a+b)\text{Subst}\Big(\int \frac{1}{\sqrt{a-b+bx^2}}}{2bf}$$

$$= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf} - \frac{(a+b)\text{Subst}\Big(\int \frac{1}{1-bx^2}\,dx, x,}{2bf}$$

$$= -\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\cosh(e+fx)\sqrt{a-b+}}{2bf}$$

Mathematica [A]
time = 0.20, size = 98, normalized size = 1.18

$$\frac{\cosh(e+fx)\sqrt{2a-b+b\cosh(2(e+fx))}}{2\sqrt{2}\,bf} - \frac{(a+b)\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])/(2*Sqrt[2]*b*f) - ((a +
 b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]
])/(2*b^(3/2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(71) = 142$.
time = 1.16, size = 204, normalized size = 2.46

method	result
default	$\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\,\left(2\sqrt{b\left(\cosh^{4}\left(fx+e\right)\right)+\left(a-b\right)\left(\cosh^{2}\left(fx+e\right)\right)}\,b^{\frac{3}{2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(2*(b*cosh(f*x+e)^4+(a-b)*cos
h(f*x+e)^2)^(1/2)*b^(3/2)-ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b
)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*b^2-a*ln(1/2*(2*b*cosh(f*x+e)^
2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*b)/b^
(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(71) = 142$.
time = 0.50, size = 2116, normalized size = 25.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((a + b)*cosh(f*x + e)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a
 + b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*
x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x +
 e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*
a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9
*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2
*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*
(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4
*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh
```

```
(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^
4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x +
 e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5
+ a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 +
a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh
(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*
a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*
x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*c
osh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3
)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x
 + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e
)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)
^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a + b)*cosh(f*
x + e)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a + b)*sinh(f*x + e)^2)
*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*si
nh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)
*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(c
osh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(2
)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2
+ b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)
^2 + 2*b^2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2), 1/8*(2*(
(a + b)*cosh(f*x + e)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a + b)*s
inh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x +
 e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f
*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*co
sh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)
^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((a + b)*cosh(f*x + e
)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a + b)*sinh(f*x + e)^2)*sqrt
(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/
(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
+ 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x
+ e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)
) + sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f
*x + e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cos
```

```
h(f*x + e)^2 + 2*b^2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2)
]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[4,0,0]%%%}+%%%{%%%{-2,[1]%%%},[2,0,0]%%%}+%%%{%%%{1,[2]%%%
},[0,0
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sinh\left(e+f\,x\right)}^3}{\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.99 $\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

```
[Out] arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[b]*f
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Mathematica [**A**]
time = 0.08, size = 49, normalized size = 1.20

$$\frac{\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/(S
qrt[b]*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(35) = 70.
time = 0.82, size = 108, normalized size = 2.63

method	result
default	$\frac{\sqrt{\left(a+b\left(\sinh^2(fx+e)\right)\right)\left(\cosh^2(fx+e)\right)}\,\ln\left(\frac{2b\left(\cosh^2(fx+e)\right)+2\sqrt{b\left(\cosh^4(fx+e)\right)+(a-b)\left(\cos}}{2\sqrt{b}}\right.}{2\sqrt{b}\cosh(fx+e)\sqrt{a+b\left(\sinh^2(fx+e)\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*ln(1/2*(2*b*cosh(f*x+e)^2+2*(
b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))/b^(1/2)/co
sh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(35) = 70$.
time = 0.49, size = 1654, normalized size = 40.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x +
 e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2
*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x +
e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2
+ b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^
3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*
x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cos
h(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(
14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3
 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(
a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x +
e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e
)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*
a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)
^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*
cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 +
3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3
 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(
f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x
 + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x +
 e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + sqrt(b)*log(-(b*cosh(f*x + e)^4 +
```

```
 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x
 + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sq
rt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*c
osh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a
- b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2)))/(b*f), -1/2*(sqrt(-b)*arctan(sqrt(2)*(a*c
osh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*s
qrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)
^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b
^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2
 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
 e))) + sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh
(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a -
b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f
*x + e) + b)))/(b*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh{\left(e + fx \right)}}{\sqrt{a + b\sinh^{2}{\left(e + fx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sinh(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{sinh}(e+f\,x)}{\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(sinh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.100 $\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 385, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[a]
*f))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
```

```
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{\sqrt{a}\,f}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 49, normalized size = 1.17

$$
-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)}{\sqrt{a}\,f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]]]/(Sqrt[a]*f))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(36) = 72$.
time = 1.15, size = 113, normalized size = 2.69

method	result
default	$-\frac{\sqrt{(a+b(\sinh^2(fx+e)))(\cosh^2(fx+e))}\,\ln\left(\frac{(a+b)(\cosh^2(fx+e))+2\sqrt{a}\sqrt{b(\cosh^4(fx+e))+(a}}{\sinh(fx+e)^2}\right.}{2\sqrt{a}\cosh(fx+e)\sqrt{a+b(\sinh^2(fx+e))}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/a^(1/2)*ln(((a+b)*cosh(f*x+e
)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^
2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(36) = 72$.
time = 0.62, size = 572, normalized size = 13.62

$$\left[\frac{\log\left(-\frac{(a+b)\cosh(fx+e)^4+4(a+b)\cosh(fx+e)\sinh(fx+e)^3+(a+b)\sinh(fx+e)^4+2(3a-b)\cosh(fx+e)^2+2\left(3(a+b)\cosh(fx+e)^2+3a-b\right)\sinh(fx+e)^2-2\sqrt{2}\left(\cosh(fx+e)^2+2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^2+1\right)\sqrt{a}\sqrt{\frac{b\cosh(fx+e)^2+b\sinh(fx+e)^2+2a-b}{\cosh(fx+e)^2-2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^2}}+4\left((a+b)\cosh(fx+e)^3+(3a-b)\cosh(fx+e)\right)\sinh(fx+e)+a+b}{\cosh(fx+e)^4+4\cosh(fx+e)\sinh(fx+e)^3+\sinh(fx+e)^4+2\left(3\cosh(fx+e)^2-1\right)\sinh(fx+e)^2-2\cosh(fx+e)^2+4\left(\cosh(fx+e)^3-\cosh(fx+e)\right)\sinh(fx+e)+1}\right)}{2\sqrt{a}f}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{2}\left(\cosh(fx+e)^2+2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^2+1\right)\sqrt{-a}\sqrt{\frac{b\cosh(fx+e)^2+b\sinh(fx+e)^2+2a-b}{\cosh(fx+e)^2-2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^2}}}{b\cosh(fx+e)^4+4b\cosh(fx+e)\sinh(fx+e)^3+b\sinh(fx+e)^4+2(2a-b)\cosh(fx+e)^2+2\left(3b\cosh(fx+e)^2+2a-b\right)\sinh(fx+e)^2+4\left(b\cosh(fx+e)^3+(2a-b)\cosh(fx+e)\right)\sinh(fx+e)+b}\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^
3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*co
sh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*c
osh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x
+ e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*co
sh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh
(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 -
 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)
)/(sqrt(a)*f), sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 +
 b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))
*sinh(f*x + e) + b))/(a*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csch(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.
time = 0.49, size = 78, normalized size = 1.86

$$\frac{2\,\arctan\left(-\frac{\sqrt{b}\,e^{(2\,fx+2\,e)}-\sqrt{be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e)}-2\,be^{(2\,fx+2\,e)}+b}-\sqrt{b}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sqrt(-a))/(sqrt(-a)*f)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sinh\left(e+f\,x\right)\,\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

3.101 $\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=89

$$\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2a^{3/2}f}-\frac{\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{2af}$$

```
[Out] 1/2*(a+b)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(3/2)/
f-1/2*coth(f*x+e)*csch(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3265, 390, 385, 212}

$$\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/
(2*a^(3/2)*f) - (Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x
])/(2*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{2af} + \frac{(a+b)\operatorname{Subst}\Big(\int}{}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{2af} + \frac{(a+b)\operatorname{Subst}\Bigg(\int}{}$$

$$= \frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\sqrt{a-b+b\cosh^2(e+fx}}{2}$$

Mathematica [A]
time = 0.23, size = 102, normalized size = 1.15

$$\frac{2(a+b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right) - \sqrt{2}\sqrt{a}\sqrt{2a-b+b\cosh(2(e+fx))}\coth(e+fx)\operatorname{csch}(e+fx)}{4a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (2*(a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[e +
 f*x]*Csch[e + f*x])/(4*a^(3/2)*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(77) = 154$.
time = 2.14, size = 234, normalized size = 2.63

method	result
default	$-\frac{\sqrt{(a+b(\sinh^2(fx+e)))(\cosh^2(fx+e))}\left(-\ln\left(\frac{(a+b)(\cosh^2(fx+e))+2\sqrt{a}\sqrt{b(\cosh^4(fx+e))+(}}{\sinh(fx+e)^2}\right.\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-ln(((a+b)*cosh(f*x+e)^2+2*
a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sin
h(f*x+e)^2*a^2-ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cos
h(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*b*sinh(f*x+e)^2*a+2*((a+b*sinh(f*x+e)
^2)*cosh(f*x+e)^2)^(1/2)*a^(3/2))/sinh(f*x+e)^2/a^(5/2)/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(77) = 154$.
time = 0.47, size = 1285, normalized size = 14.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 +
(a + b)*sinh(f*x + e)^4 - 2*(a + b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x
 + e)^2 - a - b)*sinh(f*x + e)^2 + 4*((a + b)*cosh(f*x + e)^3 - (a + b)*cos
h(f*x + e))*sinh(f*x + e) + a + b)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 +
```

```
4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a
- b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e
)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(
(a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(
cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*
cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)
^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*(a*cosh(f*x + e)^2 + 2*
a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e
)*sinh(f*x + e)^3 + a^2*f*sinh(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f
 + 2*(3*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x
+ e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((a + b)*cosh(f*x + e)^
4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 - 2*(
a + b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 - a - b)*sinh(f*x + e
)^2 + 4*((a + b)*cosh(f*x + e)^3 - (a + b)*cosh(f*x + e))*sinh(f*x + e) + a
 + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f
*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)
*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x
 + e) + b)) + sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e)
+ a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/
(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*f*sinh
(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2
- a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*
sinh(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csch(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 669 vs. 2(77) = 154.

time = 0.56, size = 669, normalized size = 7.52

$$-\frac{\left(\frac{(a+b)\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}-\sqrt{b}}{2\sqrt{-a}}\right)e^{(-4e)}}{\sqrt{-a}a}-\frac{2\left(\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{3}a+\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{3}b+5\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{2}a\sqrt{b}-3\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{2}b^{\frac{3}{2}}+4\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)a^{2}-9\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)ab+3\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)b^{2}-4a^{2}\sqrt{b}+3ab^{\frac{3}{2}}-b^{\frac{5}{2}}\right)e^{(-4e)}}{\left(\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{2}-2\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)\sqrt{b}-4a+b\right)^{2}a}\right)e^{(4e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -((a + b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sqrt(-a))*e^(-4*e)
/(sqrt(-a)*a) - 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*
e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a + (sqrt(b)*e^(2*f*x + 2*e)
- sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^
3*b + 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*sqrt(b) - 3*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2
*b^(3/2) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f
*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2 - 9*(sqrt(b)*e^(2*f*x + 2*e) - sq
rt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b
+ 3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
 - 2*b*e^(2*f*x + 2*e) + b))*b^2 - 4*a^2*sqrt(b) + 3*a*b^(3/2) - b^(5/2))*e
^(-4*e)/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
+ 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) -
 4*a + b)^2*a))*e^(4*e)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\mathrm{sinh}\left(e+f\,x\right)}^3\,\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

3.102 $\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=229

$$\frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf}+\frac{2(a+b)E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)}{3b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+\right.}{a}}}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+2/3*(a+b)*(1/(1+s
inh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(
f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b^2/f/
(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(1/(1+sinh(f*x+e)^2))^(1/2)
*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b
/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sech(f*x+e)^2*(a+b*si
nh(f*x+e)^2)/a)^(1/2)-2/3*(a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Rubi [A]
time = 0.15, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3267, 490, 545, 429, 506, 422}

$$\frac{2(a+b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^2f}+\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f) + (2*(a +
 b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(3*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
 (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(3*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*
(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b^2*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2))))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\Big(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\Big)\,\text{Subst}\Big(\int \frac{x^4}{\sqrt{1+x^2}\,\sqrt{a+bx^2}}\,dx, x, \sinh(}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{\Big(\sqrt{\cosh^2(e+fx)}\,\text{sec}}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{\Big(a\sqrt{\cosh^2(e+fx)}\,\text{se}}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{F\big(\tan^{-1}(\sinh(e+fx)}{3bf\sqrt{\frac{\text{sech}}{}}}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{2(a+b)E\big(\tan^{-1}(\sinh(}{3b^2f\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.69, size = 168, normalized size = 0.73

$$\frac{4i\sqrt{2}\,a(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2i\sqrt{2}\,a(2a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+b(2a-b+b\cosh(2(e+fx)))\sinh(2(e+fx))}{6b^2f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((4*I)*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[
I*(e + f*x), b/a] - (2*I)*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e +
 f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])*
Sinh[2*(e + f*x)])/(6*b^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.16, size = 356, normalized size = 1.55

method	result

default	$\sqrt{-\frac{b}{a}}\, b(\cosh^4(fx+e))\sinh(fx+e)+\sqrt{-\frac{b}{a}}\, a(\cosh^2(fx+e))\sinh(fx+e)-\sqrt{-\frac{b}{a}}\, b(\cosh^2(fx+e))\sinh(fx+e)+a\sqrt{\frac{b(\cosh^2(f}{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b*cosh(f*x+e)^4*sinh(f*x+e)+(-1/a*b)^(1/2)*a*cosh(f*x+e
)^2*sinh(f*x+e)-(-1/a*b)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+
e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1
/2),(a/b)^(1/2))+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*
EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*(b/a*cosh(f*x+e)^2+(a
-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))*a-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/b/(-1/a*b)^(1/2)/cosh(f*x+e
)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**F**]
time = 0.09, size = 25, normalized size = 0.11

$$\text{integral}\left(\frac{\sinh\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 804 vs. 2(237) = 474.
time = 1.15, size = 804, normalized size = 3.51

$$\frac{\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\,e^{(fx+e)}}{24bf}+$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
)*e^(f*x + e)/(b*f) + 1/24*(4*(a*sqrt(b)*e^(4*e) + b^(3/2)*e^(4*e))*log(abs
(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/2)))/b + 2*(9*a^2*e^(4*e)
 + 3*a*b*e^(4*e) + 4*b^2*e^(4*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b
*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b)
)/(sqrt(-b)*b) - 3*(6*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*e^(4*e) + 2*(sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))^3*a*b*e^(4*e) - 3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^2*e^(4*e) + 4
*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
2*b*e^(2*f*x + 2*e) + b))^2*b^(5/2)*e^(4*e) - 10*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a
^2*b*e^(4*e) + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b^2*e^(4*e) + (sqrt(b)*e^(2*f*x
 + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e
) + b))*b^3*e^(4*e) - 4*a*b^(5/2)*e^(4*e) - 2*b^(7/2)*e^(4*e))/(((sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))^2 - b)^2*b))*e^(-3*e)/(b*f^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(e+f\,x)^4}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.103 $\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{bf\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}+\frac{iaF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{bf\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(
1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(1+b*sinh(f*x+e)^2/a)^(1/2)+I*a*(cos(I*
e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))*(1+b
*sinh(f*x+e)^2/a)^(1/2)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3251, 3257, 3256, 3262, 3261}

$$\frac{ia\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\;F\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\;E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[1
+ (b*Sinh[e + f*x]^2)/a]) + (I*a*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Si
nh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{b\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} - \frac{\left(a\sqrt{1+\frac{b\sinh^2(e+}{a}}\right.}{b\sqrt{}}$$

$$= -\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{bf\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} + \frac{iaF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2}{}}}{bf\sqrt{a+b\sinh^2(e+fx}}$$

Mathematica [A]
time = 0.19, size = 89, normalized size = 0.70

$$-\frac{i\sqrt{2a-b+b\cosh(2(e+fx))}\left(E\left(i(e+fx)\middle|\frac{b}{a}\right)-F\left(i(e+fx)\middle|\frac{b}{a}\right)\right)}{bf\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*(EllipticE[I*(e + f*x), b/a] - El
lipticF[I*(e + f*x), b/a]))/(b*f*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a])
```

Maple [A]
time = 0.90, size = 113, normalized size = 0.88

method	result
default	$-\frac{\sqrt{\frac{a+b\left(\sinh^{2}(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\Bigg(\mathrm{EllipticF}\Bigg(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\Bigg)-\mathrm{EllipticE}\Bigg(\sinh(fx+e)\sqrt{-\frac{b}{a}},}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(Elli
pticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2)))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F]
time = 0.09, size = 25, normalized size = 0.20

$$\mathrm{integral}\left(\frac{\sinh\left(fx+e\right)^{2}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2{(e+fx)}}{\sqrt{a+b\sinh^2{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sinh(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{32,[4,
2,4]%%%}+%%%{%%%{-64,[1]%%%},[4,2,3]%%%}+%%%{%%%{32,[2]%%%},[4,2,2]%%%}+%%%
{%%%{-64,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{sinh}\left(e+f\,x\right)}^2}{\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.104 $\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(
1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3262, 3261}

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A]
time = 0.06, size = 68, normalized size = 1.13

$$-\frac{i\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/
(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 0.88, size = 86, normalized size = 1.43

method	result	size
default	$\frac{\sqrt{\frac{a+b\left(\sinh^2(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\,f}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(70) = 140$.
time = 0.10, size = 147, normalized size = 2.45

$$-\frac{2\left(2\,b\sqrt{\frac{a^2-ab}{b^2}}+2\,a-b\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}(\cosh\left(fx+e\right)+\sinh\left(fx+e\right))\right)\mid\frac{8\,a^2-8\,ab+b^2+4\,(2\,ab-b^2)\sqrt{\frac{a^2-ab}{b^2}}}{b^2})}{b^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)
*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*s
qrt((a^2 - a*b)/b^2))/b^2)/(b^(3/2)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(1/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.105 $\int \frac{\mathbf{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=134

$$-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{E\left(\operatorname{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] -coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-(1/(1+sinh(f*x+e)^2))^(1/2)*(1+s
inh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(
1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*
x+e)^2)/a)^(1/2)+(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a/f
```

Rubi [A]

time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3267, 491, 12, 506, 422}

$$-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f)) - (EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^
2]*Tanh[e + f*x])/(a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}\,\sqrt{a+bx^2}}\,dx, x, \sin\right.}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(b\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+f}{af}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} - \frac{E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sec}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a\right.}{}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.40, size = 150, normalized size = 1.12

$$\frac{\sqrt{2}\,(-2a+b-b\cosh(2(e+fx)))\coth(e+fx)-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{2af\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (2*I)*a*Sqrt[(
2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(2*a*f*Sqrt[
2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.92, size = 189, normalized size = 1.41

method	result
default	$-\frac{\sqrt{-\frac{b}{a}}\,b(\cosh^4(fx+e))+\left(\sqrt{-\frac{b}{a}}\,a-\sqrt{-\frac{b}{a}}\,b\right)(\cosh^2(fx+e))+\sinh(fx+e)\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{b(\cosh^2(fx+e))}{a}+}}{a\sinh(fx+e)\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\,(\sinh}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-1/a*b)^(1/2)*b*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^(1/2)*b)*cosh(f
*x+e)^2+sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*b*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-EllipticE(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))))/a/sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+
b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(140) = 280.
time = 0.13, size = 541, normalized size = 4.04

$$\frac{4\left(b\cosh(fx+e)^2+2b\cosh(fx+e)\sinh(fx+e)+b\sinh(fx+e)^2-b\right)\sqrt{b}\sqrt{\frac{2b\sqrt{\frac{a^2-ab}{b^2}}-2a+b}{b}}\sqrt{\frac{a^2-ab}{b^2}}\,F\left(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^2-ab}{b^2}}-2a+b}{b}}\left(\cosh(fx+e)+\sinh(fx+e)\right)\right)\,\middle|\,\frac{8a^2-8ab+b^2+4(2ab-b^2)\sqrt{\frac{a^2-ab}{b^2}}}{b^2}\right)+\left((2a-b)\cosh(fx+e)^2+2(2a-b)\cosh(fx+e)\sinh(fx+e)+(2a-b)\sinh(fx+e)^2-2\left(b\cosh(fx+e)^2+2b\cosh(fx+e)\sinh(fx+e)+b\sinh(fx+e)^2-b\right)\sqrt{\frac{a^2-ab}{b^2}}-2a+b\right)\sqrt{b}\sqrt{\frac{2b\sqrt{\frac{a^2-ab}{b^2}}-2a+b}{b}}\,E\left(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^2-ab}{b^2}}-2a+b}{b}}\left(\cosh(fx+e)+\sinh(fx+e)\right)\right)\,\middle|\,\frac{8a^2-8ab+b^2+4(2ab-b^2)\sqrt{\frac{a^2-ab}{b^2}}}{b^2}\right)-\sqrt{2}\left(b\cosh(fx+e)+b\sinh(fx+e)\right)\sqrt{\frac{b\cosh(fx+e)^2+b\sinh(fx+e)^2+2a-b}{\cosh(fx+e)^2-2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^2}}}{abf\cosh(fx+e)^2+2abf\cosh(fx+e)\sinh(fx+e)+abf\sinh(fx+e)^2-abf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (4*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2
 - b)*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*sqrt((a^2 - a*b
)/b^2)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cos
h(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((
a^2 - a*b)/b^2))/b^2) + ((2*a - b)*cosh(f*x + e)^2 + 2*(2*a - b)*cosh(f*x +
 e)*sinh(f*x + e) + (2*a - b)*sinh(f*x + e)^2 - 2*(b*cosh(f*x + e)^2 + 2*b*
cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((a^2 - a*b)/b^2)
- 2*a + b)*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e
(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh
(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/
b^2) - sqrt(2)*(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2
+ b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2)))/(a*b*f*cosh(f*x + e)^2 + 2*a*b*f*cosh(f*x + e)*si
nh(f*x + e) + a*b*f*sinh(f*x + e)^2 - a*b*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csch(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**A**]
time = 0.48, size = 276, normalized size = 2.06

$$-\frac{2\left(\frac{\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}-\sqrt{b}}{2\sqrt{-a}}\right)e^{(-2e)}}{\sqrt{-a}}-\frac{2\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}+\sqrt{b}\right)e^{(-2e)}}{\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^2-2\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)\sqrt{b}-4a+b}\right)e^{(3e)}}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*(arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sqrt(-a))*e^(-2*e)/sqrt(
-a) - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))*e^(-2*e)/((sqrt(b)*e^(2*f*x + 2*
e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
))^2 - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) - 4*a + b))*e^(3*e)/f^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(e+f\,x\right)^2\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

3.106 $\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=267

$$\frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}+\frac{2(a+b)}{}$$

```
[Out] 2/3*(a+b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f-1/3*coth(f*x+e)*csch(
f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a/f+2/3*(a+b)*(1/(1+sinh(f*x+e)^2))^(1/2
)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-
b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b
*sinh(f*x+e)^2)/a)^(1/2)-1/3*b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f
*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)
^(1/2)-2/3*(a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a^2/f
```

Rubi [A]
time = 0.20, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3267, 491, 597, 545, 429, 506, 422}

$$-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\operatorname{ArcTan}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{3a^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2(a+b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{3a^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}+\frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (2*(a + b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f) - (Coth[e +
 f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) + (2*(a + b)*Ell
ipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*Ell
ipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a
+ b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
```

```
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^2}\sqrt{a+bx^2}}\,dx, x, \sin\right.}{f}$$

$$= -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{s}\right.}{}$$

$$= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af}$$

$$= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af}$$

$$= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af}$$

$$= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.39, size = 201, normalized size = 0.75

$$\frac{\frac{\left(-8a^2+ab+3b^2+\left(4a^2-2ab-4b^2\right)\cosh(2(e+fx))+b(a+b)\cosh(4(e+fx))\right)\coth(e+fx)\operatorname{csch}^2(e+fx)}{\sqrt{2}}+4ia(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2ia(2a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{6a^2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (((-8*a^2 + a*b + 3*b^2 + (4*a^2 - 2*a*b - 4*b^2)*Cosh[2*(e + f*x)] + b*(a
+ b)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] + (4*I)*a*(a
 + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] -
 (2*I)*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e +
 f*x), b/a])/(6*a^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [**A**]
time = 2.08, size = 456, normalized size = 1.71

method	result
default	$2\sqrt{-\frac{b}{a}}\, ab(\sinh^6(fx+e))+2\sqrt{-\frac{b}{a}}\, b^2(\sinh^6(fx+e))+b\sqrt{\frac{a+b(\sinh^2(fx+e))}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\Big(\sinh(fx+e)$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6+2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+
b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))*a*sinh(f*x+e)^3+2*((a+b*sinh(f*x+e)^2)/a)^(1/2
)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b
^2*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3-2*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4+3*(-
1/a*b)^(1/2)*a*b*sinh(f*x+e)^4+2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+(-1/a*b)^
(1/2)*a^2*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2
)/a^2/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2131 vs. $2(271) = 542$.
time = 0.19, size = 2131, normalized size = 7.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(((2*a^2 + a*b - b^2)*cosh(f*x + e)^6 + 6*(2*a^2 + a*b - b^2)*cosh(f*x
 + e)*sinh(f*x + e)^5 + (2*a^2 + a*b - b^2)*sinh(f*x + e)^6 - 3*(2*a^2 + a*
```

```
b - b^2)*cosh(f*x + e)^4 + 3*(5*(2*a^2 + a*b - b^2)*cosh(f*x + e)^2 - 2*a^2
 - a*b + b^2)*sinh(f*x + e)^4 + 4*(5*(2*a^2 + a*b - b^2)*cosh(f*x + e)^3 -
3*(2*a^2 + a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(2*a^2 + a*b - b^2
)*cosh(f*x + e)^2 + 3*(5*(2*a^2 + a*b - b^2)*cosh(f*x + e)^4 - 6*(2*a^2 + a
*b - b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b - b^2)*sinh(f*x + e)^2 - 2*a^2 - a*
b + b^2 + 6*((2*a^2 + a*b - b^2)*cosh(f*x + e)^5 - 2*(2*a^2 + a*b - b^2)*co
sh(f*x + e)^3 + (2*a^2 + a*b - b^2)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b
+ b^2)*cosh(f*x + e)^6 + 6*(a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b
 + b^2)*sinh(f*x + e)^6 - 3*(a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a*b + b^2)*
cosh(f*x + e)^2 - a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a*b + b^2)*cosh(f*x +
e)^3 - 3*(a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(a*b + b^2)*cosh(f*
x + e)^2 + 3*(5*(a*b + b^2)*cosh(f*x + e)^4 - 6*(a*b + b^2)*cosh(f*x + e)^2
 + a*b + b^2)*sinh(f*x + e)^2 - a*b - b^2 + 6*((a*b + b^2)*cosh(f*x + e)^5
- 2*(a*b + b^2)*cosh(f*x + e)^3 + (a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))
*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/
b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*
x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2
- a*b)/b^2))/b^2) - ((2*a^2 - a*b)*cosh(f*x + e)^6 + 6*(2*a^2 - a*b)*cosh(f
*x + e)*sinh(f*x + e)^5 + (2*a^2 - a*b)*sinh(f*x + e)^6 - 3*(2*a^2 - a*b)*c
osh(f*x + e)^4 + 3*(5*(2*a^2 - a*b)*cosh(f*x + e)^2 - 2*a^2 + a*b)*sinh(f*x
 + e)^4 + 4*(5*(2*a^2 - a*b)*cosh(f*x + e)^3 - 3*(2*a^2 - a*b)*cosh(f*x + e
))*sinh(f*x + e)^3 + 3*(2*a^2 - a*b)*cosh(f*x + e)^2 + 3*(5*(2*a^2 - a*b)*c
osh(f*x + e)^4 - 6*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f*x +
e)^2 - 2*a^2 + a*b + 6*((2*a^2 - a*b)*cosh(f*x + e)^5 - 2*(2*a^2 - a*b)*cos
h(f*x + e)^3 + (2*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b + 2*b^2
)*cosh(f*x + e)^6 + 6*(a*b + 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b +
2*b^2)*sinh(f*x + e)^6 - 3*(a*b + 2*b^2)*cosh(f*x + e)^4 + 3*(5*(a*b + 2*b^
2)*cosh(f*x + e)^2 - a*b - 2*b^2)*sinh(f*x + e)^4 + 4*(5*(a*b + 2*b^2)*cosh
(f*x + e)^3 - 3*(a*b + 2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(a*b + 2*b
^2)*cosh(f*x + e)^2 + 3*(5*(a*b + 2*b^2)*cosh(f*x + e)^4 - 6*(a*b + 2*b^2)*
cosh(f*x + e)^2 + a*b + 2*b^2)*sinh(f*x + e)^2 - a*b - 2*b^2 + 6*((a*b + 2*
b^2)*cosh(f*x + e)^5 - 2*(a*b + 2*b^2)*cosh(f*x + e)^3 + (a*b + 2*b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^
2 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^
2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4
*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*((a*b + b^2)*cosh(f*x
+ e)^5 + 5*(a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^4 + (a*b + b^2)*sinh(f*x
 + e)^5 - (3*a*b + 2*b^2)*cosh(f*x + e)^3 + (10*(a*b + b^2)*cosh(f*x + e)^2
 - 3*a*b - 2*b^2)*sinh(f*x + e)^3 + b^2*cosh(f*x + e) + (10*(a*b + b^2)*cos
h(f*x + e)^3 - 3*(3*a*b + 2*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (5*(a*b +
 b^2)*cosh(f*x + e)^4 - 3*(3*a*b + 2*b^2)*cosh(f*x + e)^2 + b^2)*sinh(f*x +
 e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*b*f*cosh(f*x +
e)^6 + 6*a^2*b*f*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*b*f*sinh(f*x + e)^6 -
3*a^2*b*f*cosh(f*x + e)^4 + 3*a^2*b*f*cosh(f*x + e)^2 + 3*(5*a^2*b*f*cosh(f
```

```
*x + e)^2 - a^2*b*f)*sinh(f*x + e)^4 - a^2*b*f + 4*(5*a^2*b*f*cosh(f*x + e)
^3 - 3*a^2*b*f*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(5*a^2*b*f*cosh(f*x + e)^
4 - 6*a^2*b*f*cosh(f*x + e)^2 + a^2*b*f)*sinh(f*x + e)^2 + 6*(a^2*b*f*cosh(
f*x + e)^5 - 2*a^2*b*f*cosh(f*x + e)^3 + a^2*b*f*cosh(f*x + e))*sinh(f*x +
e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csch(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. 2(271) = 542.
time = 1.28, size = 1118, normalized size = 4.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(a + b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sqrt(-a))*e^(
-4*e)/(sqrt(-a)*a) - 2*(3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a + 3*(sqrt(b)*e^(2*f*
x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*
e) + b))^5*b - 15*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a*sqrt(b) - 15*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^4*b^(3/2) - 32*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2 - 66*(sqrt(b)*e^(2*
f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
2*e) + b))^3*a*b + 30*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^2 - 96*(sqrt(b)*e^(2*f*x
 + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e
) + b))^2*a^2*sqrt(b) + 162*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*
e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(3/2) - 30*(sqrt
(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^
(2*f*x + 2*e) + b))^2*b^(5/2) - 48*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f
*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^3 + 240*(sqrt
```

```
(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^
(2*f*x + 2*e) + b))*a^2*b - 81*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b^2 + 15*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))*b^3 + 48*a^3*sqrt(b) + 16*a^2*b^(3/2) - 3*a*b^(5/2) - 3*b^
(7/2))*e^(-4*e)/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - 2*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*s
qrt(b) - 4*a + b)^3*a))*e^(5*e)/f^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(e+f\,x\right)^4\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

3.107 $\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{b^{3/2}f}-\frac{a\cosh(e+fx)}{(a-b)bf\sqrt{a-b+b\cosh^2(e+fx)}}$$

```
[Out] arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/b^(3/2)/f-a*cosh(f
*x+e)/(a-b)/b/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3265, 393, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{b^{3/2}f}-\frac{a\cosh(e+fx)}{bf(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(b^(3/2)*f
) - (a*Cosh[e + f*x])/((a - b)*b*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{(a-b+bx^2)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)}{(a-b)bf\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\text{Subst}\Big(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx, x, \text{c}}{bf}$$

$$= -\frac{a\cosh(e+fx)}{(a-b)bf\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\text{Subst}\Bigg(\int \frac{1}{1-bx^2}\,dx, x, \frac{\text{co}}{\sqrt{a-b+}}}{bf}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\cosh(e+fx)}{(a-b)bf\sqrt{a-b+b\cosh^2(e+}}$$

Mathematica [A]
time = 0.30, size = 98, normalized size = 1.18

$$-\frac{\sqrt{2}\,a\cosh(e+fx)}{(a-b)bf\sqrt{2a-b+b\cosh(2(e+fx))}} + \frac{\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Sqrt[2]*a*Cosh[e + f*x])/((a - b)*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]
])) + Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]]]/(b^(3/2)*f)
```

Maple [**A**]
time = 1.10, size = 146, normalized size = 1.76

method	result
default	$\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(\frac{\ln\left(\frac{\frac{a}{2}+\frac{b}{2}+b\left(\sinh^2(fx+e)\right)}{\sqrt{b}}+\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cos\right.}\right.}{2b^{\frac{3}{2}}}\right.}{\cosh(fx+e)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(1/2/b^(3/2)*ln((1/2*a+1/2*b+b*si
nh(f*x+e)^2)/b^(1/2)+((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))-a/b*cosh(f*
x+e)^2/(a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1181 vs. $2(75) = 150$.
time = 0.57, size = 3038, normalized size = 36.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x +
 e)^3 + (a*b - b^2)*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)
^2 + 2*(3*(a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^
2 + a*b - b^2 + 4*((a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh
(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*
x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(
```

```
14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 +
 (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*
a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 +
 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b
- 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*c
osh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x +
e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*
x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)
^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2
 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*s
inh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 +
18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh
(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x
 + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*
b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 +
b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(
f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x
+ e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x +
 e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a*b - b^2)*
cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)
*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2*(3*(a*b - b^
2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*(
(a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x + e))*sinh(f*x
 + e))*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3
+ b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
a - b)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) +
 b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) -
4*sqrt(2)*(a*b*cosh(f*x + e)^2 + 2*a*b*cosh(f*x + e)*sinh(f*x + e) + a*b*si
nh(f*x + e)^2 + a*b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a*
b^3 - b^4)*f*cosh(f*x + e)^4 + 4*(a*b^3 - b^4)*f*cosh(f*x + e)*sinh(f*x + e
)^3 + (a*b^3 - b^4)*f*sinh(f*x + e)^4 + 2*(2*a^2*b^2 - 3*a*b^3 + b^4)*f*cos
h(f*x + e)^2 + 2*(3*(a*b^3 - b^4)*f*cosh(f*x + e)^2 + (2*a^2*b^2 - 3*a*b^3
+ b^4)*f)*sinh(f*x + e)^2 + (a*b^3 - b^4)*f + 4*((a*b^3 - b^4)*f*cosh(f*x +
 e)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(
((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3
+ (a*b - b^2)*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2
*(3*(a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*
b - b^2 + 4*((a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x +
 e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f
*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x +
```

```
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*si
nh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*
b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x
+ e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + ((a*b - b^2)*cosh(f
*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(
f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2*(3*(a*b - b^2)*cos
h(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*((a*b -
 b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))
*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh
(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sinh\left(e+f\,x\right)}^3}{{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.108 $\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\cosh(e+fx)}{(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}$$

```
[Out] cosh(f*x+e)/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3265, 197}

$$\frac{\cosh(e+fx)}{f(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Cosh[e + f*x]/((a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a-b+bx^2)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{f}\\ &= \frac{\cosh(e+fx)}{(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}\end{aligned}$$

Mathematica [**A**]
time = 0.10, size = 43, normalized size = 1.19

$$\frac{\sqrt{2}\cosh(e+fx)}{(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*Cosh[e + f*x])/((a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [**A**]
time = 0.70, size = 32, normalized size = 0.89

method	result	size
default	$\frac{\cosh(fx+e)}{(a-b)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\, f}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] cosh(f*x+e)/(a-b)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(36) = 72.
time = 0.50, size = 246, normalized size = 6.83

$$\frac{b^{2}e^{(-6\,fx-6\,e)}+2\,ab-b^{2}+(8\,a^{2}-8\,ab+3\,b^{2})e^{(-2\,fx-2\,e)}+3\,(2\,ab-b^{2})e^{(-4\,fx-4\,e)}}{2\,(a^{2}-ab)(2\,(2\,a-b)e^{(-2\,fx-2\,e)}+be^{(-4\,fx-4\,e)}+b)^{\frac{3}{2}}f}+\frac{b^{2}+3\,(2\,ab-b^{2})e^{(-2\,fx-2\,e)}+(8\,a^{2}-8\,ab+3\,b^{2})e^{(-4\,fx-4\,e)}+(2\,ab-b^{2})e^{(-6\,fx-6\,e)}}{2\,(a^{2}-ab)(2\,(2\,a-b)e^{(-2\,fx-2\,e)}+be^{(-4\,fx-4\,e)}+b)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*e^(-6*f*x - 6*e) + 2*a*b - b^2 + (8*a^2 - 8*a*b + 3*b^2)*e^(-2*f*x
 - 2*e) + 3*(2*a*b - b^2)*e^(-4*f*x - 4*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2
*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(3/2)*f) + 1/2*(b^2 + 3*(2*a*b - b^2)
*e^(-2*f*x - 2*e) + (8*a^2 - 8*a*b + 3*b^2)*e^(-4*f*x - 4*e) + (2*a*b - b^2
)*e^(-6*f*x - 6*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*
x - 4*e) + b)^(3/2)*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(34) = 68.
time = 0.45, size = 296, normalized size = 8.22

$$\frac{\sqrt{2}\left(\cosh\left(fx+e\right)^{2}+2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^{2}+1\right)\sqrt{\frac{b\cosh\left(fx+e\right)^{2}+b\sinh\left(fx+e\right)^{2}+2\,a-b}{\cosh\left(fx+e\right)^{2}-2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^{2}}}}{(ab-b^{2})f\cosh\left(fx+e\right)^{4}+4\,(ab-b^{2})f\cosh\left(fx+e\right)\sinh\left(fx+e\right)^{3}+(ab-b^{2})f\sinh\left(fx+e\right)^{4}+2\,(2\,a^{2}-3\,ab+b^{2})f\cosh\left(fx+e\right)^{2}+2\,\left(3\,(ab-b^{2})f\cosh\left(fx+e\right)^{2}+(2\,a^{2}-3\,ab+b^{2})f\right)\sinh\left(fx+e\right)^{2}+(ab-b^{2})f+4\,\left((ab-b^{2})f\cosh\left(fx+e\right)^{3}+(2\,a^{2}-3\,ab+b^{2})f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
+ 1)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*f*cosh(f
*x + e)^4 + 4*(a*b - b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*f*s
inh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a*b - b^
2)*f*cosh(f*x + e)^2 + (2*a^2 - 3*a*b + b^2)*f)*sinh(f*x + e)^2 + (a*b - b^
2)*f + 4*((a*b - b^2)*f*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*f*cosh(f*x
+ e))*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sinh(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(34) = 68.
time = 0.62, size = 99, normalized size = 2.75

$$\frac{\frac{ae^{\left(2\,fx+4\,e\right)}}{a^{2}e^{\left(2\,e\right)}-abe^{\left(2\,e\right)}}+\frac{ae^{\left(2\,e\right)}}{a^{2}e^{\left(2\,e\right)}-abe^{\left(2\,e\right)}}}{\sqrt{be^{\left(4\,fx+4\,e\right)}+4\,ae^{\left(2\,fx+2\,e\right)}-2\,be^{\left(2\,fx+2\,e\right)}+b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (a*e^(2*f*x + 4*e)/(a^2*e^(2*e) - a*b*e^(2*e)) + a*e^(2*e)/(a^2*e^(2*e) - a
*b*e^(2*e)))/(sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b)*f)
```

Mupad [**B**]
time = 0.92, size = 191, normalized size = 5.31

$$-\frac{\mathrm{e}^{e+f\,x}\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}\,\left(\frac{2\,\mathrm{e}^{e+f\,x}\,\sinh(e+f\,x)\,\left(b\,\left(2\,a-b\right)-b\,\left(4\,a-2\,b\right)\right)}{f\,\left(a\,b^2-a^2\,b\right)}+\frac{2\,b^2\,\cosh(e+f\,x)\,\mathrm{e}^{e+f\,x}}{f\,\left(a\,b^2-a^2\,b\right)}+\frac{b\,\mathrm{e}^{2\,e+2\,f\,x}\,\left(4\,a-2\,b\right)}{f\,\left(a\,b^2-a^2\,b\right)}\right)}{4\,a\,\mathrm{e}^{2\,e+2\,f\,x}-2\,b\,\mathrm{e}^{2\,e+2\,f\,x}+2\,b\,\mathrm{e}^{2\,e+2\,f\,x}\,\cosh\left(2\,e+2\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] -(exp(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)*((2*exp(e + f*x)*sinh(e + f*x)
*(b*(2*a - b) - b*(4*a - 2*b)))/(f*(a*b^2 - a^2*b)) + (2*b^2*cosh(e + f*x)*
exp(e + f*x))/(f*(a*b^2 - a^2*b)) + (b*exp(2*e + 2*f*x)*(4*a - 2*b))/(f*(a*
b^2 - a^2*b))))/(4*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) + 2*b*exp(2*e
+ 2*f*x)*cosh(2*e + 2*f*x))
```

3.109 $\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}$$

```
[Out] -arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cosh(
f*x+e)/a/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3265, 390, 385, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{3/2}f}-\frac{b\cosh(e+fx)}{af(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(a^(3/2)
*f)) - (b*Cosh[e + f*x])/(a*(a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{b\cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}}\,dx,\right.}{af}$$

$$= -\frac{b\cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cos}{\sqrt{a-b+b}}\right.}{af}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+}}$$

Mathematica [A]
time = 0.27, size = 98, normalized size = 1.17

$$\frac{-\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)-\frac{\sqrt{2}\sqrt{a}\,b\cosh(e+fx)}{(a-b)\sqrt{2a-b+b\cosh(2(e+fx))}}}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]]] - (Sqrt[2]*Sqrt[a]*b*Cosh[e + f*x])/((a - b)*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]]]))/(a^(3/2)*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(76) = 152$.
time = 1.23, size = 154, normalized size = 1.83

method	result
default	$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(-\frac{b\left(\cosh^{2}\left(fx+e\right)\right)}{a(a-b)\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right.}}\right.}{\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/a*b*cosh(f*x+e)^2/(a-b)/((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)-1/2/a^(3/2)*ln((2*a+(a+b)*sinh(f*x+e)^
2+2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2))/cosh
(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 769 vs. $2(76) = 152$.
time = 0.48, size = 1641, normalized size = 19.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x +
 e)^3 + (a*b - b^2)*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)
^2 + 2*(3*(a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^
2 + a*b - b^2 + 4*((a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)
*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh
(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*s
qrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 +
 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*c
osh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x
+ e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x
+ e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh
(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*(a*b*cosh(f*x + e)^2 + 2*a*b*cos
h(f*x + e)*sinh(f*x + e) + a*b*sinh(f*x + e)^2 + a*b)*sqrt((b*cosh(f*x + e)
^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f
*x + e) + sinh(f*x + e)^2)))/((a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 4*(a^3*
b - a^2*b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^3*b - a^2*b^2)*f*sinh(f*x
 + e)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^3*b - a
^2*b^2)*f*cosh(f*x + e)^2 + (2*a^4 - 3*a^3*b + a^2*b^2)*f)*sinh(f*x + e)^2
+ (a^3*b - a^2*b^2)*f + 4*((a^3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^4 - 3
*a^3*b + a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)), (((a*b - b^2)*cosh(f*x +
 e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x
+ e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2*(3*(a*b - b^2)*cosh(f*
x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*((a*b - b^2
)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))*sqr
t(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sin
h(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x
 + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b
)) - sqrt(2)*(a*b*cosh(f*x + e)^2 + 2*a*b*cosh(f*x + e)*sinh(f*x + e) + a*b
*sinh(f*x + e)^2 + a*b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
 b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(
(a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 4*(a^3*b - a^2*b^2)*f*cosh(f*x + e)*s
inh(f*x + e)^3 + (a^3*b - a^2*b^2)*f*sinh(f*x + e)^4 + 2*(2*a^4 - 3*a^3*b +
 a^2*b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (2
*a^4 - 3*a^3*b + a^2*b^2)*f)*sinh(f*x + e)^2 + (a^3*b - a^2*b^2)*f + 4*((a^
3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^4 - 3*a^3*b + a^2*b^2)*f*cosh(f*x +
 e))*sinh(f*x + e))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csch(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(76) = 152.
time = 0.57, size = 198, normalized size = 2.36

$$-\frac{\left(\frac{\frac{a^2be^{(2fx+4e)}}{a^4e^{(6e)}-a^3be^{(6e)}}+\frac{a^2be^{(2e)}}{a^4e^{(6e)}-a^3be^{(6e)}}}{\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}}-\frac{2\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}-\sqrt{b}}{2\sqrt{-a}}\right)e^{(-4e)}}{\sqrt{-a}a}\right)e^{(4e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^2*b*e^(2*f*x + 4*e)/(a^4*e^(6*e) - a^3*b*e^(6*e)) + a^2*b*e^(2*e)/(a^4
*e^(6*e) - a^3*b*e^(6*e)))/sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b) - 2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/
sqrt(-a))*e^(-4*e)/(sqrt(-a)*a))*e^(4*e)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(e+f\,x)\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

3.110 $\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2a^{5/2}f}-\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}-\frac{\coth(e+fx)\mathrm{cs}}{2af\sqrt{a-b+b\,\mathrm{co}}}$$

```
[Out] 1/2*(a+3*b)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(5/2
)/f-1/2*(a-3*b)*b*cosh(f*x+e)/a^2/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(1/2)-1/2*c
oth(f*x+e)*csch(f*x+e)/a/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$,
Rules used = {3265, 425, 541, 12, 385, 212}

$$\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2a^{5/2}f}-\frac{b(a-3b)\cosh(e+fx)}{2a^2f(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/(2*a^(5/2)*f) - ((a - 3*b)*b*Cosh[e + f*x])/(2*a^2*(a - b)*f*Sqrt[a - b +
 b*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x])/(2*a*f*Sqrt[a - b + b*
Cosh[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\Big(\int \frac{1}{(1-x^2)^2(a-b+bx^2)^{3/2}}\,dx, x, \cosh(e+fx)\Big)}{f}$$

$$= -\frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\text{Subst}\Big(\int \frac{a+b+2bx^2}{(1-x^2)(a-b+bx^2)^{3/2}}\,dx, x, \cosh(e}{2af}$$

$$= -\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}}$$

$$= -\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}}$$

$$= -\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}}$$

$$= \frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{(a-3b)b\cosh(e}{2a^2(a-b)f\sqrt{a-b+b}}$$

Mathematica [A]
time = 0.50, size = 134, normalized size = 0.96

$$\frac{\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)}{a^{5/2}} - \frac{(2a^2-3ab+3b^2+(a-3b)b\cosh(2(e+fx)))\coth(e+fx)\operatorname{csch}(e+fx)}{a^2(a-b)\sqrt{4a-2b+2b\cosh(2(e+fx))}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (((a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2
*(e + f*x)]]])/a^(5/2) - ((2*a^2 - 3*a*b + 3*b^2 + (a - 3*b)*b*Cosh[2*(e +
f*x)])*Coth[e + f*x]*Csch[e + f*x])/(a^2*(a - b)*Sqrt[4*a - 2*b + 2*b*Cosh[
2*(e + f*x)]]))/(2*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(123) = 246$.
time = 5.72, size = 251, normalized size = 1.81

method	result
default	$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(\frac{b^{2}\left(\cosh^{2}\left(fx+e\right)\right)}{a^{2}\left(a-b\right)\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(b^2/a^2*cosh(f*x+e)^2/(a-b)/((a+
b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)-1/2/a^2/sinh(f*x+e)^2*((a+b*sinh(f*x+
e)^2)*cosh(f*x+e)^2)^(1/2)+1/4/a^(3/2)*ln((2*a+(a+b)*sinh(f*x+e)^2+2*a^(1/2
)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2)+3/4/a^(5/2)*b*l
n((2*a+(a+b)*sinh(f*x+e)^2+2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1
/2))/sinh(f*x+e)^2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2169 vs. $2(123) = 246$.
time = 0.64, size = 4441, normalized size = 31.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^8 + 8*(a^2*b + 2*a*b^2 - 3*b
^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b + 2*a*b^2 - 3*b^3)*sinh(f*x + e)
```

```
^8 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 4*(a^3 + a^2*b - 5
*a*b^2 + 3*b^3 + 7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)
^6 + 8*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + 3*(a^3 + a^2*b - 5*a*
b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(4*a^3 + 5*a^2*b - 18*a*b^2
 + 9*b^3)*cosh(f*x + e)^4 + 2*(35*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4
 - 4*a^3 - 5*a^2*b + 18*a*b^2 - 9*b^3 + 30*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*
cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x
+ e)^5 + 10*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^3 - (4*a^3 + 5*a^
2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b + 2*a*b^2 -
3*b^3 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^2 + 4*(7*(a^2*b + 2
*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 15*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f
*x + e)^4 + a^3 + a^2*b - 5*a*b^2 + 3*b^3 - 3*(4*a^3 + 5*a^2*b - 18*a*b^2 +
 9*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b + 2*a*b^2 - 3*b^3)*cos
h(f*x + e)^7 + 3*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^5 - (4*a^3 +
 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^3 + (a^3 + a^2*b - 5*a*b^2 + 3*b
^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4
*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a -
 b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)
^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((
a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(c
osh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*c
osh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^
3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((a^2*b - 3*a*b^2)*cosh(
f*x + e)^6 + 6*(a^2*b - 3*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2*b - 3
*a*b^2)*sinh(f*x + e)^6 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + (4*
a^3 - 5*a^2*b + 3*a*b^2 + 15*(a^2*b - 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x +
e)^4 + 4*(5*(a^2*b - 3*a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 5*a^2*b + 3*a*b^2)
*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b - 3*a*b^2 + (4*a^3 - 5*a^2*b + 3*a*
b^2)*cosh(f*x + e)^2 + (15*(a^2*b - 3*a*b^2)*cosh(f*x + e)^4 + 4*a^3 - 5*a^
2*b + 3*a*b^2 + 6*(4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e
)^2 + 2*(3*(a^2*b - 3*a*b^2)*cosh(f*x + e)^5 + 2*(4*a^3 - 5*a^2*b + 3*a*b^2
)*cosh(f*x + e)^3 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e
))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - a^3*b^2)*f*c
osh(f*x + e)^8 + 8*(a^4*b - a^3*b^2)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^4
*b - a^3*b^2)*f*sinh(f*x + e)^8 + 4*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x +
e)^6 + 4*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)
*f)*sinh(f*x + e)^6 - 2*(4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^4 + 8
*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^3 + 3*(a^5 - 2*a^4*b + a^3*b^2)*f*cos
h(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^4 + 3
0*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^
2)*f)*sinh(f*x + e)^4 + 4*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^2 + 8*(
7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^5 + 10*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh
```

```
(f*x + e)^3 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^
3 + 4*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^6 + 15*(a^5 - 2*a^4*b + a^3*b^2)
*f*cosh(f*x + e)^4 - 3*(4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^2 + (a
^5 - 2*a^4*b + a^3*b^2)*f)*sinh(f*x + e)^2 + (a^4*b - a^3*b^2)*f + 8*((a^4*
b - a^3*b^2)*f*cosh(f*x + e)^7 + 3*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e
)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^3 + (a^5 - 2*a^4*b + a^
3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((a^2*b + 2*a*b^2 - 3*b^3)*co
sh(f*x + e)^8 + 8*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)*sinh(f*x + e)^7 +
 (a^2*b + 2*a*b^2 - 3*b^3)*sinh(f*x + e)^8 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b
^3)*cosh(f*x + e)^6 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3 + 7*(a^2*b + 2*a*b^2
 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b + 2*a*b^2 - 3*b^3)
*cosh(f*x + e)^3 + 3*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*
x + e)^5 - 2*(4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^4 + 2*(35*(
a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4 - 4*a^3 - 5*a^2*b + 18*a*b^2 - 9*b
^3 + 30*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 +
8*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 10*(a^3 + a^2*b - 5*a*b^2
+ 3*b^3)*cosh(f*x + e)^3 - (4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x +
e))*sinh(f*x + e)^3 + a^2*b + 2*a*b^2 - 3*b^3 + 4*(a^3 + a^2*b - 5*a*b^2 +
3*b^3)*cosh(f*x + e)^2 + 4*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 1
5*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csch(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\mathrm{sinh}\left(e+f\,x\right)}^3\,{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

3.111 $\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=341

$$-\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}}+\frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)b^2f}+\frac{(8a^2-3ab}{}$$

```
[Out] -a*cosh(f*x+e)*sinh(f*x+e)^3/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(4*a-b
)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/b^2/f+1/3*(8*a^2-
3*a*b-2*b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(
sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/(a-b)/b^3/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(
4*a-b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2
)^(1/2)/(a-b)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(8*a^2-
3*a*b-2*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/b^3/f
```

Rubi [A]
time = 0.23, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3267, 481, 596, 545, 429, 506, 422}

$$\frac{(8a^2-3ab-2b^2)\,\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3b^3f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(8a^2-3ab-2b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^3f(a-b)}-\frac{(4a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3b^2f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(4a-b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^2f(a-b)}-\frac{a\sinh^3(e+fx)\cosh(e+fx)}{bf(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Cosh[e + f*x]*Sinh[e + f*x]^3)/((a - b)*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) + ((4*a - b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(
3*(a - b)*b^2*f) + ((8*a^2 - 3*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]]
, 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*b^3*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((4*a - b)*EllipticF[ArcTa
n[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a
 - b)*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((8*a^2 -
3*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)*b^3*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\Bigg(\int \frac{x^6}{\sqrt{1+x^2}\,(a+bx^2)^{3/2}}\,dx, x, \sinh(}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\mathrm{Su}}{}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{}}{3(a-b)b^2f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{}}{3(a-b)b^2f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{}}{3(a-b)b^2f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{}}{3(a-b)b^2f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.84, size = 211, normalized size = 0.62

$$\frac{2i\sqrt{2}\,a(8a^2-3ab-2b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) - 2i\sqrt{2}\,a(8a^2-7ab-b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) - b(-8a^2+3ab-b^2+b(-a+b)\cosh(2(e+fx)))\sinh(2(e+fx))}{6(a-b)b^3f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*I)*Sqrt[2]*a*(8*a^2 - 3*a*b - 2*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)
])/a]*EllipticE[I*(e + f*x), b/a] - (2*I)*Sqrt[2]*a*(8*a^2 - 7*a*b - b^2)*S
```

```
qrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] - b*(-8*
a^2 + 3*a*b - b^2 + b*(-a + b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*(a
- b)*b^3*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.27, size = 500, normalized size = 1.47

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, ab\left(\sinh^{5}(fx+e)\right)-\sqrt{-\frac{b}{a}}\, b^{2}\left(\sinh^{5}(fx+e)\right)+4\sqrt{-\frac{b}{a}}\, a^{2}\left(\sinh^{3}(fx+e)\right)-\sqrt{-\frac{b}{a}}\, b^{2}\left(\sinh^{3}(fx+e)\right)+4a^{2}\sqrt{\frac{a+b\left(\sinh^{2}(\right.}{a}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+4*(-
1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+4*a^2*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)
*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*((a+b*sinh(f*x+e)^
2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b
)^(1/2))*b^2-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elliptic
E(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+3*((a+b*sinh(f*x+e)^2)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a*b+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)-(-1/a
*b)^(1/2)*a*b*sinh(f*x+e))/b^2/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f
*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.11, size = 55, normalized size = 0.16

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\sinh\left(fx+e\right)^{6}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^6/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1143 vs. 2(347) = 694.
time = 2.87, size = 1143, normalized size = 3.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(((a^4*b^6*e^(17*e) - 2*a^3*b^7*e^(17*e) + a^2*b^8*e^(17*e))*e^(2*f*x)
/(a^4*b^7*e^(12*e) - 2*a^3*b^8*e^(12*e) + a^2*b^9*e^(12*e)) + 2*(14*a^5*b^5
*e^(15*e) - 17*a^4*b^6*e^(15*e) + 4*a^3*b^7*e^(15*e) - a^2*b^8*e^(15*e))/(a
^4*b^7*e^(12*e) - 2*a^3*b^8*e^(12*e) + a^2*b^9*e^(12*e)))*e^(2*f*x) + (48*a
^6*b^4*e^(13*e) - 72*a^5*b^5*e^(13*e) + 25*a^4*b^6*e^(13*e) - 2*a^3*b^7*e^(
13*e) + a^2*b^8*e^(13*e))/(a^4*b^7*e^(12*e) - 2*a^3*b^8*e^(12*e) + a^2*b^9*
e^(12*e)))*e^(f*x)/(sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b)*f) + 1/24*(2*(11*a^2*sqrt(b)*e^e - 3*a*b^(3/2)*e^e - 2*b^(
5/2)*e^e)*log(abs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*
e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/2)))/(a*
b^3 - b^4) + 2*(21*a^3*e^e - 24*a^2*b*e^e - 5*a*b^2*e^e - 4*b^3*e^e)*arctan
(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))/((a*b^3 - b^4)*sqrt(-b)) - 3*(14*(sqrt
(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^
(2*f*x + 2*e) + b))^3*a^2*e^e - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*b*e^e - 3*(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b
*e^(2*f*x + 2*e) + b))^3*b^2*e^e + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4
*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(3/2)*e
^e + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
```

```
*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(5/2)*e^e - 18*(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))
*a^2*b*e^e + 6*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b^2*e^e + (sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)*b^3*e^e - 8*a*b^(5/2)*e^e - 2*b^(7/2)*e^e)/(((sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 -
 b)^2*b^3))/f^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(e+f\,x)^6}{{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.112 $\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=256

$$-\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{(2a-b)E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e}{(a-b)b^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] -a*cosh(f*x+e)*sinh(f*x+e)/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)-(2*a-b)*(1/(
1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+si
nh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-
b)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(1/(1+sinh(f*x+e)^2))^
(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2)
,(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/b/f/(sech(f*x+e
)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(2*a-b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x
+e)/(a-b)/b^2/f
```

Rubi [A]
time = 0.16, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3267, 481, 545, 429, 506, 422}

$$-\frac{(2a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{b^2f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{bf(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b^2f(a-b)}-\frac{a\sinh(e+fx)\cosh(e+fx)}{bf(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/((a - b)*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
 + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*
Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(
(a - b)*b^2*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(e+fx)}{\big(a+b\sinh^2(e+fx)\big)^{3/2}}\,dx = \frac{\Big(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\Big)\,\mathrm{Subst}\Big(\int \frac{x^4}{\sqrt{1+x^2}\,(a+bx^2)^{3/2}}\,dx, x, \sinh(}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\Big(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\Big)\,\mathrm{Su}}{}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\Big(a\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\Big)\,\mathrm{S}}{}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{F\big(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\big)\operatorname{sech}(}{(a-b)bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\,(}{}}}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(2a-b)E\big(\tan^{-1}(\sinh(e+fx))|1-}{(a-b)b^2f\sqrt{\frac{\operatorname{sech}^2(e+}{}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.72, size = 156, normalized size = 0.61

$$\frac{a\left(-2i(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+4i(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-\sqrt{2}\,b\sinh(2(e+fx))\right)}{2(a-b)b^2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*((-2*I)*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
 + f*x), b/a] + (4*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellip
ticF[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)]))/(2*(a - b)*b^2*f*Sqr
t[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.18, size = 313, normalized size = 1.22

method	result

default	$-\frac{\sqrt{-\frac{b}{a}}\, a\left(\cosh^{2}(fx+e)\right)\sinh(fx+e)+a\sqrt{\frac{b\left(\cosh^{2}(fx+e)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\text{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*((-1/a*b)^(1/2)*a*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2+(a-b)
/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+
(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sin
h(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.12, size = 55, normalized size = 0.21

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\sinh\left(fx+e\right)^{4}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Val
ue
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(e+f\,x)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.113 $\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}}+\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)bf\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}-\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+}{a}}}{bf\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] cosh(f*x+e)*sinh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)+I*(cos(I*e+I*f*x)
^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*
x+e)^2)^(1/2)/(a-b)/b/f/(1+b*sinh(f*x+e)^2/a)^(1/2)-I*(cos(I*e+I*f*x)^2)^(1
/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))*(1+b*sinh(f*x+e)^2
/a)^(1/2)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3252, 3251, 3257, 3256, 3262, 3261}

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}+\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) + (I*
EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*b*f*Sqrt[
1 + (b*Sinh[e + f*x]^2)/a]) - (I*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Si
nh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
```

```
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{a+a\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{a(a-b)}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{b} - \frac{\int \sqrt{a+}}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\sqrt{a+b\sinh^2(e+fx)}\,\int\sqrt{1+\frac{b\sinh^2(e}{a}}}{(a-b)b\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)bf\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} -$$

Mathematica [**A**]
time = 0.32, size = 151, normalized size = 0.87

$$\frac{i\sqrt{2}\,a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-i\sqrt{2}\,(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+b\sinh(2(e+fx))}{(a-b)bf\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x),
 b/a] - I*Sqrt[2]*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF
[I*(e + f*x), b/a] + b*Sinh[2*(e + f*x)])/((a - b)*b*f*Sqrt[4*a - 2*b + 2*b
*Cosh[2*(e + f*x)]])
```

Maple [**A**]
time = 1.01, size = 127, normalized size = 0.73

method	result

default	$-\frac{-\sqrt{-\frac{b}{a}}\,(\cosh^2(fx+e))\sinh(fx+e)+\sqrt{\frac{b(\cosh^2(fx+e))}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\text{EllipticE}\Bigg(\sinh(fx+e)\sqrt{-\frac{b}{a}},}{(a-b)\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-1/a*b)^(1/2)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))
/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1155 vs. $2(191) = 382$.
time = 0.10, size = 1155, normalized size = 6.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(((2*a*b - b^2)*cosh(f*x + e)^4 + 4*(2*a*b - b^2)*cosh(f*x + e)*sinh(f*x +
 e)^3 + (2*a*b - b^2)*sinh(f*x + e)^4 + 2*(4*a^2 - 4*a*b + b^2)*cosh(f*x +
e)^2 + 2*(3*(2*a*b - b^2)*cosh(f*x + e)^2 + 4*a^2 - 4*a*b + b^2)*sinh(f*x +
 e)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(f*x + e)^3 + (4*a^2 - 4*a*b + b
^2)*cosh(f*x + e))*sinh(f*x + e) - 2*(b^2*cosh(f*x + e)^4 + 4*b^2*cosh(f*x
+ e)*sinh(f*x + e)^3 + b^2*sinh(f*x + e)^4 + 2*(2*a*b - b^2)*cosh(f*x + e)^
2 + 2*(3*b^2*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 4*(b^2*
cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a
*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(
arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(
f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b
^2) - 2*((2*a*b - b^2)*cosh(f*x + e)^4 + 4*(2*a*b - b^2)*cosh(f*x + e)*sinh
(f*x + e)^3 + (2*a*b - b^2)*sinh(f*x + e)^4 + 2*(4*a^2 - 4*a*b + b^2)*cosh(
```

```
f*x + e)^2 + 2*(3*(2*a*b - b^2)*cosh(f*x + e)^2 + 4*a^2 - 4*a*b + b^2)*sinh
(f*x + e)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(f*x + e)^3 + (4*a^2 - 4*a
*b + b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/
b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a
+ b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b -
 b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*(b^2*cosh(f*x + e)^3 + 3*b^2*co
sh(f*x + e)*sinh(f*x + e)^2 + b^2*sinh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x
+ e) + (3*b^2*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e))*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2)))/((a*b^3 - b^4)*f*cosh(f*x + e)^4 + 4*(a*
b^3 - b^4)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b^3 - b^4)*f*sinh(f*x + e)^
4 + 2*(2*a^2*b^2 - 3*a*b^3 + b^4)*f*cosh(f*x + e)^2 + 2*(3*(a*b^3 - b^4)*f*
cosh(f*x + e)^2 + (2*a^2*b^2 - 3*a*b^3 + b^4)*f)*sinh(f*x + e)^2 + (a*b^3 -
 b^4)*f + 4*((a*b^3 - b^4)*f*cosh(f*x + e)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*
f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(e+f\,x)^2}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.114 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

```
[Out] -b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-I*(cos(I*e+I
*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*si
nh(f*x+e)^2)^(1/2)/a/(a-b)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3263, 21, 3257, 3256}

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*
f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
```

```
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\int \frac{-a-b\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{a(a-b)}\\
&= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{a(a-b)}\\
&= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1+\frac{b\sinh}{}}}{a(a-b)\sqrt{1+\frac{b\sinh^2(e+}{a}}}\\
&= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx}}{a(a-b)f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}
\end{aligned}$$

Mathematica [**A**]
time = 0.12, size = 100, normalized size = 0.87

$$\frac{-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-\sqrt{2}\,b\sinh(2(e+fx))}{2a(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]])
```

Maple [**A**]
time = 1.18, size = 253, normalized size = 2.20

method	result
default	$-\sqrt{-\frac{b}{a}}\, b\left(\cosh^2(fx+e)\right)\sinh(fx+e)-a\sqrt{\frac{b\left(\cosh^2(fx+e)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-1/a*b)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)-a*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh
(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/a/(a-
b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1464 vs. 2(123) = 246.
time = 0.11, size = 1464, normalized size = 12.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (((2*a*b^2 - b^3)*cosh(f*x + e)^4 + 4*(2*a*b^2 - b^3)*cosh(f*x + e)*sinh(f*
x + e)^3 + (2*a*b^2 - b^3)*sinh(f*x + e)^4 + 2*a*b^2 - b^3 + 2*(4*a^2*b - 4
*a*b^2 + b^3)*cosh(f*x + e)^2 + 2*(4*a^2*b - 4*a*b^2 + b^3 + 3*(2*a*b^2 - b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((2*a*b^2 - b^3)*cosh(f*x + e)^3 +
 (4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e) - 2*(b^3*cosh(f*x +
 e)^4 + 4*b^3*cosh(f*x + e)*sinh(f*x + e)^3 + b^3*sinh(f*x + e)^4 + b^3 + 2
*(2*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(3*b^3*cosh(f*x + e)^2 + 2*a*b^2 - b^3
```

```
)*sinh(f*x + e)^2 + 4*(b^3*cosh(f*x + e)^3 + (2*a*b^2 - b^3)*cosh(f*x + e))
*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b
^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
 b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b -
b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((2*a^2*b - a*b^2)*cosh(f*x + e)^4 + 4
*(2*a^2*b - a*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a^2*b - a*b^2)*sinh(f
*x + e)^4 + 2*a^2*b - a*b^2 + 2*(4*a^3 - 4*a^2*b + a*b^2)*cosh(f*x + e)^2 +
 2*(4*a^3 - 4*a^2*b + a*b^2 + 3*(2*a^2*b - a*b^2)*cosh(f*x + e)^2)*sinh(f*x
 + e)^2 + 4*((2*a^2*b - a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 4*a^2*b + a*b^2)*
cosh(f*x + e))*sinh(f*x + e) + 2*((a*b^2 - b^3)*cosh(f*x + e)^4 + 4*(a*b^2
- b^3)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b^2 - b^3)*sinh(f*x + e)^4 + a*b^
2 - b^3 + 2*(2*a^2*b - 3*a*b^2 + b^3)*cosh(f*x + e)^2 + 2*(2*a^2*b - 3*a*b^
2 + b^3 + 3*(a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((a*b^2 - b^
3)*cosh(f*x + e)^3 + (2*a^2*b - 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)
)*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)
/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f
*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2
 - a*b)/b^2))/b^2) - sqrt(2)*(b^3*cosh(f*x + e)^3 + 3*b^3*cosh(f*x + e)*sin
h(f*x + e)^2 + b^3*sinh(f*x + e)^3 + (2*a*b^2 - b^3)*cosh(f*x + e) + (3*b^3
*cosh(f*x + e)^2 + 2*a*b^2 - b^3)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2)))/((a^2*b^3 - a*b^4)*f*cosh(f*x + e)^4 + 4*(a^2*b^3 -
 a*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2*b^3 - a*b^4)*f*sinh(f*x + e)
^4 + 2*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^2 + 2*(3*(a^2*b^3 -
a*b^4)*f*cosh(f*x + e)^2 + (2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*f)*sinh(f*x + e)
^2 + (a^2*b^3 - a*b^4)*f + 4*((a^2*b^3 - a*b^4)*f*cosh(f*x + e)^3 + (2*a^3*
b^2 - 3*a^2*b^3 + a*b^4)*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sinh^{2}\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(-3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.115 $\int \frac{\mathbf{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=290

$$-\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f}-\frac{(a-2b)E\big(\text{ArcTan}(\sinh(}{a^2(a-b)f\sqrt{}}$$

```
[Out] -b*coth(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-(a-2*b)*coth(f*x+e)*(a+b
*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)/f-(a-2*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+si
nh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1
/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)/f/(sech(f*x+e)^2*(a+b*
sinh(f*x+e)^2)/a)^(1/2)-b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/
2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)
*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a
)^(1/2)+(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a^2/(a-b)/f
```

Rubi [A]

time = 0.21, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3267, 483, 597, 545, 429, 506, 422}

$$-\frac{b\,\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{a^2f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(a-2b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{a^2f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f(a-b)}-\frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f(a-b)}-\frac{b\coth(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Coth[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])) - ((a - 2*b)
*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)*f) - ((a - 2*b)*El
lipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
 (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + ((a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a^2*(a - b)*f
)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
```

```
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}\,(a+bx^2)^{3/2}}\,dx, x, \sinh(}{f}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Sul}}{}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e}}{a^2(a-b)f}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e}}{a^2(a-b)f}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e}}{a^2(a-b)f}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e}}{a^2(a-b)f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.90, size = 185, normalized size = 0.64

$$\frac{-((2a^2-3ab+2b^2+(a-2b)b\cosh(2(e+fx)))\coth(e+fx)) - i\sqrt{2}\,a(a-2b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) + i\sqrt{2}\,a(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{a^2(a-b)f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-((2*a^2 - 3*a*b + 2*b^2 + (a - 2*b)*b*Cosh[2*(e + f*x)])*Coth[e + f*x]) -
 I*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*
```

```
(e + f*x), b/a] + I*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/
a]*EllipticF[I*(e + f*x), b/a])/(a^2*(a - b)*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*
(e + f*x)]])
```

Maple [A]
time = 2.41, size = 284, normalized size = 0.98

method	result
default	$-\frac{\left(\sqrt{-\frac{b}{a}}\, ab-2\sqrt{-\frac{b}{a}}\, b^{2}\right)\left(\cosh^{4}(fx+e)\right)+\left(\sqrt{-\frac{b}{a}}\, a^{2}-2\sqrt{-\frac{b}{a}}\, ab+2\sqrt{-\frac{b}{a}}\, b^{2}\right)\left(\cosh^{2}(fx+e)\right)+\sinh(fx+e)\sqrt{\frac{\cosh}{}}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(((-1/a*b)^(1/2)*a*b-2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a
^2-2*(-1/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2+sinh(f*x+e)*(co
sh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(2*EllipticF(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a-2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(
a/b)^(1/2))*b-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+2*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b))/a^2/(a-b)/(-1/a*b)^(1/2)/sin
h(f*x+e)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2829 vs. $2(304) = 608$.
time = 0.14, size = 2829, normalized size = 9.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (((2*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^6 + 6*(2*a^2*b - 5*a*b^2 + 2*b^
3)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b - 5*a*b^2 + 2*b^3)*sinh(f*x + e
```

```
)^6 + (8*a^3 - 26*a^2*b + 23*a*b^2 - 6*b^3)*cosh(f*x + e)^4 + (8*a^3 - 26*a
^2*b + 23*a*b^2 - 6*b^3 + 15*(2*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^2)*s
inh(f*x + e)^4 + 4*(5*(2*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^3 + (8*a^3
 - 26*a^2*b + 23*a*b^2 - 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - 2*a^2*b + 5
*a*b^2 - 2*b^3 - (8*a^3 - 26*a^2*b + 23*a*b^2 - 6*b^3)*cosh(f*x + e)^2 + (1
5*(2*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^4 - 8*a^3 + 26*a^2*b - 23*a*b^2
 + 6*b^3 + 6*(8*a^3 - 26*a^2*b + 23*a*b^2 - 6*b^3)*cosh(f*x + e)^2)*sinh(f*
x + e)^2 + 2*(3*(2*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^5 + 2*(8*a^3 - 26
*a^2*b + 23*a*b^2 - 6*b^3)*cosh(f*x + e)^3 - (8*a^3 - 26*a^2*b + 23*a*b^2 -
 6*b^3)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b^2 - 2*b^3)*cosh(f*x + e)^6 +
 6*(a*b^2 - 2*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b^2 - 2*b^3)*sinh(f*x
 + e)^6 + (4*a^2*b - 11*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (4*a^2*b - 11*a*b^
2 + 6*b^3 + 15*(a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(a*b
^2 - 2*b^3)*cosh(f*x + e)^3 + (4*a^2*b - 11*a*b^2 + 6*b^3)*cosh(f*x + e))*s
inh(f*x + e)^3 - a*b^2 + 2*b^3 - (4*a^2*b - 11*a*b^2 + 6*b^3)*cosh(f*x + e)
^2 + (15*(a*b^2 - 2*b^3)*cosh(f*x + e)^4 - 4*a^2*b + 11*a*b^2 - 6*b^3 + 6*(
4*a^2*b - 11*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a*b^2
 - 2*b^3)*cosh(f*x + e)^5 + 2*(4*a^2*b - 11*a*b^2 + 6*b^3)*cosh(f*x + e)^3 -
 (4*a^2*b - 11*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b
)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(ar
csin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*
x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2
) + 2*((2*a^2*b - a*b^2)*cosh(f*x + e)^6 + 6*(2*a^2*b - a*b^2)*cosh(f*x + e
)*sinh(f*x + e)^5 + (2*a^2*b - a*b^2)*sinh(f*x + e)^6 + (8*a^3 - 10*a^2*b +
 3*a*b^2)*cosh(f*x + e)^4 + (8*a^3 - 10*a^2*b + 3*a*b^2 + 15*(2*a^2*b - a*b
^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(2*a^2*b - a*b^2)*cosh(f*x + e)
^3 + (8*a^3 - 10*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 2*a^2*b
+ a*b^2 - (8*a^3 - 10*a^2*b + 3*a*b^2)*cosh(f*x + e)^2 + (15*(2*a^2*b - a*b
^2)*cosh(f*x + e)^4 - 8*a^3 + 10*a^2*b - 3*a*b^2 + 6*(8*a^3 - 10*a^2*b + 3*
a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(2*a^2*b - a*b^2)*cosh(f*x +
 e)^5 + 2*(8*a^3 - 10*a^2*b + 3*a*b^2)*cosh(f*x + e)^3 - (8*a^3 - 10*a^2*b
+ 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e) + 4*((a*b^2 - b^3)*cosh(f*x + e)^6
+ 6*(a*b^2 - b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b^2 - b^3)*sinh(f*x +
e)^6 + (4*a^2*b - 7*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + (4*a^2*b - 7*a*b^2 + 3
*b^3 + 15*(a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(a*b^2 - b^
3)*cosh(f*x + e)^3 + (4*a^2*b - 7*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x +
e)^3 - a*b^2 + b^3 - (4*a^2*b - 7*a*b^2 + 3*b^3)*cosh(f*x + e)^2 + (15*(a*b
^2 - b^3)*cosh(f*x + e)^4 - 4*a^2*b + 7*a*b^2 - 3*b^3 + 6*(4*a^2*b - 7*a*b^
2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a*b^2 - b^3)*cosh(f*x +
 e)^5 + 2*(4*a^2*b - 7*a*b^2 + 3*b^3)*cosh(f*x + e)^3 - (4*a^2*b - 7*a*b^2
+ 3*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt(
(2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((
a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*
a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*((a*b^2 -
 2*b^3)*cosh(f*x + e)^5 + 5*(a*b^2 - 2*b^3)*cosh(f*x + e)*sinh(f*x + e)^4 +
```

```
 (a*b^2 - 2*b^3)*sinh(f*x + e)^5 + 4*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^
3 + 2*(2*a^2*b - 4*a*b^2 + 2*b^3 + 5*(a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(
f*x + e)^3 + 2*(5*(a*b^2 - 2*b^3)*cosh(f*x + e)^3 + 6*(a^2*b - 2*a*b^2 + b^
3)*cosh(f*x + e))*sinh(f*x + e)^2 + (3*a*b^2 - 2*b^3)*cosh(f*x + e) + (5*(a
*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 12*(a^2*b - 2*a*b^2 + b^3
)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2)))/((a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^6 + 6*(a^3*b^2 - a^2*b^3)*f*c
osh(f*x + e)*sinh(f*x + e)^5 + (a^3*b^2 - a^2*b^3)*f*sinh(f*x + e)^6 + (4*a
^4*b - 7*a^3*b^2 + 3*a^2*b^3)*f*cosh(f*x + e)^4 + (15*(a^3*b^2 - a^2*b^3)*f
*cosh(f*x + e)^2 + (4*a^4*b - 7*a^3*b^2 + 3*a^2*b^3)*f)*sinh(f*x + e)^4 - (
4*a^4*b - 7*a^3*b^2 + 3*a^2*b^3)*f*cosh(f*x + e)^2 + 4*(5*(a^3*b^2 - a^2*b^
3)*f*cosh(f*x + e)^3 + (4*a^4*b - 7*a^3*b^2 + 3*a^2*b^3)*f*cosh(f*x + e))*s
inh(f*x + e)^3 + (15*(a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^4 + 6*(4*a^4*b - 7
*a^3*b^2 + 3*a^2*b^3)*f*cosh(f*x + e)^2 - (4*a^4*b - 7*a^3*b^2 + 3*a^2*b^3)
*f)*sinh(f*x + e)^2 - (a^3*b^2 - a^2*b^3)*f + 2*(3*(a^3*b^2 - a^2*b^3)*f*co
sh(f*x + e)^5 + 2*(4*a^4*b - 7*a^3*b^2 + 3*a^2*b^3)*f*cosh(f*x + e)^3 - (4*
a^4*b - 7*a^3*b^2 + 3*a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csch(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\sinh\left(e+f\,x\right)}^2\,{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

3.116 $\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{b^{5/2}f}-\frac{a(3a-5b)\cosh(e+fx)}{3(a-b)^2b^2f\sqrt{a-b+b\cosh^2(e+fx)}}-\frac{a\cosh(e+fx)\sinh}{3(a-b)bf\,\left(a-b+b\cos\right.}$$

```
[Out] arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/b^(5/2)/f-1/3*a*co
sh(f*x+e)*sinh(f*x+e)^2/(a-b)/b/f/(a-b+b*cosh(f*x+e)^2)^(3/2)-1/3*a*(3*a-5*
b)*cosh(f*x+e)/(a-b)^2/b^2/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3265, 424, 393, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{b^{5/2}f}-\frac{a(3a-5b)\cosh(e+fx)}{3b^2f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{a\sinh^2(e+fx)\cosh(e+fx)}{3bf(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(b^(5/2)*f
) - (a*(3*a - 5*b)*Cosh[e + f*x])/(3*(a - b)^2*b^2*f*Sqrt[a - b + b*Cosh[e
+ f*x]^2]) - (a*Cosh[e + f*x]*Sinh[e + f*x]^2)/(3*(a - b)*b*f*(a - b + b*Co
sh[e + f*x]^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-b+bx^2\right)^{5/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)bf\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-a+3b+3(a-b)x^2}{\left(a-b+bx^2\right)^{3/2}}\,dx, x, \text{c}}{3(a-b)bf}$$

$$= -\frac{a(3a-5b)\cosh(e+fx)}{3(a-b)^2b^2f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{a\cosh(e+fx)\sinh^2(e+}{3(a-b)bf\left(a-b+b\cosh^2(e-\right.}$$

$$= -\frac{a(3a-5b)\cosh(e+fx)}{3(a-b)^2b^2f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{a\cosh(e+fx)\sinh^2(e+}{3(a-b)bf\left(a-b+b\cosh^2(e-\right.}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{b^{5/2}f} - \frac{a(3a-5b)\cosh(e+fx)}{3(a-b)^2b^2f\sqrt{a-b+b\cosh^2(e}}$$

Mathematica [A]
time = 0.60, size = 130, normalized size = 0.91

$$\frac{-\frac{2\sqrt{2}\,a\cosh(e+fx)\left(3a^2-7ab+3b^2+(2a-3b)b\cosh(2(e+fx))\right)}{3(a-b)^2b^2(2a-b+b\cosh(2(e+fx)))^{3/2}}+\frac{\log\left(\sqrt{2}\sqrt{b}\cosh(e+fx)+\sqrt{2a-b+b\cosh(2(e+fx))}\right)}{b^{5/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-2*Sqrt[2]*a*Cosh[e + f*x]*(3*a^2 - 7*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*
(e + f*x)]))/(3*(a - b)^2*b^2*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2)) + Log[
Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/b^(5/2
))/f
```

Maple [A]
time = 1.40, size = 230, normalized size = 1.61

method	result
default	$\frac{\sqrt{(a+b(\sinh^2(fx+e)))(\cosh^2(fx+e))}\left(\frac{\ln\left(\frac{\frac{a}{2}+\frac{b}{2}+b\left(\sinh^2(fx+e)\right)}{\sqrt{b}}+\sqrt{(a+b(\sinh^2(fx+e)))(\mathrm{co}}\right.}{2b^{\frac{5}{2}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(1/2/b^(5/2)*ln((1/2*a+1/2*b+b*si
nh(f*x+e)^2)/b^(1/2)+((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))-2*a/b^2*cos
h(f*x+e)^2/(a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)+1/3*a^2/b^2*(2*b
*sinh(f*x+e)^2+3*a-b)*cosh(f*x+e)^2/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/
2)/(a+b*sinh(f*x+e)^2)/(a^2-2*a*b+b^2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/
2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas **[B]** Leaf count of result is larger than twice the leaf count of optimal. 3631 vs. $2(129) = 258$.
time = 0.73, size = 7938, normalized size = 55.51

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3
+ b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x +
 e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*
b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2
)*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a
^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4
 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b
^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*
b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sin
h(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cos
h(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (
8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x +
 e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2
*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
 b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24
*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 +
8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a
*b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3
*b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)
)*sinh(f*x + e))*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)
*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6
+ 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*
cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b
 - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4
*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^
2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2
 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x +
 e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*
a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2
+ sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*
sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*s
inh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x +
 e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*co
sh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)
^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sq
rt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
```

```
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*
x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh
(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^
6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 +
20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6
*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 3*((a^2*b^2 - 2*a*b^3
+ b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x
 + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4
 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2
*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14
*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x
+ e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5
*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 - 2*a*
b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^5 + 10*(2*a^3*b -
5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2
 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*
x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^4 + 2*a^3
*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^
3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*
cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^5 +
 (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^3 + (2*a^
3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*log(
-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e
)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*
cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.98Error: Bad Argum
ent Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(e+f\,x)^5}{{\left(b\sinh\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sinh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.117 $\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2\cosh(e+fx)}{3(a-b)^2f\sqrt{a-b+b\cosh^2(e+fx)}}+\frac{\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)^2/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(3/2)-2/3*cosh(
f*x+e)/(a-b)^2/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3265, 386, 197}

$$\frac{\sinh^2(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}-\frac{2\cosh(e+fx)}{3f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*Cosh[e + f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2]) + (Cosh[
e + f*x]*Sinh[e + f*x]^2)/(3*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{(a-b+bx^2)^{5/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{(a-b+bx^2)^{3/2}}\,dx, x, \cosh(\right.}{3(a-b)f}$$

$$= -\frac{2\cosh(e+fx)}{3(a-b)^2 f\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\cosh(e+fx)\sinh^2(e+fx}{3(a-b)f\left(a-b+b\cosh^2(e+f\right.}$$

Mathematica [A]
time = 0.22, size = 67, normalized size = 0.77

$$\frac{\sqrt{2}\cosh(e+fx)(-5a+3b+(a-3b)\cosh(2(e+fx)))}{3(a-b)^2 f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[2]*Cosh[e + f*x]*(-5*a + 3*b + (a - 3*b)*Cosh[2*(e + f*x)]))/(3*(a -
b)^2*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [A]
time = 1.02, size = 64, normalized size = 0.74

method	result	size
default	$\frac{\left(a\left(\sinh^2(fx+e)\right)-3b\left(\sinh^2(fx+e)\right)-2a\right)\cosh(fx+e)}{3\left(a+b\left(\sinh^2(fx+e)\right)\right)^{\frac{3}{2}}\left(a^2-2ab+b^2\right)f}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a*sinh(f*x+e)^2-3*b*sinh(f*x+e)^2-2*a)*cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(3/2)/(a^2-2*a*b+b^2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 955 vs. 2(84) = 168.
time = 0.56, size = 955, normalized size = 10.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b^4*e^(-10*f*x - 10*e) - 4*a^3*b + 6*a^2*b^2 - b^4 - (16*a^4 - 32*a^
3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-2*f*x - 2*e) + 10*(2*a^3*b - 3*a^2*
b^2 + 3*a*b^3 - b^4)*e^(-4*f*x - 4*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^(-
6*f*x - 6*e) + 5*(2*a*b^3 - b^4)*e^(-8*f*x - 8*e))/((a^4 - 2*a^3*b + a^2*b^
2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) - 1/4*(
2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-2*f
*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-4*f*x
 - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e) + 5*(4*
a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10*f*x - 10
*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x
 - 4*e) + b)^(5/2)*f) - 1/4*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*
e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-4*f*x - 4*e
) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-6*f*x - 6*e)
+ 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) + (2*a^2*b^2 - 2
*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e
^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) - 1/12*(b^4 + 5*(2*a*b^3
 - b^4)*e^(-2*f*x - 2*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^(-4*f*x - 4*e)
+ 10*(2*a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*e^(-6*f*x - 6*e) - (16*a^4 - 32*
a^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-8*f*x - 8*e) - (4*a^3*b - 6*a^2*b
^2 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2
*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. $2(79) = 158$.
time = 0.55, size = 1214, normalized size = 13.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*((a - 3*b)*cosh(f*x + e)^6 + 6*(a - 3*b)*cosh(f*x + e)*sinh(f*x
 + e)^5 + (a - 3*b)*sinh(f*x + e)^6 - 3*(3*a - b)*cosh(f*x + e)^4 + 3*(5*(a
 - 3*b)*cosh(f*x + e)^2 - 3*a + b)*sinh(f*x + e)^4 + 4*(5*(a - 3*b)*cosh(f*
x + e)^3 - 3*(3*a - b)*cosh(f*x + e))*sinh(f*x + e)^3 - 3*(3*a - b)*cosh(f*
x + e)^2 + 3*(5*(a - 3*b)*cosh(f*x + e)^4 - 6*(3*a - b)*cosh(f*x + e)^2 - 3
*a + b)*sinh(f*x + e)^2 + 6*((a - 3*b)*cosh(f*x + e)^5 - 2*(3*a - b)*cosh(f
*x + e)^3 - (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt((b*cosh(
f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2))/((a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x +
 e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*
b^2 - 2*a*b^3 + b^4)*f*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
```

```
 b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^2
+ (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 24*
a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^2*b^2 -
2*a*b^3 + b^4)*f*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*
f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f
*x + e)^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^2 + (8
*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f)*sinh(f*x + e)^4 + 4*(2*
a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^2 + 8*(7*(a^2*b^2 - 2*a*
b^3 + b^4)*f*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*c
osh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(
f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^
6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^4 + 3*(8*a^4 -
 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^2 + (2*a^3*b - 5
*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*f
+ 8*((a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 +
 4*a*b^3 - b^4)*f*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b
^3 + 3*b^4)*f*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cos
h(f*x + e))*sinh(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(79) = 158.
time = 1.14, size = 276, normalized size = 3.17

$$\frac{\left(\left(\frac{\left(a^{3} e^{\left(12 \, e\right)} - 3 \, a^{2} b e^{\left(12 \, e\right)}\right) e^{\left(2 \, f x\right)}}{a^{4} e^{\left(6 \, e\right)} - 2 \, a^{3} b e^{\left(6 \, e\right)} + a^{2} b^{2} e^{\left(6 \, e\right)}} - \frac{3 \, \left(3 \, a^{3} e^{\left(10 \, e\right)} - a^{2} b e^{\left(10 \, e\right)}\right)}{a^{4} e^{\left(6 \, e\right)} - 2 \, a^{3} b e^{\left(6 \, e\right)} + a^{2} b^{2} e^{\left(6 \, e\right)}}\right) e^{\left(2 \, f x\right)} - \frac{3 \, \left(3 \, a^{3} e^{\left(8 \, e\right)} - a^{2} b e^{\left(8 \, e\right)}\right)}{a^{4} e^{\left(6 \, e\right)} - 2 \, a^{3} b e^{\left(6 \, e\right)} + a^{2} b^{2} e^{\left(6 \, e\right)}}\right) e^{\left(2 \, f x\right)} + \frac{a^{3} e^{\left(6 \, e\right)} - 3 \, a^{2} b e^{\left(6 \, e\right)}}{a^{4} e^{\left(6 \, e\right)} - 2 \, a^{3} b e^{\left(6 \, e\right)} + a^{2} b^{2} e^{\left(6 \, e\right)}}}{3 \, {\left(b e^{\left(4 \, f x + 4 \, e\right)} + 4 \, a e^{\left(2 \, f x + 2 \, e\right)} - 2 \, b e^{\left(2 \, f x + 2 \, e\right)} + b\right)}^{\frac{3}{2}} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((((a^3*e^(12*e) - 3*a^2*b*e^(12*e))*e^(2*f*x)/(a^4*e^(6*e) - 2*a^3*b*e
^(6*e) + a^2*b^2*e^(6*e)) - 3*(3*a^3*e^(10*e) - a^2*b*e^(10*e))/(a^4*e^(6*e
) - 2*a^3*b*e^(6*e) + a^2*b^2*e^(6*e)))*e^(2*f*x) - 3*(3*a^3*e^(8*e) - a^2*
b*e^(8*e))/(a^4*e^(6*e) - 2*a^3*b*e^(6*e) + a^2*b^2*e^(6*e)))*e^(2*f*x) + (
a^3*e^(6*e) - 3*a^2*b*e^(6*e))/(a^4*e^(6*e) - 2*a^3*b*e^(6*e) + a^2*b^2*e^(
6*e)))/((b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
^(3/2)*f)
```

Mupad [B]
time = 1.62, size = 148, normalized size = 1.70

$$\frac{2\,{\mathrm{e}}^{e+f\,x}\,\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)\,\sqrt{a+b\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}\,\left(a-3\,b-10\,a\,{\mathrm{e}}^{2\,e+2\,f\,x}+a\,{\mathrm{e}}^{4\,e+4\,f\,x}+6\,b\,{\mathrm{e}}^{2\,e+2\,f\,x}-3\,b\,{\mathrm{e}}^{4\,e+4\,f\,x}\right)}{3\,f\,{\left(a-b\right)}^2\,{\left(b+4\,a\,{\mathrm{e}}^{2\,e+2\,f\,x}-2\,b\,{\mathrm{e}}^{2\,e+2\,f\,x}+b\,{\mathrm{e}}^{4\,e+4\,f\,x}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] (2*exp(e + f*x)*(exp(2*e + 2*f*x) + 1)*(a + b*(exp(e + f*x)/2 - exp(- e - f
*x)/2)^2)^(1/2)*(a - 3*b - 10*a*exp(2*e + 2*f*x) + a*exp(4*e + 4*f*x) + 6*b
*exp(2*e + 2*f*x) - 3*b*exp(4*e + 4*f*x)))/(3*f*(a - b)^2*(b + 4*a*exp(2*e
+ 2*f*x) - 2*b*exp(2*e + 2*f*x) + b*exp(4*e + 4*f*x))^2)
```

3.118 $\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\cosh(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}+\frac{2\cosh(e+fx)}{3(a-b)^2f\sqrt{a-b+b\cosh^2(e+fx)}}$$

```
[Out] 1/3*cosh(f*x+e)/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(3/2)+2/3*cosh(f*x+e)/(a-b)^2
/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 198, 197}

$$\frac{2\cosh(e+fx)}{3f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{\cosh(e+fx)}{3f(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Cosh[e + f*x]/(3*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2)) + (2*Cosh[e +
 f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a-b+bx^2)^{5/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(a-b+bx^2)^{3/2}}\,dx, x, \cosh}{3(a-b)f}$$

$$= \frac{\cosh(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} + \frac{2\cosh(e+fx)}{3(a-b)^2f\sqrt{a-b+b\cosh^2(e+}}$$

Mathematica [A]
time = 0.12, size = 63, normalized size = 0.80

$$\frac{2\sqrt{2}\cosh(e+fx)(3a-2b+b\cosh(2(e+fx)))}{3(a-b)^2f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*Sqrt[2]*Cosh[e + f*x]*(3*a - 2*b + b*Cosh[2*(e + f*x)]))/(3*(a - b)^2*f*
(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [A]
time = 1.39, size = 57, normalized size = 0.72

method	result	size
default	$\frac{\left(2b\left(\sinh^2(fx+e)\right)+3a-b\right)\cosh(fx+e)}{3\left(a+b\left(\sinh^2(fx+e)\right)\right)^{\frac{3}{2}}\left(a^2-2ab+b^2\right)f}$	57
risch	Expression too large to display	394373

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*b*sinh(f*x+e)^2+3*a-b)*cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2)/(a^2-2*
a*b+b^2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 499 vs. 2(75) = 150.
time = 0.52, size = 499, normalized size = 6.32

$$\frac{2a^2b^2-2ab^3+b^4+5(4a^3b-6a^2b^2+4ab^3-b^4)e^{(-2fx-2e)}+2(24a^4-48a^3b+49a^2b^2-25ab^3+5b^4)e^{(-4fx-4e)}+10(6a^3b-9a^2b^2+5ab^3-b^4)e^{(-6fx-6e)}+5(4a^2b^2-4ab^3+b^4)e^{(-8fx-8e)}+(2ab^3-b^4)e^{(-10fx-10e)}}{3(a^4-2a^3b+a^2b^2)(2(2a-b)e^{(-2fx-2e)}+be^{(-4fx-4e)}+b)^{\frac{5}{2}}f}+\frac{2ab^3-b^4+5(4a^2b^2-4ab^3+b^4)e^{(-2fx-2e)}+10(6a^3b-9a^2b^2+5ab^3-b^4)e^{(-4fx-4e)}+2(24a^4-48a^3b+49a^2b^2-25ab^3+5b^4)e^{(-6fx-6e)}+5(4a^3b-6a^2b^2+4ab^3-b^4)e^{(-8fx-8e)}+(2a^2b^2-2ab^3+b^4)e^{(-10fx-10e)}}{3(a^4-2a^3b+a^2b^2)(2(2a-b)e^{(-2fx-2e)}+be^{(-4fx-4e)}+b)^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^
(-2*f*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-
4*f*x - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e) +
5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10*f*x
 - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-
4*f*x - 4*e) + b)^(5/2)*f) + 1/3*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 +
b^4)*e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-4*f*x
- 4*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-6*f*x -
6*e) + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) + (2*a^2*b^
2 - 2*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a -
 b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1186 vs. $2(71) = 142$.
time = 0.71, size = 1186, normalized size = 15.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(2)*(b*cosh(f*x + e)^6 + 6*b*cosh(f*x + e)*sinh(f*x + e)^5 + b*sinh
(f*x + e)^6 + 3*(2*a - b)*cosh(f*x + e)^4 + 3*(5*b*cosh(f*x + e)^2 + 2*a -
b)*sinh(f*x + e)^4 + 4*(5*b*cosh(f*x + e)^3 + 3*(2*a - b)*cosh(f*x + e))*si
nh(f*x + e)^3 + 3*(2*a - b)*cosh(f*x + e)^2 + 3*(5*b*cosh(f*x + e)^4 + 6*(2
*a - b)*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 6*(b*cosh(f*x + e)^5 +
 2*(2*a - b)*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a^2*b^2 - 2*a*b^3 + b^4)
*f*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)*sinh(f*x +
 e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*f*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cos
h(f*x + e)^2 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^6 + 2
*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^4 + 8*(
7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*
a*b^3 - b^4)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2*b^2 - 2*a*b^3 +
b^4)*f*cosh(f*x + e)^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*
x + e)^2 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f)*sinh(f*x +
 e)^4 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^2 + 8*(7*(a
^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b
^3 - b^4)*f*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3
*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*c
osh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^4
 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^2 +
```

```
 (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^2 + (a^2*b^2 - 2*a*
b^3 + b^4)*f + 8*((a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^7 + 3*(2*a^3*b
- 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2
*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3
 - b^4)*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(71) = 142.
time = 0.89, size = 254, normalized size = 3.22

$$\frac{2\left(\frac{a^{2}be^{(6\,e)}}{a^{4}e^{(6\,e)}-2\,a^{3}be^{(6\,e)}+a^{2}b^{2}e^{(6\,e)}}+\left(\left(\frac{a^{2}be^{(2\,fx+12\,e)}}{a^{4}e^{(6\,e)}-2\,a^{3}be^{(6\,e)}+a^{2}b^{2}e^{(6\,e)}}+\frac{3\,(2\,a^{3}e^{(10\,e)}-a^{2}be^{(10\,e)})}{a^{4}e^{(6\,e)}-2\,a^{3}be^{(6\,e)}+a^{2}b^{2}e^{(6\,e)}}\right)e^{(2\,fx)}+\frac{3\,(2\,a^{3}e^{(8\,e)}-a^{2}be^{(8\,e)})}{a^{4}e^{(6\,e)}-2\,a^{3}be^{(6\,e)}+a^{2}b^{2}e^{(6\,e)}}\right)e^{(2\,fx)}\right)}{3\,(be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e)}-2\,be^{(2\,fx+2\,e)}+b)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*b*e^(6*e)/(a^4*e^(6*e) - 2*a^3*b*e^(6*e) + a^2*b^2*e^(6*e)) + ((a^
2*b*e^(2*f*x + 12*e)/(a^4*e^(6*e) - 2*a^3*b*e^(6*e) + a^2*b^2*e^(6*e)) + 3*
(2*a^3*e^(10*e) - a^2*b*e^(10*e))/(a^4*e^(6*e) - 2*a^3*b*e^(6*e) + a^2*b^2*
e^(6*e)))*e^(2*f*x) + 3*(2*a^3*e^(8*e) - a^2*b*e^(8*e))/(a^4*e^(6*e) - 2*a^
3*b*e^(6*e) + a^2*b^2*e^(6*e)))*e^(2*f*x))/((b*e^(4*f*x + 4*e) + 4*a*e^(2*f
*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)^(3/2)*f)
```

Mupad [**B**]
time = 1.29, size = 133, normalized size = 1.68

$$\frac{4\,{\mathrm{e}}^{e+f\,x}\,\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)\,\sqrt{a+b\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}\,\left(b+6\,a\,{\mathrm{e}}^{2\,e+2\,f\,x}-4\,b\,{\mathrm{e}}^{2\,e+2\,f\,x}+b\,{\mathrm{e}}^{4\,e+4\,f\,x}\right)}{3\,f\,{\left(a-b\right)}^2\,{\left(b+4\,a\,{\mathrm{e}}^{2\,e+2\,f\,x}-2\,b\,{\mathrm{e}}^{2\,e+2\,f\,x}+b\,{\mathrm{e}}^{4\,e+4\,f\,x}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] (4*exp(e + f*x)*(exp(2*e + 2*f*x) + 1)*(a + b*(exp(e + f*x)/2 - exp(- e - f
*x)/2)^2)^(1/2)*(b + 6*a*exp(2*e + 2*f*x) - 4*b*exp(2*e + 2*f*x) + b*exp(4*
e + 4*f*x)))/(3*f*(a - b)^2*(b + 4*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x
) + b*exp(4*e + 4*f*x))^2)
```

3.119 $\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{a^{5/2}f}-\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}-\frac{(5a-3b)b\cosh}{3a^2(a-b)^2f\sqrt{a-b+}}$$

```
[Out] -arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*c
osh(f*x+e)/a/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(3/2)-1/3*(5*a-3*b)*b*cosh(f*x+e
)/a^2/(a-b)^2/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3265, 425, 541, 12, 385, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{5/2}f}-\frac{b(5a-3b)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{b\cosh(e+fx)}{3af(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(a^(5/2)
*f)) - (b*Cosh[e + f*x])/(3*a*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))
- ((5*a - 3*b)*b*Cosh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a - b + b*Cosh[e +
f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)^{5/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3a+b+2bx^2}{(1-x^2)(a-b+bx^2)^{3/2}}\,dx,}{3a(a-b)f}$$

$$= -\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\cosh(e+f}{3a^2(a-b)^2f\sqrt{a-b+b\cosh^2}}$$

$$= -\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\cosh(e+f}{3a^2(a-b)^2f\sqrt{a-b+b\cosh^2}}$$

$$= -\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\cosh(e+f}{3a^2(a-b)^2f\sqrt{a-b+b\cosh^2}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+\right.}$$

Mathematica [A]
time = 0.54, size = 130, normalized size = 0.96

$$\frac{-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)}{a^{5/2}} + \frac{\sqrt{2}\,b\cosh(e+fx)\left(-12a^2+13ab-3b^2+b(-5a+3b)\cosh(2(e+fx))\right)}{3a^2(a-b)^2(2a-b+b\cosh(2(e+fx)))^{3/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x
)]]]/a^(5/2)) + (Sqrt[2]*b*Cosh[e + f*x]*(-12*a^2 + 13*a*b - 3*b^2 + b*(-5*
a + 3*b)*Cosh[2*(e + f*x)]))/(3*a^2*(a - b)^2*(2*a - b + b*Cosh[2*(e + f*x)
])^(3/2)))/f
```

Maple [A]
time = 2.12, size = 236, normalized size = 1.74

method	result
default	$\frac{\sqrt{(a+b(\sinh^{2}(fx+e)))(\cosh^{2}(fx+e))}\Bigg(-\frac{b(\cosh^{2}(fx+e))}{a^{2}(a-b)\sqrt{(a+b(\sinh^{2}(fx+e)))(\cosh^{2}(fx+e}}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-b/a^2*cosh(f*x+e)^2/(a-b)/((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)-1/2/a^(5/2)*ln((2*a+(a+b)*sinh(f*x+e)^
2+2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2)-1/3/a
*b*(2*b*sinh(f*x+e)^2+3*a-b)*cosh(f*x+e)^2/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)
^2)^(1/2)/(a+b*sinh(f*x+e)^2)/(a^2-2*a*b+b^2))/cosh(f*x+e)/(a+b*sinh(f*x+e)
^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2620 vs. $2(122) = 244$.
time = 0.66, size = 5342, normalized size = 39.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 +
 b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x +
e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b
 - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)
```

```
*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^
3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4
- 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^
2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b
^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh
(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh
(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (8
*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x +
e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2*
b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*
a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8
*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*
b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*
b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))
*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x
+ e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^
2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(co
sh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x +
e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4
*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1
)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))
*sinh(f*x + e) + 1)) - 2*sqrt(2)*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^6 + 6
*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (5*a^2*b^2 - 3*a*b^3
)*sinh(f*x + e)^6 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^4 + 3*(8*
a^3*b - 7*a^2*b^2 + a*b^3 + 5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^2)*sinh(f
*x + e)^4 + 5*a^2*b^2 - 3*a*b^3 + 4*(5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^
3 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(8*a
^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^2 + 3*(5*(5*a^2*b^2 - 3*a*b^3)*cosh
(f*x + e)^4 + 8*a^3*b - 7*a^2*b^2 + a*b^3 + 6*(8*a^3*b - 7*a^2*b^2 + a*b^3)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^
5 + 2*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^3 + (8*a^3*b - 7*a^2*b^2
+ a*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
 + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)))/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^8 + 8*(a^5*b
^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^5*b^2 - 2*a^
4*b^3 + a^3*b^4)*f*sinh(f*x + e)^8 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a
^3*b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x
 + e)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^6 +
2*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^
4 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^3 + 3*(2*a^6*b - 5
*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a
^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^4 + 30*(2*a^6*b - 5*a^5*b^2 +
```

```
 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^2 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 -
14*a^4*b^3 + 3*a^3*b^4)*f)*sinh(f*x + e)^4 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4
*b^3 - a^3*b^4)*f*cosh(f*x + e)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*
cosh(f*x + e)^5 + 10*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x
 + e)^3 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f
*x + e))*sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x
+ e)^6 + 15*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^4 +
 3*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)
^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^2 + (a^5*
b^2 - 2*a^4*b^3 + a^3*b^4)*f + 8*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*
x + e)^7 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^5
+ (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^
3 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e))*sinh(f*x +
 e)), 1/3*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*
b^3 + b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f
*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*
a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x +
e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(122) = 244.
time = 0.66, size = 404, normalized size = 2.97

$$-\frac{\left(\frac{\left(\left(\left(\frac{\left(5\,a^{9}b^{2}e^{(12\,e)}-3\,a^{8}b^{3}e^{(12\,e)}\right)e^{(2\,fx)}}{a^{12}e^{(12\,e)}-2\,a^{11}be^{(12\,e)}+a^{10}b^{2}e^{(12\,e)}}+\frac{3\left(8\,a^{10}be^{(10\,e)}-7\,a^{9}b^{2}e^{(10\,e)}+a^{8}b^{3}e^{(10\,e)}\right)}{a^{12}e^{(12\,e)}-2\,a^{11}be^{(12\,e)}+a^{10}b^{2}e^{(12\,e)}}\right)e^{(2\,fx)}+\frac{3\left(8\,a^{10}be^{(8\,e)}-7\,a^{9}b^{2}e^{(8\,e)}+a^{8}b^{3}e^{(8\,e)}\right)}{a^{12}e^{(12\,e)}-2\,a^{11}be^{(12\,e)}+a^{10}b^{2}e^{(12\,e)}}\right)e^{(2\,fx)}+\frac{5\,a^{9}b^{2}e^{(6\,e)}-3\,a^{8}b^{3}e^{(6\,e)}}{a^{12}e^{(12\,e)}-2\,a^{11}be^{(12\,e)}+a^{10}b^{2}e^{(12\,e)}}\right)}{\left(be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e)}-2\,be^{(2\,fx+2\,e)}+b\right)^{\frac{3}{2}}}-\frac{6\,\arctan\left(-\frac{\sqrt{b}e^{(2\,fx+2\,e)}-\sqrt{be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e)}-2\,be^{(2\,fx+2\,e)}+b}-\sqrt{b}}{2\,\sqrt{-a}}\right)e^{(-6\,e)}}{\sqrt{-a}a^{2}}\right)e^{(6\,e)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((((5*a^9*b^2*e^(12*e) - 3*a^8*b^3*e^(12*e))*e^(2*f*x)/(a^12*e^(12*e)
 - 2*a^11*b*e^(12*e) + a^10*b^2*e^(12*e)) + 3*(8*a^10*b*e^(10*e) - 7*a^9*b^
2*e^(10*e) + a^8*b^3*e^(10*e))/(a^12*e^(12*e) - 2*a^11*b*e^(12*e) + a^10*b^
2*e^(12*e)))*e^(2*f*x) + 3*(8*a^10*b*e^(8*e) - 7*a^9*b^2*e^(8*e) + a^8*b^3*
e^(8*e))/(a^12*e^(12*e) - 2*a^11*b*e^(12*e) + a^10*b^2*e^(12*e)))*e^(2*f*x)
 + (5*a^9*b^2*e^(6*e) - 3*a^8*b^3*e^(6*e))/(a^12*e^(12*e) - 2*a^11*b*e^(12*
e) + a^10*b^2*e^(12*e)))/(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(
```

```
2*f*x + 2*e) + b)^(3/2) - 6*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/
sqrt(-a))*e^(-6*e)/(sqrt(-a)*a^2))*e^(6*e)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(e+f\,x\right)\,{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(5/2)), x)
```

3.120 $\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=344

$$-\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{2a(2a-3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(8a^2-13ab+3b^2)\,E(\text{ArcT}}{3(a-b)^2}$$

```
[Out] -1/3*a*cosh(f*x+e)*sinh(f*x+e)^3/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*a*
(2*a-3*b)*cosh(f*x+e)*sinh(f*x+e)/(a-b)^2/b^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1
/3*(8*a^2-13*a*b+3*b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)
*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(
a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/b^3/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a
)^(1/2)+2/3*(2*a-3*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*E
llipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+
b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^
(1/2)+1/3*(8*a^2-13*a*b+3*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)^
2/b^3/f
```

Rubi [A]
time = 0.25, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3267, 481, 592, 545, 429, 506, 422}

$$-\frac{(8a^2-13ab+3b^2)\,\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3b^3f(a-b)^2\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(8a^2-13ab+3b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^3f(a-b)^2}+\frac{2(2a-3b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3b^2f(a-b)^2\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2a(2a-3b)\sinh(e+fx)\cosh(e+fx)}{3b^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{a\sinh^3(e+fx)\cosh(e+fx)}{3bf(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*(a*Cosh[e + f*x]*Sinh[e + f*x]^3)/((a - b)*b*f*(a + b*Sinh[e + f*x]^2)
^(3/2)) - (2*a*(2*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^2*b^2*f*
Sqrt[a + b*Sinh[e + f*x]^2]) - ((8*a^2 - 13*a*b + 3*b^2)*EllipticE[ArcTan[S
inh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a -
b)^2*b^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*(2*a - 3
*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(3*(a - b)^2*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]
^2))/a]) + ((8*a^2 - 13*a*b + 3*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f
*x])/(3*(a - b)^2*b^3*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^6}{\sqrt{1+x^2}\,(a+bx^2)^{5/2}}\,dx, x, \sinh(\right.}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right.}{}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)\sinh(e}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+}}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)\sinh(e}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+}}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)\sinh(e}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+}}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)\sinh(e}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.40, size = 207, normalized size = 0.60

$$\frac{a\left(-2ia(8a^2-13ab+3b^2)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+2ia(8a^2-17ab+9b^2)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,b(-8a^2+17ab-7b^2+b(-5a+7b)\cosh(2(e+fx)))\sinh(2(e+fx))\right)}{6(a-b)^2b^3f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (a*((-2*I)*a*(8*a^2 - 13*a*b + 3*b^2)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(
3/2)*EllipticE[I*(e + f*x), b/a] + (2*I)*a*(8*a^2 - 17*a*b + 9*b^2)*((2*a -
 b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*
(-8*a^2 + 17*a*b - 7*b^2 + b*(-5*a + 7*b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*
x)]))/(6*(a - b)^2*b^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(400) = 800$.
time = 1.44, size = 868, normalized size = 2.52

method	result	size
default	Expression too large to display	868

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((5*(-1/a*b)^(1/2)*a^2*b-7*(-1/a*b)^(1/2)*a*b^2)*cosh(f*x+e)^4*sinh(f*
x+e)+(4*(-1/a*b)^(1/2)*a^3-11*(-1/a*b)^(1/2)*a^2*b+7*(-1/a*b)^(1/2)*a*b^2)*
cosh(f*x+e)^2*sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)
^(1/2)*b*(4*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-7*Ellipti
cF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+3*EllipticF(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*b^2-8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*a^2+13*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-3*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2+4*(b/a*cosh(f
*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)
^(1/2),(a/b)^(1/2))*a^3-11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+10*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a*b^2-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-8*(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*a^3+21*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-16*(b/a
*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)/(-1/
a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/b^2/cosh(f*x+e)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**F**]
time = 0.13, size = 71, normalized size = 0.21

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\sinh\left(fx+e\right)^{6}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^6/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.61index.cc index_m i_lex_is_greater Er
ror: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{sinh}\left(e+f\,x\right)}^6}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sinh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.121 $\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=244

$$-\frac{a\cosh(e+fx)\sinh(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{2\sqrt{a}\,(a-2b)\cosh(e+fx)E\left(\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3(a-b)^2b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}-(a-$$

```
[Out] -1/3*a*cosh(f*x+e)*sinh(f*x+e)/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(3/2)+2/3*(a-2
*b)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/2)
*EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^
(1/2))*a^(1/2)/(a-b)^2/b^(3/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2
)/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*(a-3*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh
(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2
))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^2/b/f/(sech(f*x+e)^2*(a+b*
sinh(f*x+e)^2)/a)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3267, 481, 539, 429, 422}

$$\frac{2\sqrt{a}\,(a-2b)\cosh(e+fx)E\left(\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3b^{3/2}f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{(a-3b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\operatorname{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{3abf(a-b)^2\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{a\sinh(e+fx)\cosh(e+fx)}{3bf(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*(a*Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*b*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) + (2*Sqrt[a]*(a - 2*b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e
 + f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^2*b^(3/2)*f*Sqrt[(a*Cosh[e + f*x]^2
)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - ((a - 3*b)*Ellipt
icF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^
2])/(3*a*(a - b)^2*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}\,(a+bx^2)^{5/2}}\,dx, x, \sinh(e\right.}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)}{}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left((a-3b)\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e\right.}{}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\sqrt{a}\,(a-2b)\cosh(e+fx)E\left(\tan\right.}{3(a-b)^2b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx}{a+b\sinh^2(e+f}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.13, size = 198, normalized size = 0.81

$$\frac{2ia^2(a-2b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-ia(2a^2-5ab+3b^2)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-\sqrt{2}\,b(-a^2+4ab-2b^2-(a-2b)b\cosh(2(e+fx)))\sinh(2(e+fx))}{3(a-b)^2b^2f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(a - 2*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*
(e + f*x), b/a] - I*a*(2*a^2 - 5*a*b + 3*b^2)*((2*a - b + b*Cosh[2*(e + f*x
)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] - Sqrt[2]*b*(-a^2 + 4*a*b - 2*b^2
- (a - 2*b)*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*(a - b)^2*b^2*f*(2*a
 - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(314) = 628$.
time = 1.16, size = 659, normalized size = 2.70

method	result
default	$\frac{2\sqrt{-\frac{b}{a}}\,ab(\sinh^5(fx+e))-4\sqrt{-\frac{b}{a}}\,b^2(\sinh^5(fx+e))+\sqrt{\frac{a+b(\sinh^2(fx+e))}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\text{EllipticF}\left(\sinh(fx+e)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^2-((a+b*sinh(f*x+e)^2)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
*sinh(f*x+e)^2-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^2+4*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*b^2*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-(-1/a*b)
^(1/2)*a*b*sinh(f*x+e)^3-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+((a+b*sinh(f*x+
e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(
a/b)^(1/2))*a^2-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipti
cF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*((a+b*sinh(f*x+e)^2)/a)^(1
/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))
*a^2+4*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+sinh(f*x+e)*a^2*(-1/a*b)^(1/2)-3*sinh
(f*x+e)*b*a*(-1/a*b)^(1/2))/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^
2/b/cosh(f*x+e)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4985 vs. 2(252) = 504.
time = 0.18, size = 4985, normalized size = 20.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(((2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^8 + 8*(2*a^2*b^2 - 5*a*b
^3 + 2*b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (2*a^2*b^2 - 5*a*b^3 + 2*b^4)*s
inh(f*x + e)^8 + 4*(4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*b^4)*cosh(f*x + e)^6
 + 4*(4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*b^4 + 7*(2*a^2*b^2 - 5*a*b^3 + 2*b
^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*c
osh(f*x + e)^3 + 3*(4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*b^4)*cosh(f*x + e))*
sinh(f*x + e)^5 + 2*(16*a^4 - 56*a^3*b + 62*a^2*b^2 - 31*a*b^3 + 6*b^4)*cos
```

```
h(f*x + e)^4 + 2*(35*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^4 + 16*a^4
 - 56*a^3*b + 62*a^2*b^2 - 31*a*b^3 + 6*b^4 + 30*(4*a^3*b - 12*a^2*b^2 + 9*
a*b^3 - 2*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 2*a^2*b^2 - 5*a*b^3 + 2*b
^4 + 8*(7*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^5 + 10*(4*a^3*b - 12*
a^2*b^2 + 9*a*b^3 - 2*b^4)*cosh(f*x + e)^3 + (16*a^4 - 56*a^3*b + 62*a^2*b^
2 - 31*a*b^3 + 6*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(4*a^3*b - 12*a^2*
b^2 + 9*a*b^3 - 2*b^4)*cosh(f*x + e)^2 + 4*(7*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)
*cosh(f*x + e)^6 + 15*(4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*b^4)*cosh(f*x + e
)^4 + 4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*b^4 + 3*(16*a^4 - 56*a^3*b + 62*a^
2*b^2 - 31*a*b^3 + 6*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((2*a^2*b^2
 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^7 + 3*(4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*
b^4)*cosh(f*x + e)^5 + (16*a^4 - 56*a^3*b + 62*a^2*b^2 - 31*a*b^3 + 6*b^4)*
cosh(f*x + e)^3 + (4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*b^4)*cosh(f*x + e))*s
inh(f*x + e) - 2*((a*b^3 - 2*b^4)*cosh(f*x + e)^8 + 8*(a*b^3 - 2*b^4)*cosh(
f*x + e)*sinh(f*x + e)^7 + (a*b^3 - 2*b^4)*sinh(f*x + e)^8 + 4*(2*a^2*b^2 -
 5*a*b^3 + 2*b^4)*cosh(f*x + e)^6 + 4*(2*a^2*b^2 - 5*a*b^3 + 2*b^4 + 7*(a*b
^3 - 2*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a*b^3 - 2*b^4)*cosh(f*
x + e)^3 + 3*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e))*sinh(f*x + e)^5 +
 2*(8*a^3*b - 24*a^2*b^2 + 19*a*b^3 - 6*b^4)*cosh(f*x + e)^4 + 2*(35*(a*b^3
 - 2*b^4)*cosh(f*x + e)^4 + 8*a^3*b - 24*a^2*b^2 + 19*a*b^3 - 6*b^4 + 30*(2
*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a*b^3 - 2*b^
4 + 8*(7*(a*b^3 - 2*b^4)*cosh(f*x + e)^5 + 10*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)
*cosh(f*x + e)^3 + (8*a^3*b - 24*a^2*b^2 + 19*a*b^3 - 6*b^4)*cosh(f*x + e))
*sinh(f*x + e)^3 + 4*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^2 + 4*(7*(
a*b^3 - 2*b^4)*cosh(f*x + e)^6 + 15*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x
+ e)^4 + 2*a^2*b^2 - 5*a*b^3 + 2*b^4 + 3*(8*a^3*b - 24*a^2*b^2 + 19*a*b^3 -
 6*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a*b^3 - 2*b^4)*cosh(f*x + e)
^7 + 3*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^5 + (8*a^3*b - 24*a^2*b^
2 + 19*a*b^3 - 6*b^4)*cosh(f*x + e)^3 + (2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(
f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2
 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2
) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*
(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - ((2*a^2*b^2 - 7*a*b^3 + 3*b^4)*
cosh(f*x + e)^8 + 8*(2*a^2*b^2 - 7*a*b^3 + 3*b^4)*cosh(f*x + e)*sinh(f*x +
e)^7 + (2*a^2*b^2 - 7*a*b^3 + 3*b^4)*sinh(f*x + e)^8 + 4*(4*a^3*b - 16*a^2*
b^2 + 13*a*b^3 - 3*b^4)*cosh(f*x + e)^6 + 4*(4*a^3*b - 16*a^2*b^2 + 13*a*b^
3 - 3*b^4 + 7*(2*a^2*b^2 - 7*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^
6 + 8*(7*(2*a^2*b^2 - 7*a*b^3 + 3*b^4)*cosh(f*x + e)^3 + 3*(4*a^3*b - 16*a^
2*b^2 + 13*a*b^3 - 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(16*a^4 - 72*a
^3*b + 86*a^2*b^2 - 45*a*b^3 + 9*b^4)*cosh(f*x + e)^4 + 2*(35*(2*a^2*b^2 -
7*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 16*a^4 - 72*a^3*b + 86*a^2*b^2 - 45*a*b^
3 + 9*b^4 + 30*(4*a^3*b - 16*a^2*b^2 + 13*a*b^3 - 3*b^4)*cosh(f*x + e)^2)*s
inh(f*x + e)^4 + 2*a^2*b^2 - 7*a*b^3 + 3*b^4 + 8*(7*(2*a^2*b^2 - 7*a*b^3 +
3*b^4)*cosh(f*x + e)^5 + 10*(4*a^3*b - 16*a^2*b^2 + 13*a*b^3 - 3*b^4)*cosh(
f*x + e)^3 + (16*a^4 - 72*a^3*b + 86*a^2*b^2 - 45*a*b^3 + 9*b^4)*cosh(f*x +
```

```
 e))*sinh(f*x + e)^3 + 4*(4*a^3*b - 16*a^2*b^2 + 13*a*b^3 - 3*b^4)*cosh(f*x
 + e)^2 + 4*(7*(2*a^2*b^2 - 7*a*b^3 + 3*b^4)*cosh(f*x + e)^6 + 15*(4*a^3*b
- 16*a^2*b^2 + 13*a*b^3 - 3*b^4)*cosh(f*x + e)^4 + 4*a^3*b - 16*a^2*b^2 + 1
3*a*b^3 - 3*b^4 + 3*(16*a^4 - 72*a^3*b + 86*a^2*b^2 - 45*a*b^3 + 9*b^4)*cos
h(f*x + e)^2)*sinh(f*x + e)^2 + 8*((2*a^2*b^2 - 7*a*b^3 + 3*b^4)*cosh(f*x +
 e)^7 + 3*(4*a^3*b - 16*a^2*b^2 + 13*a*b^3 - 3*b^4)*cosh(f*x + e)^5 + (16*a
^4 - 72*a^3*b + 86*a^2*b^2 - 45*a*b^3 + 9*b^4)*cosh(f*x + e)^3 + (4*a^3*b -
 16*a^2*b^2 + 13*a*b^3 - 3*b^4)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b^3 -
b^4)*cosh(f*x + e)^8 + 8*(a*b^3 - b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a*b
^3 - b^4)*sinh(f*x + e)^8 + 4*(2*a^2*b^2 - 3*a*b^3 + b^4)*cosh(f*x + e)^6 +
 4*(2*a^2*b^2 - 3*a*b^3 + b^4 + 7*(a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x +
 e)^6 + 8*(7*(a*b^3 - b^4)*cosh(f*x + e)^3 + 3*(2*a^2*b^2 - 3*a*b^3 + b^4)*
cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^3*b - 16*a^2*b^2 + 11*a*b^3 - 3*b^4
)*cosh(f*x + e)^4 + 2*(35*(a*b^3 - b^4)*cosh(f*x + e)^4 + 8*a^3*b - 16*a^2*
b^2 + 11*a*b^3 - 3*b^4 + 30*(2*a^2*b^2 - 3*a*b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(e+f\,x)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.122 $\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=241

$$\frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{i(a+b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh}}{3a(a-b)^2bf\sqrt{1+\frac{b\sinh^2(e+}{a}}}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/3*(a+b)*cos
h(f*x+e)*sinh(f*x+e)/a/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*I*(a+b)*(cos
(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(
a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^2/b/f/(1+b*sinh(f*x+e)^2/a)^(1/2)-1/3*I*(c
os(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))
*(1+b*sinh(f*x+e)^2/a)^(1/2)/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3252, 3251, 3257, 3256, 3262, 3261}

$$\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{3af(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3bf(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{i(a+b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3abf(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) +
 ((a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)^2*f*Sqrt[a + b*Sinh[e +
 f*x]^2]) + ((I/3)*(a + b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e +
f*x]^2])/(a*(a - b)^2*b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) - ((I/3)*Ellipti
cF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/((a - b)*b*f*Sqrt[a +
 b*Sinh[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\int \frac{a-a\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx}{3a(a-b)}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} +$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} +$$

Mathematica [**A**]
time = 0.96, size = 187, normalized size = 0.78

$$\frac{2ia^2(a+b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) - 2ia^2(a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) + \sqrt{2}\,b(4a^2-ab-b^2+b(a+b)\cosh(2(e+fx)))\sinh(2(e+fx))}{6a(a-b)^2bf(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(e
 + f*x), b/a] - (2*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)
*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(4*a^2 - a*b - b^2 + b*(a + b)*Cos
h[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*a*(a - b)^2*b*f*(2*a - b + b*Cosh[2*(
e + f*x)])^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(277) = 554$.
time = 1.53, size = 601, normalized size = 2.49

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, ab+\sqrt{-\frac{b}{a}}\, b^{2}\right)\left(\cosh^{4}(fx+e)\right)\sinh(fx+e)+\left(2\sqrt{-\frac{b}{a}}\, a^{2}-\sqrt{-\frac{b}{a}}\, ab-\sqrt{-\frac{b}{a}}\, b^{2}\right)\left(\cosh^{2}(fx+e)\right)\sinh(fx+e)-}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(((-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(2*(
-1/a*b)^(1/2)*a^2-(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh
(f*x+e)-(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(Elliptic
F(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a-EllipticF(sinh(f*x+e)*(-1/a*b)^
(1/2),(a/b)^(1/2))*b+EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+El
lipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)*cosh(f*x+e)^2-(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*a^2+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-(b/a*cosh(f*x
+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))*b^2-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/
2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+(b/a*cosh(f*x+e)^2
+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),
(a/b)^(1/2))*b^2)/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/a/cosh(f
*x+e)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4684 vs. 2(253) = 506.
time = 0.18, size = 4684, normalized size = 19.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*((((2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^8 + 8*(2*a^2*b^2 + a*b^3 - b
^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (2*a^2*b^2 + a*b^3 - b^4)*sinh(f*x + e)
^8 + 4*(4*a^3*b - 3*a*b^3 + b^4)*cosh(f*x + e)^6 + 4*(4*a^3*b - 3*a*b^3 + b
^4 + 7*(2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(2
*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^3 + 3*(4*a^3*b - 3*a*b^3 + b^4)*cosh(
f*x + e))*sinh(f*x + e)^5 + 2*(16*a^4 - 8*a^3*b - 10*a^2*b^2 + 11*a*b^3 - 3
*b^4)*cosh(f*x + e)^4 + 2*(35*(2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^4 + 1
6*a^4 - 8*a^3*b - 10*a^2*b^2 + 11*a*b^3 - 3*b^4 + 30*(4*a^3*b - 3*a*b^3 + b
^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 2*a^2*b^2 + a*b^3 - b^4 + 8*(7*(2*a^
2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^5 + 10*(4*a^3*b - 3*a*b^3 + b^4)*cosh(f*
x + e)^3 + (16*a^4 - 8*a^3*b - 10*a^2*b^2 + 11*a*b^3 - 3*b^4)*cosh(f*x + e)
)*sinh(f*x + e)^3 + 4*(4*a^3*b - 3*a*b^3 + b^4)*cosh(f*x + e)^2 + 4*(7*(2*a
^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^6 + 15*(4*a^3*b - 3*a*b^3 + b^4)*cosh(f
*x + e)^4 + 4*a^3*b - 3*a*b^3 + b^4 + 3*(16*a^4 - 8*a^3*b - 10*a^2*b^2 + 11
*a*b^3 - 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((2*a^2*b^2 + a*b^3 -
b^4)*cosh(f*x + e)^7 + 3*(4*a^3*b - 3*a*b^3 + b^4)*cosh(f*x + e)^5 + (16*a^
4 - 8*a^3*b - 10*a^2*b^2 + 11*a*b^3 - 3*b^4)*cosh(f*x + e)^3 + (4*a^3*b - 3
*a*b^3 + b^4)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b^3 + b^4)*cosh(f*x + e)
^8 + 8*(a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a*b^3 + b^4)*sinh(f*x
 + e)^8 + 4*(2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^2*b^2 + a*b^
3 - b^4 + 7*(a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a*b^3 +
b^4)*cosh(f*x + e)^3 + 3*(2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x
+ e)^5 + 2*(8*a^3*b - 5*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a*b^3 + b^4
)*cosh(f*x + e)^4 + 8*a^3*b - 5*a*b^3 + 3*b^4 + 30*(2*a^2*b^2 + a*b^3 - b^4
)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a*b^3 + b^4 + 8*(7*(a*b^3 + b^4)*cosh(
f*x + e)^5 + 10*(2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^3 + (8*a^3*b - 5*a*
b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2*b^2 + a*b^3 - b^4)*c
osh(f*x + e)^2 + 4*(7*(a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^2*b^2 + a*b^3
 - b^4)*cosh(f*x + e)^4 + 2*a^2*b^2 + a*b^3 - b^4 + 3*(8*a^3*b - 5*a*b^3 +
3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a*b^3 + b^4)*cosh(f*x + e)^7
+ 3*(2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^5 + (8*a^3*b - 5*a*b^3 + 3*b^4)
*cosh(f*x + e)^3 + (2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e))*
sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b
)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x
 + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 -
 a*b)/b^2))/b^2) - 4*((2*a^2*b^2 - a*b^3)*cosh(f*x + e)^8 + 8*(2*a^2*b^2 -
a*b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (2*a^2*b^2 - a*b^3)*sinh(f*x + e)^8
+ 4*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^6 + 4*(4*a^3*b - 4*a^2*b^2
+ a*b^3 + 7*(2*a^2*b^2 - a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(2*
a^2*b^2 - a*b^3)*cosh(f*x + e)^3 + 3*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x
 + e))*sinh(f*x + e)^5 + 2*(16*a^4 - 24*a^3*b + 14*a^2*b^2 - 3*a*b^3)*cosh(
f*x + e)^4 + 2*(35*(2*a^2*b^2 - a*b^3)*cosh(f*x + e)^4 + 16*a^4 - 24*a^3*b
+ 14*a^2*b^2 - 3*a*b^3 + 30*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^2)*
sinh(f*x + e)^4 + 2*a^2*b^2 - a*b^3 + 8*(7*(2*a^2*b^2 - a*b^3)*cosh(f*x + e
)^5 + 10*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^3 + (16*a^4 - 24*a^3*b
```

```
 + 14*a^2*b^2 - 3*a*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(4*a^3*b - 4*a^
2*b^2 + a*b^3)*cosh(f*x + e)^2 + 4*(7*(2*a^2*b^2 - a*b^3)*cosh(f*x + e)^6 +
 15*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^4 + 4*a^3*b - 4*a^2*b^2 + a
*b^3 + 3*(16*a^4 - 24*a^3*b + 14*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^2)*sinh(f
*x + e)^2 + 8*((2*a^2*b^2 - a*b^3)*cosh(f*x + e)^7 + 3*(4*a^3*b - 4*a^2*b^2
 + a*b^3)*cosh(f*x + e)^5 + (16*a^4 - 24*a^3*b + 14*a^2*b^2 - 3*a*b^3)*cosh
(f*x + e)^3 + (4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e) +
((a*b^3 - b^4)*cosh(f*x + e)^8 + 8*(a*b^3 - b^4)*cosh(f*x + e)*sinh(f*x + e
)^7 + (a*b^3 - b^4)*sinh(f*x + e)^8 + 4*(2*a^2*b^2 - 3*a*b^3 + b^4)*cosh(f*
x + e)^6 + 4*(2*a^2*b^2 - 3*a*b^3 + b^4 + 7*(a*b^3 - b^4)*cosh(f*x + e)^2)*
sinh(f*x + e)^6 + 8*(7*(a*b^3 - b^4)*cosh(f*x + e)^3 + 3*(2*a^2*b^2 - 3*a*b
^3 + b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^3*b - 16*a^2*b^2 + 11*a*b
^3 - 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a*b^3 - b^4)*cosh(f*x + e)^4 + 8*a^3*b
 - 16*a^2*b^2 + 11*a*b^3 - 3*b^4 + 30*(2*a^2*b^2 - 3*a*b^3 + b^4)*cosh(f*x
+ e)^2)*sinh(f*x + e)^4 + a*b^3 - b^4 + 8*(7*(a*b^3 - b^4)*cosh(f*x + e)^5
+ 10*(2*a^2*b^2 - 3*a*b^3 + b^4)*cosh(f*x + e)^3 + (8*a^3*b - 16*a^2*b^2 +
11*a*b^3 - 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2*b^2 - 3*a*b^3 +
 b^4)*cosh(f*x + e)^2 + 4*(7*(a*b^3 - b^4)*cosh(f*x + e)^6 + 15*(2*a^2*b^2
- 3*a*b^3 + b^4)*cosh(f*x + e)^4 + 2*a^2*b^2 - 3*a*b^3 + b^4 + 3*(8*a^3*b -
 16*a^2*b^2 + 11*a*b^3 - 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a*b^
3 - b^4)*cosh(f*x + e)^7 + 3*(2*a^2*b^2 - 3*a*b^3 + b^4)*cosh(f*x + e)^5 +
(8*a^3*b - 16*a^2*b^2 + 11*a*b^3 - 3*b^4)*cosh(...
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.44Error: Bad Argument Type
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(e+f\,x)^2}{{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.123 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{}}{3a^2(a-b)^2f\sqrt{1+\frac{b\,\mathrm{s}}{}}}$$

```
[Out] -1/3*b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(2*a
-b)*b*cosh(f*x+e)*sinh(f*x+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-2/3*I
*(2*a-b)*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(
b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)^2/f/(1+b*sinh(f*x+e)^2/a)^(
1/2)+1/3*I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x)
,(b/a)^(1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/
2)
```

Rubi [A]
time = 0.20, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3263, 3252, 3251, 3257, 3256, 3262, 3261}

$$-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] -1/3*(b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt
[a + b*Sinh[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]
*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)^2*f*Sqrt[1 + (b*Sinh[e + f*x]^2)
/a]) + ((I/3)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/
(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a+2b+b\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx}{3a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

Mathematica [A]
time = 0.93, size = 190, normalized size = 0.76

$$\frac{-2ia^2(2a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+ia^2(a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,b(-5a^2+5ab-b^2+b(-2a+b)\cosh(2(e+fx)))\sinh(2(e+fx))}{3a^2(a-b)^2f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] ((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*
EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a + b
)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b*Cos
h[2*(e + f*x)])^(3/2))
```

Maple [A]
time = 1.81, size = 406, normalized size = 1.62

method	result
default	$\frac{\sqrt{(a+b(\sinh^2(fx+e)))(\cosh^2(fx+e))}\left(-\frac{\sinh(fx+e)\sqrt{(a+b(\sinh^2(fx+e)))(\cosh^2(fx+e}}{3ab(a-b)\left(\sinh^2(fx+e)+\frac{a}{b}\right)^2}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/a/b/(a-b)*sinh(f*x+e)*((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+a/b)^2-2/3*b*cosh(f*x+e
)^2/a^2/(a-b)^2*sinh(f*x+e)*(-b+2*a)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1
/2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*Ellip
ticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2/3*b*(-b+2*a)/a^2/(a-b)^2/(-1
/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(
f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))))/cosh(f*x+e)/(a+b
*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5442 vs. 2(259) = 518.
time = 0.20, size = 5442, normalized size = 21.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(((4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^8 + 8*(4*a^2*b^3 - 4*a*b^4
+ b^5)*cosh(f*x + e)*sinh(f*x + e)^7 + (4*a^2*b^3 - 4*a*b^4 + b^5)*sinh(f*x
```

```
 + e)^8 + 4*(8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e)^6 + 4*(8
*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5 + 7*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(
f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e
)^3 + 3*(8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e))*sinh(f*x +
e)^5 + 4*a^2*b^3 - 4*a*b^4 + b^5 + 2*(32*a^4*b - 64*a^3*b^2 + 52*a^2*b^3 -
20*a*b^4 + 3*b^5)*cosh(f*x + e)^4 + 2*(32*a^4*b - 64*a^3*b^2 + 52*a^2*b^3 -
 20*a*b^4 + 3*b^5 + 35*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^4 + 30*(8*
a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*
(7*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^5 + 10*(8*a^3*b^2 - 12*a^2*b^3
 + 6*a*b^4 - b^5)*cosh(f*x + e)^3 + (32*a^4*b - 64*a^3*b^2 + 52*a^2*b^3 - 2
0*a*b^4 + 3*b^5)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(8*a^3*b^2 - 12*a^2*b^3
 + 6*a*b^4 - b^5)*cosh(f*x + e)^2 + 4*(7*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f
*x + e)^6 + 8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5 + 15*(8*a^3*b^2 - 12*a^2
*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e)^4 + 3*(32*a^4*b - 64*a^3*b^2 + 52*a^2*b
^3 - 20*a*b^4 + 3*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((4*a^2*b^3 - 4
*a*b^4 + b^5)*cosh(f*x + e)^7 + 3*(8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*
cosh(f*x + e)^5 + (32*a^4*b - 64*a^3*b^2 + 52*a^2*b^3 - 20*a*b^4 + 3*b^5)*c
osh(f*x + e)^3 + (8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e))*si
nh(f*x + e) - 2*((2*a*b^4 - b^5)*cosh(f*x + e)^8 + 8*(2*a*b^4 - b^5)*cosh(f
*x + e)*sinh(f*x + e)^7 + (2*a*b^4 - b^5)*sinh(f*x + e)^8 + 4*(4*a^2*b^3 -
4*a*b^4 + b^5)*cosh(f*x + e)^6 + 4*(4*a^2*b^3 - 4*a*b^4 + b^5 + 7*(2*a*b^4
- b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(2*a*b^4 - b^5)*cosh(f*x + e
)^3 + 3*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*a*b^
4 - b^5 + 2*(16*a^3*b^2 - 24*a^2*b^3 + 14*a*b^4 - 3*b^5)*cosh(f*x + e)^4 +
2*(16*a^3*b^2 - 24*a^2*b^3 + 14*a*b^4 - 3*b^5 + 35*(2*a*b^4 - b^5)*cosh(f*x
 + e)^4 + 30*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^4 +
 8*(7*(2*a*b^4 - b^5)*cosh(f*x + e)^5 + 10*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh
(f*x + e)^3 + (16*a^3*b^2 - 24*a^2*b^3 + 14*a*b^4 - 3*b^5)*cosh(f*x + e))*s
inh(f*x + e)^3 + 4*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^2 + 4*(7*(2*a*
b^4 - b^5)*cosh(f*x + e)^6 + 4*a^2*b^3 - 4*a*b^4 + b^5 + 15*(4*a^2*b^3 - 4*
a*b^4 + b^5)*cosh(f*x + e)^4 + 3*(16*a^3*b^2 - 24*a^2*b^3 + 14*a*b^4 - 3*b^
5)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((2*a*b^4 - b^5)*cosh(f*x + e)^7 +
3*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^5 + (16*a^3*b^2 - 24*a^2*b^3 +
14*a*b^4 - 3*b^5)*cosh(f*x + e)^3 + (4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x +
e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b
)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*
a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b
 - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - ((6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh
(f*x + e)^8 + 8*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)*sinh(f*x + e)
^7 + (6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*sinh(f*x + e)^8 + 4*(12*a^4*b - 16*a^3
*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^6 + 4*(12*a^4*b - 16*a^3*b^2 + 7*a^
2*b^3 - a*b^4 + 7*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^2)*sinh(f*x
 + e)^6 + 8*(7*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^3 + 3*(12*a^4*
b - 16*a^3*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*a^3*
b^2 - 5*a^2*b^3 + a*b^4 + 2*(48*a^5 - 88*a^4*b + 66*a^3*b^2 - 23*a^2*b^3 +
```

```
3*a*b^4)*cosh(f*x + e)^4 + 2*(48*a^5 - 88*a^4*b + 66*a^3*b^2 - 23*a^2*b^3 +
 3*a*b^4 + 35*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^4 + 30*(12*a^4*
b - 16*a^3*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(7
*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^5 + 10*(12*a^4*b - 16*a^3*b^
2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^3 + (48*a^5 - 88*a^4*b + 66*a^3*b^2 -
23*a^2*b^3 + 3*a*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(12*a^4*b - 16*a^3
*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^2 + 4*(7*(6*a^3*b^2 - 5*a^2*b^3 + a
*b^4)*cosh(f*x + e)^6 + 12*a^4*b - 16*a^3*b^2 + 7*a^2*b^3 - a*b^4 + 15*(12*
a^4*b - 16*a^3*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^4 + 3*(48*a^5 - 88*a^
4*b + 66*a^3*b^2 - 23*a^2*b^3 + 3*a*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 +
 8*((6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^7 + 3*(12*a^4*b - 16*a^3*
b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^5 + (48*a^5 - 88*a^4*b + 66*a^3*b^2
- 23*a^2*b^3 + 3*a*b^4)*cosh(f*x + e)^3 + (12*a^4*b - 16*a^3*b^2 + 7*a^2*b^
3 - a*b^4)*cosh(f*x + e))*sinh(f*x + e) + 2*((3*a^2*b^3 - 5*a*b^4 + 2*b^5)*
cosh(f*x + e)^8 + 8*(3*a^2*b^3 - 5*a*b^4 + 2*b^5)*cosh(f*x + e)*sinh(f*x +
e)^7 + (3*a^2*b^3 - 5*a*b^4 + 2*b^5)*sinh(f*x + e)^8 + 4*(6*a^3*b^2 - 13*a^
2*b^3 + 9*a*b^4 - 2*b^5)*cosh(f*x + e)^6 + 4*(6*a^3*b^2 - 13*a^2*b^3 + 9*a*
b^4 - 2*b^5 + 7*(3*a^2*b^3 - 5*a*b^4 + 2*b^5)*cosh(f*x + e)^2)*sinh(f*x + e
)^6 + 8*(7*(3*a^2*b^3 - 5*a*b^4 + 2*b^5)*cosh(f*x + e)^3 + 3*(6*a^3*b^2 - 1
3*a^2*b^3 + 9*a*b^4 - 2*b^5)*cosh(f*x + e))*sin...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \sinh^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(-5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.48Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(1/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.124 $\int \frac{\mathbf{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=385

$$-\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(3a^2-13ab+8b^2)\coth(e+fx}{3a^3(a-b)}$$

```
[Out] -1/3*b*coth(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(3*a-2*b)*b*coth
(f*x+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*(3*a^2-13*a*b+8*b^2)*co
th(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/(a-b)^2/f-1/3*(3*a^2-13*a*b+8*b^2)*
(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/a^3/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/3*(3*a-2*b)*b*
(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/a^3/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a^2-13*a*
b+8*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a^3/(a-b)^2/f
```

Rubi [A]
time = 0.33, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3267, 483, 593, 597, 545, 429, 506, 422}

$$-\frac{2b(3a-2b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3a^3f(a-b)^2\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2b(3a-2b)\coth(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{(3a^2-13ab+8b^2)\,\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3a^3f(a-b)^2\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(3a^2-13ab+8b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)^2}-\frac{(3a^2-13ab+8b^2)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)^2}-\frac{b\coth(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*(b*Coth[e + f*x])/(a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (2*(3*
a - 2*b)*b*Coth[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) -
 ((3*a^2 - 13*a*b + 8*b^2)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^
3*(a - b)^2*f) - ((3*a^2 - 13*a*b + 8*b^2)*EllipticE[ArcTan[Sinh[e + f*x]],
 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)^2*f*Sqr
t[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(3*a - 2*b)*b*Elliptic
F[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]
)/(3*a^3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (
(3*a^2 - 13*a*b + 8*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^3*
(a - b)^2*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))]]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}\,(a+bx^2)^{5/2}}\,dx, x, \mathrm{sin}}{f}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)}{}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 1.61, size = 234, normalized size = 0.61

$$\frac{i\left(4a^2\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}\left(\left(-3a^2+13ab-8b^2\right)E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\left(3a^2-7ab+4b^2\right)F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)\right)+2i\sqrt{2}\left(3(a-b)^2(2a-b+b\cosh(2(e+fx)))^2\coth(e+fx)-2a(a-b)b^2\sinh(2(e+fx))-(7a-5b)b^2(2a-b+b\cosh(2(e+fx)))\sinh(2(e+fx))\right)\right)}{12a^3(a-b)^2f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((I/12)*(4*a^2*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*((-3*a^2 + 13*a*b
- 8*b^2)*EllipticE[I*(e + f*x), b/a] + (3*a^2 - 7*a*b + 4*b^2)*EllipticF[I*
(e + f*x), b/a]) + (2*I)*Sqrt[2]*(3*(a - b)^2*(2*a - b + b*Cosh[2*(e + f*x)
])^2*Coth[e + f*x] - 2*a*(a - b)*b^2*Sinh[2*(e + f*x)] - (7*a - 5*b)*b^2*(2
```

```
*a - b + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)]))))/(a^3*(a - b)^2*f*(2*a -
b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [**A**]
time = 1.91, size = 747, normalized size = 1.94

method	result
default	$-\frac{\left(3\sqrt{-\frac{b}{a}}\,a^{2}b^{2}-13\sqrt{-\frac{b}{a}}\,a\,b^{3}+8\sqrt{-\frac{b}{a}}\,b^{4}\right)\left(\cosh^{6}(fx+e)\right)+\left(6\sqrt{-\frac{b}{a}}\,a^{3}b-26\sqrt{-\frac{b}{a}}\,a^{2}b^{2}+38\sqrt{-\frac{b}{a}}\,a\,b^{3}-16\sqrt{-\frac{b}{a}}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((3*(-1/a*b)^(1/2)*a^2*b^2-13*(-1/a*b)^(1/2)*a*b^3+8*(-1/a*b)^(1/2)*b^
4)*cosh(f*x+e)^6+(6*(-1/a*b)^(1/2)*a^3*b-26*(-1/a*b)^(1/2)*a^2*b^2+38*(-1/a
*b)^(1/2)*a*b^3-16*(-1/a*b)^(1/2)*b^4)*cosh(f*x+e)^4+(b/a*cosh(f*x+e)^2+(a-
b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b^2*(9*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*a^2-17*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*
b+8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-3*EllipticE(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+13*EllipticE(sinh(f*x+e)*(-1/a*b)^(1
/2),(a/b)^(1/2))*a*b-8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^
2)*cosh(f*x+e)^2*sinh(f*x+e)+(3*(-1/a*b)^(1/2)*a^4-12*(-1/a*b)^(1/2)*a^3*b+
26*(-1/a*b)^(1/2)*a^2*b^2-25*(-1/a*b)^(1/2)*a*b^3+8*(-1/a*b)^(1/2)*b^4)*cos
h(f*x+e)^2+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(9*Ell
ipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3-26*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+25*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),
(a/b)^(1/2))*a*b^2-8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-
3*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3+16*EllipticE(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-21*EllipticE(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))*a*b^2+8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))
*b^3)*sinh(f*x+e))/(a+b*sinh(f*x+e)^2)^(3/2)/sinh(f*x+e)/(-1/a*b)^(1/2)/(a-
b)^2/a^3/cosh(f*x+e)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8769 vs. $2(385) = 770$.
time $= 0.31$, size $= 8769$, normalized size $= 22.78$

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(((6*a^3*b^2 - 29*a^2*b^3 + 29*a*b^4 - 8*b^5)*cosh(f*x + e)^10 + 10*(6*
a^3*b^2 - 29*a^2*b^3 + 29*a*b^4 - 8*b^5)*cosh(f*x + e)*sinh(f*x + e)^9 + (6
*a^3*b^2 - 29*a^2*b^3 + 29*a*b^4 - 8*b^5)*sinh(f*x + e)^10 + (48*a^4*b - 26
2*a^3*b^2 + 377*a^2*b^3 - 209*a*b^4 + 40*b^5)*cosh(f*x + e)^8 + (48*a^4*b -
 262*a^3*b^2 + 377*a^2*b^3 - 209*a*b^4 + 40*b^5 + 45*(6*a^3*b^2 - 29*a^2*b^
3 + 29*a*b^4 - 8*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(6*a^3*b^2 -
 29*a^2*b^3 + 29*a*b^4 - 8*b^5)*cosh(f*x + e)^3 + (48*a^4*b - 262*a^3*b^2 +
 377*a^2*b^3 - 209*a*b^4 + 40*b^5)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(48*a
^5 - 304*a^4*b + 610*a^3*b^2 - 557*a^2*b^3 + 241*a*b^4 - 40*b^5)*cosh(f*x +
 e)^6 + 2*(48*a^5 - 304*a^4*b + 610*a^3*b^2 - 557*a^2*b^3 + 241*a*b^4 - 40*
b^5 + 105*(6*a^3*b^2 - 29*a^2*b^3 + 29*a*b^4 - 8*b^5)*cosh(f*x + e)^4 + 14*
(48*a^4*b - 262*a^3*b^2 + 377*a^2*b^3 - 209*a*b^4 + 40*b^5)*cosh(f*x + e)^2
)*sinh(f*x + e)^6 + 4*(63*(6*a^3*b^2 - 29*a^2*b^3 + 29*a*b^4 - 8*b^5)*cosh(
f*x + e)^5 + 14*(48*a^4*b - 262*a^3*b^2 + 377*a^2*b^3 - 209*a*b^4 + 40*b^5)
*cosh(f*x + e)^3 + 3*(48*a^5 - 304*a^4*b + 610*a^3*b^2 - 557*a^2*b^3 + 241*
a*b^4 - 40*b^5)*cosh(f*x + e))*sinh(f*x + e)^5 - 6*a^3*b^2 + 29*a^2*b^3 - 2
9*a*b^4 + 8*b^5 - 2*(48*a^5 - 304*a^4*b + 610*a^3*b^2 - 557*a^2*b^3 + 241*a
*b^4 - 40*b^5)*cosh(f*x + e)^4 + 2*(105*(6*a^3*b^2 - 29*a^2*b^3 + 29*a*b^4
- 8*b^5)*cosh(f*x + e)^6 - 48*a^5 + 304*a^4*b - 610*a^3*b^2 + 557*a^2*b^3 -
 241*a*b^4 + 40*b^5 + 35*(48*a^4*b - 262*a^3*b^2 + 377*a^2*b^3 - 209*a*b^4
+ 40*b^5)*cosh(f*x + e)^4 + 15*(48*a^5 - 304*a^4*b + 610*a^3*b^2 - 557*a^2*
b^3 + 241*a*b^4 - 40*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(15*(6*a^3*b
^2 - 29*a^2*b^3 + 29*a*b^4 - 8*b^5)*cosh(f*x + e)^7 + 7*(48*a^4*b - 262*a^3
*b^2 + 377*a^2*b^3 - 209*a*b^4 + 40*b^5)*cosh(f*x + e)^5 + 5*(48*a^5 - 304*
a^4*b + 610*a^3*b^2 - 557*a^2*b^3 + 241*a*b^4 - 40*b^5)*cosh(f*x + e)^3 - (
48*a^5 - 304*a^4*b + 610*a^3*b^2 - 557*a^2*b^3 + 241*a*b^4 - 40*b^5)*cosh(f
*x + e))*sinh(f*x + e)^3 - (48*a^4*b - 262*a^3*b^2 + 377*a^2*b^3 - 209*a*b^
4 + 40*b^5)*cosh(f*x + e)^2 + (45*(6*a^3*b^2 - 29*a^2*b^3 + 29*a*b^4 - 8*b^
5)*cosh(f*x + e)^8 + 28*(48*a^4*b - 262*a^3*b^2 + 377*a^2*b^3 - 209*a*b^4 +
 40*b^5)*cosh(f*x + e)^6 - 48*a^4*b + 262*a^3*b^2 - 377*a^2*b^3 + 209*a*b^4
 - 40*b^5 + 30*(48*a^5 - 304*a^4*b + 610*a^3*b^2 - 557*a^2*b^3 + 241*a*b^4
- 40*b^5)*cosh(f*x + e)^4 - 12*(48*a^5 - 304*a^4*b + 610*a^3*b^2 - 557*a^2*
b^3 + 241*a*b^4 - 40*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(6*a^3*b^
2 - 29*a^2*b^3 + 29*a*b^4 - 8*b^5)*cosh(f*x + e)^9 + 4*(48*a^4*b - 262*a^3*
b^2 + 377*a^2*b^3 - 209*a*b^4 + 40*b^5)*cosh(f*x + e)^7 + 6*(48*a^5 - 304*a
^4*b + 610*a^3*b^2 - 557*a^2*b^3 + 241*a*b^4 - 40*b^5)*cosh(f*x + e)^5 - 4*
```

```
(48*a^5 - 304*a^4*b + 610*a^3*b^2 - 557*a^2*b^3 + 241*a*b^4 - 40*b^5)*cosh(
f*x + e)^3 - (48*a^4*b - 262*a^3*b^2 + 377*a^2*b^3 - 209*a*b^4 + 40*b^5)*co
sh(f*x + e))*sinh(f*x + e) - 2*((3*a^2*b^3 - 13*a*b^4 + 8*b^5)*cosh(f*x + e
)^10 + 10*(3*a^2*b^3 - 13*a*b^4 + 8*b^5)*cosh(f*x + e)*sinh(f*x + e)^9 + (3
*a^2*b^3 - 13*a*b^4 + 8*b^5)*sinh(f*x + e)^10 + (24*a^3*b^2 - 119*a^2*b^3 +
 129*a*b^4 - 40*b^5)*cosh(f*x + e)^8 + (24*a^3*b^2 - 119*a^2*b^3 + 129*a*b^
4 - 40*b^5 + 45*(3*a^2*b^3 - 13*a*b^4 + 8*b^5)*cosh(f*x + e)^2)*sinh(f*x +
e)^8 + 8*(15*(3*a^2*b^3 - 13*a*b^4 + 8*b^5)*cosh(f*x + e)^3 + (24*a^3*b^2 -
 119*a^2*b^3 + 129*a*b^4 - 40*b^5)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(24*a
^4*b - 140*a^3*b^2 + 235*a^2*b^3 - 161*a*b^4 + 40*b^5)*cosh(f*x + e)^6 + 2*
(24*a^4*b - 140*a^3*b^2 + 235*a^2*b^3 - 161*a*b^4 + 40*b^5 + 105*(3*a^2*b^3
 - 13*a*b^4 + 8*b^5)*cosh(f*x + e)^4 + 14*(24*a^3*b^2 - 119*a^2*b^3 + 129*a
*b^4 - 40*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(3*a^2*b^3 - 13*a*b
^4 + 8*b^5)*cosh(f*x + e)^5 + 14*(24*a^3*b^2 - 119*a^2*b^3 + 129*a*b^4 - 40
*b^5)*cosh(f*x + e)^3 + 3*(24*a^4*b - 140*a^3*b^2 + 235*a^2*b^3 - 161*a*b^4
 + 40*b^5)*cosh(f*x + e))*sinh(f*x + e)^5 - 3*a^2*b^3 + 13*a*b^4 - 8*b^5 -
2*(24*a^4*b - 140*a^3*b^2 + 235*a^2*b^3 - 161*a*b^4 + 40*b^5)*cosh(f*x + e)
^4 + 2*(105*(3*a^2*b^3 - 13*a*b^4 + 8*b^5)*cosh(f*x + e)^6 - 24*a^4*b + 140
*a^3*b^2 - 235*a^2*b^3 + 161*a*b^4 - 40*b^5 + 35*(24*a^3*b^2 - 119*a^2*b^3
+ 129*a*b^4 - 40*b^5)*cosh(f*x + e)^4 + 15*(24*a^4*b - 140*a^3*b^2 + 235*a^
2*b^3 - 161*a*b^4 + 40*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(15*(3*a^2
*b^3 - 13*a*b^4 + 8*b^5)*cosh(f*x + e)^7 + 7*(24*a^3*b^2 - 119*a^2*b^3 + 12
9*a*b^4 - 40*b^5)*cosh(f*x + e)^5 + 5*(24*a^4*b - 140*a^3*b^2 + 235*a^2*b^3
 - 161*a*b^4 + 40*b^5)*cosh(f*x + e)^3 - (24*a^4*b - 140*a^3*b^2 + 235*a^2*
b^3 - 161*a*b^4 + 40*b^5)*cosh(f*x + e))*sinh(f*x + e)^3 - (24*a^3*b^2 - 11
9*a^2*b^3 + 129*a*b^4 - 40*b^5)*cosh(f*x + e)^2 + (45*(3*a^2*b^3 - 13*a*b^4
 + 8*b^5)*cosh(f*x + e)^8 + 28*(24*a^3*b^2 - 119*a^2*b^3 + 129*a*b^4 - 40*b
^5)*cosh(f*x + e)^6 - 24*a^3*b^2 + 119*a^2*b^3 - 129*a*b^4 + 40*b^5 + 30*(2
4*a^4*b - 140*a^3*b^2 + 235*a^2*b^3 - 161*a*b^4 + 40*b^5)*cosh(f*x + e)^4 -
 12*(24*a^4*b - 140*a^3*b^2 + 235*a^2*b^3 - 161*a*b^4 + 40*b^5)*cosh(f*x +
e)^2)*sinh(f*x + e)^2 + 2*(5*(3*a^2*b^3 - 13*a*...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.71Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(e+f\,x\right)^2\,{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(5/2)), x)
```

3.125 $\int \frac{1}{\sqrt{1+\sinh^2(x)}}\,dx$

Optimal. Leaf size=14

$$\frac{\text{ArcTan}(\sinh(x))\cosh(x)}{\sqrt{\cosh^2(x)}}$$

```
[Out] arctan(sinh(x))*cosh(x)/(cosh(x)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3255, 3286, 3855}

$$\frac{\cosh(x)\text{ArcTan}(\sinh(x))}{\sqrt{\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Sinh[x]^2],x]
```

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[Cosh[x]^2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{1+\sinh^2(x)}}\,dx &= \int \frac{1}{\sqrt{\cosh^2(x)}}\,dx \\
&= \frac{\cosh(x)\int \operatorname{sech}(x)\,dx}{\sqrt{\cosh^2(x)}} \\
&= \frac{\tan^{-1}(\sinh(x))\cosh(x)}{\sqrt{\cosh^2(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.36

$$\frac{2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\cosh(x)}{\sqrt{\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sinh[x]^2],x]
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x])/Sqrt[Cosh[x]^2]
```

Maple [A]
time = 0.79, size = 15, normalized size = 1.07

method	result	size
default	$\frac{\sqrt{\frac{1}{2}+\frac{\cosh(2x)}{2}}\,\arctan(\sinh(x))}{\cosh(x)}$	15
risch	$\frac{i\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\ln(\mathrm{e}^{x}+i)}{\sqrt{\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}-\frac{i\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\ln(\mathrm{e}^{x}-i)}{\sqrt{\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (cosh(x)^2)^(1/2)*arctan(sinh(x))/cosh(x)
```

Maxima [A]
time = 0.50, size = 5, normalized size = 0.36

$$2\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^x)
```

Fricas [**A**]
time = 0.45, size = 8, normalized size = 0.57

$$2\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(x) + sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\sinh^{2}\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sinh(x)**2 + 1), x)
```

Giac [**A**]
time = 0.41, size = 5, normalized size = 0.36

$$2\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.07

$$\int\frac{1}{\sqrt{{\sinh\left(x\right)}^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2 + 1)^(1/2),x)
```

```
[Out] int(1/(sinh(x)^2 + 1)^(1/2), x)
```

3.126 $\int \frac{1}{\sqrt{1-\sinh^2(x)}}\,dx$

Optimal. Leaf size=11

$$-iF(ix|-1)$$

```
[Out] -I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticF(I*sinh(x),I)
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{3261\}$

$$-iF(ix|-1)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Sinh[x]^2],x]
```

```
[Out] (-I)*EllipticF[I*x, -1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\sinh^2(x)}}\,dx = -iF(ix|-1)$$

Mathematica [A]
time = 0.03, size = 11, normalized size = 1.00

$$-iF(ix|-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Sinh[x]^2],x]
```

```
[Out] (-I)*EllipticF[I*x, -1]
```

Maple [A]
time = 0.77, size = 41, normalized size = 3.73

method	result	size
default	$\frac{\sqrt{-\left(-1+\sinh^{2}\left(x\right)\right)\left(\cosh^{2}\left(x\right)\right)}\,\sqrt{\frac{1}{2}+\frac{\cosh\left(2x\right)}{2}}\,\mathrm{EllipticF}\left(\sinh\left(x\right),i\right)}{\sqrt{1-\left(\sinh^{4}\left(x\right)\right)}\,\cosh\left(x\right)}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)/(1-sinh(x)^4)^(1/2)*Ell
ipticF(sinh(x),I)/cosh(x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-sinh(x)^2 + 1), x)
```

Fricas [**F**]
time = 0.09, size = 1, normalized size = 0.09

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{1-\sinh^{2}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - sinh(x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-sinh(x)^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{1}{\sqrt{1-\sinh\left(x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - sinh(x)^2)^(1/2),x)
```

```
[Out] int(1/(1 - sinh(x)^2)^(1/2), x)
```

3.127 $\int \frac{1}{\sqrt{-1+\sinh^2(x)}}\,dx$

Optimal. Leaf size=33

$$-\frac{iF(ix|-1)\sqrt{1-\sinh^2(x)}}{\sqrt{-1+\sinh^2(x)}}$$

```
[Out] -I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticF(I*sinh(x),I)*(1-sinh(x)^2)^(1/2)/(-1
+sinh(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3262, 3261}

$$-\frac{i\sqrt{1-\sinh^2(x)}\,F(ix|-1)}{\sqrt{\sinh^2(x)-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + Sinh[x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*x, -1]*Sqrt[1 - Sinh[x]^2])/Sqrt[-1 + Sinh[x]^2]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\sinh^2(x)}}\,dx = \frac{\sqrt{1-\sinh^2(x)}\int \frac{1}{\sqrt{1-\sinh^2(x)}}\,dx}{\sqrt{-1+\sinh^2(x)}}$$

$$= -\frac{iF(ix|-1)\sqrt{1-\sinh^2(x)}}{\sqrt{-1+\sinh^2(x)}}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 1.00

$$-\frac{i\sqrt{3-\cosh(2x)}\,F(ix|-1)}{\sqrt{-3+\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Sinh[x]^2],x]
```

```
[Out] ((-I)*Sqrt[3 - Cosh[2*x]]*EllipticF[I*x, -1])/Sqrt[-3 + Cosh[2*x]]
```

Maple [A]
time = 0.82, size = 61, normalized size = 1.85

method	result	size
default	$-\frac{i\sqrt{\left(-1+\sinh^2(x)\right)\left(\cosh^2(x)\right)}\,\sqrt{\frac{1}{2}+\frac{\cosh(2x)}{2}}\,\sqrt{1-\left(\sinh^2(x)\right)}\,\mathrm{EllipticF}(i\sinh(x),i)}{\sqrt{\sinh^4(x)-1}\,\cosh(x)\sqrt{-1+\sinh^2(x)}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*((-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)*(1-sinh(x)^2)^(1/2)/(
sinh(x)^4-1)^(1/2)*EllipticF(I*sinh(x),I)/cosh(x)/(-1+sinh(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sinh(x)^2 - 1), x)
```

Fricas [A]
time = 0.12, size = 42, normalized size = 1.27

$$-2\sqrt{2\sqrt{2}+3}\left(2\sqrt{2}-3\right)F(\arcsin\left(\sqrt{2\sqrt{2}+3}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\right)\mid -12\sqrt{2}+17)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(2*sqrt(2) + 3)*(2*sqrt(2) - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) + 3
)*(cosh(x) + sinh(x))), -12*sqrt(2) + 17)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sinh^{2}\left(x\right)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sinh(x)**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sinh(x)^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{{\sinh\left(x\right)}^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2 - 1)^(1/2),x)
```

```
[Out] int(1/(sinh(x)^2 - 1)^(1/2), x)
```

3.128 $\int \frac{1}{\sqrt{-1-\sinh^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\text{ArcTan}(\sinh(x))\cosh(x)}{\sqrt{-\cosh^2(x)}}$$

```
[Out] arctan(sinh(x))*cosh(x)/(-cosh(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3255, 3286, 3855}

$$\frac{\cosh(x)\text{ArcTan}(\sinh(x))}{\sqrt{-\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 - Sinh[x]^2],x]
```

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[-Cosh[x]^2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1-\sinh^2(x)}}\,dx &= \int \frac{1}{\sqrt{-\cosh^2(x)}}\,dx\\ &= \frac{\cosh(x)\int \operatorname{sech}(x)\,dx}{\sqrt{-\cosh^2(x)}}\\ &= \frac{\tan^{-1}(\sinh(x))\cosh(x)}{\sqrt{-\cosh^2(x)}}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 21, normalized size = 1.31

$$\frac{2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\cosh(x)}{\sqrt{-\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Sinh[x]^2],x]
```

```
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x])/Sqrt[-Cosh[x]^2]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.
time = 0.76, size = 34, normalized size = 2.12

method	result	size
default	$-\frac{\cosh(x)\sqrt{-\left(\sinh^2(x)\right)}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-\left(\sinh^2(x)\right)}}\right)}{\sinh(x)\sqrt{-\left(\cosh^2(x)\right)}}$	34
risch	$\frac{i\,\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\ln\left(\mathrm{e}^{x}+i\right)}{\sqrt{-\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}-\frac{i\,\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\ln\left(\mathrm{e}^{x}-i\right)}{\sqrt{-\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -cosh(x)*(-sinh(x)^2)^(1/2)*arctanh(1/(-sinh(x)^2)^(1/2))/sinh(x)/(-cosh(x)
^2)^(1/2)
```

Maxima [**C**] Result contains complex when optimal does not.
time = 0.53, size = 5, normalized size = 0.31

$$-2i\,\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*I*arctan(e^x)
```

Fricas [**C**] Result contains complex when optimal does not.
time = 0.45, size = 13, normalized size = 0.81

$$\log(e^x+i)-\log(e^x-i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(e^x + I) - log(e^x - I)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\sinh^2(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-sinh(x)**2 - 1), x)
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.41, size = 5, normalized size = 0.31

$$-2i\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*I*arctan(e^x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.06

$$\int\frac{1}{\sqrt{-\sinh(x)^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- sinh(x)^2 - 1)^(1/2),x)
```

```
[Out] int(1/(- sinh(x)^2 - 1)^(1/2), x)
```

3.129 $\int \frac{1}{\sqrt{a+b\sinh^2(x)}}\,dx$

Optimal. Leaf size=42

$$-\frac{iF\left(ix\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(x)}{a}}}{\sqrt{a+b\sinh^2(x)}}$$

```
[Out] -I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticF(I*sinh(x),(b/a)^(1/2))*(1+b*sinh(x)^
2/a)^(1/2)/(a+b*sinh(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3262, 3261}

$$-\frac{i\sqrt{\frac{b\sinh^2(x)}{a}+1}\;F\left(ix\middle|\frac{b}{a}\right)}{\sqrt{a+b\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*x, b/a]*Sqrt[1 + (b*Sinh[x]^2)/a])/Sqrt[a + b*Sinh[x]^2]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^2(x)}}\,dx = \frac{\sqrt{1+\frac{b\sinh^2(x)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sinh^2(x)}{a}}}\,dx}{\sqrt{a+b\sinh^2(x)}}$$
$$= -\frac{iF\left(ix\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(x)}{a}}}{\sqrt{a+b\sinh^2(x)}}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 1.26

$$-\frac{i\sqrt{\frac{2a-b+b\cosh(2x)}{a}}\,F\left(ix\middle|\frac{b}{a}\right)}{\sqrt{2a-b+b\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticF[I*x, b/a])/Sqrt[2*a - b + b
*Cosh[2*x]]
```

Maple [A]
time = 0.90, size = 63, normalized size = 1.50

method	result	size
default	$\frac{\sqrt{\frac{a+b\left(\sinh^2(x)\right)}{a}}\,\sqrt{\frac{1}{2}+\frac{\cosh(2x)}{2}}\,\mathrm{EllipticF}\left(\sinh(x)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\,\cosh(x)\sqrt{a+b\left(\sinh^2(x)\right)}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(x)^2)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh
(x)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(x)/(a+b*sinh(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(x)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(47) = 94$.
time = 0.10, size = 136, normalized size = 3.24

$$-\frac{2\left(2\,b\sqrt{\frac{a^2-ab}{b^2}}+2\,a-b\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}\,F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}\,(\cosh\left(x\right)+\sinh\left(x\right))\right)\mid\frac{8\,a^2-8\,ab+b^2+4\,(2\,ab-b^2)\,\sqrt{\frac{a^2-ab}{b^2}}}{b^2})}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)
*(cosh(x) + sinh(x))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a
*b)/b^2))/b^2)/b^(3/2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a+b\sinh^2\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sinh(x)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{b\,{\sinh\left(x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x)^2)^(1/2),x)
[Out] int(1/(a + b*sinh(x)^2)^(1/2), x)
```

3.130 $\int (d\sinh(e+fx))^m\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=128

$$\frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-}}{f}$$

```
[Out] d*AppellF1(1/2,1/2-1/2*m,-p,3/2,cosh(f*x+e)^2,-b*cosh(f*x+e)^2/(a-b))*cosh(
f*x+e)*(a-b+b*cosh(f*x+e)^2)^p*(d*sinh(f*x+e))^(-1+m)*(-sinh(f*x+e)^2)^(1/2
-1/2*m)/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Rubi [A]
time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3268, 441, 440}

$$\frac{d\cosh(e+fx)\left(-\sinh^2(e+fx)\right)^{\frac{1-m}{2}}(d\sinh(e+fx))^{m-1}\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)
/(a - b))]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*(d*Sinh[e + f*x])^(-
1 + m)*(-Sinh[e + f*x]^2)^((1 - m)/2))/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))
^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
```

```
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p\,dx &= \frac{\left(d(d\sinh(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\left(-\sinh^2(e+fx)\right)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}}{f}\\ &= \frac{\left(d\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}(d\sinh}{}\\ &= \frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+}{}\end{aligned}$$

Mathematica [F]
time = 6.25, size = 0, normalized size = 0.00

$$\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F]
time = 1.81, size = 0, normalized size = 0.00

$$\int \left(d\sinh\left(fx+e\right)\right)^m \left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)
```

Fricas [**F**]
time = 0.50, size = 27, normalized size = 0.21

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\left(d\sinh\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sinh(f*x+e))**m*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\mathrm{sinh}\left(e+f\,x\right)\right)}^m\,{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sinh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int((d*sinh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p, x)
```

3.131 $\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=226

$$-\frac{(3a+2b(2+p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}+\frac{(3a^2+4ab(1+p)+4b^2(2+3p+p^2))\cosh(}{}$$

```
[Out] -(3*a+2*b*(2+p))*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1+p)/b^2/f/(4*p^2+16*p+
15)+(3*a^2+4*a*b*(1+p)+4*b^2*(p^2+3*p+2))*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)
^p*hypergeom([1/2, -p],[3/2],-b*cosh(f*x+e)^2/(a-b))/b^2/f/(4*p^2+16*p+15)/
((1+b*cosh(f*x+e)^2/(a-b))^p)+cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1+p)*sinh(
f*x+e)^2/b/f/(5+2*p)
```

Rubi [A]
time = 0.18, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3265, 427, 396, 252, 251}

$$\frac{(3a^2+4ab(p+1)+4b^2(p^2+3p+2))\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2(e+fx)}{a-b}\right)}{b^2f(2p+3)(2p+5)}-\frac{(3a+2b(p+2))\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{p+1}}{b^2f(2p+3)(2p+5)}+\frac{\sinh^2(e+fx)\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{p+1}}{bf(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -(((3*a + 2*b*(2 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/(
b^2*f*(3 + 2*p)*(5 + 2*p))) + ((3*a^2 + 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^
2))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p,
3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Co
sh[e + f*x]^2)/(a - b))^p) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1
+ p)*Sinh[e + f*x]^2)/(b*f*(5 + 2*p))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(1-x^2\right)^2\left(a-b+bx^2\right)^p\,dx,x,\cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}\sinh^2(e+fx)}{bf(5+2p)} + \text{Su}$$

$$= -\frac{(3a+2b(2+p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} -$$

$$= -\frac{(3a+2b(2+p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} -$$

$$= -\frac{(3a+2b(2+p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} -$$

Mathematica [F]
time = 7.56, size = 0, normalized size = 0.00

$$\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [**F**]
time = 1.72, size = 0, normalized size = 0.00

$$\int \left(\sinh^{5}\left(f x+e\right)\right) \left(a+b \left(\sinh^{2}\left(f x+e\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

Fricas [**F**]
time = 0.43, size = 25, normalized size = 0.11

$$\mathrm{integral}\left(\left(b \sinh\left(f x+e\right)^{2}+a\right)^{p} \sinh\left(f x+e\right)^{5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh(e+f\,x)^5\,\left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int(sinh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^p, x)
```

3.132 $\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=137

$$\frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{bf(3+2p)}-\frac{(a+2b(1+p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\right.}{bf(3+2p)}$$

```
[Out] cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1+p)/b/f/(3+2*p)-(a+2*b*(1+p))*cosh(f*x+
e)*(a-b+b*cosh(f*x+e)^2)^p*hypergeom([1/2, -p],[3/2],-b*cosh(f*x+e)^2/(a-b)
)/b/f/(3+2*p)/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Rubi [A]
time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3265, 396, 252, 251}

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{p+1}}{bf(2p+3)}-\frac{(a+2b(p+1))\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2(e+fx)}{a-b}\right)}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a +
 2*b*(1 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1
[1/2, -p, 3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(b*f*(3 + 2*p)*(1 + (b*Cosh
[e + f*x]^2)/(a - b))^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx &= -\frac{\text{Subst}\left(\int (1-x^2)\left(a-b+bx^2\right)^p\,dx, x, \cosh(e+fx)\right)}{f}\\
&= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a+2b(1+p))\mathrm{S}}{}\\
&= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{\Big((a+2b(1+p))}{}\\
&= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a+2b(1+p))\,\mathrm{c}}{}
\end{aligned}
$$

Mathematica [F]
time = 9.47, size = 0, normalized size = 0.00

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F]
time = 1.91, size = 0, normalized size = 0.00

$$\int \left(\sinh^3(fx+e)\right)\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)
```

Fricas [F]
time = 0.42, size = 25, normalized size = 0.18

$$\mathrm{integral}\Big(\big(b\sinh\left(fx+e\right)^2+a\big)^p\sinh\left(fx+e\right)^3,x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(e+f\,x)^3\,\left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^p,x)
[Out] int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^p, x)
```

3.133 $\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^p*hypergeom([1/2, -p],[3/2],-b*cosh(f*x+e
)^2/(a-b))/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3265, 252, 251}

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, -((b*Cosh[e + f*x]^2)/(a - b))])/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\operatorname{Subst}\left(\int \left(a-b+bx^2\right)^p\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\Bigg(\left(a-b+b\cosh^2(e+fx)\right)^p \Big(1+\frac{b\cosh^2(e+fx)}{a-b}\Big)^{-p}\Bigg) \operatorname{Subst}\Big(\int \Big(}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p \Big(1+\frac{b\cosh^2(e+fx)}{a-b}\Big)^{-p}}{f}$$

Mathematica [**A**]
time = 0.16, size = 78, normalized size = 1.00

$$\frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p \left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, -((b*Cosh[e + f*x]^2)/(a - b))])/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)
```

Maple [**F**]
time = 0.70, size = 0, normalized size = 0.00

$$\int \sinh(fx+e)\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

Fricas [**F**]
time = 0.40, size = 23, normalized size = 0.29

$$\text{integral}\Big(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\sinh\left(fx+e\right),x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(e+f\,x)\,\left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^p, x)
```

3.134 $\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}}{f}$$

```
[Out] -AppellF1(1/2,1,-p,3/2,cosh(f*x+e)^2,-b*cosh(f*x+e)^2/(a-b))*cosh(f*x+e)*(a
-b+b*cosh(f*x+e)^2)^p/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Rubi [A]

time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3265, 441, 440}

$$-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b)
)]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)
/(a - b))^p))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^p}{1-x^2}\,dx, x, \cosh(e+fx)\right)}{f}\\
&= -\frac{\Bigg(\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\Bigg)\operatorname{Subst}\Bigg(\int}{f}\\
&= -\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\,\big(a}{f}
\end{aligned}
$$

Mathematica [**F**]
time = 2.32, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [**F**]
time = 1.24, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(fx+e)\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

Fricas [**F**]
time = 0.41, size = 23, normalized size = 0.26

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^p}{\sinh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x), x)
```

3.135 $\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=87

$$\frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,2,-p,3/2,cosh(f*x+e)^2,-b*cosh(f*x+e)^2/(a-b))*cosh(f*x+e)*(a-
b+b*cosh(f*x+e)^2)^p/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 441, 440}

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))]
*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)/(
a - b))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^p}{(1-x^2)^2}\,dx, x, \cosh(e+fx)\right)}{f}\\
&= \frac{\Bigg(\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\Bigg)\operatorname{Subst}\Bigg(\int}{f}\\
&= \frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\,\big(a}{f}
\end{aligned}
$$

Mathematica [F]
time = 81.55, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F]
time = 1.27, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(fx+e)^3\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.45, size = 25, normalized size = 0.29

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\operatorname{csch}\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^p}{\sinh\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^3,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^3, x)
```

3.136 $\int \mathrm{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}}{f}$$

```
[Out] -AppellF1(1/2,3,-p,3/2,cosh(f*x+e)^2,-b*cosh(f*x+e)^2/(a-b))*cosh(f*x+e)*(a
-b+b*cosh(f*x+e)^2)^p/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 441, 440}

$$-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 3, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b)
)]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)
/(a - b))^p))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^p}{(1-x^2)^3}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\Big(\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\Big)\operatorname{Subst}\Big(\int}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\,\big(a}{f}$$

Mathematica [**F**]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] $Aborted
```

Maple [**F**]
time = 1.28, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(fx+e)^5\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^5, x)
```

Fricas [**F**]
time = 0.42, size = 25, normalized size = 0.28

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\operatorname{csch}\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^5, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)^p}{{\mathrm{sinh}\left(e+f\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^5,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^5, x)
```

3.137 $\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\,\sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\Bigl(1+}{5f}$$

```
[Out] 1/5*AppellF1(5/2,1/2,-p,7/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*sinh(f*x+e)^
4*(a+b*sinh(f*x+e)^2)^p*(cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f/((1+b*sinh(f*x+
e)^2/a)^p)
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3267, 525, 524}

$$\frac{\sinh^4(e+fx)\sqrt{\cosh^2(e+fx)}\,\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[5/2, 1/2, -p, 7/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x]
)/(5*f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\big(\int \frac{x^4(a+bx^2)^p}{\sqrt{1+x^2}}\,dx, x, \mathrm{s}}{f}$$

$$= \frac{\big(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p \big(1+}{f}$$

$$= \frac{F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+f}{}$$

Mathematica [F]
time = 6.75, size = 0, normalized size = 0.00

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F]
time = 1.99, size = 0, normalized size = 0.00

$$\int \left(\sinh^4(fx+e)\right)\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.42, size = 25, normalized size = 0.24

$$\text{integral}\Big(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\sinh\left(fx+e\right)^{4},x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(e+f\,x)^4\,\left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^p, x)
```

3.138 $\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=101

$$\frac{F_1\left(\frac{3}{2};2+p,-p;\frac{5}{2};\tanh^2(e+fx),\frac{(a-b)\tanh^2(e+fx)}{a}\right)\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\tanh^3(e+fx)\left(1\right.}{3f}$$

```
[Out] 1/3*AppellF1(3/2,2+p,-p,5/2,tanh(f*x+e)^2,(a-b)*tanh(f*x+e)^2/a)*(sech(f*x+
e)^2)^p*(a+b*sinh(f*x+e)^2)^p*tanh(f*x+e)^3/f/((1-(a-b)*tanh(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3253, 525, 524}

$$\frac{\tanh^3(e+fx)\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(1-\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{-p}F_1\left(\frac{3}{2};p+2,-p;\frac{5}{2};\tanh^2(e+fx),\frac{(a-b)\tanh^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2 + p, -p, 5/2, Tanh[e + f*x]^2, ((a - b)*Tanh[e + f*x]^2)/a
]*(Sech[e + f*x]^2)^p*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x]^3)/(3*f*(1 -
((a - b)*Tanh[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3253

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff*(a + b*Sin[e + f*x]^2)^p*((Sec[e + f*x]^2)^p/(f*(a + (a + b)*Tan[e +
```

```
 f*x]^2)^p)), Subst[Int[(a + (a + b)*ff^2*x^2)^p*((A + (A + B)*ff^2*x^2)/(1
 + ff^2*x^2)^(p + 2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, A
, B}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\frac{\left(\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(a-(a-b)\tanh^2(e+\right.\right.}{}$$

$$=\frac{\left(\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(a-(a-b)\tanh^2(e+\right.\right.}{}$$

$$=\frac{F_1\left(\frac{3}{2};2+p,-p;\frac{5}{2};\tanh^2(e+fx),\frac{(a-b)\tanh^2(e+fx)}{a}\right)\operatorname{sech}^2(e+f}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 250 vs. $2(101)=$ 202.
time = 0.55, size = 250, normalized size = 2.48

$$\frac{2^{-2-p}\sqrt{\frac{b\cosh^2(e+fx)}{-a+b}}\,(2a-b+b\cosh(2(e+fx)))^{1+p}\left(-2a(2+p)F_1\left(1+p;\frac{1}{2},\frac{1}{2};2+p;\frac{2a-b+b\cosh(2(e+fx))}{2a},\frac{2a-b+b\cosh(2(e+fx))}{2(a-b)}\right)+(1+p)F_1\left(2+p;\frac{1}{2},\frac{1}{2};3+p;\frac{2a-b+b\cosh(2(e+fx))}{2a},\frac{2a-b+b\cosh(2(e+fx))}{2(a-b)}\right)(2a-b+b\cosh(2(e+fx)))\right)\operatorname{csch}(2(e+fx))\sqrt{-\frac{b\sinh^2(e+fx)}{a}}}{b^2f(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (2^(-2 - p)*Sqrt[(b*Cosh[e + f*x]^2)/(-a + b)]*(2*a - b + b*Cosh[2*(e + f*x
)])^(1 + p)*(-2*a*(2 + p)*AppellF1[1 + p, 1/2, 1/2, 2 + p, (2*a - b + b*Cos
h[2*(e + f*x)])/(2*a), (2*a - b + b*Cosh[2*(e + f*x)])/(2*(a - b))] + (1 +
p)*AppellF1[2 + p, 1/2, 1/2, 3 + p, (2*a - b + b*Cosh[2*(e + f*x)])/(2*a),
(2*a - b + b*Cosh[2*(e + f*x)])/(2*(a - b))]*(2*a - b + b*Cosh[2*(e + f*x)]
))*Csch[2*(e + f*x)]*Sqrt[-((b*Sinh[e + f*x]^2)/a)])/(b^2*f*(1 + p)*(2 + p)
)
```

Maple [**F**]
time = 1.84, size = 0, normalized size = 0.00

$$\int \left(\sinh^2\left(fx+e\right)\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.41, size = 25, normalized size = 0.25

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\sinh\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(e+f\,x)^2\,\left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^p,x)
[Out] int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^p, x)
```

3.139 $\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=99

$$-\frac{F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\left(a+b\sinh^2(e\right.}{f}$$

```
[Out] -AppellF1(-1/2,1/2,-p,1/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*csch(f*x+e)*se
ch(f*x+e)*(a+b*sinh(f*x+e)^2)^p*(cosh(f*x+e)^2)^(1/2)/f/((1+b*sinh(f*x+e)^2
/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3267, 525, 524}

$$-\frac{\sqrt{\cosh^2(e+fx)}\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1/2, -p, 1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]
*Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^
p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx &= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+bx^2)^p}{x^2\sqrt{1+x^2}}\,dx, x,\right.}{f}\\ &= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\right.\right.}{f}\\ &= -\frac{F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e}}{}\end{aligned}$$

Mathematica [**F**]
time = 2.94, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [**F**]
time = 1.08, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(fx+e)^2\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.44, size = 25, normalized size = 0.25

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\operatorname{csch}\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,\mathrm{sinh}(e+f\,x)^2+a\right)^p}{\mathrm{sinh}\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^2,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^2, x)
```

3.140 $\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$-\frac{F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\,\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\left(a+b\operatorname{sinh}\right.}{3f}$$

```
[Out] -1/3*AppellF1(-3/2,1/2,-p,-1/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*csch(f*x+
e)^3*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p*(cosh(f*x+e)^2)^(1/2)/f/((1+b*sinh(f
*x+e)^2/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3267, 525, 524}

$$-\frac{\sqrt{\cosh^2(e+fx)}\,\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -1/3*(AppellF1[-3/2, 1/2, -p, -1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)
/a)]*Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x]*(a + b*Sinh[e + f*
x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx &= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{(a+bx^2)^p}{x^4\sqrt{1+x^2}}\,dx,x,\mathrm{s}\right.}{f}\\ &= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\mathrm{s}}{}\right.\right.}{f}\\ &= -\frac{F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e}}{}\end{aligned}$$

Mathematica [**F**]
time = 5.45, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

`[In] Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]`

`[Out] Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]`

Maple [**F**]
time = 1.20, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(fx+e)^4\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)`

`[Out] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.43, size = 25, normalized size = 0.24

$$\operatorname{integral}\left(\left(b \sinh\left(fx+e\right)^{2}+a\right)^{p} \operatorname{csch}\left(fx+e\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b \sinh(e+f\,x)^{2}+a\right)^{p}}{\sinh\left(e+f\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^4,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^4, x)
```

3.141 $\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=106

$$\frac{3ax}{8}-\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{d}-\frac{3b\cosh^5(c+dx)}{5d}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3a\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a}{}$$

```
[Out] 3/8*a*x-b*cosh(d*x+c)/d+b*cosh(d*x+c)^3/d-3/5*b*cosh(d*x+c)^5/d+1/7*b*cosh(
d*x+c)^7/d-3/8*a*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cosh(d*x+c)*sinh(d*x+c)^3/
d
```

Rubi [A]

time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3299, 2715, 8, 2713}

$$\frac{a\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3a\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3ax}{8}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3b\cosh^5(c+dx)}{5d}+\frac{b\cosh^3(c+dx)}{d}-\frac{b\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (3*a*x)/8 - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/d - (3*b*Cosh[c + d*x
]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d) - (3*a*Cosh[c + d*x]*Sinh[c + d*x])/
(8*d) + (a*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
```

```
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx &= \int \left(a\sinh^4(c+dx)+b\sinh^7(c+dx)\right)\,dx\\
&= a\int \sinh^4(c+dx)\,dx+b\int \sinh^7(c+dx)\,dx\\
&= \frac{a\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{4}(3a)\int \sinh^2(c+dx)\,dx-\\
&= -\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{d}-\frac{3b\cosh^5(c+dx)}{5d}+\\
&= \frac{3ax}{8}-\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{d}-\frac{3b\cosh^5(c+dx)}{5d}+
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 81, normalized size = 0.76

$$\frac{840ac+840adx-1225b\cosh(c+dx)+245b\cosh(3(c+dx))-49b\cosh(5(c+dx))+5b\cosh(7(c+dx))-560a\sinh(2(c+dx))+70a\sinh(4(c+dx))}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (840*a*c + 840*a*d*x - 1225*b*Cosh[c + d*x] + 245*b*Cosh[3*(c + d*x)] - 49*
b*Cosh[5*(c + d*x)] + 5*b*Cosh[7*(c + d*x)] - 560*a*Sinh[2*(c + d*x)] + 70*
a*Sinh[4*(c + d*x)])/(2240*d)
```

Maple [A]
time = 1.33, size = 93, normalized size = 0.88

method	result
default	$\frac{3ax}{8}-\frac{35b\cosh(dx+c)}{64d}+\frac{7b\cosh(3dx+3c)}{64d}-\frac{7b\cosh(5dx+5c)}{320d}+\frac{b\cosh(7dx+7c)}{448d}-\frac{a\sinh(2dx+2c)}{4d}+\frac{a\sinh(4dx+4c)}{32d}$
risch	$\frac{3ax}{8}+\frac{b\,\mathrm{e}^{7dx+7c}}{896d}-\frac{7b\,\mathrm{e}^{5dx+5c}}{640d}+\frac{a\,\mathrm{e}^{4dx+4c}}{64d}+\frac{7b\,\mathrm{e}^{3dx+3c}}{128d}-\frac{a\,\mathrm{e}^{2dx+2c}}{8d}-\frac{35b\,\mathrm{e}^{dx+c}}{128d}-\frac{35b\,\mathrm{e}^{-dx-c}}{128d}+\frac{a\,\mathrm{e}^{-2dx-2c}}{8d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a*x-35/64*b*cosh(d*x+c)/d+7/64*b/d*cosh(3*d*x+3*c)-7/320*b/d*cosh(5*d*x
+5*c)+1/448*b/d*cosh(7*d*x+7*c)-1/4*a*sinh(2*d*x+2*c)/d+1/32*a*sinh(4*d*x+4
*c)/d
```

Maxima [A]

time = 0.29, size = 164, normalized size = 1.55

$$\frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{4480}\,b\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/4480*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x -
 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) -
 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d)
```

Fricas [A]

time = 0.43, size = 188, normalized size = 1.77

$$\frac{5\,b\cosh\left(dx+c\right)^{7}+35\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}-49\,b\cosh\left(dx+c\right)^{5}+280\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+35\,\left(5\,b\cosh\left(dx+c\right)^{3}-7\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}+245\,b\cosh\left(dx+c\right)^{3}+840\,adx+35\,\left(3\,b\cosh\left(dx+c\right)^{5}-14\,b\cosh\left(dx+c\right)^{3}+21\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}-1225\,b\cosh\left(dx+c\right)+280\,\left(a\cosh\left(dx+c\right)^{3}-4\,a\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b*cosh(d*x + c)^7 + 35*b*cosh(d*x + c)*sinh(d*x + c)^6 - 49*b*cos
h(d*x + c)^5 + 280*a*cosh(d*x + c)*sinh(d*x + c)^3 + 35*(5*b*cosh(d*x + c)^
3 - 7*b*cosh(d*x + c))*sinh(d*x + c)^4 + 245*b*cosh(d*x + c)^3 + 840*a*d*x
+ 35*(3*b*cosh(d*x + c)^5 - 14*b*cosh(d*x + c)^3 + 21*b*cosh(d*x + c))*sinh
(d*x + c)^2 - 1225*b*cosh(d*x + c) + 280*(a*cosh(d*x + c)^3 - 4*a*cosh(d*x
+ c))*sinh(d*x + c))/d
```

Sympy [A]

time = 0.65, size = 192, normalized size = 1.81

$$\begin{cases}\frac{3ax\sinh^{4}\left(c+dx\right)}{8}-\frac{3ax\sinh^{2}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{4}+\frac{3ax\cosh^{4}\left(c+dx\right)}{8}+\frac{5a\sinh^{3}\left(c+dx\right)\cosh\left(c+dx\right)}{8d}-\frac{3a\sinh\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{8d}+\frac{b\sinh^{6}\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2b\sinh^{4}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{d}+\frac{8b\sinh^{2}\left(c+dx\right)\cosh^{5}\left(c+dx\right)}{5d}-\frac{16b\cosh^{7}\left(c+dx\right)}{35d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^{3}\left(c\right)\right)\sinh^{4}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
- 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + b*sinh(c + d*x)**6*cosh(c + d*
x)/d - 2*b*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 8*b*sinh(c + d*x)**2*cosh(
c + d*x)**5/(5*d) - 16*b*cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh
(c)**3)*sinh(c)**4, True))
```

Giac [A]

time = 0.41, size = 182, normalized size = 1.72

$$\frac{3}{8}\,ax+\frac{be^{(7\,dx+7\,c)}}{896\,d}-\frac{7\,be^{(5\,dx+5\,c)}}{640\,d}+\frac{ae^{(4\,dx+4\,c)}}{64\,d}+\frac{7\,be^{(3\,dx+3\,c)}}{128\,d}-\frac{ae^{(2\,dx+2\,c)}}{8\,d}-\frac{35\,be^{(dx+c)}}{128\,d}-\frac{35\,be^{(-dx-c)}}{128\,d}+\frac{ae^{(-2\,dx-2\,c)}}{8\,d}+\frac{7\,be^{(-3\,dx-3\,c)}}{128\,d}-\frac{ae^{(-4\,dx-4\,c)}}{64\,d}-\frac{7\,be^{(-5\,dx-5\,c)}}{640\,d}+\frac{be^{(-7\,dx-7\,c)}}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 3/8*a*x + 1/896*b*e^(7*d*x + 7*c)/d - 7/640*b*e^(5*d*x + 5*c)/d + 1/64*a*e^
(4*d*x + 4*c)/d + 7/128*b*e^(3*d*x + 3*c)/d - 1/8*a*e^(2*d*x + 2*c)/d - 35/
128*b*e^(d*x + c)/d - 35/128*b*e^(-d*x - c)/d + 1/8*a*e^(-2*d*x - 2*c)/d +
7/128*b*e^(-3*d*x - 3*c)/d - 1/64*a*e^(-4*d*x - 4*c)/d - 7/640*b*e^(-5*d*x
- 5*c)/d + 1/896*b*e^(-7*d*x - 7*c)/d
```

Mupad [B]
time = 0.26, size = 85, normalized size = 0.80

$$\frac{280\,b\cosh(c+d\,x)^3-280\,b\cosh(c+d\,x)-168\,b\cosh(c+d\,x)^5+40\,b\cosh(c+d\,x)^7-175\,a\cosh(c+d\,x)\,\sinh(c+d\,x)+105\,a\,d\,x+70\,a\cosh(c+d\,x)^3\sinh(c+d\,x)}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^3),x)
```

```
[Out] (280*b*cosh(c + d*x)^3 - 280*b*cosh(c + d*x) - 168*b*cosh(c + d*x)^5 + 40*b
*cosh(c + d*x)^7 - 175*a*cosh(c + d*x)*sinh(c + d*x) + 105*a*d*x + 70*a*cos
h(c + d*x)^3*sinh(c + d*x))/(280*d)
```

3.142 $\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=99

$$-\frac{5bx}{16}-\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{5b\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{5b\cosh(c+dx)\sinh^3(c+dx)}{24d}+b\cos$$

```
[Out] -5/16*b*x-a*cosh(d*x+c)/d+1/3*a*cosh(d*x+c)^3/d+5/16*b*cosh(d*x+c)*sinh(d*x
+c)/d-5/24*b*cosh(d*x+c)*sinh(d*x+c)^3/d+1/6*b*cosh(d*x+c)*sinh(d*x+c)^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3299, 2713, 2715, 8}

$$\frac{a\cosh^3(c+dx)}{3d}-\frac{a\cosh(c+dx)}{d}+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}-\frac{5b\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{5b\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{5bx}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-5*b*x)/16 - (a*Cosh[c + d*x])/d + (a*Cosh[c + d*x]^3)/(3*d) + (5*b*Cosh[c
 + d*x]*Sinh[c + d*x])/(16*d) - (5*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24*d)
+ (b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
```

```
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx &= i\int \left(-ia\sinh^3(c+dx)-ib\sinh^6(c+dx)\right)\,dx\\
&= a\int \sinh^3(c+dx)\,dx+b\int \sinh^6(c+dx)\,dx\\
&= \frac{b\cosh(c+dx)\sinh^5(c+dx)}{6d}-\frac{1}{6}(5b)\int \sinh^4(c+dx)\,dx-\frac{a\mathrm{S}}{}\\
&= -\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}-\frac{5b\cosh(c+dx)\sinh^3(c}{24d}\\
&= -\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{5b\cosh(c+dx)\sinh(c+}{16d}\\
&= -\frac{5bx}{16}-\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{5b\cosh(c+dx)\mathrm{si}}{16d}
\end{aligned}
$$

Mathematica [**A**]
time = 0.10, size = 66, normalized size = 0.67

$$\frac{-144a\cosh(c+dx)+16a\cosh(3(c+dx))+b(-60c-60dx+45\sinh(2(c+dx))-9\sinh(4(c+dx))+\sinh(6(c+dx)))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-144*a*Cosh[c + d*x] + 16*a*Cosh[3*(c + d*x)] + b*(-60*c - 60*d*x + 45*Sin
h[2*(c + d*x)] - 9*Sinh[4*(c + d*x)] + Sinh[6*(c + d*x)]))/(192*d)
```

Maple [**A**]
time = 1.31, size = 78, normalized size = 0.79

method	result
default	$-\frac{5bx}{16}-\frac{3a\cosh(dx+c)}{4d}+\frac{15b\sinh(2dx+2c)}{64d}-\frac{3b\sinh(4dx+4c)}{64d}+\frac{b\sinh(6dx+6c)}{192d}+\frac{a\cosh(3dx+3c)}{12d}$
risch	$-\frac{5bx}{16}+\frac{b\,\mathrm{e}^{6dx+6c}}{384d}-\frac{3b\,\mathrm{e}^{4dx+4c}}{128d}+\frac{a\,\mathrm{e}^{3dx+3c}}{24d}+\frac{15b\,\mathrm{e}^{2dx+2c}}{128d}-\frac{3a\,\mathrm{e}^{dx+c}}{8d}-\frac{3a\,\mathrm{e}^{-dx-c}}{8d}-\frac{15b\,\mathrm{e}^{-2dx-2c}}{128d}+\frac{a\,\mathrm{e}^{-3dx-3c}}{24d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -5/16*b*x-3/4*a*cosh(d*x+c)/d+15/64*b*sinh(2*d*x+2*c)/d-3/64*b*sinh(4*d*x+4
*c)/d+1/192*b*sinh(6*d*x+6*c)/d+1/12*a/d*cosh(3*d*x+3*c)
```

Maxima [A]
time = 0.28, size = 143, normalized size = 1.44

$$-\frac{1}{384}b\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}}{d}\right)+\frac{1}{24}a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/384*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d
+ 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x -
 6*c))/d) + 1/24*a*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d
+ e^(-3*d*x - 3*c)/d)
```

Fricas [A]
time = 0.43, size = 135, normalized size = 1.36

$$\frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+8\,a\cosh\left(dx+c\right)^{3}+24\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+2\,\left(5\,b\cosh\left(dx+c\right)^{3}-9\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}-30\,bdx-72\,a\cosh\left(dx+c\right)+3\,\left(b\cosh\left(dx+c\right)^{5}-6\,b\cosh\left(dx+c\right)^{3}+15\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 8*a*cosh(d*x + c)^3 + 24*a*cosh(d
*x + c)*sinh(d*x + c)^2 + 2*(5*b*cosh(d*x + c)^3 - 9*b*cosh(d*x + c))*sinh(
d*x + c)^3 - 30*b*d*x - 72*a*cosh(d*x + c) + 3*(b*cosh(d*x + c)^5 - 6*b*cos
h(d*x + c)^3 + 15*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(92) = 184$.
time = 0.44, size = 194, normalized size = 1.96

$$\begin{cases} \frac{a\sinh^{2}\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2a\cosh^{3}\left(c+dx\right)}{3d}+\frac{5bx\sinh^{6}\left(c+dx\right)}{16}-\frac{15bx\sinh^{4}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{16}+\frac{15bx\sinh^{2}\left(c+dx\right)\cosh^{4}\left(c+dx\right)}{16}-\frac{5bx\cosh^{6}\left(c+dx\right)}{16}+\frac{11b\sinh^{5}\left(c+dx\right)\cosh\left(c+dx\right)}{16d}-\frac{5b\sinh^{3}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{6d}+\frac{5b\sinh\left(c+dx\right)\cosh^{5}\left(c+dx\right)}{16d} & \text{for}\ d\neq 0 \\ x\left(a+b\sinh^{3}\left(c\right)\right)\sinh^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d)
+ 5*b*x*sinh(c + d*x)**6/16 - 15*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 +
 15*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b*x*cosh(c + d*x)**6/16 +
11*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b*sinh(c + d*x)**3*cosh(c +
d*x)**3/(6*d) + 5*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a
 + b*sinh(c)**3)*sinh(c)**3, True))
```

Giac [A]
time = 0.44, size = 152, normalized size = 1.54

$$-\frac{5}{16}\,bx+\frac{be^{(6\,dx+6\,c)}}{384\,d}-\frac{3\,be^{(4\,dx+4\,c)}}{128\,d}+\frac{ae^{(3\,dx+3\,c)}}{24\,d}+\frac{15\,be^{(2\,dx+2\,c)}}{128\,d}-\frac{3\,ae^{(dx+c)}}{8\,d}-\frac{3\,ae^{(-dx-c)}}{8\,d}-\frac{15\,be^{(-2\,dx-2\,c)}}{128\,d}+\frac{ae^{(-3\,dx-3\,c)}}{24\,d}+\frac{3\,be^{(-4\,dx-4\,c)}}{128\,d}-\frac{be^{(-6\,dx-6\,c)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -5/16*b*x + 1/384*b*e^(6*d*x + 6*c)/d - 3/128*b*e^(4*d*x + 4*c)/d + 1/24*a*
e^(3*d*x + 3*c)/d + 15/128*b*e^(2*d*x + 2*c)/d - 3/8*a*e^(d*x + c)/d - 3/8*
a*e^(-d*x - c)/d - 15/128*b*e^(-2*d*x - 2*c)/d + 1/24*a*e^(-3*d*x - 3*c)/d
+ 3/128*b*e^(-4*d*x - 4*c)/d - 1/384*b*e^(-6*d*x - 6*c)/d
```

Mupad [B]
time = 0.45, size = 67, normalized size = 0.68

$$\frac{\frac{a\cosh(3\,c+3\,d\,x)}{12}-\frac{3\,a\cosh(c+d\,x)}{4}+\frac{15\,b\sinh(2\,c+2\,d\,x)}{64}-\frac{3\,b\sinh(4\,c+4\,d\,x)}{64}+\frac{b\sinh(6\,c+6\,d\,x)}{192}}{d}-\frac{5\,b\,x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^3),x)
```

```
[Out] ((a*cosh(3*c + 3*d*x))/12 - (3*a*cosh(c + d*x))/4 + (15*b*sinh(2*c + 2*d*x)
)/64 - (3*b*sinh(4*c + 4*d*x))/64 + (b*sinh(6*c + 6*d*x))/192)/d - (5*b*x)/
16
```

3.143 $\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=70

$$-\frac{ax}{2}+\frac{b\cosh(c+dx)}{d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}+\frac{a\cosh(c+dx)\sinh(c+dx)}{2d}$$

```
[Out] -1/2*a*x+b*cosh(d*x+c)/d-2/3*b*cosh(d*x+c)^3/d+1/5*b*cosh(d*x+c)^5/d+1/2*a*
cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]

time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {3299, 2715, 8, 2713}

$$\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{ax}{2}+\frac{b\cosh^5(c+dx)}{5d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -1/2*(a*x) + (b*Cosh[c + d*x])/d - (2*b*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c
+ d*x]^5)/(5*d) + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx &= -\int \left(-a\sinh^2(c+dx)-b\sinh^5(c+dx)\right)\,dx\\ &= a\int \sinh^2(c+dx)\,dx+b\int \sinh^5(c+dx)\,dx\\ &= \frac{a\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{1}{2}a\int 1\,dx+\frac{b\text{Subst}\big(\int (1-2x^2}{}\\ &= -\frac{ax}{2}+\frac{b\cosh(c+dx)}{d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}+\end{aligned}$$

Mathematica [A]
time = 0.07, size = 79, normalized size = 1.13

$$\frac{a(-c-dx)}{2d}+\frac{5b\cosh(c+dx)}{8d}-\frac{5b\cosh(3(c+dx))}{48d}+\frac{b\cosh(5(c+dx))}{80d}+\frac{a\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (a*(-c - d*x))/(2*d) + (5*b*Cosh[c + d*x])/(8*d) - (5*b*Cosh[3*(c + d*x)])/
(48*d) + (b*Cosh[5*(c + d*x)])/(80*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.98, size = 63, normalized size = 0.90

method	result	siz
default	$-\frac{ax}{2}+\frac{5b\cosh(dx+c)}{8d}-\frac{5b\cosh(3dx+3c)}{48d}+\frac{b\cosh(5dx+5c)}{80d}+\frac{a\sinh(2dx+2c)}{4d}$	63
risch	$-\frac{ax}{2}+\frac{b\,\mathrm{e}^{5dx+5c}}{160d}-\frac{5b\,\mathrm{e}^{3dx+3c}}{96d}+\frac{a\,\mathrm{e}^{2dx+2c}}{8d}+\frac{5b\,\mathrm{e}^{dx+c}}{16d}+\frac{5b\,\mathrm{e}^{-dx-c}}{16d}-\frac{a\,\mathrm{e}^{-2dx-2c}}{8d}-\frac{5b\,\mathrm{e}^{-3dx-3c}}{96d}+\frac{b\,\mathrm{e}^{-5dx-5c}}{160d}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*x+5/8*b*cosh(d*x+c)/d-5/48*b/d*cosh(3*d*x+3*c)+1/80*b/d*cosh(5*d*x+5
*c)+1/4*a*sinh(2*d*x+2*c)/d
```

Maxima [A]
time = 0.28, size = 120, normalized size = 1.71

$$-\frac{1}{8}\,a\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+\frac{1}{480}\,b\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b*(3*e^(5*d*x
 + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d -
 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

Fricas [A]
time = 0.44, size = 105, normalized size = 1.50

$$\frac{3\,b\cosh\left(dx+c\right)^{5}+15\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}-25\,b\cosh\left(dx+c\right)^{3}-120\,adx+120\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+15\,\left(2\,b\cosh\left(dx+c\right)^{3}-5\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+150\,b\cosh\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 - 25*b*cosh
(d*x + c)^3 - 120*a*d*x + 120*a*cosh(d*x + c)*sinh(d*x + c) + 15*(2*b*cosh(
d*x + c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c)^2 + 150*b*cosh(d*x + c))/d
```

Sympy [A]
time = 0.27, size = 117, normalized size = 1.67

$$\begin{cases} \frac{ax\sinh^{2}\left(c+dx\right)}{2}-\frac{ax\cosh^{2}\left(c+dx\right)}{2}+\frac{a\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d}+\frac{b\sinh^{4}\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{4b\sinh^{2}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{3d}+\frac{8b\cosh^{5}\left(c+dx\right)}{15d} & \text{for}\: d\neq 0 \\ x\left(a+b\sinh^{3}\left(c\right)\right)\sinh^{2}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x
)*cosh(c + d*x)/(2*d) + b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sinh(c + d
*x)**2*cosh(c + d*x)**3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*
(a + b*sinh(c)**3)*sinh(c)**2, True))
```

Giac [A]
time = 0.42, size = 122, normalized size = 1.74

$$-\frac{1}{2}\,ax+\frac{be^{\left(5\,dx+5\,c\right)}}{160\,d}-\frac{5\,be^{\left(3\,dx+3\,c\right)}}{96\,d}+\frac{ae^{\left(2\,dx+2\,c\right)}}{8\,d}+\frac{5\,be^{\left(dx+c\right)}}{16\,d}+\frac{5\,be^{\left(-dx-c\right)}}{16\,d}-\frac{ae^{\left(-2\,dx-2\,c\right)}}{8\,d}-\frac{5\,be^{\left(-3\,dx-3\,c\right)}}{96\,d}+\frac{be^{\left(-5\,dx-5\,c\right)}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/2*a*x + 1/160*b*e^(5*d*x + 5*c)/d - 5/96*b*e^(3*d*x + 3*c)/d + 1/8*a*e^(
2*d*x + 2*c)/d + 5/16*b*e^(d*x + c)/d + 5/16*b*e^(-d*x - c)/d - 1/8*a*e^(-2
*d*x - 2*c)/d - 5/96*b*e^(-3*d*x - 3*c)/d + 1/160*b*e^(-5*d*x - 5*c)/d
```

Mupad [B]
time = 0.12, size = 55, normalized size = 0.79

$$\frac{b\cosh(c+d\,x)-\frac{2\,b\,{\cosh(c+d\,x)}^3}{3}+\frac{b\,{\cosh(c+d\,x)}^5}{5}+\frac{a\,\cosh(c+d\,x)\,\sinh(c+d\,x)}{2}}{d}-\frac{a\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^3),x)
```

```
[Out] (b*cosh(c + d*x) - (2*b*cosh(c + d*x)^3)/3 + (b*cosh(c + d*x)^5)/5 + (a*cos
h(c + d*x)*sinh(c + d*x))/2)/d - (a*x)/2
```

3.144 $\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=60

$$\frac{3bx}{8}+\frac{a\cosh(c+dx)}{d}-\frac{3b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}$$

```
[Out] 3/8*b*x+a*cosh(d*x+c)/d-3/8*b*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b*cosh(d*x+c)*s
inh(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.210$, Rules used = {3299, 2718, 2715, 8}

$$\frac{a\cosh(c+dx)}{d}+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (3*b*x)/8 + (a*Cosh[c + d*x])/d - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) +
 (b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx &= -\left(i\int \left(ia\sinh(c+dx)+ib\sinh^4(c+dx)\right)\,dx\right)\\
&= a\int \sinh(c+dx)\,dx+b\int \sinh^4(c+dx)\,dx\\
&= \frac{a\cosh(c+dx)}{d}+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{4}(3b)\int \sinh^2(\\
&= \frac{a\cosh(c+dx)}{d}-\frac{3b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)}{4}\\
&= \frac{3bx}{8}+\frac{a\cosh(c+dx)}{d}-\frac{3b\cosh(c+dx)\sinh(c+dx)}{8d}+b\cosh(
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 0.75

$$\frac{32a\cosh(c+dx)+b(12(c+dx)-8\sinh(2(c+dx))+\sinh(4(c+dx)))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (32*a*Cosh[c + d*x] + b*(12*(c + d*x) - 8*Sinh[2*(c + d*x)] + Sinh[4*(c + d
*x)]))/(32*d)
```

Maple [A]
time = 1.06, size = 47, normalized size = 0.78

method	result	size
default	$\frac{a\cosh(dx+c)}{d}+\frac{3bx}{8}-\frac{b\sinh(2dx+2c)}{4d}+\frac{b\sinh(4dx+4c)}{32d}$	47
risch	$\frac{3bx}{8}+\frac{b\,e^{4dx+4c}}{64d}-\frac{b\,e^{2dx+2c}}{8d}+\frac{a\,e^{dx+c}}{2d}+\frac{a\,e^{-dx-c}}{2d}+\frac{b\,e^{-2dx-2c}}{8d}-\frac{b\,e^{-4dx-4c}}{64d}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] a*cosh(d*x+c)/d+3/8*b*x-1/4*b*sinh(2*d*x+2*c)/d+1/32*b*sinh(4*d*x+4*c)/d
```

Maxima [A]
time = 0.28, size = 74, normalized size = 1.23

$$\frac{1}{64}\,b\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + a*cosh(d*x + c)/d
```

Fricas [A]
time = 0.47, size = 63, normalized size = 1.05

$$\frac{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+3\,bdx+8\,a\cosh\left(dx+c\right)+\left(b\cosh\left(dx+c\right)^{3}-4\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + 3*b*d*x + 8*a*cosh(d*x + c) + (b*cos
h(d*x + c)^3 - 4*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(56) = 112$.
time = 0.19, size = 121, normalized size = 2.02

$$\begin{cases} \frac{a\cosh\left(c+dx\right)}{d}+\frac{3bx\sinh^{4}\left(c+dx\right)}{8}-\frac{3bx\sinh^{2}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{4}+\frac{3bx\cosh^{4}\left(c+dx\right)}{8}+\frac{5b\sinh^{3}\left(c+dx\right)\cosh\left(c+dx\right)}{8d}-\frac{3b\sinh\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{8d} & \text{for}\: d\neq 0 \\ x\left(a+b\sinh^{3}\left(c\right)\right)\sinh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Piecewise((a*cosh(c + d*x)/d + 3*b*x*sinh(c + d*x)**4/8 - 3*b*x*sinh(c + d*
x)**2*cosh(c + d*x)**2/4 + 3*b*x*cosh(c + d*x)**4/8 + 5*b*sinh(c + d*x)**3*
cosh(c + d*x)/(8*d) - 3*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)),
(x*(a + b*sinh(c)**3)*sinh(c), True))
```

Giac [A]
time = 0.40, size = 92, normalized size = 1.53

$$\frac{3}{8}\,bx+\frac{be^{\left(4\,dx+4\,c\right)}}{64\,d}-\frac{be^{\left(2\,dx+2\,c\right)}}{8\,d}+\frac{ae^{\left(dx+c\right)}}{2\,d}+\frac{ae^{\left(-dx-c\right)}}{2\,d}+\frac{be^{\left(-2\,dx-2\,c\right)}}{8\,d}-\frac{be^{\left(-4\,dx-4\,c\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/64*b*e^(4*d*x + 4*c)/d - 1/8*b*e^(2*d*x + 2*c)/d + 1/2*a*e^(d*x
 + c)/d + 1/2*a*e^(-d*x - c)/d + 1/8*b*e^(-2*d*x - 2*c)/d - 1/64*b*e^(-4*d*
x - 4*c)/d
```

Mupad [B]
time = 0.20, size = 42, normalized size = 0.70

$$\frac{3\,b\,x}{8}+\frac{a\cosh(c+d\,x)-\frac{b\sinh(2\,c+2\,d\,x)}{4}+\frac{b\sinh(4\,c+4\,d\,x)}{32}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^3),x)
```

```
[Out] (3*b*x)/8 + (a*cosh(c + d*x) - (b*sinh(2*c + 2*d*x))/4 + (b*sinh(4*c + 4*d*
x))/32)/d
```

3.145 $\int \left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=32

$$ax-\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}$$

```
[Out] a*x-b*cosh(d*x+c)/d+1/3*b*cosh(d*x+c)^3/d
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = $\{2713\}$

$$ax+\frac{b\cosh^3(c+dx)}{3d}-\frac{b\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sinh[c + d*x]^3,x]
```

```
[Out] a*x - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/(3*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^3(c+dx)\right)\,dx &= ax+b\int \sinh^3(c+dx)\,dx\\ &= ax-\frac{b\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= ax-\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.06

$$ax-\frac{3b\cosh(c+dx)}{4d}+\frac{b\cosh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sinh[c + d*x]^3,x]
```

```
[Out] a*x - (3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])/(12*d)
```

Maple [A]
time = 0.85, size = 33, normalized size = 1.03

method	result	size
default	$ax+b\left(-\frac{3\cosh\left(dx+c\right)}{4d}+\frac{\cosh\left(3dx+3c\right)}{12d}\right)$	33
risch	$ax+\frac{b\,{\mathrm e}^{3dx+3c}}{24d}-\frac{3b\,{\mathrm e}^{dx+c}}{8d}-\frac{3b\,{\mathrm e}^{-dx-c}}{8d}+\frac{b\,{\mathrm e}^{-3dx-3c}}{24d}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*(-3/4/d*cosh(d*x+c)+1/12/d*cosh(3*d*x+3*c))
```

Maxima [A]
time = 0.27, size = 59, normalized size = 1.84

$$ax+\frac{1}{24}\,b\left(\frac{e^{\left(3\,dx+3\,c\right)}}{d}-\frac{9\,e^{\left(dx+c\right)}}{d}-\frac{9\,e^{\left(-dx-c\right)}}{d}+\frac{e^{\left(-3\,dx-3\,c\right)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] a*x + 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-
3*d*x - 3*c)/d)
```

Fricas [A]
time = 0.45, size = 47, normalized size = 1.47

$$\frac{b\cosh\left(dx+c\right)^{3}+3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+12\,adx-9\,b\cosh\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + 12*a*d*x - 9*
b*cosh(d*x + c))/d
```

Sympy [A]
time = 0.11, size = 41, normalized size = 1.28

$$ax+b\left(\begin{cases}\frac{\sinh^{2}\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2\cosh^{3}\left(c+dx\right)}{3d} & \text{for}\ d\neq 0\\ x\sinh^{3}\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)**3,x)
```

```
[Out] a*x + b*Piecewise((sinh(c + d*x)**2*cosh(c + d*x)/d - 2*cosh(c + d*x)**3/(3
*d), Ne(d, 0)), (x*sinh(c)**3, True))
```

Giac [**A**]
time = 0.41, size = 59, normalized size = 1.84

$$ax + \frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] a*x + 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-
3*d*x - 3*c)/d)
```

Mupad [**B**]
time = 0.62, size = 29, normalized size = 0.91

$$a\,x - \frac{b\cosh(c+d\,x) - \frac{b\cosh(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sinh(c + d*x)^3,x)
```

```
[Out] a*x - (b*cosh(c + d*x) - (b*cosh(c + d*x)^3)/3)/d
```

3.146 $\int \operatorname{csch}(c+dx) \left(a+b\sinh^3(c+dx)\right) dx$

Optimal. Leaf size=40

$$-\frac{bx}{2}-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}$$

```
[Out] -1/2*b*x-a*arctanh(cosh(d*x+c))/d+1/2*b*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3299, 3855, 2715, 8}

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -1/2*(b*x) - (a*ArcTanh[Cosh[c + d*x]])/d + (b*Cosh[c + d*x]*Sinh[c + d*x])
/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx &= i\int\left(-ia\operatorname{csch}(c+dx)-ib\sinh^2(c+dx)\right)\,dx\\ &= a\int \operatorname{csch}(c+dx)\,dx+b\int \sinh^2(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{1}{2}b\int\\ &= -\frac{bx}{2}-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 1.80

$$\frac{b(-c-dx)}{2d}-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (b*(-c - d*x))/(2*d) - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[Sinh[c/2 + (
d*x)/2]])/d + (b*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 1.55, size = 40, normalized size = 1.00

method	result	size
derivativedivides	$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	40
default	$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$	40
risch	$-\frac{bx}{2}+\frac{b\,\mathrm{e}^{2dx+2c}}{8d}-\frac{b\,\mathrm{e}^{-2dx-2c}}{8d}+\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*(1/2*sinh(d*x+c)*cosh(d*x+c)-1/2*d*x-1/2*c)
)
```

Maxima [A]
time = 0.26, size = 50, normalized size = 1.25

$$-\frac{1}{8}\,b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a*log(tanh(1/2*d*x
+ 1/2*c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(36) = 72$.
time = 0.56, size = 258, normalized size = 6.45

$$-\frac{4\,bdx\cosh\left(dx+c\right)^{2}-b\cosh\left(dx+c\right)^{4}-4\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}-b\sinh\left(dx+c\right)^{4}+2\,\left(2\,bdx-3\,b\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)^{2}+8\,\left(a\cosh\left(dx+c\right)^{2}+2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^{2}\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)-8\,\left(a\cosh\left(dx+c\right)^{2}+2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^{2}\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)+4\,\left(2\,bdx\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)^{3}\right)\sinh\left(dx+c\right)+b}{8\,\left(d\cosh\left(dx+c\right)^{2}+2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+d\sinh\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*b*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 8*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d
*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 8*(a*cosh(d*x + c)^2 +
2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*log(cosh(d*x + c) + si
nh(d*x + c) - 1) + 4*(2*b*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*x +
 c) + b)/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x
+ c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^{3}\left(c + dx\right)\right)\operatorname{csch}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**3)*csch(c + d*x), x)
```

Giac [**A**]
time = 0.41, size = 62, normalized size = 1.55

$$-\frac{4\,(dx+c)b - be^{(2\,dx+2\,c)} + be^{(-2\,dx-2\,c)} + 8\,a\log\left(e^{(dx+c)}+1\right) - 8\,a\log\left(\left|e^{(dx+c)}-1\right|\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/8*(4*(d*x + c)*b - b*e^(2*d*x + 2*c) + b*e^(-2*d*x - 2*c) + 8*a*log(e^(d
*x + c) + 1) - 8*a*log(abs(e^(d*x + c) - 1)))/d
```

Mupad [B]
time = 0.69, size = 73, normalized size = 1.82

$$\frac{b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}-\frac{b\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}-\frac{b\,x}{2}-\frac{2\,\mathrm{atan}\left(\frac{a\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)/sinh(c + d*x),x)
```

```
[Out] (b*exp(2*c + 2*d*x))/(8*d) - (b*exp(- 2*c - 2*d*x))/(8*d) - (b*x)/2 - (2*at
an((a*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-d^2)^(1
/2)
```

3.147 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=24

$$\frac{b\cosh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}$$

```
[Out] b*cosh(d*x+c)/d-a*coth(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {3299, 3852, 8, 2718}

$$\frac{b\cosh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (b*Cosh[c + d*x])/d - (a*Coth[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx &= -\int \left(-a\operatorname{csch}^2(c+dx)-b\sinh(c+dx)\right)\,dx\\
&= a\int \operatorname{csch}^2(c+dx)\,dx+b\int \sinh(c+dx)\,dx\\
&= \frac{b\cosh(c+dx)}{d}-\frac{(ia)\text{Subst}(\int 1\,dx,x,-i\coth(c+dx))}{d}\\
&= \frac{b\cosh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 1.46

$$\frac{b\cosh(c)\cosh(dx)}{d}-\frac{a\coth(c+dx)}{d}+\frac{b\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (b*Cosh[c]*Cosh[d*x])/d - (a*Coth[c + d*x])/d + (b*Sinh[c]*Sinh[d*x])/d
```

Maple [A]
time = 1.83, size = 48, normalized size = 2.00

method	result	size
risch	$\frac{b\,\mathrm{e}^{dx+c}}{2d}+\frac{b\,\mathrm{e}^{-dx-c}}{2d}-\frac{2a}{d\left(\mathrm{e}^{2dx+2c}-1\right)}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/d*exp(d*x+c)+1/2*b/d*exp(-d*x-c)-2*a/d/(exp(2*d*x+2*c)-1)
```

Maxima [A]
time = 0.28, size = 47, normalized size = 1.96

$$\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{2\,a}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [**A**]
time = 0.44, size = 40, normalized size = 1.67

$$-\frac{a\cosh(dx+c)-(b\cosh(dx+c)+a)\sinh(dx+c)}{d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -(a*cosh(d*x + c) - (b*cosh(d*x + c) + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^3(c+dx)\right)\operatorname{csch}^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**3)*csch(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(24) = 48.
time = 0.41, size = 59, normalized size = 2.46

$$\frac{be^{(dx+c)}+\frac{be^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}-b}{e^{(3\,dx+3\,c)}-e^{(dx+c)}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/2*(b*e^(d*x + c) + (b*e^(2*d*x + 2*c) - 4*a*e^(d*x + c) - b)/(e^(3*d*x +
3*c) - e^(d*x + c)))/d
```

Mupad [**B**]
time = 0.09, size = 47, normalized size = 1.96

$$\frac{b\,\mathrm{e}^{-c-d\,x}}{2\,d}-\frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}+\frac{b\,\mathrm{e}^{c+d\,x}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)/sinh(c + d*x)^2,x)
```

```
[Out] (b*exp(- c - d*x))/(2*d) - (2*a)/(d*(exp(2*c + 2*d*x) - 1)) + (b*exp(c + d*
x))/(2*d)
```

3.148 $\int \operatorname{csch}^3(c+dx) \left(a+b\sinh^3(c+dx)\right) \, dx$

Optimal. Leaf size=39

$$bx+\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

```
[Out] b*x+1/2*a*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{3299, 3853, 3855\}$

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] b*x + (a*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/(2
*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx &= -\left(i\int \left(ib+ia\operatorname{csch}^3(c+dx)\right)\,dx\right)\\ &= bx+a\int \operatorname{csch}^3(c+dx)\,dx\\ &= bx-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{1}{2}a\int \operatorname{csch}(c+dx)\,dx\\ &= bx+\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 1.62

$$bx-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] b*x - (a*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tanh[(c + d*x)/2]])/(2*d) - (a
*Sech[(c + d*x)/2]^2)/(8*d)
```

Maple [A]
time = 2.06, size = 71, normalized size = 1.82

method	result	size
risch	$bx-\frac{a\,e^{dx+c}\left(1+e^{2dx+2c}\right)}{d\left(e^{2dx+2c}-1\right)^2}+\frac{a\ln\left(e^{dx+c}+1\right)}{2d}-\frac{a\ln\left(e^{dx+c}-1\right)}{2d}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] b*x-a*exp(d*x+c)*(1+exp(2*d*x+2*c))/d/(exp(2*d*x+2*c)-1)^2+1/2*a/d*ln(exp(d
*x+c)+1)-1/2*a/d*ln(exp(d*x+c)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(35) = 70$.
time = 0.28, size = 91, normalized size = 2.33

$$bx+\frac{1}{2}a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] b*x + 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x
 - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(35) = 70$.
time = 0.45, size = 521, normalized size = 13.36

$$\frac{2\,bdx\cosh\left(dx+c\right)^{4}+2\,bdx\sinh\left(dx+c\right)^{4}-4\,bdx\cosh\left(dx+c\right)^{2}-2\,a\cosh\left(dx+c\right)^{3}+2\,\left(4\,bdx\cosh\left(dx+c\right)-a\right)\sinh\left(dx+c\right)^{3}+2\,bdx+2\,\left(6\,bdx\cosh\left(dx+c\right)^{2}-2\,bdx-3\,a\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}-2\,a\cosh\left(dx+c\right)+\left(a\cosh\left(dx+c\right)^{4}+4\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+a\sinh\left(dx+c\right)^{4}-2\,a\cosh\left(dx+c\right)^{2}+2\,\left(3\,a\cosh\left(dx+c\right)^{2}-a\right)\sinh\left(dx+c\right)^{2}+4\,\left(a\cosh\left(dx+c\right)^{3}-a\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+a\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)-\left(a\cosh\left(dx+c\right)^{4}+4\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+a\sinh\left(dx+c\right)^{4}-2\,a\cosh\left(dx+c\right)^{2}+2\,\left(3\,a\cosh\left(dx+c\right)^{2}-a\right)\sinh\left(dx+c\right)^{2}+4\,\left(a\cosh\left(dx+c\right)^{3}-a\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+a\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)+2\,\left(4\,bdx\cosh\left(dx+c\right)^{3}-4\,bdx\cosh\left(dx+c\right)-3\,a\cosh\left(dx+c\right)^{2}-a\right)\sinh\left(dx+c\right)}{2\,\left(d\cosh\left(dx+c\right)^{4}+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{4}-2\,d\cosh\left(dx+c\right)^{2}+2\,\left(3\,d\cosh\left(dx+c\right)^{2}-d\right)\sinh\left(dx+c\right)^{2}+4\,\left(d\cosh\left(dx+c\right)^{3}-d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x*cosh(d*x + c)^4 + 2*b*d*x*sinh(d*x + c)^4 - 4*b*d*x*cosh(d*x +
 c)^2 - 2*a*cosh(d*x + c)^3 + 2*(4*b*d*x*cosh(d*x + c) - a)*sinh(d*x + c)^3
 + 2*b*d*x + 2*(6*b*d*x*cosh(d*x + c)^2 - 2*b*d*x - 3*a*cosh(d*x + c))*sinh
(d*x + c)^2 - 2*a*cosh(d*x + c) + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*si
nh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x +
 c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*cosh(d*x + c))*sinh(d
*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d*x + c)^4 +
4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2
 + 2*(3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*c
osh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2
*(4*b*d*x*cosh(d*x + c)^3 - 4*b*d*x*cosh(d*x + c) - 3*a*cosh(d*x + c)^2 - a
)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d
*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d
*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.
time = 0.42, size = 73, normalized size = 1.87

$$\frac{2\,(dx+c)b+a\log\left(e^{(dx+c)}+1\right)-a\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{2\,\left(ae^{(3\,dx+3\,c)}+ae^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*b + a*log(e^(d*x + c) + 1) - a*log(abs(e^(d*x + c) - 1)) -
 2*(a*e^(3*d*x + 3*c) + a*e^(d*x + c))/(e^(2*d*x + 2*c) - 1)^2)/d
```

Mupad [B]
time = 0.11, size = 102, normalized size = 2.62

$$b\,x+\frac{\mathrm{atan}\left(\frac{a\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}-\frac{a\,{\mathrm{e}}^{c+d\,x}}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a\,{\mathrm{e}}^{c+d\,x}}{d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)/sinh(c + d*x)^3,x)
```

```
[Out] b*x + (atan((a*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/
(-d^2)^(1/2) - (a*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a*exp(c + d
*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.149 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=41

$$-\frac{b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}$$

```
[Out] -b*arctanh(cosh(d*x+c))/d+a*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3299, 3855, 3852}

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}-\frac{b\tanh^{-1}(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -((b*ArcTanh[Cosh[c + d*x]])/d) + (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)
/(3*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx &= \int \left(b\operatorname{csch}(c+dx)+a\operatorname{csch}^4(c+dx)\right)\,dx \\
&= a\int \operatorname{csch}^4(c+dx)\,dx + b\int \operatorname{csch}(c+dx)\,dx \\
&= -\frac{b\tanh^{-1}(\cosh(c+dx))}{d} + \frac{(ia)\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\,\text{cot}\right.}{d} \\
&= -\frac{b\tanh^{-1}(\cosh(c+dx))}{d} + \frac{a\coth(c+dx)}{d} - \frac{a\coth^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 76, normalized size = 1.85

$$\frac{2a\coth(c+dx)}{3d} - \frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d} - \frac{b\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{b\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (b*Lo
g[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d
```

Maple [A]
time = 2.05, size = 63, normalized size = 1.54

method	result	size
risch	$-\frac{4a\left(3\,\mathrm{e}^{2dx+2c}-1\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^3} + \frac{\ln\left(\mathrm{e}^{dx+c}-1\right)b}{d} - \frac{\ln\left(\mathrm{e}^{dx+c}+1\right)b}{d}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*a*(3*exp(2*d*x+2*c)-1)/d/(exp(2*d*x+2*c)-1)^3+1/d*ln(exp(d*x+c)-1)*b-1
/d*ln(exp(d*x+c)+1)*b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(39) = 78.
time = 0.28, size = 131, normalized size = 3.20

$$-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right) + \frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)} - \frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a*(3*e^(-2*d*x
 - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)
) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(39) = 78$.
time = 0.47, size = 652, normalized size = 15.90

$$-\frac{12\,a\cosh\left(dx+c\right)^{2}+24\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+12\,a\sinh\left(dx+c\right)^{2}+3\,\left(b\cosh\left(dx+c\right)^{6}+6\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+b\sinh\left(dx+c\right)^{6}-3\,b\cosh\left(dx+c\right)^{4}+3\,\left(5\,b\cosh\left(dx+c\right)^{2}-b\right)\sinh\left(dx+c\right)^{4}+4\,\left(5\,b\cosh\left(dx+c\right)^{3}-3\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+3\,b\cosh\left(dx+c\right)^{2}+3\,\left(5\,b\cosh\left(dx+c\right)^{4}-6\,b\cosh\left(dx+c\right)^{2}+b\right)\sinh\left(dx+c\right)^{2}+6\,\left(b\cosh\left(dx+c\right)^{5}-2\,b\cosh\left(dx+c\right)^{3}+b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)-b\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)-3\,\left(b\cosh\left(dx+c\right)^{6}+6\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+b\sinh\left(dx+c\right)^{6}-3\,b\cosh\left(dx+c\right)^{4}+3\,\left(5\,b\cosh\left(dx+c\right)^{2}-b\right)\sinh\left(dx+c\right)^{4}+4\,\left(5\,b\cosh\left(dx+c\right)^{3}-3\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+3\,b\cosh\left(dx+c\right)^{2}+3\,\left(5\,b\cosh\left(dx+c\right)^{4}-6\,b\cosh\left(dx+c\right)^{2}+b\right)\sinh\left(dx+c\right)^{2}+6\,\left(b\cosh\left(dx+c\right)^{5}-2\,b\cosh\left(dx+c\right)^{3}+b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)-b\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)-4\,a}{3\,\left(d\cosh\left(dx+c\right)^{6}+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+d\sinh\left(dx+c\right)^{6}-3\,d\cosh\left(dx+c\right)^{4}+3\,\left(5\,d\cosh\left(dx+c\right)^{2}-d\right)\sinh\left(dx+c\right)^{4}+4\,\left(5\,d\cosh\left(dx+c\right)^{3}-3\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)^{2}+3\,\left(5\,d\cosh\left(dx+c\right)^{4}-6\,d\cosh\left(dx+c\right)^{2}+d\right)\sinh\left(dx+c\right)^{2}+6\,\left(d\cosh\left(dx+c\right)^{5}-2\,d\cosh\left(dx+c\right)^{3}+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*cosh(d*x + c)^2 + 24*a*cosh(d*x + c)*sinh(d*x + c) + 12*a*sinh(d
*x + c)^2 + 3*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*si
nh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*
cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*
x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*si
nh(d*x + c) - b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*(b*cosh(d*x + c
)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x
+ c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)
^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh
(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)
^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*log(cosh(d*x
 + c) + sinh(d*x + c) - 1) - 4*a)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*si
nh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x +
 c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x
 + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 +
 d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**A**]
time = 0.41, size = 62, normalized size = 1.51

$$-\frac{3\,b\log\left(e^{\left(dx+c\right)}+1\right)-3\,b\log\left(\left|e^{\left(dx+c\right)}-1\right|\right)+\frac{4\,\left(3\,ae^{\left(2\,dx+2\,c\right)}-a\right)}{\left(e^{\left(2\,dx+2\,c\right)}-1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/3*(3*b*log(e^(d*x + c) + 1) - 3*b*log(abs(e^(d*x + c) - 1)) + 4*(3*a*e^(
2*d*x + 2*c) - a)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B]
time = 0.69, size = 110, normalized size = 2.68

$$-\frac{4\,a}{d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,a}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}-\frac{2\,\mathrm{atan}\left(\frac{b\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,\sqrt{-d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)/sinh(c + d*x)^4,x)
```

```
[Out] - (4*a)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a)/(3*d*(3*exp
(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (2*atan((b*ex
p(d*x)*exp(c)*(-d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))/(-d^2)^(1/2)
```

3.150 $\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=192

$$-\frac{5}{8}abx-\frac{a^2\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}-\frac{4b^2\cosh^3(c+dx)}{3d}+\frac{6b^2\cosh^5(c+dx)}{5d}-$$

```
[Out] -5/8*a*b*x-a^2*cosh(d*x+c)/d+b^2*cosh(d*x+c)/d+1/3*a^2*cosh(d*x+c)^3/d-4/3*
b^2*cosh(d*x+c)^3/d+6/5*b^2*cosh(d*x+c)^5/d-4/7*b^2*cosh(d*x+c)^7/d+1/9*b^2
*cosh(d*x+c)^9/d+5/8*a*b*cosh(d*x+c)*sinh(d*x+c)/d-5/12*a*b*cosh(d*x+c)*sin
h(d*x+c)^3/d+1/3*a*b*cosh(d*x+c)*sinh(d*x+c)^5/d
```

Rubi [A]
time = 0.13, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3299, 2713, 2715, 8}

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a^2\cosh(c+dx)}{d}+\frac{ab\sinh^5(c+dx)\cosh(c+dx)}{3d}-\frac{5ab\sinh^3(c+dx)\cosh(c+dx)}{12d}+\frac{5ab\sinh(c+dx)\cosh(c+dx)}{8d}-\frac{5abx}{8}+\frac{b^2\cosh^9(c+dx)}{9d}-\frac{4b^2\cosh^7(c+dx)}{7d}+\frac{6b^2\cosh^5(c+dx)}{5d}-\frac{4b^2\cosh^3(c+dx)}{3d}+\frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-5*a*b*x)/8 - (a^2*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x])/d + (a^2*Cosh[c
+ d*x]^3)/(3*d) - (4*b^2*Cosh[c + d*x]^3)/(3*d) + (6*b^2*Cosh[c + d*x]^5)/(
5*d) - (4*b^2*Cosh[c + d*x]^7)/(7*d) + (b^2*Cosh[c + d*x]^9)/(9*d) + (5*a*b
*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) - (5*a*b*Cosh[c + d*x]*Sinh[c + d*x]^3)
/(12*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx &= i\int\left(-ia^2\sinh^3(c+dx)-2iab\sinh^6(c+dx)-ib^2\sinh^9(c+\right.\\
&= a^2\int\sinh^3(c+dx)\,dx+(2ab)\int\sinh^6(c+dx)\,dx+b^2\int\sinh\\
&= \frac{ab\cosh(c+dx)\sinh^5(c+dx)}{3d}-\frac{1}{3}(5ab)\int\sinh^4(c+dx)\,dx-\\
&= -\frac{a^2\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}-4b^2\\
&= -\frac{a^2\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}-4b^2\\
&= -\frac{5}{8}abx-\frac{a^2\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+d}{3d}\end{aligned}$$

Mathematica [A]
time = 0.50, size = 125, normalized size = 0.65

$$\frac{-1890(32a^2-21b^2)\cosh(c+dx)+420(16a^2-21b^2)\cosh(3(c+dx))+b(2268b\cosh(5(c+dx))-405b\cosh(7(c+dx))+35b\cosh(9(c+dx))-840a(60c+60dx-45\sinh(2(c+dx))+9\sinh(4(c+dx))-\sinh(6(c+dx))))}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-1890*(32*a^2 - 21*b^2)*Cosh[c + d*x] + 420*(16*a^2 - 21*b^2)*Cosh[3*(c +
d*x)] + b*(2268*b*Cosh[5*(c + d*x)] - 405*b*Cosh[7*(c + d*x)] + 35*b*Cosh[9
*(c + d*x)] - 840*a*(60*c + 60*d*x - 45*Sinh[2*(c + d*x)] + 9*Sinh[4*(c + d
*x)] - Sinh[6*(c + d*x)])))/(80640*d)
```

Maple [A]
time = 1.42, size = 152, normalized size = 0.79

method	result
default	$\frac{\left(-\frac{21b^2}{64}+\frac{a^2}{4}\right)\cosh(3dx+3c)}{3d}+\frac{\left(\frac{63b^2}{128}-\frac{3a^2}{4}\right)\cosh(dx+c)}{d}-\frac{5abx}{8}+\frac{9b^2\cosh(5dx+5c)}{320d}-\frac{9b^2\cosh(7dx+7c)}{1792d}+\frac{b^2\cosh(9dx+9}{2304d}$
risch	$-\frac{5abx}{8}+\frac{b^2e^{9dx+9c}}{4608d}-\frac{9b^2e^{7dx+7c}}{3584d}+\frac{ab\,e^{6dx+6c}}{192d}+\frac{9b^2e^{5dx+5c}}{640d}-\frac{3\,e^{4dx+4c}ab}{64d}+\frac{e^{3dx+3c}a^2}{24d}-\frac{7\,e^{3dx+3c}b^2}{128d}+\frac{15\,e^{2dx+2c}}{64d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-21/64*b^2+1/4*a^2)/d*cosh(3*d*x+3*c)+(63/128*b^2-3/4*a^2)/d*cosh(d*x+
c)-5/8*a*b*x+9/320*b^2*cosh(5*d*x+5*c)/d-9/1792*b^2/d*cosh(7*d*x+7*c)+1/230
4*b^2/d*cosh(9*d*x+9*c)+15/32*a*b*sinh(2*d*x+2*c)/d-3/32*a*b*sinh(4*d*x+4*c
)/d+1/96*a*b*sinh(6*d*x+6*c)/d
```

Maxima [**A**]
time = 0.29, size = 272, normalized size = 1.42

$$-\frac{1}{161280}\,b^{2}\left(\frac{\left(405\,e^{\left(-2\,dx-2\,c\right)}-2268\,e^{\left(-4\,dx-4\,c\right)}+8820\,e^{\left(-6\,dx-6\,c\right)}-39690\,e^{\left(-8\,dx-8\,c\right)}-35\right)e^{\left(9\,dx+9\,c\right)}}{d}-\frac{39690\,e^{\left(-dx-c\right)}-8820\,e^{\left(-3\,dx-3\,c\right)}+2268\,e^{\left(-5\,dx-5\,c\right)}-405\,e^{\left(-7\,dx-7\,c\right)}+35\,e^{\left(-9\,dx-9\,c\right)}}{d}\right)-\frac{1}{192}\,ab\left(\frac{\left(9\,e^{\left(-2\,dx-2\,c\right)}-45\,e^{\left(-4\,dx-4\,c\right)}-1\right)e^{\left(6\,dx+6\,c\right)}}{d}+\frac{120\,\left(dx+c\right)}{d}+\frac{45\,e^{\left(-2\,dx-2\,c\right)}-9\,e^{\left(-4\,dx-4\,c\right)}+e^{\left(-6\,dx-6\,c\right)}}{d}\right)+\frac{1}{24}\,a^{2}\left(\frac{e^{\left(3\,dx+3\,c\right)}}{d}-\frac{9\,e^{\left(dx+c\right)}}{d}-\frac{9\,e^{\left(-dx-c\right)}}{d}+\frac{e^{\left(-3\,dx-3\,c\right)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/161280*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d
*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x
 - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c
) + 35*e^(-9*d*x - 9*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x
 - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*
e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(174) = 348$.
time = 0.44, size = 355, normalized size = 1.85

$$\frac{35\,b^{2}\cosh\left(dx+c\right)^{9}+315\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{8}-405\,b^{2}\cosh\left(dx+c\right)^{7}+5040\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+2268\,b^{2}\cosh\left(dx+c\right)^{5}+105\,\left(28\,b^{2}\cosh\left(dx+c\right)^{3}-27\,b^{2}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{6}+315\,\left(14\,b^{2}\cosh\left(dx+c\right)^{5}-45\,b^{2}\cosh\left(dx+c\right)^{3}+36\,b^{2}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}-50400\,abdx+420\,\left(16\,a^{2}-21\,b^{2}\right)\cosh\left(dx+c\right)^{3}+3360\,\left(5\,ab\cosh\left(dx+c\right)^{3}-9\,ab\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+315\,\left(4\,b^{2}\cosh\left(dx+c\right)^{7}-27\,b^{2}\cosh\left(dx+c\right)^{5}+72\,b^{2}\cosh\left(dx+c\right)^{3}+4\,\left(16\,a^{2}-21\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}-1890\,\left(32\,a^{2}-21\,b^{2}\right)\cosh\left(dx+c\right)+5040\,\left(ab\cosh\left(dx+c\right)^{5}-6\,ab\cosh\left(dx+c\right)^{3}+15\,ab\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^2*cosh(d*x + c)^9 + 315*b^2*cosh(d*x + c)*sinh(d*x + c)^8 - 4
05*b^2*cosh(d*x + c)^7 + 5040*a*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2268*b^2*
cosh(d*x + c)^5 + 105*(28*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c))*sinh(
d*x + c)^6 + 315*(14*b^2*cosh(d*x + c)^5 - 45*b^2*cosh(d*x + c)^3 + 36*b^2*
cosh(d*x + c))*sinh(d*x + c)^4 - 50400*a*b*d*x + 420*(16*a^2 - 21*b^2)*cosh
(d*x + c)^3 + 3360*(5*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))*sinh(d*x +
 c)^3 + 315*(4*b^2*cosh(d*x + c)^7 - 27*b^2*cosh(d*x + c)^5 + 72*b^2*cosh(d
*x + c)^3 + 4*(16*a^2 - 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 1890*(32*a
^2 - 21*b^2)*cosh(d*x + c) + 5040*(a*b*cosh(d*x + c)^5 - 6*a*b*cosh(d*x + c
)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A]
time = 1.40, size = 325, normalized size = 1.69

$$\begin{cases} \frac{a^2 \sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^2\cosh^3(c+dx)}{3d} + \frac{5abx\sinh^6(c+dx)}{8} - \frac{15abx\sinh^4(c+dx)\cosh^2(c+dx)}{8} + \frac{15abx\sinh^2(c+dx)\cosh^4(c+dx)}{8} - \frac{5abx\cosh^6(c+dx)}{8} + \frac{11ab\sinh^5(c+dx)\cosh(c+dx)}{8d} - \frac{5ab\sinh^3(c+dx)\cosh^3(c+dx)}{3d} + \frac{5ab\sinh(c+dx)\cosh^5(c+dx)}{8d} + \frac{b^2\sinh^8(c+dx)\cosh(c+dx)}{d} - \frac{8b^2\sinh^6(c+dx)\cosh^3(c+dx)}{3d} + \frac{16b^2\sinh^4(c+dx)\cosh^5(c+dx)}{5d} - \frac{64b^2\sinh^2(c+dx)\cosh^7(c+dx)}{35d} + \frac{128b^2\cosh^9(c+dx)}{315d} & \text{for } d \neq 0 \\ x\left(a + b\sinh^3(c)\right)^2\sinh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3/
(3*d) + 5*a*b*x*sinh(c + d*x)**6/8 - 15*a*b*x*sinh(c + d*x)**4*cosh(c + d*x
)**2/8 + 15*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/8 - 5*a*b*x*cosh(c + d*
x)**6/8 + 11*a*b*sinh(c + d*x)**5*cosh(c + d*x)/(8*d) - 5*a*b*sinh(c + d*x)
**3*cosh(c + d*x)**3/(3*d) + 5*a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d) + b
**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**2*sinh(c + d*x)**6*cosh(c + d*x
)**3/(3*d) + 16*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 64*b**2*sinh
(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**2*cosh(c + d*x)**9/(315*d), N
e(d, 0)), (x*(a + b*sinh(c)**3)**2*sinh(c)**3, True))
```

Giac [A]
time = 0.45, size = 301, normalized size = 1.57

$$-\frac{5}{8}\,abx + \frac{b^2e^{(9\,dx+9\,c)}}{4608\,d} - \frac{9\,b^2e^{(7\,dx+7\,c)}}{3584\,d} + \frac{abe^{(6\,dx+6\,c)}}{192\,d} + \frac{9\,b^2e^{(5\,dx+5\,c)}}{640\,d} - \frac{3\,abe^{(4\,dx+4\,c)}}{64\,d} + \frac{15\,abe^{(2\,dx+2\,c)}}{64\,d} - \frac{15\,abe^{(-2\,dx-2\,c)}}{64\,d} + \frac{3\,abe^{(-4\,dx-4\,c)}}{64\,d} + \frac{9\,b^2e^{(-5\,dx-5\,c)}}{640\,d} - \frac{abe^{(-6\,dx-6\,c)}}{192\,d} - \frac{9\,b^2e^{(-7\,dx-7\,c)}}{3584\,d} + \frac{b^2e^{(-9\,dx-9\,c)}}{4608\,d} + \frac{(16\,a^2-21\,b^2)e^{(3\,dx+3\,c)}}{384\,d} - \frac{3\,(32\,a^2-21\,b^2)e^{(dx+c)}}{256\,d} - \frac{3\,(32\,a^2-21\,b^2)e^{(-dx-c)}}{256\,d} + \frac{(16\,a^2-21\,b^2)e^{(-3\,dx-3\,c)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -5/8*a*b*x + 1/4608*b^2*e^(9*d*x + 9*c)/d - 9/3584*b^2*e^(7*d*x + 7*c)/d +
1/192*a*b*e^(6*d*x + 6*c)/d + 9/640*b^2*e^(5*d*x + 5*c)/d - 3/64*a*b*e^(4*d
*x + 4*c)/d + 15/64*a*b*e^(2*d*x + 2*c)/d - 15/64*a*b*e^(-2*d*x - 2*c)/d +
3/64*a*b*e^(-4*d*x - 4*c)/d + 9/640*b^2*e^(-5*d*x - 5*c)/d - 1/192*a*b*e^(-
6*d*x - 6*c)/d - 9/3584*b^2*e^(-7*d*x - 7*c)/d + 1/4608*b^2*e^(-9*d*x - 9*c
)/d + 1/384*(16*a^2 - 21*b^2)*e^(3*d*x + 3*c)/d - 3/256*(32*a^2 - 21*b^2)*e
^(d*x + c)/d - 3/256*(32*a^2 - 21*b^2)*e^(-d*x - c)/d + 1/384*(16*a^2 - 21*
b^2)*e^(-3*d*x - 3*c)/d
```

Mupad [B]
time = 0.93, size = 149, normalized size = 0.78

$$\frac{\frac{a^2\cosh(c+d\,x)^3}{3} - a^2\cosh(c+d\,x) + \frac{\sinh(c+d\,x)\,a\,b\cosh(c+d\,x)^5}{3} - \frac{13\sinh(c+d\,x)\,a\,b\cosh(c+d\,x)^3}{12} + \frac{11\sinh(c+d\,x)\,a\,b\cosh(c+d\,x)}{8} - \frac{5\,d\,x\,a\,b}{8} + \frac{b^2\cosh(c+d\,x)^9}{9} - \frac{4\,b^2\cosh(c+d\,x)^7}{7} + \frac{6\,b^2\cosh(c+d\,x)^5}{5} - \frac{4\,b^2\cosh(c+d\,x)^3}{3} + b^2\cosh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^3)^2,x)
```

```
[Out] (b^2*cosh(c + d*x) - a^2*cosh(c + d*x) + (a^2*cosh(c + d*x)^3)/3 - (4*b^2*c
osh(c + d*x)^3)/3 + (6*b^2*cosh(c + d*x)^5)/5 - (4*b^2*cosh(c + d*x)^7)/7 +
 (b^2*cosh(c + d*x)^9)/9 - (13*a*b*cosh(c + d*x)^3*sinh(c + d*x))/12 + (a*b
*cosh(c + d*x)^5*sinh(c + d*x))/3 + (11*a*b*cosh(c + d*x)*sinh(c + d*x))/8
- (5*a*b*d*x)/8)/d
```

3.151 $\int \sinh^2(c+dx) \left(a+b\sinh^3(c+dx)\right)^2 \, dx$

Optimal. Leaf size=180

$$-\frac{a^2x}{2}+\frac{35b^2x}{128}+\frac{2ab\cosh(c+dx)}{d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{2ab\cosh^5(c+dx)}{5d}+\frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d}-35b$$

```
[Out] -1/2*a^2*x+35/128*b^2*x+2*a*b*cosh(d*x+c)/d-4/3*a*b*cosh(d*x+c)^3/d+2/5*a*b
*cosh(d*x+c)^5/d+1/2*a^2*cosh(d*x+c)*sinh(d*x+c)/d-35/128*b^2*cosh(d*x+c)*s
inh(d*x+c)/d+35/192*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d-7/48*b^2*cosh(d*x+c)*si
nh(d*x+c)^5/d+1/8*b^2*cosh(d*x+c)*sinh(d*x+c)^7/d
```

Rubi [A]

time = 0.12, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = $\{3299, 2715, 8, 2713\}$

$$\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^2x}{2}+\frac{2ab\cosh^5(c+dx)}{5d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^7(c+dx)\cosh(c+dx)}{8d}-\frac{7b^2\sinh^5(c+dx)\cosh(c+dx)}{48d}+\frac{35b^2\sinh^3(c+dx)\cosh(c+dx)}{192d}-\frac{35b^2\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{35b^2x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] -1/2*(a^2*x) + (35*b^2*x)/128 + (2*a*b*Cosh[c + d*x])/d - (4*a*b*Cosh[c + d
*x]^3)/(3*d) + (2*a*b*Cosh[c + d*x]^5)/(5*d) + (a^2*Cosh[c + d*x]*Sinh[c +
d*x])/(2*d) - (35*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (35*b^2*Cosh[c
 + d*x]*Sinh[c + d*x]^3)/(192*d) - (7*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(4
8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^7)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx &= -\int \left(-a^2\sinh^2(c+dx) - 2ab\sinh^5(c+dx) - b^2\sinh^8(c+dx\right. \\
&= a^2\int \sinh^2(c+dx)\,dx + (2ab)\int \sinh^5(c+dx)\,dx + b^2\int \sinh \\
&= \frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2\cosh(c+dx)\sinh^7(c+dx)}{8d} \\
&= -\frac{a^2x}{2} + \frac{2ab\cosh(c+dx)}{d} - \frac{4ab\cosh^3(c+dx)}{3d} + \frac{2ab\cosh^5(c}{5d} \\
&= -\frac{a^2x}{2} + \frac{2ab\cosh(c+dx)}{d} - \frac{4ab\cosh^3(c+dx)}{3d} + \frac{2ab\cosh^5(c}{5d} \\
&= -\frac{a^2x}{2} + \frac{2ab\cosh(c+dx)}{d} - \frac{4ab\cosh^3(c+dx)}{3d} + \frac{2ab\cosh^5(c}{5d} \\
&= -\frac{a^2x}{2} + \frac{35b^2x}{128} + \frac{2ab\cosh(c+dx)}{d} - \frac{4ab\cosh^3(c+dx)}{3d} + 2a
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 133, normalized size = 0.74

$$\frac{-7680a^2c + 4200b^2c - 7680a^2dx + 4200b^2dx + 19200ab\cosh(c+dx) - 3200ab\cosh(3(c+dx)) + 384ab\cosh(5(c+dx)) + 3840a^2\sinh(2(c+dx)) - 3360b^2\sinh(2(c+dx)) + 840b^2\sinh(4(c+dx)) - 160b^2\sinh(6(c+dx)) + 15b^2\sinh(8(c+dx))}{15360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-7680*a^2*c + 4200*b^2*c - 7680*a^2*d*x + 4200*b^2*d*x + 19200*a*b*Cosh[c
 + d*x] - 3200*a*b*Cosh[3*(c + d*x)] + 384*a*b*Cosh[5*(c + d*x)] + 3840*a^2*
Sinh[2*(c + d*x)] - 3360*b^2*Sinh[2*(c + d*x)] + 840*b^2*Sinh[4*(c + d*x)]
 - 160*b^2*Sinh[6*(c + d*x)] + 15*b^2*Sinh[8*(c + d*x)])/(15360*d)
```

Maple [A]
time = 1.53, size = 135, normalized size = 0.75

method	result
default	$\frac{\left(-\frac{7b^2}{16}+\frac{a^2}{2}\right)\sinh(2dx+2c)}{2d} - \frac{a^2x}{2} + \frac{35b^2x}{128} + \frac{7b^2\sinh(4dx+4c)}{128d} - \frac{b^2\sinh(6dx+6c)}{96d} + \frac{b^2\sinh(8dx+8c)}{1024d} + \frac{ab\cosh(5dx+5c)}{40d}$

risch	$\frac{35b^2x}{128}-\frac{a^2x}{2}+\frac{b^2\mathrm{e}^{8dx+8c}}{2048d}-\frac{b^2\mathrm{e}^{6dx+6c}}{192d}+\frac{ab\,\mathrm{e}^{5dx+5c}}{80d}+\frac{7\,\mathrm{e}^{4dx+4c}b^2}{256d}-\frac{5ab\,\mathrm{e}^{3dx+3c}}{48d}+\frac{\mathrm{e}^{2dx+2c}a^2}{8d}-\frac{7\,\mathrm{e}^{2dx+2c}b^2}{64d}+\frac{5ab\,\mathrm{e}^{d}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-7/16*b^2+1/2*a^2)*sinh(2*d*x+2*c)/d-1/2*a^2*x+35/128*b^2*x+7/128*b^2*
sinh(4*d*x+4*c)/d-1/96*b^2*sinh(6*d*x+6*c)/d+1/1024*b^2*sinh(8*d*x+8*c)/d+1
/40*a*b/d*cosh(5*d*x+5*c)+5/4*a*b*cosh(d*x+c)/d-5/24*a*b/d*cosh(3*d*x+3*c)
```

Maxima [A]
time = 0.28, size = 237, normalized size = 1.32

$$-\frac{1}{8}\,a^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{6144}\,b^2\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+672\,e^{(-6\,dx-6\,c)}-3\right)e^{(8\,dx+8\,c)}}{d}-\frac{1680\,(dx+c)}{d}-\frac{672\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+32\,e^{(-6\,dx-6\,c)}-3\,e^{(-8\,dx-8\,c)}}{d}\right)+\frac{1}{240}\,ab\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/6144*b^2*((32*e
^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x
 + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c)
 + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5
*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*
e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

Fricas [A]
time = 0.42, size = 274, normalized size = 1.52

$$\frac{15\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+48\,ab\cosh\left(dx+c\right)^5+240\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+15\left(7\,b^2\cosh\left(dx+c\right)^3-8\,b^2\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^5-400\,ab\cosh\left(dx+c\right)^3+5\left(21\,b^2\cosh\left(dx+c\right)^5-80\,b^2\cosh\left(dx+c\right)^3+84\,b^2\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3-15\,(64\,a^2-35\,b^2)dx+2400\,ab\cosh\left(dx+c\right)+240\left(2\,ab\cosh\left(dx+c\right)^3-5\,ab\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2+15\left(b^2\cosh\left(dx+c\right)^7-8\,b^2\cosh\left(dx+c\right)^5+28\,b^2\cosh\left(dx+c\right)^3+8\,(8\,a^2-7\,b^2)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/1920*(15*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 48*a*b*cosh(d*x + c)^5 + 240
*a*b*cosh(d*x + c)*sinh(d*x + c)^4 + 15*(7*b^2*cosh(d*x + c)^3 - 8*b^2*cosh
(d*x + c))*sinh(d*x + c)^5 - 400*a*b*cosh(d*x + c)^3 + 5*(21*b^2*cosh(d*x +
 c)^5 - 80*b^2*cosh(d*x + c)^3 + 84*b^2*cosh(d*x + c))*sinh(d*x + c)^3 - 15
*(64*a^2 - 35*b^2)*d*x + 2400*a*b*cosh(d*x + c) + 240*(2*a*b*cosh(d*x + c)^
3 - 5*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + 15*(b^2*cosh(d*x + c)^7 - 8*b^2*
cosh(d*x + c)^5 + 28*b^2*cosh(d*x + c)^3 + 8*(8*a^2 - 7*b^2)*cosh(d*x + c))
*sinh(d*x + c))/d
```

Sympy [A]
time = 0.98, size = 340, normalized size = 1.89

$$\begin{cases}\frac{a^2x\sinh^2(c+dx)}{2}-\frac{a^2x\cosh^2(c+dx)}{2}+\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{2ab\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{8ab\sinh^2(c+dx)\cosh^3(c+dx)}{3d}+\frac{16ab\cosh^5(c+dx)}{15d}+\frac{35b^2x\sinh^8(c+dx)}{128}-\frac{35b^2x\sinh^6(c+dx)\cosh^2(c+dx)}{32}+\frac{105b^2x\sinh^4(c+dx)\cosh^4(c+dx)}{64}-\frac{35b^2x\sinh^2(c+dx)\cosh^6(c+dx)}{32}+\frac{35b^2x\cosh^8(c+dx)}{128}+\frac{93b^2\sinh^7(c+dx)\cosh(c+dx)}{128d}-\frac{511b^2\sinh^5(c+dx)\cosh^3(c+dx)}{384d}+\frac{385b^2\sinh^3(c+dx)\cosh^5(c+dx)}{384d}-\frac{35b^2\sinh(c+dx)\cosh^7(c+dx)}{128d} & \text{for}\ d\neq 0\\ x\left(a+b\sinh^3(c)\right)^2\sinh^2(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 8
*a*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15*
d) + 35*b**2*x*sinh(c + d*x)**8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d
*x)**2/32 + 105*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sin
h(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**
2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c
 + d*x)**3/(384*d) + 385*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 3
5*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)
**3)**2*sinh(c)**2, True))
```

Giac [**A**]
time = 0.43, size = 260, normalized size = 1.44

$$-\frac{1}{128}\left(64\,a^2-35\,b^2\right)x+\frac{b^2e^{(8\,dx+8\,c)}}{2048\,d}-\frac{b^2e^{(6\,dx+6\,c)}}{192\,d}+\frac{abe^{(5\,dx+5\,c)}}{80\,d}+\frac{7\,b^2e^{(4\,dx+4\,c)}}{256\,d}-\frac{5\,abe^{(3\,dx+3\,c)}}{48\,d}+\frac{5\,abe^{(dx+c)}}{8\,d}+\frac{5\,abe^{(-dx-c)}}{8\,d}-\frac{5\,abe^{(-3\,dx-3\,c)}}{48\,d}-\frac{7\,b^2e^{(-4\,dx-4\,c)}}{256\,d}+\frac{abe^{(-5\,dx-5\,c)}}{80\,d}+\frac{b^2e^{(-6\,dx-6\,c)}}{192\,d}-\frac{b^2e^{(-8\,dx-8\,c)}}{2048\,d}+\frac{(8\,a^2-7\,b^2)e^{(2\,dx+2\,c)}}{64\,d}-\frac{(8\,a^2-7\,b^2)e^{(-2\,dx-2\,c)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/128*(64*a^2 - 35*b^2)*x + 1/2048*b^2*e^(8*d*x + 8*c)/d - 1/192*b^2*e^(6*
d*x + 6*c)/d + 1/80*a*b*e^(5*d*x + 5*c)/d + 7/256*b^2*e^(4*d*x + 4*c)/d - 5
/48*a*b*e^(3*d*x + 3*c)/d + 5/8*a*b*e^(d*x + c)/d + 5/8*a*b*e^(-d*x - c)/d
 - 5/48*a*b*e^(-3*d*x - 3*c)/d - 7/256*b^2*e^(-4*d*x - 4*c)/d + 1/80*a*b*e^(
-5*d*x - 5*c)/d + 1/192*b^2*e^(-6*d*x - 6*c)/d - 1/2048*b^2*e^(-8*d*x - 8*c
)/d + 1/64*(8*a^2 - 7*b^2)*e^(2*d*x + 2*c)/d - 1/64*(8*a^2 - 7*b^2)*e^(-2*d
*x - 2*c)/d
```

Mupad [**B**]
time = 1.63, size = 126, normalized size = 0.70

$$\frac{480\,a^2\,\mathrm{sinh}\left(2\,c+2\,d\,x\right)-420\,b^2\,\mathrm{sinh}\left(2\,c+2\,d\,x\right)+105\,b^2\,\mathrm{sinh}\left(4\,c+4\,d\,x\right)-20\,b^2\,\mathrm{sinh}\left(6\,c+6\,d\,x\right)+\frac{15\,b^2\,\mathrm{sinh}\left(8\,c+8\,d\,x\right)}{8}+2400\,a\,b\,\mathrm{cosh}\left(c+d\,x\right)-400\,a\,b\,\mathrm{cosh}\left(3\,c+3\,d\,x\right)+48\,a\,b\,\mathrm{cosh}\left(5\,c+5\,d\,x\right)-960\,a^2\,d\,x+525\,b^2\,d\,x}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^3)^2,x)
```

```
[Out] (480*a^2*sinh(2*c + 2*d*x) - 420*b^2*sinh(2*c + 2*d*x) + 105*b^2*sinh(4*c +
 4*d*x) - 20*b^2*sinh(6*c + 6*d*x) + (15*b^2*sinh(8*c + 8*d*x))/8 + 2400*a*
b*cosh(c + d*x) - 400*a*b*cosh(3*c + 3*d*x) + 48*a*b*cosh(5*c + 5*d*x) - 96
0*a^2*d*x + 525*b^2*d*x)/(1920*d)
```

3.152 $\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=130

$$\frac{3abx}{4}+\frac{a^2\cosh(c+dx)}{d}-\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{d}-\frac{3b^2\cosh^5(c+dx)}{5d}+\frac{b^2\cosh^7(c+dx)}{7d}-3ab\cosh(c$$

```
[Out] 3/4*a*b*x+a^2*cosh(d*x+c)/d-b^2*cosh(d*x+c)/d+b^2*cosh(d*x+c)^3/d-3/5*b^2*c
osh(d*x+c)^5/d+1/7*b^2*cosh(d*x+c)^7/d-3/4*a*b*cosh(d*x+c)*sinh(d*x+c)/d+1/
2*a*b*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3299, 2718, 2715, 8, 2713}

$$\frac{a^2\cosh(c+dx)}{d}+\frac{ab\sinh^3(c+dx)\cosh(c+dx)}{2d}-\frac{3ab\sinh(c+dx)\cosh(c+dx)}{4d}+\frac{3abx}{4}+\frac{b^2\cosh^7(c+dx)}{7d}-\frac{3b^2\cosh^5(c+dx)}{5d}+\frac{b^2\cosh^3(c+dx)}{d}-\frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (3*a*b*x)/4 + (a^2*Cosh[c + d*x])/d - (b^2*Cosh[c + d*x])/d + (b^2*Cosh[c +
 d*x]^3)/d - (3*b^2*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]^7)/(7*d) -
(3*a*b*Cosh[c + d*x]*Sinh[c + d*x])/(4*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x
]^3)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx &= -\left(i\int \left(ia^2\sinh(c+dx)+2iab\sinh^4(c+dx)+ib^2\sinh^7(c+d\right.\right.\\ &= a^2\int \sinh(c+dx)\,dx+(2ab)\int \sinh^4(c+dx)\,dx+b^2\int \sinh^7(\\ &= \frac{a^2\cosh(c+dx)}{d}+\frac{ab\cosh(c+dx)\sinh^3(c+dx)}{2d}-\frac{1}{2}(3ab)\int \mathrm{si}\\ &= \frac{a^2\cosh(c+dx)}{d}-\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{d}-\frac{3b^2\,\mathrm{co}}{}\\ &= \frac{3abx}{4}+\frac{a^2\cosh(c+dx)}{d}-\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 92, normalized size = 0.71

$$\frac{35(64a^2-35b^2)\cosh(c+dx)+b(245b\cosh(3(c+dx))-49b\cosh(5(c+dx))+5b\cosh(7(c+dx))+140a(12(c+dx)-8\sinh(2(c+dx))+\sinh(4(c+dx))))}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (35*(64*a^2 - 35*b^2)*Cosh[c + d*x] + b*(245*b*Cosh[3*(c + d*x)] - 49*b*Cos
h[5*(c + d*x)] + 5*b*Cosh[7*(c + d*x)] + 140*a*(12*(c + d*x) - 8*Sinh[2*(c
+ d*x)] + Sinh[4*(c + d*x)])))/(2240*d)
```

Maple [A]
time = 1.18, size = 109, normalized size = 0.84

method	result
default	$\frac{\left(-\frac{35b^2}{64}+a^2\right)\cosh(dx+c)}{d}+\frac{3abx}{4}+\frac{7b^2\cosh(3dx+3c)}{64d}-\frac{7b^2\cosh(5dx+5c)}{320d}+\frac{b^2\cosh(7dx+7c)}{448d}-\frac{ab\sinh(2dx+2c)}{2d}+ab\,\mathrm{si}$
risch	$\frac{3abx}{4}+\frac{b^2e^{7dx+7c}}{896d}-\frac{7b^2e^{5dx+5c}}{640d}+\frac{e^{4dx+4c}ab}{32d}+\frac{7\,e^{3dx+3c}b^2}{128d}-\frac{e^{2dx+2c}ab}{4d}+\frac{e^{dx+c}a^2}{2d}-\frac{35\,e^{dx+c}b^2}{128d}+\frac{e^{-dx-c}a^2}{2d}-35$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-35/64*b^2+a^2)/d*cosh(d*x+c)+3/4*a*b*x+7/64*b^2/d*cosh(3*d*x+3*c)-7/320*b
^2*cosh(5*d*x+5*c)/d+1/448*b^2/d*cosh(7*d*x+7*c)-1/2*a*b*sinh(2*d*x+2*c)/d+
1/16*a*b*sinh(4*d*x+4*c)/d
```

Maxima [A]
time = 0.27, size = 180, normalized size = 1.38

$$\frac{1}{32}\,ab\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{4480}\,b^2\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d}\right)+\frac{a^2\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d
*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x -
c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) +
a^2*cosh(d*x + c)/d
```

Fricas [A]
time = 0.41, size = 220, normalized size = 1.69

$$\frac{5\,b^2\cosh\left(dx+c\right)^7+35\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6-49\,b^2\cosh\left(dx+c\right)^5+560\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+245\,b^2\cosh\left(dx+c\right)^3+35\left(5\,b^2\cosh\left(dx+c\right)^3-7\,b^2\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^4+1680\,abdx+35\left(3\,b^2\cosh\left(dx+c\right)^5-14\,b^2\cosh\left(dx+c\right)^3+21\,b^2\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2+35\left(64\,a^2-35\,b^2\right)\cosh\left(dx+c\right)+560\left(ab\cosh\left(dx+c\right)^3-4\,ab\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b^2*cosh(d*x + c)^7 + 35*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 49*b
^2*cosh(d*x + c)^5 + 560*a*b*cosh(d*x + c)*sinh(d*x + c)^3 + 245*b^2*cosh(d
*x + c)^3 + 35*(5*b^2*cosh(d*x + c)^3 - 7*b^2*cosh(d*x + c))*sinh(d*x + c)^
4 + 1680*a*b*d*x + 35*(3*b^2*cosh(d*x + c)^5 - 14*b^2*cosh(d*x + c)^3 + 21*
b^2*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(64*a^2 - 35*b^2)*cosh(d*x + c) + 5
60*(a*b*cosh(d*x + c)^3 - 4*a*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A]
time = 0.70, size = 219, normalized size = 1.68

$$\begin{cases}\frac{a^2\cosh\left(c+dx\right)}{d}+\frac{3abx\sinh^4\left(c+dx\right)}{4}-\frac{3abx\sinh^2\left(c+dx\right)\cosh^2\left(c+dx\right)}{2}+\frac{3abx\cosh^4\left(c+dx\right)}{4}+\frac{5ab\sinh^3\left(c+dx\right)\cosh\left(c+dx\right)}{4d}-\frac{3ab\sinh\left(c+dx\right)\cosh^3\left(c+dx\right)}{4d}+\frac{b^2\sinh^6\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2b^2\sinh^4\left(c+dx\right)\cosh^3\left(c+dx\right)}{d}+\frac{8b^2\sinh^2\left(c+dx\right)\cosh^5\left(c+dx\right)}{5d}-\frac{16b^2\cosh^7\left(c+dx\right)}{35d} & \text{for}\ d\neq 0\\ x\left(a+b\sinh^3\left(c\right)\right)^2\sinh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*cosh(c + d*x)/d + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh
(c + d*x)**2*cosh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c
 + d*x)**3*cosh(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d)
```

```
 + b**2*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b**2*sinh(c + d*x)**4*cosh(c +
 d*x)**3/d + 8*b**2*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**2*cosh(
c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2*sinh(c), True))
```

Giac [A]

time = 0.43, size = 219, normalized size = 1.68

$$\frac{3}{4}\,abx + \frac{b^{2}e^{\left(7\,dx+7\,c\right)}}{896\,d} - \frac{7\,b^{2}e^{\left(5\,dx+5\,c\right)}}{640\,d} + \frac{abe^{\left(4\,dx+4\,c\right)}}{32\,d} + \frac{7\,b^{2}e^{\left(3\,dx+3\,c\right)}}{128\,d} - \frac{abe^{\left(2\,dx+2\,c\right)}}{4\,d} + \frac{abe^{\left(-2\,dx-2\,c\right)}}{4\,d} + \frac{7\,b^{2}e^{\left(-3\,dx-3\,c\right)}}{128\,d} - \frac{abe^{\left(-4\,dx-4\,c\right)}}{32\,d} - \frac{7\,b^{2}e^{\left(-5\,dx-5\,c\right)}}{640\,d} + \frac{b^{2}e^{\left(-7\,dx-7\,c\right)}}{896\,d} + \frac{\left(64\,a^{2}-35\,b^{2}\right)e^{\left(dx+c\right)}}{128\,d} + \frac{\left(64\,a^{2}-35\,b^{2}\right)e^{\left(-dx-c\right)}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 3/4*a*b*x + 1/896*b^2*e^(7*d*x + 7*c)/d - 7/640*b^2*e^(5*d*x + 5*c)/d + 1/3
2*a*b*e^(4*d*x + 4*c)/d + 7/128*b^2*e^(3*d*x + 3*c)/d - 1/4*a*b*e^(2*d*x +
2*c)/d + 1/4*a*b*e^(-2*d*x - 2*c)/d + 7/128*b^2*e^(-3*d*x - 3*c)/d - 1/32*a
*b*e^(-4*d*x - 4*c)/d - 7/640*b^2*e^(-5*d*x - 5*c)/d + 1/896*b^2*e^(-7*d*x
- 7*c)/d + 1/128*(64*a^2 - 35*b^2)*e^(d*x + c)/d + 1/128*(64*a^2 - 35*b^2)*
e^(-d*x - c)/d
```

Mupad [B]

time = 0.25, size = 104, normalized size = 0.80

$$\frac{a^2\,\mathrm{cosh}\left(c+d\,x\right)+\frac{\mathrm{sinh}\left(c+d\,x\right)\,a\,b\,{\mathrm{cosh}\left(c+d\,x\right)}^3}{2}-\frac{5\,\mathrm{sinh}\left(c+d\,x\right)\,a\,b\,\mathrm{cosh}\left(c+d\,x\right)}{4}+\frac{3\,d\,x\,a\,b}{4}+\frac{b^2\,{\mathrm{cosh}\left(c+d\,x\right)}^7}{7}-\frac{3\,b^2\,{\mathrm{cosh}\left(c+d\,x\right)}^5}{5}+b^2\,{\mathrm{cosh}\left(c+d\,x\right)}^3-b^2\,\mathrm{cosh}\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^3)^2,x)
```

```
[Out] (a^2*cosh(c + d*x) - b^2*cosh(c + d*x) + b^2*cosh(c + d*x)^3 - (3*b^2*cosh(
c + d*x)^5)/5 + (b^2*cosh(c + d*x)^7)/7 + (a*b*cosh(c + d*x)^3*sinh(c + d*x
))/2 - (5*a*b*cosh(c + d*x)*sinh(c + d*x))/4 + (3*a*b*d*x)/4)/d
```

3.153 $\int \left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=114

$$a^2x-\frac{5b^2x}{16}-\frac{2ab\cosh(c+dx)}{d}+\frac{2ab\cosh^3(c+dx)}{3d}+\frac{5b^2\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{5b^2\cosh(c+dx)\sinh^3(c+}{24d}$$

```
[Out] a^2*x-5/16*b^2*x-2*a*b*cosh(d*x+c)/d+2/3*a*b*cosh(d*x+c)^3/d+5/16*b^2*cosh(
d*x+c)*sinh(d*x+c)/d-5/24*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d+1/6*b^2*cosh(d*x+
c)*sinh(d*x+c)^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$,
Rules used = {3292, 2713, 2715, 8}

$$a^2x+\frac{2ab\cosh^3(c+dx)}{3d}-\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^5(c+dx)\cosh(c+dx)}{6d}-\frac{5b^2\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{5b^2\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{5b^2x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] a^2*x - (5*b^2*x)/16 - (2*a*b*Cosh[c + d*x])/d + (2*a*b*Cosh[c + d*x]^3)/(3
*d) + (5*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*b^2*Cosh[c + d*x]*Sin
h[c + d*x]^3)/(24*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
```

```
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^3(c+dx)\right)^2\,dx &= \int \left(a^2+2ab\sinh^3(c+dx)+b^2\sinh^6(c+dx)\right)\,dx\\ &= a^2x+(2ab)\int \sinh^3(c+dx)\,dx+b^2\int \sinh^6(c+dx)\,dx\\ &= a^2x+\frac{b^2\cosh(c+dx)\sinh^5(c+dx)}{6d}-\frac{1}{6}\left(5b^2\right)\int \sinh^4(c+dx)\,dx-\frac{(2ab)\mathrm{S}}{}\\ &= a^2x-\frac{2ab\cosh(c+dx)}{d}+\frac{2ab\cosh^3(c+dx)}{3d}-\frac{5b^2\cosh(c+dx)\sinh^3(c+d}{24d}\\ &= a^2x-\frac{2ab\cosh(c+dx)}{d}+\frac{2ab\cosh^3(c+dx)}{3d}+\frac{5b^2\cosh(c+dx)\sinh(c+dx}{16d}\\ &= a^2x-\frac{5b^2x}{16}-\frac{2ab\cosh(c+dx)}{d}+\frac{2ab\cosh^3(c+dx)}{3d}+\frac{5b^2\cosh(c+dx)\sinh}{16d}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 94, normalized size = 0.82

$$\frac{192a^2c-60b^2c+192a^2dx-60b^2dx-288ab\cosh(c+dx)+32ab\cosh(3(c+dx))+45b^2\sinh(2(c+dx))-9b^2\sinh(4(c+dx))+b^2\sinh(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (192*a^2*c - 60*b^2*c + 192*a^2*d*x - 60*b^2*d*x - 288*a*b*Cosh[c + d*x] +
32*a*b*Cosh[3*(c + d*x)] + 45*b^2*Sinh[2*(c + d*x)] - 9*b^2*Sinh[4*(c + d*x
)] + b^2*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 1.32, size = 93, normalized size = 0.82

method	result
default	$a^2x-\frac{5b^2x}{16}+\frac{15b^2\sinh(2dx+2c)}{64d}-\frac{3b^2\sinh(4dx+4c)}{64d}+\frac{b^2\sinh(6dx+6c)}{192d}-\frac{3ab\cosh(dx+c)}{2d}+\frac{ab\cosh(3dx+3c)}{6d}$
risch	$a^2x-\frac{5b^2x}{16}+\frac{b^2\mathrm{e}^{6dx+6c}}{384d}-\frac{3\,\mathrm{e}^{4dx+4c}b^2}{128d}+\frac{ab\,\mathrm{e}^{3dx+3c}}{12d}+\frac{15\,\mathrm{e}^{2dx+2c}b^2}{128d}-\frac{3ab\,\mathrm{e}^{dx+c}}{4d}-\frac{3ab\,\mathrm{e}^{-dx-c}}{4d}-\frac{15\,\mathrm{e}^{-2dx-2c}b^2}{128d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x-5/16*b^2*x+15/64*b^2*sinh(2*d*x+2*c)/d-3/64*b^2*sinh(4*d*x+4*c)/d+1/1
92*b^2*sinh(6*d*x+6*c)/d-3/2*a*b*cosh(d*x+c)/d+1/6*a*b/d*cosh(3*d*x+3*c)
```

Maxima [A]
time = 0.27, size = 151, normalized size = 1.32

$$a^{2}x - \frac{1}{384}\,b^{2}\left(\frac{\left(9\,e^{(-2\,dx-2\,c)} - 45\,e^{(-4\,dx-4\,c)} - 1\right)e^{(6\,dx+6\,c)}}{d} + \frac{120\,(dx+c)}{d} + \frac{45\,e^{(-2\,dx-2\,c)} - 9\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)}}{d}\right) + \frac{1}{12}\,ab\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 1/384*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x
+ 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^
(-6*d*x - 6*c))/d) + 1/12*a*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-
d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [A]
time = 0.42, size = 160, normalized size = 1.40

$$\frac{3\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5} + 16\,ab\cosh\left(dx+c\right)^{3} + 48\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2} + 2\,\left(5\,b^{2}\cosh\left(dx+c\right)^{3} - 9\,b^{2}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3} + 6\,\left(16\,a^{2} - 5\,b^{2}\right)dx - 144\,ab\cosh\left(dx+c\right) + 3\,\left(b^{2}\cosh\left(dx+c\right)^{5} - 6\,b^{2}\cosh\left(dx+c\right)^{3} + 15\,b^{2}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 16*a*b*cosh(d*x + c)^3 + 48*a*b
*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(5*b^2*cosh(d*x + c)^3 - 9*b^2*cosh(d*x
+ c))*sinh(d*x + c)^3 + 6*(16*a^2 - 5*b^2)*d*x - 144*a*b*cosh(d*x + c) + 3*
(b^2*cosh(d*x + c)^5 - 6*b^2*cosh(d*x + c)^3 + 15*b^2*cosh(d*x + c))*sinh(d
*x + c))/d
```

Sympy [A]
time = 0.44, size = 212, normalized size = 1.86

$$\begin{cases} a^{2}x + \frac{2ab\sinh^{2}(c+dx)\cosh(c+dx)}{d} - \frac{4ab\cosh^{3}(c+dx)}{3d} + \frac{5b^{2}x\sinh^{6}(c+dx)}{16} - \frac{15b^{2}x\sinh^{4}(c+dx)\cosh^{2}(c+dx)}{16} + \frac{15b^{2}x\sinh^{2}(c+dx)\cosh^{4}(c+dx)}{16} - \frac{5b^{2}x\cosh^{6}(c+dx)}{16} + \frac{11b^{2}\sinh^{5}(c+dx)\cosh(c+dx)}{16d} - \frac{5b^{2}\sinh^{3}(c+dx)\cosh^{3}(c+dx)}{6d} + \frac{5b^{2}\sinh(c+dx)\cosh^{5}(c+dx)}{16d} & \text{for}\ d \neq 0 \\ x\left(a + b\sinh^{3}(c)\right)^{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*sinh(c + d*x)**2*cosh(c + d*x)/d - 4*a*b*cosh(c +
 d*x)**3/(3*d) + 5*b**2*x*sinh(c + d*x)**6/16 - 15*b**2*x*sinh(c + d*x)**4*
cosh(c + d*x)**2/16 + 15*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b*
*2*x*cosh(c + d*x)**6/16 + 11*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) -
5*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b**2*sinh(c + d*x)*cosh(
c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2, True))
```

Giac [A]
time = 0.42, size = 178, normalized size = 1.56

$$\frac{1}{16}\left(16\,a^{2} - 5\,b^{2}\right)x + \frac{b^{2}e^{(6\,dx+6\,c)}}{384\,d} - \frac{3\,b^{2}e^{(4\,dx+4\,c)}}{128\,d} + \frac{abe^{(3\,dx+3\,c)}}{12\,d} + \frac{15\,b^{2}e^{(2\,dx+2\,c)}}{128\,d} - \frac{3\,abe^{(dx+c)}}{4\,d} - \frac{3\,abe^{(-dx-c)}}{4\,d} - \frac{15\,b^{2}e^{(-2\,dx-2\,c)}}{128\,d} + \frac{abe^{(-3\,dx-3\,c)}}{12\,d} + \frac{3\,b^{2}e^{(-4\,dx-4\,c)}}{128\,d} - \frac{b^{2}e^{(-6\,dx-6\,c)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/16*(16*a^2 - 5*b^2)*x + 1/384*b^2*e^(6*d*x + 6*c)/d - 3/128*b^2*e^(4*d*x
+ 4*c)/d + 1/12*a*b*e^(3*d*x + 3*c)/d + 15/128*b^2*e^(2*d*x + 2*c)/d - 3/4*
a*b*e^(d*x + c)/d - 3/4*a*b*e^(-d*x - c)/d - 15/128*b^2*e^(-2*d*x - 2*c)/d
+ 1/12*a*b*e^(-3*d*x - 3*c)/d + 3/128*b^2*e^(-4*d*x - 4*c)/d - 1/384*b^2*e^
(-6*d*x - 6*c)/d
```

Mupad [B]
time = 0.46, size = 85, normalized size = 0.75

$$\frac{\frac{45\,b^2\,\sinh(2\,c+2\,d\,x)}{4}-\frac{9\,b^2\,\sinh(4\,c+4\,d\,x)}{4}+\frac{b^2\,\sinh(6\,c+6\,d\,x)}{4}-72\,a\,b\cosh(c+d\,x)+8\,a\,b\cosh(3\,c+3\,d\,x)+48\,a^2\,d\,x-15\,b^2\,d\,x}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2,x)
```

```
[Out] ((45*b^2*sinh(2*c + 2*d*x))/4 - (9*b^2*sinh(4*c + 4*d*x))/4 + (b^2*sinh(6*c
 + 6*d*x))/4 - 72*a*b*cosh(c + d*x) + 8*a*b*cosh(3*c + 3*d*x) + 48*a^2*d*x
- 15*b^2*d*x)/(48*d)
```

3.154 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=88

$$-abx-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\cosh(c+dx)}{d}-\frac{2b^2\cosh^3(c+dx)}{3d}+\frac{b^2\cosh^5(c+dx)}{5d}+\frac{ab\cosh(c+dx)\sinh}{d}$$

```
[Out] -a*b*x-a^2*arctanh(cosh(d*x+c))/d+b^2*cosh(d*x+c)/d-2/3*b^2*cosh(d*x+c)^3/d
+1/5*b^2*cosh(d*x+c)^5/d+a*b*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3299, 3855, 2715, 8, 2713}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{ab\sinh(c+dx)\cosh(c+dx)}{d}-abx+\frac{b^2\cosh^5(c+dx)}{5d}-\frac{2b^2\cosh^3(c+dx)}{3d}+\frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] -(a*b*x) - (a^2*ArcTanh[Cosh[c + d*x]])/d + (b^2*Cosh[c + d*x])/d - (2*b^2*
Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^5)/(5*d) + (a*b*Cosh[c + d*x]*S
inh[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
```

```
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx &= i\int\left(-ia^2\operatorname{csch}(c+dx)-2iab\sinh^2(c+dx)-ib^2\sinh^5(c+dx\right.\\ &= a^2\int\operatorname{csch}(c+dx)\,dx+(2ab)\int\sinh^2(c+dx)\,dx+b^2\int\sinh^5(\\ &= -\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{ab\cosh(c+dx)\sinh(c+dx)}{d}-(c\\ &= -abx-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\cosh(c+dx)}{d}-\frac{2b^2\cosh}{3}\end{aligned}$$

Mathematica [**A**]
time = 0.23, size = 96, normalized size = 1.09

$$\frac{150b^2\cosh(c+dx)-25b^2\cosh(3(c+dx))+3b^2\cosh(5(c+dx))+120a\left(-2\left(bc+bdx+a\log\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)\right)-a\log\left(\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)\right)+b\sinh(2(c+dx))\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (150*b^2*Cosh[c + d*x] - 25*b^2*Cosh[3*(c + d*x)] + 3*b^2*Cosh[5*(c + d*x)]
 + 120*a*(-2*(b*c + b*d*x + a*Log[Cosh[(c + d*x)/2]] - a*Log[Sinh[(c + d*x)
/2]]) + b*Sinh[2*(c + d*x)]))/(240*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(84) = 168$.
time = 2.00, size = 171, normalized size = 1.94

method	result
risch	$-abx+\frac{b^2\mathrm{e}^{5dx+5c}}{160d}-\frac{5\,\mathrm{e}^{3dx+3c}b^2}{96d}+\frac{\mathrm{e}^{2dx+2c}ab}{4d}+\frac{5\,\mathrm{e}^{dx+c}b^2}{16d}+\frac{5\,\mathrm{e}^{-dx-c}b^2}{16d}-\frac{\mathrm{e}^{-2dx-2c}ab}{4d}-\frac{5\,\mathrm{e}^{-3dx-3c}b^2}{96d}+\frac{b^2\mathrm{e}^{-5dx-}}{160d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*b*x+1/160*b^2/d*exp(5*d*x+5*c)-5/96/d*exp(3*d*x+3*c)*b^2+1/4/d*exp(2*d*x
+2*c)*a*b+5/16/d*exp(d*x+c)*b^2+5/16/d*exp(-d*x-c)*b^2-1/4/d*exp(-2*d*x-2*c
```

```
)*a*b-5/96/d*exp(-3*d*x-3*c)*b^2+1/160*b^2/d*exp(-5*d*x-5*c)+a^2/d*ln(exp(d
*x+c)-1)-a^2/d*ln(exp(d*x+c)+1)
```

Maxima [A]
time = 0.26, size = 140, normalized size = 1.59

$$-\frac{1}{4}ab\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+\frac{1}{480}b^2\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)+\frac{a^2\log\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b^2*(3*e^(5
*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)
/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^2*log(tanh(1/2*d*x +
 1/2*c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(84) = 168$.
time = 0.46, size = 1052, normalized size = 11.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^2*cosh(d*x + c)^10 + 30*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^
2*sinh(d*x + c)^10 - 25*b^2*cosh(d*x + c)^8 - 480*a*b*d*x*cosh(d*x + c)^5 +
 120*a*b*cosh(d*x + c)^7 + 5*(27*b^2*cosh(d*x + c)^2 - 5*b^2)*sinh(d*x + c)
^8 + 150*b^2*cosh(d*x + c)^6 + 40*(9*b^2*cosh(d*x + c)^3 - 5*b^2*cosh(d*x +
 c) + 3*a*b)*sinh(d*x + c)^7 + 10*(63*b^2*cosh(d*x + c)^4 - 70*b^2*cosh(d*x
 + c)^2 + 84*a*b*cosh(d*x + c) + 15*b^2)*sinh(d*x + c)^6 + 150*b^2*cosh(d*x
 + c)^4 + 4*(189*b^2*cosh(d*x + c)^5 - 350*b^2*cosh(d*x + c)^3 - 120*a*b*d*
x + 630*a*b*cosh(d*x + c)^2 + 225*b^2*cosh(d*x + c))*sinh(d*x + c)^5 - 120*
a*b*cosh(d*x + c)^3 + 10*(63*b^2*cosh(d*x + c)^6 - 175*b^2*cosh(d*x + c)^4
 - 240*a*b*d*x*cosh(d*x + c) + 420*a*b*cosh(d*x + c)^3 + 225*b^2*cosh(d*x +
c)^2 + 15*b^2)*sinh(d*x + c)^4 - 25*b^2*cosh(d*x + c)^2 + 40*(9*b^2*cosh(d*
x + c)^7 - 35*b^2*cosh(d*x + c)^5 - 120*a*b*d*x*cosh(d*x + c)^2 + 105*a*b*c
osh(d*x + c)^4 + 75*b^2*cosh(d*x + c)^3 + 15*b^2*cosh(d*x + c) - 3*a*b)*sin
h(d*x + c)^3 + 5*(27*b^2*cosh(d*x + c)^8 - 140*b^2*cosh(d*x + c)^6 - 960*a*
b*d*x*cosh(d*x + c)^3 + 504*a*b*cosh(d*x + c)^5 + 450*b^2*cosh(d*x + c)^4 +
 180*b^2*cosh(d*x + c)^2 - 72*a*b*cosh(d*x + c) - 5*b^2)*sinh(d*x + c)^2 +
3*b^2 - 480*(a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^4*sinh(d*x + c) + 10
*a^2*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^2*cosh(d*x + c)^2*sinh(d*x + c)
^3 + 5*a^2*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*sinh(d*x + c)^5)*log(cosh(d*
x + c) + sinh(d*x + c) + 1) + 480*(a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c
)^4*sinh(d*x + c) + 10*a^2*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^2*cosh(d*
```

```
x + c)^2*sinh(d*x + c)^3 + 5*a^2*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*sinh(d
*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 10*(3*b^2*cosh(d*x + c)
^9 - 20*b^2*cosh(d*x + c)^7 - 240*a*b*d*x*cosh(d*x + c)^4 + 84*a*b*cosh(d*x
 + c)^6 + 90*b^2*cosh(d*x + c)^5 + 60*b^2*cosh(d*x + c)^3 - 36*a*b*cosh(d*x
 + c)^2 - 5*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh
(d*x + c)^4*sinh(d*x + c) + 10*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*d*cos
h(d*x + c)^2*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d
*x + c)^5)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**A**]
time = 0.43, size = 154, normalized size = 1.75

$$-\frac{480\,(dx+c)ab-3\,b^2e^{(5\,dx+5\,c)}+25\,b^2e^{(3\,dx+3\,c)}-120\,abe^{(2\,dx+2\,c)}-150\,b^2e^{(dx+c)}+480\,a^2\log\left(e^{(dx+c)}+1\right)-480\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right)-\left(150\,b^2e^{(4\,dx+4\,c)}-120\,abe^{(3\,dx+3\,c)}-25\,b^2e^{(2\,dx+2\,c)}+3\,b^2\right)e^{(-5\,dx-5\,c)}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/480*(480*(d*x + c)*a*b - 3*b^2*e^(5*d*x + 5*c) + 25*b^2*e^(3*d*x + 3*c)
- 120*a*b*e^(2*d*x + 2*c) - 150*b^2*e^(d*x + c) + 480*a^2*log(e^(d*x + c) +
 1) - 480*a^2*log(abs(e^(d*x + c) - 1)) - (150*b^2*e^(4*d*x + 4*c) - 120*a*
b*e^(3*d*x + 3*c) - 25*b^2*e^(2*d*x + 2*c) + 3*b^2)*e^(-5*d*x - 5*c))/d
```

Mupad [**B**]
time = 0.21, size = 177, normalized size = 2.01

$$\frac{5\,b^2\,\mathrm{e}^{c+d\,x}}{16\,d}-\frac{2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}-a\,b\,x+\frac{5\,b^2\,\mathrm{e}^{-c-d\,x}}{16\,d}-\frac{5\,b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{96\,d}-\frac{5\,b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{96\,d}+\frac{b^2\,\mathrm{e}^{-5\,c-5\,d\,x}}{160\,d}+\frac{b^2\,\mathrm{e}^{5\,c+5\,d\,x}}{160\,d}-\frac{a\,b\,\mathrm{e}^{-2\,c-2\,d\,x}}{4\,d}+\frac{a\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x),x)
```

```
[Out] (5*b^2*exp(c + d*x))/(16*d) - (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d
*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^2)^(1/2) - a*b*x + (5*b^2*exp(- c - d*x))/(
16*d) - (5*b^2*exp(- 3*c - 3*d*x))/(96*d) - (5*b^2*exp(3*c + 3*d*x))/(96*d)
 + (b^2*exp(- 5*c - 5*d*x))/(160*d) + (b^2*exp(5*c + 5*d*x))/(160*d) - (a*b
*exp(- 2*c - 2*d*x))/(4*d) + (a*b*exp(2*c + 2*d*x))/(4*d)
```

3.155 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx$

Optimal. Leaf size=82

$$\frac{3b^2x}{8}+\frac{2ab\cosh(c+dx)}{d}-\frac{a^2\coth(c+dx)}{d}-\frac{3b^2\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^2\cosh(c+dx)\sinh^3(c+dx)}{4d}$$

```
[Out] 3/8*b^2*x+2*a*b*cosh(d*x+c)/d-a^2*coth(d*x+c)/d-3/8*b^2*cosh(d*x+c)*sinh(d*
x+c)/d+1/4*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3299, 3852, 8, 2718, 2715}

$$-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b^2\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3b^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (3*b^2*x)/8 + (2*a*b*Cosh[c + d*x])/d - (a^2*Coth[c + d*x])/d - (3*b^2*Cosh
[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx &= -\int \left(-a^2\operatorname{csch}^2(c+dx)-2ab\sinh(c+dx)-b^2\sinh^4(c+dx)\right.\\ &= a^2\int \operatorname{csch}^2(c+dx)\,dx+(2ab)\int \sinh(c+dx)\,dx+b^2\int \sinh^4\\ &= \frac{2ab\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{4}\left(3b^2\right)\int\\ &= \frac{2ab\cosh(c+dx)}{d}-\frac{a^2\coth(c+dx)}{d}-\frac{3b^2\cosh(c+dx)\sinh(c}{8d}\\ &= \frac{3b^2x}{8}+\frac{2ab\cosh(c+dx)}{d}-\frac{a^2\coth(c+dx)}{d}-\frac{3b^2\cosh(c+d}{8}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 92, normalized size = 1.12

$$\frac{3b^2(c+dx)}{8d}+\frac{2ab\cosh(c)\cosh(dx)}{d}-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\sinh(c)\sinh(dx)}{d}-\frac{b^2\sinh(2(c+dx))}{4d}+\frac{b^2\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (3*b^2*(c + d*x))/(8*d) + (2*a*b*Cosh[c]*Cosh[d*x])/d - (a^2*Coth[c + d*x])
/d + (2*a*b*Sinh[c]*Sinh[d*x])/d - (b^2*Sinh[2*(c + d*x)])/(4*d) + (b^2*Sin
h[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 1.99, size = 124, normalized size = 1.51

method	result	size
risch	$\frac{3b^2x}{8}+\frac{e^{4dx+4c}b^2}{64d}-\frac{e^{2dx+2c}b^2}{8d}+\frac{ab\,e^{dx+c}}{d}+\frac{ab\,e^{-dx-c}}{d}+\frac{e^{-2dx-2c}b^2}{8d}-\frac{e^{-4dx-4c}b^2}{64d}-\frac{2a^2}{d\left(e^{2dx+2c}-1\right)}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*b^2*x+1/64/d*exp(4*d*x+4*c)*b^2-1/8/d*exp(2*d*x+2*c)*b^2+a*b/d*exp(d*x+
c)+a*b/d*exp(-d*x-c)+1/8/d*exp(-2*d*x-2*c)*b^2-1/64/d*exp(-4*d*x-4*c)*b^2-2
*a^2/d/(exp(2*d*x+2*c)-1)
```

Maxima [A]
time = 0.28, size = 113, normalized size = 1.38

$$\frac{1}{64}\,b^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+ab\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{2\,a^2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + a*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a^2/(
d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [A]
time = 0.44, size = 142, normalized size = 1.73

$$\frac{b^2\cosh\left(dx+c\right)^5+5\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-9\,b^2\cosh\left(dx+c\right)^3+\left(10\,b^2\cosh\left(dx+c\right)^3-27\,b^2\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2-8\,\left(8\,a^2-b^2\right)\cosh\left(dx+c\right)+8\,\left(3\,b^2dx+16\,ab\cosh\left(dx+c\right)+8\,a^2\right)\sinh\left(dx+c\right)}{64\,d\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(b^2*cosh(d*x + c)^5 + 5*b^2*cosh(d*x + c)*sinh(d*x + c)^4 - 9*b^2*cos
h(d*x + c)^3 + (10*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c))*sinh(d*x + c
)^2 - 8*(8*a^2 - b^2)*cosh(d*x + c) + 8*(3*b^2*d*x + 16*a*b*cosh(d*x + c) +
 8*a^2)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.46, size = 149, normalized size = 1.82

$$\frac{24\,(dx+c)b^2+b^2e^{(4\,dx+4\,c)}-8\,b^2e^{(2\,dx+2\,c)}+64\,abe^{(dx+c)}+\frac{\left(64\,abe^{(5\,dx+5\,c)}-64\,abe^{(3\,dx+3\,c)}-9\,b^2e^{(2\,dx+2\,c)}+b^2-8\,\left(16\,a^2-b^2\right)e^{(4\,dx+4\,c)}\right)e^{(-4\,dx-4\,c)}}{\left(e^{(dx+c)}+1\right)\left(e^{(dx+c)}-1\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/64*(24*(d*x + c)*b^2 + b^2*e^(4*d*x + 4*c) - 8*b^2*e^(2*d*x + 2*c) + 64*a
*b*e^(d*x + c) + (64*a*b*e^(5*d*x + 5*c) - 64*a*b*e^(3*d*x + 3*c) - 9*b^2*e
^(2*d*x + 2*c) + b^2 - 8*(16*a^2 - b^2)*e^(4*d*x + 4*c))*e^(-4*d*x - 4*c)/(
(e^(d*x + c) + 1)*(e^(d*x + c) - 1)))/d
```

Mupad [B]
time = 0.74, size = 123, normalized size = 1.50

$$\frac{3\,b^2\,x}{8}-\frac{2\,a^2}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}+\frac{b^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}-\frac{b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}-\frac{b^2\,{\mathrm{e}}^{-4\,c-4\,d\,x}}{64\,d}+\frac{b^2\,{\mathrm{e}}^{4\,c+4\,d\,x}}{64\,d}+\frac{a\,b\,{\mathrm{e}}^{c+d\,x}}{d}+\frac{a\,b\,{\mathrm{e}}^{-c-d\,x}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^2,x)
```

```
[Out] (3*b^2*x)/8 - (2*a^2)/(d*(exp(2*c + 2*d*x) - 1)) + (b^2*exp(- 2*c - 2*d*x))
/(8*d) - (b^2*exp(2*c + 2*d*x))/(8*d) - (b^2*exp(- 4*c - 4*d*x))/(64*d) + (
b^2*exp(4*c + 4*d*x))/(64*d) + (a*b*exp(c + d*x))/d + (a*b*exp(- c - d*x))/
d
```

3.156 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=77

$$2abx+\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

```
[Out] 2*a*b*x+1/2*a^2*arctanh(cosh(d*x+c))/d-b^2*cosh(d*x+c)/d+1/3*b^2*cosh(d*x+c
)^3/d-1/2*a^2*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3299, 3853, 3855, 2713}

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+2abx+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] 2*a*b*x + (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^2*Cosh[c + d*x])/d + (b^2
*Cosh[c + d*x]^3)/(3*d) - (a^2*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx &= -\left(i\int\left(2iab+ia^2\operatorname{csch}^3(c+dx)+ib^2\sinh^3(c+dx)\right)\,dx\right)\\ &= 2abx+a^2\int\operatorname{csch}^3(c+dx)\,dx+b^2\int\sinh^3(c+dx)\,dx\\ &= 2abx-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{1}{2}a^2\int\operatorname{csch}(c+dx)\,dx\\ &= 2abx+\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\cosh^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 105, normalized size = 1.36

$$2abx-\frac{3b^2\cosh(c+dx)}{4d}+\frac{b^2\cosh(3(c+dx))}{12d}-\frac{a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] 2*a*b*x - (3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cosh[3*(c + d*x)])/(12*d) - (a
^2*Csch[(c + d*x)/2]^2)/(8*d) - (a^2*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^2*S
ech[(c + d*x)/2]^2)/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.
time = 2.16, size = 144, normalized size = 1.87

method	result
risch	$2abx+\frac{\mathrm{e}^{3dx+3c}b^2}{24d}-\frac{3\,\mathrm{e}^{dx+c}b^2}{8d}-\frac{3\,\mathrm{e}^{-dx-c}b^2}{8d}+\frac{\mathrm{e}^{-3dx-3c}b^2}{24d}-\frac{a^2\mathrm{e}^{dx+c}\left(1+\mathrm{e}^{2dx+2c}\right)}{d\left(\mathrm{e}^{2dx+2c}-1\right)^2}+\frac{a^2\ln\left(\mathrm{e}^{dx+c}+1\right)}{2d}-\frac{a^2\ln(\mathrm{e}^{dx+c}-}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b*x+1/24/d*exp(3*d*x+3*c)*b^2-3/8/d*exp(d*x+c)*b^2-3/8/d*exp(-d*x-c)*b^
2+1/24/d*exp(-3*d*x-3*c)*b^2-a^2*exp(d*x+c)*(1+exp(2*d*x+2*c))/d/(exp(2*d*x
+2*c)-1)^2+1/2*a^2/d*ln(exp(d*x+c)+1)-1/2*a^2/d*ln(exp(d*x+c)-1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(71) = 142$.
time = 0.27, size = 152, normalized size = 1.97

$$2\,abx + \frac{1}{24}\,b^2\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right) + \frac{1}{2}\,a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\,\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*x + 1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d
+ e^(-3*d*x - 3*c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c)
 - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-
4*d*x - 4*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1616 vs. $2(71) = 142$.
time = 0.48, size = 1616, normalized size = 20.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(b^2*cosh(d*x + c)^10 + 10*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + b^2*sin
h(d*x + c)^10 + 48*a*b*d*x*cosh(d*x + c)^7 - 11*b^2*cosh(d*x + c)^8 - 96*a*
b*d*x*cosh(d*x + c)^5 + (45*b^2*cosh(d*x + c)^2 - 11*b^2)*sinh(d*x + c)^8 +
 8*(15*b^2*cosh(d*x + c)^3 + 6*a*b*d*x - 11*b^2*cosh(d*x + c))*sinh(d*x + c
)^7 + 48*a*b*d*x*cosh(d*x + c)^3 - 2*(12*a^2 - 5*b^2)*cosh(d*x + c)^6 + 2*(
105*b^2*cosh(d*x + c)^4 + 168*a*b*d*x*cosh(d*x + c) - 154*b^2*cosh(d*x + c)
^2 - 12*a^2 + 5*b^2)*sinh(d*x + c)^6 + 4*(63*b^2*cosh(d*x + c)^5 + 252*a*b*
d*x*cosh(d*x + c)^2 - 154*b^2*cosh(d*x + c)^3 - 24*a*b*d*x - 3*(12*a^2 - 5*
b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(12*a^2 - 5*b^2)*cosh(d*x + c)^4 +
2*(105*b^2*cosh(d*x + c)^6 + 840*a*b*d*x*cosh(d*x + c)^3 - 385*b^2*cosh(d*x
 + c)^4 - 240*a*b*d*x*cosh(d*x + c) - 15*(12*a^2 - 5*b^2)*cosh(d*x + c)^2 -
 12*a^2 + 5*b^2)*sinh(d*x + c)^4 - 11*b^2*cosh(d*x + c)^2 + 8*(15*b^2*cosh(
d*x + c)^7 + 210*a*b*d*x*cosh(d*x + c)^4 - 77*b^2*cosh(d*x + c)^5 - 120*a*b
*d*x*cosh(d*x + c)^2 + 6*a*b*d*x - 5*(12*a^2 - 5*b^2)*cosh(d*x + c)^3 - (12
*a^2 - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (45*b^2*cosh(d*x + c)^8 + 10
08*a*b*d*x*cosh(d*x + c)^5 - 308*b^2*cosh(d*x + c)^6 - 960*a*b*d*x*cosh(d*x
 + c)^3 + 144*a*b*d*x*cosh(d*x + c) - 30*(12*a^2 - 5*b^2)*cosh(d*x + c)^4 -
 12*(12*a^2 - 5*b^2)*cosh(d*x + c)^2 - 11*b^2)*sinh(d*x + c)^2 + b^2 + 12*(
a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)*sinh(d*x + c)^6 + a^2*sinh(d*x +
c)^7 - 2*a^2*cosh(d*x + c)^5 + (21*a^2*cosh(d*x + c)^2 - 2*a^2)*sinh(d*x +
c)^5 + a^2*cosh(d*x + c)^3 + 5*(7*a^2*cosh(d*x + c)^3 - 2*a^2*cosh(d*x + c)
)*sinh(d*x + c)^4 + (35*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(d*x + c)^2 + a^2)
```

```
*sinh(d*x + c)^3 + (21*a^2*cosh(d*x + c)^5 - 20*a^2*cosh(d*x + c)^3 + 3*a^2
*cosh(d*x + c))*sinh(d*x + c)^2 + (7*a^2*cosh(d*x + c)^6 - 10*a^2*cosh(d*x
+ c)^4 + 3*a^2*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
 + c) + 1) - 12*(a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)*sinh(d*x + c)^6
+ a^2*sinh(d*x + c)^7 - 2*a^2*cosh(d*x + c)^5 + (21*a^2*cosh(d*x + c)^2 - 2
*a^2)*sinh(d*x + c)^5 + a^2*cosh(d*x + c)^3 + 5*(7*a^2*cosh(d*x + c)^3 - 2*
a^2*cosh(d*x + c))*sinh(d*x + c)^4 + (35*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(
d*x + c)^2 + a^2)*sinh(d*x + c)^3 + (21*a^2*cosh(d*x + c)^5 - 20*a^2*cosh(d
*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^2 + (7*a^2*cosh(d*x + c)^6 -
 10*a^2*cosh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*
x + c) + sinh(d*x + c) - 1) + 2*(5*b^2*cosh(d*x + c)^9 + 168*a*b*d*x*cosh(d
*x + c)^6 - 44*b^2*cosh(d*x + c)^7 - 240*a*b*d*x*cosh(d*x + c)^4 + 72*a*b*d
*x*cosh(d*x + c)^2 - 6*(12*a^2 - 5*b^2)*cosh(d*x + c)^5 - 4*(12*a^2 - 5*b^2
)*cosh(d*x + c)^3 - 11*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^7
 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 - 2*d*cosh(d*x + c
)^5 + (21*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3
 - 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x
+ c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5
- 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*d*cosh(d*x
 + c)^6 - 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(71) = 142.
time = 0.44, size = 162, normalized size = 2.10

$$\frac{48\,(dx+c)ab+b^2e^{(3\,dx+3\,c)}-9\,b^2e^{(dx+c)}+12\,a^2\log\left(e^{(dx+c)}+1\right)-12\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{\left(11\,b^2e^{(2\,dx+2\,c)}-b^2+3\,(8\,a^2+3\,b^2)e^{(6\,dx+6\,c)}+(24\,a^2-19\,b^2)e^{(4\,dx+4\,c)}\right)e^{(-3\,dx-3\,c)}}{\left(e^{(dx+c)}+1\right)^2\left(e^{(dx+c)}-1\right)^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/24*(48*(d*x + c)*a*b + b^2*e^(3*d*x + 3*c) - 9*b^2*e^(d*x + c) + 12*a^2*l
og(e^(d*x + c) + 1) - 12*a^2*log(abs(e^(d*x + c) - 1)) - (11*b^2*e^(2*d*x +
 2*c) - b^2 + 3*(8*a^2 + 3*b^2)*e^(6*d*x + 6*c) + (24*a^2 - 19*b^2)*e^(4*d*
x + 4*c))*e^(-3*d*x - 3*c)/((e^(d*x + c) + 1)^2*(e^(d*x + c) - 1)^2))/d
```

Mupad [B]
time = 0.71, size = 175, normalized size = 2.27

$$\frac{\operatorname{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}-\frac{3\,b^2\,\mathrm{e}^{c+d\,x}}{8\,d}+2\,a\,b\,x-\frac{3\,b^2\,\mathrm{e}^{-c-d\,x}}{8\,d}+\frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}+\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{a^2\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^2\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^3,x)
```

```
[Out] (atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^
2)^(1/2) - (3*b^2*exp(c + d*x))/(8*d) + 2*a*b*x - (3*b^2*exp(- c - d*x))/(8
*d) + (b^2*exp(- 3*c - 3*d*x))/(24*d) + (b^2*exp(3*c + 3*d*x))/(24*d) - (a^
2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^2*exp(c + d*x))/(d*(exp(4
*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.157 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=76

$$-\frac{b^2x}{2}-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}+\frac{a^2\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

```
[Out] -1/2*b^2*x-2*a*b*arctanh(cosh(d*x+c))/d+a^2*coth(d*x+c)/d-1/3*a^2*coth(d*x+
c)^3/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]

time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3299, 3855, 3852, 2715, 8}

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] -1/2*(b^2*x) - (2*a*b*ArcTanh[Cosh[c + d*x]])/d + (a^2*Coth[c + d*x])/d - (
a^2*Coth[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx &= \int \left(2ab\operatorname{csch}(c+dx)+a^2\operatorname{csch}^4(c+dx)+b^2\sinh^2(c+dx)\right)\,dx\\
&= a^2\int \operatorname{csch}^4(c+dx)\,dx+(2ab)\int \operatorname{csch}(c+dx)\,dx+b^2\int \sinh^2(\\
&= -\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}-\\
&= -\frac{b^2x}{2}-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}+\frac{a^2\coth(c+dx)}{d}-a^2\coth
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 81, normalized size = 1.07

$$\frac{-4a^2\coth(c+dx)\left(-2+\operatorname{csch}^2(c+dx)\right)+3b\left(-2\left(bc+bdx+4a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-4a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\sinh(2(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-4*a^2*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + 3*b*(-2*(b*c + b*d*x + 4*a*L
og[Cosh[(c + d*x)/2]] - 4*a*Log[Sinh[(c + d*x)/2]]) + b*Sinh[2*(c + d*x)]))
/(12*d)
```

Maple [A]
time = 2.25, size = 108, normalized size = 1.42

method	result	size
risch	$-\frac{b^2x}{2}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}b^2}{8d}-\frac{4a^2\left(3\,e^{2dx+2c}-1\right)}{3d\left(e^{2dx+2c}-1\right)^3}+\frac{2ab\ln\left(e^{dx+c}-1\right)}{d}-\frac{2ab\ln\left(e^{dx+c}+1\right)}{d}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2*x+1/8/d*exp(2*d*x+2*c)*b^2-1/8/d*exp(-2*d*x-2*c)*b^2-4/3*a^2*(3*ex
p(2*d*x+2*c)-1)/d/(exp(2*d*x+2*c)-1)^3+2*a*b/d*ln(exp(d*x+c)-1)-2*a*b/d*ln(
exp(d*x+c)+1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(70) = 140$.
time = 0.29, size = 170, normalized size = 2.24

$$-\frac{1}{8}\,b^{2}\left(4\,x-\frac{e^{\left(2\,dx+2\,c\right)}}{d}+\frac{e^{\left(-2\,dx-2\,c\right)}}{d}\right)-2\,ab\left(\frac{\log\left(e^{\left(-dx-c\right)}+1\right)}{d}-\frac{\log\left(e^{\left(-dx-c\right)}-1\right)}{d}\right)+\frac{4}{3}\,a^{2}\left(\frac{3\,e^{\left(-2\,dx-2\,c\right)}}{d\left(3\,e^{\left(-2\,dx-2\,c\right)}-3\,e^{\left(-4\,dx-4\,c\right)}+e^{\left(-6\,dx-6\,c\right)}-1\right)}-\frac{1}{d\left(3\,e^{\left(-2\,dx-2\,c\right)}-3\,e^{\left(-4\,dx-4\,c\right)}+e^{\left(-6\,dx-6\,c\right)}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 2*a*b*(log(e^(-d*
x - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*
e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1748 vs. $2(70) = 140$.
time = 0.48, size = 1748, normalized size = 23.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^2*cosh(d*x + c)^10 + 30*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^2
*sinh(d*x + c)^10 - 3*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^8 - 3*(4*b^2*d*x -
45*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(d*x + c)^8 + 24*(15*b^2*cosh(d*x + c)^
3 - (4*b^2*d*x + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 6*(6*b^2*d*x + b^2
)*cosh(d*x + c)^6 + 6*(105*b^2*cosh(d*x + c)^4 + 6*b^2*d*x - 14*(4*b^2*d*x
+ 3*b^2)*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^6 + 12*(63*b^2*cosh(d*x + c)^
5 - 14*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^3 + 3*(6*b^2*d*x + b^2)*cosh(d*x +
 c))*sinh(d*x + c)^5 - 6*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c)^4 + 6*(10
5*b^2*cosh(d*x + c)^6 - 35*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^4 - 6*b^2*d*x
+ 15*(6*b^2*d*x + b^2)*cosh(d*x + c)^2 - 16*a^2 + b^2)*sinh(d*x + c)^4 + 24
*(15*b^2*cosh(d*x + c)^7 - 7*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^5 + 5*(6*b^2
*d*x + b^2)*cosh(d*x + c)^3 - (6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c))*sin
h(d*x + c)^3 + (12*b^2*d*x + 32*a^2 - 9*b^2)*cosh(d*x + c)^2 + (135*b^2*cos
h(d*x + c)^8 - 84*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^6 + 90*(6*b^2*d*x + b^2
)*cosh(d*x + c)^4 + 12*b^2*d*x - 36*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c
)^2 + 32*a^2 - 9*b^2)*sinh(d*x + c)^2 + 3*b^2 - 48*(a*b*cosh(d*x + c)^8 + 8
*a*b*cosh(d*x + c)*sinh(d*x + c)^7 + a*b*sinh(d*x + c)^8 - 3*a*b*cosh(d*x +
 c)^6 + (28*a*b*cosh(d*x + c)^2 - 3*a*b)*sinh(d*x + c)^6 + 3*a*b*cosh(d*x +
 c)^4 + 2*(28*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))*sinh(d*x + c)^5 +
(70*a*b*cosh(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2 + 3*a*b)*sinh(d*x + c)^4 -
 a*b*cosh(d*x + c)^2 + 4*(14*a*b*cosh(d*x + c)^5 - 15*a*b*cosh(d*x + c)^3 +
 3*a*b*cosh(d*x + c))*sinh(d*x + c)^3 + (28*a*b*cosh(d*x + c)^6 - 45*a*b*co
```

```
sh(d*x + c)^4 + 18*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^2 + 2*(4*a*b*co
sh(d*x + c)^7 - 9*a*b*cosh(d*x + c)^5 + 6*a*b*cosh(d*x + c)^3 - a*b*cosh(d*
x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 48*(a*b*cos
h(d*x + c)^8 + 8*a*b*cosh(d*x + c)*sinh(d*x + c)^7 + a*b*sinh(d*x + c)^8 -
3*a*b*cosh(d*x + c)^6 + (28*a*b*cosh(d*x + c)^2 - 3*a*b)*sinh(d*x + c)^6 +
3*a*b*cosh(d*x + c)^4 + 2*(28*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))*si
nh(d*x + c)^5 + (70*a*b*cosh(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2 + 3*a*b)*s
inh(d*x + c)^4 - a*b*cosh(d*x + c)^2 + 4*(14*a*b*cosh(d*x + c)^5 - 15*a*b*c
osh(d*x + c)^3 + 3*a*b*cosh(d*x + c))*sinh(d*x + c)^3 + (28*a*b*cosh(d*x +
c)^6 - 45*a*b*cosh(d*x + c)^4 + 18*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)
^2 + 2*(4*a*b*cosh(d*x + c)^7 - 9*a*b*cosh(d*x + c)^5 + 6*a*b*cosh(d*x + c)
^3 - a*b*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) -
1) + 2*(15*b^2*cosh(d*x + c)^9 - 12*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^7 + 1
8*(6*b^2*d*x + b^2)*cosh(d*x + c)^5 - 12*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*
x + c)^3 + (12*b^2*d*x + 32*a^2 - 9*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*c
osh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 3*
d*cosh(d*x + c)^6 + (28*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 + 2*(28*d*
cosh(d*x + c)^3 - 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 3*d*cosh(d*x + c)^4
+ (70*d*cosh(d*x + c)^4 - 45*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 4*(
14*d*cosh(d*x + c)^5 - 15*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
 c)^3 - d*cosh(d*x + c)^2 + (28*d*cosh(d*x + c)^6 - 45*d*cosh(d*x + c)^4 +
18*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 2*(4*d*cosh(d*x + c)^7 - 9*d*co
sh(d*x + c)^5 + 6*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(70) = 140.
time = 0.44, size = 151, normalized size = 1.99

$$-\frac{12\,(dx+c)b^2-3\,b^2e^{(2\,dx+2\,c)}+48\,ab\log\left(e^{(dx+c)}+1\right)-48\,ab\log\left(\left|e^{(dx+c)}-1\right|\right)+\frac{\left(3\,b^2e^{(6\,dx+6\,c)}-3\,b^2+3\,(32\,a^2-3\,b^2)e^{(4\,dx+4\,c)}-(32\,a^2-9\,b^2)e^{(2\,dx+2\,c)}\right)e^{(-2\,dx-2\,c)}}{\left(e^{(dx+c)}+1\right)^3\left(e^{(dx+c)}-1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(d*x + c)*b^2 - 3*b^2*e^(2*d*x + 2*c) + 48*a*b*log(e^(d*x + c) +
1) - 48*a*b*log(abs(e^(d*x + c) - 1)) + (3*b^2*e^(6*d*x + 6*c) - 3*b^2 + 3*
```

```
(32*a^2 - 3*b^2)*e^(4*d*x + 4*c) - (32*a^2 - 9*b^2)*e^(2*d*x + 2*c))*e^(-2*
d*x - 2*c)/((e^(d*x + c) + 1)^3*(e^(d*x + c) - 1)^3))/d
```

Mupad [B]
time = 0.13, size = 163, normalized size = 2.14

$$\frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}-\frac{4\,a^2}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{4\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}-\frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}-\frac{b^2\,x}{2}-\frac{8\,a^2}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^4,x)
```

```
[Out] (b^2*exp(2*c + 2*d*x))/(8*d) - (4*a^2)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2
*d*x) + 1)) - (4*atan((a*b*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)
))*(a^2*b^2)^(1/2))/(-d^2)^(1/2) - (b^2*exp(- 2*c - 2*d*x))/(8*d) - (b^2*x)
/2 - (8*a^2)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*
x) - 1))
```

3.158 $\int \textbf{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=90

$$-\frac{3a^2\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{b^2\cosh(c+dx)}{d}-\frac{2ab\coth(c+dx)}{d}+\frac{3a^2\coth(c+dx)\text{csch}(c+dx)}{8d}-a^2\coth(c+$$

```
[Out] -3/8*a^2*arctanh(cosh(d*x+c))/d+b^2*cosh(d*x+c)/d-2*a*b*coth(d*x+c)/d+3/8*a
^2*coth(d*x+c)*csch(d*x+c)/d-1/4*a^2*coth(d*x+c)*csch(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3299, 3852, 8, 3853, 3855, 2718}

$$-\frac{3a^2\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^2\coth(c+dx)\text{csch}^3(c+dx)}{4d}+\frac{3a^2\coth(c+dx)\text{csch}(c+dx)}{8d}-\frac{2ab\coth(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-3*a^2*ArcTanh[Cosh[c + d*x]])/(8*d) + (b^2*Cosh[c + d*x])/d - (2*a*b*Coth
[c + d*x])/d + (3*a^2*Coth[c + d*x]*Csch[c + d*x])/(8*d) - (a^2*Coth[c + d*
x]*Csch[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx &= i\int \left(-2iab\operatorname{csch}^2(c+dx)-ia^2\operatorname{csch}^5(c+dx)-ib^2\sinh(c+dx\\ &= a^2\int \operatorname{csch}^5(c+dx)\,dx+(2ab)\int \operatorname{csch}^2(c+dx)\,dx+b^2\int \sinh\\ &= \frac{b^2\cosh(c+dx)}{d}-\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}-\frac{1}{4}\left(3a^2\right)\int \\ &= \frac{b^2\cosh(c+dx)}{d}-\frac{2ab\coth(c+dx)}{d}+\frac{3a^2\coth(c+dx)\operatorname{csch}(c}{8d}\\ &= -\frac{3a^2\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{b^2\cosh(c+dx)}{d}-\frac{2ab\coth(c+}{d}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 149, normalized size = 1.66

$$\frac{b^2\cosh(c)\cosh(dx)}{d}-\frac{2ab\coth(c+dx)}{d}+\frac{3a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{b^2\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (b^2*Cosh[c]*Cosh[d*x])/d - (2*a*b*Coth[c + d*x])/d + (3*a^2*Csch[(c + d*x)
/2]^2)/(32*d) - (a^2*Csch[(c + d*x)/2]^4)/(64*d) + (3*a^2*Log[Tanh[(c + d*x
)/2]])/(8*d) + (3*a^2*Sech[(c + d*x)/2]^2)/(32*d) + (a^2*Sech[(c + d*x)/2]^
4)/(64*d) + (b^2*Sinh[c]*Sinh[d*x])/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(84) = 168$.
time = 2.23, size = 171, normalized size = 1.90

method	result
risch	$\frac{e^{dx+c}b^2}{2d} + \frac{e^{-dx-c}b^2}{2d} + \frac{a\left(3a\,e^{7dx+7c}-16b\,e^{6dx+6c}-11a\,e^{5dx+5c}+48b\,e^{4dx+4c}-11a\,e^{3dx+3c}-48b\,e^{2dx+2c}+3a\,e^{dx+c}+16b\right)}{4d\left(e^{2dx+2c}-1\right)^4} - 3a^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*exp(d*x+c)*b^2+1/2/d*exp(-d*x-c)*b^2+1/4*a*(3*a*exp(7*d*x+7*c)-16*b*e
xp(6*d*x+6*c)-11*a*exp(5*d*x+5*c)+48*b*exp(4*d*x+4*c)-11*a*exp(3*d*x+3*c)-4
8*b*exp(2*d*x+2*c)+3*a*exp(d*x+c)+16*b)/d/(exp(2*d*x+2*c)-1)^4-3/8*a^2/d*ln
(exp(d*x+c)+1)+3/8*a^2/d*ln(exp(d*x+c)-1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(84) = 168$.
time = 0.27, size = 188, normalized size = 2.09

$$\frac{1}{2}\,b^2\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right) - \frac{1}{8}\,a^2\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\,\left(3\,e^{(-dx-c)} - 11\,e^{(-3\,dx-3\,c)} - 11\,e^{(-5\,dx-5\,c)} + 3\,e^{(-7\,dx-7\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)} - 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} - e^{(-8\,dx-8\,c)} - 1)}\right) + \frac{4\,ab}{d(e^{(-2\,dx-2\,c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) - 1/8*a^2*(3*log(e^(-d*x - c) + 1)
/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) -
11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 4*a*b/(d*(e^(-2
*d*x - 2*c) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2119 vs. $2(84) = 168$.
time = 0.59, size = 2119, normalized size = 23.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^2*cosh(d*x + c)^10 + 40*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 4*b^2*
sinh(d*x + c)^10 - 32*a*b*cosh(d*x + c)^7 + 6*(a^2 - 2*b^2)*cosh(d*x + c)^8
 + 6*(30*b^2*cosh(d*x + c)^2 + a^2 - 2*b^2)*sinh(d*x + c)^8 + 16*(30*b^2*co
sh(d*x + c)^3 - 2*a*b + 3*(a^2 - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 96
*a*b*cosh(d*x + c)^5 - 2*(11*a^2 - 4*b^2)*cosh(d*x + c)^6 + 2*(420*b^2*cosh
(d*x + c)^4 - 112*a*b*cosh(d*x + c) + 84*(a^2 - 2*b^2)*cosh(d*x + c)^2 - 11
*a^2 + 4*b^2)*sinh(d*x + c)^6 + 12*(84*b^2*cosh(d*x + c)^5 - 56*a*b*cosh(d*
x + c)^2 + 28*(a^2 - 2*b^2)*cosh(d*x + c)^3 + 8*a*b - (11*a^2 - 4*b^2)*cosh
```

```
(d*x + c))*sinh(d*x + c)^5 - 96*a*b*cosh(d*x + c)^3 - 2*(11*a^2 - 4*b^2)*co
sh(d*x + c)^4 + 2*(420*b^2*cosh(d*x + c)^6 - 560*a*b*cosh(d*x + c)^3 + 210*
(a^2 - 2*b^2)*cosh(d*x + c)^4 + 240*a*b*cosh(d*x + c) - 15*(11*a^2 - 4*b^2)
*cosh(d*x + c)^2 - 11*a^2 + 4*b^2)*sinh(d*x + c)^4 + 8*(60*b^2*cosh(d*x + c
)^7 - 140*a*b*cosh(d*x + c)^4 + 42*(a^2 - 2*b^2)*cosh(d*x + c)^5 + 120*a*b*
cosh(d*x + c)^2 - 5*(11*a^2 - 4*b^2)*cosh(d*x + c)^3 - 12*a*b - (11*a^2 - 4
*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 32*a*b*cosh(d*x + c) + 6*(a^2 - 2*b^
2)*cosh(d*x + c)^2 + 6*(30*b^2*cosh(d*x + c)^8 - 112*a*b*cosh(d*x + c)^5 +
28*(a^2 - 2*b^2)*cosh(d*x + c)^6 + 160*a*b*cosh(d*x + c)^3 - 5*(11*a^2 - 4*
b^2)*cosh(d*x + c)^4 - 48*a*b*cosh(d*x + c) - 2*(11*a^2 - 4*b^2)*cosh(d*x +
 c)^2 + a^2 - 2*b^2)*sinh(d*x + c)^2 + 4*b^2 - 3*(a^2*cosh(d*x + c)^9 + 9*a
^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 - 4*a^2*cosh(d*x + c
)^7 + 4*(9*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^7 + 6*a^2*cosh(d*x + c)
^5 + 28*(3*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(21
*a^2*cosh(d*x + c)^4 - 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^5 - 4*a^
2*cosh(d*x + c)^3 + 2*(63*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x + c)^3 + 15
*a^2*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*a^2*cosh(d*x + c)^6 - 35*a^2*co
sh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^3 + a^2*cosh(d*
x + c) + 12*(3*a^2*cosh(d*x + c)^7 - 7*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x
 + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c)^2 + (9*a^2*cosh(d*x + c)^8 - 28*
a^2*cosh(d*x + c)^6 + 30*a^2*cosh(d*x + c)^4 - 12*a^2*cosh(d*x + c)^2 + a^2
)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*(a^2*cosh(d*x +
 c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 - 4*a^2*c
osh(d*x + c)^7 + 4*(9*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^7 + 6*a^2*co
sh(d*x + c)^5 + 28*(3*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c
)^6 + 6*(21*a^2*cosh(d*x + c)^4 - 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x +
c)^5 - 4*a^2*cosh(d*x + c)^3 + 2*(63*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x
+ c)^3 + 15*a^2*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*a^2*cosh(d*x + c)^6
- 35*a^2*cosh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^3 +
a^2*cosh(d*x + c) + 12*(3*a^2*cosh(d*x + c)^7 - 7*a^2*cosh(d*x + c)^5 + 5*a
^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c)^2 + (9*a^2*cosh(d*x +
 c)^8 - 28*a^2*cosh(d*x + c)^6 + 30*a^2*cosh(d*x + c)^4 - 12*a^2*cosh(d*x +
 c)^2 + a^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(10*
b^2*cosh(d*x + c)^9 - 56*a*b*cosh(d*x + c)^6 + 12*(a^2 - 2*b^2)*cosh(d*x +
c)^7 + 120*a*b*cosh(d*x + c)^4 - 3*(11*a^2 - 4*b^2)*cosh(d*x + c)^5 - 72*a*
b*cosh(d*x + c)^2 - 2*(11*a^2 - 4*b^2)*cosh(d*x + c)^3 + 8*a*b + 3*(a^2 - 2
*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*
sinh(d*x + c)^8 + d*sinh(d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 4*(9*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*s
inh(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cosh(d*x + c)^4 - 14*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x + c)^5 - 70*d*cosh(d*x +
 c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 - 4*d*cosh(d*x + c)^3 + 4*(21*d
*cosh(d*x + c)^6 - 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*
x + c)^3 + 12*(3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c
)^3 - d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (9*d*cosh(d*x +
```

```
c)^8 - 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 +
 d)*sinh(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(84) = 168.
time = 0.46, size = 172, normalized size = 1.91

$$\frac{4\,b^2 e^{(dx+c)} + 4\,b^2 e^{(-dx-c)} - 3\,a^2 \log\left(e^{(dx+c)} + 1\right) + 3\,a^2 \log\left(\left|e^{(dx+c)} - 1\right|\right) + \frac{2\,\left(3\,a^2 e^{(7\,dx+7\,c)} - 16\,abe^{(6\,dx+6\,c)} - 11\,a^2 e^{(5\,dx+5\,c)} + 48\,abe^{(4\,dx+4\,c)} - 11\,a^2 e^{(3\,dx+3\,c)} - 48\,abe^{(2\,dx+2\,c)} + 3\,a^2 e^{(dx+c)} + 16\,ab\right)}{\left(e^{(2\,dx+2\,c)} - 1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*e^(d*x + c) + 4*b^2*e^(-d*x - c) - 3*a^2*log(e^(d*x + c) + 1) +
3*a^2*log(abs(e^(d*x + c) - 1)) + 2*(3*a^2*e^(7*d*x + 7*c) - 16*a*b*e^(6*d*
x + 6*c) - 11*a^2*e^(5*d*x + 5*c) + 48*a*b*e^(4*d*x + 4*c) - 11*a^2*e^(3*d*
x + 3*c) - 48*a*b*e^(2*d*x + 2*c) + 3*a^2*e^(d*x + c) + 16*a*b)/(e^(2*d*x +
 2*c) - 1)^4)/d
```

Mupad [B]
time = 0.14, size = 355, normalized size = 3.94

$$\frac{\frac{3\,a^2\,\mathrm{e}^{c+d\,x}}{4\,d}-\frac{2\,a\,b}{d}}{\mathrm{e}^{2\,c+2\,d\,x}-1}-\frac{\frac{4\,a^2\,\mathrm{e}^{3\,c+3\,d\,x}}{d}-\frac{a\,b}{d}+\frac{3\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{d}-\frac{3\,a\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{d}+\frac{a\,b\,\mathrm{e}^{6\,c+6\,d\,x}}{d}}{6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1}-\frac{\frac{2\,a^2\,\mathrm{e}^{c+d\,x}}{d}+\frac{a\,b}{d}-\frac{2\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{d}+\frac{a\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}+\frac{b^2\,\mathrm{e}^{c+d\,x}}{2\,d}-\frac{3\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{4\,\sqrt{-d^2}}+\frac{b^2\,\mathrm{e}^{-c-d\,x}}{2\,d}-\frac{a^2\,\mathrm{e}^{c+d\,x}}{2\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^5,x)
```

```
[Out] ((3*a^2*exp(c + d*x))/(4*d) - (2*a*b)/d)/(exp(2*c + 2*d*x) - 1) - ((4*a^2*e
xp(3*c + 3*d*x))/d - (a*b)/d + (3*a*b*exp(2*c + 2*d*x))/d - (3*a*b*exp(4*c
+ 4*d*x))/d + (a*b*exp(6*c + 6*d*x))/d)/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2
*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*a^2*exp(c + d*x))/
d + (a*b)/d - (2*a*b*exp(2*c + 2*d*x))/d + (a*b*exp(4*c + 4*d*x))/d)/(3*exp
(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + (b^2*exp(c + d
*x))/(2*d) - (3*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/2)))*(a
^4)^(1/2))/(4*(-d^2)^(1/2)) + (b^2*exp(- c - d*x))/(2*d) - (a^2*exp(c + d*x
))/(2*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.159 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=88

$$b^2x+\frac{ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{a^2\coth(c+dx)}{d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a^2\coth^5(c+dx)}{5d}-\frac{ab\coth(c+dx)\operatorname{csch}}{d}$$

```
[Out] b^2*x+a*b*arctanh(cosh(d*x+c))/d-a^2*coth(d*x+c)/d+2/3*a^2*coth(d*x+c)^3/d-
1/5*a^2*coth(d*x+c)^5/d-a*b*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3299, 3853, 3855, 3852}

$$-\frac{a^2\coth^5(c+dx)}{5d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{ab\coth(c+dx)\operatorname{csch}(c+dx)}{d}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] b^2*x + (a*b*ArcTanh[Cosh[c + d*x]])/d - (a^2*Coth[c + d*x])/d + (2*a^2*Cot
h[c + d*x]^3)/(3*d) - (a^2*Coth[c + d*x]^5)/(5*d) - (a*b*Coth[c + d*x]*Csch
[c + d*x])/d
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx &= -\int\left(-b^2-2ab\operatorname{csch}^3(c+dx)-a^2\operatorname{csch}^6(c+dx)\right)\,dx\\ &= b^2x+a^2\int\operatorname{csch}^6(c+dx)\,dx+(2ab)\int\operatorname{csch}^3(c+dx)\,dx\\ &= b^2x-\frac{ab\coth(c+dx)\operatorname{csch}(c+dx)}{d}-(ab)\int\operatorname{csch}(c+dx)\,dx-\\ &= b^2x+\frac{ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{a^2\coth(c+dx)}{d}+\frac{2a^2\coth^3(}{3d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(88) = 176.
time = 0.67, size = 197, normalized size = 2.24

$$\frac{-256a^2\coth\left(\frac{1}{2}(c+dx)\right)-240ab\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+19a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)\sinh(c+dx)-3a^2\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)\sinh(c+dx)+16\left(60b^2c+60b^2dx-60ab\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-15ab\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)-19a^2\operatorname{csch}^3(c+dx)\sinh^4\left(\frac{1}{2}(c+dx)\right)-12a^2\operatorname{csch}^5(c+dx)\sinh^6\left(\frac{1}{2}(c+dx)\right)-16a^2\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-256*a^2*Coth[(c + d*x)/2] - 240*a*b*Csch[(c + d*x)/2]^2 + 19*a^2*Csch[(c
+ d*x)/2]^4*Sinh[c + d*x] - 3*a^2*Csch[(c + d*x)/2]^6*Sinh[c + d*x] + 16*(6
0*b^2*c + 60*b^2*d*x - 60*a*b*Log[Tanh[(c + d*x)/2]] - 15*a*b*Sech[(c + d*x
)/2]^2 - 19*a^2*Csch[c + d*x]^3*Sinh[(c + d*x)/2]^4 - 12*a^2*Csch[c + d*x]^
5*Sinh[(c + d*x)/2]^6 - 16*a^2*Tanh[(c + d*x)/2]))/(960*d)
```

Maple [A]
time = 2.28, size = 130, normalized size = 1.48

method	result
risch	$b^2x-\frac{2a\left(15b\,\mathrm{e}^{9dx+9c}-30b\,\mathrm{e}^{7dx+7c}+80a\,\mathrm{e}^{4dx+4c}+30b\,\mathrm{e}^{3dx+3c}-40a\,\mathrm{e}^{2dx+2c}-15b\,\mathrm{e}^{dx+c}+8a\right)}{15d\left(\mathrm{e}^{2dx+2c}-1\right)^5}+\frac{ab\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}-\frac{ab\ln\left(\mathrm{e}^{dx+c}-\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x-2/15*a*(15*b*exp(9*d*x+9*c)-30*b*exp(7*d*x+7*c)+80*a*exp(4*d*x+4*c)+3
0*b*exp(3*d*x+3*c)-40*a*exp(2*d*x+2*c)-15*b*exp(d*x+c)+8*a)/d/(exp(2*d*x+2*
c)-1)^5+a*b/d*ln(exp(d*x+c)+1)-a*b/d*ln(exp(d*x+c)-1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(84) = 168$.
time = 0.28, size = 303, normalized size = 3.44

$$b^2x + ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)}\right) - \frac{16}{15}\,a^2\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)} - 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} - 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} - 1)} - \frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)} - 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} - 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} - 1)} - \frac{1}{d(5\,e^{(-2\,dx-2\,c)} - 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} - 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} - 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] b^2*x + a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x
 - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
- 16/15*a^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c
) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1
0*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*
d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*
d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2310 vs. $2(84) = 168$.
time = 0.47, size = 2310, normalized size = 26.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(15*b^2*d*x*cosh(d*x + c)^10 + 15*b^2*d*x*sinh(d*x + c)^10 - 75*b^2*d*
x*cosh(d*x + c)^8 - 30*a*b*cosh(d*x + c)^9 + 150*b^2*d*x*cosh(d*x + c)^6 +
30*(5*b^2*d*x*cosh(d*x + c) - a*b)*sinh(d*x + c)^9 + 60*a*b*cosh(d*x + c)^7
 + 15*(45*b^2*d*x*cosh(d*x + c)^2 - 5*b^2*d*x - 18*a*b*cosh(d*x + c))*sinh(
d*x + c)^8 + 60*(30*b^2*d*x*cosh(d*x + c)^3 - 10*b^2*d*x*cosh(d*x + c) - 18
*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^7 + 30*(105*b^2*d*x*cosh(d*x + c)
^4 - 70*b^2*d*x*cosh(d*x + c)^2 - 84*a*b*cosh(d*x + c)^3 + 5*b^2*d*x + 14*a
*b*cosh(d*x + c))*sinh(d*x + c)^6 + 60*(63*b^2*d*x*cosh(d*x + c)^5 - 70*b^2
*d*x*cosh(d*x + c)^3 - 63*a*b*cosh(d*x + c)^4 + 15*b^2*d*x*cosh(d*x + c) +
21*a*b*cosh(d*x + c)^2)*sinh(d*x + c)^5 - 60*a*b*cosh(d*x + c)^3 - 10*(15*b
^2*d*x + 16*a^2)*cosh(d*x + c)^4 + 10*(315*b^2*d*x*cosh(d*x + c)^6 - 525*b^
2*d*x*cosh(d*x + c)^4 - 378*a*b*cosh(d*x + c)^5 + 225*b^2*d*x*cosh(d*x + c)
^2 + 210*a*b*cosh(d*x + c)^3 - 15*b^2*d*x - 16*a^2)*sinh(d*x + c)^4 - 15*b^
2*d*x + 20*(90*b^2*d*x*cosh(d*x + c)^7 - 210*b^2*d*x*cosh(d*x + c)^5 - 126*
a*b*cosh(d*x + c)^6 + 150*b^2*d*x*cosh(d*x + c)^3 + 105*a*b*cosh(d*x + c)^4
 - 3*a*b - 2*(15*b^2*d*x + 16*a^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*a*b*
cosh(d*x + c) + 5*(15*b^2*d*x + 16*a^2)*cosh(d*x + c)^2 + 5*(135*b^2*d*x*co
sh(d*x + c)^8 - 420*b^2*d*x*cosh(d*x + c)^6 - 216*a*b*cosh(d*x + c)^7 + 450
```

```
*b^2*d*x*cosh(d*x + c)^4 + 252*a*b*cosh(d*x + c)^5 + 15*b^2*d*x - 36*a*b*co
sh(d*x + c) - 12*(15*b^2*d*x + 16*a^2)*cosh(d*x + c)^2 + 16*a^2)*sinh(d*x +
 c)^2 - 16*a^2 + 15*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x + c)*sinh(d*x +
 c)^9 + a*b*sinh(d*x + c)^10 - 5*a*b*cosh(d*x + c)^8 + 5*(9*a*b*cosh(d*x +
c)^2 - a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(3*a*b*cosh(d*x +
 c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cosh(d*x + c)^4 - 1
4*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^6 - 10*a*b*cosh(d*x + c)^4 + 4*(
63*a*b*cosh(d*x + c)^5 - 70*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sin
h(d*x + c)^5 + 10*(21*a*b*cosh(d*x + c)^6 - 35*a*b*cosh(d*x + c)^4 + 15*a*b
*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b
*cosh(d*x + c)^7 - 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 - a*b*cosh
(d*x + c))*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 - 28*a*b*cosh(d*x + c
)^6 + 30*a*b*cosh(d*x + c)^4 - 12*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^
2 - a*b + 10*(a*b*cosh(d*x + c)^9 - 4*a*b*cosh(d*x + c)^7 + 6*a*b*cosh(d*x
+ c)^5 - 4*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c))*log(cosh
(d*x + c) + sinh(d*x + c) + 1) - 15*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x
 + c)*sinh(d*x + c)^9 + a*b*sinh(d*x + c)^10 - 5*a*b*cosh(d*x + c)^8 + 5*(9
*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(
3*a*b*cosh(d*x + c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cos
h(d*x + c)^4 - 14*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^6 - 10*a*b*cosh(
d*x + c)^4 + 4*(63*a*b*cosh(d*x + c)^5 - 70*a*b*cosh(d*x + c)^3 + 15*a*b*co
sh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a*b*cosh(d*x + c)^6 - 35*a*b*cosh(d*x
 + c)^4 + 15*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x +
c)^2 + 40*(3*a*b*cosh(d*x + c)^7 - 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x +
 c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 - 28*
a*b*cosh(d*x + c)^6 + 30*a*b*cosh(d*x + c)^4 - 12*a*b*cosh(d*x + c)^2 + a*b
)*sinh(d*x + c)^2 - a*b + 10*(a*b*cosh(d*x + c)^9 - 4*a*b*cosh(d*x + c)^7 +
 6*a*b*cosh(d*x + c)^5 - 4*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*
x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 10*(15*b^2*d*x*cosh(d*x +
c)^9 - 60*b^2*d*x*cosh(d*x + c)^7 - 27*a*b*cosh(d*x + c)^8 + 90*b^2*d*x*cos
h(d*x + c)^5 + 42*a*b*cosh(d*x + c)^6 - 18*a*b*cosh(d*x + c)^2 - 4*(15*b^2*
d*x + 16*a^2)*cosh(d*x + c)^3 + 3*a*b + (15*b^2*d*x + 16*a^2)*cosh(d*x + c)
)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 +
 d*sinh(d*x + c)^10 - 5*d*cosh(d*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 - d)*sin
h(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^7 +
 10*d*cosh(d*x + c)^6 + 10*(21*d*cosh(d*x + c)^4 - 14*d*cosh(d*x + c)^2 + d
)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*c
osh(d*x + c))*sinh(d*x + c)^5 - 10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x +
c)^6 - 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4
0*(3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh
(d*x + c))*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 -
 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + d)*si
nh(d*x + c)^2 + 10*(d*cosh(d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x
+ c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 141, normalized size = 1.60

$$\frac{15\,(dx+c)b^2+15\,ab\log\left(e^{(dx+c)}+1\right)-15\,ab\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{2\,\left(15\,abe^{(9\,dx+9\,c)}-30\,abe^{(7\,dx+7\,c)}+80\,a^2e^{(4\,dx+4\,c)}+30\,abe^{(3\,dx+3\,c)}-40\,a^2e^{(2\,dx+2\,c)}-15\,abe^{(dx+c)}+8\,a^2\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)*b^2 + 15*a*b*log(e^(d*x + c) + 1) - 15*a*b*log(abs(e^(d*
x + c) - 1)) - 2*(15*a*b*e^(9*d*x + 9*c) - 30*a*b*e^(7*d*x + 7*c) + 80*a^2*
e^(4*d*x + 4*c) + 30*a*b*e^(3*d*x + 3*c) - 40*a^2*e^(2*d*x + 2*c) - 15*a*b*
e^(d*x + c) + 8*a^2)/(e^(2*d*x + 2*c) - 1)^5)/d
```

Mupad [**B**]
time = 0.65, size = 351, normalized size = 3.99

$$b^2\,x-\frac{\frac{32\,a^2\,\mathrm{e}^{4\,c+4\,d\,x}}{5\,d}-\frac{8\,a\,b\,\mathrm{e}^{c+d\,x}}{5\,d}+\frac{24\,a\,b\,\mathrm{e}^{3\,c+3\,d\,x}}{5\,d}-\frac{24\,a\,b\,\mathrm{e}^{5\,c+5\,d\,x}}{5\,d}+\frac{8\,a\,b\,\mathrm{e}^{7\,c+7\,d\,x}}{5\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1}+\frac{2\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}-\frac{64\,a^2}{15\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{16\,a^2}{5\,d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}-\frac{2\,a\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{12\,a\,b\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^6,x)
```

```
[Out] b^2*x - ((32*a^2*exp(4*c + 4*d*x))/(5*d) - (8*a*b*exp(c + d*x))/(5*d) + (24
*a*b*exp(3*c + 3*d*x))/(5*d) - (24*a*b*exp(5*c + 5*d*x))/(5*d) + (8*a*b*exp
(7*c + 7*d*x))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*
c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) + (2*atan((a*b*ex
p(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(-d^2)^(1
/2) - (64*a^2)/(15*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6
*d*x) - 1)) - (16*a^2)/(5*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*ex
p(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (2*a*b*exp(c + d*x))/(d*(exp(2*c
+ 2*d*x) - 1)) - (12*a*b*exp(c + d*x))/(5*d*(exp(4*c + 4*d*x) - 2*exp(2*c +
 2*d*x) + 1))
```

3.160 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=133

$$\frac{5a^2\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{2ab\coth(c+dx)}{d}-\frac{2ab\coth^3(c+dx)}{3d}-\frac{5a^2\coth(c+dx}{16}$$

```
[Out] 5/16*a^2*arctanh(cosh(d*x+c))/d-b^2*arctanh(cosh(d*x+c))/d+2*a*b*coth(d*x+c
)/d-2/3*a*b*coth(d*x+c)^3/d-5/16*a^2*coth(d*x+c)*csch(d*x+c)/d+5/24*a^2*cot
h(d*x+c)*csch(d*x+c)^3/d-1/6*a^2*coth(d*x+c)*csch(d*x+c)^5/d
```

Rubi [A]
time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3299, 3855, 3852, 3853}

$$\frac{5a^2\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}-\frac{5a^2\coth(c+dx)\operatorname{csch}(c+dx)}{16d}-\frac{2ab\coth^3(c+dx)}{3d}+\frac{2ab\coth(c+dx)}{d}-\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (5*a^2*ArcTanh[Cosh[c + d*x]])/(16*d) - (b^2*ArcTanh[Cosh[c + d*x]])/d + (2
*a*b*Coth[c + d*x])/d - (2*a*b*Coth[c + d*x]^3)/(3*d) - (5*a^2*Coth[c + d*x
]*Csch[c + d*x])/(16*d) + (5*a^2*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a
^2*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^7(c+dx)\,(a+b\sinh^3(c+dx))^2\,dx &= -(i\int (ib^2\operatorname{csch}(c+dx)+2iab\operatorname{csch}^4(c+dx)+ia^2\operatorname{csch}^7(c+d\\ &= a^2\int \operatorname{csch}^7(c+dx)\,dx+(2ab)\int \operatorname{csch}^4(c+dx)\,dx+b^2\int \operatorname{csch}\\ &= -\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}-\\ &= -\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{2ab\coth(c+dx)}{d}-\frac{2ab\coth^3(c}{3d}\\ &= -\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{2ab\coth(c+dx)}{d}-\frac{2ab\coth^3(c}{3d}\\ &= \frac{5a^2\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}+2ab\,\mathrm{co}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 235, normalized size = 1.77

$$\frac{4ab\coth(c+dx)}{3d}-\frac{5a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a^2\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{2ab\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{b^2\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b^2\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{5a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{5a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a^2\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (4*a*b*Coth[c + d*x])/(3*d) - (5*a^2*Csch[(c + d*x)/2]^2)/(64*d) + (a^2*Csc
h[(c + d*x)/2]^4)/(64*d) - (a^2*Csch[(c + d*x)/2]^6)/(384*d) - (2*a*b*Coth[
c + d*x]*Csch[c + d*x]^2)/(3*d) - (b^2*Log[Cosh[c/2 + (d*x)/2]])/d + (b^2*L
og[Sinh[c/2 + (d*x)/2]])/d - (5*a^2*Log[Tanh[(c + d*x)/2]])/(16*d) - (5*a^2
*Sech[(c + d*x)/2]^2)/(64*d) - (a^2*Sech[(c + d*x)/2]^4)/(64*d) - (a^2*Sech
[(c + d*x)/2]^6)/(384*d)
```

Maple [A]
time = 2.27, size = 209, normalized size = 1.57

method	result
risch	$-\frac{a(15a\,\mathrm{e}^{11dx+11c}-85a\,\mathrm{e}^{9dx+9c}+192b\,\mathrm{e}^{8dx+8c}+198a\,\mathrm{e}^{7dx+7c}-640b\,\mathrm{e}^{6dx+6c}+198a\,\mathrm{e}^{5dx+5c}+768b\,\mathrm{e}^{4dx+4c}-85a\,\mathrm{e}^{3dx+3c}-384b\,\mathrm{e}^{2dx}}{24d(\mathrm{e}^{2dx+2c}-1)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*a*(15*a*exp(11*d*x+11*c)-85*a*exp(9*d*x+9*c)+192*b*exp(8*d*x+8*c)+198
*a*exp(7*d*x+7*c)-640*b*exp(6*d*x+6*c)+198*a*exp(5*d*x+5*c)+768*b*exp(4*d*x
+4*c)-85*a*exp(3*d*x+3*c)-384*b*exp(2*d*x+2*c)+15*a*exp(d*x+c)+64*b)/d/(exp
(2*d*x+2*c)-1)^6-5/16*a^2/d*ln(exp(d*x+c)-1)+1/d*ln(exp(d*x+c)-1)*b^2+5/16*
a^2/d*ln(exp(d*x+c)+1)-1/d*ln(exp(d*x+c)+1)*b^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(123) = 246$.
time = 0.28, size = 316, normalized size = 2.38

$$\frac{1}{48}\,a^2\left(\frac{15\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15\,e^{(-dx-c)}-85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}+198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9\,dx-9\,c)}+15\,e^{(-11\,dx-11\,c)}\right)}{d(6\,e^{(-2\,dx-2\,c)}-15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}-15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}-e^{(-12\,dx-12\,c)}-1)}\right)-b^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)+\frac{8}{3}\,ab\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/48*a^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e
^(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x -
7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c)
- 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-1
0*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) - b^2*(log(e^(-d*x - c) + 1)/d -
log(e^(-d*x - c) - 1)/d) + 8/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*
c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c)
 - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3607 vs. $2(123) = 246$.
time = 0.47, size = 3607, normalized size = 27.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(30*a^2*cosh(d*x + c)^11 + 330*a^2*cosh(d*x + c)*sinh(d*x + c)^10 + 3
0*a^2*sinh(d*x + c)^11 - 170*a^2*cosh(d*x + c)^9 + 384*a*b*cosh(d*x + c)^8
+ 10*(165*a^2*cosh(d*x + c)^2 - 17*a^2)*sinh(d*x + c)^9 + 396*a^2*cosh(d*x
+ c)^7 + 6*(825*a^2*cosh(d*x + c)^3 - 255*a^2*cosh(d*x + c) + 64*a*b)*sinh(
d*x + c)^8 - 1280*a*b*cosh(d*x + c)^6 + 12*(825*a^2*cosh(d*x + c)^4 - 510*a
^2*cosh(d*x + c)^2 + 256*a*b*cosh(d*x + c) + 33*a^2)*sinh(d*x + c)^7 + 396*
a^2*cosh(d*x + c)^5 + 4*(3465*a^2*cosh(d*x + c)^5 - 3570*a^2*cosh(d*x + c)^
```

```
3 + 2688*a*b*cosh(d*x + c)^2 + 693*a^2*cosh(d*x + c) - 320*a*b)*sinh(d*x +
c)^6 + 1536*a*b*cosh(d*x + c)^4 + 12*(1155*a^2*cosh(d*x + c)^6 - 1785*a^2*c
osh(d*x + c)^4 + 1792*a*b*cosh(d*x + c)^3 + 693*a^2*cosh(d*x + c)^2 - 640*a
*b*cosh(d*x + c) + 33*a^2)*sinh(d*x + c)^5 - 170*a^2*cosh(d*x + c)^3 + 12*(
825*a^2*cosh(d*x + c)^7 - 1785*a^2*cosh(d*x + c)^5 + 2240*a*b*cosh(d*x + c)
^4 + 1155*a^2*cosh(d*x + c)^3 - 1600*a*b*cosh(d*x + c)^2 + 165*a^2*cosh(d*x
 + c) + 128*a*b)*sinh(d*x + c)^4 - 768*a*b*cosh(d*x + c)^2 + 2*(2475*a^2*co
sh(d*x + c)^8 - 7140*a^2*cosh(d*x + c)^6 + 10752*a*b*cosh(d*x + c)^5 + 6930
*a^2*cosh(d*x + c)^4 - 12800*a*b*cosh(d*x + c)^3 + 1980*a^2*cosh(d*x + c)^2
 + 3072*a*b*cosh(d*x + c) - 85*a^2)*sinh(d*x + c)^3 + 30*a^2*cosh(d*x + c)
+ 6*(275*a^2*cosh(d*x + c)^9 - 1020*a^2*cosh(d*x + c)^7 + 1792*a*b*cosh(d*x
 + c)^6 + 1386*a^2*cosh(d*x + c)^5 - 3200*a*b*cosh(d*x + c)^4 + 660*a^2*cos
h(d*x + c)^3 + 1536*a*b*cosh(d*x + c)^2 - 85*a^2*cosh(d*x + c) - 128*a*b)*s
inh(d*x + c)^2 + 128*a*b - 3*((5*a^2 - 16*b^2)*cosh(d*x + c)^12 + 12*(5*a^2
 - 16*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a^2 - 16*b^2)*sinh(d*x + c)^
12 - 6*(5*a^2 - 16*b^2)*cosh(d*x + c)^10 + 6*(11*(5*a^2 - 16*b^2)*cosh(d*x
+ c)^2 - 5*a^2 + 16*b^2)*sinh(d*x + c)^10 + 20*(11*(5*a^2 - 16*b^2)*cosh(d*
x + c)^3 - 3*(5*a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(5*a^2 -
16*b^2)*cosh(d*x + c)^8 + 15*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 - 18*(5*a
^2 - 16*b^2)*cosh(d*x + c)^2 + 5*a^2 - 16*b^2)*sinh(d*x + c)^8 + 24*(33*(5*
a^2 - 16*b^2)*cosh(d*x + c)^5 - 30*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 + 5*(5*
a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(5*a^2 - 16*b^2)*cosh(d*x
 + c)^6 + 4*(231*(5*a^2 - 16*b^2)*cosh(d*x + c)^6 - 315*(5*a^2 - 16*b^2)*co
sh(d*x + c)^4 + 105*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 - 25*a^2 + 80*b^2)*sin
h(d*x + c)^6 + 24*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 - 63*(5*a^2 - 16*b^2
)*cosh(d*x + c)^5 + 35*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 - 5*(5*a^2 - 16*b^2
)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 + 15
*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^8 - 84*(5*a^2 - 16*b^2)*cosh(d*x + c)^6
 + 70*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 - 20*(5*a^2 - 16*b^2)*cosh(d*x + c)^
2 + 5*a^2 - 16*b^2)*sinh(d*x + c)^4 + 20*(11*(5*a^2 - 16*b^2)*cosh(d*x + c)
^9 - 36*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 + 42*(5*a^2 - 16*b^2)*cosh(d*x + c
)^5 - 20*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 + 3*(5*a^2 - 16*b^2)*cosh(d*x + c
))*sinh(d*x + c)^3 - 6*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 + 6*(11*(5*a^2 - 16
*b^2)*cosh(d*x + c)^10 - 45*(5*a^2 - 16*b^2)*cosh(d*x + c)^8 + 70*(5*a^2 -
16*b^2)*cosh(d*x + c)^6 - 50*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 + 15*(5*a^2 -
 16*b^2)*cosh(d*x + c)^2 - 5*a^2 + 16*b^2)*sinh(d*x + c)^2 + 5*a^2 - 16*b^2
 + 12*((5*a^2 - 16*b^2)*cosh(d*x + c)^11 - 5*(5*a^2 - 16*b^2)*cosh(d*x + c)
^9 + 10*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 - 10*(5*a^2 - 16*b^2)*cosh(d*x + c
)^5 + 5*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 - (5*a^2 - 16*b^2)*cosh(d*x + c))*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*((5*a^2 - 16*b^2)
*cosh(d*x + c)^12 + 12*(5*a^2 - 16*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (5
*a^2 - 16*b^2)*sinh(d*x + c)^12 - 6*(5*a^2 - 16*b^2)*cosh(d*x + c)^10 + 6*(
11*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 - 5*a^2 + 16*b^2)*sinh(d*x + c)^10 + 20
*(11*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 - 3*(5*a^2 - 16*b^2)*cosh(d*x + c))*s
inh(d*x + c)^9 + 15*(5*a^2 - 16*b^2)*cosh(d*x + c)^8 + 15*(33*(5*a^2 - 16*b
```

```
^2)*cosh(d*x + c)^4 - 18*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 + 5*a^2 - 16*b^2)
*sinh(d*x + c)^8 + 24*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^5 - 30*(5*a^2 - 16
*b^2)*cosh(d*x + c)^3 + 5*(5*a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 -
 20*(5*a^2 - 16*b^2)*cosh(d*x + c)^6 + 4*(231*(5*a^2 - 16*b^2)*cosh(d*x + c
)^6 - 315*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 + 105*(5*a^2 - 16*b^2)*cosh(d*x
+ c)^2 - 25*a^2 + 80*b^2)*sinh(d*x + c)^6 + 24*(33*(5*a^2 - 16*b^2)*cosh(d*
x + c)^7 - 63*(5*a^2 - 16*b^2)*cosh(d*x + c)^5 + 35*(5*a^2 - 16*b^2)*cosh(d
*x + c)^3 - 5*(5*a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(5*a^2 -
 16*b^2)*cosh(d*x + c)^4 + 15*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^8 - 84*(5*
a^2 - 16*b^2)*cosh(d*x + c)^6 + 70*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 - 20*(5
*a^2 - 16*b^2)*cosh(d*x + c)^2 + 5*a^2 - 16*b^2)*sinh(d*x + c)^4 + 20*(11*(
5*a^2 - 16*b^2)*cosh(d*x + c)^9 - 36*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 + 42*
(5*a^2 - 16*b^2)*cosh(d*x + c)^5 - 20*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 + 3*
(5*a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(5*a^2 - 16*b^2)*cosh(d
*x + c)^2 + 6*(11*(5*a^2 - 16*b^2)*cosh(d*x + c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.50, size = 204, normalized size = 1.53

$$\frac{3\,(5\,a^2-16\,b^2)\log\left(e^{(dx+c)}+1\right)-3\,(5\,a^2-16\,b^2)\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{2\,\left(15\,a^2e^{(11\,dx+11\,c)}-85\,a^2e^{(9\,dx+9\,c)}+192\,abe^{(8\,dx+8\,c)}+198\,a^2e^{(7\,dx+7\,c)}-640\,abe^{(6\,dx+6\,c)}+198\,a^2e^{(5\,dx+5\,c)}+768\,abe^{(4\,dx+4\,c)}-85\,a^2e^{(3\,dx+3\,c)}-384\,abe^{(2\,dx+2\,c)}+15\,a^2e^{(dx+c)}+64\,ab\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^6}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^2 - 16*b^2)*log(e^(d*x + c) + 1) - 3*(5*a^2 - 16*b^2)*log(abs(
e^(d*x + c) - 1)) - 2*(15*a^2*e^(11*d*x + 11*c) - 85*a^2*e^(9*d*x + 9*c) +
192*a*b*e^(8*d*x + 8*c) + 198*a^2*e^(7*d*x + 7*c) - 640*a*b*e^(6*d*x + 6*c)
 + 198*a^2*e^(5*d*x + 5*c) + 768*a*b*e^(4*d*x + 4*c) - 85*a^2*e^(3*d*x + 3*
c) - 384*a*b*e^(2*d*x + 2*c) + 15*a^2*e^(d*x + c) + 64*a*b)/(e^(2*d*x + 2*c
) - 1)^6)/d
```

Mupad [**B**]
time = 0.16, size = 434, normalized size = 3.26

$$\frac{\frac{5\,a^2\,\mathrm{e}^{c+d\,x}}{12\,d}-\frac{8\,a\,b}{d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1}-\frac{\frac{a^2\,\mathrm{e}^{c+d\,x}}{3\,d}+\frac{16\,a\,b}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\left(5\,a^2\,\sqrt{-d^2}-16\,b^2\,\sqrt{-d^2}\right)}{d\,\sqrt{25\,a^4-160\,a^2\,b^2+256\,b^4}}\right)\,\sqrt{25\,a^4-160\,a^2\,b^2+256\,b^4}}{8\,\sqrt{-d^2}}-\frac{18\,a^2\,\mathrm{e}^{c+d\,x}}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}-\frac{80\,a^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1\right)}-\frac{32\,a^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(15\,\mathrm{e}^{4\,c+4\,d\,x}-6\,\mathrm{e}^{2\,c+2\,d\,x}-20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}-6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)}-\frac{5\,a^2\,\mathrm{e}^{c+d\,x}}{8\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^7,x)
```

```
[Out] ((5*a^2*exp(c + d*x))/(12*d) - (8*a*b)/d)/(exp(4*c + 4*d*x) - 2*exp(2*c + 2
*d*x) + 1) - ((a^2*exp(c + d*x))/(3*d) + (16*a*b)/(3*d))/(3*exp(2*c + 2*d*x
) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + (atan((exp(d*x)*exp(c)*(5*
a^2*(-d^2)^(1/2) - 16*b^2*(-d^2)^(1/2)))/(d*(25*a^4 + 256*b^4 - 160*a^2*b^2
)^(1/2)))*(25*a^4 + 256*b^4 - 160*a^2*b^2)^(1/2))/(8*(-d^2)^(1/2)) - (18*a^
2*exp(c + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6
*d*x) + exp(8*c + 8*d*x) + 1)) - (80*a^2*exp(c + d*x))/(3*d*(5*exp(2*c + 2*
d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp
(10*c + 10*d*x) - 1)) - (32*a^2*exp(c + d*x))/(3*d*(15*exp(4*c + 4*d*x) - 6
*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c
+ 10*d*x) + exp(12*c + 12*d*x) + 1)) - (5*a^2*exp(c + d*x))/(8*d*(exp(2*c +
 2*d*x) - 1))
```

3.161 $\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=291

$$-\frac{a^3x}{2}+\frac{105}{128}ab^2x+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{5b^3\cosh^3(c+dx)}{3d}+\frac{3a^2b\cosh^5(c}{5d}$$

```
[Out] -1/2*a^3*x+105/128*a*b^2*x+3*a^2*b*cosh(d*x+c)/d-b^3*cosh(d*x+c)/d-2*a^2*b*
cosh(d*x+c)^3/d+5/3*b^3*cosh(d*x+c)^3/d+3/5*a^2*b*cosh(d*x+c)^5/d-2*b^3*cos
h(d*x+c)^5/d+10/7*b^3*cosh(d*x+c)^7/d-5/9*b^3*cosh(d*x+c)^9/d+1/11*b^3*cosh
(d*x+c)^11/d+1/2*a^3*cosh(d*x+c)*sinh(d*x+c)/d-105/128*a*b^2*cosh(d*x+c)*si
nh(d*x+c)/d+35/64*a*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d-7/16*a*b^2*cosh(d*x+c)*
sinh(d*x+c)^5/d+3/8*a*b^2*cosh(d*x+c)*sinh(d*x+c)^7/d
```

Rubi [A]
time = 0.15, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3299, 2715, 8, 2713}

$$\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^3x}{2}+\frac{3a^2b\cosh^5(c+dx)}{5d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{3a^2b\cosh(c+dx)}{d}+\frac{3ab^2\sinh^7(c+dx)\cosh(c+dx)}{8d}-\frac{7ab^2\sinh^5(c+dx)\cosh(c+dx)}{16d}+\frac{35ab^2\sinh^3(c+dx)\cosh(c+dx)}{64d}-\frac{105ab^2\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{105}{128}ab^2x+\frac{b^3\cosh^{11}(c+dx)}{11d}-\frac{5b^3\cosh^9(c+dx)}{9d}+\frac{10b^3\cosh^7(c+dx)}{7d}-\frac{2b^3\cosh^5(c+dx)}{d}+\frac{5b^3\cosh^3(c+dx)}{3d}-\frac{b^3\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] -1/2*(a^3*x) + (105*a*b^2*x)/128 + (3*a^2*b*Cosh[c + d*x])/d - (b^3*Cosh[c
+ d*x])/d - (2*a^2*b*Cosh[c + d*x]^3)/d + (5*b^3*Cosh[c + d*x]^3)/(3*d) + (
3*a^2*b*Cosh[c + d*x]^5)/(5*d) - (2*b^3*Cosh[c + d*x]^5)/d + (10*b^3*Cosh[c
 + d*x]^7)/(7*d) - (5*b^3*Cosh[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]^11)/(
11*d) + (a^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (105*a*b^2*Cosh[c + d*x]*
Sinh[c + d*x])/(128*d) + (35*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(64*d) -
(7*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(16*d) + (3*a*b^2*Cosh[c + d*x]*Sin
h[c + d*x]^7)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx &= -\int \left(-a^3\sinh^2(c+dx)-3a^2b\sinh^5(c+dx)-3ab^2\sinh^8(c+\right.\\
&= a^3\int \sinh^2(c+dx)\,dx+\left(3a^2b\right)\int \sinh^5(c+dx)\,dx+\left(3ab^2\right)\\
&= \frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{3ab^2\cosh(c+dx)\sinh^7(c+d}{8d}\\
&= -\frac{a^3x}{2}+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}-\frac{2a^2b\cosh^3(c}{d}\\
&= -\frac{a^3x}{2}+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}-\frac{2a^2b\cosh^3(c}{d}\\
&= -\frac{a^3x}{2}+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}-\frac{2a^2b\cosh^3(c}{d}\\
&= -\frac{a^3x}{2}+\frac{105}{128}ab^2x+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}-2a
\end{aligned}
$$

Mathematica [A]
time = 0.37, size = 194, normalized size = 0.67

$$\frac{-27720a(64a^2-105b^2)(c+dx)-20790b(-320a^2+77b^2)\cosh(c+dx)+34650b(-32a^2+11b^2)\cosh(3(c+dx))-2079b(-64a^2+55b^2)\cosh(5(c+dx))+27225b^3\cosh(7(c+dx))-4235b^3\cosh(9(c+dx))+315b^3\cosh(11(c+dx))+110880a(8a^2-21b^2)\sinh(2(c+dx))+582120ab^2\sinh(4(c+dx))-110880ab^2\sinh(6(c+dx))+10395ab^2\sinh(8(c+dx))}{3548160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-27720*a*(64*a^2 - 105*b^2)*(c + d*x) - 20790*b*(-320*a^2 + 77*b^2)*Cosh[c
 + d*x] + 34650*b*(-32*a^2 + 11*b^2)*Cosh[3*(c + d*x)] - 2079*b*(-64*a^2 +
55*b^2)*Cosh[5*(c + d*x)] + 27225*b^3*Cosh[7*(c + d*x)] - 4235*b^3*Cosh[9*(
c + d*x)] + 315*b^3*Cosh[11*(c + d*x)] + 110880*a*(8*a^2 - 21*b^2)*Sinh[2*(
c + d*x)] + 582120*a*b^2*Sinh[4*(c + d*x)] - 110880*a*b^2*Sinh[6*(c + d*x)]
 + 10395*a*b^2*Sinh[8*(c + d*x)])/(3548160*d)
```

Maple [A]
time = 1.76, size = 220, normalized size = 0.76

method	result
default	$\frac{\left(-\frac{231}{512}b^3+\frac{15}{8}a^2b\right)\cosh(dx+c)}{d}+\frac{\left(-\frac{165}{1024}b^3+\frac{3}{16}a^2b\right)\cosh(5dx+5c)}{5d}+\frac{\left(\frac{165}{512}b^3-\frac{15}{16}a^2b\right)\cosh(3dx+3c)}{3d}+\frac{\left(-\frac{21}{16}ab^2+\frac{1}{2}a^3\right)\sinh(2dx-}{2d}$
risch	$-\frac{5b\,\mathrm{e}^{-3dx-3c}a^2}{32d}+\frac{55b^3\mathrm{e}^{-7dx-7c}}{14336d}-\frac{5b\,\mathrm{e}^{3dx+3c}a^2}{32d}-\frac{21a\,\mathrm{e}^{2dx+2c}b^2}{64d}+\frac{15b\,\mathrm{e}^{dx+c}a^2}{16d}+\frac{3a\,b^2\mathrm{e}^{8dx+8c}}{2048d}-\frac{a\,b^2\mathrm{e}^{6dx+6c}}{64d}+\frac{21a\,b^2}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-231/512*b^3+15/8*a^2*b)/d*cosh(d*x+c)+1/5*(-165/1024*b^3+3/16*a^2*b)/d*co
sh(5*d*x+5*c)+1/3*(165/512*b^3-15/16*a^2*b)/d*cosh(3*d*x+3*c)+1/2*(-21/16*a
*b^2+1/2*a^3)*sinh(2*d*x+2*c)/d-1/2*a^3*x+105/128*a*b^2*x+55/7168*b^3/d*cos
h(7*d*x+7*c)-11/9216*b^3/d*cosh(9*d*x+9*c)+1/11264*b^3/d*cosh(11*d*x+11*c)+
21/128*a*b^2*sinh(4*d*x+4*c)/d-1/32*a*b^2*sinh(6*d*x+6*c)/d+3/1024*a*b^2*si
nh(8*d*x+8*c)/d
```

Maxima [A]
time = 0.27, size = 387, normalized size = 1.33

$$-\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{1419264}b^3\left(\frac{\left(847e^{(-2dx-2c)}-5445e^{(-4dx-4c)}+22869e^{(-6dx-6c)}-76230e^{(-8dx-8c)}+320166e^{(-10dx-10c)}-63\right)e^{(11dx+11c)}}{d}+\frac{320166e^{(-dx-c)}-76230e^{(-3dx-3c)}+22869e^{(-5dx-5c)}-5445e^{(-7dx-7c)}+847e^{(-9dx-9c)}-63e^{(-11dx-11c)}}{d}\right)-\frac{1}{2048}ab^2\left(\frac{\left(32e^{(-2dx-2c)}-168e^{(-4dx-4c)}+672e^{(-6dx-6c)}-3\right)e^{(8dx+8c)}}{d}-\frac{1680(dx+c)}{d}-\frac{672e^{(-2dx-2c)}-168e^{(-4dx-4c)}+32e^{(-6dx-6c)}-3e^{(-8dx-8c)}}{d}\right)+\frac{1}{160}a^2b\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/1419264*b^3*((8
47*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-6*d*x - 6*c) - 7623
0*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^(11*d*x + 11*c)/d +
(320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e^(-5*d*x - 5*c) - 54
45*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*d*x - 11*c))/d) - 1/
2048*a*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6
*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168
*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 1/160*a^
2*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e
^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 568 vs. 2(267) = 534.
time = 0.43, size = 568, normalized size = 1.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/3548160*(315*b^3*cosh(d*x + c)^11 + 3465*b^3*cosh(d*x + c)*sinh(d*x + c)^
10 - 4235*b^3*cosh(d*x + c)^9 + 83160*a*b^2*cosh(d*x + c)*sinh(d*x + c)^7 +
 27225*b^3*cosh(d*x + c)^7 + 3465*(15*b^3*cosh(d*x + c)^3 - 11*b^3*cosh(d*x
 + c))*sinh(d*x + c)^8 + 1155*(126*b^3*cosh(d*x + c)^5 - 308*b^3*cosh(d*x +
 c)^3 + 165*b^3*cosh(d*x + c))*sinh(d*x + c)^6 + 2079*(64*a^2*b - 55*b^3)*c
osh(d*x + c)^5 + 83160*(7*a*b^2*cosh(d*x + c)^3 - 8*a*b^2*cosh(d*x + c))*si
nh(d*x + c)^5 + 3465*(30*b^3*cosh(d*x + c)^7 - 154*b^3*cosh(d*x + c)^5 + 27
5*b^3*cosh(d*x + c)^3 + 3*(64*a^2*b - 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^
4 - 34650*(32*a^2*b - 11*b^3)*cosh(d*x + c)^3 + 27720*(21*a*b^2*cosh(d*x +
c)^5 - 80*a*b^2*cosh(d*x + c)^3 + 84*a*b^2*cosh(d*x + c))*sinh(d*x + c)^3 -
 27720*(64*a^3 - 105*a*b^2)*d*x + 3465*(5*b^3*cosh(d*x + c)^9 - 44*b^3*cosh
(d*x + c)^7 + 165*b^3*cosh(d*x + c)^5 + 6*(64*a^2*b - 55*b^3)*cosh(d*x + c)
^3 - 30*(32*a^2*b - 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 20790*(320*a^2
*b - 77*b^3)*cosh(d*x + c) + 27720*(3*a*b^2*cosh(d*x + c)^7 - 24*a*b^2*cosh
(d*x + c)^5 + 84*a*b^2*cosh(d*x + c)^3 + 8*(8*a^3 - 21*a*b^2)*cosh(d*x + c)
)*sinh(d*x + c))/d
```

Sympy [**A**]
time = 3.05, size = 498, normalized size = 1.71

$$\begin{cases} \frac{a^3 x \sinh^2(c+dx)}{2} - \frac{a^3 x \cosh^2(c+dx)}{2} + \frac{a^3 \sinh(c+dx)\cosh(c+dx)}{2d} + \frac{3a^2 b \sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4a^2 b \sinh^2(c+dx)\cosh^3(c+dx)}{d} + \frac{8a^2 b \cosh^5(c+dx)}{5d} + \frac{105ab^2 x \sinh^8(c+dx)}{128} - \frac{105ab^2 x \sinh^6(c+dx)\cosh^2(c+dx)}{32} + \frac{315ab^2 x \sinh^4(c+dx)\cosh^4(c+dx)}{64} - \frac{105ab^2 x \sinh^2(c+dx)\cosh^6(c+dx)}{32} + \frac{105ab^2 x \cosh^8(c+dx)}{128} + \frac{279ab^2 \sinh^7(c+dx)\cosh(c+dx)}{128d} - \frac{511ab^2 \sinh^5(c+dx)\cosh^3(c+dx)}{128d} + \frac{385ab^2 \sinh^3(c+dx)\cosh^5(c+dx)}{128d} - \frac{105ab^2 \sinh(c+dx)\cosh^7(c+dx)}{128d} + \frac{b^3 \sinh^{10}(c+dx)\cosh(c+dx)}{d} - \frac{10b^3 \sinh^8(c+dx)\cosh^3(c+dx)}{3d} + \frac{16b^3 \sinh^6(c+dx)\cosh^5(c+dx)}{3d} - \frac{32b^3 \sinh^4(c+dx)\cosh^7(c+dx)}{7d} + \frac{128b^3 \sinh^2(c+dx)\cosh^9(c+dx)}{63d} - \frac{256b^3 \cosh^{11}(c+dx)}{693d} & \text{for}\ d \neq 0 \\ x\left(a + b\sinh^3(c)\right)^3 \sinh^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d
- 4*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5/
(5*d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)**6*c
osh(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 10
5*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c + d*x
)**8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a*b**2*s
inh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)**3*cosh
(c + d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) +
b**3*sinh(c + d*x)**10*cosh(c + d*x)/d - 10*b**3*sinh(c + d*x)**8*cosh(c +
d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)**6*cosh(c + d*x)**5/(3*d) - 32*b**3*s
inh(c + d*x)**4*cosh(c + d*x)**7/(7*d) + 128*b**3*sinh(c + d*x)**2*cosh(c +
 d*x)**9/(63*d) - 256*b**3*cosh(c + d*x)**11/(693*d), Ne(d, 0)), (x*(a + b*
sinh(c)**3)**3*sinh(c)**2, True))
```

Giac [**A**]
time = 0.47, size = 431, normalized size = 1.48

$$\frac{b^3 e^{(11\,dx+11\,c)}}{22528\,d} - \frac{11\,b^3 e^{(9\,dx+9\,c)}}{18432\,d} + \frac{3\,ab^2 e^{(8\,dx+8\,c)}}{2048\,d} + \frac{55\,b^3 e^{(7\,dx+7\,c)}}{14336\,d} - \frac{ab^2 e^{(6\,dx+6\,c)}}{64\,d} + \frac{21\,ab^2 e^{(4\,dx+4\,c)}}{256\,d} - \frac{21\,ab^2 e^{(-4\,dx-4\,c)}}{256\,d} + \frac{ab^2 e^{(-6\,dx-6\,c)}}{64\,d} + \frac{55\,b^3 e^{(-7\,dx-7\,c)}}{14336\,d} - \frac{3\,ab^2 e^{(-8\,dx-8\,c)}}{2048\,d} - \frac{11\,b^3 e^{(-9\,dx-9\,c)}}{18432\,d} + \frac{b^3 e^{(-11\,dx-11\,c)}}{22528\,d} - \frac{1}{128}\left(64\,a^3 - 105\,ab^2\right)x + \frac{3\,(64\,a^2 b - 55\,b^3)e^{(5\,dx+5\,c)}}{10240\,d} - \frac{5\,(32\,a^2 b - 11\,b^3)e^{(3\,dx+3\,c)}}{1024\,d} + \frac{(8\,a^3 - 21\,ab^2)e^{(2\,dx+2\,c)}}{64\,d} + \frac{3\,(320\,a^2 b - 77\,b^3)e^{(dx+c)}}{1024\,d} + \frac{3\,(320\,a^2 b - 77\,b^3)e^{(-dx-c)}}{1024\,d} - \frac{(8\,a^3 - 21\,ab^2)e^{(-2\,dx-2\,c)}}{64\,d} - \frac{5\,(32\,a^2 b - 11\,b^3)e^{(-3\,dx-3\,c)}}{1024\,d} + \frac{3\,(64\,a^2 b - 55\,b^3)e^{(-5\,dx-5\,c)}}{10240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/22528*b^3*e^(11*d*x + 11*c)/d - 11/18432*b^3*e^(9*d*x + 9*c)/d + 3/2048*a
*b^2*e^(8*d*x + 8*c)/d + 55/14336*b^3*e^(7*d*x + 7*c)/d - 1/64*a*b^2*e^(6*d
*x + 6*c)/d + 21/256*a*b^2*e^(4*d*x + 4*c)/d - 21/256*a*b^2*e^(-4*d*x - 4*c
)/d + 1/64*a*b^2*e^(-6*d*x - 6*c)/d + 55/14336*b^3*e^(-7*d*x - 7*c)/d - 3/2
048*a*b^2*e^(-8*d*x - 8*c)/d - 11/18432*b^3*e^(-9*d*x - 9*c)/d + 1/22528*b^
3*e^(-11*d*x - 11*c)/d - 1/128*(64*a^3 - 105*a*b^2)*x + 3/10240*(64*a^2*b -
 55*b^3)*e^(5*d*x + 5*c)/d - 5/1024*(32*a^2*b - 11*b^3)*e^(3*d*x + 3*c)/d +
 1/64*(8*a^3 - 21*a*b^2)*e^(2*d*x + 2*c)/d + 3/1024*(320*a^2*b - 77*b^3)*e^
(d*x + c)/d + 3/1024*(320*a^2*b - 77*b^3)*e^(-d*x - c)/d - 1/64*(8*a^3 - 21
*a*b^2)*e^(-2*d*x - 2*c)/d - 5/1024*(32*a^2*b - 11*b^3)*e^(-3*d*x - 3*c)/d
+ 3/10240*(64*a^2*b - 55*b^3)*e^(-5*d*x - 5*c)/d
```

Mupad [B]
time = 1.09, size = 231, normalized size = 0.79

$$\frac{\frac{\sinh(c+d\,x)\,a^3\cosh(c+d\,x)}{2}-\frac{d\,x\,a^3}{2}+\frac{3\,a^2\,b\cosh(c+d\,x)^5}{5}-2\,a^2\,b\cosh\left(c+d\,x\right)^3+3\,a^2\,b\cosh\left(c+d\,x\right)+\frac{3\sinh(c+d\,x)\,a\,b^2\cosh(c+d\,x)^7}{8}-\frac{25\sinh(c+d\,x)\,a\,b^2\cosh(c+d\,x)^5}{16}+\frac{163\sinh(c+d\,x)\,a\,b^2\cosh(c+d\,x)^3}{64}-\frac{279\sinh(c+d\,x)\,a\,b^2\cosh(c+d\,x)}{128}+\frac{105\,d\,x\,a\,b^2}{128}+\frac{b^3\cosh(c+d\,x)^{11}}{11}-\frac{5\,b^3\cosh(c+d\,x)^9}{9}+\frac{10\,b^3\cosh(c+d\,x)^7}{7}-2\,b^3\cosh\left(c+d\,x\right)^5+\frac{5\,b^3\cosh(c+d\,x)^3}{3}-b^3\cosh\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^3)^3,x)
```

```
[Out] ((5*b^3*cosh(c + d*x)^3)/3 - b^3*cosh(c + d*x) - 2*b^3*cosh(c + d*x)^5 + (1
0*b^3*cosh(c + d*x)^7)/7 - (5*b^3*cosh(c + d*x)^9)/9 + (b^3*cosh(c + d*x)^1
1)/11 - 2*a^2*b*cosh(c + d*x)^3 + (3*a^2*b*cosh(c + d*x)^5)/5 + 3*a^2*b*cos
h(c + d*x) + (a^3*cosh(c + d*x)*sinh(c + d*x))/2 - (a^3*d*x)/2 - (279*a*b^2
*cosh(c + d*x)*sinh(c + d*x))/128 + (105*a*b^2*d*x)/128 + (163*a*b^2*cosh(c
 + d*x)^3*sinh(c + d*x))/64 - (25*a*b^2*cosh(c + d*x)^5*sinh(c + d*x))/16 +
 (3*a*b^2*cosh(c + d*x)^7*sinh(c + d*x))/8)/d
```

3.162 $\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=267

$$\frac{9}{8}a^2bx-\frac{63b^3x}{256}+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+dx)}{d}-\frac{9ab^2\cosh^5(c+dx)}{5d}+\frac{3ab^2\cosh^7(}{7d}$$

```
[Out] 9/8*a^2*b*x-63/256*b^3*x+a^3*cosh(d*x+c)/d-3*a*b^2*cosh(d*x+c)/d+3*a*b^2*co
sh(d*x+c)^3/d-9/5*a*b^2*cosh(d*x+c)^5/d+3/7*a*b^2*cosh(d*x+c)^7/d-9/8*a^2*b
*cosh(d*x+c)*sinh(d*x+c)/d+63/256*b^3*cosh(d*x+c)*sinh(d*x+c)/d+3/4*a^2*b*c
osh(d*x+c)*sinh(d*x+c)^3/d-21/128*b^3*cosh(d*x+c)*sinh(d*x+c)^3/d+21/160*b^
3*cosh(d*x+c)*sinh(d*x+c)^5/d-9/80*b^3*cosh(d*x+c)*sinh(d*x+c)^7/d+1/10*b^3
*cosh(d*x+c)*sinh(d*x+c)^9/d
```

Rubi [A]
time = 0.17, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3299, 2718, 2715, 8, 2713}

$$\frac{a^3\cosh(c+dx)}{d}+\frac{3a^2b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{9a^2b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{9}{8}a^2bx+\frac{3ab^2\cosh^7(c+dx)}{7d}-\frac{9ab^2\cosh^5(c+dx)}{5d}+\frac{3ab^2\cosh^3(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{b^3\sinh^9(c+dx)\cosh(c+dx)}{10d}-\frac{9b^3\sinh^7(c+dx)\cosh(c+dx)}{80d}+\frac{21b^3\sinh^5(c+dx)\cosh(c+dx)}{160d}-\frac{21b^3\sinh^3(c+dx)\cosh(c+dx)}{128d}+\frac{63b^3\sinh(c+dx)\cosh(c+dx)}{256d}-\frac{63b^3x}{256}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (9*a^2*b*x)/8 - (63*b^3*x)/256 + (a^3*Cosh[c + d*x])/d - (3*a*b^2*Cosh[c +
d*x])/d + (3*a*b^2*Cosh[c + d*x]^3)/d - (9*a*b^2*Cosh[c + d*x]^5)/(5*d) + (
3*a*b^2*Cosh[c + d*x]^7)/(7*d) - (9*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d
) + (63*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) + (3*a^2*b*Cosh[c + d*x]*S
inh[c + d*x]^3)/(4*d) - (21*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(128*d) + (2
1*b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(160*d) - (9*b^3*Cosh[c + d*x]*Sinh[c
+ d*x]^7)/(80*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^9)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx &= -\left(i\int \left(ia^3\sinh(c+dx)+3ia^2b\sinh^4(c+dx)+3iab^2\sinh^7(c+\right.\right.\\
&= a^3\int \sinh(c+dx)\,dx+\left(3a^2b\right)\int \sinh^4(c+dx)\,dx+\left(3ab^2\right)\int \mathrm{si}\\
&= \frac{a^3\cosh(c+dx)}{d}+\frac{3a^2b\cosh(c+dx)\sinh^3(c+dx)}{4d}+\frac{b^3\cosh(c+}{}\\
&= \frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+dx)}{d}-\frac{9a}{}\\
&= \frac{9}{8}a^2bx+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+}{d}\\
&= \frac{9}{8}a^2bx+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+}{d}\\
&= \frac{9}{8}a^2bx+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+}{d}\\
&= \frac{9}{8}a^2bx-\frac{63b^3x}{256}+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\,\mathrm{c}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.32, size = 184, normalized size = 0.69

$$\frac{1120a(64a^2-105b^2)\cosh(c+dx)+b(80640a^2c-17640b^2c+80640a^2dx-17640b^2dx+23520ab\cosh(3(c+dx))-4704ab\cosh(5(c+dx))+480ab\cosh(7(c+dx))-53760a^2\sinh(2(c+dx))+14700b^2\sinh(2(c+dx))+6720a^2\sinh(4(c+dx))-4200b^2\sinh(4(c+dx))+1050b^2\sinh(6(c+dx))-175b^2\sinh(8(c+dx))+14b^2\sinh(10(c+dx)))}{71680d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (1120*a*(64*a^2 - 105*b^2)*Cosh[c + d*x] + b*(80640*a^2*c - 17640*b^2*c + 8
0640*a^2*d*x - 17640*b^2*d*x + 23520*a*b*Cosh[3*(c + d*x)] - 4704*a*b*Cosh[
5*(c + d*x)] + 480*a*b*Cosh[7*(c + d*x)] - 53760*a^2*Sinh[2*(c + d*x)] + 14
700*b^2*Sinh[2*(c + d*x)] + 6720*a^2*Sinh[4*(c + d*x)] - 4200*b^2*Sinh[4*(c
 + d*x)] + 1050*b^2*Sinh[6*(c + d*x)] - 175*b^2*Sinh[8*(c + d*x)] + 14*b^2*
Sinh[10*(c + d*x)]))/(71680*d)
```

Maple [A]
time = 1.79, size = 192, normalized size = 0.72

method	result
default	$\frac{\left(-\frac{15}{64}b^3+\frac{3}{8}a^2b\right)\sinh(4dx+4c)}{4d}+\frac{\left(\frac{105}{256}b^3-\frac{3}{2}a^2b\right)\sinh(2dx+2c)}{2d}+\frac{\left(-\frac{105}{64}a\,b^2+a^3\right)\cosh(dx+c)}{d}-\frac{63b^3x}{256}+\frac{9a^2bx}{8}+\frac{15b^3\sinh(}{102}$
risch	$\frac{9a^2bx}{8}-\frac{21a\,b^2\mathrm{e}^{5dx+5c}}{640d}+\frac{21a\,b^2\mathrm{e}^{3dx+3c}}{128d}-\frac{3b\,\mathrm{e}^{2dx+2c}a^2}{8d}+\frac{3a\,b^2\mathrm{e}^{-7dx-7c}}{896d}+\frac{a^3\mathrm{e}^{dx+c}}{2d}+\frac{a^3\mathrm{e}^{-dx-c}}{2d}-\frac{105b^3\mathrm{e}^{-2dx-2c}}{1024d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-15/64*b^3+3/8*a^2*b)*sinh(4*d*x+4*c)/d+1/2*(105/256*b^3-3/2*a^2*b)*si
nh(2*d*x+2*c)/d+(-105/64*a*b^2+a^3)/d*cosh(d*x+c)-63/256*b^3*x+9/8*a^2*b*x+
15/1024*b^3*sinh(6*d*x+6*c)/d-5/2048*b^3*sinh(8*d*x+8*c)/d+1/5120*b^3*sinh(
10*d*x+10*c)/d+21/64*a*b^2/d*cosh(3*d*x+3*c)-21/320*a*b^2/d*cosh(5*d*x+5*c)
+3/448*a*b^2/d*cosh(7*d*x+7*c)
```

Maxima [A]
time = 0.27, size = 318, normalized size = 1.19

$$\frac{3}{64}a^2b\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{20480}b^3\left(\frac{\left(25e^{(-2dx-2c)}-150e^{(-4dx-4c)}+600e^{(-6dx-6c)}-2100e^{(-8dx-8c)}-2\right)e^{(10dx+10c)}}{d}+\frac{5040(dx+c)}{d}+\frac{2100e^{(-2dx-2c)}-600e^{(-4dx-4c)}+150e^{(-6dx-6c)}-25e^{(-8dx-8c)}+2e^{(-10dx-10c)}}{d}\right)-\frac{3}{4480}ab^2\left(\frac{\left(49e^{(-2dx-2c)}-245e^{(-4dx-4c)}+1225e^{(-6dx-6c)}-5\right)e^{(7dx+7c)}}{d}+\frac{1225e^{(-dx-c)}-245e^{(-3dx-3c)}+49e^{(-5dx-5c)}-5e^{(-7dx-7c)}}{d}\right)+\frac{a^3\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((25*e^(-2*d*x - 2*c) - 150*e^(-
4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x
+ 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c
) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) -
 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x
 - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c)
+ 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + a^3*cosh(d*x + c)/d
```

Fricas [A]
time = 0.45, size = 453, normalized size = 1.70

$$\frac{35b^3\cosh(dx+c)\sinh(dx+c)^9+120ab^2\cosh(dx+c)^7+840ab^2\cosh(dx+c)\sinh(dx+c)^6-1176ab^2\cosh(dx+c)^5+70\left(6b^3\cosh(dx+c)^3-5b^3\cosh(dx+c)\right)\sinh(dx+c)^7+5880ab^2\cosh(dx+c)^3+7\left(126b^3\cosh(dx+c)^5-350b^3\cosh(dx+c)^3+225b^3\cosh(dx+c)\right)\sinh(dx+c)^5+840\left(5ab^2\cosh(dx+c)^3-7ab^2\cosh(dx+c)\right)\sinh(dx+c)^4+70\left(6b^3\cosh(dx+c)^7-35b^3\cosh(dx+c)^5+75b^3\cosh(dx+c)^3+12\left(8a^2b-5b^3\right)\cosh(dx+c)\right)\sinh(dx+c)^3+630\left(32a^2b-7b^3\right)dx+840\left(3ab^2\cosh(dx+c)^5-14ab^2\cosh(dx+c)^3+21ab^2\cosh(dx+c)\right)\sinh(dx+c)^2+280\left(64a^3-105ab^2\right)\cosh(dx+c)+35\left(b^3\cosh(dx+c)^9-10b^3\cosh(dx+c)^7+45b^3\cosh(dx+c)^5+24\left(8a^2b-5b^3\right)\cosh(dx+c)^3-6\left(128a^2b-35b^3\right)\cosh(dx+c)\right)\sinh(dx+c)}{17920d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/17920*(35*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 120*a*b^2*cosh(d*x + c)^7 +
 840*a*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 1176*a*b^2*cosh(d*x + c)^5 + 70*
(6*b^3*cosh(d*x + c)^3 - 5*b^3*cosh(d*x + c))*sinh(d*x + c)^7 + 5880*a*b^2*
cosh(d*x + c)^3 + 7*(126*b^3*cosh(d*x + c)^5 - 350*b^3*cosh(d*x + c)^3 + 22
5*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 840*(5*a*b^2*cosh(d*x + c)^3 - 7*a*b
^2*cosh(d*x + c))*sinh(d*x + c)^4 + 70*(6*b^3*cosh(d*x + c)^7 - 35*b^3*cosh
(d*x + c)^5 + 75*b^3*cosh(d*x + c)^3 + 12*(8*a^2*b - 5*b^3)*cosh(d*x + c))*
sinh(d*x + c)^3 + 630*(32*a^2*b - 7*b^3)*d*x + 840*(3*a*b^2*cosh(d*x + c)^5
 - 14*a*b^2*cosh(d*x + c)^3 + 21*a*b^2*cosh(d*x + c))*sinh(d*x + c)^2 + 280
*(64*a^3 - 105*a*b^2)*cosh(d*x + c) + 35*(b^3*cosh(d*x + c)^9 - 10*b^3*cosh
(d*x + c)^7 + 45*b^3*cosh(d*x + c)^5 + 24*(8*a^2*b - 5*b^3)*cosh(d*x + c)^3
 - 6*(128*a^2*b - 35*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy **[A]**
time = 2.28, size = 496, normalized size = 1.86

$$\begin{cases} \frac{a^3 \cosh(c+dx)}{d} + \frac{9a^2bx\sinh^4(c+dx)}{8} - \frac{9a^2bx\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{9a^2bx\cosh^4(c+dx)}{8} + \frac{15a^2b\sinh^3(c+dx)\cosh(c+dx)}{8d} - \frac{9a^2b\sinh(c+dx)\cosh^3(c+dx)}{8d} + \frac{3ab^2\sinh^6(c+dx)\cosh(c+dx)}{d} - \frac{6ab^2\sinh^4(c+dx)\cosh^3(c+dx)}{d} + \frac{24ab^2\sinh^2(c+dx)\cosh^5(c+dx)}{5d} - \frac{48ab^2\cosh^7(c+dx)}{35d} + \frac{63b^3x\sinh^{10}(c+dx)}{256} - \frac{315b^3x\sinh^8(c+dx)\cosh^2(c+dx)}{256} + \frac{315b^3x\sinh^6(c+dx)\cosh^4(c+dx)}{128} - \frac{315b^3x\sinh^4(c+dx)\cosh^6(c+dx)}{128} + \frac{315b^3x\sinh^2(c+dx)\cosh^8(c+dx)}{256} - \frac{63b^3x\cosh^{10}(c+dx)}{256} + \frac{193b^3\sinh^9(c+dx)\cosh(c+dx)}{256d} - \frac{237b^3\sinh^7(c+dx)\cosh^3(c+dx)}{128d} + \frac{21b^3\sinh^5(c+dx)\cosh^5(c+dx)}{10d} - \frac{147b^3\sinh^3(c+dx)\cosh^7(c+dx)}{128d} + \frac{63b^3\sinh(c+dx)\cosh^9(c+dx)}{256d} & \text{for}\ d \neq 0 \\ x\left(a+b\sinh^3(c)\right)^3\sinh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Piecewise((a**3*cosh(c + d*x)/d + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b*
x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 15*
a**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cosh(c
 + d*x)**3/(8*d) + 3*a*b**2*sinh(c + d*x)**6*cosh(c + d*x)/d - 6*a*b**2*sin
h(c + d*x)**4*cosh(c + d*x)**3/d + 24*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)
**5/(5*d) - 48*a*b**2*cosh(c + d*x)**7/(35*d) + 63*b**3*x*sinh(c + d*x)**10
/256 - 315*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 315*b**3*x*sinh(c
 + d*x)**6*cosh(c + d*x)**4/128 - 315*b**3*x*sinh(c + d*x)**4*cosh(c + d*x)
**6/128 + 315*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256 - 63*b**3*x*cosh
(c + d*x)**10/256 + 193*b**3*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) - 237*b
**3*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + 21*b**3*sinh(c + d*x)**5*co
sh(c + d*x)**5/(10*d) - 147*b**3*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d)
+ 63*b**3*sinh(c + d*x)*cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh
(c)**3)**3*sinh(c), True))
```

Giac **[A]**
time = 0.48, size = 379, normalized size = 1.42

$$\frac{b^3e^{(10\,dx+10\,c)}}{10240\,d} - \frac{5\,b^3e^{(8\,dx+8\,c)}}{4096\,d} + \frac{3\,ab^2e^{(7\,dx+7\,c)}}{896\,d} + \frac{15\,b^3e^{(6\,dx+6\,c)}}{2048\,d} - \frac{21\,ab^2e^{(5\,dx+5\,c)}}{640\,d} + \frac{21\,ab^2e^{(3\,dx+3\,c)}}{128\,d} + \frac{21\,ab^2e^{(-3\,dx-3\,c)}}{128\,d} - \frac{21\,ab^2e^{(-5\,dx-5\,c)}}{640\,d} - \frac{15\,b^3e^{(-6\,dx-6\,c)}}{2048\,d} + \frac{3\,ab^2e^{(-7\,dx-7\,c)}}{896\,d} + \frac{5\,b^3e^{(-8\,dx-8\,c)}}{4096\,d} - \frac{b^3e^{(-10\,dx-10\,c)}}{10240\,d} + \frac{9}{256}\,(32\,a^2b - 7\,b^3)x + \frac{3\,(8\,a^2b - 5\,b^3)e^{(4\,dx+4\,c)}}{512\,d} - \frac{3\,(128\,a^2b - 35\,b^3)e^{(2\,dx+2\,c)}}{1024\,d} + \frac{(64\,a^3 - 105\,ab^2)e^{(dx+c)}}{128\,d} + \frac{(64\,a^3 - 105\,ab^2)e^{(-dx-c)}}{128\,d} + \frac{3\,(128\,a^2b - 35\,b^3)e^{(-2\,dx-2\,c)}}{1024\,d} - \frac{3\,(8\,a^2b - 5\,b^3)e^{(-4\,dx-4\,c)}}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/10240*b^3*e^(10*d*x + 10*c)/d - 5/4096*b^3*e^(8*d*x + 8*c)/d + 3/896*a*b^
2*e^(7*d*x + 7*c)/d + 15/2048*b^3*e^(6*d*x + 6*c)/d - 21/640*a*b^2*e^(5*d*x
 + 5*c)/d + 21/128*a*b^2*e^(3*d*x + 3*c)/d + 21/128*a*b^2*e^(-3*d*x - 3*c)/
d - 21/640*a*b^2*e^(-5*d*x - 5*c)/d - 15/2048*b^3*e^(-6*d*x - 6*c)/d + 3/89
6*a*b^2*e^(-7*d*x - 7*c)/d + 5/4096*b^3*e^(-8*d*x - 8*c)/d - 1/10240*b^3*e^
(-10*d*x - 10*c)/d + 9/256*(32*a^2*b - 7*b^3)*x + 3/512*(8*a^2*b - 5*b^3)*e
^(4*d*x + 4*c)/d - 3/1024*(128*a^2*b - 35*b^3)*e^(2*d*x + 2*c)/d + 1/128*(6
4*a^3 - 105*a*b^2)*e^(d*x + c)/d + 1/128*(64*a^3 - 105*a*b^2)*e^(-d*x - c)/
d + 3/1024*(128*a^2*b - 35*b^3)*e^(-2*d*x - 2*c)/d - 3/512*(8*a^2*b - 5*b^3
)*e^(-4*d*x - 4*c)/d
```

Mupad [B]
time = 2.95, size = 189, normalized size = 0.71

$$\frac{8960\,a^3\cosh(c+d\,x)+\frac{3675\,b^3\sinh(2\,c+2\,d\,x)}{2}-525\,b^3\sinh(4\,c+4\,d\,x)+\frac{525\,b^3\sinh(6\,c+6\,d\,x)}{4}-\frac{175\,b^3\sinh(8\,c+8\,d\,x)}{8}+\frac{7\,b^3\sinh(10\,c+10\,d\,x)}{4}+2940\,a\,b^2\cosh(3\,c+3\,d\,x)-588\,a\,b^2\cosh(5\,c+5\,d\,x)+60\,a\,b^2\cosh(7\,c+7\,d\,x)-6720\,a^2\,b\sinh(2\,c+2\,d\,x)+840\,a^2\,b\sinh(4\,c+4\,d\,x)-14700\,a\,b^2\cosh(c+d\,x)-2205\,b^3\,d\,x+10080\,a^2\,b\,d\,x}{8960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^3)^3,x)
```

```
[Out] (8960*a^3*cosh(c + d*x) + (3675*b^3*sinh(2*c + 2*d*x))/2 - 525*b^3*sinh(4*c
 + 4*d*x) + (525*b^3*sinh(6*c + 6*d*x))/4 - (175*b^3*sinh(8*c + 8*d*x))/8 +
 (7*b^3*sinh(10*c + 10*d*x))/4 + 2940*a*b^2*cosh(3*c + 3*d*x) - 588*a*b^2*c
osh(5*c + 5*d*x) + 60*a*b^2*cosh(7*c + 7*d*x) - 6720*a^2*b*sinh(2*c + 2*d*x
) + 840*a^2*b*sinh(4*c + 4*d*x) - 14700*a*b^2*cosh(c + d*x) - 2205*b^3*d*x
+ 10080*a^2*b*d*x)/(8960*d)
```

3.163 $\int \left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=204

$$a^3x-\frac{15}{16}ab^2x-\frac{3a^2b\cosh(c+dx)}{d}+\frac{b^3\cosh(c+dx)}{d}+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{4b^3\cosh^3(c+dx)}{3d}+\frac{6b^3\cosh^5(c+dx)}{5d}$$

```
[Out] a^3*x-15/16*a*b^2*x-3*a^2*b*cosh(d*x+c)/d+b^3*cosh(d*x+c)/d+a^2*b*cosh(d*x+
c)^3/d-4/3*b^3*cosh(d*x+c)^3/d+6/5*b^3*cosh(d*x+c)^5/d-4/7*b^3*cosh(d*x+c)^
7/d+1/9*b^3*cosh(d*x+c)^9/d+15/16*a*b^2*cosh(d*x+c)*sinh(d*x+c)/d-5/8*a*b^2
*cosh(d*x+c)*sinh(d*x+c)^3/d+1/2*a*b^2*cosh(d*x+c)*sinh(d*x+c)^5/d
```

Rubi [A]
time = 0.09, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {3292, 2713, 2715, 8}

$$a^3x+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{3a^2b\cosh(c+dx)}{d}+\frac{ab^2\sinh^5(c+dx)\cosh(c+dx)}{2d}-\frac{5ab^2\sinh^3(c+dx)\cosh(c+dx)}{8d}+\frac{15ab^2\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{15}{16}ab^2x+\frac{b^3\cosh^9(c+dx)}{9d}-\frac{4b^3\cosh^7(c+dx)}{7d}+\frac{6b^3\cosh^5(c+dx)}{5d}-\frac{4b^3\cosh^3(c+dx)}{3d}+\frac{b^3\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] a^3*x - (15*a*b^2*x)/16 - (3*a^2*b*Cosh[c + d*x])/d + (b^3*Cosh[c + d*x])/d
 + (a^2*b*Cosh[c + d*x]^3)/d - (4*b^3*Cosh[c + d*x]^3)/(3*d) + (6*b^3*Cosh[
c + d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(
9*d) + (15*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*a*b^2*Cosh[c + d*
x]*Sinh[c + d*x]^3)/(8*d) + (a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sinh^3(c+dx)\right)^3\,dx &= \int \left(a^3+3a^2b\sinh^3(c+dx)+3ab^2\sinh^6(c+dx)+b^3\sinh^9(c+dx)\right)\,dx \\
&= a^3x+\left(3a^2b\right)\int \sinh^3(c+dx)\,dx+\left(3ab^2\right)\int \sinh^6(c+dx)\,dx+b^3\int \sinh^9(\\
&= a^3x+\frac{ab^2\cosh(c+dx)\sinh^5(c+dx)}{2d}-\frac{1}{2}\left(5ab^2\right)\int \sinh^4(c+dx)\,dx-\frac{(3a^2}{} \\
&= a^3x-\frac{3a^2b\cosh(c+dx)}{d}+\frac{b^3\cosh(c+dx)}{d}+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{4b^3\cosh^3}{3} \\
&= a^3x-\frac{3a^2b\cosh(c+dx)}{d}+\frac{b^3\cosh(c+dx)}{d}+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{4b^3\cosh^3}{3} \\
&= a^3x-\frac{15}{16}ab^2x-\frac{3a^2b\cosh(c+dx)}{d}+\frac{b^3\cosh(c+dx)}{d}+\frac{a^2b\cosh^3(c+dx)}{d}-
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 159, normalized size = 0.78

$$\frac{80640a^3c-75600ab^2c+80640a^3dx-75600ab^2dx+5670b\left(-32a^2+7b^2\right)\cosh(c+dx)+1260\left(16a^2b-7b^3\right)\cosh(3(c+dx))+2268b^3\cosh(5(c+dx))-405b^3\cosh(7(c+dx))+35b^3\cosh(9(c+dx))+56700ab^2\sinh(2(c+dx))-11340ab^2\sinh(4(c+dx))+1260ab^2\sinh(6(c+dx))}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (80640*a^3*c - 75600*a*b^2*c + 80640*a^3*d*x - 75600*a*b^2*d*x + 5670*b*(-3
2*a^2 + 7*b^2)*Cosh[c + d*x] + 1260*(16*a^2*b - 7*b^3)*Cosh[3*(c + d*x)] +
2268*b^3*Cosh[5*(c + d*x)] - 405*b^3*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(c +
 d*x)] + 56700*a*b^2*Sinh[2*(c + d*x)] - 11340*a*b^2*Sinh[4*(c + d*x)] + 12
60*a*b^2*Sinh[6*(c + d*x)])/(80640*d)
```

Maple [A]
time = 1.33, size = 167, normalized size = 0.82

method	result
default	$a^3x+\frac{\left(-\frac{21}{64}b^3+\frac{3}{4}a^2b\right)\cosh(3dx+3c)}{3d}+\frac{\left(\frac{63}{128}b^3-\frac{9}{4}a^2b\right)\cosh(dx+c)}{d}-\frac{15a\,b^2x}{16}+\frac{9b^3\cosh(5dx+5c)}{320d}-\frac{9b^3\cosh(7dx+7c)}{1792d}+$
risch	$a^3x-\frac{15a\,b^2x}{16}+\frac{b^3\mathrm{e}^{9dx+9c}}{4608d}-\frac{9b^3\mathrm{e}^{7dx+7c}}{3584d}+\frac{a\,b^2\mathrm{e}^{6dx+6c}}{128d}+\frac{9b^3\mathrm{e}^{5dx+5c}}{640d}-\frac{9a\,b^2\mathrm{e}^{4dx+4c}}{128d}+\frac{b\,\mathrm{e}^{3dx+3c}a^2}{8d}-\frac{7b^3\mathrm{e}^{3dx+3c}}{128d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+1/3*(-21/64*b^3+3/4*a^2*b)/d*cosh(3*d*x+3*c)+(63/128*b^3-9/4*a^2*b)/d
*cosh(d*x+c)-15/16*a*b^2*x+9/320*b^3/d*cosh(5*d*x+5*c)-9/1792*b^3/d*cosh(7*
d*x+7*c)+1/2304*b^3/d*cosh(9*d*x+9*c)+45/64*a*b^2*sinh(2*d*x+2*c)/d-9/64*a*
b^2*sinh(4*d*x+4*c)/d+1/64*a*b^2*sinh(6*d*x+6*c)/d
```

Maxima [**A**]
time = 0.28, size = 280, normalized size = 1.37

$$a^3x - \frac{1}{161280}b^3\left(\frac{\left(405\,e^{(-2\,dx-2\,c)} - 2268\,e^{(-4\,dx-4\,c)} + 8820\,e^{(-6\,dx-6\,c)} - 39690\,e^{(-8\,dx-8\,c)} - 35\right)e^{(9\,dx+9\,c)}}{d} - \frac{39690\,e^{(-dx-c)} - 8820\,e^{(-3\,dx-3\,c)} + 2268\,e^{(-5\,dx-5\,c)} - 405\,e^{(-7\,dx-7\,c)} + 35\,e^{(-9\,dx-9\,c)}}{d}\right) - \frac{1}{128}ab^2\left(\frac{\left(9\,e^{(-2\,dx-2\,c)} - 45\,e^{(-4\,dx-4\,c)} - 1\right)e^{(6\,dx+6\,c)}}{d} + \frac{120\,(dx+c)}{d} + \frac{45\,e^{(-2\,dx-2\,c)} - 9\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)}}{d}\right) + \frac{1}{8}a^2b\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*
e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*
e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*
x - 7*c) + 35*e^(-9*d*x - 9*c))/d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*
e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x -
 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 1/8*a^2*b*(e^(3*d*x + 3
*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(188) = 376$.
time = 0.40, size = 380, normalized size = 1.86

$$\frac{35\,b^3\cosh(dx+c)^9 + 315\,b^3\cosh(dx+c)\sinh(dx+c)^8 - 405\,b^3\cosh(dx+c)^7 + 7560\,ab^2\cosh(dx+c)\sinh(dx+c)^5 + 2268\,b^3\cosh(dx+c)^5 + 105\left(28\,b^3\cosh(dx+c)^3 - 27\,b^3\cosh(dx+c)\right)\sinh(dx+c)^6 + 315\left(14\,b^3\cosh(dx+c)^5 - 45\,b^3\cosh(dx+c)^3 + 36\,b^3\cosh(dx+c)\right)\sinh(dx+c)^4 + 1260\left(16\,a^2b - 7\,b^3\right)\cosh(dx+c)^3 + 5040\left(5\,ab^2\cosh(dx+c)^3 - 9\,ab^2\cosh(dx+c)\right)\sinh(dx+c)^3 + 5040\left(16\,a^3 - 15\,ab^2\right)dx + 315\left(4\,b^3\cosh(dx+c)^7 - 27\,b^3\cosh(dx+c)^5 + 72\,b^3\cosh(dx+c)^3 + 12\left(16\,a^2b - 7\,b^3\right)\cosh(dx+c)\right)\sinh(dx+c)^2 - 5670\left(32\,a^2b - 7\,b^3\right)\cosh(dx+c) + 7560\left(ab^2\cosh(dx+c)^5 - 6\,ab^2\cosh(dx+c)^3 + 15\,ab^2\cosh(dx+c)\right)\sinh(dx+c)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^3*cosh(d*x + c)^9 + 315*b^3*cosh(d*x + c)*sinh(d*x + c)^8 - 4
05*b^3*cosh(d*x + c)^7 + 7560*a*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2268*b^
3*cosh(d*x + c)^5 + 105*(28*b^3*cosh(d*x + c)^3 - 27*b^3*cosh(d*x + c))*sin
h(d*x + c)^6 + 315*(14*b^3*cosh(d*x + c)^5 - 45*b^3*cosh(d*x + c)^3 + 36*b^
3*cosh(d*x + c))*sinh(d*x + c)^4 + 1260*(16*a^2*b - 7*b^3)*cosh(d*x + c)^3
+ 5040*(5*a*b^2*cosh(d*x + c)^3 - 9*a*b^2*cosh(d*x + c))*sinh(d*x + c)^3 +
5040*(16*a^3 - 15*a*b^2)*d*x + 315*(4*b^3*cosh(d*x + c)^7 - 27*b^3*cosh(d*x
 + c)^5 + 72*b^3*cosh(d*x + c)^3 + 12*(16*a^2*b - 7*b^3)*cosh(d*x + c))*sin
h(d*x + c)^2 - 5670*(32*a^2*b - 7*b^3)*cosh(d*x + c) + 7560*(a*b^2*cosh(d*x
 + c)^5 - 6*a*b^2*cosh(d*x + c)^3 + 15*a*b^2*cosh(d*x + c))*sinh(d*x + c))/
d
```

Sympy [A]
time = 1.53, size = 340, normalized size = 1.67

$$\begin{cases} a^{3}x + \frac{3a^{2}b\sinh^{2}(c+dx)\cosh(c+dx)}{d} - \frac{2a^{2}b\cosh^{3}(c+dx)}{d} + \frac{15ab^{2}x\sinh^{6}(c+dx)}{16} - \frac{45ab^{2}x\sinh^{4}(c+dx)\cosh^{2}(c+dx)}{16} + \frac{45ab^{2}x\sinh^{2}(c+dx)\cosh^{4}(c+dx)}{16} - \frac{15ab^{2}x\cosh^{6}(c+dx)}{16} + \frac{33ab^{2}\sinh^{5}(c+dx)\cosh(c+dx)}{16d} - \frac{5ab^{2}\sinh^{3}(c+dx)\cosh^{3}(c+dx)}{2d} + \frac{15ab^{2}\sinh(c+dx)\cosh^{5}(c+dx)}{16d} + \frac{b^{3}\sinh^{8}(c+dx)\cosh(c+dx)}{d} - \frac{8b^{3}\sinh^{6}(c+dx)\cosh^{3}(c+dx)}{3d} + \frac{16b^{3}\sinh^{4}(c+dx)\cosh^{5}(c+dx)}{5d} - \frac{64b^{3}\sinh^{2}(c+dx)\cosh^{7}(c+dx)}{35d} + \frac{128b^{3}\cosh^{9}(c+dx)}{315d} & \text{for } d \neq 0 \\ x\left(a + b\sinh^{3}(c)\right)^{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*b*co
sh(c + d*x)**3/d + 15*a*b**2*x*sinh(c + d*x)**6/16 - 45*a*b**2*x*sinh(c + d
*x)**4*cosh(c + d*x)**2/16 + 45*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/
16 - 15*a*b**2*x*cosh(c + d*x)**6/16 + 33*a*b**2*sinh(c + d*x)**5*cosh(c +
d*x)/(16*d) - 5*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) + 15*a*b**2*
sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + b**3*sinh(c + d*x)**8*cosh(c + d*x)
/d - 8*b**3*sinh(c + d*x)**6*cosh(c + d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)
**4*cosh(c + d*x)**5/(5*d) - 64*b**3*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*
d) + 128*b**3*cosh(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**3
, True))
```

Giac [A]
time = 0.43, size = 327, normalized size = 1.60

$$\frac{b^{3}e^{(9\,dx+9\,c)}}{4608\,d} - \frac{9\,b^{3}e^{(7\,dx+7\,c)}}{3584\,d} + \frac{ab^{2}e^{(6\,dx+6\,c)}}{128\,d} + \frac{9\,b^{3}e^{(5\,dx+5\,c)}}{640\,d} - \frac{9\,ab^{2}e^{(4\,dx+4\,c)}}{128\,d} + \frac{45\,ab^{2}e^{(2\,dx+2\,c)}}{128\,d} - \frac{45\,ab^{2}e^{(-2\,dx-2\,c)}}{128\,d} + \frac{9\,ab^{2}e^{(-4\,dx-4\,c)}}{128\,d} + \frac{9\,b^{3}e^{(-5\,dx-5\,c)}}{640\,d} - \frac{ab^{2}e^{(-6\,dx-6\,c)}}{128\,d} - \frac{9\,b^{3}e^{(-7\,dx-7\,c)}}{3584\,d} + \frac{b^{3}e^{(-9\,dx-9\,c)}}{4608\,d} + \frac{1}{16}\,(16\,a^{3} - 15\,ab^{2})x + \frac{(16\,a^{2}b - 7\,b^{3})e^{(3\,dx+3\,c)}}{128\,d} - \frac{9\,(32\,a^{2}b - 7\,b^{3})e^{(dx+c)}}{256\,d} - \frac{9\,(32\,a^{2}b - 7\,b^{3})e^{(-dx-c)}}{256\,d} + \frac{(16\,a^{2}b - 7\,b^{3})e^{(-3\,dx-3\,c)}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/4608*b^3*e^(9*d*x + 9*c)/d - 9/3584*b^3*e^(7*d*x + 7*c)/d + 1/128*a*b^2*e
^(6*d*x + 6*c)/d + 9/640*b^3*e^(5*d*x + 5*c)/d - 9/128*a*b^2*e^(4*d*x + 4*c
)/d + 45/128*a*b^2*e^(2*d*x + 2*c)/d - 45/128*a*b^2*e^(-2*d*x - 2*c)/d + 9/
128*a*b^2*e^(-4*d*x - 4*c)/d + 9/640*b^3*e^(-5*d*x - 5*c)/d - 1/128*a*b^2*e
^(-6*d*x - 6*c)/d - 9/3584*b^3*e^(-7*d*x - 7*c)/d + 1/4608*b^3*e^(-9*d*x -
9*c)/d + 1/16*(16*a^3 - 15*a*b^2)*x + 1/128*(16*a^2*b - 7*b^3)*e^(3*d*x + 3
*c)/d - 9/256*(32*a^2*b - 7*b^3)*e^(d*x + c)/d - 9/256*(32*a^2*b - 7*b^3)*e
^(-d*x - c)/d + 1/128*(16*a^2*b - 7*b^3)*e^(-3*d*x - 3*c)/d
```

Mupad [B]
time = 0.93, size = 164, normalized size = 0.80

$$\frac{d\,x\,a^{3} + a^{2}\,b\cosh(c+d\,x)^{3} - 3\,a^{2}\,b\cosh(c+d\,x) + \frac{\sinh(c+d\,x)\,a\,b^{2}\cosh(c+d\,x)^{5}}{2} - \frac{13\sinh(c+d\,x)\,a\,b^{2}\cosh(c+d\,x)^{3}}{8} + \frac{33\sinh(c+d\,x)\,a\,b^{2}\cosh(c+d\,x)}{16} - \frac{15\,d\,x\,a\,b^{2}}{16} + \frac{b^{3}\cosh(c+d\,x)^{9}}{9} - \frac{4\,b^{3}\cosh(c+d\,x)^{7}}{7} + \frac{6\,b^{3}\cosh(c+d\,x)^{5}}{5} - \frac{4\,b^{3}\cosh(c+d\,x)^{3}}{3} + b^{3}\cosh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3,x)
```

```
[Out] (b^3*cosh(c + d*x) - (4*b^3*cosh(c + d*x)^3)/3 + (6*b^3*cosh(c + d*x)^5)/5
- (4*b^3*cosh(c + d*x)^7)/7 + (b^3*cosh(c + d*x)^9)/9 + a^2*b*cosh(c + d*x)
^3 - 3*a^2*b*cosh(c + d*x) + a^3*d*x + (33*a*b^2*cosh(c + d*x)*sinh(c + d*x
))/16 - (15*a*b^2*d*x)/16 - (13*a*b^2*cosh(c + d*x)^3*sinh(c + d*x))/8 + (a
*b^2*cosh(c + d*x)^5*sinh(c + d*x))/2)/d
```

3.164 $\int \mathbf{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=201

$$-\frac{3}{2}a^2bx+\frac{35b^3x}{128}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{2ab^2\cosh^3(c+dx)}{d}+\frac{3ab^2\cosh^5(c+dx)}{5d}+\frac{3a}{}$$

```
[Out] -3/2*a^2*b*x+35/128*b^3*x-a^3*arctanh(cosh(d*x+c))/d+3*a*b^2*cosh(d*x+c)/d-
2*a*b^2*cosh(d*x+c)^3/d+3/5*a*b^2*cosh(d*x+c)^5/d+3/2*a^2*b*cosh(d*x+c)*sin
h(d*x+c)/d-35/128*b^3*cosh(d*x+c)*sinh(d*x+c)/d+35/192*b^3*cosh(d*x+c)*sinh
(d*x+c)^3/d-7/48*b^3*cosh(d*x+c)*sinh(d*x+c)^5/d+1/8*b^3*cosh(d*x+c)*sinh(d
*x+c)^7/d
```

Rubi [A]
time = 0.14, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3299, 3855, 2715, 8, 2713}

$$-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3}{2}a^2bx+\frac{3ab^2\cosh^5(c+dx)}{5d}-\frac{2ab^2\cosh^3(c+dx)}{d}+\frac{3ab^2\cosh(c+dx)}{d}+\frac{b^3\sinh^7(c+dx)\cosh(c+dx)}{8d}-\frac{7b^3\sinh^5(c+dx)\cosh(c+dx)}{48d}+\frac{35b^3\sinh^3(c+dx)\cosh(c+dx)}{192d}-\frac{35b^3\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{35b^3x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-3*a^2*b*x)/2 + (35*b^3*x)/128 - (a^3*ArcTanh[Cosh[c + d*x]])/d + (3*a*b^2
*Cosh[c + d*x])/d - (2*a*b^2*Cosh[c + d*x]^3)/d + (3*a*b^2*Cosh[c + d*x]^5)
/(5*d) + (3*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (35*b^3*Cosh[c + d*x
]*Sinh[c + d*x])/(128*d) + (35*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(192*d) -
 (7*b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(48*d) + (b^3*Cosh[c + d*x]*Sinh[c +
 d*x]^7)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx &= i\int\left(-ia^3\operatorname{csch}(c+dx)-3ia^2b\sinh^2(c+dx)-3iab^2\sinh^5(c+\right.\\
&= a^3\int\operatorname{csch}(c+dx)\,dx+\left(3a^2b\right)\int\sinh^2(c+dx)\,dx+\left(3ab^2\right)\int\\
&= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\cosh(c+dx)\sinh(c+dx)}{2d}+\\
&= -\frac{3}{2}a^2bx-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{2ab}{}\\
&= -\frac{3}{2}a^2bx-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{2ab}{}\\
&= -\frac{3}{2}a^2bx-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{2ab}{}\\
&= -\frac{3}{2}a^2bx+\frac{35b^3x}{128}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+}{d}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 158, normalized size = 0.79

$$\frac{-23040a^2bc+4200b^3c-23040a^2bdx+4200b^3dx+28800ab^2\cosh(c+dx)-4800ab^2\cosh(3(c+dx))+576ab^2\cosh(5(c+dx))+15360a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+11520a^2b\sinh(2(c+dx))-3360b^3\sinh(2(c+dx))+840b^3\sinh(4(c+dx))-160b^3\sinh(6(c+dx))+15b^3\sinh(8(c+dx))}{15360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-23040*a^2*b*c + 4200*b^3*c - 23040*a^2*b*d*x + 4200*b^3*d*x + 28800*a*b^2
*Cosh[c + d*x] - 4800*a*b^2*Cosh[3*(c + d*x)] + 576*a*b^2*Cosh[5*(c + d*x)]
 + 15360*a^3*Log[Tanh[(c + d*x)/2]] + 11520*a^2*b*Sinh[2*(c + d*x)] - 3360*
b^3*Sinh[2*(c + d*x)] + 840*b^3*Sinh[4*(c + d*x)] - 160*b^3*Sinh[6*(c + d*x
)] + 15*b^3*Sinh[8*(c + d*x)])/(15360*d)
```

Maple [**A**]
time = 2.08, size = 325, normalized size = 1.62

method	result
risch	$-\frac{3a^2bx}{2}+\frac{35b^3x}{128}+\frac{b^3\mathrm{e}^{8dx+8c}}{2048d}-\frac{b^3\mathrm{e}^{6dx+6c}}{192d}+\frac{3a\,b^2\mathrm{e}^{5dx+5c}}{160d}+\frac{7b^3\mathrm{e}^{4dx+4c}}{256d}-\frac{5a\,b^2\mathrm{e}^{3dx+3c}}{32d}+\frac{3b\,\mathrm{e}^{2dx+2c}a^2}{8d}-\frac{7b^3\mathrm{e}^{2dx+2c}}{64d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*a^2*b*x+35/128*b^3*x+1/2048*b^3/d*exp(8*d*x+8*c)-1/192*b^3/d*exp(6*d*x
+6*c)+3/160*a*b^2/d*exp(5*d*x+5*c)+7/256*b^3/d*exp(4*d*x+4*c)-5/32*a*b^2/d*
exp(3*d*x+3*c)+3/8*b/d*exp(2*d*x+2*c)*a^2-7/64*b^3/d*exp(2*d*x+2*c)+15/16*a
/d*exp(d*x+c)*b^2+15/16*a/d*exp(-d*x-c)*b^2-3/8*b/d*exp(-2*d*x-2*c)*a^2+7/6
4*b^3/d*exp(-2*d*x-2*c)-5/32*a*b^2/d*exp(-3*d*x-3*c)-7/256*b^3/d*exp(-4*d*x
-4*c)+3/160*a*b^2/d*exp(-5*d*x-5*c)+1/192*b^3/d*exp(-6*d*x-6*c)-1/2048*b^3/
d*exp(-8*d*x-8*c)+a^3/d*ln(exp(d*x+c)-1)-a^3/d*ln(exp(d*x+c)+1)
```

Maxima [**A**]
time = 0.29, size = 257, normalized size = 1.28

$$-\frac{3}{8}\,a^2b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{6144}\,b^3\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+672\,e^{(-6\,dx-6\,c)}-3\right)e^{(8\,dx+8\,c)}}{d}-\frac{1680\,(dx+c)}{d}-\frac{672\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+32\,e^{(-6\,dx-6\,c)}-3\,e^{(-8\,dx-8\,c)}}{d}\right)+\frac{1}{160}\,ab^2\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{a^3\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/6144*b^3*((32
*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d
*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*
c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 1/160*a*b^2*(3*e^(5*d*x
 + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d -
 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^3*log(tanh(1/2*d*x + 1/2
*c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2609 vs. $2(185) = 370$.
time = 0.45, size = 2609, normalized size = 12.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/30720*(15*b^3*cosh(d*x + c)^16 + 240*b^3*cosh(d*x + c)*sinh(d*x + c)^15 +
 15*b^3*sinh(d*x + c)^16 - 160*b^3*cosh(d*x + c)^14 + 576*a*b^2*cosh(d*x +
```

```
c)^13 + 840*b^3*cosh(d*x + c)^12 + 40*(45*b^3*cosh(d*x + c)^2 - 4*b^3)*sinh
(d*x + c)^14 - 4800*a*b^2*cosh(d*x + c)^11 + 16*(525*b^3*cosh(d*x + c)^3 -
140*b^3*cosh(d*x + c) + 36*a*b^2)*sinh(d*x + c)^13 + 4*(6825*b^3*cosh(d*x +
 c)^4 - 3640*b^3*cosh(d*x + c)^2 + 1872*a*b^2*cosh(d*x + c) + 210*b^3)*sinh
(d*x + c)^12 + 28800*a*b^2*cosh(d*x + c)^9 + 16*(4095*b^3*cosh(d*x + c)^5 -
 3640*b^3*cosh(d*x + c)^3 + 2808*a*b^2*cosh(d*x + c)^2 + 630*b^3*cosh(d*x +
 c) - 300*a*b^2)*sinh(d*x + c)^11 - 240*(192*a^2*b - 35*b^3)*d*x*cosh(d*x +
 c)^8 + 480*(24*a^2*b - 7*b^3)*cosh(d*x + c)^10 + 8*(15015*b^3*cosh(d*x + c
)^6 - 20020*b^3*cosh(d*x + c)^4 + 20592*a*b^2*cosh(d*x + c)^3 + 6930*b^3*co
sh(d*x + c)^2 - 6600*a*b^2*cosh(d*x + c) + 1440*a^2*b - 420*b^3)*sinh(d*x +
 c)^10 + 28800*a*b^2*cosh(d*x + c)^7 + 80*(2145*b^3*cosh(d*x + c)^7 - 4004*
b^3*cosh(d*x + c)^5 + 5148*a*b^2*cosh(d*x + c)^4 + 2310*b^3*cosh(d*x + c)^3
 - 3300*a*b^2*cosh(d*x + c)^2 + 360*a*b^2 + 60*(24*a^2*b - 7*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^9 + 6*(32175*b^3*cosh(d*x + c)^8 - 80080*b^3*cosh(d*x +
 c)^6 + 123552*a*b^2*cosh(d*x + c)^5 + 69300*b^3*cosh(d*x + c)^4 - 132000*a
*b^2*cosh(d*x + c)^3 + 43200*a*b^2*cosh(d*x + c) - 40*(192*a^2*b - 35*b^3)*
d*x + 3600*(24*a^2*b - 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 - 4800*a*b^2
*cosh(d*x + c)^5 + 48*(3575*b^3*cosh(d*x + c)^9 - 11440*b^3*cosh(d*x + c)^7
 + 20592*a*b^2*cosh(d*x + c)^6 + 13860*b^3*cosh(d*x + c)^5 - 33000*a*b^2*co
sh(d*x + c)^4 + 21600*a*b^2*cosh(d*x + c)^2 - 40*(192*a^2*b - 35*b^3)*d*x*c
osh(d*x + c) + 1200*(24*a^2*b - 7*b^3)*cosh(d*x + c)^3 + 600*a*b^2)*sinh(d*
x + c)^7 - 840*b^3*cosh(d*x + c)^4 - 480*(24*a^2*b - 7*b^3)*cosh(d*x + c)^6
 + 24*(5005*b^3*cosh(d*x + c)^10 - 20020*b^3*cosh(d*x + c)^8 + 41184*a*b^2*
cosh(d*x + c)^7 + 32340*b^3*cosh(d*x + c)^6 - 92400*a*b^2*cosh(d*x + c)^5 +
 100800*a*b^2*cosh(d*x + c)^3 - 280*(192*a^2*b - 35*b^3)*d*x*cosh(d*x + c)^
2 + 4200*(24*a^2*b - 7*b^3)*cosh(d*x + c)^4 + 8400*a*b^2*cosh(d*x + c) - 48
0*a^2*b + 140*b^3)*sinh(d*x + c)^6 + 576*a*b^2*cosh(d*x + c)^3 + 16*(4095*b
^3*cosh(d*x + c)^11 - 20020*b^3*cosh(d*x + c)^9 + 46332*a*b^2*cosh(d*x + c)
^8 + 41580*b^3*cosh(d*x + c)^7 - 138600*a*b^2*cosh(d*x + c)^6 + 226800*a*b^
2*cosh(d*x + c)^4 - 840*(192*a^2*b - 35*b^3)*d*x*cosh(d*x + c)^3 + 7560*(24
*a^2*b - 7*b^3)*cosh(d*x + c)^5 + 37800*a*b^2*cosh(d*x + c)^2 - 300*a*b^2 -
 180*(24*a^2*b - 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 160*b^3*cosh(d*x +
 c)^2 + 20*(1365*b^3*cosh(d*x + c)^12 - 8008*b^3*cosh(d*x + c)^10 + 20592*a
*b^2*cosh(d*x + c)^9 + 20790*b^3*cosh(d*x + c)^8 - 79200*a*b^2*cosh(d*x + c
)^7 + 181440*a*b^2*cosh(d*x + c)^5 - 840*(192*a^2*b - 35*b^3)*d*x*cosh(d*x
+ c)^4 + 5040*(24*a^2*b - 7*b^3)*cosh(d*x + c)^6 + 50400*a*b^2*cosh(d*x + c
)^3 - 1200*a*b^2*cosh(d*x + c) - 42*b^3 - 360*(24*a^2*b - 7*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^4 + 16*(525*b^3*cosh(d*x + c)^13 - 3640*b^3*cosh(d*x +
 c)^11 + 10296*a*b^2*cosh(d*x + c)^10 + 11550*b^3*cosh(d*x + c)^9 - 49500*a
*b^2*cosh(d*x + c)^8 + 151200*a*b^2*cosh(d*x + c)^6 - 840*(192*a^2*b - 35*b
^3)*d*x*cosh(d*x + c)^5 + 3600*(24*a^2*b - 7*b^3)*cosh(d*x + c)^7 + 63000*a
*b^2*cosh(d*x + c)^4 - 3000*a*b^2*cosh(d*x + c)^2 - 210*b^3*cosh(d*x + c) -
 600*(24*a^2*b - 7*b^3)*cosh(d*x + c)^3 + 36*a*b^2)*sinh(d*x + c)^3 - 15*b^
3 + 8*(225*b^3*cosh(d*x + c)^14 - 1820*b^3*cosh(d*x + c)^12 + 5616*a*b^2*co
sh(d*x + c)^11 + 6930*b^3*cosh(d*x + c)^10 - 33000*a*b^2*cosh(d*x + c)^9 +
```

```
129600*a*b^2*cosh(d*x + c)^7 - 840*(192*a^2*b - 35*b^3)*d*x*cosh(d*x + c)^6
 + 2700*(24*a^2*b - 7*b^3)*cosh(d*x + c)^8 + 75600*a*b^2*cosh(d*x + c)^5 -
6000*a*b^2*cosh(d*x + c)^3 - 630*b^3*cosh(d*x + c)^2 - 900*(24*a^2*b - 7*b^
3)*cosh(d*x + c)^4 + 216*a*b^2*cosh(d*x + c) + 20*b^3)*sinh(d*x + c)^2 - 30
720*(a^3*cosh(d*x + c)^8 + 8*a^3*cosh(d*x + c)^7*sinh(d*x + c) + 28*a^3*cos
h(d*x + c)^6*sinh(d*x + c)^2 + 56*a^3*cosh(d*x + c)^5*sinh(d*x + c)^3 + 70*
a^3*cosh(d*x + c)^4*sinh(d*x + c)^4 + 56*a^3*cosh(d*x + c)^3*sinh(d*x + c)^
5 + 28*a^3*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*a^3*cosh(d*x + c)*sinh(d*x +
 c)^7 + a^3*sinh(d*x + c)^8)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 30720
*(a^3*cosh(d*x + c)^8 + 8*a^3*cosh(d*x + c)^7*sinh(d*x + c) + 28*a^3*cosh(d
*x + c)^6*sinh(d*x + c)^2 + 56*a^3*cosh(d*x + c)^5*sinh(d*x + c)^3 + 70*a^3
*cosh(d*x + c)^4*sinh(d*x + c)^4 + 56*a^3*cosh(d*x + c)^3*sinh(d*x + c)^5 +
 28*a^3*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*a^3*cosh(d*x + c)*sinh(d*x + c)
^7 + a^3*sinh(d*x + c)^8)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 16*(15*b
^3*cosh(d*x + c)^15 - 140*b^3*cosh(d*x + c)^13 + 468*a*b^2*cosh(d*x + c)^12
 + 630*b^3*cosh(d*x + c)^11 - 3300*a*b^2*cosh(d*x + c)^10 + 16200*a*b^2*cos
h(d*x + c)^8 - 120*(192*a^2*b - 35*b^3)*d*x*cosh(d*x + c)^7 + 300*(24*a^2*b
 - 7*b^3)*cosh(d*x + c)^9 + 12600*a*b^2*cosh(d*x + c)^6 - 1500*a*b^2*cosh(d
*x + c)^4 - 210*b^3*cosh(d*x + c)^3 - 180*(24*a^2*b - 7*b^3)*cosh(d*x + c)^
5 + 108*a*b^2*cosh(d*x + c)^2 + 20*b^3*cosh(d*x...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 279, normalized size = 1.39

$$\frac{15\,b^{3}e^{(8\,dx+8\,c)} - 160\,b^{3}e^{(6\,dx+6\,c)} + 576\,ab^{2}e^{(5\,dx+5\,c)} + 840\,b^{3}e^{(4\,dx+4\,c)} - 4800\,ab^{2}e^{(3\,dx+3\,c)} + 11520\,a^{2}be^{(2\,dx+2\,c)} - 3360\,b^{3}e^{(2\,dx+2\,c)} + 28800\,ab^{2}e^{(dx+c)} - 30720\,a^{3}\log\left(e^{(dx+c)}+1\right) + 30720\,a^{3}\log\left(\left|e^{(dx+c)}-1\right|\right) - 240\,(192\,a^{2}b - 35\,b^{3})(dx+c) + \left(28800\,ab^{2}e^{(7\,dx+7\,c)} - 4800\,ab^{2}e^{(5\,dx+5\,c)} - 840\,b^{3}e^{(4\,dx+4\,c)} + 576\,ab^{2}e^{(3\,dx+3\,c)} + 160\,b^{3}e^{(2\,dx+2\,c)} - 15\,b^{3} - 480\,(24\,a^{2}b - 7\,b^{3})e^{(6\,dx+6\,c)}\right)e^{(-8\,dx-8\,c)}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/30720*(15*b^3*e^(8*d*x + 8*c) - 160*b^3*e^(6*d*x + 6*c) + 576*a*b^2*e^(5*
d*x + 5*c) + 840*b^3*e^(4*d*x + 4*c) - 4800*a*b^2*e^(3*d*x + 3*c) + 11520*a
^2*b*e^(2*d*x + 2*c) - 3360*b^3*e^(2*d*x + 2*c) + 28800*a*b^2*e^(d*x + c) -
 30720*a^3*log(e^(d*x + c) + 1) + 30720*a^3*log(abs(e^(d*x + c) - 1)) - 240
*(192*a^2*b - 35*b^3)*(d*x + c) + (28800*a*b^2*e^(7*d*x + 7*c) - 4800*a*b^2
*e^(5*d*x + 5*c) - 840*b^3*e^(4*d*x + 4*c) + 576*a*b^2*e^(3*d*x + 3*c) + 16
```

```
0*b^3*e^(2*d*x + 2*c) - 15*b^3 - 480*(24*a^2*b - 7*b^3)*e^(6*d*x + 6*c))*e^
(-8*d*x - 8*c))/d
```

Mupad [B]
time = 0.50, size = 315, normalized size = 1.57

$$\frac{7\,b^3\,\mathrm{e}^{4\,c+4\,d\,x}}{256\,d}-\frac{2\,\mathrm{atan}\left(\frac{a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}}-\frac{7\,b^3\,\mathrm{e}^{-4\,c-4\,d\,x}}{256\,d}-x\,\left(\frac{3\,a^2\,b}{2}-\frac{35\,b^3}{128}\right)+\frac{b^3\,\mathrm{e}^{-6\,c-6\,d\,x}}{192\,d}-\frac{b^3\,\mathrm{e}^{6\,c+6\,d\,x}}{192\,d}-\frac{b^3\,\mathrm{e}^{-8\,c-8\,d\,x}}{2048\,d}+\frac{b^3\,\mathrm{e}^{8\,c+8\,d\,x}}{2048\,d}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,\left(24\,a^2\,b-7\,b^3\right)}{64\,d}+\frac{\mathrm{e}^{2\,c+2\,d\,x}\,\left(24\,a^2\,b-7\,b^3\right)}{64\,d}+\frac{15\,a\,b^2\,\mathrm{e}^{-c-d\,x}}{16\,d}-\frac{5\,a\,b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{32\,d}-\frac{5\,a\,b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{32\,d}+\frac{3\,a\,b^2\,\mathrm{e}^{-5\,c-5\,d\,x}}{160\,d}+\frac{3\,a\,b^2\,\mathrm{e}^{5\,c+5\,d\,x}}{160\,d}+\frac{15\,a\,b^2\,\mathrm{e}^{c+d\,x}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x),x)
```

```
[Out] (7*b^3*exp(4*c + 4*d*x))/(256*d) - (2*atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2
))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/2) - (7*b^3*exp(- 4*c - 4*d*x))/
(256*d) - x*((3*a^2*b)/2 - (35*b^3)/128) + (b^3*exp(- 6*c - 6*d*x))/(192*d)
 - (b^3*exp(6*c + 6*d*x))/(192*d) - (b^3*exp(- 8*c - 8*d*x))/(2048*d) + (b^
3*exp(8*c + 8*d*x))/(2048*d) - (exp(- 2*c - 2*d*x)*(24*a^2*b - 7*b^3))/(64*
d) + (exp(2*c + 2*d*x)*(24*a^2*b - 7*b^3))/(64*d) + (15*a*b^2*exp(- c - d*x
))/(16*d) - (5*a*b^2*exp(- 3*c - 3*d*x))/(32*d) - (5*a*b^2*exp(3*c + 3*d*x)
)/(32*d) + (3*a*b^2*exp(- 5*c - 5*d*x))/(160*d) + (3*a*b^2*exp(5*c + 5*d*x)
)/(160*d) + (15*a*b^2*exp(c + d*x))/(16*d)
```

3.165 $\int \mathrm{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=152

$$\frac{9}{8}ab^2x+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{d}-\frac{3b^3\cosh^5(c+dx)}{5d}+\frac{b^3\cosh^7(c+dx)}{7d}-a^3\,\mathrm{cot}$$

```
[Out] 9/8*a*b^2*x+3*a^2*b*cosh(d*x+c)/d-b^3*cosh(d*x+c)/d+b^3*cosh(d*x+c)^3/d-3/5
*b^3*cosh(d*x+c)^5/d+1/7*b^3*cosh(d*x+c)^7/d-a^3*coth(d*x+c)/d-9/8*a*b^2*co
sh(d*x+c)*sinh(d*x+c)/d+3/4*a*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Rubi [A]

time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3299, 3852, 8, 2718, 2715, 2713}

$$-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\cosh(c+dx)}{d}+\frac{3ab^2\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{9ab^2\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{9}{8}ab^2x+\frac{b^3\cosh^7(c+dx)}{7d}-\frac{3b^3\cosh^5(c+dx)}{5d}+\frac{b^3\cosh^3(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (9*a*b^2*x)/8 + (3*a^2*b*Cosh[c + d*x])/d - (b^3*Cosh[c + d*x])/d + (b^3*Co
sh[c + d*x]^3)/d - (3*b^3*Cosh[c + d*x]^5)/(5*d) + (b^3*Cosh[c + d*x]^7)/(7
*d) - (a^3*Coth[c + d*x])/d - (9*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) +
 (3*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx &= -\int \left(-a^3\operatorname{csch}^2(c+dx)-3a^2b\sinh(c+dx)-3ab^2\sinh^4(c+\\ &= a^3\int \operatorname{csch}^2(c+dx)\,dx+\left(3a^2b\right)\int \sinh(c+dx)\,dx+\left(3ab^2\right)\int\\ &= \frac{3a^2b\cosh(c+dx)}{d}+\frac{3ab^2\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{4}(9ab\\ &= \frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{d}-3b\\ &= \frac{9}{8}ab^2x+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+}{d}\end{aligned}$$

Mathematica [**A**]
time = 0.77, size = 140, normalized size = 0.92

$$\frac{2520ab^2c+2520ab^2dx+35b(192a^2-35b^2)\cosh(c+dx)+245b^3\cosh(3(c+dx))-49b^3\cosh(5(c+dx))+5b^3\cosh(7(c+dx))-1120a^3\coth\left(\frac{1}{2}(c+dx)\right)-1680ab^2\sinh(2(c+dx))+210ab^2\sinh(4(c+dx))-1120a^3\tanh\left(\frac{1}{2}(c+dx)\right)}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (2520*a*b^2*c + 2520*a*b^2*d*x + 35*b*(192*a^2 - 35*b^2)*Cosh[c + d*x] + 24
5*b^3*Cosh[3*(c + d*x)] - 49*b^3*Cosh[5*(c + d*x)] + 5*b^3*Cosh[7*(c + d*x)
] - 1120*a^3*Coth[(c + d*x)/2] - 1680*a*b^2*Sinh[2*(c + d*x)] + 210*a*b^2*S
inh[4*(c + d*x)] - 1120*a^3*Tanh[(c + d*x)/2])/(2240*d)
```

Maple [**A**]
time = 1.90, size = 268, normalized size = 1.76

method	result
risch	$\frac{9a\,b^2x}{8}+\frac{b^3\mathrm{e}^{7dx+7c}}{896d}-\frac{7b^3\mathrm{e}^{5dx+5c}}{640d}+\frac{3a\,b^2\mathrm{e}^{4dx+4c}}{64d}+\frac{7b^3\mathrm{e}^{3dx+3c}}{128d}-\frac{3a\,\mathrm{e}^{2dx+2c}b^2}{8d}+\frac{3b\,\mathrm{e}^{dx+c}a^2}{2d}-\frac{35b^3\mathrm{e}^{dx+c}}{128d}+\frac{3b\,\mathrm{e}^{-dx-c}}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 9/8*a*b^2*x+1/896*b^3/d*exp(7*d*x+7*c)-7/640*b^3/d*exp(5*d*x+5*c)+3/64*a*b^
2/d*exp(4*d*x+4*c)+7/128*b^3/d*exp(3*d*x+3*c)-3/8*a/d*exp(2*d*x+2*c)*b^2+3/
2*b/d*exp(d*x+c)*a^2-35/128*b^3/d*exp(d*x+c)+3/2*b/d*exp(-d*x-c)*a^2-35/128
*b^3/d*exp(-d*x-c)+3/8*a/d*exp(-2*d*x-2*c)*b^2+7/128*b^3/d*exp(-3*d*x-3*c)-
3/64*a*b^2/d*exp(-4*d*x-4*c)-7/640*b^3/d*exp(-5*d*x-5*c)+1/896*b^3/d*exp(-7
*d*x-7*c)-2*a^3/d/(exp(2*d*x+2*c)-1)
```

Maxima [A]
time = 0.29, size = 220, normalized size = 1.45

$$\frac{3}{64}\,ab^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{4480}\,b^3\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d}\right)+\frac{3}{2}\,a^2b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{2\,a^3}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4
*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x
- c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d)
+ 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a^3/(d*(e^(-2*d*x - 2*c) -
 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(142) = 284.
time = 0.49, size = 302, normalized size = 1.99

$$\frac{20\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+105\,ab^2\cosh\left(dx+c\right)^5+525\,ab^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-945\,ab^2\cosh\left(dx+c\right)^3+2\,\left(70\,b^3\cosh\left(dx+c\right)^3-81\,b^3\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^5+4\,\left(35\,b^3\cosh\left(dx+c\right)^5-135\,b^3\cosh\left(dx+c\right)^3+147\,b^3\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3+105\,\left(10\,ab^2\cosh\left(dx+c\right)^3-27\,ab^2\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2-280\,\left(8\,a^3-3\,ab^2\right)\cosh\left(dx+c\right)+2\,\left(10\,b^3\cosh\left(dx+c\right)^7-81\,b^3\cosh\left(dx+c\right)^5+294\,b^3\cosh\left(dx+c\right)^3+1260\,ab^2dx+1120\,a^3+105\,\left(32\,a^2b-7\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{2240\,d\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(20*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 105*a*b^2*cosh(d*x + c)^5 +
525*a*b^2*cosh(d*x + c)*sinh(d*x + c)^4 - 945*a*b^2*cosh(d*x + c)^3 + 2*(70
*b^3*cosh(d*x + c)^3 - 81*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(35*b^3*co
sh(d*x + c)^5 - 135*b^3*cosh(d*x + c)^3 + 147*b^3*cosh(d*x + c))*sinh(d*x +
 c)^3 + 105*(10*a*b^2*cosh(d*x + c)^3 - 27*a*b^2*cosh(d*x + c))*sinh(d*x +
c)^2 - 280*(8*a^3 - 3*a*b^2)*cosh(d*x + c) + 2*(10*b^3*cosh(d*x + c)^7 - 81
```

```
*b^3*cosh(d*x + c)^5 + 294*b^3*cosh(d*x + c)^3 + 1260*a*b^2*d*x + 1120*a^3
+ 105*(32*a^2*b - 7*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.50, size = 276, normalized size = 1.82

$$\frac{5040\,(dx+c)ab^2+5\,b^3e^{(7\,dx+7\,c)}-49\,b^3e^{(5\,dx+5\,c)}+210\,ab^2e^{(4\,dx+4\,c)}+245\,b^3e^{(3\,dx+3\,c)}-1680\,ab^2e^{(2\,dx+2\,c)}+6720\,a^2be^{(dx+c)}-1225\,b^3e^{(dx+c)}-\frac{\left(1890\,ab^2e^{(5\,dx+5\,c)}+294\,b^3e^{(4\,dx+4\,c)}-210\,ab^2e^{(3\,dx+3\,c)}-54\,b^3e^{(2\,dx+2\,c)}+5\,b^3-35\,(192\,a^2b-35\,b^3)e^{(8\,dx+8\,c)}+560\,(16\,a^3-3\,ab^2)e^{(7\,dx+7\,c)}+210\,(32\,a^2b-7\,b^3)e^{(6\,dx+6\,c)}\right)e^{(-7\,dx-7\,c)}}{\left(e^{(dx+c)}+1\right)\left(e^{(dx+c)}-1\right)}}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5040*(d*x + c)*a*b^2 + 5*b^3*e^(7*d*x + 7*c) - 49*b^3*e^(5*d*x + 5*
c) + 210*a*b^2*e^(4*d*x + 4*c) + 245*b^3*e^(3*d*x + 3*c) - 1680*a*b^2*e^(2*
d*x + 2*c) + 6720*a^2*b*e^(d*x + c) - 1225*b^3*e^(d*x + c) - (1890*a*b^2*e^
(5*d*x + 5*c) + 294*b^3*e^(4*d*x + 4*c) - 210*a*b^2*e^(3*d*x + 3*c) - 54*b^
3*e^(2*d*x + 2*c) + 5*b^3 - 35*(192*a^2*b - 35*b^3)*e^(8*d*x + 8*c) + 560*(
16*a^3 - 3*a*b^2)*e^(7*d*x + 7*c) + 210*(32*a^2*b - 7*b^3)*e^(6*d*x + 6*c))
*e^(-7*d*x - 7*c)/((e^(d*x + c) + 1)*(e^(d*x + c) - 1)))/d
```

Mupad [**B**]
time = 0.40, size = 252, normalized size = 1.66

$$\frac{\mathrm{e}^{c+d\,x}\,\left(192\,a^2\,b-35\,b^3\right)}{128\,d}-\frac{2\,a^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}+\frac{7\,b^3\,{\mathrm{e}}^{-3\,c-3\,d\,x}}{128\,d}+\frac{7\,b^3\,{\mathrm{e}}^{3\,c+3\,d\,x}}{128\,d}-\frac{7\,b^3\,{\mathrm{e}}^{-5\,c-5\,d\,x}}{640\,d}-\frac{7\,b^3\,{\mathrm{e}}^{5\,c+5\,d\,x}}{640\,d}+\frac{b^3\,{\mathrm{e}}^{-7\,c-7\,d\,x}}{896\,d}+\frac{b^3\,{\mathrm{e}}^{7\,c+7\,d\,x}}{896\,d}+\frac{{\mathrm{e}}^{-c-d\,x}\,\left(192\,a^2\,b-35\,b^3\right)}{128\,d}+\frac{9\,a\,b^2\,x}{8}+\frac{3\,a\,b^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}-\frac{3\,a\,b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}-\frac{3\,a\,b^2\,{\mathrm{e}}^{-4\,c-4\,d\,x}}{64\,d}+\frac{3\,a\,b^2\,{\mathrm{e}}^{4\,c+4\,d\,x}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^2,x)
```

```
[Out] (exp(c + d*x)*(192*a^2*b - 35*b^3))/(128*d) - (2*a^3)/(d*(exp(2*c + 2*d*x)
- 1)) + (7*b^3*exp(- 3*c - 3*d*x))/(128*d) + (7*b^3*exp(3*c + 3*d*x))/(128*
d) - (7*b^3*exp(- 5*c - 5*d*x))/(640*d) - (7*b^3*exp(5*c + 5*d*x))/(640*d)
+ (b^3*exp(- 7*c - 7*d*x))/(896*d) + (b^3*exp(7*c + 7*d*x))/(896*d) + (exp(
- c - d*x)*(192*a^2*b - 35*b^3))/(128*d) + (9*a*b^2*x)/8 + (3*a*b^2*exp(- 2
*c - 2*d*x))/(8*d) - (3*a*b^2*exp(2*c + 2*d*x))/(8*d) - (3*a*b^2*exp(- 4*c
- 4*d*x))/(64*d) + (3*a*b^2*exp(4*c + 4*d*x))/(64*d)
```

3.166 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=156

$$3a^2bx-\frac{5b^3x}{16}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{ab^2\cosh^3(c+dx)}{d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

```
[Out] 3*a^2*b*x-5/16*b^3*x+1/2*a^3*arctanh(cosh(d*x+c))/d-3*a*b^2*cosh(d*x+c)/d+a
*b^2*cosh(d*x+c)^3/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d+5/16*b^3*cosh(d*x+c)
*sinh(d*x+c)/d-5/24*b^3*cosh(d*x+c)*sinh(d*x+c)^3/d+1/6*b^3*cosh(d*x+c)*sin
h(d*x+c)^5/d
```

Rubi [A]

time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3299, 3853, 3855, 2713, 2715, 8}

$$\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+3a^2bx+\frac{ab^2\cosh^3(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{b^3\sinh^5(c+dx)\cosh(c+dx)}{6d}-\frac{5b^3\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{5b^3\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{5b^3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] 3*a^2*b*x - (5*b^3*x)/16 + (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) - (3*a*b^2*Co
sh[c + d*x])/d + (a*b^2*Cosh[c + d*x]^3)/d - (a^3*Coth[c + d*x]*Csch[c + d*
x])/(2*d) + (5*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*b^3*Cosh[c + d*
x]*Sinh[c + d*x]^3)/(24*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx &= -\Bigg(i\int\left(3ia^2b+ia^3\operatorname{csch}^3(c+dx)+3iab^2\sinh^3(c+dx)+ib^3\,\mathrm{s}\right.\\
&= 3a^2bx+a^3\int\operatorname{csch}^3(c+dx)\,dx+\left(3ab^2\right)\int\sinh^3(c+dx)\,dx+\\
&= 3a^2bx-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b^3\cosh(c+dx)\sinh^5(}{6d}\\
&= 3a^2bx+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{ab^2\,\mathrm{c}}{}\\
&= 3a^2bx+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{ab^2\,\mathrm{c}}{}\\
&= 3a^2bx-\frac{5b^3x}{16}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{3ab^2\cosh(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 2.29, size = 150, normalized size = 0.96

$$\frac{576a^2bc-60b^3c+576a^2bdx-60b^3dx-432ab^2\cosh(c+dx)+48ab^2\cosh(3(c+dx))-24a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)-96a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-24a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+45b^3\sinh(2(c+dx))-9b^3\sinh(4(c+dx))+b^3\sinh(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (576*a^2*b*c - 60*b^3*c + 576*a^2*b*d*x - 60*b^3*d*x - 432*a*b^2*Cosh[c + d
*x] + 48*a*b^2*Cosh[3*(c + d*x)] - 24*a^3*Csch[(c + d*x)/2]^2 - 96*a^3*Log[
```

```
Tanh[(c + d*x)/2]] - 24*a^3*Sech[(c + d*x)/2]^2 + 45*b^3*Sinh[2*(c + d*x)]
- 9*b^3*Sinh[4*(c + d*x)] + b^3*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 2.00, size = 258, normalized size = 1.65

method	result
risch	$3a^2bx - \frac{5b^3x}{16} + \frac{b^3 e^{6dx+6c}}{384d} - \frac{3b^3 e^{4dx+4c}}{128d} + \frac{a\,b^2 e^{3dx+3c}}{8d} + \frac{15b^3 e^{2dx+2c}}{128d} - \frac{9a\,e^{dx+c}b^2}{8d} - \frac{9a\,e^{-dx-c}b^2}{8d} - \frac{15b^3 e^{-2dx-2c}}{128d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a^2*b*x-5/16*b^3*x+1/384*b^3/d*exp(6*d*x+6*c)-3/128*b^3/d*exp(4*d*x+4*c)+
1/8*a*b^2/d*exp(3*d*x+3*c)+15/128*b^3/d*exp(2*d*x+2*c)-9/8*a/d*exp(d*x+c)*b
^2-9/8*a/d*exp(-d*x-c)*b^2-15/128*b^3/d*exp(-2*d*x-2*c)+1/8*a*b^2/d*exp(-3*
d*x-3*c)+3/128*b^3/d*exp(-4*d*x-4*c)-1/384*b^3/d*exp(-6*d*x-6*c)-a^3*exp(d*
x+c)*(1+exp(2*d*x+2*c))/d/(exp(2*d*x+2*c)-1)^2+1/2*a^3/d*ln(exp(d*x+c)+1)-1
/2*a^3/d*ln(exp(d*x+c)-1)
```

Maxima [A]
time = 0.29, size = 244, normalized size = 1.56

$$3\,a^2bx - \frac{1}{384}\,b^3\left(\frac{\left(9\,e^{(-2\,dx-2\,c)} - 45\,e^{(-4\,dx-4\,c)} - 1\right)e^{(6\,dx+6\,c)}}{d} + \frac{120\,(dx+c)}{d} + \frac{45\,e^{(-2\,dx-2\,c)} - 9\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)}}{d}\right) + \frac{1}{8}\,ab^2\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right) + \frac{1}{2}\,a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3*a^2*b*x - 1/384*b^3*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*
d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c)
+ e^(-6*d*x - 6*c))/d) + 1/8*a*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9
*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 1/2*a^3*(log(e^(-d*x - c) + 1)/d -
log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3627 vs. $2(144) = 288$.
time = 0.50, size = 3627, normalized size = 23.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(b^3*cosh(d*x + c)^16 + 16*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + b^3*s
inh(d*x + c)^16 - 11*b^3*cosh(d*x + c)^14 + 48*a*b^2*cosh(d*x + c)^13 + 64*
```

```
b^3*cosh(d*x + c)^12 + (120*b^3*cosh(d*x + c)^2 - 11*b^3)*sinh(d*x + c)^14
- 528*a*b^2*cosh(d*x + c)^11 + 2*(280*b^3*cosh(d*x + c)^3 - 77*b^3*cosh(d*x
 + c) + 24*a*b^2)*sinh(d*x + c)^13 + (1820*b^3*cosh(d*x + c)^4 - 1001*b^3*c
osh(d*x + c)^2 + 624*a*b^2*cosh(d*x + c) + 64*b^3)*sinh(d*x + c)^12 + 4*(10
92*b^3*cosh(d*x + c)^5 - 1001*b^3*cosh(d*x + c)^3 + 936*a*b^2*cosh(d*x + c)
^2 + 192*b^3*cosh(d*x + c) - 132*a*b^2)*sinh(d*x + c)^11 - 48*(48*a^2*b - 5
*b^3)*d*x*cosh(d*x + c)^8 - 3*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x
+ c)^10 + (8008*b^3*cosh(d*x + c)^6 - 11011*b^3*cosh(d*x + c)^4 + 13728*a*b
^2*cosh(d*x + c)^3 + 4224*b^3*cosh(d*x + c)^2 - 5808*a*b^2*cosh(d*x + c) -
99*b^3 + 24*(48*a^2*b - 5*b^3)*d*x)*sinh(d*x + c)^10 - 96*(4*a^3 - 5*a*b^2)
*cosh(d*x + c)^9 + 2*(5720*b^3*cosh(d*x + c)^7 - 11011*b^3*cosh(d*x + c)^5
+ 17160*a*b^2*cosh(d*x + c)^4 + 7040*b^3*cosh(d*x + c)^3 - 14520*a*b^2*cosh
(d*x + c)^2 - 192*a^3 + 240*a*b^2 - 15*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*
cosh(d*x + c))*sinh(d*x + c)^9 + 3*(4290*b^3*cosh(d*x + c)^8 - 11011*b^3*co
sh(d*x + c)^6 + 20592*a*b^2*cosh(d*x + c)^5 + 10560*b^3*cosh(d*x + c)^4 - 2
9040*a*b^2*cosh(d*x + c)^3 - 16*(48*a^2*b - 5*b^3)*d*x - 45*(33*b^3 - 8*(48
*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^2 - 288*(4*a^3 - 5*a*b^2)*cosh(d*x + c))
*sinh(d*x + c)^8 - 528*a*b^2*cosh(d*x + c)^5 - 96*(4*a^3 - 5*a*b^2)*cosh(d*
x + c)^7 + 8*(1430*b^3*cosh(d*x + c)^9 - 4719*b^3*cosh(d*x + c)^7 + 10296*a
*b^2*cosh(d*x + c)^6 + 6336*b^3*cosh(d*x + c)^5 - 21780*a*b^2*cosh(d*x + c)
^4 - 48*(48*a^2*b - 5*b^3)*d*x*cosh(d*x + c) - 45*(33*b^3 - 8*(48*a^2*b - 5
*b^3)*d*x)*cosh(d*x + c)^3 - 48*a^3 + 60*a*b^2 - 432*(4*a^3 - 5*a*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^7 - 64*b^3*cosh(d*x + c)^4 + 3*(33*b^3 + 8*(48*a
^2*b - 5*b^3)*d*x)*cosh(d*x + c)^6 + (8008*b^3*cosh(d*x + c)^10 - 33033*b^3
*cosh(d*x + c)^8 + 82368*a*b^2*cosh(d*x + c)^7 + 59136*b^3*cosh(d*x + c)^6
- 243936*a*b^2*cosh(d*x + c)^5 - 1344*(48*a^2*b - 5*b^3)*d*x*cosh(d*x + c)^
2 - 630*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^4 - 8064*(4*a^3 -
 5*a*b^2)*cosh(d*x + c)^3 + 99*b^3 + 24*(48*a^2*b - 5*b^3)*d*x - 672*(4*a^3
 - 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 48*a*b^2*cosh(d*x + c)^3 + 2*(
2184*b^3*cosh(d*x + c)^11 - 11011*b^3*cosh(d*x + c)^9 + 30888*a*b^2*cosh(d*
x + c)^8 + 25344*b^3*cosh(d*x + c)^7 - 121968*a*b^2*cosh(d*x + c)^6 - 1344*
(48*a^2*b - 5*b^3)*d*x*cosh(d*x + c)^3 - 378*(33*b^3 - 8*(48*a^2*b - 5*b^3)
*d*x)*cosh(d*x + c)^5 - 6048*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^4 - 264*a*b^2
- 1008*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^2 + 9*(33*b^3 + 8*(48*a^2*b - 5*b^3)
*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 11*b^3*cosh(d*x + c)^2 + (1820*b^3*c
osh(d*x + c)^12 - 11011*b^3*cosh(d*x + c)^10 + 34320*a*b^2*cosh(d*x + c)^9
+ 31680*b^3*cosh(d*x + c)^8 - 174240*a*b^2*cosh(d*x + c)^7 - 3360*(48*a^2*b
 - 5*b^3)*d*x*cosh(d*x + c)^4 - 630*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cos
h(d*x + c)^6 - 12096*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^5 - 2640*a*b^2*cosh(d*
x + c) - 3360*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^3 - 64*b^3 + 45*(33*b^3 + 8*(
48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(140*b^3*cosh(d
*x + c)^13 - 1001*b^3*cosh(d*x + c)^11 + 3432*a*b^2*cosh(d*x + c)^10 + 3520
*b^3*cosh(d*x + c)^9 - 21780*a*b^2*cosh(d*x + c)^8 - 672*(48*a^2*b - 5*b^3)
*d*x*cosh(d*x + c)^5 - 90*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)
^7 - 2016*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^6 - 1320*a*b^2*cosh(d*x + c)^2 -
```

```
840*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^4 - 64*b^3*cosh(d*x + c) + 15*(33*b^3 +
 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^3 + 12*a*b^2)*sinh(d*x + c)^3 - b^
3 + (120*b^3*cosh(d*x + c)^14 - 1001*b^3*cosh(d*x + c)^12 + 3744*a*b^2*cosh
(d*x + c)^11 + 4224*b^3*cosh(d*x + c)^10 - 29040*a*b^2*cosh(d*x + c)^9 - 13
44*(48*a^2*b - 5*b^3)*d*x*cosh(d*x + c)^6 - 135*(33*b^3 - 8*(48*a^2*b - 5*b
^3)*d*x)*cosh(d*x + c)^8 - 3456*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^7 - 5280*a*
b^2*cosh(d*x + c)^3 - 2016*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^5 - 384*b^3*cosh
(d*x + c)^2 + 45*(33*b^3 + 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^4 + 144*
a*b^2*cosh(d*x + c) + 11*b^3)*sinh(d*x + c)^2 + 192*(a^3*cosh(d*x + c)^10 +
 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*sinh(d*x + c)^10 - 2*a^3*cosh(d
*x + c)^8 + a^3*cosh(d*x + c)^6 + (45*a^3*cosh(d*x + c)^2 - 2*a^3)*sinh(d*x
 + c)^8 + 8*(15*a^3*cosh(d*x + c)^3 - 2*a^3*cosh(d*x + c))*sinh(d*x + c)^7
+ (210*a^3*cosh(d*x + c)^4 - 56*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6
+ 2*(126*a^3*cosh(d*x + c)^5 - 56*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c)
)*sinh(d*x + c)^5 + 5*(42*a^3*cosh(d*x + c)^6 - 28*a^3*cosh(d*x + c)^4 + 3*
a^3*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(30*a^3*cosh(d*x + c)^7 - 28*a^3*c
osh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^3)*sinh(d*x + c)^3 + (45*a^3*cosh(d*x
+ c)^8 - 56*a^3*cosh(d*x + c)^6 + 15*a^3*cosh(d*x + c)^4)*sinh(d*x + c)^2 +
 2*(5*a^3*cosh(d*x + c)^9 - 8*a^3*cosh(d*x + c)^7 + 3*a^3*cosh(d*x + c)^5)*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(144) = 288.
time = 0.52, size = 289, normalized size = 1.85

$$\frac{b^3e^{(6\,dx+6\,c)}-9\,b^3e^{(4\,dx+4\,c)}+48\,ab^2e^{(3\,dx+3\,c)}+45\,b^3e^{(2\,dx+2\,c)}-432\,ab^2e^{(dx+c)}+192\,a^3\log\left(e^{(dx+c)}+1\right)-192\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right)+24\,(48\,a^2b-5\,b^3)(dx+c)-\frac{(45\,b^3e^{(8\,dx+8\,c)}-99\,b^3e^{(6\,dx+6\,c)}+528\,ab^2e^{(5\,dx+5\,c)}+64\,b^3e^{(4\,dx+4\,c)}-48\,ab^2e^{(3\,dx+3\,c)}-11\,b^3e^{(2\,dx+2\,c)}+b^3+48\,(8\,a^3+9\,ab^2)e^{(9\,dx+9\,c)}+48\,(8\,a^3-19\,ab^2)e^{(7\,dx+7\,c)})e^{(-6\,dx-6\,c)}}{\left(e^{(dx+c)}+1\right)^2\left(e^{(dx+c)}-1\right)^2}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/384*(b^3*e^(6*d*x + 6*c) - 9*b^3*e^(4*d*x + 4*c) + 48*a*b^2*e^(3*d*x + 3*
c) + 45*b^3*e^(2*d*x + 2*c) - 432*a*b^2*e^(d*x + c) + 192*a^3*log(e^(d*x +
c) + 1) - 192*a^3*log(abs(e^(d*x + c) - 1)) + 24*(48*a^2*b - 5*b^3)*(d*x +
c) - (45*b^3*e^(8*d*x + 8*c) - 99*b^3*e^(6*d*x + 6*c) + 528*a*b^2*e^(5*d*x
+ 5*c) + 64*b^3*e^(4*d*x + 4*c) - 48*a*b^2*e^(3*d*x + 3*c) - 11*b^3*e^(2*d*
x + 2*c) + b^3 + 48*(8*a^3 + 9*a*b^2)*e^(9*d*x + 9*c) + 48*(8*a^3 - 19*a*b^
```

```
2)*e^(7*d*x + 7*c))*e^(-6*d*x - 6*c)/((e^(d*x + c) + 1)^2*(e^(d*x + c) - 1)
^2))/d
```

Mupad [B]

time = 0.93, size = 290, normalized size = 1.86

$$x\left(3\,a^2\,b-\frac{5\,b^3}{16}\right)+\frac{\mathrm{atan}\left(\frac{a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}}-\frac{15\,b^3\,\mathrm{e}^{-2\,c-2\,d\,x}}{128\,d}+\frac{15\,b^3\,\mathrm{e}^{2\,c+2\,d\,x}}{128\,d}+\frac{3\,b^3\,\mathrm{e}^{-4\,c-4\,d\,x}}{128\,d}-\frac{3\,b^3\,\mathrm{e}^{4\,c+4\,d\,x}}{128\,d}-\frac{b^3\,\mathrm{e}^{-6\,c-6\,d\,x}}{384\,d}+\frac{b^3\,\mathrm{e}^{6\,c+6\,d\,x}}{384\,d}-\frac{9\,a\,b^2\,\mathrm{e}^{-c-d\,x}}{8\,d}+\frac{a\,b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{8\,d}+\frac{a\,b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{8\,d}-\frac{9\,a\,b^2\,\mathrm{e}^{c+d\,x}}{8\,d}-\frac{a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^3,x)
```

```
[Out] x*(3*a^2*b - (5*b^3)/16) + (atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6
)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/2) - (15*b^3*exp(- 2*c - 2*d*x))/(128*d) +
 (15*b^3*exp(2*c + 2*d*x))/(128*d) + (3*b^3*exp(- 4*c - 4*d*x))/(128*d) - (
3*b^3*exp(4*c + 4*d*x))/(128*d) - (b^3*exp(- 6*c - 6*d*x))/(384*d) + (b^3*e
xp(6*c + 6*d*x))/(384*d) - (9*a*b^2*exp(- c - d*x))/(8*d) + (a*b^2*exp(- 3*
c - 3*d*x))/(8*d) + (a*b^2*exp(3*c + 3*d*x))/(8*d) - (9*a*b^2*exp(c + d*x))
/(8*d) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(c + d*x
))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.167 $\int \mathbf{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=129

$$-\frac{3}{2}ab^2x-\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^3\cosh(c+dx)}{d}-\frac{2b^3\cosh^3(c+dx)}{3d}+\frac{b^3\cosh^5(c+dx)}{5d}+\frac{a^3\coth(c+dx}{d}$$

```
[Out] -3/2*a*b^2*x-3*a^2*b*arctanh(cosh(d*x+c))/d+b^3*cosh(d*x+c)/d-2/3*b^3*cosh(
d*x+c)^3/d+1/5*b^3*cosh(d*x+c)^5/d+a^3*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/
d+3/2*a*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = $\{3299, 3855, 3852, 2715, 8, 2713\}$

$$-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3}{2}ab^2x+\frac{b^3\cosh^5(c+dx)}{5d}-\frac{2b^3\cosh^3(c+dx)}{3d}+\frac{b^3\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cosh[c + d*x]])/d + (b^3*Cosh[c + d*x])/d
 - (2*b^3*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d) + (a^3*Coth[
c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) + (3*a*b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
```

```
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx &= \int \left(3a^2b\operatorname{csch}(c+dx)+a^3\operatorname{csch}^4(c+dx)+3ab^2\sinh^2(c+dx)\right.\\ &= a^3\int \operatorname{csch}^4(c+dx)\,dx+\left(3a^2b\right)\int \operatorname{csch}(c+dx)\,dx+\left(3ab^2\right)\\ &= -\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)\sinh(c+dx}{2d}\\ &= -\frac{3}{2}ab^2x-\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^3\cosh(c+dx)}{d}-\end{aligned}$$

Mathematica [A]
time = 0.31, size = 169, normalized size = 1.31

$$\frac{-360ab^2c-360ab^2dx+150b^3\cosh(c+dx)-25b^3\cosh(3(c+dx))+3b^3\cosh(5(c+dx))+80a^3\coth\left(\frac{1}{2}(c+dx)\right)+720a^2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+80a^3\operatorname{csch}^3(c+dx)\sinh^4\left(\frac{1}{2}(c+dx)\right)-5a^3\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)\sinh(c+dx)+180ab^2\sinh(2(c+dx))+80a^3\tanh\left(\frac{1}{2}(c+dx)\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-360*a*b^2*c - 360*a*b^2*d*x + 150*b^3*Cosh[c + d*x] - 25*b^3*Cosh[3*(c +
d*x)] + 3*b^3*Cosh[5*(c + d*x)] + 80*a^3*Coth[(c + d*x)/2] + 720*a^2*b*Log[
Tanh[(c + d*x)/2]] + 80*a^3*Csch[c + d*x]^3*Sinh[(c + d*x)/2]^4 - 5*a^3*Csc
h[(c + d*x)/2]^4*Sinh[c + d*x] + 180*a*b^2*Sinh[2*(c + d*x)] + 80*a^3*Tanh[
(c + d*x)/2])/(240*d)
```

Maple [A]
time = 2.08, size = 214, normalized size = 1.66

method	result
risch	$-\frac{3a\,b^2x}{2}+\frac{b^3\mathrm{e}^{5dx+5c}}{160d}-\frac{5b^3\mathrm{e}^{3dx+3c}}{96d}+\frac{3a\,\mathrm{e}^{2dx+2c}b^2}{8d}+\frac{5b^3\mathrm{e}^{dx+c}}{16d}+\frac{5b^3\mathrm{e}^{-dx-c}}{16d}-\frac{3a\,\mathrm{e}^{-2dx-2c}b^2}{8d}-\frac{5b^3\mathrm{e}^{-3dx-3c}}{96d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*a*b^2*x+1/160*b^3/d*exp(5*d*x+5*c)-5/96*b^3/d*exp(3*d*x+3*c)+3/8*a/d*e
xp(2*d*x+2*c)*b^2+5/16*b^3/d*exp(d*x+c)+5/16*b^3/d*exp(-d*x-c)-3/8*a/d*exp(
-2*d*x-2*c)*b^2-5/96*b^3/d*exp(-3*d*x-3*c)+1/160*b^3/d*exp(-5*d*x-5*c)-4/3*
a^3*(3*exp(2*d*x+2*c)-1)/d/(exp(2*d*x+2*c)-1)^3+3*a^2*b/d*ln(exp(d*x+c)-1)-
3*a^2*b/d*ln(exp(d*x+c)+1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(119) = 238$.
time = 0.27, size = 260, normalized size = 2.02

$$-\frac{3}{8}ab^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+\frac{1}{480}b^3\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)-3a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)+\frac{4}{3}a^3\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b^3*(3*e^
(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x -
c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) - 3*a^2*b*(log(e^(-d*x
 - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3
*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e
^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3801 vs. $2(119) = 238$.
time = 0.47, size = 3801, normalized size = 29.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^3*cosh(d*x + c)^16 + 48*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + 3*b
^3*sinh(d*x + c)^16 - 34*b^3*cosh(d*x + c)^14 + 180*a*b^2*cosh(d*x + c)^13
+ 234*b^3*cosh(d*x + c)^12 + 2*(180*b^3*cosh(d*x + c)^2 - 17*b^3)*sinh(d*x
+ c)^14 + 4*(420*b^3*cosh(d*x + c)^3 - 119*b^3*cosh(d*x + c) + 45*a*b^2)*si
nh(d*x + c)^13 - 378*b^3*cosh(d*x + c)^10 + 26*(210*b^3*cosh(d*x + c)^4 - 1
19*b^3*cosh(d*x + c)^2 + 90*a*b^2*cosh(d*x + c) + 9*b^3)*sinh(d*x + c)^12 -
```

```
 180*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^11 + 4*(3276*b^3*cosh(d*x + c)^5
 - 3094*b^3*cosh(d*x + c)^3 - 180*a*b^2*d*x + 3510*a*b^2*cosh(d*x + c)^2 +
702*b^3*cosh(d*x + c) - 135*a*b^2)*sinh(d*x + c)^11 + 2*(12012*b^3*cosh(d*x
 + c)^6 - 17017*b^3*cosh(d*x + c)^4 + 25740*a*b^2*cosh(d*x + c)^3 + 7722*b^
3*cosh(d*x + c)^2 - 189*b^3 - 990*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c))*si
nh(d*x + c)^10 + 360*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^9 + 4*(8580*b^3*co
sh(d*x + c)^7 - 17017*b^3*cosh(d*x + c)^5 + 32175*a*b^2*cosh(d*x + c)^4 + 1
2870*b^3*cosh(d*x + c)^3 + 540*a*b^2*d*x - 945*b^3*cosh(d*x + c) + 90*a*b^2
 - 2475*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 378*b^3*
cosh(d*x + c)^6 + 6*(6435*b^3*cosh(d*x + c)^8 - 17017*b^3*cosh(d*x + c)^6 +
 38610*a*b^2*cosh(d*x + c)^5 + 19305*b^3*cosh(d*x + c)^4 - 2835*b^3*cosh(d*
x + c)^2 - 4950*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^3 + 540*(6*a*b^2*d*x
+ a*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 - 120*(18*a*b^2*d*x + 16*a^3 - 3*a*
b^2)*cosh(d*x + c)^7 + 24*(1430*b^3*cosh(d*x + c)^9 - 4862*b^3*cosh(d*x + c
)^7 + 12870*a*b^2*cosh(d*x + c)^6 + 7722*b^3*cosh(d*x + c)^5 - 1890*b^3*cos
h(d*x + c)^3 - 90*a*b^2*d*x - 2475*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^4
- 80*a^3 + 15*a*b^2 + 540*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^2)*sinh(d*x +
 c)^7 - 234*b^3*cosh(d*x + c)^4 + 6*(4004*b^3*cosh(d*x + c)^10 - 17017*b^3*
cosh(d*x + c)^8 + 51480*a*b^2*cosh(d*x + c)^7 + 36036*b^3*cosh(d*x + c)^6 -
 13230*b^3*cosh(d*x + c)^4 - 13860*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^5
+ 5040*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^3 + 63*b^3 - 140*(18*a*b^2*d*x +
 16*a^3 - 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 180*a*b^2*cosh(d*x + c)
^3 + 20*(36*a*b^2*d*x + 32*a^3 - 27*a*b^2)*cosh(d*x + c)^5 + 4*(3276*b^3*co
sh(d*x + c)^11 - 17017*b^3*cosh(d*x + c)^9 + 57915*a*b^2*cosh(d*x + c)^8 +
46332*b^3*cosh(d*x + c)^7 - 23814*b^3*cosh(d*x + c)^5 - 20790*(4*a*b^2*d*x
+ 3*a*b^2)*cosh(d*x + c)^6 + 180*a*b^2*d*x + 11340*(6*a*b^2*d*x + a*b^2)*co
sh(d*x + c)^4 + 567*b^3*cosh(d*x + c) + 160*a^3 - 135*a*b^2 - 630*(18*a*b^2
*d*x + 16*a^3 - 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 34*b^3*cosh(d*x
 + c)^2 + 2*(2730*b^3*cosh(d*x + c)^12 - 17017*b^3*cosh(d*x + c)^10 + 64350
*a*b^2*cosh(d*x + c)^9 + 57915*b^3*cosh(d*x + c)^8 - 39690*b^3*cosh(d*x + c
)^6 - 29700*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^7 + 22680*(6*a*b^2*d*x +
a*b^2)*cosh(d*x + c)^5 + 2835*b^3*cosh(d*x + c)^2 - 2100*(18*a*b^2*d*x + 16
*a^3 - 3*a*b^2)*cosh(d*x + c)^3 - 117*b^3 + 50*(36*a*b^2*d*x + 32*a^3 - 27*
a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(420*b^3*cosh(d*x + c)^13 - 3094*
b^3*cosh(d*x + c)^11 + 12870*a*b^2*cosh(d*x + c)^10 + 12870*b^3*cosh(d*x +
c)^9 - 11340*b^3*cosh(d*x + c)^7 - 7425*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x +
c)^8 + 7560*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^6 + 1890*b^3*cosh(d*x + c)^
3 - 1050*(18*a*b^2*d*x + 16*a^3 - 3*a*b^2)*cosh(d*x + c)^4 - 234*b^3*cosh(d
*x + c) + 45*a*b^2 + 50*(36*a*b^2*d*x + 32*a^3 - 27*a*b^2)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 - 3*b^3 + 2*(180*b^3*cosh(d*x + c)^14 - 1547*b^3*cosh(d*x
+ c)^12 + 7020*a*b^2*cosh(d*x + c)^11 + 7722*b^3*cosh(d*x + c)^10 - 8505*b^
3*cosh(d*x + c)^8 - 4950*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^9 + 6480*(6*
a*b^2*d*x + a*b^2)*cosh(d*x + c)^7 + 2835*b^3*cosh(d*x + c)^4 - 1260*(18*a*
b^2*d*x + 16*a^3 - 3*a*b^2)*cosh(d*x + c)^5 - 702*b^3*cosh(d*x + c)^2 + 270
*a*b^2*cosh(d*x + c) + 100*(36*a*b^2*d*x + 32*a^3 - 27*a*b^2)*cosh(d*x + c)
```

```
^3 + 17*b^3)*sinh(d*x + c)^2 - 1440*(a^2*b*cosh(d*x + c)^11 + 11*a^2*b*cosh
(d*x + c)*sinh(d*x + c)^10 + a^2*b*sinh(d*x + c)^11 - 3*a^2*b*cosh(d*x + c)
^9 + 3*a^2*b*cosh(d*x + c)^7 + (55*a^2*b*cosh(d*x + c)^2 - 3*a^2*b)*sinh(d*
x + c)^9 + 3*(55*a^2*b*cosh(d*x + c)^3 - 9*a^2*b*cosh(d*x + c))*sinh(d*x +
c)^8 - a^2*b*cosh(d*x + c)^5 + 3*(110*a^2*b*cosh(d*x + c)^4 - 36*a^2*b*cosh
(d*x + c)^2 + a^2*b)*sinh(d*x + c)^7 + 21*(22*a^2*b*cosh(d*x + c)^5 - 12*a^
2*b*cosh(d*x + c)^3 + a^2*b*cosh(d*x + c))*sinh(d*x + c)^6 + (462*a^2*b*cos
h(d*x + c)^6 - 378*a^2*b*cosh(d*x + c)^4 + 63*a^2*b*cosh(d*x + c)^2 - a^2*b
)*sinh(d*x + c)^5 + (330*a^2*b*cosh(d*x + c)^7 - 378*a^2*b*cosh(d*x + c)^5
+ 105*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^4 + (165
*a^2*b*cosh(d*x + c)^8 - 252*a^2*b*cosh(d*x + c)^6 + 105*a^2*b*cosh(d*x + c
)^4 - 10*a^2*b*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*a^2*b*cosh(d*x + c)^9
 - 108*a^2*b*cosh(d*x + c)^7 + 63*a^2*b*cosh(d*x + c)^5 - 10*a^2*b*cosh(d*x
 + c)^3)*sinh(d*x + c)^2 + (11*a^2*b*cosh(d*x + c)^10 - 27*a^2*b*cosh(d*x +
 c)^8 + 21*a^2*b*cosh(d*x + c)^6 - 5*a^2*b*cosh(d*x + c)^4)*sinh(d*x + c))*
log(cosh(d*x + c) + sinh(d*x + c) + 1) + 1440*(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(119) = 238.
time = 0.50, size = 285, normalized size = 2.21

$$-\frac{720\,(dx+c)ab^{2}-3\,b^{3}e^{(5\,dx+5\,c)}+25\,b^{3}e^{(3\,dx+3\,c)}-180\,ab^{2}e^{(2\,dx+2\,c)}-150\,b^{3}e^{(dx+c)}+1440\,a^{2}b\log\left(e^{(dx+c)}+1\right)-1440\,a^{2}b\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{\left(150\,b^{3}e^{(10\,dx+10\,c)}-180\,ab^{2}e^{(9\,dx+9\,c)}-475\,b^{3}e^{(8\,dx+8\,c)}+528\,b^{3}e^{(6\,dx+6\,c)}-234\,b^{3}e^{(4\,dx+4\,c)}+180\,ab^{2}e^{(3\,dx+3\,c)}+34\,b^{3}e^{(2\,dx+2\,c)}-3\,b^{3}-60\,(32\,a^{3}-9\,ab^{2})e^{(7\,dx+7\,c)}+20\,(32\,a^{3}-27\,ab^{2})e^{(5\,dx+5\,c)}\right)e^{(-5\,dx-5\,c)}}{\left(e^{(dx+c)}+1\right)^{3}\left(e^{(dx+c)}-1\right)^{3}}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/480*(720*(d*x + c)*a*b^2 - 3*b^3*e^(5*d*x + 5*c) + 25*b^3*e^(3*d*x + 3*c
) - 180*a*b^2*e^(2*d*x + 2*c) - 150*b^3*e^(d*x + c) + 1440*a^2*b*log(e^(d*x
 + c) + 1) - 1440*a^2*b*log(abs(e^(d*x + c) - 1)) - (150*b^3*e^(10*d*x + 10
*c) - 180*a*b^2*e^(9*d*x + 9*c) - 475*b^3*e^(8*d*x + 8*c) + 528*b^3*e^(6*d*
x + 6*c) - 234*b^3*e^(4*d*x + 4*c) + 180*a*b^2*e^(3*d*x + 3*c) + 34*b^3*e^(
2*d*x + 2*c) - 3*b^3 - 60*(32*a^3 - 9*a*b^2)*e^(7*d*x + 7*c) + 20*(32*a^3 -
 27*a*b^2)*e^(5*d*x + 5*c))*e^(-5*d*x - 5*c)/((e^(d*x + c) + 1)^3*(e^(d*x +
 c) - 1)^3))/d
```

Mupad [B]
time = 0.88, size = 267, normalized size = 2.07

$$\frac{5\,b^3\,\mathrm{e}^{c+d\,x}}{16\,d}-\frac{4\,a^3}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{5\,b^3\,\mathrm{e}^{-c-d\,x}}{16\,d}-\frac{5\,b^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{96\,d}-\frac{5\,b^3\,\mathrm{e}^{3\,c+3\,d\,x}}{96\,d}+\frac{b^3\,\mathrm{e}^{-5\,c-5\,d\,x}}{160\,d}+\frac{b^3\,\mathrm{e}^{5\,c+5\,d\,x}}{160\,d}-\frac{8\,a^3}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{6\,\mathrm{atan}\left(\frac{a^2\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^4\,b^2}}\right)\,\sqrt{a^4\,b^2}}{\sqrt{-d^2}}-\frac{3\,a\,b^2\,x}{2}-\frac{3\,a\,b^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}+\frac{3\,a\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^4,x)
```

```
[Out] (5*b^3*exp(c + d*x))/(16*d) - (4*a^3)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*
d*x) + 1)) + (5*b^3*exp(- c - d*x))/(16*d) - (5*b^3*exp(- 3*c - 3*d*x))/(96
*d) - (5*b^3*exp(3*c + 3*d*x))/(96*d) + (b^3*exp(- 5*c - 5*d*x))/(160*d) +
(b^3*exp(5*c + 5*d*x))/(160*d) - (8*a^3)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4
*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (6*atan((a^2*b*exp(d*x)*exp(c)*(-d^2
)^(1/2))/(d*(a^4*b^2)^(1/2)))*(a^4*b^2)^(1/2))/(-d^2)^(1/2) - (3*a*b^2*x)/2
 - (3*a*b^2*exp(- 2*c - 2*d*x))/(8*d) + (3*a*b^2*exp(2*c + 2*d*x))/(8*d)
```

3.168 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=148

$$\frac{3b^3x}{8}-\frac{3a^3\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{3a^2b\coth(c+dx)}{d}+\frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{a^3}{}$$

```
[Out] 3/8*b^3*x-3/8*a^3*arctanh(cosh(d*x+c))/d+3*a*b^2*cosh(d*x+c)/d-3*a^2*b*coth
(d*x+c)/d+3/8*a^3*coth(d*x+c)*csch(d*x+c)/d-1/4*a^3*coth(d*x+c)*csch(d*x+c)
^3/d-3/8*b^3*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b^3*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Rubi [A]

time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = $\{3299, 3852, 8, 3853, 3855, 2718, 2715\}$

$$-\frac{3a^3\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{3a^2b\coth(c+dx)}{d}+\frac{3ab^2\cosh(c+dx)}{d}+\frac{b^3\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b^3\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3b^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (3*b^3*x)/8 - (3*a^3*ArcTanh[Cosh[c + d*x]])/(8*d) + (3*a*b^2*Cosh[c + d*x]
)/d - (3*a^2*b*Coth[c + d*x])/d + (3*a^3*Coth[c + d*x]*Csch[c + d*x])/(8*d)
 - (a^3*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d) - (3*b^3*Cosh[c + d*x]*Sinh[c
+ d*x])/(8*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
```

```
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx &= i\int \left(-3ia^2b\operatorname{csch}^2(c+dx)-ia^3\operatorname{csch}^5(c+dx)-3iab^2\sinh(c+\right.\\
&= a^3\int \operatorname{csch}^5(c+dx)\,dx+\left(3a^2b\right)\int \operatorname{csch}^2(c+dx)\,dx+\left(3ab^2\right)\\
&= \frac{3ab^2\cosh(c+dx)}{d}-\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{b^3\cosh(c}{}\\
&= \frac{3ab^2\cosh(c+dx)}{d}-\frac{3a^2b\coth(c+dx)}{d}+\frac{3a^3\coth(c+dx)\operatorname{csc}}{8d}\\
&= \frac{3b^3x}{8}-\frac{3a^3\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{3a^2b}{}
\end{aligned}$$

Mathematica [A]
time = 6.11, size = 218, normalized size = 1.47

$$\frac{3b^3(c+dx)}{8d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{3a^2b\coth\left(\frac{1}{2}(c+dx)\right)}{2d}+\frac{3a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a^3\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a^3\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{b^3\sinh(2(c+dx))}{4d}+\frac{b^3\sinh(4(c+dx))}{32d}-\frac{3a^2b\tanh\left(\frac{1}{2}(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (3*b^3*(c + d*x))/(8*d) + (3*a*b^2*Cosh[c + d*x])/d - (3*a^2*b*Coth[(c + d*
x)/2])/(2*d) + (3*a^3*Csch[(c + d*x)/2]^2)/(32*d) - (a^3*Csch[(c + d*x)/2]^
```

```
4)/(64*d) + (3*a^3*Log[Tanh[(c + d*x)/2]])/(8*d) + (3*a^3*Sech[(c + d*x)/2]
^2)/(32*d) + (a^3*Sech[(c + d*x)/2]^4)/(64*d) - (b^3*Sinh[2*(c + d*x)])/(4*
d) + (b^3*Sinh[4*(c + d*x)])/(32*d) - (3*a^2*b*Tanh[(c + d*x)/2])/(2*d)
```

Maple [A]
time = 2.10, size = 249, normalized size = 1.68

method	result
risch	$\frac{3b^3x}{8}+\frac{b^3\mathrm{e}^{4dx+4c}}{64d}-\frac{b^3\mathrm{e}^{2dx+2c}}{8d}+\frac{3a\,\mathrm{e}^{dx+c}b^2}{2d}+\frac{3a\,\mathrm{e}^{-dx-c}b^2}{2d}+\frac{b^3\mathrm{e}^{-2dx-2c}}{8d}-\frac{b^3\mathrm{e}^{-4dx-4c}}{64d}+\frac{a^2\left(3a\,\mathrm{e}^{7dx+7c}-24b\,\mathrm{e}^{6dx+6c}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*b^3*x+1/64*b^3/d*exp(4*d*x+4*c)-1/8*b^3/d*exp(2*d*x+2*c)+3/2*a/d*exp(d*
x+c)*b^2+3/2*a/d*exp(-d*x-c)*b^2+1/8*b^3/d*exp(-2*d*x-2*c)-1/64*b^3/d*exp(-
4*d*x-4*c)+1/4*a^2*(3*a*exp(7*d*x+7*c)-24*b*exp(6*d*x+6*c)-11*a*exp(5*d*x+5
*c)+72*b*exp(4*d*x+4*c)-11*a*exp(3*d*x+3*c)-72*b*exp(2*d*x+2*c)+3*a*exp(d*x
+c)+24*b)/d/(exp(2*d*x+2*c)-1)^4+3/8*a^3/d*ln(exp(d*x+c)-1)-3/8*a^3/d*ln(ex
p(d*x+c)+1)
```

Maxima [A]
time = 0.28, size = 255, normalized size = 1.72

$$\frac{1}{64}\,b^3\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{3}{2}\,ab^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)-\frac{1}{8}\,a^3\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,\left(3\,e^{(-dx-c)}-11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}+3\,e^{(-7\,dx-7\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}\right)+\frac{6\,a^2b}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/64*b^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/2*a*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) - 1
/8*a^3*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*
x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d
*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x
- 8*c) - 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4541 vs. $2(136) = 272$.
time = 0.48, size = 4541, normalized size = 30.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(b^3*cosh(d*x + c)^16 + 16*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + b^3*si
nh(d*x + c)^16 - 12*b^3*cosh(d*x + c)^14 + 96*a*b^2*cosh(d*x + c)^13 + 12*(
```

```
10*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^14 + 8*(70*b^3*cosh(d*x + c)^3
 - 21*b^3*cosh(d*x + c) + 12*a*b^2)*sinh(d*x + c)^13 + 2*(12*b^3*d*x + 19*b^
3)*cosh(d*x + c)^12 + 2*(910*b^3*cosh(d*x + c)^4 + 12*b^3*d*x - 546*b^3*cos
h(d*x + c)^2 + 624*a*b^2*cosh(d*x + c) + 19*b^3)*sinh(d*x + c)^12 + 48*(a^3
 - 6*a*b^2)*cosh(d*x + c)^11 + 24*(182*b^3*cosh(d*x + c)^5 - 182*b^3*cosh(d
*x + c)^3 + 312*a*b^2*cosh(d*x + c)^2 + 2*a^3 - 12*a*b^2 + (12*b^3*d*x + 19
*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 4*(24*b^3*d*x + 96*a^2*b + 11*b^3)*
cosh(d*x + c)^10 + 4*(2002*b^3*cosh(d*x + c)^6 - 3003*b^3*cosh(d*x + c)^4 +
 6864*a*b^2*cosh(d*x + c)^3 - 24*b^3*d*x - 96*a^2*b - 11*b^3 + 33*(12*b^3*d
*x + 19*b^3)*cosh(d*x + c)^2 + 132*(a^3 - 6*a*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^10 - 16*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^9 + 8*(1430*b^3*cosh(d*x + c
)^7 - 3003*b^3*cosh(d*x + c)^5 + 8580*a*b^2*cosh(d*x + c)^4 + 55*(12*b^3*d*
x + 19*b^3)*cosh(d*x + c)^3 - 22*a^3 + 24*a*b^2 + 330*(a^3 - 6*a*b^2)*cosh(
d*x + c)^2 - 5*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c))*sinh(d*x + c
)^9 + 144*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^8 + 18*(715*b^3*cosh(d*x + c)^8
 - 2002*b^3*cosh(d*x + c)^6 + 6864*a*b^2*cosh(d*x + c)^5 + 8*b^3*d*x + 55*(
12*b^3*d*x + 19*b^3)*cosh(d*x + c)^4 + 440*(a^3 - 6*a*b^2)*cosh(d*x + c)^3
+ 64*a^2*b - 10*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^2 - 8*(11*a^
3 - 12*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 - 16*(11*a^3 - 12*a*b^2)*cosh(
d*x + c)^7 + 16*(715*b^3*cosh(d*x + c)^9 - 2574*b^3*cosh(d*x + c)^7 + 10296
*a*b^2*cosh(d*x + c)^6 + 99*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^5 + 990*(a^
3 - 6*a*b^2)*cosh(d*x + c)^4 - 30*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x
 + c)^3 - 11*a^3 + 12*a*b^2 - 36*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^2 + 72*(
b^3*d*x + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(24*b^3*d*x + 288*a^2
*b - 11*b^3)*cosh(d*x + c)^6 + 4*(2002*b^3*cosh(d*x + c)^10 - 9009*b^3*cosh
(d*x + c)^8 + 41184*a*b^2*cosh(d*x + c)^7 + 462*(12*b^3*d*x + 19*b^3)*cosh(
d*x + c)^6 + 5544*(a^3 - 6*a*b^2)*cosh(d*x + c)^5 - 24*b^3*d*x - 210*(24*b^
3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^4 - 336*(11*a^3 - 12*a*b^2)*cosh(d
*x + c)^3 - 288*a^2*b + 11*b^3 + 1008*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^2 -
 28*(11*a^3 - 12*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 96*a*b^2*cosh(d*x
+ c)^3 + 48*(a^3 - 6*a*b^2)*cosh(d*x + c)^5 + 24*(182*b^3*cosh(d*x + c)^11
- 1001*b^3*cosh(d*x + c)^9 + 5148*a*b^2*cosh(d*x + c)^8 + 66*(12*b^3*d*x +
19*b^3)*cosh(d*x + c)^7 + 924*(a^3 - 6*a*b^2)*cosh(d*x + c)^6 - 42*(24*b^3*
d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^5 - 84*(11*a^3 - 12*a*b^2)*cosh(d*x
+ c)^4 + 336*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^3 + 2*a^3 - 12*a*b^2 - 14*(1
1*a^3 - 12*a*b^2)*cosh(d*x + c)^2 - (24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(
d*x + c))*sinh(d*x + c)^5 + 12*b^3*cosh(d*x + c)^2 + 2*(12*b^3*d*x + 192*a^
2*b - 19*b^3)*cosh(d*x + c)^4 + 2*(910*b^3*cosh(d*x + c)^12 - 6006*b^3*cosh
(d*x + c)^10 + 34320*a*b^2*cosh(d*x + c)^9 + 495*(12*b^3*d*x + 19*b^3)*cosh
(d*x + c)^8 + 7920*(a^3 - 6*a*b^2)*cosh(d*x + c)^7 - 420*(24*b^3*d*x + 96*a
^2*b + 11*b^3)*cosh(d*x + c)^6 - 1008*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^5 +
 12*b^3*d*x + 5040*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^4 - 280*(11*a^3 - 12*a
*b^2)*cosh(d*x + c)^3 + 192*a^2*b - 19*b^3 - 30*(24*b^3*d*x + 288*a^2*b - 1
1*b^3)*cosh(d*x + c)^2 + 120*(a^3 - 6*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4
 + 8*(70*b^3*cosh(d*x + c)^13 - 546*b^3*cosh(d*x + c)^11 + 3432*a*b^2*cosh(
```

```
d*x + c)^10 + 55*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^9 + 990*(a^3 - 6*a*b^2
)*cosh(d*x + c)^8 - 60*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^7 - 1
68*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^6 + 1008*(b^3*d*x + 8*a^2*b)*cosh(d*x
+ c)^5 - 70*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^4 - 10*(24*b^3*d*x + 288*a^2*
b - 11*b^3)*cosh(d*x + c)^3 + 12*a*b^2 + 60*(a^3 - 6*a*b^2)*cosh(d*x + c)^2
 + (12*b^3*d*x + 192*a^2*b - 19*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - b^3 +
 12*(10*b^3*cosh(d*x + c)^14 - 91*b^3*cosh(d*x + c)^12 + 624*a*b^2*cosh(d*x
 + c)^11 + 11*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^10 + 220*(a^3 - 6*a*b^2)*
cosh(d*x + c)^9 - 15*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^8 - 48*
(11*a^3 - 12*a*b^2)*cosh(d*x + c)^7 + 336*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)
^6 - 28*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^5 - 5*(24*b^3*d*x + 288*a^2*b - 1
1*b^3)*cosh(d*x + c)^4 + 24*a*b^2*cosh(d*x + c) + 40*(a^3 - 6*a*b^2)*cosh(d
*x + c)^3 + b^3 + (12*b^3*d*x + 192*a^2*b - 19*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 - 24*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^1
1 + a^3*sinh(d*x + c)^12 - 4*a^3*cosh(d*x + c)^10 + 6*a^3*cosh(d*x + c)^8 +
 2*(33*a^3*cosh(d*x + c)^2 - 2*a^3)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x
+ c)^3 - 2*a^3*cosh(d*x + c))*sinh(d*x + c)^9 - 4*a^3*cosh(d*x + c)^6 + 3*(
165*a^3*cosh(d*x + c)^4 - 60*a^3*cosh(d*x + c)^2 + 2*a^3)*sinh(d*x + c)^8 +
 24*(33*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c)^3 + 2*a^3*cosh(d*x + c))
*sinh(d*x + c)^7 + a^3*cosh(d*x + c)^4 + 4*(231...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(136) = 272.
time = 0.54, size = 329, normalized size = 2.22

$$\frac{24\,(dx+c)b^3+b^3e^{(4\,dx+4\,c)}-8\,b^3e^{(2\,dx+2\,c)}+96\,ab^2e^{(dx+c)}-24\,a^3\log\left(e^{(dx+c)}+1\right)+24\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right)+\frac{\left(96\,ab^2e^{(3\,dx+3\,c)}+12\,b^3e^{(2\,dx+2\,c)}-b^3+48\,(a^3+2\,ab^2)e^{(11\,dx+11\,c)}-8\,(48\,a^2b-b^3)e^{(10\,dx+10\,c)}-16\,(11\,a^3+24\,ab^2)e^{(9\,dx+9\,c)}+3\,(384\,a^2b-11\,b^3)e^{(8\,dx+8\,c)}-16\,(11\,a^3-36\,ab^2)e^{(7\,dx+7\,c)}-4\,(288\,a^2b-13\,b^3)e^{(6\,dx+6\,c)}+48\,(a^3-8\,ab^2)e^{(5\,dx+5\,c)}+2\,(192\,a^2b-19\,b^3)e^{(4\,dx+4\,c)}\right)e^{(-4\,dx-4\,c)}}{\left(e^{(dx+c)}+1\right)^4\left(e^{(dx+c)}-1\right)^4}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/64*(24*(d*x + c)*b^3 + b^3*e^(4*d*x + 4*c) - 8*b^3*e^(2*d*x + 2*c) + 96*a
*b^2*e^(d*x + c) - 24*a^3*log(e^(d*x + c) + 1) + 24*a^3*log(abs(e^(d*x + c)
 - 1)) + (96*a*b^2*e^(3*d*x + 3*c) + 12*b^3*e^(2*d*x + 2*c) - b^3 + 48*(a^3
 + 2*a*b^2)*e^(11*d*x + 11*c) - 8*(48*a^2*b - b^3)*e^(10*d*x + 10*c) - 16*(
11*a^3 + 24*a*b^2)*e^(9*d*x + 9*c) + 3*(384*a^2*b - 11*b^3)*e^(8*d*x + 8*c)
 - 16*(11*a^3 - 36*a*b^2)*e^(7*d*x + 7*c) - 4*(288*a^2*b - 13*b^3)*e^(6*d*x
```

```
 + 6*c) + 48*(a^3 - 8*a*b^2)*e^(5*d*x + 5*c) + 2*(192*a^2*b - 19*b^3)*e^(4*
d*x + 4*c))*e^(-4*d*x - 4*c)/((e^(d*x + c) + 1)^4*(e^(d*x + c) - 1)^4))/d
```

Mupad [B]
time = 0.87, size = 451, normalized size = 3.05

$$\frac{3\,b^3\,x}{8}-\frac{\frac{3\,a^2\,b}{2\,d}+\frac{2\,a^3\,{\mathrm{e}}^{c+d\,x}}{d}-\frac{3\,a^2\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{d}+\frac{3\,a^2\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{2\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{\frac{4\,a^3\,{\mathrm{e}}^{3\,c+3\,d\,x}}{d}-\frac{3\,a^2\,b}{2\,d}+\frac{9\,a^2\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{2\,d}-\frac{9\,a^2\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{2\,d}+\frac{3\,a^2\,b\,{\mathrm{e}}^{6\,c+6\,d\,x}}{2\,d}}{6\,{\mathrm{e}}^{4\,c+4\,d\,x}-4\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{3\,a^2\,b}{d}-\frac{3\,a^3\,{\mathrm{e}}^{c+d\,x}}{4\,d}}{{\mathrm{e}}^{2\,c+2\,d\,x}-1}-\frac{3\,\mathrm{atan}\left(\frac{a^3\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{4\,\sqrt{-d^2}}+\frac{b^3\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}-\frac{b^3\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}-\frac{b^3\,{\mathrm{e}}^{-4\,c-4\,d\,x}}{64\,d}+\frac{b^3\,{\mathrm{e}}^{4\,c+4\,d\,x}}{64\,d}+\frac{3\,a\,b^2\,{\mathrm{e}}^{-c-d\,x}}{2\,d}+\frac{3\,a\,b^2\,{\mathrm{e}}^{c+d\,x}}{2\,d}-\frac{a^3\,{\mathrm{e}}^{c+d\,x}}{2\,d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^5,x)
```

```
[Out] (3*b^3*x)/8 - ((3*a^2*b)/(2*d) + (2*a^3*exp(c + d*x))/d - (3*a^2*b*exp(2*c
+ 2*d*x))/d + (3*a^2*b*exp(4*c + 4*d*x))/(2*d))/(3*exp(2*c + 2*d*x) - 3*exp
(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((4*a^3*exp(3*c + 3*d*x))/d - (3*a^
2*b)/(2*d) + (9*a^2*b*exp(2*c + 2*d*x))/(2*d) - (9*a^2*b*exp(4*c + 4*d*x))/
(2*d) + (3*a^2*b*exp(6*c + 6*d*x))/(2*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c +
 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((3*a^2*b)/d - (3*a^
3*exp(c + d*x))/(4*d))/(exp(2*c + 2*d*x) - 1) - (3*atan((a^3*exp(d*x)*exp(c
)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(4*(-d^2)^(1/2)) + (b^3*exp(-
 2*c - 2*d*x))/(8*d) - (b^3*exp(2*c + 2*d*x))/(8*d) - (b^3*exp(- 4*c - 4*d*
x))/(64*d) + (b^3*exp(4*c + 4*d*x))/(64*d) + (3*a*b^2*exp(- c - d*x))/(2*d)
 + (3*a*b^2*exp(c + d*x))/(2*d) - (a^3*exp(c + d*x))/(2*d*(exp(4*c + 4*d*x)
 - 2*exp(2*c + 2*d*x) + 1))
```

3.169 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=131

$$3ab^2x+\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{b^3\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}-\frac{a^3\coth(c+dx)}{d}+\frac{2a^3\coth^3(c+dx)}{3d}-$$

```
[Out] 3*a*b^2*x+3/2*a^2*b*arctanh(cosh(d*x+c))/d-b^3*cosh(d*x+c)/d+1/3*b^3*cosh(d
*x+c)^3/d-a^3*coth(d*x+c)/d+2/3*a^3*coth(d*x+c)^3/d-1/5*a^3*coth(d*x+c)^5/d
-3/2*a^2*b*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$,
Rules used = {3299, 3853, 3855, 3852, 2713}

$$-\frac{a^3\coth^5(c+dx)}{5d}+\frac{2a^3\coth^3(c+dx)}{3d}-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{3a^2b\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+3ab^2x+\frac{b^3\cosh^3(c+dx)}{3d}-\frac{b^3\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] 3*a*b^2*x + (3*a^2*b*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^3*Cosh[c + d*x])/d
+ (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x])/d + (2*a^3*Coth[c + d*x
]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d) - (3*a^2*b*Coth[c + d*x]*Csch[c +
d*x])/(2*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx &= -\int\left(-3ab^2-3a^2b\operatorname{csch}^3(c+dx)-a^3\operatorname{csch}^6(c+dx)-b^3\sinh\right.\\ &= 3ab^2x+a^3\int\operatorname{csch}^6(c+dx)\,dx+\left(3a^2b\right)\int\operatorname{csch}^3(c+dx)\,dx+\\ &= 3ab^2x-\frac{3a^2b\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{1}{2}\left(3a^2b\right)\int\operatorname{csch}(c+\\ &= 3ab^2x+\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{b^3\cosh(c+dx)}{d}+\frac{b^3\operatorname{co}}{}\end{aligned}$$

Mathematica [A]
time = 1.27, size = 225, normalized size = 1.72

$$\frac{-360b^3\cosh(c+dx)+40b^3\cosh(3(c+dx))+\frac{1}{2}a\left(-256a^2\coth\left(\frac{1}{2}(c+dx)\right)-360ab\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+19a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)\sinh(c+dx)-3a^2\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)\sinh(c+dx)+8\left(180b\left(2b(c+dx)-a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)-45ab\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)-38a^2\operatorname{csch}^3(c+dx)\sinh^4\left(\frac{1}{2}(c+dx)\right)-24a^2\operatorname{csch}^5(c+dx)\sinh^6\left(\frac{1}{2}(c+dx)\right)-32a^2\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-360*b^3*Cosh[c + d*x] + 40*b^3*Cosh[3*(c + d*x)] + (a*(-256*a^2*Coth[(c +
 d*x)/2] - 360*a*b*Csch[(c + d*x)/2]^2 + 19*a^2*Csch[(c + d*x)/2]^4*Sinh[c
+ d*x] - 3*a^2*Csch[(c + d*x)/2]^6*Sinh[c + d*x] + 8*(180*b*(2*b*(c + d*x)
- a*Log[Tanh[(c + d*x)/2]]) - 45*a*b*Sech[(c + d*x)/2]^2 - 38*a^2*Csch[c +
d*x]^3*Sinh[(c + d*x)/2]^4 - 24*a^2*Csch[c + d*x]^5*Sinh[(c + d*x)/2]^6 - 3
2*a^2*Tanh[(c + d*x)/2])))/2)/(480*d)
```

Maple [A]
time = 2.08, size = 204, normalized size = 1.56

method	result

risch	$3a\,b^{2}x+\frac{b^{3}\mathrm{e}^{3dx+3c}}{24d}-\frac{3b^{3}\mathrm{e}^{dx+c}}{8d}-\frac{3b^{3}\mathrm{e}^{-dx-c}}{8d}+\frac{b^{3}\mathrm{e}^{-3dx-3c}}{24d}-\frac{a^{2}\left(45b\,\mathrm{e}^{9dx+9c}-90b\,\mathrm{e}^{7dx+7c}+160a\,\mathrm{e}^{4dx+4c}+90b\,\mathrm{e}^{3dx+3c}-8}{15d\left(\mathrm{e}^{2dx+2c}-1\right)^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a*b^2*x+1/24*b^3/d*exp(3*d*x+3*c)-3/8*b^3/d*exp(d*x+c)-3/8*b^3/d*exp(-d*x
-c)+1/24*b^3/d*exp(-3*d*x-3*c)-1/15*a^2*(45*b*exp(9*d*x+9*c)-90*b*exp(7*d*x
+7*c)+160*a*exp(4*d*x+4*c)+90*b*exp(3*d*x+3*c)-80*a*exp(2*d*x+2*c)-45*b*exp
(d*x+c)+16*a)/d/(exp(2*d*x+2*c)-1)^5-3/2*a^2*b/d*ln(exp(d*x+c)-1)+3/2*a^2*b
/d*ln(exp(d*x+c)+1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(121) = 242$.
time = 0.28, size = 365, normalized size = 2.79

$$3\,ab^{2}x+\frac{1}{24}\,b^{3}\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3}{2}\,a^{2}b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)-\frac{16}{15}\,a^{3}\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1)}-\frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1)}-\frac{1}{d(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3*a*b^2*x + 1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/
d + e^(-3*d*x - 3*c)/d) + 3/2*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x
- c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) -
e^(-4*d*x - 4*c) - 1))) - 16/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2
*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-
10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-
4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c
) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6
*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4629 vs. $2(121) = 242$.
time = 0.48, size = 4629, normalized size = 35.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(5*b^3*cosh(d*x + c)^16 + 80*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + 5*b
^3*sinh(d*x + c)^16 + 360*a*b^2*d*x*cosh(d*x + c)^13 - 70*b^3*cosh(d*x + c)
^14 - 1800*a*b^2*d*x*cosh(d*x + c)^11 + 10*(60*b^3*cosh(d*x + c)^2 - 7*b^3)
*sinh(d*x + c)^14 + 3600*a*b^2*d*x*cosh(d*x + c)^9 + 20*(140*b^3*cosh(d*x +
```

```
 c)^3 + 18*a*b^2*d*x - 49*b^3*cosh(d*x + c))*sinh(d*x + c)^13 - 10*(36*a^2*
b - 23*b^3)*cosh(d*x + c)^12 + 10*(910*b^3*cosh(d*x + c)^4 + 468*a*b^2*d*x*
cosh(d*x + c) - 637*b^3*cosh(d*x + c)^2 - 36*a^2*b + 23*b^3)*sinh(d*x + c)^
12 + 40*(546*b^3*cosh(d*x + c)^5 + 702*a*b^2*d*x*cosh(d*x + c)^2 - 637*b^3*
cosh(d*x + c)^3 - 45*a*b^2*d*x - 3*(36*a^2*b - 23*b^3)*cosh(d*x + c))*sinh(
d*x + c)^11 + 90*(8*a^2*b - 3*b^3)*cosh(d*x + c)^10 + 10*(4004*b^3*cosh(d*x
 + c)^6 + 10296*a*b^2*d*x*cosh(d*x + c)^3 - 7007*b^3*cosh(d*x + c)^4 - 1980
*a*b^2*d*x*cosh(d*x + c) + 72*a^2*b - 27*b^3 - 66*(36*a^2*b - 23*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^10 + 20*(2860*b^3*cosh(d*x + c)^7 + 12870*a*b^2*d
*x*cosh(d*x + c)^4 - 7007*b^3*cosh(d*x + c)^5 - 4950*a*b^2*d*x*cosh(d*x + c
)^2 + 180*a*b^2*d*x - 110*(36*a^2*b - 23*b^3)*cosh(d*x + c)^3 + 45*(8*a^2*b
 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 30*(2145*b^3*cosh(d*x + c)^8 + 1
5444*a*b^2*d*x*cosh(d*x + c)^5 - 7007*b^3*cosh(d*x + c)^6 - 9900*a*b^2*d*x*
cosh(d*x + c)^3 + 1080*a*b^2*d*x*cosh(d*x + c) - 165*(36*a^2*b - 23*b^3)*co
sh(d*x + c)^4 + 135*(8*a^2*b - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 - 80
*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^7 + 80*(715*b^3*cosh(d*x + c)^9 + 77
22*a*b^2*d*x*cosh(d*x + c)^6 - 3003*b^3*cosh(d*x + c)^7 - 7425*a*b^2*d*x*co
sh(d*x + c)^4 + 1620*a*b^2*d*x*cosh(d*x + c)^2 - 99*(36*a^2*b - 23*b^3)*cos
h(d*x + c)^5 - 45*a*b^2*d*x + 135*(8*a^2*b - 3*b^3)*cosh(d*x + c)^3 - 16*a^
3)*sinh(d*x + c)^7 - 90*(8*a^2*b - 3*b^3)*cosh(d*x + c)^6 + 10*(4004*b^3*co
sh(d*x + c)^10 + 61776*a*b^2*d*x*cosh(d*x + c)^7 - 21021*b^3*cosh(d*x + c)^
8 - 83160*a*b^2*d*x*cosh(d*x + c)^5 + 30240*a*b^2*d*x*cosh(d*x + c)^3 - 924
*(36*a^2*b - 23*b^3)*cosh(d*x + c)^6 + 1890*(8*a^2*b - 3*b^3)*cosh(d*x + c)
^4 - 72*a^2*b + 27*b^3 - 56*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c))*sinh(d*x
 + c)^6 + 40*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^5 + 20*(1092*b^3*cosh(d*
x + c)^11 + 23166*a*b^2*d*x*cosh(d*x + c)^8 - 7007*b^3*cosh(d*x + c)^9 - 41
580*a*b^2*d*x*cosh(d*x + c)^6 + 22680*a*b^2*d*x*cosh(d*x + c)^4 - 396*(36*a
^2*b - 23*b^3)*cosh(d*x + c)^7 + 1134*(8*a^2*b - 3*b^3)*cosh(d*x + c)^5 + 9
0*a*b^2*d*x + 32*a^3 - 84*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^2 - 27*(8*a
^2*b - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 70*b^3*cosh(d*x + c)^2 + 10*
(36*a^2*b - 23*b^3)*cosh(d*x + c)^4 + 10*(910*b^3*cosh(d*x + c)^12 + 25740*
a*b^2*d*x*cosh(d*x + c)^9 - 7007*b^3*cosh(d*x + c)^10 - 59400*a*b^2*d*x*cos
h(d*x + c)^7 + 45360*a*b^2*d*x*cosh(d*x + c)^5 - 495*(36*a^2*b - 23*b^3)*co
sh(d*x + c)^8 + 1890*(8*a^2*b - 3*b^3)*cosh(d*x + c)^6 - 280*(45*a*b^2*d*x
+ 16*a^3)*cosh(d*x + c)^3 + 36*a^2*b - 23*b^3 - 135*(8*a^2*b - 3*b^3)*cosh(
d*x + c)^2 + 20*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 8*
(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^3 + 8*(350*b^3*cosh(d*x + c)^13 + 128
70*a*b^2*d*x*cosh(d*x + c)^10 - 3185*b^3*cosh(d*x + c)^11 - 37125*a*b^2*d*x
*cosh(d*x + c)^8 + 37800*a*b^2*d*x*cosh(d*x + c)^6 - 275*(36*a^2*b - 23*b^3
)*cosh(d*x + c)^9 + 1350*(8*a^2*b - 3*b^3)*cosh(d*x + c)^7 - 45*a*b^2*d*x -
 350*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^4 - 225*(8*a^2*b - 3*b^3)*cosh(d
*x + c)^3 - 16*a^3 + 50*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^2 + 5*(36*a^2
*b - 23*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*b^3 + 2*(300*b^3*cosh(d*x +
 c)^14 + 14040*a*b^2*d*x*cosh(d*x + c)^11 - 3185*b^3*cosh(d*x + c)^12 - 495
00*a*b^2*d*x*cosh(d*x + c)^9 + 64800*a*b^2*d*x*cosh(d*x + c)^7 - 330*(36*a^
```

```
2*b - 23*b^3)*cosh(d*x + c)^10 + 2025*(8*a^2*b - 3*b^3)*cosh(d*x + c)^8 - 8
40*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^5 - 675*(8*a^2*b - 3*b^3)*cosh(d*x
 + c)^4 + 200*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^3 + 35*b^3 + 30*(36*a^2
*b - 23*b^3)*cosh(d*x + c)^2 - 12*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c))*si
nh(d*x + c)^2 + 180*(a^2*b*cosh(d*x + c)^13 + 13*a^2*b*cosh(d*x + c)*sinh(d
*x + c)^12 + a^2*b*sinh(d*x + c)^13 - 5*a^2*b*cosh(d*x + c)^11 + 10*a^2*b*c
osh(d*x + c)^9 + (78*a^2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^11 + 11
*(26*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^10 - 10*a
^2*b*cosh(d*x + c)^7 + 5*(143*a^2*b*cosh(d*x + c)^4 - 55*a^2*b*cosh(d*x + c
)^2 + 2*a^2*b)*sinh(d*x + c)^9 + 3*(429*a^2*b*cosh(d*x + c)^5 - 275*a^2*b*c
osh(d*x + c)^3 + 30*a^2*b*cosh(d*x + c))*sinh(d*x + c)^8 + 5*a^2*b*cosh(d*x
 + c)^5 + 2*(858*a^2*b*cosh(d*x + c)^6 - 825*a^2*b*cosh(d*x + c)^4 + 180*a^
2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^7 + 2*(858*a^2*b*cosh(d*x + c)
^7 - 1155*a^2*b*cosh(d*x + c)^5 + 420*a^2*b*cosh(d*x + c)^3 - 35*a^2*b*cosh
(d*x + c))*sinh(d*x + c)^6 - a^2*b*cosh(d*x + c)^3 + (1287*a^2*b*cosh(d*x +
 c)^8 - 2310*a^2*b*cosh(d*x + c)^6 + 1260*a^2*b*cosh(d*x + c)^4 - 210*a^2*b
*cosh(d*x + c)^2 + 5*a^2*b)*sinh(d*x + c)^5 + 5*(143*a^2*b*cosh(d*x + c)^9
- 330*a^2*b*cosh(d*x + c)^7 + 252*a^2*b*cosh(d*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(121) = 242.
time = 0.51, size = 270, normalized size = 2.06

$$\frac{360\,(dx+c)ab^2+5\,b^3e^{(3\,dx+3\,c)}-45\,b^3e^{(dx+c)}+180\,a^2b\log\left(e^{(dx+c)}+1\right)-180\,a^2b\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{\left(475\,b^3e^{(8\,dx+8\,c)}+1280\,a^3e^{(7\,dx+7\,c)}-640\,a^3e^{(5\,dx+5\,c)}+128\,a^3e^{(3\,dx+3\,c)}-70\,b^3e^{(2\,dx+2\,c)}+5\,b^3+45\,(8\,a^2b+b^3)e^{(12\,dx+12\,c)}-10\,(72\,a^2b+23\,b^3)e^{(10\,dx+10\,c)}+20\,(36\,a^2b-25\,b^3)e^{(6\,dx+6\,c)}-5\,(72\,a^2b-55\,b^3)e^{(4\,dx+4\,c)}\right)e^{(-3\,dx-3\,c)}}{\left(e^{(dx+c)}+1\right)^5\left(e^{(dx+c)}-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/120*(360*(d*x + c)*a*b^2 + 5*b^3*e^(3*d*x + 3*c) - 45*b^3*e^(d*x + c) + 1
80*a^2*b*log(e^(d*x + c) + 1) - 180*a^2*b*log(abs(e^(d*x + c) - 1)) - (475*
b^3*e^(8*d*x + 8*c) + 1280*a^3*e^(7*d*x + 7*c) - 640*a^3*e^(5*d*x + 5*c) +
128*a^3*e^(3*d*x + 3*c) - 70*b^3*e^(2*d*x + 2*c) + 5*b^3 + 45*(8*a^2*b + b^
3)*e^(12*d*x + 12*c) - 10*(72*a^2*b + 23*b^3)*e^(10*d*x + 10*c) + 20*(36*a^
2*b - 25*b^3)*e^(6*d*x + 6*c) - 5*(72*a^2*b - 55*b^3)*e^(4*d*x + 4*c))*e^(-
3*d*x - 3*c)/((e^(d*x + c) + 1)^5*(e^(d*x + c) - 1)^5))/d
```

Mupad [B]
time = 0.81, size = 432, normalized size = 3.30

$$\frac{b^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{3\,b^3\,\mathrm{e}^{c+d\,x}}{8\,d}-\frac{3\,b^3\,\mathrm{e}^{-c-d\,x}}{8\,d}-\frac{\frac{32\,a^3\,\mathrm{e}^{4\,c+4\,d\,x}}{5\,d}+\frac{36\,a^2\,b\,\mathrm{e}^{3\,c+3\,d\,x}}{5\,d}-\frac{36\,a^2\,b\,\mathrm{e}^{5\,c+5\,d\,x}}{5\,d}+\frac{12\,a^2\,b\,\mathrm{e}^{7\,c+7\,d\,x}}{5\,d}-\frac{12\,a^2\,b\,\mathrm{e}^{c+d\,x}}{5\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1}+\frac{b^3\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{64\,a^3}{15\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{16\,a^3}{5\,d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}+\frac{3\,\mathrm{atan}\left(\frac{a^2\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^4\,b^2}}\right)\,\sqrt{a^4\,b^2}}{\sqrt{-d^2}}+3\,a\,b^2\,x-\frac{3\,a^2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{18\,a^2\,b\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^6,x)
```

```
[Out] (b^3*exp(- 3*c - 3*d*x))/(24*d) - (3*b^3*exp(c + d*x))/(8*d) - (3*b^3*exp(-
 c - d*x))/(8*d) - ((32*a^3*exp(4*c + 4*d*x))/(5*d) + (36*a^2*b*exp(3*c + 3
*d*x))/(5*d) - (36*a^2*b*exp(5*c + 5*d*x))/(5*d) + (12*a^2*b*exp(7*c + 7*d*
x))/(5*d) - (12*a^2*b*exp(c + d*x))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c
 + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) -
 1) + (b^3*exp(3*c + 3*d*x))/(24*d) - (64*a^3)/(15*d*(3*exp(2*c + 2*d*x) -
3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (16*a^3)/(5*d*(6*exp(4*c + 4*
d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (
3*atan((a^2*b*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4*b^2)^(1/2)))*(a^4*b^2)^
(1/2))/(-d^2)^(1/2) + 3*a*b^2*x - (3*a^2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*
x) - 1)) - (18*a^2*b*exp(c + d*x))/(5*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d
*x) + 1))
```

3.170 $\int \mathrm{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=166

$$-\frac{b^3x}{2}+\frac{5a^3\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\coth(c+dx)}{d}-\frac{a^2b\coth^3(c+dx)}{d}-5a^3\,\mathrm{c}$$

```
[Out] -1/2*b^3*x+5/16*a^3*arctanh(cosh(d*x+c))/d-3*a*b^2*arctanh(cosh(d*x+c))/d+3
*a^2*b*coth(d*x+c)/d-a^2*b*coth(d*x+c)^3/d-5/16*a^3*coth(d*x+c)*csch(d*x+c)
/d+5/24*a^3*coth(d*x+c)*csch(d*x+c)^3/d-1/6*a^3*coth(d*x+c)*csch(d*x+c)^5/d
+1/2*b^3*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3299, 3855, 3852, 3853, 2715, 8}

$$\frac{5a^3\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^3\coth(c+dx)\mathrm{csch}^5(c+dx)}{6d}+\frac{5a^3\coth(c+dx)\mathrm{csch}^3(c+dx)}{24d}-\frac{5a^3\coth(c+dx)\mathrm{csch}(c+dx)}{16d}-\frac{a^2b\coth^3(c+dx)}{d}+\frac{3a^2b\coth(c+dx)}{d}-\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^3\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] -1/2*(b^3*x) + (5*a^3*ArcTanh[Cosh[c + d*x]])/(16*d) - (3*a*b^2*ArcTanh[Cos
h[c + d*x]])/d + (3*a^2*b*Coth[c + d*x])/d - (a^2*b*Coth[c + d*x]^3)/d - (5
*a^3*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^3*Coth[c + d*x]*Csch[c + d*
x]^3)/(24*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d) + (b^3*Cosh[c + d*
x]*Sinh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx &= -\left(i\int\left(3iab^2\operatorname{csch}(c+dx)+3ia^2b\operatorname{csch}^4(c+dx)+ia^3\operatorname{csch}^7(c\right.\right.\\
&= a^3\int\operatorname{csch}^7(c+dx)\,dx+\left(3a^2b\right)\int\operatorname{csch}^4(c+dx)\,dx+\left(3ab^2\right)\\
&= -\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}\\
&= -\frac{b^3x}{2}-\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\coth(c+dx)}{d}-\frac{a^2}{}\\
&= -\frac{b^3x}{2}-\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\coth(c+dx)}{d}-\frac{a^2}{}\\
&= -\frac{b^3x}{2}+\frac{5a^3\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{3ab^2\tanh^{-1}(\cosh(c+dx}{d}
\end{aligned}$$

Mathematica [A]
time = 1.13, size = 236, normalized size = 1.42

$$-\frac{192b^3c+192b^3dx-384a^2b\coth\left(\frac{1}{2}(c+dx)\right)+30a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+a^3\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)+120a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-1152ab^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+30a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+6a^3\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)+a^3\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)-384a^2b\operatorname{csch}^3(c+dx)\sinh^4\left(\frac{1}{2}(c+dx)\right)-6a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)(a-4b\sinh(c+dx))-96b^3\sinh(2(c+dx))-384a^2b\tanh\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] -1/384*(192*b^3*c + 192*b^3*d*x - 384*a^2*b*Coth[(c + d*x)/2] + 30*a^3*Csch
[(c + d*x)/2]^2 + a^3*Csch[(c + d*x)/2]^6 + 120*a^3*Log[Tanh[(c + d*x)/2]]
```

```
- 1152*a*b^2*Log[Tanh[(c + d*x)/2]] + 30*a^3*Sech[(c + d*x)/2]^2 + 6*a^3*Se
ch[(c + d*x)/2]^4 + a^3*Sech[(c + d*x)/2]^6 - 384*a^2*b*Csch[c + d*x]^3*Sin
h[(c + d*x)/2]^4 - 6*a^2*Csch[(c + d*x)/2]^4*(a - 4*b*Sinh[c + d*x]) - 96*b
^3*Sinh[2*(c + d*x)] - 384*a^2*b*Tanh[(c + d*x)/2])/d
```

Maple [A]
time = 2.13, size = 254, normalized size = 1.53

method	result
risch	$-\frac{b^3 x}{2}+\frac{b^3 e^{2dx+2c}}{8d}-\frac{b^3 e^{-2dx-2c}}{8d}-\frac{a^2(15a\,e^{11dx+11c}-85a\,e^{9dx+9c}+288b\,e^{8dx+8c}+198a\,e^{7dx+7c}-960b\,e^{6dx+6c}+198a\,e^{5dx+5c}+}{24d\left(e^{2dx+2c}-1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^3*x+1/8*b^3/d*exp(2*d*x+2*c)-1/8*b^3/d*exp(-2*d*x-2*c)-1/24*a^2*(15*
a*exp(11*d*x+11*c)-85*a*exp(9*d*x+9*c)+288*b*exp(8*d*x+8*c)+198*a*exp(7*d*x
+7*c)-960*b*exp(6*d*x+6*c)+198*a*exp(5*d*x+5*c)+1152*b*exp(4*d*x+4*c)-85*a*
exp(3*d*x+3*c)-576*b*exp(2*d*x+2*c)+15*a*exp(d*x+c)+96*b)/d/(exp(2*d*x+2*c)
-1)^6+5/16*a^3/d*ln(exp(d*x+c)+1)-3*a/d*ln(exp(d*x+c)+1)*b^2-5/16*a^3/d*ln(
exp(d*x+c)-1)+3*a/d*ln(exp(d*x+c)-1)*b^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(154) = 308.
time = 0.28, size = 355, normalized size = 2.14

$$-\frac{1}{8}b^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+\frac{1}{48}a^3\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15e^{(-dx-c)}-85e^{(-3dx-3c)}+198e^{(-5dx-5c)}+198e^{(-7dx-7c)}-85e^{(-9dx-9c)}+15e^{(-11dx-11c)}\right)}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(-10dx-10c)}-e^{(-12dx-12c)}-1)}\right)-3ab^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)+4a^2b\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/48*a^3*(15*log(
e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*
e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d
*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x -
4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e
^(-12*d*x - 12*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x -
 c) - 1)/d) + 4*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6210 vs. 2(154) = 308.
time = 0.51, size = 6210, normalized size = 37.41

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(6*b^3*cosh(d*x + c)^16 + 96*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + 6*b^
3*sinh(d*x + c)^16 - 30*a^3*cosh(d*x + c)^13 - 12*(2*b^3*d*x + 3*b^3)*cosh(
d*x + c)^14 - 12*(2*b^3*d*x - 60*b^3*cosh(d*x + c)^2 + 3*b^3)*sinh(d*x + c)
^14 + 170*a^3*cosh(d*x + c)^11 + 6*(560*b^3*cosh(d*x + c)^3 - 5*a^3 - 28*(2
*b^3*d*x + 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 + 12*(12*b^3*d*x + 7*b^3)
*cosh(d*x + c)^12 + 6*(1820*b^3*cosh(d*x + c)^4 + 24*b^3*d*x - 65*a^3*cosh(
d*x + c) + 14*b^3 - 182*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
12 - 396*a^3*cosh(d*x + c)^9 + 2*(13104*b^3*cosh(d*x + c)^5 - 1170*a^3*cosh
(d*x + c)^2 - 2184*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^3 + 85*a^3 + 72*(12*b^
3*d*x + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 12*(30*b^3*d*x + 48*a^2*b
+ 7*b^3)*cosh(d*x + c)^10 + 2*(24024*b^3*cosh(d*x + c)^6 - 4290*a^3*cosh(d*
x + c)^3 - 180*b^3*d*x - 6006*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^4 + 935*a^3
*cosh(d*x + c) - 288*a^2*b - 42*b^3 + 396*(12*b^3*d*x + 7*b^3)*cosh(d*x + c
)^2)*sinh(d*x + c)^10 - 396*a^3*cosh(d*x + c)^7 + 2*(34320*b^3*cosh(d*x + c
)^7 - 10725*a^3*cosh(d*x + c)^4 - 12012*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^5
 + 4675*a^3*cosh(d*x + c)^2 + 1320*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^3 - 1
98*a^3 - 60*(30*b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^9
+ 480*(b^3*d*x + 4*a^2*b)*cosh(d*x + c)^8 + 6*(12870*b^3*cosh(d*x + c)^8 -
6435*a^3*cosh(d*x + c)^5 - 6006*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^6 + 4675*
a^3*cosh(d*x + c)^3 + 80*b^3*d*x + 990*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^4
 - 594*a^3*cosh(d*x + c) + 320*a^2*b - 90*(30*b^3*d*x + 48*a^2*b + 7*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^8 + 170*a^3*cosh(d*x + c)^5 + 12*(5720*b^3*co
sh(d*x + c)^9 - 4290*a^3*cosh(d*x + c)^6 - 3432*(2*b^3*d*x + 3*b^3)*cosh(d*
x + c)^7 + 4675*a^3*cosh(d*x + c)^4 + 792*(12*b^3*d*x + 7*b^3)*cosh(d*x + c
)^5 - 1188*a^3*cosh(d*x + c)^2 - 120*(30*b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d
*x + c)^3 - 33*a^3 + 320*(b^3*d*x + 4*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7
 - 12*(30*b^3*d*x + 192*a^2*b - 7*b^3)*cosh(d*x + c)^6 + 12*(4004*b^3*cosh(
d*x + c)^10 - 4290*a^3*cosh(d*x + c)^7 - 3003*(2*b^3*d*x + 3*b^3)*cosh(d*x
+ c)^8 + 6545*a^3*cosh(d*x + c)^5 + 924*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^
6 - 2772*a^3*cosh(d*x + c)^3 - 30*b^3*d*x - 210*(30*b^3*d*x + 48*a^2*b + 7*
b^3)*cosh(d*x + c)^4 - 231*a^3*cosh(d*x + c) - 192*a^2*b + 7*b^3 + 1120*(b^
3*d*x + 4*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 30*a^3*cosh(d*x + c)^3
+ 2*(13104*b^3*cosh(d*x + c)^11 - 19305*a^3*cosh(d*x + c)^8 - 12012*(2*b^3*
d*x + 3*b^3)*cosh(d*x + c)^9 + 39270*a^3*cosh(d*x + c)^6 + 4752*(12*b^3*d*x
 + 7*b^3)*cosh(d*x + c)^7 - 24948*a^3*cosh(d*x + c)^4 - 1512*(30*b^3*d*x +
48*a^2*b + 7*b^3)*cosh(d*x + c)^5 - 4158*a^3*cosh(d*x + c)^2 + 13440*(b^3*d
*x + 4*a^2*b)*cosh(d*x + c)^3 + 85*a^3 - 36*(30*b^3*d*x + 192*a^2*b - 7*b^3
)*cosh(d*x + c))*sinh(d*x + c)^5 + 12*(12*b^3*d*x + 96*a^2*b - 7*b^3)*cosh(
d*x + c)^4 + 2*(5460*b^3*cosh(d*x + c)^12 - 10725*a^3*cosh(d*x + c)^9 - 600
6*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^10 + 28050*a^3*cosh(d*x + c)^7 + 2970*(
12*b^3*d*x + 7*b^3)*cosh(d*x + c)^8 - 24948*a^3*cosh(d*x + c)^5 - 1260*(30*
```

```
b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d*x + c)^6 - 6930*a^3*cosh(d*x + c)^3 + 72
*b^3*d*x + 16800*(b^3*d*x + 4*a^2*b)*cosh(d*x + c)^4 + 425*a^3*cosh(d*x + c
) + 576*a^2*b - 42*b^3 - 90*(30*b^3*d*x + 192*a^2*b - 7*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^4 + 2*(1680*b^3*cosh(d*x + c)^13 - 4290*a^3*cosh(d*x + c)^
10 - 2184*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^11 + 14025*a^3*cosh(d*x + c)^8
+ 1320*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^9 - 16632*a^3*cosh(d*x + c)^6 - 7
20*(30*b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d*x + c)^7 - 6930*a^3*cosh(d*x + c)
^4 + 13440*(b^3*d*x + 4*a^2*b)*cosh(d*x + c)^5 + 850*a^3*cosh(d*x + c)^2 -
120*(30*b^3*d*x + 192*a^2*b - 7*b^3)*cosh(d*x + c)^3 - 15*a^3 + 24*(12*b^3*
d*x + 96*a^2*b - 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*b^3 - 12*(2*b^3*
d*x + 16*a^2*b - 3*b^3)*cosh(d*x + c)^2 + 2*(360*b^3*cosh(d*x + c)^14 - 117
0*a^3*cosh(d*x + c)^11 - 546*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^12 + 4675*a^
3*cosh(d*x + c)^9 + 396*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^10 - 7128*a^3*co
sh(d*x + c)^7 - 270*(30*b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d*x + c)^8 - 4158*
a^3*cosh(d*x + c)^5 + 6720*(b^3*d*x + 4*a^2*b)*cosh(d*x + c)^6 + 850*a^3*co
sh(d*x + c)^3 - 12*b^3*d*x - 90*(30*b^3*d*x + 192*a^2*b - 7*b^3)*cosh(d*x +
 c)^4 - 45*a^3*cosh(d*x + c) - 96*a^2*b + 18*b^3 + 36*(12*b^3*d*x + 96*a^2*
b - 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((5*a^3 - 48*a*b^2)*cosh(d*
x + c)^14 + 14*(5*a^3 - 48*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^13 + (5*a^3 -
 48*a*b^2)*sinh(d*x + c)^14 - 6*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^12 - (30*a
^3 - 288*a*b^2 - 91*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^12 +
4*(91*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 - 18*(5*a^3 - 48*a*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^11 + 15*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^10 + (1001*(5*a
^3 - 48*a*b^2)*cosh(d*x + c)^4 + 75*a^3 - 720*a*b^2 - 396*(5*a^3 - 48*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*(5*a^3 - 48*a*b^2)*cosh(d*x +
 c)^5 - 660*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(154) = 308.
time = 0.52, size = 327, normalized size = 1.97

$$-\frac{24\,(dx+c)b^3 - 6\,b^3e^{(2\,dx+2\,c)} - 3\,(5\,a^3 - 48\,ab^2)\log\left(e^{(dx+c)}+1\right) + 3\,(5\,a^3 - 48\,ab^2)\log\left(\left|e^{(dx+c)}-1\right|\right) + \frac{2\,\left(15\,a^3e^{(13\,dx+13\,c)}+3\,b^3e^{(12\,dx+12\,c)}-85\,a^3e^{(11\,dx+11\,c)}+198\,a^3e^{(9\,dx+9\,c)}+198\,a^3e^{(7\,dx+7\,c)}-85\,a^3e^{(5\,dx+5\,c)}+15\,a^3e^{(3\,dx+3\,c)}+3\,b^3+18\,(16\,a^2b-b^3)e^{(10\,dx+10\,c)}-15\,(64\,a^2b-3\,b^3)e^{(8\,dx+8\,c)}+12\,(96\,a^2b-5\,b^3)e^{(6\,dx+6\,c)}-9\,(64\,a^2b-5\,b^3)e^{(4\,dx+4\,c)}+6\,(16\,a^2b-3\,b^3)e^{(2\,dx+2\,c)}\right)e^{(-2\,dx-2\,c)}}{\left(e^{(dx+c)}+1\right)^6\left(e^{(dx+c)}-1\right)^6}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/48*(24*(d*x + c)*b^3 - 6*b^3*e^(2*d*x + 2*c) - 3*(5*a^3 - 48*a*b^2)*log(
e^(d*x + c) + 1) + 3*(5*a^3 - 48*a*b^2)*log(abs(e^(d*x + c) - 1)) + 2*(15*a
^3*e^(13*d*x + 13*c) + 3*b^3*e^(12*d*x + 12*c) - 85*a^3*e^(11*d*x + 11*c) +
 198*a^3*e^(9*d*x + 9*c) + 198*a^3*e^(7*d*x + 7*c) - 85*a^3*e^(5*d*x + 5*c)
 + 15*a^3*e^(3*d*x + 3*c) + 3*b^3 + 18*(16*a^2*b - b^3)*e^(10*d*x + 10*c) -
 15*(64*a^2*b - 3*b^3)*e^(8*d*x + 8*c) + 12*(96*a^2*b - 5*b^3)*e^(6*d*x + 6
*c) - 9*(64*a^2*b - 5*b^3)*e^(4*d*x + 4*c) + 6*(16*a^2*b - 3*b^3)*e^(2*d*x
+ 2*c))*e^(-2*d*x - 2*c)/((e^(d*x + c) + 1)^6*(e^(d*x + c) - 1)^6))/d
```

Mupad [B]
time = 0.29, size = 486, normalized size = 2.93

$$\frac{\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\left(5\,a^3\,\sqrt{-d^2}-48\,a\,b^2\,\sqrt{-d^2}\right)}{d\,\sqrt{25\,a^6-480\,a^4\,b^2+2304\,a^2\,b^4}}\right)\,\sqrt{25\,a^6-480\,a^4\,b^2+2304\,a^2\,b^4}}{8\,\sqrt{-d^2}}-\frac{b^3\,x}{2}-\frac{\frac{12\,a^2\,b}{d}-\frac{5\,a^3\,{\mathrm{e}}^{c+d\,x}}{12\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{\frac{8\,a^2\,b}{d}+\frac{a^3\,{\mathrm{e}}^{c+d\,x}}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{b^3\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}+\frac{b^3\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}-\frac{18\,a^3\,{\mathrm{e}}^{c+d\,x}}{d\,\left(6\,{\mathrm{e}}^{4\,c+4\,d\,x}-4\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1\right)}-\frac{80\,a^3\,{\mathrm{e}}^{c+d\,x}}{3\,d\,\left(5\,{\mathrm{e}}^{2\,c+2\,d\,x}-10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}-5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}-1\right)}-\frac{32\,a^3\,{\mathrm{e}}^{c+d\,x}}{3\,d\,\left(15\,{\mathrm{e}}^{4\,c+4\,d\,x}-6\,{\mathrm{e}}^{2\,c+2\,d\,x}-20\,{\mathrm{e}}^{6\,c+6\,d\,x}+15\,{\mathrm{e}}^{8\,c+8\,d\,x}-6\,{\mathrm{e}}^{10\,c+10\,d\,x}+{\mathrm{e}}^{12\,c+12\,d\,x}+1\right)}-\frac{5\,a^3\,{\mathrm{e}}^{c+d\,x}}{8\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^7,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(5*a^3*(-d^2)^(1/2) - 48*a*b^2*(-d^2)^(1/2)))/(d*(25
*a^6 + 2304*a^2*b^4 - 480*a^4*b^2)^(1/2)))*(25*a^6 + 2304*a^2*b^4 - 480*a^4
*b^2)^(1/2))/(8*(-d^2)^(1/2)) - (b^3*x)/2 - ((12*a^2*b)/d - (5*a^3*exp(c +
d*x))/(12*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((8*a^2*b)/d +
(a^3*exp(c + d*x))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*
c + 6*d*x) - 1) - (b^3*exp(- 2*c - 2*d*x))/(8*d) + (b^3*exp(2*c + 2*d*x))/(
8*d) - (18*a^3*exp(c + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) -
4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (80*a^3*exp(c + d*x))/(3*d*(5
*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c +
 8*d*x) + exp(10*c + 10*d*x) - 1)) - (32*a^3*exp(c + d*x))/(3*d*(15*exp(4*c
 + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x)
- 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (5*a^3*exp(c + d*x))/(8
*d*(exp(2*c + 2*d*x) - 1))
```

3.171 $\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=328

$$-\frac{ax}{b^2}-\frac{2(-1)^{2/3}a^{4/3}\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^2d}-\frac{2(-1)^{2/3}a^{4/3}\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}\right.}{\sqrt{\sqrt[3]{-1}}}\right.}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

```
[Out] -a*x/b^2-cosh(d*x+c)/b/d+1/3*cosh(d*x+c)^3/b/d-2/3*(-1)^(2/3)*a^(4/3)*arcta
n((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)
*a^(2/3)-b^(2/3))^(1/2))/b^2/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)-2/3*a^(4/
3)*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/b
^2/d/(a^(2/3)+b^(2/3))^(1/2)-2/3*(-1)^(2/3)*a^(4/3)*arctan((-1)^(1/6)*((-1)
^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^(2/3
)*b^(2/3))^(1/2))/b^2/d/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A]

time = 0.63, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3299, 2713, 3292, 2739, 632, 210}

$$-\frac{2(-1)^{2/3}a^{4/3}\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}a^{4/3}\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^2d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}-\frac{2a^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}+b^{2/3}}}-\frac{ax}{b^2}+\frac{\cosh^3(c+dx)}{3bd}-\frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -((a*x)/b^2) - (2*(-1)^(2/3)*a^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3)
 + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/
3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^2*d) - (2*(-1)^(2/
3)*a^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x
)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^
(2/3)]*b^2*d) - (2*a^(4/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sq
rt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^2*d) - Cosh[c + d*x]/(
b*d) + Cosh[c + d*x]^3/(3*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\int \left(\frac{a}{b^2} - \frac{\sinh^3(c+dx)}{b} - \frac{a^2}{b^2\left(a+b\sinh^3(c+dx)\right)}\right)dx$$

$$= -\frac{ax}{b^2} + \frac{a^2\int \frac{1}{a+b\sinh^3(c+dx)}\,dx}{b^2} + \frac{\int \sinh^3(c+dx)\,dx}{b}$$

$$= -\frac{ax}{b^2} + \frac{a^2\int \left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a} - i\sqrt[3]{b}\sinh(c+dx)\right)} + \frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a} + \sqrt[6]{-1}\sqrt[3]{b}\sinh(c\right.}\right.}{b^2}$$

$$= -\frac{ax}{b^2} - \frac{\cosh(c+dx)}{bd} + \frac{\cosh^3(c+dx)}{3bd} + \frac{\left(\sqrt[6]{-1}\,a^{4/3}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a} - i\sqrt[3]{b}\sinh(c+dx)}}{3b^2}$$

$$= -\frac{ax}{b^2} - \frac{\cosh(c+dx)}{bd} + \frac{\cosh^3(c+dx)}{3bd} - \frac{\left(2(-1)^{2/3}a^{4/3}\right)\text{Subst}\left(\int \frac{}{\sqrt[6]{-1}\sqrt[3]{a} - 2\sqrt[3]{}}\right.}{3}$$

$$= -\frac{ax}{b^2} - \frac{\cosh(c+dx)}{bd} + \frac{\cosh^3(c+dx)}{3bd} + \frac{\left(4(-1)^{2/3}a^{4/3}\right)\text{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}\,a^{2/3} - \right.}\right.}{}$$

$$= -\frac{ax}{b^2} + \frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b} - (-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3} - b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}\,a^{2/3} - b^{2/3}}\,b^2 d} - \frac{2(-1)^{2/3}a^{4/3}\tan^{-1}}{3\sqrt{\sqrt[3]{}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.26, size = 168, normalized size = 0.51

$$\frac{-12ac - 12adx - 9b\cosh(c+dx) + b\cosh(3(c+dx)) + 8a^2\text{RootSum}\left[-b + 3b\#1^2 + 8a\#1^3 - 3b\#1^4 + b\#1^6\&, \frac{c\#1 + dx\#1 + 2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) + \cosh\left(\frac{1}{2}(c+dx)\right)\#1 - \sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1}{b + 4a\#1 - 2b\#1^2 + b\#1^4}\&\right]}{12b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-12*a*c - 12*a*d*x - 9*b*Cosh[c + d*x] + b*Cosh[3*(c + d*x)] + 8*a^2*RootS
um[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1 + d*x*#1 + 2*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d
*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(12*b^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.30, size = 236, normalized size = 0.72

method	result
derivativedivides	$\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^{2}}-\frac{a^{2}\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}-3a\_\right.}\right.}{}$
default	$\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^{2}}-\frac{a^{2}\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}-3a\_\right.}\right.}{}$
risch	$-\frac{ax}{b^{2}}+\frac{\mathrm{e}^{3dx+3c}}{24bd}-\frac{3\,\mathrm{e}^{dx+c}}{8bd}-\frac{3\,\mathrm{e}^{-dx-c}}{8bd}+\frac{\mathrm{e}^{-3dx-3c}}{24bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^{2}b^{12}d^{6}+729b^{14}d^{6}\right)\_Z^{6}-243a^{4}b^{8}d\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3/b/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/b/
(tanh(1/2*d*x+1/2*c)+1)-a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/3*a^2/b^2*sum((_R
^4-2*_R^2+1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=
RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))-1/3/b/(tanh(1/2*d*x+1/2*c)-1)^
3-1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/b/(tanh(1/2*d*x+1/2*c)-1)+a/b^2*ln(ta
nh(1/2*d*x+1/2*c)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 8*a^2*integrate(e^(3*d*x + 3*c)/(b^3*e^(6*d*x + 6*c) - 3*b^3*e^(4*d*x + 4*c
) + 8*a*b^2*e^(3*d*x + 3*c) + 3*b^3*e^(2*d*x + 2*c) - b^3), x) - 1/24*(24*a
*d*x*e^(3*d*x + 3*c) - b*e^(6*d*x + 6*c) + 9*b*e^(4*d*x + 4*c) + 9*b*e^(2*d
*x + 2*c) - b)*e^(-3*d*x - 3*c)/(b^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28816 vs. $2(237) = 474$.
time = 5.59, size = 28816, normalized size = 87.85

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/24*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x +
c)^6 - 24*a*d*x*cosh(d*x + c)^3 - 9*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c
)^2 - 3*b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - 6*a*d*x - 9*b*cosh(d*
x + c))*sinh(d*x + c)^3 + 12*sqrt(2/3)*sqrt(1/6)*(b^2*d*cosh(d*x + c)^3 + 3
*b^2*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^2*d*cosh(d*x + c)*sinh(d*x + c)^
2 + b^2*d*sinh(d*x + c)^3)*sqrt((6*a^4 - (a^2*b^4 + b^6)*(2*a^4/(a^2*b^4*d^
2 + b^6*d^2) - 2*(1/2)^(2/3)*(a^8/(a^2*b^4*d^2 + b^6*d^2)^2 - a^6/(a^2*b^8*
d^4 + b^10*d^4))*(-I*sqrt(3) + 1)/(2*a^12/(a^2*b^4*d^2 + b^6*d^2)^3 - 3*a^1
0/((a^2*b^8*d^4 + b^10*d^4)*(a^2*b^4*d^2 + b^6*d^2)) + a^8/(a^2*b^12*d^6 +
b^14*d^6) + a^8/((a^2 + b^2)^2*b^10*d^6))^(1/3) - (1/2)^(1/3)*(2*a^12/(a^2*
b^4*d^2 + b^6*d^2)^3 - 3*a^10/((a^2*b^8*d^4 + b^10*d^4)*(a^2*b^4*d^2 + b^6*
d^2)) + a^8/(a^2*b^12*d^6 + b^14*d^6) + a^8/((a^2 + b^2)^2*b^10*d^6))^(1/3)
*(I*sqrt(3) + 1))*d^2 - 3*sqrt(1/3)*(a^2*b^4 + b^6)*d^2*sqrt(-(4*a^8 + 16*a
^6*b^2 + (a^4*b^8 + 2*a^2 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(sinh(c + d*x)**6/(a + b*sinh(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^6/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [B]
time = 10.80, size = 1579, normalized size = 4.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^6/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log((294912*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b
^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^2*a^7*b^5*d^2 - 98304*a^10*exp
(d*x)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^
4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)) + 1327104*root(729*a^2*b^12*d^6*z^6 +
729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^3*
a^6*b^7*d^3 + 2654208*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^
4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^4*a^5*b^9*d^4 + 1990656*roo
t(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^
4*d^2*z^2 - a^8, z, k)^5*a^4*b^11*d^5 + 24576*root(729*a^2*b^12*d^6*z^6 + 7
29*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)*a^8
*b^3*d + 589824*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*
d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^2*a^8*b^4*d^2*exp(d*x)*exp(root(7
29*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d
^2*z^2 - a^8, z, k)) + 5308416*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6
 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^3*a^7*b^6*d^3*exp(
d*x)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4
 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)) - 663552*root(729*a^2*b^12*d^6*z^6 + 72
9*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^4*a^
4*b^10*d^4*exp(d*x)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*
a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)) + 2654208*root(729*a^2*b
^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 -
 a^8, z, k)^4*a^6*b^8*d^4*exp(d*x)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14
*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)) - 9953280
*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^
6*b^4*d^2*z^2 - a^8, z, k)^5*a^3*b^12*d^5*exp(d*x)*exp(root(729*a^2*b^12*d^
6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8,
z, k)) - 7962624*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8
*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^5*a^5*b^10*d^5*exp(d*x)*exp(root
(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4
*d^2*z^2 - a^8, z, k)) - 491520*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^
6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)*a^9*b^2*d*exp(d*x
)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 +
27*a^6*b^4*d^2*z^2 - a^8, z, k)))/b^15)*root(729*a^2*b^12*d^6*z^6 + 729*b^1
4*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k), k, 1, 6)
 - (3*exp(c + d*x))/(8*b*d) - (3*exp(- c - d*x))/(8*b*d) + exp(- 3*c - 3*d*
x)/(24*b*d) + exp(3*c + 3*d*x)/(24*b*d) - (a*x)/b^2
```

3.172 $\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=295

$$-\frac{x}{2b}+\frac{2a\text{ArcTan}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\,\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,b^{5/3}d}+\frac{2a\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,b^{5/3}d}$$

```
[Out] -1/2*x/b+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d+2/3*a*arctan((-1)^(1/6)*((-1)^(5/6
)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)
)/b^(5/3)/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)+2/3*a*arctan((-1)^(5/6)*((-1
)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)-b^(2/3)
)^(1/2))/b^(5/3)/d/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2)+2/3*a*arctanh((b^(1/
3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/b^(5/3)/d/(a^(2/3)
+b^(2/3))^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = $\{3299, 2715, 8, 2739, 632, 212, 210\}$

$$\frac{2a\text{ArcTan}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\,\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}+\frac{2a\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}+\frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -1/2*x/b + (2*a*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c
+ d*x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^
(2/3)) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3)
+ I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqr
t[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3) - a^(1/3
)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b
^(5/3)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx &= -\left(i\int\left(\frac{i\sinh^2(c+dx)}{b}-\frac{ia\sinh^2(c+dx)}{b\left(a+b\sinh^3(c+dx)\right)}\right)dx\right)\\
&= \frac{\int\sinh^2(c+dx)\,dx}{b}-\frac{a\int\frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx}{b}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd}-\frac{\int 1\,dx}{2b}+\frac{a\int\left(\frac{i}{3b^{2/3}\left(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{}{3b^{2/3}\left(\right.}\right.}{}\\
&= -\frac{x}{2b}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}+\frac{(ia)\int\frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{5/3}}+\frac{(ia)\int\frac{}{\sqrt[6]{-}}}{}\\
&= -\frac{x}{2b}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}+\frac{(2a)\text{Subst}\left(\int\frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{b}\,x-i\sqrt[3]{a}\,x^2}\,dx,x,\text{ta}\right.}{3b^{5/3}d}\\
&= -\frac{x}{2b}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}-\frac{(4a)\text{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}\,a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-\right.}{3b^{5/3}}\\
&= -\frac{x}{2b}-\frac{2a\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\,\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,b^{5/3}d}-\frac{2a\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\,\tan}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-}}\right.}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.23, size = 299, normalized size = 1.01

$$\frac{-6(c+dx)-2a\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{c+dx+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)-2c\#1^2-2dx\#1^2-4\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2+c\#1^4+dx\#1^4+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4}{b\#1+4a\#1^2-2b\#1^3+b\#1^5}\&\right]+3\sinh(2(c+dx))}{12bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-6*(c + d*x) - 2*a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &
, (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/
2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*
x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1
^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cos
h[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3
 + b*#1^5) & ] + 3*Sinh[2*(c + d*x)])/(12*b*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.21, size = 190, normalized size = 0.64

method	result
derivativedivides	$\frac{\frac{4a\left(\sum_{\_\_R=\mathrm{RootOf}\left(a\_\_Z^{6}-3a\_\_Z^{4}-8b\_\_Z^{3}+3a\_\_Z^{2}-a\right)}\frac{\_\_R^{2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_\_R\right)}{\_\_R^{5}a-2\_\_R^{3}a-4\_\_R^{2}b+\_\_Ra}\right)}{3b}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{}{2b(\mathrm{ta}}}{d}$
default	$\frac{\frac{4a\left(\sum_{\_\_R=\mathrm{RootOf}\left(a\_\_Z^{6}-3a\_\_Z^{4}-8b\_\_Z^{3}+3a\_\_Z^{2}-a\right)}\frac{\_\_R^{2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_\_R\right)}{\_\_R^{5}a-2\_\_R^{3}a-4\_\_R^{2}b+\_\_Ra}\right)}{3b}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{}{2b(\mathrm{ta}}}{d}$
risch	$-\frac{x}{2b}+\frac{\mathrm{e}^{2dx+2c}}{8bd}-\frac{\mathrm{e}^{-2dx-2c}}{8bd}+\left(\sum_{\_\_R=\mathrm{RootOf}\left(\left(729a^{2}b^{10}d^{6}+729b^{12}d^{6}\right)\_\_Z^{6}-243a^{2}b^{8}d^{4}\_\_Z^{4}+27a^{4}d^{2}\_\_Z^{2}b^{4}-a^{6}\right)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4/3*a/b*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c
)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))-1/2/b/(tanh(1/2*d*x+1
/2*c)+1)^2+1/2/b/(tanh(1/2*d*x+1/2*c)+1)-1/2/b*ln(tanh(1/2*d*x+1/2*c)+1)+1/
2/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/b/(tanh(1/2*d*x+1/2*c)-1)+1/2/b*ln(tanh(1
/2*d*x+1/2*c)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/8*(4*d*x*e^(2*d*x + 2*c) - e^(4*d*x + 4*c) + 1)*e^(-2*d*x - 2*c)/(b*d) -
 1/32*integrate(64*(a*e^(5*d*x + 5*c) - 2*a*e^(3*d*x + 3*c) + a*e^(d*x + c)
)/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c) + 8*a*b*e^(3*d*x + 3*c) + 3*
b^2*e^(2*d*x + 2*c) - b^2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28427 vs. 2(210) = 420.
time = 3.34, size = 28427, normalized size = 96.36

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/24*(12*d*x*cosh(d*x + c)^2 - 3*cosh(d*x + c)^4 - 12*cosh(d*x + c)*sinh(d
*x + c)^3 - 3*sinh(d*x + c)^4 - 12*sqrt(2/3)*sqrt(1/6)*(b*d*cosh(d*x + c)^2
 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2)*sqrt(-((a^2*b^2
 + b^4)*(2*(1/2)^(2/3)*(a^4/(a^2*b^6*d^4 + b^8*d^4) - a^4/(a^2*b^2*d^2 + b^
4*d^2)^2)*(-I*sqrt(3) + 1)/(a^6/(a^2*b^10*d^6 + b^12*d^6) - 3*a^6/((a^2*b^6
*d^4 + b^8*d^4)*(a^2*b^2*d^2 + b^4*d^2)) + 2*a^6/(a^2*b^2*d^2 + b^4*d^2)^3
+ a^8/((a^2 + b^2)^2*b^10*d^6))^(1/3) - (1/2)^(1/3)*(a^6/(a^2*b^10*d^6 + b^
12*d^6) - 3*a^6/((a^2*b^6*d^4 + b^8*d^4)*(a^2*b^2*d^2 + b^4*d^2)) + 2*a^6/(
a^2*b^2*d^2 + b^4*d^2)^3 + a^8/((a^2 + b^2)^2*b^10*d^6))^(1/3)*(I*sqrt(3) +
 1) + 2*a^2/(a^2*b^2*d^2 + b^4*d^2))*d^2 + 3*sqrt(1/3)*(a^2*b^2 + b^4)*d^2*
sqrt(-((a^4*b^6 + 2*a^2*b^8 + b^10)*(2*(1/2)^(2/3)*(a^4/(a^2*b^6*d^4 + b^8*
d^4) - a^4/(a^2*b^2*d^2 + b^4*d^2)^2)*(-I*sqrt(3) + 1)/(a^6/(a^2*b^10*d^6 +
 b^12*d^6) - 3*a^6/((a^2*b^6*d^4 + b^8*d^4)*(a^2*b^2*d^2 + b^4*d^2)) + 2*a^
6/(a^2*b^2*d^2 + b^4*d^2) ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(sinh(c + d*x)**5/(a + b*sinh(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^5/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [B]
time = 11.48, size = 1114, normalized size = 3.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^5/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log(- root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4
*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k)*(root(729*a^2*b^10*d^6*z^6 + 729*b^1
2*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k)*(root(729
*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2
*z^2 - a^6, z, k)*(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b
^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k)*((663552*(8*a^6*d^4 + 4*a^4*b^
2*d^4 - 5*a^5*b*d^4*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z
^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b^7 + (199065
6*root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a
^4*b^4*d^2*z^2 - a^6, z, k)*(4*a^5*d^5*exp(d*x)*exp(root(729*a^2*b^10*d^6*z
^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z,
k)) - a^4*b*d^5 + 5*a^3*b^2*d^5*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 72
9*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b
^5) + (442368*(4*a^6*b*d^3 + 8*a^7*d^3*exp(d*x)*exp(root(729*a^2*b^10*d^6*z
^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z,
k)) - 5*a^5*b^2*d^3*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z
^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b^9) - (29491
2*a^6*d^2*(2*b - 5*a*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*
z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b^10) - (245
76*a^7*d*(8*a - 5*b*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z
^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b^12) - (3276
8*a^8*(b - 4*a*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 -
243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b^14)*root(729*a^2
*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2
 - a^6, z, k), k, 1, 6) - x/(2*b) - exp(- 2*c - 2*d*x)/(8*b*d) + exp(2*c +
2*d*x)/(8*b*d)
```

3.173 $\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=303

$$-\frac{2a^{2/3}\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{4/3}d}+\frac{2\sqrt[3]{-1}a^{2/3}\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh(\frac{1}{2}(c}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,b^{4/3}d}$$

```
[Out] cosh(d*x+c)/b/d+2/3*(-1)^(1/3)*a^(2/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3
)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/b^(4/3
)/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)-2/3*a^(2/3)*arctanh((b^(1/3)-a^(1/3)
*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)+b^(2/3))^
(1/2)-2/3*a^(2/3)*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*
d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b^(4/3)/d/((-1)^
(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3299, 2718, 2739, 632, 210}

$$-\frac{2a^{2/3}\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[3]{-1}a^{2/3}\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}-\frac{2a^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-2*a^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*
x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*
a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(4/3)*d) + (2*(-1)^(1/3)*a^(2/3)*ArcTan[((-
1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3
)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(4/3)*d) - (
2*a^(2/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2
/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^(4/3)*d) + Cosh[c + d*x]/(b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx &= \int \left(\frac{\sinh(c+dx)}{b}-\frac{a\sinh(c+dx)}{b\left(a+b\sinh^3(c+dx)\right)}\right)dx \\
&= \frac{\int \sinh(c+dx)\,dx}{b}-\frac{a\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx}{b} \\
&= \frac{\cosh(c+dx)}{bd}+\frac{(ia)\int \left(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}-\frac{}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\right.}\right.}{} \\
&= \frac{\cosh(c+dx)}{bd}-\frac{\left(ia^{2/3}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{4/3}}+\frac{\left(\sqrt[6]{-1}\,a^{2/3}\right)\int \frac{}{\sqrt[6]{-}}}{} \\
&= \frac{\cosh(c+dx)}{bd}-\frac{\left(2a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}\,dx,x,\tan\right.}{3b^{4/3}d} \\
&= \frac{\cosh(c+dx)}{bd}+\frac{\left(4a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}\right)-x^2}\,dx,x,2\sqrt[3]{-1}\sqrt[3]{b}\right.}{3b^{4/3}d} \\
&= -\frac{2\sqrt[3]{-1}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,b^{4/3}d}-\frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(}{\sqrt{\sqrt[3]{-1}\,a}}\right.}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}-(}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.24, size = 214, normalized size = 0.71

$$\frac{3\cosh(c+dx)-a\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{-c-dx-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)+c\#1^2+dx\#1^2+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2}{b+4a\#1-2b\#1^2+b\#1^4}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (3*Cosh[c + d*x] - a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &
 , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x
)/2]*#1 - Sinh[(c + d*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cosh[(c + d*x)
/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2
)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(3*b*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.14, size = 123, normalized size = 0.41

method	result
derivativedivides	$\frac{\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3b}-}{d}$
default	$\frac{\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3b}-}{d}$
risch	$\frac{\mathrm{e}^{dx+c}}{2bd}+\frac{\mathrm{e}^{-dx-c}}{2bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^8d^6+729b^{10}d^6\right)\_Z^6+243a^2b^6d^4\_Z^4-a^4\right)}\_R\ln\left(\mathrm{e}^{dx+c}+\left(-\frac{243}{a^4}\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b/(tanh(1/2*d*x+1/2*c)+1)-2/3*a/b*sum((_R^3-_R)/(_R^5*a-2*_R^3*a-4*_
R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3
*_Z^2*a-a))-1/b/(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/16*integrate(64*(a*e^(4*d*
x + 4*c) - a*e^(2*d*x + 2*c))/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c)
+ 8*a*b*e^(3*d*x + 3*c) + 3*b^2*e^(2*d*x + 2*c) - b^2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 20941 vs. 2(210) = 420.
time = 1.51, size = 20941, normalized size = 69.11

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2/3)*sqrt(1/6)*(b*d*cosh(d*x + c) + b*d*sinh(d*x + c))*sqrt(((a^
2*b^2 + b^4)*(2*(1/2)^(2/3)*a^4*(-I*sqrt(3) + 1)/((a^2*b^2*d^2 + b^4*d^2)^2
*(a^4/(a^2*b^8*d^6 + b^10*d^6) - 2*a^6/(a^2*b^2*d^2 + b^4*d^2)^3 - (a^2 - b
```

```
^2)*a^4/((a^2 + b^2)^2*b^8*d^6))^(1/3)) + (1/2)^(1/3)*(a^4/(a^2*b^8*d^6 + b
^10*d^6) - 2*a^6/(a^2*b^2*d^2 + b^4*d^2)^3 - (a^2 - b^2)*a^4/((a^2 + b^2)^2
*b^8*d^6))^(1/3)*(I*sqrt(3) + 1) + 2*a^2/(a^2*b^2*d^2 + b^4*d^2))*d^2 + 3*s
qrt(1/3)*(a^2*b^2 + b^4)*d^2*sqrt(-((a^4*b^4 + 2*a^2*b^6 + b^8)*(2*(1/2)^(2
/3)*a^4*(-I*sqrt(3) + 1)/((a^2*b^2*d^2 + b^4*d^2)^2*(a^4/(a^2*b^8*d^6 + b^1
0*d^6) - 2*a^6/(a^2*b^2*d^2 + b^4*d^2)^3 - (a^2 - b^2)*a^4/((a^2 + b^2)^2*b
^8*d^6))^(1/3)) + (1/2)^(1/3)*(a^4/(a^2*b^8*d^6 + b^10*d^6) - 2*a^6/(a^2*b^
2*d^2 + b^4*d^2)^3 - (a^2 - b^2)*a^4/((a^2 + b^2)^2*b^8*d^6))^(1/3)*(I*sqrt
(3) + 1) + 2*a^2/(a^2*b^2*d^2 + b^4*d^2))^2*d^4 - 12*a^4 - 4*(a^4*b^2 + a^2
*b^4)*(2*(1/2)^(2/3)*a^4*(-I*sqrt(3) + 1)/((a^2*b^2*d^2 + b^4*d^2)^2*(a^4/(
a^2*b^8*d^6 + b^10*d^6) - ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(sinh(c + d*x)**4/(a + b*sinh(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^4/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [B]
time = 23.50, size = 906, normalized size = 2.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log((8192*a^6*(8*a - b*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b
^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k))))/b^12 - root(729*a^2*b^8*d
^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)*(root(729*a^2*
b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)*(root(729
*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)*(roo
```

```
t(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)
*((663552*(4*a^5*b*d^4 + 16*a^6*d^4*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 +
 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)) + 11*a^4*b^2*d^4*exp(
d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4
- a^4, z, k))))/b^7 + (1990656*root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6
+ 243*a^2*b^6*d^4*z^4 - a^4, z, k)*(4*a^5*d^5*exp(d*x)*exp(root(729*a^2*b^8
*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)) - a^4*b*d^5
 + 5*a^3*b^2*d^5*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 +
 243*a^2*b^6*d^4*z^4 - a^4, z, k))))/b^5) - (221184*(8*a^6*d^3 + a^4*b^2*d^
3 - 25*a^5*b*d^3*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 +
 243*a^2*b^6*d^4*z^4 - a^4, z, k))))/b^8) - (294912*a^5*d^2*(b - 7*a*exp(d*
x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 -
a^4, z, k))))/b^9) - (196608*a^6*d*(b - 2*a*exp(d*x)*exp(root(729*a^2*b^8*d
^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k))))/b^11))*root
(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k),
 k, 1, 6) + exp(c + d*x)/(2*b*d) + exp(- c - d*x)/(2*b*d)
```

3.174 $\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{x}{b}+\frac{2(-1)^{2/3}\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\,\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}\,bd}+\frac{2(-1)^{2/3}\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+\right.}{\sqrt{\sqrt[3]{-1}\,a^{2}}}\right.}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,b}$$

```
[Out] x/b+2/3*(-1)^(2/3)*a^(1/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*
tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/b/d/((-1)^(1/3)*a^
(2/3)-b^(2/3))^(1/2)+2/3*a^(1/3)*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*
c))/(a^(2/3)+b^(2/3))^(1/2))/b/d/(a^(2/3)+b^(2/3))^(1/2)+2/3*(-1)^(2/3)*a^(
1/3)*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/(
(-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b/d/((-1)^(1/3)*a^(2/3)-(-1)^
(2/3)*b^(2/3))^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3299, 3292, 2739, 632, 210}

$$\frac{2(-1)^{2/3}\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\,\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2(-1)^{2/3}\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3bd\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{a}\,\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+b^{2/3}}}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] x/b + (2*(-1)^(2/3)*a^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1
/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*
Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b*d) + (2*(-1)^(2/3)*a^(1/3)*
ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt
[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b*d)
 + (2*a^(1/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) +
b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx &= i\int\left(-\frac{i}{b}+\frac{ia}{b\left(a+b\sinh^3(c+dx)\right)}\right)dx \\
&= \frac{x}{b}-\frac{a\int\frac{1}{a+b\sinh^3(c+dx)}\,dx}{b} \\
&= \frac{x}{b}-\frac{a\int\left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\,\sqrt[3]{a}-i\sqrt[3]{b}\,\sinh(c+dx)\right)}+\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\,\sqrt[3]{a}+\sqrt[6]{-1}\,\sqrt[3]{b}\,\sinh(c+dx)\right.}\right.}{b} \\
&= \frac{x}{b}-\frac{\left(\sqrt[6]{-1}\,\sqrt[3]{a}\right)\int\frac{1}{\sqrt[6]{-1}\,\sqrt[3]{a}-i\sqrt[3]{b}\,\sinh(c+dx)}\,dx}{3b}-\frac{\left(\sqrt[6]{-1}\,\sqrt[3]{a}\right)\int\frac{}{\sqrt[6]{-1}\,\sqrt[3]{a}+\sqrt[6]{-}}}{3b} \\
&= \frac{x}{b}+\frac{\left(2(-1)^{2/3}\sqrt[3]{a}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt[6]{-1}\,\sqrt[3]{a}-2\sqrt[3]{b}\,x+\sqrt[6]{-1}\,\sqrt[3]{a}\,x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+ic\right.\right.}{3bd} \\
&= \frac{x}{b}-\frac{\left(4(-1)^{2/3}\sqrt[3]{a}\right)\operatorname{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}\,a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}+2\sqrt[6]{-1}\,\sqrt[3]{a}\right.}{3bd} \\
&= \frac{x}{b}-\frac{2(-1)^{2/3}\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,bd}+\frac{2(-1)^{2/3}\sqrt[3]{a}\,\tan^{-1}\left(\right.}{3\sqrt{\sqrt[3]{-}}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.15, size = 145, normalized size = 0.49

$$\frac{3c+3dx-2a\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{c\#1+dx\#1+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1}{b+4a\#1-2b\#1^2+b\#1^4}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (3*c + 3*d*x - 2*a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & ,
 (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ]
)/(3*b*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.10, size = 124, normalized size = 0.42

method	result
derivativedivides	$\frac{\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}}{d}$
default	$\frac{\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}}{d}$
risch	$\frac{x}{b}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(729a^2b^6d^6+729b^8d^6\right)\_Z^6-243a^2b^4d^4\_Z^4+27a^2b^2d^2\_Z^2-a^2\right)}\_R\ln\left(e^{dx+c}+\left(486ab\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a/b*sum((_R^4-2*_R^2+1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/
2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))-1/b*ln(tan
h(1/2*d*x+1/2*c)-1)+1/b*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -8*a*integrate(e^(3*d*x + 3*c)/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c)
 + 8*a*b*e^(3*d*x + 3*c) + 3*b^2*e^(2*d*x + 2*c) - b^2), x) + x/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27931 vs. 2(205) = 410.
time = 1.24, size = 27931, normalized size = 95.00

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2/3)*sqrt(1/6)*b*sqrt(((a^2*b^2 + b^4)*(2*(1/2)^(2/3)*(a^4/(a^2*
b^2*d^2 + b^4*d^2)^2 - a^2/(a^2*b^4*d^4 + b^6*d^4))*(-I*sqrt(3) + 1)/(2*a^6
/(a^2*b^2*d^2 + b^4*d^2)^3 - 3*a^4/((a^2*b^4*d^4 + b^6*d^4)*(a^2*b^2*d^2 +
b^4*d^2)) + a^2/(a^2*b^6*d^6 + b^8*d^6) + a^2/((a^2 + b^2)^2*b^4*d^6))^(1/3
```

```
) + (1/2)^(1/3)*(2*a^6/(a^2*b^2*d^2 + b^4*d^2)^3 - 3*a^4/((a^2*b^4*d^4 + b^
6*d^4)*(a^2*b^2*d^2 + b^4*d^2)) + a^2/(a^2*b^6*d^6 + b^8*d^6) + a^2/((a^2 +
 b^2)^2*b^4*d^6))^(1/3)*(I*sqrt(3) + 1) - 2*a^2/(a^2*b^2*d^2 + b^4*d^2))*d^
2 + 3*sqrt(1/3)*(a^2*b^2 + b^4)*d^2*sqrt(-((a^4*b^4 + 2*a^2*b^6 + b^8)*(2*(
1/2)^(2/3)*(a^4/(a^2*b^2*d^2 + b^4*d^2)^2 - a^2/(a^2*b^4*d^4 + b^6*d^4))*(-
I*sqrt(3) + 1)/(2*a^6/(a^2*b^2*d^2 + b^4*d^2)^3 - 3*a^4/((a^2*b^4*d^4 + b^6
*d^4)*(a^2*b^2*d^2 + b^4*d^2)) + a^2/(a^2*b^6*d^6 + b^8*d^6) + a^2/((a^2 +
b^2)^2*b^4*d^6))^(1/3) + (1/2)^(1/3)*(2*a^6/(a^2*b^2*d^2 + b^4*d^2)^3 - 3*a
^4/((a^2*b^4*d^4 + b^6*d^4)*(a^2*b^2*d^2 + b^4*d^2)) + a^2/(a^2*b^6*d^6 + b
^8*d^6) + a^2/((a^2 + b^2 ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(sinh(c + d*x)**3/(a + b*sinh(c + d*x)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^3/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [**B**]
time = 10.76, size = 1498, normalized size = 5.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log(-(24576*a^3*(405*root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 24
3*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^5*b^7*d^5*exp(root(729*
a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^
2 - a^2, z, k) + d*x) - root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^
2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)*a*b^2*d - 27*root(729*a^2*b
^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a
```

```
^2, z, k)^4*b^6*d^4*exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^
2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k) + d*x) - 4*a^2*exp(root(729
*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z
^2 - a^2, z, k) + d*x) + 12*root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 24
3*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^2*a*b^3*d^2 - 54*root(7
29*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2
*z^2 - a^2, z, k)^3*a*b^4*d^3 + 108*root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*
z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^4*a*b^5*d^4 - 8
1*root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2
*b^2*d^2*z^2 - a^2, z, k)^5*a*b^6*d^5 + 20*root(729*a^2*b^6*d^6*z^6 + 729*b
^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)*a^2*b*d*
exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a
^2*b^2*d^2*z^2 - a^2, z, k) + d*x) + 24*root(729*a^2*b^6*d^6*z^6 + 729*b^8*
d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^2*a^2*b^2*d
^2*exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 2
7*a^2*b^2*d^2*z^2 - a^2, z, k) + d*x) - 216*root(729*a^2*b^6*d^6*z^6 + 729*
b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^3*a^2*b
^3*d^3*exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4
 + 27*a^2*b^2*d^2*z^2 - a^2, z, k) + d*x) + 108*root(729*a^2*b^6*d^6*z^6 +
729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^4*a
^2*b^4*d^4*exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4
*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k) + d*x) + 324*root(729*a^2*b^6*d^6*z^
6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)
^5*a^2*b^5*d^5*exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4
*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k) + d*x)))/b^10)*root(729*a^2*b^6*
d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2,
 z, k), k, 1, 6) + x/b
```

3.175 $\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=262

$$-\frac{2\text{ArcTan}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\,\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,b^{2/3}d}-\frac{2\text{ArcTan}\left(\frac{\sqrt[6]{-1}\,\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,b^{2/3}d}-\frac{2\,\text{ta}}{}$$

```
[Out] -2/3*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/(
(-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/b^(2/3)/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(
1/2)-2/3*arctan((-1)^(5/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c
))/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2))/b^(2/3)/d/(-(-1)^(2/3)*a^(2/3)-b^(2
/3))^(1/2)-2/3*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/
3))^(1/2))/b^(2/3)/d/(a^(2/3)+b^(2/3))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3299, 2739, 632, 212, 210}

$$-\frac{2\text{ArcTan}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\,\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-\frac{2\text{ArcTan}\left(\frac{\sqrt[6]{-1}\,\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}+b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-2*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/
Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(
2/3)]*b^(2/3)*d) - (2*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Ta
nh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a
^(2/3) - b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)
/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^(2/3)*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\int \left(\frac{i}{3b^{2/3}\left(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{i}{3b^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c\right.}\right.$$

$$= -\frac{i\int \frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{2/3}}-\frac{i\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{2/3}}-\frac{i\int \frac{}{(-1)^{5/6}\sqrt[3]{}}}{}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{b}\,x-i\sqrt[3]{a}\,x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{2/3}d}-\frac{2\text{Subst}\left(\int \frac{}{\sqrt[6]{-1}}\right.}{}$$

$$= \frac{4\text{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}\,a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{2/3}d}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,b^{2/3}d}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right.}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right.}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,b^{2/3}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.11, size = 275, normalized size = 1.05

$$\frac{\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{c+dx+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)-2c\#1^2-2dx\#1^2-4\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2+c\#1^4+dx\#1^4+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4}{b\#1+4a\#1^2-2b\#1^3+b\#1^5}\&\right]}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c + d*x + 2*Log[-
Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*
x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*
x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1
^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3 + b*#1^5) & ]/(6*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.02, size = 78, normalized size = 0.30

method	result
derivativedivides	$-\frac{4\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3d}$
default	$-\frac{4\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3d}$
risch	$\sum_{\_R=\text{RootOf}\left(-1+\left(729a^2b^4d^6+729b^6d^6\right)\_Z^6-243b^4d^4\_Z^4+27b^2d^2\_Z^2\right)}\_R\ln\Bigl(\mathrm{e}^{dx+c}+\left(-243b^3d^5a^2-24\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3/d*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),
_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maple [F]

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 24063 vs. $2(181) = 362$.
time = 1.67, size = 24063, normalized size = 91.84

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2/3)*sqrt(1/6)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a^2*b^2
*d^4 + b^4*d^4) - 1/(a^2*d^2 + b^2*d^2)^2)/(1/(a^2*b^4*d^6 + b^6*d^6) - 3/(
(a^2*b^2*d^4 + b^4*d^4)*(a^2*d^2 + b^2*d^2)) + 2/(a^2*d^2 + b^2*d^2)^3 + a^
2/((a^2 + b^2)^2*b^4*d^6))^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(a^2*b^4*
d^6 + b^6*d^6) - 3/((a^2*b^2*d^4 + b^4*d^4)*(a^2*d^2 + b^2*d^2)) + 2/(a^2*d
^2 + b^2*d^2)^3 + a^2/((a^2 + b^2)^2*b^4*d^6))^(1/3) + 2/(a^2*d^2 + b^2*d^2
))*(a^2 + b^2)*d^2 + 3*sqrt(1/3)*(a^2 + b^2)*d^2*sqrt(-((a^4*b^2 + 2*a^2*b^
4 + b^6)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a^2*b^2*d^4 + b^4*d^4) - 1/(a^
2*d^2 + b^2*d^2)^2)/(1/(a^2*b^4*d^6 + b^6*d^6) - 3/((a^2*b^2*d^4 + b^4*d^4)
*(a^2*d^2 + b^2*d^2)) + 2/(a^2*d^2 + b^2*d^2)^3 + a^2/((a^2 + b^2)^2*b^4*d^
6))^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(a^2*b^4*d^6 + b^6*d^6) - 3/((a^
2*b^2*d^4 + b^4*d^4)*(a^2*d^2 + b^2*d^2)) + 2/(a^2*d^2 + b^2*d^2)^3 + a^2/(
(a^2 + b^2)^2*b^4*d^6))^(1/3) + 2/(a^2*d^2 + b^2*d^2))^2*d^4 - 4*(a^2*b^2 +
 b^4)*(2*(1/2)^(2/3)*(-I* ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(sinh(c + d*x)**2/(a + b*sinh(c + d*x)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [B]
time = 11.13, size = 932, normalized size = 3.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 2
7*b^2*d^2*z^2 - 1, z, k)*(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*
b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)*(root(729*a^2*b^4*d^6*z^6 + 729*b^6
*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)*(root(729*a^2*b^4*d^
6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)*((663
552*(8*a^5*d^4 + 4*a^3*b^2*d^4 - 5*a^4*b*d^4*exp(d*x)*exp(root(729*a^2*b^4*
d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k))))/
b^6 - (1990656*root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4
 + 27*b^2*d^2*z^2 - 1, z, k)*(4*a^5*d^5*exp(d*x)*exp(root(729*a^2*b^4*d^6*z
^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)) - a^4*b
*d^5 + 5*a^3*b^2*d^5*exp(d*x)*exp(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^
6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k))))/b^5) - (442368*(4*a^4*b*
d^3 + 8*a^5*d^3*exp(d*x)*exp(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 2
43*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)) - 5*a^3*b^2*d^3*exp(d*x)*exp(ro
ot(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2
 - 1, z, k))))/b^7) - (294912*a^3*d^2*(2*b - 5*a*exp(d*x)*exp(root(729*a^2*
b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)
)))/b^7) + (24576*a^3*d*(8*a - 5*b*exp(d*x)*exp(root(729*a^2*b^4*d^6*z^6 +
729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k))))/b^8) + (32
768*a^3*(b - 4*a*exp(d*x)*exp(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 -
243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k))))/b^9)*root(729*a^2*b^4*d^6*z^
6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k), k, 1, 6)
```

3.176 $\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=290

$$\frac{2\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}\sqrt[3]{b}\,d}-\frac{2\sqrt[3]{-1}\,\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\sqrt[3]{b}\,d}$$

```
[Out] -2/3*(-1)^(1/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*
x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a^(1/3)/b^(1/3)/d/((-1)^(1/3)
*a^(2/3)-b^(2/3))^(1/2)+2/3*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(
a^(2/3)+b^(2/3))^(1/2))/a^(1/3)/b^(1/3)/d/(a^(2/3)+b^(2/3))^(1/2)+2/3*arcta
n((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)
*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(1/3)/b^(1/3)/d/((-1)^(1/3)*a^(2/3)-(
-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3299, 2739, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\,d\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\sqrt[3]{-1}\,\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\,d\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\,d\sqrt{a^{2/3}+b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (2*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/S
qrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a
^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(1/3)*d) - (2*(-1)^(1/3)*ArcTan[((-1)^(1/6)*
((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3)
 - b^(2/3)]])/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(1/3)*d) + (2
*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3
*a^(1/3)*Sqrt[a^(2/3) + b^(2/3)]*b^(1/3)*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\Bigg(i\int\Bigg(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[6]{-1}\,\sqrt[3]{a}-i\sqrt[3]{b}\,\sinh(c+dx)\right)}-\frac{(-}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\Big(\sqrt[6]{-1}\,\sqrt[3]{a}+}$$

$$=\frac{i\int\frac{1}{\sqrt[6]{-1}\,\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\,\sinh(c+dx)}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}}-\frac{\sqrt[6]{-1}\,\int\frac{1}{\sqrt[6]{-1}\,\sqrt[3]{a}+\sqrt[6]{-1}\,\sqrt[3]{b}\,\sinh(c+dx)}\,d}{3\sqrt[3]{a}\,\sqrt[3]{b}}$$

$$=\frac{2\text{Subst}\left(\int\frac{1}{\sqrt[6]{-1}\,\sqrt[3]{a}+2\sqrt[3]{-1}\,\sqrt[3]{b}\,x+\sqrt[6]{-1}\,\sqrt[3]{a}\,x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,d}-\Big(2$$

$$=-\frac{4\text{Subst}\Bigg(\int\frac{1}{-4\left(\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}\right)-x^2}\,dx,x,2\sqrt[3]{-1}\,\sqrt[3]{b}+2\sqrt[6]{-1}\,\sqrt[3]{a}\,\tan\big(\tfrac{1}{2}(}{3\sqrt[3]{a}\,\sqrt[3]{b}\,d}$$

$$=\frac{2\sqrt[3]{-1}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\,\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,\sqrt[3]{b}\,d}+\frac{2\tan^{-1}\Bigg(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1}}}{3\sqrt[3]{a}\,\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.14, size = 199, normalized size = 0.69

$$\frac{\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{-c-dx-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)+c\#1^2+dx\#1^2+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2}{b+4a\#1-2b\#1^2+b\#1^4}\&\right]}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (-c - d*x - 2*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d
*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/
2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2)/(b + 4*a*#1 - 2*b*#
1^2 + b*#1^4) & ]/(3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.19, size = 82, normalized size = 0.28

method	result
derivativedivides	$\frac{2\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3d}$
default	$\frac{2\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3d}$
risch	$\sum_{\_R=\text{RootOf}\left(-1+(729a^4b^2d^6+729a^2b^4d^6)\_Z^6+243a^2b^2d^4\_Z^4\right)}\_R\ln\left(\mathrm{e}^{dx+c}+\left(\frac{243d^5b^2a^5}{a^2-b^2}+\frac{243d^5b^4a^3}{a^2-b^2}\right)\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*sum((_R^3-_R)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-
_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(d*x + c)/(b*sinh(d*x + c)^3 + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 18312 vs. 2(197) = 394.
time = 1.26, size = 18312, normalized size = 63.14

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2/3)*sqrt(1/6)*sqrt(((a^2 + b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(
a^4*b^2*d^6 + a^2*b^4*d^6) - 2/(a^2*d^2 + b^2*d^2)^3 - (a^2 - b^2)/((a^2 +
b^2)^2*a^2*b^2*d^6))^(1/3) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/((a^2*d^2 + b^2
*d^2)^2*(1/(a^4*b^2*d^6 + a^2*b^4*d^6) - 2/(a^2*d^2 + b^2*d^2)^3 - (a^2 - b
^2)/((a^2 + b^2)^2*a^2*b^2*d^6))^(1/3)) + 2/(a^2*d^2 + b^2*d^2))*d^2 + 3*sq
rt(1/3)*(a^2 + b^2)*d^2*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*((1/2)^(1/3)*(I*sqrt
(3) + 1)*(1/(a^4*b^2*d^6 + a^2*b^4*d^6) - 2/(a^2*d^2 + b^2*d^2)^3 - (a^2 -
b^2)/((a^2 + b^2)^2*a^2*b^2*d^6))^(1/3) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/((
a^2*d^2 + b^2*d^2)^2*(1/(a^4*b^2*d^6 + a^2*b^4*d^6) - 2/(a^2*d^2 + b^2*d^2)
^3 - (a^2 - b^2)/((a^2 + b^2)^2*a^2*b^2*d^6))^(1/3)) + 2/(a^2*d^2 + b^2*d^2
))^2*d^4 - 4*(a^2 + b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(a^4*b^2*d^6 + a^2
*b^4*d^6) - 2/(a^2*d^2 + b^2*d^2)^3 - (a^2 - b^2)/((a^2 + b^2)^2*a^2*b^2*d^
6))^(1/3) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/((a^2*d^2 + b^2*d^2)^2*(1/(a^4*b
^2*d^6 + a^2*b^4*d^6) - 2 ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh{\left(c + dx \right)}}{a + b \sinh^{3}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(sinh(c + d*x)/(a + b*sinh(c + d*x)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [**B**]
time = 21.84, size = 857, normalized size = 2.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4
*z^4 - 1, z, k)*(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b
^2*d^4*z^4 - 1, z, k)*(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243
*a^2*b^2*d^4*z^4 - 1, z, k)*(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6
 + 243*a^2*b^2*d^4*z^4 - 1, z, k)*((663552*(4*a^4*b*d^4 + 16*a^5*d^4*exp(d*
x)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4
 - 1, z, k)) + 11*a^3*b^2*d^4*exp(d*x)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a
^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))/b^6 - (1990656*root(729*
a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k)*(4*a
^5*d^5*exp(d*x)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^
2*b^2*d^4*z^4 - 1, z, k)) - a^4*b*d^5 + 5*a^3*b^2*d^5*exp(d*x)*exp(root(729
*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))/
b^5) + (221184*(8*a^4*d^3 + a^2*b^2*d^3 - 25*a^3*b*d^3*exp(d*x)*exp(root(72
9*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))
/b^6) - (294912*a^2*d^2*(b - 7*a*exp(d*x)*exp(root(729*a^4*b^2*d^6*z^6 + 72
9*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))/b^6) + (196608*a^2*d*
(b - 2*a*exp(d*x)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*
a^2*b^2*d^4*z^4 - 1, z, k))))/b^7) - (8192*a*(8*a - b*exp(d*x)*exp(root(729
*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))/
b^7)*root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 -
 1, z, k), k, 1, 6)
```

3.177 $\int \frac{1}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=280

$$-\frac{2(-1)^{2/3}\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}-\frac{2(-1)^{2/3}\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh(\frac{1}{2}\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3a^{2/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d}$$

```
[Out] -2/3*(-1)^(2/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*
x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a^(2/3)/d/((-1)^(1/3)*a^(2/3)
-b^(2/3))^(1/2)-2/3*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+
b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)+b^(2/3))^(1/2)-2/3*(-1)^(2/3)*arctan((-1
)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2
/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/d/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(
2/3))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$,
Rules used = {3292, 2739, 632, 210}

$$-\frac{2(-1)^{2/3}\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^3)^(-1),x]
```

```
[Out] (-2*(-1)^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c +
 d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[
(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) - (2*(-1)^(2/3)*ArcTan[((-1)^(1
/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(
2/3) - b^(2/3)]])/(3*a^(2/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - (2*Arc
Tanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*a^(
2/3)*Sqrt[a^(2/3) + b^(2/3)]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\sinh^3(c+dx)}\,dx = \int \left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\,\sqrt[3]{a}-i\sqrt[3]{b}\,\sinh(c+dx)\right)}+\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\,\sqrt[3]{a}+\sqrt[6]{-1}\,\sqrt[3]{b}\,\mathrm{s}\right.}\right.$$

$$=\frac{\sqrt[6]{-1}\int\frac{1}{\sqrt[6]{-1}\,\sqrt[3]{a}-i\sqrt[3]{b}\,\sinh(c+dx)}\,dx}{3a^{2/3}}+\frac{\sqrt[6]{-1}\int\frac{1}{\sqrt[6]{-1}\,\sqrt[3]{a}+\sqrt[6]{-1}\,\sqrt[3]{b}\,\sinh(c+dx)}}{3a^{2/3}}$$

$$=-\frac{\left(2(-1)^{2/3}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt[6]{-1}\,\sqrt[3]{a}-2\sqrt[3]{b}\,x+\sqrt[6]{-1}\,\sqrt[3]{a}\,x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3a^{2/3}d}$$

$$=\frac{\left(4(-1)^{2/3}\right)\mathrm{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}\,a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}+2\sqrt[6]{-1}\,\sqrt[3]{a}\,\tan\left(\frac{1}{2}(i\right.\right.}{3a^{2/3}d}$$

$$=\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,d}-\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}+}{\sqrt{\sqrt[3]{-1}}}\right.}{3a^{2/3}\sqrt{\sqrt[3]{-1}\,a^{2/3}-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.11, size = 131, normalized size = 0.47

$$\frac{2\mathrm{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{c\#1+dx\#1+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1}{b+4a\#1-2b\#1^2+b\#1^4}\&\right]}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^3)^(-1),x]
```

```
[Out] (2*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1 + d*x*#1
+ 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sin
h[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.96, size = 87, normalized size = 0.31

method	result
derivativedivides	$\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{6}-3a\_Z^{4}-8b\_Z^{3}+3a\_Z^{2}-a\right)}\frac{\left(-\_R^{4}+2\_R^{2}-1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a-2\_R^{3}a-4\_R^{2}b+\_Ra}}{3d}$
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{6}-3a\_Z^{4}-8b\_Z^{3}+3a\_Z^{2}-a\right)}\frac{\left(-\_R^{4}+2\_R^{2}-1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a-2\_R^{3}a-4\_R^{2}b+\_Ra}}{3d}$
risch	$\sum_{\_R=\operatorname{RootOf}(-1+(729a^6d^6+729a^4b^2d^6)\_Z^6-243a^4d^4\_Z^4+27a^2d^2\_Z^2)}\_R\ln\left(e^{dx+c}+\left(-\frac{486d^5a^6}{b}-486b\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*sum((-_R^4+2*_R^2-1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+
1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sinh(d*x + c)^3 + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 24084 vs. 2(191) = 382.
time = 1.22, size = 24084, normalized size = 86.01

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2/3)*sqrt(1/6)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a^4*d^4
+ a^2*b^2*d^4) - 1/(a^2*d^2 + b^2*d^2)^2)/(1/(a^6*d^6 + a^4*b^2*d^6) - 3/((
a^4*d^4 + a^2*b^2*d^4)*(a^2*d^2 + b^2*d^2)) + 2/(a^2*d^2 + b^2*d^2)^3 + b^2
/((a^2 + b^2)^2*a^4*d^6))^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(a^6*d^6 +
 a^4*b^2*d^6) - 3/((a^4*d^4 + a^2*b^2*d^4)*(a^2*d^2 + b^2*d^2)) + 2/(a^2*d^
2 + b^2*d^2)^3 + b^2/((a^2 + b^2)^2*a^4*d^6))^(1/3) + 2/(a^2*d^2 + b^2*d^2)
)*(a^2 + b^2)*d^2 + 3*sqrt(1/3)*(a^2 + b^2)*d^2*sqrt(-((a^6 + 2*a^4*b^2 + a
^2*b^4)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a^4*d^4 + a^2*b^2*d^4) - 1/(a^2
*d^2 + b^2*d^2)^2)/(1/(a^6*d^6 + a^4*b^2*d^6) - 3/((a^4*d^4 + a^2*b^2*d^4)*
(a^2*d^2 + b^2*d^2)) + 2/(a^2*d^2 + b^2*d^2)^3 + b^2/((a^2 + b^2)^2*a^4*d^6
))^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(a^6*d^6 + a^4*b^2*d^6) - 3/((a^4
*d^4 + a^2*b^2*d^4)*(a^2*d^2 + b^2*d^2)) + 2/(a^2*d^2 + b^2*d^2)^3 + b^2/((
a^2 + b^2)^2*a^4*d^6))^(1/3) + 2/(a^2*d^2 + b^2*d^2))^2*d^4 - 4*(a^4 + a^2*
b^2)*(2*(1/2)^(2/3)*(-I*s ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sinh^3{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(1/(a + b*sinh(c + d*x)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [**B**]
time = 9.49, size = 1261, normalized size = 4.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log((24576*(root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4
*z^4 + 27*a^2*d^2*z^2 - 1, z, k)*b*d - 4*exp(root(729*a^4*b^2*d^6*z^6 + 729
```

```
*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) + 12*root
(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 -
 1, z, k)^2*a*b*d^2 - 20*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a
^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)*a*d*exp(root(729*a^4*b^2*d^6*z^6 + 7
29*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) + 24*ro
ot(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2
 - 1, z, k)^2*a^2*d^2*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*
a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) + 216*root(729*a^4*b^2*d^6*z
^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)^3*a^3*d^
3*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2
*d^2*z^2 - 1, z, k) + d*x) + 108*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6
 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)^4*a^4*d^4*exp(root(729*a^4*b
^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)
+ d*x) - 324*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 +
 27*a^2*d^2*z^2 - 1, z, k)^5*a^5*d^5*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^6
*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) + 54*root(729
*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1,
z, k)^3*a^2*b*d^3 + 108*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^
4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)^4*a^3*b*d^4 + 81*root(729*a^4*b^2*d^6
*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)^5*a^4*
b*d^5 - 27*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 2
7*a^2*d^2*z^2 - 1, z, k)^4*a^2*b^2*d^4*exp(root(729*a^4*b^2*d^6*z^6 + 729*a
^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) - 405*root(
729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 -
1, z, k)^5*a^3*b^2*d^5*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243
*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x)))/b^5)*root(729*a^4*b^2*d^6
*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k), k, 1,
 6)
```

3.178 $\int \frac{\mathbf{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=286

$$\frac{2\sqrt[3]{b}\,\text{ArcTan}\Bigg(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\,\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\Bigg)}{3a\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,d}+\frac{2\sqrt[3]{b}\,\text{ArcTan}\Bigg(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}}{3a\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,d}$$

```
[Out] -arctanh(cosh(d*x+c))/a/d+2/3*b^(1/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)
+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a/d/((-
1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)+2/3*b^(1/3)*arctan((-1)^(5/6)*((-1)^(1/6)*b
^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2))/
a/d/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2)+2/3*b^(1/3)*arctanh((b^(1/3)-a^(1/3
)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/a/d/(a^(2/3)+b^(2/3))^(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3299, 3855, 2739, 632, 212, 210}

$$\frac{2\sqrt[3]{b}\,\text{ArcTan}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\,\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{b}\,\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}+b^{2/3}}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (2*b^(1/3)*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x
)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*a*Sqrt[-((-1)^(2/3)*a^(2/
3)) - b^(2/3)]*d) + (2*b^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a
^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*a*Sqrt[(
-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - ArcTanh[Cosh[c + d*x]]/(a*d) + (2*b^(1/3)
*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3
*a*Sqrt[a^(2/3) + b^(2/3)]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx = i\int\left(-\frac{i\operatorname{csch}(c+dx)}{a}+\frac{ib\sinh^2(c+dx)}{a\left(a+b\sinh^3(c+dx)\right)}\right)dx$$

$$=\frac{\int\operatorname{csch}(c+dx)\,dx}{a}-\frac{b\int\frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx}{a}$$

$$=-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{b\int\left(\frac{i}{3b^{2/3}\left(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{i}{3b^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-\right.}\right.}{}$$

$$=-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\left(i\sqrt[3]{b}\right)\int\frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3a}+\frac{\left(i\sqrt[3]{b}\right)\int\frac{}{\sqrt[6]{-}}}{}$$

$$=-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\left(2\sqrt[3]{b}\right)\operatorname{Subst}\left(\int\frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{b}\,x-i\sqrt[3]{a}\,x^2}\,dx,x,\tan\left(\frac{1}{2}\right.\right.}{3ad}$$

$$=-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}-\frac{\left(4\sqrt[3]{b}\right)\operatorname{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}\,a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{}\right.}{3ad}$$

$$=-\frac{2\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\,\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,d}-\frac{2\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\,\mathrm{t}}{\sqrt{\sqrt[3]{-1}\,a^{2/3}}}\right.}{3a\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.16, size = 295, normalized size = 1.03

$$\frac{6\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-b\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{c+dx+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)-2c\#1^2-2dx\#1^2-4\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2+c\#1^4+dx\#1^4+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4}{b\#1+4a\#1^2-2b\#1^3+b\#1^5}\&\right]}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (6*Log[Tanh[(c + d*x)/2]] - b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 +
 b*#1^6 & , (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[
(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2
- 2*b*#1^3 + b*#1^5) & ])/(6*a*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.59, size = 98, normalized size = 0.34

method	result
derivativedivides	$\frac{\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}-3a\_Z^{4}-8b\_Z^{3}+3a\_Z^{2}-a\right)}\frac{\_R^{2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a-2\_R^{3}a-4\_R^{2}b+\_Ra}\right)}{3a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}-3a\_Z^{4}-8b\_Z^{3}+3a\_Z^{2}-a\right)}\frac{\_R^{2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a-2\_R^{3}a-4\_R^{2}b+\_Ra}\right)}{3a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)}{da}+2\left(\sum_{\_R=\mathrm{RootOf}\left(\left(46656a^{8}d^{6}+46656a^{6}b^{2}d^{6}\right)\_Z^{6}-3888a^{4}b^{2}d^{4}\_Z^{4}+108a^{2}b^{2}d^{2}\_Z^{2}-b^{2}\right)}\_R\ln\left(\mathrm{e}^{d}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4/3/a*b*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c
)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))+1/a*ln(tanh(1/2*d*x+1
/2*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(5*d*x + 5*c) - 2*b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a*
b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c) + 8*a^2*e^(3*d*x + 3*c) + 3*a*b*e
^(2*d*x + 2*c) - a*b), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28005 vs. 2(205) = 410.
time = 2.62, size = 28005, normalized size = 97.92

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2/3)*sqrt(1/6)*a*d*sqrt(((a^4 + a^2*b^2)*(2*(1/2)^(2/3)*(b^4/(a^
4*d^2 + a^2*b^2*d^2)^2 - b^2/(a^6*d^4 + a^4*b^2*d^4))*(-I*sqrt(3) + 1)/(2*b
```

```
^6/(a^4*d^2 + a^2*b^2*d^2)^3 - 3*b^4/((a^6*d^4 + a^4*b^2*d^4)*(a^4*d^2 + a^
2*b^2*d^2)) + b^2/(a^8*d^6 + a^6*b^2*d^6) + b^2/((a^2 + b^2)^2*a^4*d^6))^(1
/3) + (1/2)^(1/3)*(2*b^6/(a^4*d^2 + a^2*b^2*d^2)^3 - 3*b^4/((a^6*d^4 + a^4*
b^2*d^4)*(a^4*d^2 + a^2*b^2*d^2)) + b^2/(a^8*d^6 + a^6*b^2*d^6) + b^2/((a^2
 + b^2)^2*a^4*d^6))^(1/3)*(I*sqrt(3) + 1) - 2*b^2/(a^4*d^2 + a^2*b^2*d^2))*
d^2 + 3*sqrt(1/3)*(a^4 + a^2*b^2)*d^2*sqrt(-((a^8 + 2*a^6*b^2 + a^4*b^4)*(2
*(1/2)^(2/3)*(b^4/(a^4*d^2 + a^2*b^2*d^2)^2 - b^2/(a^6*d^4 + a^4*b^2*d^4))*
(-I*sqrt(3) + 1)/(2*b^6/(a^4*d^2 + a^2*b^2*d^2)^3 - 3*b^4/((a^6*d^4 + a^4*b
^2*d^4)*(a^4*d^2 + a^2*b^2*d^2)) + b^2/(a^8*d^6 + a^6*b^2*d^6) + b^2/((a^2
+ b^2)^2*a^4*d^6))^(1/3) + (1/2)^(1/3)*(2*b^6/(a^4*d^2 + a^2*b^2*d^2)^3 - 3
*b^4/((a^6*d^4 + a^4*b^2*d^4)*(a^4*d^2 + a^2*b^2*d^2)) + b^2/(a^8*d^6 + a^6
*b^2*d^6) + b^2/((a^2 + b ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [**B**]
time = 55.94, size = 2500, normalized size = 8.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^3)),x)
```

```
[Out] symsum(log(-(2147483648*a*b*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6
- 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) - 1073741824
*b^2 - 86973087744*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2
*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^6*d^4 + 86973087744*root(729
*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z
```

```
^2 - b^2, z, k)^6*a^8*d^6 + 134217728*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^
6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*b^3*d + 32212
25472*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27
*a^2*b^2*d^2*z^2 - b^2, z, k)*a^2*b*d + 18589155328*root(729*a^6*b^2*d^6*z^
6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)
^2*a^2*b^2*d^2 - 2818572288*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 24
3*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^2*b^3*d^3 - 8818104
7296*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*
a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^4*b^2*d^4 + 18119393280*root(729*a^6*b^2*d
^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2,
z, k)^5*a^4*b^3*d^5 + 70665633792*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^
6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^6*b^2*d^6 - 3
2614907904*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4
 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^6*b^3*d^7 - 57982058496*root(729*a^6
*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 -
 b^2, z, k)^3*a^5*d^3*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*
a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) - 333396836352*roo
t(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*
d^2*z^2 - b^2, z, k)^5*a^7*d^5*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z
^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) + 3913788
94848*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27
*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^9*d^7*exp(root(729*a^6*b^2*d^6*z^6 + 729*
a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x)
- 17716740096*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*
z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^4*b*d^3 + 30802968576*root(729*a^
6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2
- b^2, z, k)^5*a^6*b*d^5 - 40768634880*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d
^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^8*b*d^7
+ 268435456*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^
4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a*b^3*d^2*exp(root(729*a^6*b^2*d^6*z^
6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)
 + d*x) - 16642998272*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*
b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^3*b*d^2*exp(root(729*a^6*
b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 -
b^2, z, k) + d*x) + 36238786560*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6
- 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^5*b*d^4*exp(roo
t(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*
d^2*z^2 - b^2, z, k) + d*x) + 2717908992*root(729*a^6*b^2*d^6*z^6 + 729*a^8
*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^7*b*d^
6*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27
*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) - 5637144576*root(729*a^6*b^2*d^6*z^6
+ 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*a
*b^2*d*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4
 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) + 100763959296*root(729*a^6*b^2*d
^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2,
```

```
z, k)^3*a^3*b^2*d^3*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^
4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) - 4831838208*root(72
9*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*
z^2 - b^2, z, k)^4*a^3*b^3*d^4*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z
^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) - 4946594
36544*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27
*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^5*b^2*d^5*exp(root(729*a^6*b^2*d^6*z^6 +
729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d
*x) + 21743271936*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*
d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^5*b^3*d^6*exp(root(729*a^6*b^
2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^
2, z, k) + d*x) + 399532621824*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 -
 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^7*b^2*d^7*exp(ro
ot(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2
*d^2*z^2 - b^2, z, k) + d*x))/b^9)*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z
^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k), k, 1, 6) + log(
exp(d*x + 1/(a*d)) - 1)/(a*d) - log(exp(d*x - 1...
```

3.179 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=304

$$-\frac{2b^{2/3}\operatorname{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}+\frac{2\sqrt[3]{-1}b^{2/3}\operatorname{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c\right.\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3a^{4/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d}$$

```
[Out] -coth(d*x+c)/a/d+2/3*(-1)^(1/3)*b^(2/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/
3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a^(4/
3)/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)-2/3*b^(2/3)*arctanh((b^(1/3)-a^(1/3
)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/a^(4/3)/d/(a^(2/3)+b^(2/3))
^(1/2)-2/3*b^(2/3)*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2
*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(4/3)/d/((-1)
^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3299, 3852, 8, 2739, 632, 210}

$$-\frac{2b^{2/3}\operatorname{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[3]{-1}b^{2/3}\operatorname{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}-\frac{2b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-2*b^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*
x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[(-1
)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) + (2*(-1)^(1/3)*b^(2/3)*ArcTan[((-
1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3
)*a^(2/3) - b^(2/3)]])/(3*a^(4/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - (
2*b^(2/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2
/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) + b^(2/3)]*d) - Coth[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\int \left(-\frac{\operatorname{csch}^2(c+dx)}{a}+\frac{b\sinh(c+dx)}{a\left(a+b\sinh^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \operatorname{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx}{a}$$

$$= \frac{(ib)\int \left(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[6]{-1}\,\sqrt[3]{a}-i\sqrt[3]{b}\,\sinh(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[6]{-1}\,\sqrt[3]{a}+\sqrt[6]{-1}\,\sqrt[3]{b}\,\mathrm{si}\right.}\right.}{a}$$

$$= -\frac{\coth(c+dx)}{ad} - \frac{\left(ib^{2/3}\right)\int \frac{1}{\sqrt[6]{-1}\,\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\,\sinh(c+dx)}\,dx}{3a^{4/3}} + \frac{\left(\sqrt[6]{-1}\,b^{2/3}\right)\int \frac{}{\sqrt[6]{}}}{}$$

$$= -\frac{\coth(c+dx)}{ad} - \frac{\left(2b^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\,\sqrt[3]{a}+2\sqrt[3]{-1}\,\sqrt[3]{b}\,x+\sqrt[6]{-1}\,\sqrt[3]{a}\,x^2}\,dx, x, \mathrm{ta}\right.}{3a^{4/3}d}$$

$$= -\frac{\coth(c+dx)}{ad} + \frac{\left(4b^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{-1}\,\sqrt[3]{b}\right.}{3a^{4/3}d}$$

$$= -\frac{2\sqrt[3]{-1}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,d} - \frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}+(-}{\sqrt{\sqrt[3]{-1}\,a}}\right.}{3a^{4/3}\sqrt{\sqrt[3]{-1}\,a^{2/3}-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.30, size = 230, normalized size = 0.76

$$-\frac{3\coth\left(\frac{1}{2}(c+dx)\right)+2b\operatorname{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&, \frac{-c-dx-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)+c\#1^2+dx\#1^2+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2}{b+4a\#1-2b\#1^2+b\#1^4}\&\right]+3\tanh\left(\frac{1}{2}(c+dx)\right)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -1/6*(3*Coth[(c + d*x)/2] + 2*b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4
 + b*#1^6 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Co
sh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cos
h[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/
2]*#1]*#1^2)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ] + 3*Tanh[(c + d*x)/2])/(a
*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.66, size = 118, normalized size = 0.39

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{\_R^5a-2\_R^3a-4\_R^2b+\_}\right.}{3a}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{\_R^5a-2\_R^3a-4\_R^2b+\_}\right.}{3a}}{d}$
risch	$-\frac{2}{da\left(e^{2dx+2c}-1\right)}+4\left(\sum_{\_R=\mathrm{RootOf}\left(\left(2985984a^{10}d^6+2985984a^8b^2d^6\right)\_Z^6+62208a^6b^2d^4\_Z^4-b^4\right)}\_R\ln\left(e^{dx+}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-1/2/a/tanh(1/2*d*x+1/2*c)-2/3/a*b*sum((_R^3
-_R)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_
Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(4*d*x + 4*c) - b*e^(2*d*
x + 2*c))/(a*b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c) + 8*a^2*e^(3*d*x + 3
*c) + 3*a*b*e^(2*d*x + 2*c) - a*b), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 21133 vs. 2(211) = 422.
time = 1.24, size = 21133, normalized size = 69.52

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2/3)*sqrt(1/6)*(a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d
*x + c) + a*d*sinh(d*x + c)^2 - a*d)*sqrt(((a^4 + a^2*b^2)*(2*(1/2)^(2/3)*b
^4*(-I*sqrt(3) + 1)/((a^4*d^2 + a^2*b^2*d^2)^2*(b^4/(a^10*d^6 + a^8*b^2*d^6
```

```
) - 2*b^6/(a^4*d^2 + a^2*b^2*d^2)^3 - (a^2 - b^2)*b^4/((a^2 + b^2)^2*a^8*d^
6))^(1/3)) + (1/2)^(1/3)*(b^4/(a^10*d^6 + a^8*b^2*d^6) - 2*b^6/(a^4*d^2 + a
^2*b^2*d^2)^3 - (a^2 - b^2)*b^4/((a^2 + b^2)^2*a^8*d^6))^(1/3)*(I*sqrt(3) +
 1) + 2*b^2/(a^4*d^2 + a^2*b^2*d^2))*d^2 + 3*sqrt(1/3)*(a^4 + a^2*b^2)*d^2*
sqrt(-((a^8 + 2*a^6*b^2 + a^4*b^4)*(2*(1/2)^(2/3)*b^4*(-I*sqrt(3) + 1)/((a^
4*d^2 + a^2*b^2*d^2)^2*(b^4/(a^10*d^6 + a^8*b^2*d^6) - 2*b^6/(a^4*d^2 + a^2
*b^2*d^2)^3 - (a^2 - b^2)*b^4/((a^2 + b^2)^2*a^8*d^6))^(1/3)) + (1/2)^(1/3)
*(b^4/(a^10*d^6 + a^8*b^2*d^6) - 2*b^6/(a^4*d^2 + a^2*b^2*d^2)^3 - (a^2 - b
^2)*b^4/((a^2 + b^2)^2*a^8*d^6))^(1/3)*(I*sqrt(3) + 1) + 2*b^2/(a^4*d^2 + a
^2*b^2*d^2))^2*d^4 - 12*b^4 - 4*(a^4*b^2 + a^2*b^4)*(2*(1/2)^(2/3)*b^4*(-I*
sqrt(3) + 1)/((a^4*d^2 +  ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [**B**]
time = 23.06, size = 1293, normalized size = 4.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^3)),x)
```

```
[Out] symsum(log(-(8192*b^4*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243
*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x) - 65536*a*b^3 - 294912*root(729*a^8*b^
2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^2*a^3*b^3*d
^2 - 221184*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z
^4 - b^4, z, k)^3*a^4*b^3*d^3 - 196608*root(729*a^8*b^2*d^6*z^6 + 729*a^10*
d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)*a^2*b^3*d + 10616832*root(729*a^
```

```
8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^4*a^8*d
^4*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 -
b^4, z, k) + d*x) + 7962624*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 2
43*a^6*b^2*d^4*z^4 - b^4, z, k)^5*a^9*d^5*exp(root(729*a^8*b^2*d^6*z^6 + 72
9*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x) - 1769472*root(729
*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^3*a^
6*b*d^3 + 2654208*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2
*d^4*z^4 - b^4, z, k)^4*a^7*b*d^4 - 1990656*root(729*a^8*b^2*d^6*z^6 + 729*
a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^5*a^8*b*d^5 + 2064384*root(
729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^2
*a^4*b^2*d^2*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*
d^4*z^4 - b^4, z, k) + d*x) + 5529600*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d
^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^3*a^5*b^2*d^3*exp(root(729*a^8*b^
2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x) + 72
99072*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b
^4, z, k)^4*a^6*b^2*d^4*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 2
43*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x) + 9953280*root(729*a^8*b^2*d^6*z^6 +
 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^5*a^7*b^2*d^5*exp(root
(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)
+ d*x) + 393216*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d
^4*z^4 - b^4, z, k)*a^3*b^2*d*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z
^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x))/(a^4*b^5))*root(729*a^8*b^2*d
^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k), k, 1, 6) + 2/
(a*d - a*d*exp(2*c + 2*d*x))
```

3.180 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

<u>**Optimal**</u>. Leaf size=322

$$\frac{2(-1)^{2/3}b\operatorname{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}+\frac{2(-1)^{2/3}b\operatorname{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}\right.\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3a^{5/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d}$$

```
[Out] 1/2*arctanh(cosh(d*x+c))/a/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d+2/3*(-1)^(2/3)
*b*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-
1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a^(5/3)/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/
2)+2/3*b*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1
/2))/a^(5/3)/d/(a^(2/3)+b^(2/3))^(1/2)+2/3*(-1)^(2/3)*b*arctan((-1)^(1/6)*(
(-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^
(2/3)*b^(2/3))^(1/2))/a^(5/3)/d/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/
2)
```

<u>**Rubi**</u> [A]

time = 0.43, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3299, 3853, 3855, 3292, 2739, 632, 210}

$$\frac{2(-1)^{2/3}b\operatorname{ArcTan}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2(-1)^{2/3}b\operatorname{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}+\frac{2b\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (2*(-1)^(2/3)*b*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c
 + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(5/3)*Sqrt
[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) + (2*(-1)^(2/3)*b*ArcTan[((-1)
^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*
a^(2/3) - b^(2/3)]])/(3*a^(5/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) + Arc
Tanh[Cosh[c + d*x]]/(2*a*d) + (2*b*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x
)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*a^(5/3)*Sqrt[a^(2/3) + b^(2/3)]*d) - (Co
th[c + d*x]*Csch[c + d*x])/(2*a*d)
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\left(i\int\left(\frac{i\operatorname{csch}^3(c+dx)}{a}-\frac{ib}{a\left(a+b\sinh^3(c+dx)\right)}\right)dx\right)$$

$$= \frac{\int \operatorname{csch}^3(c+dx)\,dx}{a}-\frac{b\int \frac{1}{a+b\sinh^3(c+dx)}\,dx}{a}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}-\frac{\int \operatorname{csch}(c+dx)\,dx}{2a}-\frac{b\int\left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\,\sqrt[3]{a}-i\sqrt[3]{b}\ \mathrm{si}\right.}\right.}{}$$

$$= \frac{\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}-\frac{\left(\sqrt[6]{-1}\,b\right)\int\frac{1}{\sqrt[6]{-1}\,\sqrt[3]{a}-i\sqrt[3]{}}}{3a^{5/3}}$$

$$= \frac{\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}+\frac{\left(2(-1)^{2/3}b\right)\operatorname{Subst}\left(\int\frac{}{\sqrt[6]{-}}\right.}{}$$

$$= \frac{\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}-\frac{\left(4(-1)^{2/3}b\right)\operatorname{Subst}\left(\int\frac{}{-4\left(\right.}\right.}{}$$

$$= -\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,d}+\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{}}{\sqrt{\sqrt[3]{-}}}\right.}{3a^{5/3}\sqrt{\sqrt[3]{-1}\,a^{2/3}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.36, size = 178, normalized size = 0.55

$$-\frac{16b\operatorname{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{c\#1+dx\#1+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1}{b+4a\#1-2b\#1^2+b\#1^4}\&\right]+3\left(\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+4\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\right)}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -1/24*(16*b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1
+ d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]
*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ] + 3*(C
sch[(c + d*x)/2]^2 + 4*Log[Tanh[(c + d*x)/2]] + Sech[(c + d*x)/2]^2))/(a*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.87, size = 138, normalized size = 0.43

method	result
derivativedivides	$\frac{\frac{\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}-3a\_Z^{4}-8b\_Z^{3}+3a\_Z^{2}-a\right)}\frac{\left(\_R^{4}-2\_R^{2}+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a-2\_R^{3}a-4\_R^{2}b+\_Ra}\right)}{3a}-\frac{}{8a}}{d}$
default	$\frac{\frac{\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}-3a\_Z^{4}-8b\_Z^{3}+3a\_Z^{2}-a\right)}\frac{\left(\_R^{4}-2\_R^{2}+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a-2\_R^{3}a-4\_R^{2}b+\_Ra}\right)}{3a}-\frac{}{8a}}{d}$
risch	$-\frac{\mathrm{e}^{dx+c}\left(1+\mathrm{e}^{2dx+2c}\right)}{da\left(\mathrm{e}^{2dx+2c}-1\right)^{2}}+8\left(\sum_{\_R=\mathrm{RootOf}\left(\left(191102976a^{12}d^{6}+191102976a^{10}b^{2}d^{6}\right)\_Z^{6}-995328a^{8}b^{2}d^{4}\_Z^{4}+1728a^{4}d^{2}\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a+1/3/a*b*sum((_R^4-2*_R^2+1)/(_R^5*a-2*_R^3
*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z
^3*b+3*_Z^2*a-a))-1/8/a/tanh(1/2*d*x+1/2*c)^2-1/2/a*ln(tanh(1/2*d*x+1/2*c))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -8*b*integrate(e^(3*d*x + 3*c)/(a*b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c)
 + 8*a^2*e^(3*d*x + 3*c) + 3*a*b*e^(2*d*x + 2*c) - a*b), x) - (e^(3*d*x + 3
*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*c) + a*d) + 1/
2*log((e^(d*x + c) + 1)*e^(-c))/(a*d) - 1/2*log((e^(d*x + c) - 1)*e^(-c))/(
a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 29179 vs. 2(229) = 458.
time = 8.96, size = 29179, normalized size = 90.62

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(2/3)*sqrt(1/6)*(a*d*cosh(d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh
(d*x + c)^3 + a*d*sinh(d*x + c)^4 - 2*a*d*cosh(d*x + c)^2 + 2*(3*a*d*cosh(d
*x + c)^2 - a*d)*sinh(d*x + c)^2 + a*d + 4*(a*d*cosh(d*x + c)^3 - a*d*cosh(
d*x + c))*sinh(d*x + c))*sqrt(-((a^4 + a^2*b^2)*(2*(1/2)^(2/3)*(b^4/(a^8*d^
4 + a^6*b^2*d^4) - b^4/(a^4*d^2 + a^2*b^2*d^2)^2)*(-I*sqrt(3) + 1)/(b^6/(a^
12*d^6 + a^10*b^2*d^6) - 3*b^6/((a^8*d^4 + a^6*b^2*d^4)*(a^4*d^2 + a^2*b^2*
d^2)) + 2*b^6/(a^4*d^2 + a^2*b^2*d^2)^3 + b^8/((a^2 + b^2)^2*a^10*d^6))^(1/
3) - (1/2)^(1/3)*(b^6/(a^12*d^6 + a^10*b^2*d^6) - 3*b^6/((a^8*d^4 + a^6*b^2
*d^4)*(a^4*d^2 + a^2*b^2*d^2)) + 2*b^6/(a^4*d^2 + a^2*b^2*d^2)^3 + b^8/((a^
2 + b^2)^2*a^10*d^6))^(1/3)*(I*sqrt(3) + 1) + 2*b^2/(a^4*d^2 + a^2*b^2*d^2)
)*d^2 + 3*sqrt(1/3)*(a^4 + a^2*b^2)*d^2*sqrt(-((a^10 + 2*a^8*b^2 + a^6*b^4)
*(2*(1/2)^(2/3)*(b^4/(a^8*d^4 + a^6*b^2*d^4) - b^4/(a^4*d^2 + a^2*b^2*d^2)^
2)*(-I*sqrt(3) + 1)/(b^6/(a^12*d^6 + a^10*b^2*d^6) - 3*b^6/((a^8*d^4 + a^6*
b^2*d^4)*(a^4*d^2 + a^2*b ...
```

Sympy **[F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac **[F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)^3/(b*sinh(d*x + c)^3 + a), x)
```

Mupad **[B]**
time = 89.25, size = 2500, normalized size = 7.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^3)),x)
```

```
[Out] symsum(log(-(16777216*b^7*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12
*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 5033164
8*a*b^6 + 33554432*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b
```

```
^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)*a*b^7*d + 671088640*a^2*b^5*ex
p(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z
^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) + 201326592*root(729*a^10*b^2*d^6*z^6
 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)
^2*a^3*b^6*d^2 - 1509949440*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 -
243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^2*a^5*b^4*d^2 - 27179
08992*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 +
27*a^4*b^4*d^2*z^2 - b^6, z, k)^3*a^5*b^5*d^3 + 2717908992*root(729*a^10*b^
2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b
^6, z, k)^3*a^7*b^3*d^3 + 6039797760*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d
^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^4*a^7*b^4*d^
4 - 4076863488*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d
^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^4*a^9*b^2*d^4 - 679477248*root(729
*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2
*z^2 - b^6, z, k)^5*a^9*b^3*d^5 + 16307453952*root(729*a^10*b^2*d^6*z^6 + 7
29*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^6*a
^11*b^2*d^6 - 32614907904*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 24
3*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^7*a^11*b^3*d^7 + 452984
832*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27
*a^4*b^4*d^2*z^2 - b^6, z, k)*a^3*b^5*d + 4076863488*root(729*a^10*b^2*d^6*
z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z,
 k)^5*a^11*b*d^5 - 40768634880*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6
 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^7*a^13*b*d^7 - 978
44723712*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4
 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^5*a^12*d^5*exp(d*x)*exp(root(729*a^10*b^
2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b
^6, z, k)) + 391378894848*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 24
3*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^7*a^14*d^7*exp(d*x)*exp
(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^
4*b^4*d^2*z^2 - b^6, z, k)) + 10871635968*root(729*a^10*b^2*d^6*z^6 + 729*a
^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^4*a^10*
b*d^4*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b
^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 55717134336*root(729*a^10*b
^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 -
b^6, z, k)^6*a^12*b*d^6*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d
^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 306184192
0*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a
^4*b^4*d^2*z^2 - b^6, z, k)^2*a^4*b^5*d^2*exp(d*x)*exp(root(729*a^10*b^2*d^
6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6,
z, k)) - 7247757312*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*
b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^2*a^6*b^3*d^2*exp(d*x)*exp(ro
ot(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b
^4*d^2*z^2 - b^6, z, k)) + 9688842240*root(729*a^10*b^2*d^6*z^6 + 729*a^12*
d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^3*a^6*b^4*d
^3*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*
```

```
d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) + 36238786560*root(729*a^10*b^2*
d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6
, z, k)^3*a^8*b^2*d^3*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6
*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 301989888*r
oot(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*
b^4*d^2*z^2 - b^6, z, k)^4*a^6*b^5*d^4*exp(d*x)*exp(root(729*a^10*b^2*d^6*z
^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z,
k)) + 48695869440*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^
2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^4*a^8*b^3*d^4*exp(d*x)*exp(root
(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4
*d^2*z^2 - b^6, z, k)) + 6341787648*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^
6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^5*a^8*b^4*d^5
*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^
4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 244838301696*root(729*a^10*b^2*d
^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6,
 z, k)^5*a^10*b^2*d^5*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6
*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 74742497280
*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^
4*b^4*d^2*z^2 - b^6, z, k)^6*a^10*b^3*d^6*exp(d...
```

3.181 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=317

$$-\frac{2b^{4/3}\operatorname{ArcTan}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,d}-\frac{2b^{4/3}\operatorname{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right.}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right.}{3a^2\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,d}$$

```
[Out] b*arctanh(cosh(d*x+c))/a^2/d+coth(d*x+c)/a/d-1/3*coth(d*x+c)^3/a/d-2/3*b^(4
/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((
-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a^2/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)-
2/3*b^(4/3)*arctan((-1)^(5/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/
2*c))/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2))/a^2/d/(-(-1)^(2/3)*a^(2/3)-b^(2/
3))^(1/2)-2/3*b^(4/3)*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3
)+b^(2/3))^(1/2))/a^2/d/(a^(2/3)+b^(2/3))^(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3299, 3855, 3852, 2739, 632, 212, 210}

$$\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{2b^{4/3}\operatorname{ArcTan}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\operatorname{ArcTan}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^2d\sqrt{a^{2/3}+b^{2/3}}}-\frac{\coth^3(c+dx)}{3ad}+\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-2*b^(4/3)*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*
x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*a^2*Sqrt[-((-1)^(2/3)*a^
(2/3)) - b^(2/3)]*d) - (2*b^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) +
I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*a^2*S
qrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) + (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) -
 (2*b^(4/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^
(2/3)]])/(3*a^2*Sqrt[a^(2/3) + b^(2/3)]*d) + Coth[c + d*x]/(a*d) - Coth[c +
 d*x]^3/(3*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \left(-\frac{b\operatorname{csch}(c+dx)}{a^2}+\frac{\operatorname{csch}^4(c+dx)}{a}-\frac{b^2\sinh^2(c+dx)}{a^2\left(-a-b\sinh^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \operatorname{csch}^4(c+dx)\,dx}{a}-\frac{b\int \operatorname{csch}(c+dx)\,dx}{a^2}-\frac{b^2\int \frac{\sinh^2(c+dx)}{-a-b\sinh^3(c+dx)}\,dx}{a^2}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{b^2\int \Bigg(-\frac{i}{3b^{2/3}\left(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}-\frac{\quad}{3b^{2/3}\Big(\sqrt[6]{-1}\sqrt[3]{\quad}}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{\left(ib^{4/3}\right)\int \frac{\quad}{-i\sqrt[3]{a}-i\sqrt[3]{\quad}}}{3a^2}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{\left(2b^{4/3}\right)\operatorname{Subst}\Big(\int \frac{\quad}{-}}{\quad}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}+\frac{\left(4b^{4/3}\right)\operatorname{Subst}\Bigg(\int \frac{\quad}{-}}{\quad}$$

$$= \frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,d}+\frac{2b^{4/3}\tan^{-1}\Bigg(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-}}}{3a^2\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.91, size = 370, normalized size = 1.17

$$\frac{8a\coth\left(\frac{1}{2}(c+dx)\right)-24b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+4b^2\text{RootSum}\left[-b+3b\#1^2+8a\#1^3-3b\#1^4+b\#1^6\&,\frac{c+dx+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)-2c\#1^2-2dx\#1^2-4\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2+c\#1^4+dx\#1^4+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4}{b\#1+4a\#1^2-2b\#1^3+b\#1^5}\&\right]+8a\operatorname{csch}^3(c+dx)\sinh^4\left(\frac{1}{2}(c+dx)\right)-\frac{1}{2}a\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)\sinh(c+dx)+8a\tanh\left(\frac{1}{2}(c+dx)\right)}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (8*a*Coth[(c + d*x)/2] - 24*b*Log[Tanh[(c + d*x)/2]] + 4*b^2*RootSum[-b + 3
*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c + d*x + 2*Log[-Cosh[(c + d*x)
/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*
c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] + 8*a*Csch[c + d*x]
```

```
^3*Sinh[(c + d*x)/2]^4 - (a*Csch[(c + d*x)/2]^4*Sinh[c + d*x])/2 + 8*a*Tanh
[(c + d*x)/2])/(24*a^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.80, size = 164, normalized size = 0.52

method	result
derivativedivides	$\frac{-\frac{\frac{\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{3}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}}-\frac{4b^{2}\Bigg(_{\_R=\mathrm{RootOf}\left(a\_Z^{6}-3a\right.}}{}}{d}$
default	$\frac{-\frac{\frac{\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{3}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{2}}-\frac{4b^{2}\Bigg(_{\_R=\mathrm{RootOf}\left(a\_Z^{6}-3a\right.}}{}}{d}$
risch	$-\frac{4\left(3\,\mathrm{e}^{2dx+2c}-1\right)}{3ad\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}+\frac{b\ln\left(\mathrm{e}^{dx+c}+1\right)}{a^{2}d}-\frac{b\ln\left(\mathrm{e}^{dx+c}-1\right)}{a^{2}d}+16\Bigg(_{\_R=\mathrm{RootOf}\left(\left(12230590464a^{14}d^{6}+12230590464a^{12}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/a*(1/3*tanh(1/2*d*x+1/2*c)^3-3*tanh(1/2*d*x+1/2*c))-1/24/a/tanh(1
/2*d*x+1/2*c)^3+3/8/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(1/2*d*x+1/2*c))-4
/3*b^2/a^2*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-
_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*e^(6*d*x + 6*c) - 3*a*d*e^(4*d*x + 4*c) +
 3*a*d*e^(2*d*x + 2*c) - a*d) + b*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - b
*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 16*integrate(1/8*(b^2*e^(5*d*x + 5
*c) - 2*b^2*e^(3*d*x + 3*c) + b^2*e^(d*x + c))/(a^2*b*e^(6*d*x + 6*c) - 3*a
^2*b*e^(4*d*x + 4*c) + 8*a^3*e^(3*d*x + 3*c) + 3*a^2*b*e^(2*d*x + 2*c) - a^
2*b), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30233 vs. 2(234) = 468.
time = 6.68, size = 30233, normalized size = 95.37

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(2/3)*sqrt(1/6)*(a^2*d*cosh(d*x + c)^6 + 6*a^2*d*cosh(d*x + c)*
sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x + c)^4 + 3*a^2*d
*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(d*x + c)^4 + 4*
(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x + c)^3 - a^2*d +
 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2*d)*sinh(d*x + c
)^2 + 6*(a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)^3 + a^2*d*cosh(d*x +
 c))*sinh(d*x + c))*sqrt((6*b^4 - (a^6 + a^4*b^2)*(2*b^4/(a^6*d^2 + a^4*b^2
*d^2) - 2*(1/2)^(2/3)*(b^8/(a^6*d^2 + a^4*b^2*d^2)^2 - b^6/(a^10*d^4 + a^8*
b^2*d^4))*(-I*sqrt(3) + 1)/(2*b^12/(a^6*d^2 + a^4*b^2*d^2)^3 - 3*b^10/((a^1
0*d^4 + a^8*b^2*d^4)*(a^6*d^2 + a^4*b^2*d^2)) + b^8/(a^14*d^6 + a^12*b^2*d^
6) + b^8/((a^2 + b^2)^2*a^10*d^6))^(1/3) - (1/2)^(1/3)*(2*b^12/(a^6*d^2 + a
^4*b^2*d^2)^3 - 3*b^10/((a^10*d^4 + a^8*b^2*d^4)*(a^6*d^2 + a^4*b^2*d^2)) +
 b^8/(a^14*d^6 + a^12*b^2*d^6) + b^8/((a^2 + b^2)^2*a^10*d^6))^(1/3)*(I*sqr
t(3) + 1))*d^2 + 3*sqrt(1 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)^4/(b*sinh(d*x + c)^3 + a), x)
```

Mupad [**B**]
time = 59.69, size = 2500, normalized size = 7.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^3)),x)
```

```
[Out] symsum(log((18589155328*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*
a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^2*a^4*b^7*d^2 - 134217728
*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^
4*b^6*d^2*z^2 - b^8, z, k)*a*b^9*d - 1073741824*b^9 + 2818572288*root(729*a
^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z
^2 - b^8, z, k)^3*a^5*b^7*d^3 + 17716740096*root(729*a^12*b^2*d^6*z^6 + 729
*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^3*a^7
*b^5*d^3 - 88181047296*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a
^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^4*a^8*b^5*d^4 - 8697308774
4*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a
^4*b^6*d^2*z^2 - b^8, z, k)^4*a^10*b^3*d^4 - 18119393280*root(729*a^12*b^2*
d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8
, z, k)^5*a^9*b^5*d^5 - 30802968576*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^
6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^5*a^11*b^3*d^
5 + 70665633792*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*
d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^6*a^12*b^3*d^6 + 32614907904*root
(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6
*d^2*z^2 - b^8, z, k)^7*a^13*b^3*d^7 - 3221225472*root(729*a^12*b^2*d^6*z^6
 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)
*a^3*b^7*d + 2147483648*a*b^8*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*
a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) + 869
73087744*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4
 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^6*a^14*b*d^6 + 40768634880*root(729*a^12
*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2
 - b^8, z, k)^7*a^15*b*d^7 - 391378894848*root(729*a^12*b^2*d^6*z^6 + 729*a^
14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^7*a^16*d
^7*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*
d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) + 268435456*root(729*a^12*b^2*d^
6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8,
z, k)^2*a^3*b^8*d^2*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z
^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) - 16642998272*r
oot(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*
b^6*d^2*z^2 - b^8, z, k)^2*a^5*b^6*d^2*exp(d*x)*exp(root(729*a^12*b^2*d^6*z
^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z,
k)) - 100763959296*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b
^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^3*a^6*b^6*d^3*exp(d*x)*exp(roo
t(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^
6*d^2*z^2 - b^8, z, k)) + 57982058496*root(729*a^12*b^2*d^6*z^6 + 729*a^14*
d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^3*a^8*b^4*d
^3*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*
d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) - 4831838208*root(729*a^12*b^2*d
^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8,
 z, k)^4*a^7*b^6*d^4*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*
z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) + 36238786560*
root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4
```

```
*b^6*d^2*z^2 - b^8, z, k)^4*a^9*b^4*d^4*exp(d*x)*exp(root(729*a^12*b^2*d^6*
z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z,
 k)) + 494659436544*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*
b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^5*a^10*b^4*d^5*exp(d*x)*exp(r
oot(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*
b^6*d^2*z^2 - b^8, z, k)) + 333396836352*root(729*a^12*b^2*d^6*z^6 + 729*a^
14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^5*a^12*b
^2*d^5*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*
b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) + 21743271936*root(729*a^12*
b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 -
 b^8, z, k)^6*a^11*b^4*d^6*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^1
4*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) + 271790
8992*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 2
7*a^4*b^6*d^2*z^2 - b^8, z, k)^6*a^13*b^2*d^6*exp(d*x)*exp(root(729*a^12*b^
2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b
^8, z, k)) - 399532621824*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 24
3*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^7*a^14*b^2*d^7*exp(d*x)
*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 2
7*a^4*b^6*d^2*z^2 - b^8, z, k)) + 5637144576*root(729*a^12*b^2*d^6*z^6 + 72
9*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)*a^2*
b^8*d*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b
^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)))/...
```

3.182 $\int \frac{1}{1+\sinh^3(x)}\,dx$

Optimal. Leaf size=139

$$-\frac{2\sqrt[6]{-1}\,\text{ArcTan}\left(\frac{i+\sqrt[6]{-1}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[3]{-1}}}\right)}{3\sqrt{1-\sqrt[3]{-1}}}-\frac{1}{3}\sqrt{2}\,\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{1}{3}\sqrt[6]{-1}\,\log\left(1+(-1)^{5/6}-\sqrt[6]{-1}\,\tanh\right.$$

```
[Out] -1/3*(-1)^(1/6)*ln(1+(-1)^(5/6)-(-1)^(1/6)*tanh(1/2*x))+1/3*(-1)^(1/6)*ln(1
+(-1)^(1/6)+(-1)^(1/3)*tanh(1/2*x))-1/3*arctanh(1/2*(1-tanh(1/2*x))*2^(1/2)
)*2^(1/2)-2/3*(-1)^(1/6)*arctan((I+(-1)^(1/6)*tanh(1/2*x))/(1-(-1)^(1/3))^(
1/2))/(1-(-1)^(1/3))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3292, 2739, 632, 210, 631, 212, 630, 31}

$$-\frac{2\sqrt[6]{-1}\,\text{ArcTan}\left(\frac{\sqrt[6]{-1}\,\tanh\left(\frac{x}{2}\right)+i}{\sqrt{1-\sqrt[3]{-1}}}\right)}{3\sqrt{1-\sqrt[3]{-1}}}-\frac{1}{3}\sqrt{2}\,\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{1}{3}\sqrt[6]{-1}\,\log\left(-\sqrt[6]{-1}\,\tanh\left(\frac{x}{2}\right)+(-1)^{5/6}+1\right)+\frac{1}{3}\sqrt[6]{-1}\,\log\left(\sqrt[3]{-1}\,\tanh\left(\frac{x}{2}\right)+\sqrt[6]{-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^3)^(-1),x]
```

```
[Out] (-2*(-1)^(1/6)*ArcTan[(I + (-1)^(1/6)*Tanh[x/2])/Sqrt[1 - (-1)^(1/3)]])/(3*
Sqrt[1 - (-1)^(1/3)]) - (Sqrt[2]*ArcTanh[(1 - Tanh[x/2])/Sqrt[2]])/3 - ((-1
)^(1/6)*Log[1 + (-1)^(5/6) - (-1)^(1/6)*Tanh[x/2]])/3 + ((-1)^(1/6)*Log[1 +
 (-1)^(1/6) + (-1)^(1/3)*Tanh[x/2]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sinh^3(x)}\,dx &= \int \left(\frac{\sqrt[6]{-1}}{3\left(\sqrt[6]{-1}-i\sinh(x)\right)}+\frac{\sqrt[6]{-1}}{3\left(\sqrt[6]{-1}+\sqrt[6]{-1}\sinh(x)\right)}+\frac{\sqrt[6]{-1}}{3\left(\sqrt[6]{-1}+(-1)^{5/6}\sinh(x)\right)}\right.\\ &= \frac{1}{3}\sqrt[6]{-1}\int \frac{1}{\sqrt[6]{-1}-i\sinh(x)}\,dx+\frac{1}{3}\sqrt[6]{-1}\int \frac{1}{\sqrt[6]{-1}+\sqrt[6]{-1}\sinh(x)}\,dx+\frac{1}{3}\sqrt[6]{-1}\int\\ &= \frac{1}{3}\left(2\sqrt[6]{-1}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{-1}-2ix-\sqrt[6]{-1}x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{3}\left(2\sqrt[6]{-1}\right)\text{Subst}\left(\right.\\ &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,1-\tanh\left(\frac{x}{2}\right)\right)\right)-\frac{1}{3}\left(4\sqrt[6]{-1}\right)\text{Subst}\left(\int \frac{1}{-4\left(1-\sqrt[3]{-1}\right)}\right.\\ &= -\frac{2\sqrt[6]{-1}\tan^{-1}\left(\frac{i+\sqrt[6]{-1}\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[3]{-1}}}\right)}{3\sqrt{1-\sqrt[3]{-1}}}-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{1}{3}\sqrt[6]{-1}\log\left(1\right.\end{aligned}$$

Mathematica [A]
time = 1.05, size = 156, normalized size = 1.12

$$\frac{i\sqrt{-1-i\sqrt{3}}\left(i+\sqrt{3}\right)\text{ArcTan}\left(\frac{2+\left(1-i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2+2i\sqrt{3}}}\right)+\left(-1-i\sqrt{3}\right)\sqrt{-1+i\sqrt{3}}\,\text{ArcTan}\left(\frac{2+\left(1+i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2-2i\sqrt{3}}}\right)+2\tanh^{-1}\left(\frac{-1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^3)^(-1),x]
```

```
[Out] (I*Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[(2 + (1 - I*Sqrt[3])*Tanh[x/2]
)/Sqrt[-2 + (2*I)*Sqrt[3]]] + (-1 - I*Sqrt[3])*Sqrt[-1 + I*Sqrt[3]]*ArcTan[
(2 + (1 + I*Sqrt[3])*Tanh[x/2])/Sqrt[-2 - (2*I)*Sqrt[3]]] + 2*ArcTanh[(-1 +
 Tanh[x/2])/Sqrt[2]])/(3*Sqrt[2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.64, size = 82, normalized size = 0.59

method	result
risch	$\left(\sum_{\_R=\text{RootOf}\left(81\_Z^4-9\_Z^2+1\right)}\_R\ln\left(-9\_R^2+3\_R+\mathrm{e}^x\right)\right)+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^x+1-\sqrt{2}\right)}{6}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^x+1+\sqrt{2}\right.}{6}$
default	$\frac{2\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+2\_Z^3+2\_Z^2-2\_Z+1\right)}\frac{\left(-\_R^2-\_R+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3+3\_R^2+2\_R-1}\right)}{3}+\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*sum((-_R^2-_R+1)/(2*_R^3+3*_R^2+2*_R-1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z
^4+2*_Z^3+2*_Z^2-2*_Z+1))+1/3*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2)
)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^3),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) - integrate(2/3*(
e^(3*x) - 4*e^(2*x) - e^x)/(e^(4*x) - 2*e^(3*x) + 2*e^(2*x) + 2*e^x + 1), x
)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(92) = 184$.
time = 0.44, size = 185, normalized size = 1.33

$$-\frac{1}{6}\sqrt{3}\log\left(-4\left(\sqrt{3}+1\right)e^{x}+4\sqrt{3}+4e^{(2x)}+8\right)+\frac{1}{6}\sqrt{3}\log\left(4\left(\sqrt{3}-1\right)e^{x}-4\sqrt{3}+4e^{(2x)}+8\right)+\frac{1}{6}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-1\right)e^{x}+2\sqrt{2}-e^{(2x)}-3}{e^{(2x)}+2e^{x}-1}\right)+\frac{2}{3}\arctan\left(-\left(\sqrt{3}+1\right)e^{x}+\sqrt{\left(\sqrt{3}-1\right)e^{x}-\sqrt{3}+e^{(2x)}+2}\left(\sqrt{3}+1\right)-1\right)-\frac{2}{3}\arctan\left(-\left(\sqrt{3}-1\right)e^{x}+\frac{1}{2}\sqrt{-4\left(\sqrt{3}+1\right)e^{x}+4\sqrt{3}+4e^{(2x)}+8}\left(\sqrt{3}-1\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*log(-4*(sqrt(3) + 1)*e^x + 4*sqrt(3) + 4*e^(2*x) + 8) + 1/6*sq
rt(3)*log(4*(sqrt(3) - 1)*e^x - 4*sqrt(3) + 4*e^(2*x) + 8) + 1/6*sqrt(2)*lo
g(-(2*(sqrt(2) - 1)*e^x + 2*sqrt(2) - e^(2*x) - 3)/(e^(2*x) + 2*e^x - 1)) +
 2/3*arctan(-(sqrt(3) + 1)*e^x + sqrt((sqrt(3) - 1)*e^x - sqrt(3) + e^(2*x)
 + 2)*(sqrt(3) + 1) - 1) - 2/3*arctan(-(sqrt(3) - 1)*e^x + 1/2*sqrt(-4*(sqr
t(3) + 1)*e^x + 4*sqrt(3) + 4*e^(2*x) + 8)*(sqrt(3) - 1) + 1)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5697 vs. $2(133) = 266$.
time = 32.87, size = 5697, normalized size = 40.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**3),x)
```

```
[Out] -2730935734518397297302171298494629987458472657311991598275400*sqrt(3)*I*sq
rt(1 + sqrt(3)*I)*log(tanh(x/2) - 1 + sqrt(2))/(567614333326955414950687115
47884217702675164237235096719281544 + 4013639441941715127028842203182095006
8139251222039410549591998*sqrt(2) - 115863790611757427927687110921905889558
98140233336817863353300*sqrt(3)*I*sqrt(1 + sqrt(3)*I) - 8192807203555191891
906513895483889962375417971935974794826200*sqrt(6)*I*sqrt(1 + sqrt(3)*I) +
34759137183527228378306133276571766867694420700010453590059900*sqrt(1 + sqr
t(3)*I) + 24578421610665575675719541686451669887126253915807924384478600*sq
rt(2)*sqrt(1 + sqrt(3)*I)) - 1931063176862623798794785182031764825983023372
22280297225550*sqrt(6)*I*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - 1 + sqrt(2))/
(56761433332695541495068711547884217702675164237235096719281544 + 401363944
19417151270288422031820950068139251222039410549591998*sqrt(2) - 11586379061
175742792768711092190588955898140233336817863353300*sqrt(3)*I*sqrt(1 + sqrt
(3)*I) - 8192807203555191891906513895483889962375417971935974794826200*sqrt
(6)*I*sqrt(1 + sqrt(3)*I) + 34759137183527228378306133276571766867694420700
010453590059900*sqrt(1 + sqrt(3)*I) + 2457842161066557567571954168645166988
7126253915807924384478600*sqrt(2)*sqrt(1 + sqrt(3)*I)) + 133787981398057170
90096140677273650022713083740679803516530666*log(tanh(x/2) - 1 + sqrt(2))/(
56761433332695541495068711547884217702675164237235096719281544 + 4013639441
9417151270288422031820950068139251222039410549591998*sqrt(2) - 115863790611
75742792768711092190588955898140233336817863353300*sqrt(3)*I*sqrt(1 + sqrt(
3)*I) - 8192807203555191891906513895483889962375417971935974794826200*sqrt(
6)*I*sqrt(1 + sqrt(3)*I) + 347591371835272283783061332765717668676944207000
10453590059900*sqrt(1 + sqrt(3)*I) + 24578421610665575675719541686451669887
126253915807924384478600*sqrt(2)*sqrt(1 + sqrt(3)*I)) + 9460238888782590249
178118591314036283779194039539182786546924*sqrt(2)*log(tanh(x/2) - 1 + sqrt
(2))/(56761433332695541495068711547884217702675164237235096719281544 + 4013
6394419417151270288422031820950068139251222039410549591998*sqrt(2) - 115863
79061175742792768711092190588955898140233336817863353300*sqrt(3)*I*sqrt(1 +
 sqrt(3)*I) - 8192807203555191891906513895483889962375417971935974794826200
*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 347591371835272283783061332765717668676944
20700010453590059900*sqrt(1 + sqrt(3)*I) + 24578421610665575675719541686451
669887126253915807924384478600*sqrt(2)*sqrt(1 + sqrt(3)*I)) + 8192807203555
191891906513895483889962375417971935974794826200*sqrt(1 + sqrt(3)*I)*log(ta
nh(x/2) - 1 + sqrt(2))/(567614333326955414950687115478842177026751642372350
96719281544 + 4013639441941715127028842203182095006813925122203941054959199
8*sqrt(2) - 11586379061175742792768711092190588955898140233336817863353300*
sqrt(3)*I*sqrt(1 + sqrt(3)*I) - 8192807203555191891906513895483889962375417
971935974794826200*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 347591371835272283783061
33276571766867694420700010453590059900*sqrt(1 + sqrt(3)*I) + 24578421610665
575675719541686451669887126253915807924384478600*sqrt(2)*sqrt(1 + sqrt(3)*I
)) + 5793189530587871396384355546095294477949070116668408931676650*sqrt(2)*
sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - 1 + sqrt(2))/(5676143333269554149506871
1547884217702675164237235096719281544 + 40136394419417151270288422031820950
068139251222039410549591998*sqrt(2) - 1158637906117574279276871109219058895
```

```
5898140233336817863353300*sqrt(3)*I*sqrt(1 + sqrt(3)*I) - 81928072035551918
919065138954838899623754179719359747948262OO*sqrt(6)*I*sqrt(1 + sqrt(3)*I)
+ 34759137183527228378306133276571766867694420700010453590059900*sqrt(1 + s
qrt(3)*I) + 2457842161066557567571954168645166988712625391580792438447860O*
sqrt(2)*sqrt(1 + sqrt(3)*I)) - 819280720355519189190651389548388996237541797
71935974794826200*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - sqrt(2) - 1)/(5676143
333269554149506871154788421770267516423723509671928154 4 + 40136394419417151
2702884220318209500681392512220394105495919 98*sqrt(2) - 1158637906117574279
2768711092190588955898140233336817863353300*sqrt(3)*I*sqrt(1 + sqrt(3)*I) -
 819280720355519189190651389548388996237541797193597479482620O*sqrt(6)*I*sq
rt(1 + sqrt(3)*I) + 347591371835272283783061332765717668676944207000104535 9
0059900*sqrt(1 + sqrt(3)*I) + 2457842161066557567571954168645166988712625 39
15807924384478600*sqrt(2)*sqrt(1 + sqrt(3)*I)) - 579318953058787139638435 55
46095294477949070116668408931676650*sqrt(2)*sqrt(1 + sqrt(3)*I)*log(tanh(x/
2) - sqrt(2) - 1)/(567614333326955414950687115478842177026751642372350967 19
281544 + 401363944194171512702884220318209500681392512220394105495919 98*sqr
t(2) - 1158637906117574279276871109219058895589814023333681786335330 0*sqrt(
3)*I*sqrt(1 + sqrt(3)*I) - 819280720355519189190651389548388996237541797 193
5974794826200*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 347591371835272283783061332 76
571766867694420700010453590059900*sqrt(1 + sqrt(3)*I) + 245784216106655756 7
5719541686451669887126253915807924384478600*sqrt(2)*sqrt(1 + sqrt(3)*I)) -
13378798139805717090096140677273650022713083740...
```

Giac [A]
time = 0.41, size = 102, normalized size = 0.73

$$\frac{1}{6}\pi+\frac{1}{6}\sqrt{3}\log\left(\left(\sqrt{3}+e^{x}-1\right)^{2}+e^{(2x)}\right)-\frac{1}{6}\sqrt{3}\log\left(\left(\sqrt{3}-e^{x}+1\right)^{2}+e^{(2x)}\right)+\frac{1}{6}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2e^{x}+2\right|}{2\left(\sqrt{2}+e^{x}+1\right)}\right)+\frac{1}{3}\arctan\left(-\left(\sqrt{3}+1\right)e^{x}-1\right)+\frac{1}{3}\arctan\left(\left(\sqrt{3}-1\right)e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^3),x, algorithm="giac")
```

```
[Out] 1/6*pi + 1/6*sqrt(3)*log((sqrt(3) + e^x - 1)^2 + e^(2*x)) - 1/6*sqrt(3)*log
((sqrt(3) - e^x + 1)^2 + e^(2*x)) + 1/6*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*
e^x + 2)/(sqrt(2) + e^x + 1)) + 1/3*arctan(-(sqrt(3) + 1)*e^x - 1) + 1/3*ar
ctan((sqrt(3) - 1)*e^x - 1)
```

Mupad [B]
time = 1.77, size = 203, normalized size = 1.46

$$\frac{\mathrm{atan}\left(\frac{77824\,{\mathrm{e}}^x-32768\,\sqrt{3}-45056\,\sqrt{3}\,{\mathrm{e}}^x+57344}{77824\,{\mathrm{e}}^x-45056\,\sqrt{3}\,{\mathrm{e}}^x}\right)}{3}-\frac{\mathrm{atan}\left(\frac{77824\,{\mathrm{e}}^x+45056\,\sqrt{3}\,{\mathrm{e}}^x}{77824\,{\mathrm{e}}^x+32768\,\sqrt{3}+45056\,\sqrt{3}\,{\mathrm{e}}^x+57344}\right)}{3}-\frac{\sqrt{2}\,\ln\left(41984\,\sqrt{2}\,{\mathrm{e}}^x-17408\,\sqrt{2}-59392\,{\mathrm{e}}^x+24576\right)}{6}+\frac{\sqrt{2}\,\ln\left(17408\,\sqrt{2}-59392\,{\mathrm{e}}^x-41984\,\sqrt{2}\,{\mathrm{e}}^x+24576\right)}{6}-\frac{\sqrt{3}\,\ln\left({\left(77824\,{\mathrm{e}}^x-32768\,\sqrt{3}-45056\,\sqrt{3}\,{\mathrm{e}}^x+57344\right)}^2+{\left(77824\,{\mathrm{e}}^x-45056\,\sqrt{3}\,{\mathrm{e}}^x\right)}^2\right)}{6}+\frac{\sqrt{3}\,\ln\left({\left(77824\,{\mathrm{e}}^x+32768\,\sqrt{3}+45056\,\sqrt{3}\,{\mathrm{e}}^x+57344\right)}^2+{\left(77824\,{\mathrm{e}}^x+45056\,\sqrt{3}\,{\mathrm{e}}^x\right)}^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3 + 1),x)
```

```
[Out] atan((77824*exp(x) - 32768*3^(1/2) - 45056*3^(1/2)*exp(x) + 57344)/(77824*e
xp(x) - 45056*3^(1/2)*exp(x)))/3 - atan((77824*exp(x) + 45056*3^(1/2)*exp(x
```

```
))/(77824*exp(x) + 32768*3^(1/2) + 45056*3^(1/2)*exp(x) + 57344))/3 - (2^(1
/2)*log(41984*2^(1/2)*exp(x) - 17408*2^(1/2) - 59392*exp(x) + 24576))/6 + (
2^(1/2)*log(17408*2^(1/2) - 59392*exp(x) - 41984*2^(1/2)*exp(x) + 24576))/6
 - (3^(1/2)*log((77824*exp(x) - 32768*3^(1/2) - 45056*3^(1/2)*exp(x) + 5734
4)^2 + (77824*exp(x) - 45056*3^(1/2)*exp(x))^2))/6 + (3^(1/2)*log((77824*ex
p(x) + 32768*3^(1/2) + 45056*3^(1/2)*exp(x) + 57344)^2 + (77824*exp(x) + 45
056*3^(1/2)*exp(x))^2))/6
```

3.183 $\int \frac{1}{1-\sinh^3(x)}\,dx$

Optimal. Leaf size=133

$$\frac{2(-1)^{5/6}\text{ArcTan}\left(\frac{i-(-1)^{5/6}\tanh\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{2/3}}}\right)}{3\sqrt{1+(-1)^{2/3}}}+\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{1}{3}(-1)^{5/6}\log\left(1+(-1)^{5/6}+(-1)^{2/3}\right.$$

```
[Out] -1/3*(-1)^(5/6)*ln(1+(-1)^(5/6)+(-1)^(2/3)*tanh(1/2*x))+1/3*(-1)^(5/6)*ln(1
+(-1)^(1/6)+(-1)^(5/6)*tanh(1/2*x))+1/3*arctanh(1/2*(1+tanh(1/2*x))*2^(1/2)
)*2^(1/2)+2/3*(-1)^(5/6)*arctan((I-(-1)^(5/6)*tanh(1/2*x))/(1+(-1)^(2/3))^(
1/2))/(1+(-1)^(2/3))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3292, 2739, 632, 210, 630, 31, 631, 212}

$$\frac{2(-1)^{5/6}\text{ArcTan}\left(\frac{-(-1)^{5/6}\tanh\left(\frac{x}{2}\right)+i}{\sqrt{1+(-1)^{2/3}}}\right)}{3\sqrt{1+(-1)^{2/3}}}+\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)-\frac{1}{3}(-1)^{5/6}\log\left((-1)^{2/3}\tanh\left(\frac{x}{2}\right)+(-1)^{5/6}+1\right)+\frac{1}{3}(-1)^{5/6}\log\left((-1)^{5/6}\tanh\left(\frac{x}{2}\right)+\sqrt[6]{-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^3)^(-1),x]
```

```
[Out] (2*(-1)^(5/6)*ArcTan[(I - (-1)^(5/6)*Tanh[x/2])/Sqrt[1 + (-1)^(2/3)]])/(3*S
qrt[1 + (-1)^(2/3)]) + (Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[2]])/3 - ((-1)
^(5/6)*Log[1 + (-1)^(5/6) + (-1)^(2/3)*Tanh[x/2]])/3 + ((-1)^(5/6)*Log[1 +
(-1)^(1/6) + (-1)^(5/6)*Tanh[x/2]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sinh^3(x)}\,dx &= \int \left(-\frac{(-1)^{5/6}}{3\left(-(-1)^{5/6}-i\sinh(x)\right)}-\frac{(-1)^{5/6}}{3\left(-(-1)^{5/6}+\sqrt[6]{-1}\,\sinh(x)\right)}-\frac{(-1)^{5}}{3\left(-(-1)^{5/6}+(-\right.}\right.\\ &= -\left(\frac{1}{3}(-1)^{5/6}\int \frac{1}{-(-1)^{5/6}-i\sinh(x)}\,dx\right)-\frac{1}{3}(-1)^{5/6}\int \frac{1}{-(-1)^{5/6}+\sqrt[6]{-1}\,\sinh(x)}\,d\\ &= -\left(\frac{1}{3}\left(2(-1)^{5/6}\right)\text{Subst}\left(\int \frac{1}{-(-1)^{5/6}-2ix+(-1)^{5/6}x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\right)-\frac{1}{3}(2(-\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,1+\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{3}(-1)^{2/3}\text{Subst}\left(\int \frac{1}{-1+\sqrt[6]{-1}+(-1)^{5/}}\right.\\ &= \frac{2(-1)^{5/6}\tan^{-1}\left(\frac{i-(-1)^{5/6}\tanh\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{2/3}}}\right)}{3\sqrt{1+(-1)^{2/3}}}+\frac{1}{3}\sqrt{2}\,\tanh^{-1}\left(\frac{1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{1}{3}(-1)^{5/6}\log\end{aligned}$$

Mathematica [A]
time = 0.91, size = 156, normalized size = 1.17

$$\frac{\sqrt{-1+i\sqrt{3}}\left(1+i\sqrt{3}\right)\text{ArcTan}\left(\frac{2+\left(-1-i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2-2i\sqrt{3}}}\right)+\sqrt{-1-i\sqrt{3}}\left(1-i\sqrt{3}\right)\text{ArcTan}\left(\frac{2+i\left(i+\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2+2i\sqrt{3}}}\right)+2\tanh^{-1}\left(\frac{1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^3)^(-1),x]
```

```
[Out] (Sqrt[-1 + I*Sqrt[3]]*(1 + I*Sqrt[3])*ArcTan[(2 + (-1 - I*Sqrt[3])*Tanh[x/2
])/Sqrt[-2 - (2*I)*Sqrt[3]]] + Sqrt[-1 - I*Sqrt[3]]*(1 - I*Sqrt[3])*ArcTan[
(2 + I*(I + Sqrt[3])*Tanh[x/2])/Sqrt[-2 + (2*I)*Sqrt[3]]] + 2*ArcTanh[(1 +
Tanh[x/2])/Sqrt[2]])/(3*Sqrt[2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.64, size = 80, normalized size = 0.60

method	result
risch	$\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{x}+\sqrt{2}-1\right)}{6}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{x}-1-\sqrt{2}\right)}{6}+\left(\sum_{\_R=\mathrm{RootOf}\left(81\_Z^{4}-9\_Z^{2}+1\right)}\_R\ln\left(9\_R^{2}-3\_R+\mathrm{e}^{x}\right)\right)$
default	$\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{3}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-2\_Z^{3}+2\_Z^{2}+2\_Z+1\right)}\frac{\left(-\_R^{2}+\_R+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^{3}-3\_R^{2}+2\_R+1}\right)}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))+2/3*sum((-_R^2+_R+1)/(2*
_R^3-3*_R^2+2*_R+1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^4-2*_Z^3+2*_Z^2+2*_Z+1)
)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*log(-(sqrt(2) - e^x + 1)/(sqrt(2) + e^x - 1)) + integrate(2/3*
(e^(3*x) + 4*e^(2*x) - e^x)/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1),
x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(88) = 176.
time = 0.44, size = 180, normalized size = 1.35

$$-\frac{1}{6}\sqrt{3}\log\left(4\left(\sqrt{3}+1\right)e^{x}+4\sqrt{3}+4e^{(2x)}+8\right)+\frac{1}{6}\sqrt{3}\log\left(-4\left(\sqrt{3}-1\right)e^{x}-4\sqrt{3}+4e^{(2x)}+8\right)+\frac{1}{6}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}-1\right)e^{x}-2\sqrt{2}+e^{(2x)}+3}{e^{(2x)}-2e^{x}-1}\right)-\frac{2}{3}\arctan\left(-\left(\sqrt{3}+1\right)e^{x}+\frac{1}{2}\sqrt{-4\left(\sqrt{3}-1\right)e^{x}-4\sqrt{3}+4e^{(2x)}+8}\left(\sqrt{3}+1\right)+1\right)+\frac{2}{3}\arctan\left(-\left(\sqrt{3}-1\right)e^{x}+\sqrt{\left(\sqrt{3}+1\right)e^{x}+\sqrt{3}+e^{(2x)}+2}\left(\sqrt{3}-1\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*log(4*(sqrt(3) + 1)*e^x + 4*sqrt(3) + 4*e^(2*x) + 8) + 1/6*sqr
t(3)*log(-4*(sqrt(3) - 1)*e^x - 4*sqrt(3) + 4*e^(2*x) + 8) + 1/6*sqrt(2)*lo
g((2*(sqrt(2) - 1)*e^x - 2*sqrt(2) + e^(2*x) + 3)/(e^(2*x) - 2*e^x - 1)) -
2/3*arctan(-(sqrt(3) + 1)*e^x + 1/2*sqrt(-4*(sqrt(3) - 1)*e^x - 4*sqrt(3) +
 4*e^(2*x) + 8)*(sqrt(3) + 1) + 1) + 2/3*arctan(-(sqrt(3) - 1)*e^x + sqrt((
sqrt(3) + 1)*e^x + sqrt(3) + e^(2*x) + 2)*(sqrt(3) - 1) - 1)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 40.70, size = 5697, normalized size = 42.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**3),x)
```

```
[Out] 1993064208509905040030222651967612223670550485947450269720751264236*sqrt(1
+ sqrt(3)*I)*log(tanh(x/2) + 1 + sqrt(2))/(-13808353887544825059365004867017
9166982382068319622001807473872588 40 + 976398067090657147495138524480159906
16395345985506319298307157515 30*sqrt(2) - 845585530306531772145617860950579
39221797314463241809903104074138 22*sqrt(1 + sqrt(3)*I) - 19930642085099050
40030222651967612223670550485947450269720751264236*sqrt(6)*I*sqrt(1 + sqrt(
3)*I) + 2818618434355105907152059536501931307393243815441393663436802471274
*sqrt(3)*I*sqrt(1 + sqrt(3)*I) + 597919262552971512009066795590283667101165
1457842350809162253792708*sqrt(2)*sqrt(1 + sqrt(3)*I)) + 46976973905918431 7
858676589416988551232207302573565610572800411879*sqrt(6)*I*sqrt(1 + sqrt(3)
*I)*log(tanh(x/2) + 1 + sqrt(2))/(-13808353887544825059365004867017916698 23
82068319622001807473872588 40 + 97639806709065714749513852448015990616395345
98550631929830715751530*sqrt(2) - 8455855303065317721456178609505793922179 7
31446324180990310407413822*sqrt(1 + sqrt(3)*I) - 1993064208509905040030222 6
519676122236705504859474502697207512642 36*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 2
818618434355105907152059536501931307393243815441393663436802471274*sqrt(3)*
I*sqrt(1 + sqrt(3)*I) + 5979192625529715120090667955902836671011651457842 35
0809162253792708*sqrt(2)*sqrt(1 + sqrt(3)*I)) + 32546602236355238249837950 8
1600533020546511532850210643276905250510*log(tanh(x/2) + 1 + sqrt(2))/(-138
0835388754482505936500486701791669823820683196220018074738725884 0 + 9763980
67090657147495138524480159906163953459855063192983071575153 0*sqrt(2) - 8455
85530306531772145617860950579392217973144632418099031040741382 2*sqrt(1 + sq
rt(3)*I) - 19930642085099050400302226519676122236705504859474502697207512 64
236*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 281861843435510590715205953650193130739
3243815441393663436802471274*sqrt(3)*I*sqrt(1 + sqrt(3)*I) + 59791926255297
15120090667955902836671011651457842350809162253792708*sqrt(2)*sqrt(1 + sqrt
(3)*I)) - 23013923145908041765608341445029861163730344719937000301245645431
40*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))/(-1380835388754482505936500486701 79
16698238206831962200180747387258840 + 976398067090657147495138524480159906 1
6395345985506319298307157515 30*sqrt(2) - 845585530306531772145617860950579 3
92217973144632418099031040741382 2*sqrt(1 + sqrt(3)*I) - 1993064208509905040
0302226519676122236705504859474502697207512642 36*sqrt(6)*I*sqrt(1 + sqrt(3)
*I) + 2818618434355105907152059536501931307393243815441393663436802471274*s
qrt(3)*I*sqrt(1 + sqrt(3)*I) + 59791926255297151200906679559028366710116514
5784235080916225379270 8*sqrt(2)*sqrt(1 + sqrt(3)*I)) - 664354736169968346 67
674088398920407455685016198248342324025042141 2*sqrt(3)*I*sqrt(1 + sqrt(3)*I
)*log(tanh(x/2) + 1 + sqrt(2))/(-1380835388754482505936500486701791669823 82
068319622001807473872588 40 + 97639806709065714749513852448015990616395345 98
5506319298307157515 30*sqrt(2) - 8455855303065317721456178609505793922179731
4463241809903104074138 22*sqrt(1 + sqrt(3)*I) - 19930642085099050400302226 51
967612223670550485947450269720751264236*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 281
8618434355105907152059536501931307393243815441393663436802471274*sqrt(3)*I*
sqrt(1 + sqrt(3)*I) + 5979192625529715120090667955902836671011651457842350 8
09162253792708*sqrt(2)*sqrt(1 + sqrt(3)*I)) - 14093092171775529535760297682
509656536966219077206968317184012356 37*sqrt(2)*sqrt(1 + sqrt(3)*I)*log(tanh
```

```
(x/2) + 1 + sqrt(2))/(-1380835388754482505936500486701791669823820683196220
0180747387258840 + 97639806709065714749513852448015990616395345985506319298
30715751530*sqrt(2) - 84558553030653177214561786095057939221797314463241809
90310407413822*sqrt(1 + sqrt(3)*I) - 19930642085099050400302226519676122236
705504859474502697207512642 36*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 2818618434355
105907152059536501931307393243815441393663436802471274*sqrt(3)*I*sqrt(1 + s
qrt(3)*I) + 597919262552971512009066795590283667101165145784235080916225379
2708*sqrt(2)*sqrt(1 + sqrt(3)*I)) + 140930921717755295357602976825096565369
6621907720696831718401235637*sqrt(2)*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - sq
rt(2) + 1)/(-13808353887544825059365004867017916698238206831962200180747387
258840 + 976398067090657147495138524480159906163953459855063192983071575153
0*sqrt(2) - 845585530306531772145617860950579392217973144632418099031040741
3822*sqrt(1 + sqrt(3)*I) - 199306420850990504003022265196761222367055048594
7450269720751264236*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 281861843435510590715 20
59536501931307393243815441393663436802471274*sqrt(3)*I*sqrt(1 + sqrt(3)*I)
+ 5979192625529715120090667955902836671011651457842350809162253792708*sqrt(
2)*sqrt(1 + sqrt(3)*I)) + 6643547361699683466767408839892040745568501619824
83423240250421412*sqrt(3)*I*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - sqrt(2) + 1
)/(-13808353887544825059365004867017916698238206831962200180747387258840 +
9763980670906571474951385244801599061639534598550631929830715751530*sqrt(2)
 - 8455855303065317721456178609505793922179731446324180990310407413822*sqrt
(1 + sqrt(3)*I) - 199306420850990504003022265 19...
```

Giac [**A**]
time = 0.41, size = 106, normalized size = 0.80

$$-\frac{1}{6}\pi - \frac{1}{6}\sqrt{3}\log\left(\left(\sqrt{3}+e^{x}+1\right)^{2}+e^{(2x)}\right)+\frac{1}{6}\sqrt{3}\log\left(\left(\sqrt{3}-e^{x}-1\right)^{2}+e^{(2x)}\right)-\frac{1}{6}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2e^{x}-2\right|}{\left|2\sqrt{2}+2e^{x}-2\right|}\right)-\frac{1}{3}\arctan\left(-\left(\sqrt{3}+1\right)e^{x}+1\right)-\frac{1}{3}\arctan\left(\left(\sqrt{3}-1\right)e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^3),x, algorithm="giac")
```

```
[Out] -1/6*pi - 1/6*sqrt(3)*log((sqrt(3) + e^x + 1)^2 + e^(2*x)) + 1/6*sqrt(3)*lo
g((sqrt(3) - e^x - 1)^2 + e^(2*x)) - 1/6*sqrt(2)*log(abs(-2*sqrt(2) + 2*e^x
 - 2)/abs(2*sqrt(2) + 2*e^x - 2)) - 1/3*arctan(-(sqrt(3) + 1)*e^x + 1) - 1/
3*arctan((sqrt(3) - 1)*e^x + 1)
```

Mupad [**B**]
time = 2.07, size = 225, normalized size = 1.69

$$\frac{\mathrm{atan}\left(\frac{77824\,e^{x}+32768\,\sqrt{3}-45056\,\sqrt{3}\,e^{x}-57344}{77824\,e^{x}-45056\,\sqrt{3}\,e^{x}}\right)}{3}+\frac{\mathrm{atan}\left(\frac{77824\,e^{x}-32768\,\sqrt{3}+45056\,\sqrt{3}\,e^{x}-57344}{77824\,e^{x}+45056\,\sqrt{3}\,e^{x}}\right)}{3}+\frac{\pi\,\mathrm{sign}\left(77824\,e^{x}+32768\,\sqrt{3}-45056\,\sqrt{3}\,e^{x}-57344\right)}{3}-\frac{\sqrt{2}\,\ln\left(59392\,e^{x}-17408\,\sqrt{2}-41984\,\sqrt{2}\,e^{x}+24576\right)}{6}+\frac{\sqrt{2}\,\ln\left(59392\,e^{x}+17408\,\sqrt{2}+41984\,\sqrt{2}\,e^{x}+24576\right)}{6}+\frac{\sqrt{3}\,\ln\left(\left(77824\,e^{x}-32768\,\sqrt{3}+45056\,\sqrt{3}\,e^{x}-57344\right)^{2}+\left(77824\,e^{x}+45056\,\sqrt{3}\,e^{x}\right)^{2}\right)}{6}-\frac{\sqrt{3}\,\ln\left(\left(77824\,e^{x}+32768\,\sqrt{3}-45056\,\sqrt{3}\,e^{x}-57344\right)^{2}+\left(77824\,e^{x}-45056\,\sqrt{3}\,e^{x}\right)^{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^3 - 1),x)
```

```
[Out] atan((77824*exp(x) + 32768*3^(1/2) - 45056*3^(1/2)*exp(x) - 57344)/(77824*e
xp(x) - 45056*3^(1/2)*exp(x)))/3 + atan((77824*exp(x) - 32768*3^(1/2) + 450
```

```
56*3^(1/2)*exp(x) - 57344)/(77824*exp(x) + 45056*3^(1/2)*exp(x)))/3 + (pi*s
ign(77824*exp(x) + 32768*3^(1/2) - 45056*3^(1/2)*exp(x) - 57344))/3 - (2^(1
/2)*log(59392*exp(x) - 17408*2^(1/2) - 41984*2^(1/2)*exp(x) + 24576))/6 + (
2^(1/2)*log(59392*exp(x) + 17408*2^(1/2) + 41984*2^(1/2)*exp(x) + 24576))/6
 + (3^(1/2)*log((77824*exp(x) - 32768*3^(1/2) + 45056*3^(1/2)*exp(x) - 5734
4)^2 + (77824*exp(x) + 45056*3^(1/2)*exp(x))^2))/6 - (3^(1/2)*log((77824*ex
p(x) + 32768*3^(1/2) - 45056*3^(1/2)*exp(x) - 57344)^2 + (77824*exp(x) - 45
056*3^(1/2)*exp(x))^2))/6
```

3.184 $\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=111

$$\frac{1}{128}(48a+35b)x-\frac{(80a+93b)\cosh(c+dx)\sinh(c+dx)}{128d}+\frac{(48a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d}-25b\cosh$$

```
[Out] 1/128*(48*a+35*b)*x-1/128*(80*a+93*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/192*(48*a
+163*b)*cosh(d*x+c)^3*sinh(d*x+c)/d-25/48*b*cosh(d*x+c)^5*sinh(d*x+c)/d+1/8
*b*cosh(d*x+c)^7*sinh(d*x+c)/d
```

Rubi [A]

time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = $\{3296, 1271, 1828, 1171, 393, 212\}$

$$\frac{(48a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{(80a+93b)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x(48a+35b)+\frac{b\sinh(c+dx)\cosh^7(c+dx)}{8d}-\frac{25b\sinh(c+dx)\cosh^5(c+dx)}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] ((48*a + 35*b)*x)/128 - ((80*a + 93*b)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d)
 + ((48*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) - (25*b*Cosh[c +
d*x]^5*Sinh[c + d*x])/(48*d) + (b*Cosh[c + d*x]^7*Sinh[c + d*x])/(8*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a-2ax^2+(a+b)x^4\right)}{(1-x^2)^5}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^7(c+dx)\sinh(c+dx)}{8d} - \frac{\text{Subst}\left(\int \frac{b+8bx^2-8(a-b)x^4+8(a+b)x^6}{(1-x^2)^4}\right.}{8d}$$

$$= -\frac{25b\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b\cosh^7(c+dx)\sinh(c+dx)}{8d}$$

$$= \frac{(48a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d} - \frac{25b\cosh^5(c+dx)\sin}{48d}$$

$$= -\frac{(80a+93b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{(48a+163b)\cosh^3(c}{192}$$

$$= \frac{1}{128}(48a+35b)x - \frac{(80a+93b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{(48}{}$$

Mathematica [A]
time = 0.11, size = 82, normalized size = 0.74

$$\frac{1152ac+840bc+1152adx+840bdx-96(8a+7b)\sinh(2(c+dx))+24(4a+7b)\sinh(4(c+dx))-32b\sinh(6(c+dx))+3b\sinh(8(c+dx))}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (1152*a*c + 840*b*c + 1152*a*d*x + 840*b*d*x - 96*(8*a + 7*b)*Sinh[2*(c + d
*x)] + 24*(4*a + 7*b)*Sinh[4*(c + d*x)] - 32*b*Sinh[6*(c + d*x)] + 3*b*Sinh
[8*(c + d*x)])/(3072*d)
```

Maple [A]
time = 1.13, size = 82, normalized size = 0.74

method	result
default	$\frac{\left(-\frac{7b}{16}-\frac{a}{2}\right)\sinh(2dx+2c)}{2d}+\frac{\left(\frac{7b}{32}+\frac{a}{8}\right)\sinh(4dx+4c)}{4d}+\frac{3ax}{8}+\frac{35bx}{128}-\frac{b\sinh(6dx+6c)}{96d}+\frac{b\sinh(8dx+8c)}{1024d}$
risch	$\frac{35bx}{128}+\frac{3ax}{8}+\frac{b\,e^{8dx+8c}}{2048d}-\frac{b\,e^{6dx+6c}}{192d}+\frac{7\,e^{4dx+4c}b}{256d}+\frac{e^{4dx+4c}a}{64d}-\frac{7b\,e^{2dx+2c}}{64d}-\frac{e^{2dx+2c}a}{8d}+\frac{7\,e^{-2dx-2c}b}{64d}+\frac{e^{-2dx-2c}a}{8d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-7/16*b-1/2*a)*sinh(2*d*x+2*c)/d+1/4*(7/32*b+1/8*a)*sinh(4*d*x+4*c)/d+
3/8*a*x+35/128*b*x-1/96*b*sinh(6*d*x+6*c)/d+1/1024*b*sinh(8*d*x+8*c)/d
```

Maxima [A]
time = 0.27, size = 175, normalized size = 1.58

$$\frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{6144}\,b\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+672\,e^{(-6\,dx-6\,c)}-3\right)e^{(8\,dx+8\,c)}}{d}-\frac{1680\,(dx+c)}{d}-\frac{672\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+32\,e^{(-6\,dx-6\,c)}-3\,e^{(-8\,dx-8\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/6144*b*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x -
 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (6
72*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*
d*x - 8*c))/d)
```

Fricas [A]
time = 0.44, size = 174, normalized size = 1.57

$$\frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{7}+3\,\left(7\,b\cosh\left(dx+c\right)^{3}-8\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{5}+\left(21\,b\cosh\left(dx+c\right)^{5}-80\,b\cosh\left(dx+c\right)^{3}+12\,(4\,a+7\,b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+3\,(48\,a+35\,b)dx+3\,\left(b\cosh\left(dx+c\right)^{7}-8\,b\cosh\left(dx+c\right)^{5}+4\,(4\,a+7\,b)\cosh\left(dx+c\right)^{3}-8\,(8\,a+7\,b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/384*(3*b*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b*cosh(d*x + c)^3 - 8*b*cos
h(d*x + c))*sinh(d*x + c)^5 + (21*b*cosh(d*x + c)^5 - 80*b*cosh(d*x + c)^3
+ 12*(4*a + 7*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(48*a + 35*b)*d*x + 3*(
b*cosh(d*x + c)^7 - 8*b*cosh(d*x + c)^5 + 4*(4*a + 7*b)*cosh(d*x + c)^3 - 8
*(8*a + 7*b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(104) = 208$.
time = 1.22, size = 306, normalized size = 2.76

$$\begin{cases} \frac{3ax\sinh^{4}\left(c+dx\right)}{8}-\frac{3ax\sinh^{2}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{4}+\frac{3ax\cosh^{4}\left(c+dx\right)}{8}+\frac{5a\sinh^{3}\left(c+dx\right)\cosh\left(c+dx\right)}{8d}-\frac{3a\sinh\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{8d}+\frac{35bx\sinh^{8}\left(c+dx\right)}{128}-\frac{35bx\sinh^{6}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{32}+\frac{105bx\sinh^{4}\left(c+dx\right)\cosh^{4}\left(c+dx\right)}{64}-\frac{35bx\sinh^{2}\left(c+dx\right)\cosh^{6}\left(c+dx\right)}{32}+\frac{35bx\cosh^{8}\left(c+dx\right)}{128}+\frac{93b\sinh^{7}\left(c+dx\right)\cosh\left(c+dx\right)}{128d}-\frac{511b\sinh^{5}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{384d}+\frac{385b\sinh^{3}\left(c+dx\right)\cosh^{5}\left(c+dx\right)}{384d}-\frac{35b\sinh\left(c+dx\right)\cosh^{7}\left(c+dx\right)}{128d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^{4}\left(c\right)\right)\sinh^{4}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
- 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 35*b*x*sinh(c + d*x)**8/128 -
35*b*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b*x*sinh(c + d*x)**4*cosh
(c + d*x)**4/64 - 35*b*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b*x*cosh
(c + d*x)**8/128 + 93*b*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*b*sinh
(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b*sinh(c + d*x)**3*cosh(c + d*x
)**5/(384*d) - 35*b*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(
a + b*sinh(c)**4)*sinh(c)**4, True))
```

Giac [A]
time = 0.44, size = 155, normalized size = 1.40

$$\frac{1}{128}\left(48\,a+35\,b\right)x+\frac{be^{\left(8\,dx+8\,c\right)}}{2048\,d}-\frac{be^{\left(6\,dx+6\,c\right)}}{192\,d}+\frac{\left(4\,a+7\,b\right)e^{\left(4\,dx+4\,c\right)}}{256\,d}-\frac{\left(8\,a+7\,b\right)e^{\left(2\,dx+2\,c\right)}}{64\,d}+\frac{\left(8\,a+7\,b\right)e^{\left(-2\,dx-2\,c\right)}}{64\,d}-\frac{\left(4\,a+7\,b\right)e^{\left(-4\,dx-4\,c\right)}}{256\,d}+\frac{be^{\left(-6\,dx-6\,c\right)}}{192\,d}-\frac{be^{\left(-8\,dx-8\,c\right)}}{2048\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/128*(48*a + 35*b)*x + 1/2048*b*e^(8*d*x + 8*c)/d - 1/192*b*e^(6*d*x + 6*c
)/d + 1/256*(4*a + 7*b)*e^(4*d*x + 4*c)/d - 1/64*(8*a + 7*b)*e^(2*d*x + 2*c
)/d + 1/64*(8*a + 7*b)*e^(-2*d*x - 2*c)/d - 1/256*(4*a + 7*b)*e^(-4*d*x - 4
*c)/d + 1/192*b*e^(-6*d*x - 6*c)/d - 1/2048*b*e^(-8*d*x - 8*c)/d
```

Mupad [B]
time = 0.92, size = 88, normalized size = 0.79

$$\frac{12\,a\sinh\left(4\,c+4\,d\,x\right)-96\,a\sinh\left(2\,c+2\,d\,x\right)-84\,b\sinh\left(2\,c+2\,d\,x\right)+21\,b\sinh\left(4\,c+4\,d\,x\right)-4\,b\sinh\left(6\,c+6\,d\,x\right)+\frac{3\,b\sinh\left(8\,c+8\,d\,x\right)}{8}+144\,a\,d\,x+105\,b\,d\,x}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^4),x)
```

```
[Out] (12*a*sinh(4*c + 4*d*x) - 96*a*sinh(2*c + 2*d*x) - 84*b*sinh(2*c + 2*d*x) +
 21*b*sinh(4*c + 4*d*x) - 4*b*sinh(6*c + 6*d*x) + (3*b*sinh(8*c + 8*d*x))/8
 + 144*a*d*x + 105*b*d*x)/(384*d)
```

3.185 $\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=67

$$-\frac{(a+b)\cosh(c+dx)}{d}+\frac{(a+3b)\cosh^3(c+dx)}{3d}-\frac{3b\cosh^5(c+dx)}{5d}+\frac{b\cosh^7(c+dx)}{7d}$$

```
[Out] -(a+b)*cosh(d*x+c)/d+1/3*(a+3*b)*cosh(d*x+c)^3/d-3/5*b*cosh(d*x+c)^5/d+1/7*
b*cosh(d*x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3294, 1167}

$$\frac{(a+3b)\cosh^3(c+dx)}{3d}-\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3b\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -(((a + b)*Cosh[c + d*x])/d) + ((a + 3*b)*Cosh[c + d*x]^3)/(3*d) - (3*b*Cos
h[c + d*x]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d)
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+b-2bx^2+bx^4\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(a\left(1+\frac{b}{a}\right)-(a+3b)x^2+3bx^4-bx^6\right)\,dx,x,\cosh(c\right.}{d}\\ &= -\frac{(a+b)\cosh(c+dx)}{d}+\frac{(a+3b)\cosh^3(c+dx)}{3d}-\frac{3b\cosh^5(c+}{5d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 93, normalized size = 1.39

$$-\frac{3a\cosh(c+dx)}{4d}-\frac{35b\cosh(c+dx)}{64d}+\frac{a\cosh(3(c+dx))}{12d}+\frac{7b\cosh(3(c+dx))}{64d}-\frac{7b\cosh(5(c+dx))}{320d}+\frac{b\cosh(7(c+dx))}{448d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) - (35*b*Cosh[c + d*x])/(64*d) + (a*Cosh[3*(c + d
*x)])/(12*d) + (7*b*Cosh[3*(c + d*x)])/(64*d) - (7*b*Cosh[5*(c + d*x)])/(32
0*d) + (b*Cosh[7*(c + d*x)])/(448*d)
```

Maple [A]
time = 0.70, size = 70, normalized size = 1.04

method	result
default	$\frac{\left(-\frac{35b}{64}-\frac{3a}{4}\right)\cosh(dx+c)}{d}+\frac{\left(\frac{21b}{64}+\frac{a}{4}\right)\cosh(3dx+3c)}{3d}-\frac{7b\cosh(5dx+5c)}{320d}+\frac{b\cosh(7dx+7c)}{448d}$
risch	$\frac{b\,\mathrm{e}^{7dx+7c}}{896d}-\frac{7b\,\mathrm{e}^{5dx+5c}}{640d}+\frac{7b\,\mathrm{e}^{3dx+3c}}{128d}+\frac{\mathrm{e}^{3dx+3c}a}{24d}-\frac{3a\,\mathrm{e}^{dx+c}}{8d}-\frac{35b\,\mathrm{e}^{dx+c}}{128d}-\frac{3\,\mathrm{e}^{-dx-c}a}{8d}-\frac{35\,\mathrm{e}^{-dx-c}b}{128d}+\frac{7\,\mathrm{e}^{-3dx-3c}b}{128d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] (-35/64*b-3/4*a)/d*cosh(d*x+c)+1/3*(21/64*b+1/4*a)/d*cosh(3*d*x+3*c)-7/320*
b/d*cosh(5*d*x+5*c)+1/448*b/d*cosh(7*d*x+7*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(61) = 122$.
time = 0.27, size = 157, normalized size = 2.34

$$-\frac{1}{4480}\,b\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d}\right)+\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/4480*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x - 6
*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) + 49
*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/24*a*(e^(3*d*x + 3*c)/d - 9*
e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(61) = 122$.
time = 0.41, size = 155, normalized size = 2.31

$$\frac{15\,b\cosh\left(dx+c\right)^{7}+105\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}-147\,b\cosh\left(dx+c\right)^{5}+105\,\left(5\,b\cosh\left(dx+c\right)^{3}-7\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}+35\,(16\,a+21\,b)\cosh\left(dx+c\right)^{3}+105\,\left(3\,b\cosh\left(dx+c\right)^{5}-14\,b\cosh\left(dx+c\right)^{3}+(16\,a+21\,b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}-105\,(48\,a+35\,b)\cosh\left(dx+c\right)}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/6720*(15*b*cosh(d*x + c)^7 + 105*b*cosh(d*x + c)*sinh(d*x + c)^6 - 147*b*
cosh(d*x + c)^5 + 105*(5*b*cosh(d*x + c)^3 - 7*b*cosh(d*x + c))*sinh(d*x +
c)^4 + 35*(16*a + 21*b)*cosh(d*x + c)^3 + 105*(3*b*cosh(d*x + c)^5 - 14*b*c
osh(d*x + c)^3 + (16*a + 21*b)*cosh(d*x + c))*sinh(d*x + c)^2 - 105*(48*a +
 35*b)*cosh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(56) = 112$.
time = 0.63, size = 128, normalized size = 1.91

$$\begin{cases} \frac{a\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a\cosh^3(c+dx)}{3d} + \frac{b\sinh^6(c+dx)\cosh(c+dx)}{d} - \frac{2b\sinh^4(c+dx)\cosh^3(c+dx)}{d} + \frac{8b\sinh^2(c+dx)\cosh^5(c+dx)}{5d} - \frac{16b\cosh^7(c+dx)}{35d} & \text{for } d \neq 0 \\ x\left(a + b\sinh^4(c)\right)\sinh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d)
+ b*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b*sinh(c + d*x)**4*cosh(c + d*x)**
3/d + 8*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b*cosh(c + d*x)**7/(
35*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*sinh(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(61) = 122$.
time = 0.43, size = 142, normalized size = 2.12

$$\frac{be^{(7\,dx+7\,c)}}{896\,d} - \frac{7\,be^{(5\,dx+5\,c)}}{640\,d} + \frac{(16\,a+21\,b)e^{(3\,dx+3\,c)}}{384\,d} - \frac{(48\,a+35\,b)e^{(dx+c)}}{128\,d} - \frac{(48\,a+35\,b)e^{(-dx-c)}}{128\,d} + \frac{(16\,a+21\,b)e^{(-3\,dx-3\,c)}}{384\,d} - \frac{7\,be^{(-5\,dx-5\,c)}}{640\,d} + \frac{be^{(-7\,dx-7\,c)}}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/896*b*e^(7*d*x + 7*c)/d - 7/640*b*e^(5*d*x + 5*c)/d + 1/384*(16*a + 21*b)
*e^(3*d*x + 3*c)/d - 1/128*(48*a + 35*b)*e^(d*x + c)/d - 1/128*(48*a + 35*b
)*e^(-d*x - c)/d + 1/384*(16*a + 21*b)*e^(-3*d*x - 3*c)/d - 7/640*b*e^(-5*d
*x - 5*c)/d + 1/896*b*e^(-7*d*x - 7*c)/d
```

Mupad [B]
time = 0.79, size = 66, normalized size = 0.99

$$-\frac{a\cosh(c+d\,x) + b\cosh(c+d\,x) - \frac{a\cosh(c+d\,x)^3}{3} - b\cosh(c+d\,x)^3 + \frac{3\,b\cosh(c+d\,x)^5}{5} - \frac{b\cosh(c+d\,x)^7}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^4),x)
```

```
[Out] -(a*cosh(c + d*x) + b*cosh(c + d*x) - (a*cosh(c + d*x)^3)/3 - b*cosh(c + d*
x)^3 + (3*b*cosh(c + d*x)^5)/5 - (b*cosh(c + d*x)^7)/7)/d
```

3.186 $\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=83

$$-\frac{1}{16}(8a+5b)x+\frac{(8a+11b)\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{13b\cosh^3(c+dx)\sinh(c+dx)}{24d}+\frac{b\cosh^5(c+dx)\sinh(}{6d}$$

```
[Out] -1/16*(8*a+5*b)*x+1/16*(8*a+11*b)*cosh(d*x+c)*sinh(d*x+c)/d-13/24*b*cosh(d*
x+c)^3*sinh(d*x+c)/d+1/6*b*cosh(d*x+c)^5*sinh(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3296, 1271, 1171, 393, 212}

$$\frac{(8a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}x(8a+5b)+\frac{b\sinh(c+dx)\cosh^5(c+dx)}{6d}-\frac{13b\sinh(c+dx)\cosh^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/16*((8*a + 5*b)*x) + ((8*a + 11*b)*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) -
 (13*b*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d) + (b*Cosh[c + d*x]^5*Sinh[c +
d*x])/(6*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{x^2\left(a-2ax^2+(a+b)x^4\right)}{\left(1-x^2\right)^4}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{b\cosh^5(c+dx)\sinh(c+dx)}{6d}+\frac{\text{Subst}\left(\int \frac{-b+6(a-b)x^2-6(a+b)x^4}{\left(1-x^2\right)^3}\,dx}{6d}\\
&= -\frac{13b\cosh^3(c+dx)\sinh(c+dx)}{24d}+\frac{b\cosh^5(c+dx)\sinh(c+dx}{6d}\\
&= \frac{(8a+11b)\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{13b\cosh^3(c+dx)\sinh(}{24d}\\
&= -\frac{1}{16}(8a+5b)x+\frac{(8a+11b)\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{13b}{}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.76

$$\frac{-96ac-60bc-96adx-60bdx+(48a+45b)\sinh(2(c+dx))-9b\sinh(4(c+dx))+b\sinh(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-96*a*c - 60*b*c - 96*a*d*x - 60*b*d*x + (48*a + 45*b)*Sinh[2*(c + d*x)] -
 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 1.03, size = 61, normalized size = 0.73

method	result
default	$\frac{\left(\frac{15b}{32}+\frac{a}{2}\right)\sinh(2dx+2c)}{2d}-\frac{ax}{2}-\frac{5bx}{16}-\frac{3b\sinh(4dx+4c)}{64d}+\frac{b\sinh(6dx+6c)}{192d}$
risch	$-\frac{5bx}{16}-\frac{ax}{2}+\frac{b\,\mathrm{e}^{6dx+6c}}{384d}-\frac{3\,\mathrm{e}^{4dx+4c}b}{128d}+\frac{15b\,\mathrm{e}^{2dx+2c}}{128d}+\frac{\mathrm{e}^{2dx+2c}a}{8d}-\frac{15\,\mathrm{e}^{-2dx-2c}b}{128d}-\frac{\mathrm{e}^{-2dx-2c}a}{8d}+\frac{3\,\mathrm{e}^{-4dx-4c}b}{128d}-\frac{b\,\mathrm{e}^{-6d}}{38}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(15/32*b+1/2*a)*sinh(2*d*x+2*c)/d-1/2*a*x-5/16*b*x-3/64*b*sinh(4*d*x+4*
c)/d+1/192*b*sinh(6*d*x+6*c)/d
```

Maxima [A]
time = 0.27, size = 122, normalized size = 1.47

$$-\frac{1}{8}\,a\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{384}\,b\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b*((9*e^(-2*d
*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d +
(45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Fricas [A]
time = 0.42, size = 109, normalized size = 1.31

$$\frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+2\left(5\,b\cosh\left(dx+c\right)^{3}-9\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}-6\,(8\,a+5\,b)dx+3\left(b\cosh\left(dx+c\right)^{5}-6\,b\cosh\left(dx+c\right)^{3}+(16\,a+15\,b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 - 9*b*cosh
(d*x + c))*sinh(d*x + c)^3 - 6*(8*a + 5*b)*d*x + 3*(b*cosh(d*x + c)^5 - 6*b
*cosh(d*x + c)^3 + (16*a + 15*b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(76) = 152$.
time = 0.46, size = 206, normalized size = 2.48

$$\begin{cases}\frac{ax\sinh^{2}(c+dx)}{2}-\frac{ax\cosh^{2}(c+dx)}{2}+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{5bx\sinh^{6}(c+dx)}{16}-\frac{15bx\sinh^{4}(c+dx)\cosh^{2}(c+dx)}{16}+\frac{15bx\sinh^{2}(c+dx)\cosh^{4}(c+dx)}{16}-\frac{5bx\cosh^{6}(c+dx)}{16}+\frac{11b\sinh^{5}(c+dx)\cosh(c+dx)}{16d}-\frac{5b\sinh^{3}(c+dx)\cosh^{3}(c+dx)}{6d}+\frac{5b\sinh(c+dx)\cosh^{5}(c+dx)}{16d} & \text{for}\ d\neq 0\\ x\left(a+b\sinh^{4}(c)\right)\sinh^{2}(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x
)*cosh(c + d*x)/(2*d) + 5*b*x*sinh(c + d*x)**6/16 - 15*b*x*sinh(c + d*x)**4
*cosh(c + d*x)**2/16 + 15*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b*x*
cosh(c + d*x)**6/16 + 11*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b*sinh
(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*sinh(c + d*x)*cosh(c + d*x)**5/(1
6*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*sinh(c)**2, True))
```

Giac [A]
time = 0.42, size = 113, normalized size = 1.36

$$-\frac{1}{16}\,(8\,a+5\,b)x+\frac{be^{(6\,dx+6\,c)}}{384\,d}-\frac{3\,be^{(4\,dx+4\,c)}}{128\,d}+\frac{(16\,a+15\,b)e^{(2\,dx+2\,c)}}{128\,d}-\frac{(16\,a+15\,b)e^{(-2\,dx-2\,c)}}{128\,d}+\frac{3\,be^{(-4\,dx-4\,c)}}{128\,d}-\frac{be^{(-6\,dx-6\,c)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/16*(8*a + 5*b)*x + 1/384*b*e^(6*d*x + 6*c)/d - 3/128*b*e^(4*d*x + 4*c)/d
 + 1/128*(16*a + 15*b)*e^(2*d*x + 2*c)/d - 1/128*(16*a + 15*b)*e^(-2*d*x -
2*c)/d + 3/128*b*e^(-4*d*x - 4*c)/d - 1/384*b*e^(-6*d*x - 6*c)/d
```

Mupad [B]
time = 0.16, size = 64, normalized size = 0.77

$$\frac{12\,a\,\sinh(2\,c+2\,d\,x)+\frac{45\,b\,\sinh(2\,c+2\,d\,x)}{4}-\frac{9\,b\,\sinh(4\,c+4\,d\,x)}{4}+\frac{b\,\sinh(6\,c+6\,d\,x)}{4}-24\,a\,d\,x-15\,b\,d\,x}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^4),x)
```

```
[Out] (12*a*sinh(2*c + 2*d*x) + (45*b*sinh(2*c + 2*d*x))/4 - (9*b*sinh(4*c + 4*d*
x))/4 + (b*sinh(6*c + 6*d*x))/4 - 24*a*d*x - 15*b*d*x)/(48*d)
```

3.187 $\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=46

$$\frac{(a+b)\cosh(c+dx)}{d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}$$

```
[Out] (a+b)*cosh(d*x+c)/d-2/3*b*cosh(d*x+c)^3/d+1/5*b*cosh(d*x+c)^5/d
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = $\{3294\}$

$$\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\cosh^5(c+dx)}{5d}-\frac{2b\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] ((a + b)*Cosh[c + d*x])/d - (2*b*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c + d*x]^
5)/(5*d)
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx&=\frac{\operatorname{Subst}\left(\int \left(a+b-2bx^2+bx^4\right)\,dx,x,\cosh(c+dx)\right)}{d}\\&=\frac{(a+b)\cosh(c+dx)}{d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 69, normalized size = 1.50

$$\frac{a\cosh(c)\cosh(dx)}{d}+\frac{5b\cosh(c+dx)}{8d}-\frac{5b\cosh(3(c+dx))}{48d}+\frac{b\cosh(5(c+dx))}{80d}+\frac{a\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d + (5*b*Cosh[c + d*x])/(8*d) - (5*b*Cosh[3*(c + d*x)
])/(48*d) + (b*Cosh[5*(c + d*x)])/(80*d) + (a*Sinh[c]*Sinh[d*x])/d
```

Maple [A]
time = 0.71, size = 47, normalized size = 1.02

method	result	size
default	$\frac{\left(\frac{5b}{8}+a\right)\cosh(dx+c)}{d}-\frac{5b\cosh(3dx+3c)}{48d}+\frac{b\cosh(5dx+5c)}{80d}$	47
risch	$\frac{b\,\mathrm{e}^{5dx+5c}}{160d}-\frac{5b\,\mathrm{e}^{3dx+3c}}{96d}+\frac{a\,\mathrm{e}^{dx+c}}{2d}+\frac{5b\,\mathrm{e}^{dx+c}}{16d}+\frac{\mathrm{e}^{-dx-c}a}{2d}+\frac{5\,\mathrm{e}^{-dx-c}b}{16d}-\frac{5\,\mathrm{e}^{-3dx-3c}b}{96d}+\frac{b\,\mathrm{e}^{-5dx-5c}}{160d}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] (5/8*b+a)/d*cosh(d*x+c)-5/48*b/d*cosh(3*d*x+3*c)+1/80*b/d*cosh(5*d*x+5*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(42) = 84$.
time = 0.26, size = 97, normalized size = 2.11

$$\frac{1}{480}\,b\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/480*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 1
50*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a*cosh(
d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(42) = 84$.
time = 0.57, size = 91, normalized size = 1.98

$$\frac{3\,b\cosh\left(dx+c\right)^{5}+15\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}-25\,b\cosh\left(dx+c\right)^{3}+15\,\left(2\,b\cosh\left(dx+c\right)^{3}-5\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+30\,\left(8\,a+5\,b\right)\cosh\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 - 25*b*cosh
(d*x + c)^3 + 15*(2*b*cosh(d*x + c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c)^2
+ 30*(8*a + 5*b)*cosh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(39) = 78$.
time = 0.28, size = 80, normalized size = 1.74

$$\begin{cases} \frac{a\cosh\left(c+dx\right)}{d} + \frac{b\sinh^{4}\left(c+dx\right)\cosh\left(c+dx\right)}{d} - \frac{4b\sinh^{2}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{3d} + \frac{8b\cosh^{5}\left(c+dx\right)}{15d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^{4}\left(c\right)\right)\sinh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Piecewise((a*cosh(c + d*x)/d + b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sin
h(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0
)), (x*(a + b*sinh(c)**4)*sinh(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(42) = 84$.
time = 0.42, size = 100, normalized size = 2.17

$$\frac{be^{(5\,dx+5\,c)}}{160\,d} - \frac{5\,be^{(3\,dx+3\,c)}}{96\,d} + \frac{(8\,a+5\,b)e^{(dx+c)}}{16\,d} + \frac{(8\,a+5\,b)e^{(-dx-c)}}{16\,d} - \frac{5\,be^{(-3\,dx-3\,c)}}{96\,d} + \frac{be^{(-5\,dx-5\,c)}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/160*b*e^(5*d*x + 5*c)/d - 5/96*b*e^(3*d*x + 3*c)/d + 1/16*(8*a + 5*b)*e^(
d*x + c)/d + 1/16*(8*a + 5*b)*e^(-d*x - c)/d - 5/96*b*e^(-3*d*x - 3*c)/d +
1/160*b*e^(-5*d*x - 5*c)/d
```

Mupad [B]
time = 0.10, size = 46, normalized size = 1.00

$$\frac{15\,a\cosh(c+d\,x)+15\,b\cosh(c+d\,x)-10\,b\cosh(c+d\,x)^3+3\,b\cosh(c+d\,x)^5}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^4),x)
```

```
[Out] (15*a*cosh(c + d*x) + 15*b*cosh(c + d*x) - 10*b*cosh(c + d*x)^3 + 3*b*cosh(
c + d*x)^5)/(15*d)
```

3.188 $\int \left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=52

$$ax+\frac{3bx}{8}-\frac{3b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}$$

```
[Out] a*x+3/8*b*x-3/8*b*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b*cosh(d*x+c)*sinh(d*x+c)^3
/d
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {2715, 8}

$$ax+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sinh[c + d*x]^4,x]
```

```
[Out] a*x + (3*b*x)/8 - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c + d*x
]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^4(c+dx)\right)\,dx &= ax+b\int \sinh^4(c+dx)\,dx\\ &= ax+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{4}(3b)\int \sinh^2(c+dx)\,dx\\ &= ax-\frac{3b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}+\frac{1}{8}(3b)\int\\ &= ax+\frac{3bx}{8}-\frac{3b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.94

$$ax+\frac{3b(c+dx)}{8d}-\frac{b\sinh(2(c+dx))}{4d}+\frac{b\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sinh[c + d*x]^4,x]
```

```
[Out] a*x + (3*b*(c + d*x))/(8*d) - (b*Sinh[2*(c + d*x)])/(4*d) + (b*Sinh[4*(c +
d*x)])/(32*d)
```

Maple [A]
time = 1.04, size = 39, normalized size = 0.75

method	result	size
default	$ax+b\left(\frac{3x}{8}-\frac{\sinh(2dx+2c)}{4d}+\frac{\sinh(4dx+4c)}{32d}\right)$	39
risch	$ax+\frac{3bx}{8}+\frac{e^{4dx+4c}b}{64d}-\frac{b\,e^{2dx+2c}}{8d}+\frac{e^{-2dx-2c}b}{8d}-\frac{e^{-4dx-4c}b}{64d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sinh(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*(3/8*x-1/4*sinh(2*d*x+2*c)/d+1/32*sinh(4*d*x+4*c)/d)
```

Maxima [A]
time = 0.26, size = 66, normalized size = 1.27

$$\frac{1}{64}\,b\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + a*x
```

Fricas [A]
time = 0.38, size = 59, normalized size = 1.13

$$\frac{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\left(8\,a+3\,b\right)dx+\left(b\cosh\left(dx+c\right)^{3}-4\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + (8*a + 3*b)*d*x + (b*cosh(d*x + c)^3
 - 4*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A]
time = 0.17, size = 100, normalized size = 1.92

$$ax + b\left(\begin{cases} \frac{3x\sinh^4(c+dx)}{8} - \frac{3x\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3x\cosh^4(c+dx)}{8} + \frac{5\sinh^3(c+dx)\cosh(c+dx)}{8d} - \frac{3\sinh(c+dx)\cosh^3(c+dx)}{8d} & \text{for}\ d \neq 0 \\ x\sinh^4(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)**4,x)
```

```
[Out] a*x + b*Piecewise((3*x*sinh(c + d*x)**4/8 - 3*x*sinh(c + d*x)**2*cosh(c + d
*x)**2/4 + 3*x*cosh(c + d*x)**4/8 + 5*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
- 3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*sinh(c)**4, True))
```

Giac [A]
time = 0.42, size = 66, normalized size = 1.27

$$\frac{1}{64}\,b\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + a*x
```

Mupad [B]
time = 0.70, size = 38, normalized size = 0.73

$$a\,x + \frac{3\,b\,x}{8} - \frac{\frac{b\sinh(2\,c+2\,d\,x)}{4} - \frac{b\sinh(4\,c+4\,d\,x)}{32}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sinh(c + d*x)^4,x)
```

```
[Out] a*x + (3*b*x)/8 - ((b*sinh(2*c + 2*d*x))/4 - (b*sinh(4*c + 4*d*x))/32)/d
```

3.189 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=42

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}$$

```
[Out] -a*arctanh(cosh(d*x+c))/d-b*cosh(d*x+c)/d+1/3*b*cosh(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3294, 1167, 212}

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh^3(c+dx)}{3d}-\frac{b\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -((a*ArcTanh[Cosh[c + d*x]])/d) - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)
/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{1-x^2}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\operatorname{Subst}\left(\int \left(b-bx^2+\frac{a}{1-x^2}\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}-\frac{a\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cosh}{d}\\ &= -\frac{a\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 1.67

$$-\frac{3b\cosh(c+dx)}{4d}+\frac{b\cosh(3(c+dx))}{12d}-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])/(12*d) - (a*Log[Cosh[c/2
 + (d*x)/2]])/d + (a*Log[Sinh[c/2 + (d*x)/2]])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(40) = 80$.
time = 1.10, size = 88, normalized size = 2.10

method	result	size
risch	$\frac{b\,\mathrm{e}^{3dx+3c}}{24d}-\frac{3b\,\mathrm{e}^{dx+c}}{8d}-\frac{3\,\mathrm{e}^{-dx-c}b}{8d}+\frac{\mathrm{e}^{-3dx-3c}b}{24d}+\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*b/d*exp(3*d*x+3*c)-3/8*b/d*exp(d*x+c)-3/8/d*exp(-d*x-c)*b+1/24/d*exp(-
3*d*x-3*c)*b+a/d*ln(exp(d*x+c)-1)-a/d*ln(exp(d*x+c)+1)
```

Maxima [A]
time = 0.26, size = 71, normalized size = 1.69

$$\frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
- 3*c)/d) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(40) = 80$.
time = 0.40, size = 395, normalized size = 9.40

$$\frac{b\cosh\left(dx+c\right)^{6}+6\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+b\sinh\left(dx+c\right)^{6}-9\,b\cosh\left(dx+c\right)^{4}+3\,\left(5\,b\cosh\left(dx+c\right)^{2}-3\,b\right)\sinh\left(dx+c\right)^{4}+4\,\left(5\,b\cosh\left(dx+c\right)^{3}-9\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}-9\,b\cosh\left(dx+c\right)^{2}+3\,\left(5\,b\cosh\left(dx+c\right)^{4}-18\,b\cosh\left(dx+c\right)^{2}-3\,b\right)\sinh\left(dx+c\right)^{2}-24\,\left(a\cosh\left(dx+c\right)^{3}+3\,a\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)+3\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+a\sinh\left(dx+c\right)^{3}\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)+24\,\left(a\cosh\left(dx+c\right)^{3}+3\,a\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)+3\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+a\sinh\left(dx+c\right)^{3}\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)+6\,\left(b\cosh\left(dx+c\right)^{5}-6\,b\cosh\left(dx+c\right)^{3}-3\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+b}{24\,\left(d\cosh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+d\sinh\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/24*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x +
c)^6 - 9*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - 3*b)*sinh(d*x + c)^4
+ 4*(5*b*cosh(d*x + c)^3 - 9*b*cosh(d*x + c))*sinh(d*x + c)^3 - 9*b*cosh(d*
x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 18*b*cosh(d*x + c)^2 - 3*b)*sinh(d*x +
c)^2 - 24*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*cosh
(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*log(cosh(d*x + c) + sinh(d*x
 + c) + 1) + 24*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*
a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*log(cosh(d*x + c) + si
nh(d*x + c) - 1) + 6*(b*cosh(d*x + c)^5 - 6*b*cosh(d*x + c)^3 - 3*b*cosh(d*
x + c))*sinh(d*x + c) + b)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*
x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^{4}\left(c + dx\right)\right)\operatorname{csch}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**4)*csch(c + d*x), x)
```

Giac [A]
time = 0.43, size = 78, normalized size = 1.86

$$\frac{be^{\left(3\,dx+3\,c\right)}-9\,be^{\left(dx+c\right)}-\left(9\,be^{\left(2\,dx+2\,c\right)}-b\right)e^{\left(-3\,dx-3\,c\right)}-24\,a\log\left(e^{\left(dx+c\right)}+1\right)+24\,a\log\left({\left|e^{\left(dx+c\right)}-1\right|}\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/24*(b*e^(3*d*x + 3*c) - 9*b*e^(d*x + c) - (9*b*e^(2*d*x + 2*c) - b)*e^(-3
*d*x - 3*c) - 24*a*log(e^(d*x + c) + 1) + 24*a*log(abs(e^(d*x + c) - 1)))/d
```

Mupad [B]
time = 0.13, size = 96, normalized size = 2.29

$$\frac{b\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{3\,b\,\mathrm{e}^{-c-d\,x}}{8\,d}+\frac{b\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{3\,b\,\mathrm{e}^{c+d\,x}}{8\,d}-\frac{2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x),x)
```

```
[Out] (b*exp(- 3*c - 3*d*x))/(24*d) - (3*b*exp(- c - d*x))/(8*d) + (b*exp(3*c + 3
*d*x))/(24*d) - (3*b*exp(c + d*x))/(8*d) - (2*atan((a*exp(d*x)*exp(c)*(-d^2
)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-d^2)^(1/2)
```

3.190 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=39

$$-\frac{bx}{2}-\frac{a\coth(c+dx)}{d}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}$$

```
[Out] -1/2*b*x-a*coth(d*x+c)/d+1/2*b*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3296, 1273, 464, 212}

$$-\frac{a\coth(c+dx)}{d}+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/2*(b*x) - (a*Coth[c + d*x])/d + (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a-2ax^2+(a+b)x^4}{x^2(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{b\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{-2a+(2a+b)x^2}{x^2(1-x^2)}\,dx, x, \tanh\right.}{2d}\\
&= -\frac{a\coth(c+dx)}{d} + \frac{b\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{b\text{Subst}\left(\int \frac{1}{1-}\right.}{}\\
&= -\frac{bx}{2} - \frac{a\coth(c+dx)}{d} + \frac{b\cosh(c+dx)\sinh(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 1.15

$$\frac{b(-c-dx)}{2d} - \frac{a\coth(c+dx)}{d} + \frac{b\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (b*(-c - d*x))/(2*d) - (a*Coth[c + d*x])/d + (b*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 1.09, size = 55, normalized size = 1.41

method	result	size
risch	$-\frac{bx}{2}+\frac{b\,{\rm e}^{2dx+2c}}{8d}-\frac{{\rm e}^{-2dx-2c}b}{8d}-\frac{2a}{d\left({\rm e}^{2dx+2c}-1\right)}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*x+1/8*b/d*exp(2*d*x+2*c)-1/8/d*exp(-2*d*x-2*c)*b-2*a/d/(exp(2*d*x+2*
c)-1)
```

Maxima [**A**]
time = 0.27, size = 54, normalized size = 1.38

$$-\frac{1}{8}\,b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+\frac{2\,a}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a/(d*(e^(-2*d*x -
 2*c) - 1))
```

Fricas [**A**]
time = 0.39, size = 70, normalized size = 1.79

$$\frac{b\cosh\left(dx+c\right)^{3}+3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}-\left(8\,a+b\right)\cosh\left(dx+c\right)-4\left(bdx-2\,a\right)\sinh\left(dx+c\right)}{8\,d\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 - (8*a + b)*cosh
(d*x + c) - 4*(b*d*x - 2*a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(35) = 70$.
time = 0.43, size = 88, normalized size = 2.26

$$-\frac{4\left(dx+c\right)b-be^{(2\,dx+2\,c)}-\frac{be^{(4\,dx+4\,c)}-16\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b}{e^{(4\,dx+4\,c)}-e^{(2\,dx+2\,c)}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/8*(4*(d*x + c)*b - b*e^(2*d*x + 2*c) - (b*e^(4*d*x + 4*c) - 16*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)/(e^(4*d*x + 4*c) - e^(2*d*x + 2*c)))/d
```

Mupad [B]
time = 0.72, size = 54, normalized size = 1.38

$$\frac{b\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}-\frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{b\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}-\frac{b\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^2,x)
```

```
[Out] (b*exp(2*c + 2*d*x))/(8*d) - (2*a)/(d*(exp(2*c + 2*d*x) - 1)) - (b*exp(- 2*
c - 2*d*x))/(8*d) - (b*x)/2
```

3.191 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=47

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b\cosh(c+dx)}{d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

```
[Out] 1/2*a*arctanh(cosh(d*x+c))/d+b*cosh(d*x+c)/d-1/2*a*coth(d*x+c)*csch(d*x+c)/
d
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3294, 1171, 396, 212}

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (a*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*Cosh[c + d*x])/d - (a*Coth[c + d*x]*C
sch[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)dx &= \frac{\text{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{(1-x^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= -\frac{a\coth(c+dx)\text{csch}(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{-a-2b+2bx^2}{1-x^2}\,dx, x, \cosh\right.}{2d}\\ &= \frac{b\cosh(c+dx)}{d} - \frac{a\coth(c+dx)\text{csch}(c+dx)}{2d} + \frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{}\\ &= \frac{a\tanh^{-1}(\cosh(c+dx))}{2d} + \frac{b\cosh(c+dx)}{d} - \frac{a\coth(c+dx)\text{csc}}{2d}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 82, normalized size = 1.74

$$\frac{b\cosh(c)\cosh(dx)}{d} - \frac{a\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (b*Cosh[c]*Cosh[d*x])/d - (a*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tanh[(c +
d*x)/2]])/(2*d) - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sinh[c]*Sinh[d*x])/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(43) = 86$.
time = 1.25, size = 95, normalized size = 2.02

method	result	size
risch	$\frac{b\,\mathrm{e}^{dx+c}}{2d} + \frac{\mathrm{e}^{-dx-c}b}{2d} - \frac{a\,\mathrm{e}^{dx+c}\left(1+\mathrm{e}^{2dx+2c}\right)}{d\left(\mathrm{e}^{2dx+2c}-1\right)^2} + \frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{2d} - \frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{2d}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/d*exp(d*x+c)+1/2/d*exp(-d*x-c)*b-a*exp(d*x+c)*(1+exp(2*d*x+2*c))/d/(e
xp(2*d*x+2*c)-1)^2+1/2*a/d*ln(exp(d*x+c)+1)-1/2*a/d*ln(exp(d*x+c)-1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(43) = 86$.
time = 0.27, size = 115, normalized size = 2.45

$$\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(log(e^(-d*x - c) + 1)/d - l
og(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*
x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(43) = 86$.
time = 0.41, size = 690, normalized size = 14.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c
)^6 - (2*a + b)*cosh(d*x + c)^4 + (15*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x
 + c)^4 + 4*(5*b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c)^3
 - (2*a + b)*cosh(d*x + c)^2 + (15*b*cosh(d*x + c)^4 - 6*(2*a + b)*cosh(d*x
 + c)^2 - 2*a - b)*sinh(d*x + c)^2 + (a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)
*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + 2*(5*a*cosh(d*
x + c)^2 - a)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))
*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x +
c)^2 + a)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d
*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 - 2*a*cos
h(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*
x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh
(d*x + c)^4 - 6*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*log(cosh(d*x + c) + s
inh(d*x + c) - 1) + 2*(3*b*cosh(d*x + c)^5 - 2*(2*a + b)*cosh(d*x + c)^3 -
(2*a + b)*cosh(d*x + c))*sinh(d*x + c) + b)/(d*cosh(d*x + c)^5 + 5*d*cosh(d
*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 + 2*(5*d*
cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*
x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 - 6*d*cosh
(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(43) = 86.
time = 0.44, size = 107, normalized size = 2.28

$$\frac{2\,b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+a\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-a\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*(e^(d*x + c) + e^(-d*x - c)) + a*log(e^(d*x + c) + e^(-d*x - c) +
2) - a*log(e^(d*x + c) + e^(-d*x - c) - 2) - 4*a*(e^(d*x + c) + e^(-d*x - c
))/((e^(d*x + c) + e^(-d*x - c))^2 - 4))/d
```

Mupad [**B**]
time = 0.72, size = 126, normalized size = 2.68

$$\frac{b\,e^{-c-d\,x}}{2\,d}+\frac{b\,e^{c+d\,x}}{2\,d}+\frac{\mathrm{atan}\left(\frac{a\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}-\frac{a\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^3,x)
```

```
[Out] (b*exp(- c - d*x))/(2*d) + (b*exp(c + d*x))/(2*d) + (atan((a*exp(d*x)*exp(c
)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-d^2)^(1/2) - (a*exp(c + d*x
))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2
*exp(2*c + 2*d*x) + 1))
```

3.192 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$bx+\frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}$$

```
[Out] b*x+a*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3296, 1275, 213}

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] b*x + (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a-2ax^2+(a+b)x^4}{x^4(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{x^4}-\frac{a}{x^2}-\frac{b}{-1+x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \mathrm{tan}\right.}{d} \\
&= bx+\frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.29

$$bx+\frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] b*x + (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d)
```

Maple [A]
time = 1.39, size = 37, normalized size = 1.19

method	result	size
risch	$bx-\frac{4a\left(3\,\mathrm{e}^{2dx+2c}-1\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] b*x-4/3*a*(3*exp(2*d*x+2*c)-1)/d/(exp(2*d*x+2*c)-1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(29) = 58.
time = 0.26, size = 97, normalized size = 3.13

$$bx+\frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] b*x + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) +
 e^(-6*d*x - 6*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(29) = 58$.

time = 0.42, size = 129, normalized size = 4.16

$$\frac{2\,a\cosh\left(dx+c\right)^{3}+6\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+\left(3\,bdx-2\,a\right)\sinh\left(dx+c\right)^{3}-6\,a\cosh\left(dx+c\right)-3\left(3\,bdx-\left(3\,bdx-2\,a\right)\cosh\left(dx+c\right)^{2}-2\,a\right)\sinh\left(dx+c\right)}{3\left(d\sinh\left(dx+c\right)^{3}+3\left(d\cosh\left(dx+c\right)^{2}-d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + (3*b*d*x - 2
*a)*sinh(d*x + c)^3 - 6*a*cosh(d*x + c) - 3*(3*b*d*x - (3*b*d*x - 2*a)*cosh
(d*x + c)^2 - 2*a)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2
 - d)*sinh(d*x + c))
```

Sympy [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**A**]

time = 0.45, size = 45, normalized size = 1.45

$$\frac{3\left(dx+c\right)b-\frac{4\left(3\,ae^{\left(2\,dx+2\,c\right)}-a\right)}{\left(e^{\left(2\,dx+2\,c\right)}-1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*b - 4*(3*a*e^(2*d*x + 2*c) - a)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [**B**]

time = 0.70, size = 81, normalized size = 2.61

$$\frac{4\,a-12\,a\,\mathrm{e}^{2\,c+2\,d\,x}-3\,b\,d\,x+9\,b\,d\,x\,\mathrm{e}^{2\,c+2\,d\,x}-9\,b\,d\,x\,\mathrm{e}^{4\,c+4\,d\,x}+3\,b\,d\,x\,\mathrm{e}^{6\,c+6\,d\,x}}{3\,d\,{\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^4,x)
```

```
[Out] (4*a - 12*a*exp(2*c + 2*d*x) - 3*b*d*x + 9*b*d*x*exp(2*c + 2*d*x) - 9*b*d*x
*exp(4*c + 4*d*x) + 3*b*d*x*exp(6*c + 6*d*x))/(3*d*(exp(2*c + 2*d*x) - 1)^3
)
```

3.193 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=64

$$-\frac{(3a+8b)\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{3a\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{a\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}$$

```
[Out] -1/8*(3*a+8*b)*arctanh(cosh(d*x+c))/d+3/8*a*coth(d*x+c)*csch(d*x+c)/d-1/4*a
*coth(d*x+c)*csch(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3294, 1171, 393, 212}

$$-\frac{(3a+8b)\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a\coth(c+dx)\operatorname{csch}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/8*((3*a + 8*b)*ArcTanh[Cosh[c + d*x]])/d + (3*a*Coth[c + d*x]*Csch[c + d
*x])/(8*d) - (a*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{(1-x^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{-3a-4b+4bx^2}{(1-x^2)^2}\,dx, x, \cosh\right.}{4d}$$

$$= \frac{3a\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} -$$

$$= -\frac{(3a+8b)\tanh^{-1}(\cosh(c+dx))}{8d} + \frac{3a\coth(c+dx)\operatorname{csch}(c+dx}{8d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 139 vs. 2(64) = 128.
time = 0.03, size = 139, normalized size = 2.17

$$\frac{3a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{b\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{b\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{3a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{3a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (3*a*Csch[(c + d*x)/2]^2)/(32*d) - (a*Csch[(c + d*x)/2]^4)/(64*d) - (b*Log[
Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d + (3*a*Log[Tanh[(c
 + d*x)/2]])/(8*d) + (3*a*Sech[(c + d*x)/2]^2)/(32*d) + (a*Sech[(c + d*x)/2
]^4)/(64*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(58) = 116.
time = 1.28, size = 121, normalized size = 1.89

method	result	size
risch	$\frac{a\,e^{dx+c}\left(3\,e^{6dx+6c}-11\,e^{4dx+4c}-11\,e^{2dx+2c}+3\right)}{4d\left(e^{2dx+2c}-1\right)^4}-\frac{3a\ln\left(e^{dx+c}+1\right)}{8d}-\frac{\ln\left(e^{dx+c}+1\right)b}{d}+\frac{3a\ln\left(e^{dx+c}-1\right)}{8d}+\frac{\ln\left(e^{dx+c}-1\right)b}{d}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*exp(d*x+c)*(3*exp(6*d*x+6*c)-11*exp(4*d*x+4*c)-11*exp(2*d*x+2*c)+3)/d
/(exp(2*d*x+2*c)-1)^4-3/8*a/d*ln(exp(d*x+c)+1)-1/d*ln(exp(d*x+c)+1)*b+3/8*a
/d*ln(exp(d*x+c)-1)+1/d*ln(exp(d*x+c)-1)*b
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(58) = 116$.
time = 0.28, size = 174, normalized size = 2.72

$$-\frac{1}{8}\,a\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3\,e^{(-dx-c)}-11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}+3\,e^{(-7\,dx-7\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}\right)-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*
x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d
*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x
- 8*c) - 1))) - b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1476 vs. $2(58) = 116$.
time = 0.47, size = 1476, normalized size = 23.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(6*a*cosh(d*x + c)^7 + 42*a*cosh(d*x + c)*sinh(d*x + c)^6 + 6*a*sinh(d*
x + c)^7 - 22*a*cosh(d*x + c)^5 + 2*(63*a*cosh(d*x + c)^2 - 11*a)*sinh(d*x
+ c)^5 + 10*(21*a*cosh(d*x + c)^3 - 11*a*cosh(d*x + c))*sinh(d*x + c)^4 - 2
2*a*cosh(d*x + c)^3 + 2*(105*a*cosh(d*x + c)^4 - 110*a*cosh(d*x + c)^2 - 11
*a)*sinh(d*x + c)^3 + 2*(63*a*cosh(d*x + c)^5 - 110*a*cosh(d*x + c)^3 - 33*
a*cosh(d*x + c))*sinh(d*x + c)^2 + 6*a*cosh(d*x + c) - ((3*a + 8*b)*cosh(d*
x + c)^8 + 8*(3*a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + 8*b)*sinh(d
*x + c)^8 - 4*(3*a + 8*b)*cosh(d*x + c)^6 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^
2 - 3*a - 8*b)*sinh(d*x + c)^6 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^3 - 3*(3*a
+ 8*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a + 8*b)*cosh(d*x + c)^4 + 2*(
35*(3*a + 8*b)*cosh(d*x + c)^4 - 30*(3*a + 8*b)*cosh(d*x + c)^2 + 9*a + 24*
b)*sinh(d*x + c)^4 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^5 - 10*(3*a + 8*b)*cosh
(d*x + c)^3 + 3*(3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a + 8*b)*
```

```
cosh(d*x + c)^2 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^6 - 15*(3*a + 8*b)*cosh(d*
x + c)^4 + 9*(3*a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*sinh(d*x + c)^2 + 8*(
(3*a + 8*b)*cosh(d*x + c)^7 - 3*(3*a + 8*b)*cosh(d*x + c)^5 + 3*(3*a + 8*b)
*cosh(d*x + c)^3 - (3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) + 3*a + 8*b)*lo
g(cosh(d*x + c) + sinh(d*x + c) + 1) + ((3*a + 8*b)*cosh(d*x + c)^8 + 8*(3*
a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + 8*b)*sinh(d*x + c)^8 - 4*(3
*a + 8*b)*cosh(d*x + c)^6 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*s
inh(d*x + c)^6 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^3 - 3*(3*a + 8*b)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 6*(3*a + 8*b)*cosh(d*x + c)^4 + 2*(35*(3*a + 8*b)*c
osh(d*x + c)^4 - 30*(3*a + 8*b)*cosh(d*x + c)^2 + 9*a + 24*b)*sinh(d*x + c)
^4 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^5 - 10*(3*a + 8*b)*cosh(d*x + c)^3 + 3*
(3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a + 8*b)*cosh(d*x + c)^2
+ 4*(7*(3*a + 8*b)*cosh(d*x + c)^6 - 15*(3*a + 8*b)*cosh(d*x + c)^4 + 9*(3*
a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*sinh(d*x + c)^2 + 8*((3*a + 8*b)*cosh
(d*x + c)^7 - 3*(3*a + 8*b)*cosh(d*x + c)^5 + 3*(3*a + 8*b)*cosh(d*x + c)^3
 - (3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) + 3*a + 8*b)*log(cosh(d*x + c)
+ sinh(d*x + c) - 1) + 2*(21*a*cosh(d*x + c)^6 - 55*a*cosh(d*x + c)^4 - 33*
a*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x +
 c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh
(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x +
c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*
cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh
(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*
(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh
(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c
)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(58) = 116.
time = 0.43, size = 124, normalized size = 1.94

$$-\frac{(3\,a+8\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(3\,a+8\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(3\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{3}-20\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{2}-4\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/16*((3*a + 8*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (3*a + 8*b)*log(e^
(d*x + c) + e^(-d*x - c) - 2) - 4*(3*a*(e^(d*x + c) + e^(-d*x - c))^3 - 20*
a*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^2)/d
```

Mupad [B]
time = 0.74, size = 242, normalized size = 3.78

$$\frac{3\,a\,\mathrm{e}^{c+d\,x}}{4\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(3\,a\,\sqrt{-d^2}+8\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{9\,a^2+48\,a\,b+64\,b^2}}\right)\,\sqrt{9\,a^2+48\,a\,b+64\,b^2}}{4\,\sqrt{-d^2}}-\frac{a\,\mathrm{e}^{c+d\,x}}{2\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{6\,a\,\mathrm{e}^{c+d\,x}}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{4\,a\,\mathrm{e}^{c+d\,x}}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^5,x)
```

```
[Out] (3*a*exp(c + d*x))/(4*d*(exp(2*c + 2*d*x) - 1)) - (atan((exp(d*x)*exp(c)*(3
*a*(-d^2)^(1/2) + 8*b*(-d^2)^(1/2)))/(d*(48*a*b + 9*a^2 + 64*b^2)^(1/2)))*(
48*a*b + 9*a^2 + 64*b^2)^(1/2))/(4*(-d^2)^(1/2)) - (a*exp(c + d*x))/(2*d*(e
xp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (6*a*exp(c + d*x))/(d*(3*exp(2
*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*a*exp(c + d*
x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(
8*c + 8*d*x) + 1))
```

3.194 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=47

$$-\frac{(a+b)\coth(c+dx)}{d}+\frac{2a\coth^3(c+dx)}{3d}-\frac{a\coth^5(c+dx)}{5d}$$

```
[Out] -(a+b)*coth(d*x+c)/d+2/3*a*coth(d*x+c)^3/d-1/5*a*coth(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{3296, 14\}$

$$-\frac{(a+b)\coth(c+dx)}{d}-\frac{a\coth^5(c+dx)}{5d}+\frac{2a\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -(((a + b)*Coth[c + d*x])/d) + (2*a*Coth[c + d*x]^3)/(3*d) - (a*Coth[c + d*
x]^5)/(5*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a-2ax^2+(a+b)x^4}{x^6}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{x^6}-\frac{2a}{x^4}+\frac{a+b}{x^2}\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= -\frac{(a+b)\coth(c+dx)}{d}+\frac{2a\coth^3(c+dx)}{3d}-\frac{a\coth^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 1.51

$$-\frac{8a\coth(c+dx)}{15d}-\frac{b\coth(c+dx)}{d}+\frac{4a\coth(c+dx)\operatorname{csch}^2(c+dx)}{15d}-\frac{a\coth(c+dx)\operatorname{csch}^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-8*a*Coth[c + d*x])/(15*d) - (b*Coth[c + d*x])/d + (4*a*Coth[c + d*x]*Csch
[c + d*x]^2)/(15*d) - (a*Coth[c + d*x]*Csch[c + d*x]^4)/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(43) = 86$.
time = 1.20, size = 98, normalized size = 2.09

method	result	size
risch	$-\frac{2\left(15b\,\mathrm{e}^{8dx+8c}-60b\,\mathrm{e}^{6dx+6c}+80a\,\mathrm{e}^{4dx+4c}+90b\,\mathrm{e}^{4dx+4c}-40a\,\mathrm{e}^{2dx+2c}-60b\,\mathrm{e}^{2dx+2c}+8a+15b\right)}{15d\left(\mathrm{e}^{2dx+2c}-1\right)^5}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(15*b*exp(8*d*x+8*c)-60*b*exp(6*d*x+6*c)+80*a*exp(4*d*x+4*c)+90*b*exp
(4*d*x+4*c)-40*a*exp(2*d*x+2*c)-60*b*exp(2*d*x+2*c)+8*a+15*b)/d/(exp(2*d*x+
2*c)-1)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(43) = 86$.
time = 0.27, size = 228, normalized size = 4.85

$$-\frac{16}{15}\,a\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1)}-\frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1)}-\frac{1}{d(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1)}\right)+\frac{2\,b}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -16/15*a*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e
^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x
 - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(43) = 86$.
time = 0.59, size = 333, normalized size = 7.09

$$-\frac{4\left(\left(4\,a+15\,b\right)\cosh\left(dx+c\right)^4-16\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\left(4\,a+15\,b\right)\sinh\left(dx+c\right)^4-20\left(a+3\,b\right)\cosh\left(dx+c\right)^2+2\left(3\left(4\,a+15\,b\right)\cosh\left(dx+c\right)^2-10\,a-30\,b\right)\sinh\left(dx+c\right)^2-8\left(2\,a\cosh\left(dx+c\right)^3-5\,a\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+40\,a+45\,b\right)}{15\left(d\cosh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+d\sinh\left(dx+c\right)^6-6\,d\cosh\left(dx+c\right)^4+3\left(5\,d\cosh\left(dx+c\right)^2-2\,d\right)\sinh\left(dx+c\right)^4+4\left(5\,d\cosh\left(dx+c\right)^3-4\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3+15\,d\cosh\left(dx+c\right)^2+3\left(5\,d\cosh\left(dx+c\right)^4-12\,d\cosh\left(dx+c\right)^2+5\,d\right)\sinh\left(dx+c\right)^2+2\left(3\,d\cosh\left(dx+c\right)^5-8\,d\cosh\left(dx+c\right)^3+5\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)-10\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -4/15*((4*a + 15*b)*cosh(d*x + c)^4 - 16*a*cosh(d*x + c)*sinh(d*x + c)^3 +
(4*a + 15*b)*sinh(d*x + c)^4 - 20*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(4*a + 1
5*b)*cosh(d*x + c)^2 - 10*a - 30*b)*sinh(d*x + c)^2 - 8*(2*a*cosh(d*x + c)^
3 - 5*a*cosh(d*x + c))*sinh(d*x + c) + 40*a + 45*b)/(d*cosh(d*x + c)^6 + 6*
d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 6*d*cosh(d*x + c)^4 +
 3*(5*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 4
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x
+ c)^4 - 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)
^5 - 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c) - 10*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(43) = 86.
time = 0.43, size = 97, normalized size = 2.06

$$-\frac{2\left(15\,be^{(8\,dx+8\,c)}-60\,be^{(6\,dx+6\,c)}+80\,ae^{(4\,dx+4\,c)}+90\,be^{(4\,dx+4\,c)}-40\,ae^{(2\,dx+2\,c)}-60\,be^{(2\,dx+2\,c)}+8\,a+15\,b\right)}{15\,d\left(e^{(2\,dx+2\,c)}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -2/15*(15*b*e^(8*d*x + 8*c) - 60*b*e^(6*d*x + 6*c) + 80*a*e^(4*d*x + 4*c) +
 90*b*e^(4*d*x + 4*c) - 40*a*e^(2*d*x + 2*c) - 60*b*e^(2*d*x + 2*c) + 8*a +
 15*b)/(d*(e^(2*d*x + 2*c) - 1)^5)
```

Mupad [**B**]
time = 0.72, size = 337, normalized size = 7.17

$$\frac{\frac{2\,b}{5\,d}+\frac{6\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{5\,d}-\frac{2\,b\,\mathrm{e}^{6\,c+6\,d\,x}}{5\,d}-\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(8\,a+3\,b)}{5\,d}}{6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1}-\frac{\frac{2\,(8\,a+3\,b)}{15\,d}-\frac{4\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{5\,d}+\frac{2\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{5\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}-\frac{\frac{2\,b}{5\,d}-\frac{8\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{5\,d}-\frac{8\,b\,\mathrm{e}^{6\,c+6\,d\,x}}{5\,d}+\frac{2\,b\,\mathrm{e}^{8\,c+8\,d\,x}}{5\,d}+\frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,(8\,a+3\,b)}{5\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1}-\frac{4\,b}{5\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^6,x)
```

```
[Out] ((2*b)/(5*d) + (6*b*exp(4*c + 4*d*x))/(5*d) - (2*b*exp(6*c + 6*d*x))/(5*d)
- (2*exp(2*c + 2*d*x)*(8*a + 3*b))/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c +
```

```
 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(8*a + 3*b))/(15
*d) - (4*b*exp(2*c + 2*d*x))/(5*d) + (2*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2
*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((2*b)/(5*d) - (
8*b*exp(2*c + 2*d*x))/(5*d) - (8*b*exp(6*c + 6*d*x))/(5*d) + (2*b*exp(8*c +
 8*d*x))/(5*d) + (4*exp(4*c + 4*d*x)*(8*a + 3*b))/(5*d))/(5*exp(2*c + 2*d*x
) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10
*c + 10*d*x) - 1) - (4*b)/(5*d*(exp(2*c + 2*d*x) - 1))
```

3.195 $\int \textbf{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=92

$$\frac{(5a+8b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{(5a+8b)\coth(c+dx)\mathrm{csch}(c+dx)}{16d}+\frac{5a\coth(c+dx)\mathrm{csch}^3(c+dx)}{24d}-a\coth$$

```
[Out] 1/16*(5*a+8*b)*arctanh(cosh(d*x+c))/d-1/16*(5*a+8*b)*coth(d*x+c)*csch(d*x+c
)/d+5/24*a*coth(d*x+c)*csch(d*x+c)^3/d-1/6*a*coth(d*x+c)*csch(d*x+c)^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3294, 1171, 393, 205, 212}

$$\frac{(5a+8b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{(5a+8b)\coth(c+dx)\mathrm{csch}(c+dx)}{16d}-\frac{a\coth(c+dx)\mathrm{csch}^5(c+dx)}{6d}+\frac{5a\coth(c+dx)\mathrm{csch}^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] ((5*a + 8*b)*ArcTanh[Cosh[c + d*x]])/(16*d) - ((5*a + 8*b)*Coth[c + d*x]*Cs
ch[c + d*x])/(16*d) + (5*a*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a*Coth[
c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \text{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{(1-x^2)^4}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\coth(c+dx)\text{csch}^5(c+dx)}{6d} - \frac{\text{Subst}\Big(\int \frac{-5a-6b+6bx^2}{(1-x^2)^3}\,dx, x, \text{co}}{6d}$$

$$= \frac{5a\coth(c+dx)\text{csch}^3(c+dx)}{24d} - \frac{a\coth(c+dx)\text{csch}^5(c+dx)}{6d} +$$

$$= -\frac{(5a+8b)\coth(c+dx)\text{csch}(c+dx)}{16d} + \frac{5a\coth(c+dx)\text{csch}^3(}{24d}$$

$$= \frac{(5a+8b)\tanh^{-1}(\cosh(c+dx))}{16d} - \frac{(5a+8b)\coth(c+dx)\text{csch}(}{16d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 199 vs. 2(92) = 184.
time = 0.03, size = 199, normalized size = 2.16

$$-\frac{5a\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{b\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\text{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a\text{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{5a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{5a\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{b\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\text{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a\text{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-5*a*Csch[(c + d*x)/2]^2)/(64*d) - (b*Csch[(c + d*x)/2]^2)/(8*d) + (a*Csch
[(c + d*x)/2]^4)/(64*d) - (a*Csch[(c + d*x)/2]^6)/(384*d) - (5*a*Log[Tanh[(
c + d*x)/2]])/(16*d) - (b*Log[Tanh[(c + d*x)/2]])/(2*d) - (5*a*Sech[(c + d*
x)/2]^2)/(64*d) - (b*Sech[(c + d*x)/2]^2)/(8*d) - (a*Sech[(c + d*x)/2]^4)/(
64*d) - (a*Sech[(c + d*x)/2]^6)/(384*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(84) = 168$.
time = 1.29, size = 213, normalized size = 2.32

method	result
risch	$-\frac{\mathrm{e}^{dx+c}(15a\,\mathrm{e}^{10dx+10c}+24b\,\mathrm{e}^{10dx+10c}-85a\,\mathrm{e}^{8dx+8c}-72b\,\mathrm{e}^{8dx+8c}+198a\,\mathrm{e}^{6dx+6c}+48b\,\mathrm{e}^{6dx+6c}+198a\,\mathrm{e}^{4dx+4c}+48b\,\mathrm{e}^{4dx+4c}-85a\,\mathrm{e}^{2dx}}{24d\left(\mathrm{e}^{2dx+2c}-1\right)^{6}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*exp(d*x+c)*(15*a*exp(10*d*x+10*c)+24*b*exp(10*d*x+10*c)-85*a*exp(8*d*
x+8*c)-72*b*exp(8*d*x+8*c)+198*a*exp(6*d*x+6*c)+48*b*exp(6*d*x+6*c)+198*a*e
xp(4*d*x+4*c)+48*b*exp(4*d*x+4*c)-85*a*exp(2*d*x+2*c)-72*b*exp(2*d*x+2*c)+1
5*a+24*b)/d/(exp(2*d*x+2*c)-1)^6-5/16*a/d*ln(exp(d*x+c)-1)-1/2/d*ln(exp(d*x
+c)-1)*b+5/16*a/d*ln(exp(d*x+c)+1)+1/2/d*ln(exp(d*x+c)+1)*b
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(84) = 168$.
time = 0.29, size = 268, normalized size = 2.91

$$\frac{1}{48}\,a\left(\frac{15\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15\,e^{(-dx-c)}-85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}+198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9\,dx-9\,c)}+15\,e^{(-11\,dx-11\,c)}\right)}{d(6\,e^{(-2\,dx-2\,c)}-15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}-15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}-e^{(-12\,dx-12\,c)}-1)}\right)+\frac{1}{2}\,b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/48*a*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(
-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*
c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) -
15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*
d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 1/2*b*(log(e^(-d*x - c) + 1)/d -
log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3115 vs. $2(84) = 168$.
time = 0.42, size = 3115, normalized size = 33.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/48*(6*(5*a + 8*b)*cosh(d*x + c)^11 + 66*(5*a + 8*b)*cosh(d*x + c)*sinh(d
*x + c)^10 + 6*(5*a + 8*b)*sinh(d*x + c)^11 - 2*(85*a + 72*b)*cosh(d*x + c)
^9 + 2*(165*(5*a + 8*b)*cosh(d*x + c)^2 - 85*a - 72*b)*sinh(d*x + c)^9 + 18
*(55*(5*a + 8*b)*cosh(d*x + c)^3 - (85*a + 72*b)*cosh(d*x + c))*sinh(d*x +
c)^8 + 12*(33*a + 8*b)*cosh(d*x + c)^7 + 12*(165*(5*a + 8*b)*cosh(d*x + c)^
4 - 6*(85*a + 72*b)*cosh(d*x + c)^2 + 33*a + 8*b)*sinh(d*x + c)^7 + 84*(33*
(5*a + 8*b)*cosh(d*x + c)^5 - 2*(85*a + 72*b)*cosh(d*x + c)^3 + (33*a + 8*b
)*cosh(d*x + c))*sinh(d*x + c)^6 + 12*(33*a + 8*b)*cosh(d*x + c)^5 + 12*(23
1*(5*a + 8*b)*cosh(d*x + c)^6 - 21*(85*a + 72*b)*cosh(d*x + c)^4 + 21*(33*a
 + 8*b)*cosh(d*x + c)^2 + 33*a + 8*b)*sinh(d*x + c)^5 + 12*(165*(5*a + 8*b)
*cosh(d*x + c)^7 - 21*(85*a + 72*b)*cosh(d*x + c)^5 + 35*(33*a + 8*b)*cosh(
d*x + c)^3 + 5*(33*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(85*a + 72*b
)*cosh(d*x + c)^3 + 2*(495*(5*a + 8*b)*cosh(d*x + c)^8 - 84*(85*a + 72*b)*c
osh(d*x + c)^6 + 210*(33*a + 8*b)*cosh(d*x + c)^4 + 60*(33*a + 8*b)*cosh(d*
x + c)^2 - 85*a - 72*b)*sinh(d*x + c)^3 + 6*(55*(5*a + 8*b)*cosh(d*x + c)^9
 - 12*(85*a + 72*b)*cosh(d*x + c)^7 + 42*(33*a + 8*b)*cosh(d*x + c)^5 + 20*
(33*a + 8*b)*cosh(d*x + c)^3 - (85*a + 72*b)*cosh(d*x + c))*sinh(d*x + c)^2
 + 6*(5*a + 8*b)*cosh(d*x + c) - 3*((5*a + 8*b)*cosh(d*x + c)^12 + 12*(5*a
+ 8*b)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a + 8*b)*sinh(d*x + c)^12 - 6*(5
*a + 8*b)*cosh(d*x + c)^10 + 6*(11*(5*a + 8*b)*cosh(d*x + c)^2 - 5*a - 8*b)
*sinh(d*x + c)^10 + 20*(11*(5*a + 8*b)*cosh(d*x + c)^3 - 3*(5*a + 8*b)*cosh
(d*x + c))*sinh(d*x + c)^9 + 15*(5*a + 8*b)*cosh(d*x + c)^8 + 15*(33*(5*a +
 8*b)*cosh(d*x + c)^4 - 18*(5*a + 8*b)*cosh(d*x + c)^2 + 5*a + 8*b)*sinh(d*
x + c)^8 + 24*(33*(5*a + 8*b)*cosh(d*x + c)^5 - 30*(5*a + 8*b)*cosh(d*x + c
)^3 + 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(5*a + 8*b)*cosh(d*
x + c)^6 + 4*(231*(5*a + 8*b)*cosh(d*x + c)^6 - 315*(5*a + 8*b)*cosh(d*x +
c)^4 + 105*(5*a + 8*b)*cosh(d*x + c)^2 - 25*a - 40*b)*sinh(d*x + c)^6 + 24*
(33*(5*a + 8*b)*cosh(d*x + c)^7 - 63*(5*a + 8*b)*cosh(d*x + c)^5 + 35*(5*a
+ 8*b)*cosh(d*x + c)^3 - 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*
(5*a + 8*b)*cosh(d*x + c)^4 + 15*(33*(5*a + 8*b)*cosh(d*x + c)^8 - 84*(5*a
+ 8*b)*cosh(d*x + c)^6 + 70*(5*a + 8*b)*cosh(d*x + c)^4 - 20*(5*a + 8*b)*co
sh(d*x + c)^2 + 5*a + 8*b)*sinh(d*x + c)^4 + 20*(11*(5*a + 8*b)*cosh(d*x +
c)^9 - 36*(5*a + 8*b)*cosh(d*x + c)^7 + 42*(5*a + 8*b)*cosh(d*x + c)^5 - 20
*(5*a + 8*b)*cosh(d*x + c)^3 + 3*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^3
 - 6*(5*a + 8*b)*cosh(d*x + c)^2 + 6*(11*(5*a + 8*b)*cosh(d*x + c)^10 - 45*
(5*a + 8*b)*cosh(d*x + c)^8 + 70*(5*a + 8*b)*cosh(d*x + c)^6 - 50*(5*a + 8*
b)*cosh(d*x + c)^4 + 15*(5*a + 8*b)*cosh(d*x + c)^2 - 5*a - 8*b)*sinh(d*x +
 c)^2 + 12*((5*a + 8*b)*cosh(d*x + c)^11 - 5*(5*a + 8*b)*cosh(d*x + c)^9 +
10*(5*a + 8*b)*cosh(d*x + c)^7 - 10*(5*a + 8*b)*cosh(d*x + c)^5 + 5*(5*a +
8*b)*cosh(d*x + c)^3 - (5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) + 5*a + 8*b
)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*((5*a + 8*b)*cosh(d*x + c)^12
+ 12*(5*a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a + 8*b)*sinh(d*x + c)
^12 - 6*(5*a + 8*b)*cosh(d*x + c)^10 + 6*(11*(5*a + 8*b)*cosh(d*x + c)^2 -
```

```
5*a - 8*b)*sinh(d*x + c)^10 + 20*(11*(5*a + 8*b)*cosh(d*x + c)^3 - 3*(5*a +
 8*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(5*a + 8*b)*cosh(d*x + c)^8 + 15*
(33*(5*a + 8*b)*cosh(d*x + c)^4 - 18*(5*a + 8*b)*cosh(d*x + c)^2 + 5*a + 8*
b)*sinh(d*x + c)^8 + 24*(33*(5*a + 8*b)*cosh(d*x + c)^5 - 30*(5*a + 8*b)*co
sh(d*x + c)^3 + 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(5*a + 8*
b)*cosh(d*x + c)^6 + 4*(231*(5*a + 8*b)*cosh(d*x + c)^6 - 315*(5*a + 8*b)*c
osh(d*x + c)^4 + 105*(5*a + 8*b)*cosh(d*x + c)^2 - 25*a - 40*b)*sinh(d*x +
c)^6 + 24*(33*(5*a + 8*b)*cosh(d*x + c)^7 - 63*(5*a + 8*b)*cosh(d*x + c)^5
+ 35*(5*a + 8*b)*cosh(d*x + c)^3 - 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x +
c)^5 + 15*(5*a + 8*b)*cosh(d*x + c)^4 + 15*(33*(5*a + 8*b)*cosh(d*x + c)^8
- 84*(5*a + 8*b)*cosh(d*x + c)^6 + 70*(5*a + 8*b)*cosh(d*x + c)^4 - 20*(5*a
 + 8*b)*cosh(d*x + c)^2 + 5*a + 8*b)*sinh(d*x + c)^4 + 20*(11*(5*a + 8*b)*c
osh(d*x + c)^9 - 36*(5*a + 8*b)*cosh(d*x + c)^7 + 42*(5*a + 8*b)*cosh(d*x +
 c)^5 - 20*(5*a + 8*b)*cosh(d*x + c)^3 + 3*(5*a + 8*b)*cosh(d*x + c))*sinh(
d*x + c)^3 - 6*(5*a + 8*b)*cosh(d*x + c)^2 + 6*(11*(5*a + 8*b)*cosh(d*x + c
)^10 - 45*(5*a + 8*b)*cosh(d*x + c)^8 + 70*(5*a + 8*b)*cosh(d*x + c)^6 - 50
*(5*a + 8*b)*cosh(d*x + c)^4 + 15*(5*a + 8*b)*cosh(d*x + c)^2 - 5*a - 8*b)*
sinh(d*x + c)^2 + 12*((5*a + 8*b)*cosh(d*x + c)^11 - 5*(5*a + 8*b)*cosh(d*x
 + c)^9 + 10*(5*a + 8*b)*cosh(d*x + c)^7 - 10*(5*a + 8*b)*cosh(d*x + c)^5 +
 5*(5*a + 8*b)*cosh(d*x + c)^3 - (5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) +
 5*a + 8*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(11*(5*a + 8*b)*cosh
(d*x + c)^10 - 3*(85*a + 72*b)*cosh(d*x + c)^8 + 14*(33*a + 8*b)*cosh(d*x +
 c)^6 + 10*(33*a + 8*b)*cosh(d*x + c)^4 - (85*a + 72*b)*cosh(d*x + c)^2 + 5
*a + 8*b)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(84) = 168.
time = 0.45, size = 207, normalized size = 2.25

$$\frac{3\,(5\,a+8\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\,(5\,a+8\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\,\left(15\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+24\,b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5-160\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-192\,b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+528\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)+384\,b\left(e^{(dx+c)}+e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\right)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/96*(3*(5*a + 8*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*(5*a + 8*b)*log
(e^(d*x + c) + e^(-d*x - c) - 2) - 4*(15*a*(e^(d*x + c) + e^(-d*x - c))^5 +
```

```
 24*b*(e^(d*x + c) + e^(-d*x - c))^5 - 160*a*(e^(d*x + c) + e^(-d*x - c))^3
 - 192*b*(e^(d*x + c) + e^(-d*x - c))^3 + 528*a*(e^(d*x + c) + e^(-d*x - c)
) + 384*b*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4
)^3)/d
```

Mupad [B]

time = 0.77, size = 472, normalized size = 5.13

$$\frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(5\,a\,\sqrt{-d^2}+8\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{25\,a^2+80\,a\,b+64\,b^2}}\right)\,\sqrt{25\,a^2+80\,a\,b+64\,b^2}}{8\,\sqrt{-d^2}}-\frac{\frac{2\,b\,e^{9\,c+9\,d\,x}}{3\,d}-\frac{8\,b\,e^{7\,c+7\,d\,x}}{3\,d}-\frac{8\,b\,e^{3\,c+3\,d\,x}}{3\,d}+\frac{4\,e^{5\,c+5\,d\,x}\,\left(8\,a+3\,b\right)}{3\,d}+\frac{2\,b\,e^{c+d\,x}}{3\,d}}{15\,e^{4\,c+4\,d\,x}-6\,e^{2\,c+2\,d\,x}-20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}-6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1}+\frac{e^{c+d\,x}\,\left(5\,a-16\,b\right)}{12\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{a\,e^{c+d\,x}}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}-\frac{22\,a\,e^{c+d\,x}}{3\,d\,\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}-\frac{e^{c+d\,x}\,\left(5\,a+8\,b\right)}{8\,d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{16\,a\,e^{c+d\,x}}{3\,d\,\left(5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^7,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(5*a*(-d^2)^(1/2) + 8*b*(-d^2)^(1/2)))/(d*(80*a*b +
25*a^2 + 64*b^2)^(1/2)))*(80*a*b + 25*a^2 + 64*b^2)^(1/2))/(8*(-d^2)^(1/2))
 - ((2*b*exp(9*c + 9*d*x))/(3*d) - (8*b*exp(7*c + 7*d*x))/(3*d) - (8*b*exp(
3*c + 3*d*x))/(3*d) + (4*exp(5*c + 5*d*x)*(8*a + 3*b))/(3*d) + (2*b*exp(c +
 d*x))/(3*d))/(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*
x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) +
 (exp(c + d*x)*(5*a - 16*b))/(12*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) +
 1)) - (a*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp
(6*c + 6*d*x) - 1)) - (22*a*exp(c + d*x))/(3*d*(6*exp(4*c + 4*d*x) - 4*exp(
2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (exp(c + d*x)*
(5*a + 8*b))/(8*d*(exp(2*c + 2*d*x) - 1)) - (16*a*exp(c + d*x))/(3*d*(5*exp
(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d
*x) + exp(10*c + 10*d*x) - 1))
```

3.196 $\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=120

$$-\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{(a+b)(a+5b)\cosh^3(c+dx)}{3d}-\frac{2b(3a+5b)\cosh^5(c+dx)}{5d}+\frac{2b(a+5b)\cosh^7(c+dx}{7d}$$

```
[Out] -(a+b)^2*cosh(d*x+c)/d+1/3*(a+b)*(a+5*b)*cosh(d*x+c)^3/d-2/5*b*(3*a+5*b)*co
sh(d*x+c)^5/d+2/7*b*(a+5*b)*cosh(d*x+c)^7/d-5/9*b^2*cosh(d*x+c)^9/d+1/11*b^
2*cosh(d*x+c)^11/d
```

Rubi [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3294, 1167}

$$\frac{2b(a+5b)\cosh^7(c+dx)}{7d}-\frac{2b(3a+5b)\cosh^5(c+dx)}{5d}+\frac{(a+b)(a+5b)\cosh^3(c+dx)}{3d}-\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^{11}(c+dx)}{11d}-\frac{5b^2\cosh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(((a + b)^2*Cosh[c + d*x])/d) + ((a + b)*(a + 5*b)*Cosh[c + d*x]^3)/(3*d)
- (2*b*(3*a + 5*b)*Cosh[c + d*x]^5)/(5*d) + (2*b*(a + 5*b)*Cosh[c + d*x]^7)
/(7*d) - (5*b^2*Cosh[c + d*x]^9)/(9*d) + (b^2*Cosh[c + d*x]^11)/(11*d)
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx &= -\frac{\text{Subst}\Big(\int\left(1-x^2\right)\left(a+b-2bx^2+bx^4\right)^2\,dx,x,\cosh(c+dx)}{d}\\ &= -\frac{\text{Subst}\big(\int\left((a+b)^2+(-a-5b)(a+b)x^2+2b(3a+5b)x^4-}{d}\\ &= -\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{(a+b)(a+5b)\cosh^3(c+dx)}{3d}-\frac{2b}{}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 207, normalized size = 1.72

$$-\frac{3a^2\cosh(c+dx)}{4d}-\frac{35ab\cosh(c+dx)}{32d}-\frac{231b^2\cosh(c+dx)}{512d}+\frac{a^2\cosh(3(c+dx))}{12d}+\frac{7ab\cosh(3(c+dx))}{32d}+\frac{55b^2\cosh(3(c+dx))}{512d}-\frac{7ab\cosh(5(c+dx))}{160d}-\frac{33b^2\cosh(5(c+dx))}{1024d}+\frac{ab\cosh(7(c+dx))}{224d}+\frac{55b^2\cosh(7(c+dx))}{7168d}-\frac{11b^2\cosh(9(c+dx))}{9216d}+\frac{b^2\cosh(11(c+dx))}{11264d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-3*a^2*Cosh[c + d*x])/(4*d) - (35*a*b*Cosh[c + d*x])/(32*d) - (231*b^2*Cos
h[c + d*x])/(512*d) + (a^2*Cosh[3*(c + d*x)])/(12*d) + (7*a*b*Cosh[3*(c + d
*x)])/(32*d) + (55*b^2*Cosh[3*(c + d*x)])/(512*d) - (7*a*b*Cosh[5*(c + d*x)
])/(160*d) - (33*b^2*Cosh[5*(c + d*x)])/(1024*d) + (a*b*Cosh[7*(c + d*x)])/
(224*d) + (55*b^2*Cosh[7*(c + d*x)])/(7168*d) - (11*b^2*Cosh[9*(c + d*x)])/
(9216*d) + (b^2*Cosh[11*(c + d*x)])/(11264*d)
```

Maple [A]
time = 0.85, size = 138, normalized size = 1.15

method	result
default	$\frac{\left(-\frac{165}{1024}b^2-\frac{7}{32}ab\right)\cosh(5dx+5c)}{5d}+\frac{\left(\frac{55}{1024}b^2+\frac{1}{32}ab\right)\cosh(7dx+7c)}{7d}+\frac{\left(-\frac{231}{512}b^2-\frac{35}{32}ab-\frac{3}{4}a^2\right)\cosh(dx+c)}{d}+\frac{\left(\frac{165}{512}b^2+\frac{21}{32}ab+\frac{1}{4}a^2\right)}{3d}$
risch	$\frac{b^2\mathrm{e}^{11dx+11c}}{22528d}-\frac{11b^2\mathrm{e}^{9dx+9c}}{18432d}+\frac{55b^2\mathrm{e}^{7dx+7c}}{14336d}+\frac{b\,\mathrm{e}^{7dx+7c}a}{448d}-\frac{33b^2\mathrm{e}^{5dx+5c}}{2048d}-\frac{7b\,\mathrm{e}^{5dx+5c}a}{320d}+\frac{\mathrm{e}^{3dx+3c}a^2}{24d}+\frac{7\,\mathrm{e}^{3dx+3c}ab}{64d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(-165/1024*b^2-7/32*a*b)/d*cosh(5*d*x+5*c)+1/7*(55/1024*b^2+1/32*a*b)/d
*cosh(7*d*x+7*c)+(-231/512*b^2-35/32*a*b-3/4*a^2)/d*cosh(d*x+c)+1/3*(165/51
2*b^2+21/32*a*b+1/4*a^2)/d*cosh(3*d*x+3*c)-11/9216*b^2/d*cosh(9*d*x+9*c)+1/
11264*b^2/d*cosh(11*d*x+11*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(110) = 220$.
time = 0.27, size = 307, normalized size = 2.56

$$-\frac{1}{1419264}b^2\left(\frac{\left(847\,e^{(-2\,dx-2\,c)}-5445\,e^{(-4\,dx-4\,c)}+22869\,e^{(-6\,dx-6\,c)}-76230\,e^{(-8\,dx-8\,c)}+320166\,e^{(-10\,dx-10\,c)}-63\right)e^{(11\,dx+11\,c)}}{d}+\frac{320166\,e^{(-dx-c)}-76230\,e^{(-3\,dx-3\,c)}+22869\,e^{(-5\,dx-5\,c)}-5445\,e^{(-7\,dx-7\,c)}+847\,e^{(-9\,dx-9\,c)}-63\,e^{(-11\,dx-11\,c)}}{d}\right)-\frac{1}{2240}ab\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d}\right)+\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/1419264*b^2*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-6
*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^(1
1*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e^(
-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*d*
x - 11*c))/d) - 1/2240*a*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1
225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-
3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/24*a^2*(e^(
3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(110) = 220$.
time = 0.38, size = 404, normalized size = 3.37

$$\frac{315 \, b^{2} \cosh\left(dx + c\right)^{11} + 3465 \, b^{2} \cosh\left(dx + c\right) \sinh\left(dx + c\right)^{10} - 4235 \, b^{2} \cosh\left(dx + c\right)^{9} + 3465 \, {\left(15 \, b^{2} \cosh\left(dx + c\right)^{3} - 11 \, b^{2} \cosh\left(dx + c\right)\right)} \sinh\left(dx + c\right)^{8} + 495 \, {\left(32 \, ab + 55 \, b^{2}\right)} \cosh\left(dx + c\right)^{7} + 1155 \, {\left(126 \, b^{2} \cosh\left(dx + c\right)^{5} - 308 \, b^{2} \cosh\left(dx + c\right)^{3} + 3 \, {\left(32 \, ab + 55 \, b^{2}\right)} \cosh\left(dx + c\right)\right)} \sinh\left(dx + c\right)^{6} - 693 \, {\left(224 \, ab + 165 \, b^{2}\right)} \cosh\left(dx + c\right)^{5} + 3465 \, {\left(30 \, b^{2} \cosh\left(dx + c\right)^{7} - 154 \, b^{2} \cosh\left(dx + c\right)^{5} + 5 \, {\left(32 \, ab + 55 \, b^{2}\right)} \cosh\left(dx + c\right)^{3} - {\left(224 \, ab + 165 \, b^{2}\right)} \cosh\left(dx + c\right)\right)} \sinh\left(dx + c\right)^{4} + 2310 \, {\left(128 \, a^{2} + 336 \, ab + 165 \, b^{2}\right)} \cosh\left(dx + c\right)^{3} + 3465 \, {\left(5 \, b^{2} \cosh\left(dx + c\right)^{9} - 44 \, b^{2} \cosh\left(dx + c\right)^{7} + 3 \, {\left(32 \, ab + 55 \, b^{2}\right)} \cosh\left(dx + c\right)^{5} - 2 \, {\left(224 \, ab + 165 \, b^{2}\right)} \cosh\left(dx + c\right)^{3} + 2 \, {\left(128 \, a^{2} + 336 \, ab + 165 \, b^{2}\right)} \cosh\left(dx + c\right)\right)} \sinh\left(dx + c\right)^{2} - 6930 \, {\left(384 \, a^{2} + 560 \, ab + 231 \, b^{2}\right)} \cosh\left(dx + c\right)}{3548160 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/3548160*(315*b^2*cosh(d*x + c)^11 + 3465*b^2*cosh(d*x + c)*sinh(d*x + c)^
10 - 4235*b^2*cosh(d*x + c)^9 + 3465*(15*b^2*cosh(d*x + c)^3 - 11*b^2*cosh(
d*x + c))*sinh(d*x + c)^8 + 495*(32*a*b + 55*b^2)*cosh(d*x + c)^7 + 1155*(1
26*b^2*cosh(d*x + c)^5 - 308*b^2*cosh(d*x + c)^3 + 3*(32*a*b + 55*b^2)*cosh
(d*x + c))*sinh(d*x + c)^6 - 693*(224*a*b + 165*b^2)*cosh(d*x + c)^5 + 3465
*(30*b^2*cosh(d*x + c)^7 - 154*b^2*cosh(d*x + c)^5 + 5*(32*a*b + 55*b^2)*co
sh(d*x + c)^3 - (224*a*b + 165*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 2310*(
128*a^2 + 336*a*b + 165*b^2)*cosh(d*x + c)^3 + 3465*(5*b^2*cosh(d*x + c)^9
- 44*b^2*cosh(d*x + c)^7 + 3*(32*a*b + 55*b^2)*cosh(d*x + c)^5 - 2*(224*a*b
 + 165*b^2)*cosh(d*x + c)^3 + 2*(128*a^2 + 336*a*b + 165*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^2 - 6930*(384*a^2 + 560*a*b + 231*b^2)*cosh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(109) = 218$.
time = 2.99, size = 280, normalized size = 2.33

$$\begin{cases} \frac{a^{2} \sinh^{2}{\left(c + dx \right)} \cosh{\left(c + dx \right)}}{d} - \frac{2 a^{2} \cosh^{3}{\left(c + dx \right)}}{3 d} + \frac{2 ab \sinh^{6}{\left(c + dx \right)} \cosh{\left(c + dx \right)}}{d} - \frac{4 ab \sinh^{4}{\left(c + dx \right)} \cosh^{3}{\left(c + dx \right)}}{d} + \frac{16 ab \sinh^{2}{\left(c + dx \right)} \cosh^{5}{\left(c + dx \right)}}{5 d} - \frac{32 ab \cosh^{7}{\left(c + dx \right)}}{35 d} + \frac{b^{2} \sinh^{10}{\left(c + dx \right)} \cosh{\left(c + dx \right)}}{d} - \frac{10 b^{2} \sinh^{8}{\left(c + dx \right)} \cosh^{3}{\left(c + dx \right)}}{3 d} + \frac{16 b^{2} \sinh^{6}{\left(c + dx \right)} \cosh^{5}{\left(c + dx \right)}}{3 d} - \frac{32 b^{2} \sinh^{4}{\left(c + dx \right)} \cosh^{7}{\left(c + dx \right)}}{7 d} + \frac{128 b^{2} \sinh^{2}{\left(c + dx \right)} \cosh^{9}{\left(c + dx \right)}}{63 d} - \frac{256 b^{2} \cosh^{11}{\left(c + dx \right)}}{693 d} & \text{for}\: d \neq 0 \\ x \left(a + b \sinh^{4}{\left(c \right)}\right)^{2} \sinh^{3}{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3/
(3*d) + 2*a*b*sinh(c + d*x)**6*cosh(c + d*x)/d - 4*a*b*sinh(c + d*x)**4*cos
h(c + d*x)**3/d + 16*a*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 32*a*b*c
```

```
osh(c + d*x)**7/(35*d) + b**2*sinh(c + d*x)**10*cosh(c + d*x)/d - 10*b**2*s
inh(c + d*x)**8*cosh(c + d*x)**3/(3*d) + 16*b**2*sinh(c + d*x)**6*cosh(c +
d*x)**5/(3*d) - 32*b**2*sinh(c + d*x)**4*cosh(c + d*x)**7/(7*d) + 128*b**2*
sinh(c + d*x)**2*cosh(c + d*x)**9/(63*d) - 256*b**2*cosh(c + d*x)**11/(693*
d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c)**3, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(110) = 220.
time = 0.43, size = 278, normalized size = 2.32

$$\frac{b^{2}e^{(11\,dx+11\,c)}}{22528\,d}-\frac{11\,b^{2}e^{(9\,dx+9\,c)}}{18432\,d}-\frac{11\,b^{2}e^{(-9\,dx-9\,c)}}{18432\,d}+\frac{b^{2}e^{(-11\,dx-11\,c)}}{22528\,d}+\frac{(32\,ab+55\,b^{2})e^{(7\,dx+7\,c)}}{14336\,d}-\frac{(224\,ab+165\,b^{2})e^{(5\,dx+5\,c)}}{10240\,d}+\frac{(128\,a^{2}+336\,ab+165\,b^{2})e^{(3\,dx+3\,c)}}{3072\,d}-\frac{(384\,a^{2}+560\,ab+231\,b^{2})e^{(dx+c)}}{1024\,d}-\frac{(384\,a^{2}+560\,ab+231\,b^{2})e^{(-dx-c)}}{1024\,d}+\frac{(128\,a^{2}+336\,ab+165\,b^{2})e^{(-3\,dx-3\,c)}}{3072\,d}-\frac{(224\,ab+165\,b^{2})e^{(-5\,dx-5\,c)}}{10240\,d}+\frac{(32\,ab+55\,b^{2})e^{(-7\,dx-7\,c)}}{14336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/22528*b^2*e^(11*d*x + 11*c)/d - 11/18432*b^2*e^(9*d*x + 9*c)/d - 11/18432
*b^2*e^(-9*d*x - 9*c)/d + 1/22528*b^2*e^(-11*d*x - 11*c)/d + 1/14336*(32*a*
b + 55*b^2)*e^(7*d*x + 7*c)/d - 1/10240*(224*a*b + 165*b^2)*e^(5*d*x + 5*c)
/d + 1/3072*(128*a^2 + 336*a*b + 165*b^2)*e^(3*d*x + 3*c)/d - 1/1024*(384*a
^2 + 560*a*b + 231*b^2)*e^(d*x + c)/d - 1/1024*(384*a^2 + 560*a*b + 231*b^2
)*e^(-d*x - c)/d + 1/3072*(128*a^2 + 336*a*b + 165*b^2)*e^(-3*d*x - 3*c)/d
- 1/10240*(224*a*b + 165*b^2)*e^(-5*d*x - 5*c)/d + 1/14336*(32*a*b + 55*b^2
)*e^(-7*d*x - 7*c)/d
```

Mupad [**B**]
time = 0.36, size = 150, normalized size = 1.25

$$-\frac{-\frac{a^{2}\cosh(c+d\,x)^{3}}{3}+a^{2}\cosh(c+d\,x)-\frac{2\,a\,b\cosh(c+d\,x)^{7}}{7}+\frac{6\,a\,b\cosh(c+d\,x)^{5}}{5}-2\,a\,b\cosh(c+d\,x)^{3}+2\,a\,b\cosh(c+d\,x)-\frac{b^{2}\cosh(c+d\,x)^{11}}{11}+\frac{5\,b^{2}\cosh(c+d\,x)^{9}}{9}-\frac{10\,b^{2}\cosh(c+d\,x)^{7}}{7}+2\,b^{2}\cosh(c+d\,x)^{5}-\frac{5\,b^{2}\cosh(c+d\,x)^{3}}{3}+b^{2}\cosh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^4)^2,x)
```

```
[Out] -(a^2*cosh(c + d*x) + b^2*cosh(c + d*x) - (a^2*cosh(c + d*x)^3)/3 - (5*b^2*
cosh(c + d*x)^3)/3 + 2*b^2*cosh(c + d*x)^5 - (10*b^2*cosh(c + d*x)^7)/7 + (
5*b^2*cosh(c + d*x)^9)/9 - (b^2*cosh(c + d*x)^11)/11 + 2*a*b*cosh(c + d*x)
- 2*a*b*cosh(c + d*x)^3 + (6*a*b*cosh(c + d*x)^5)/5 - (2*a*b*cosh(c + d*x)^
7)/7)/d
```

3.197 $\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=161

$$-\frac{1}{256}\left(128a^2+160ab+63b^2\right)x+\frac{\left(128a^2+352ab+193b^2\right)\cosh(c+dx)\sinh(c+dx)}{256d}-\frac{b(416a+447b)\cosh^3(}{384}$$

```
[Out] -1/256*(128*a^2+160*a*b+63*b^2)*x+1/256*(128*a^2+352*a*b+193*b^2)*cosh(d*x+
c)*sinh(d*x+c)/d-1/384*b*(416*a+447*b)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/480*b*
(160*a+513*b)*cosh(d*x+c)^5*sinh(d*x+c)/d-41/80*b^2*cosh(d*x+c)^7*sinh(d*x+
c)/d+1/10*b^2*cosh(d*x+c)^9*sinh(d*x+c)/d
```

Rubi [A]
time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.261$, Rules used = {3296, 1271, 1828, 1171, 393, 212}

$$\frac{\left(128a^2+352ab+193b^2\right)\sinh(c+dx)\cosh(c+dx)}{256d}-\frac{1}{256}x\left(128a^2+160ab+63b^2\right)+\frac{b(160a+513b)\sinh(c+dx)\cosh^5(c+dx)}{480d}-\frac{b(416a+447b)\sinh(c+dx)\cosh^3(c+dx)}{384d}+\frac{b^2\sinh(c+dx)\cosh^9(c+dx)}{10d}-\frac{41b^2\sinh(c+dx)\cosh^7(c+dx)}{80d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/256*((128*a^2 + 160*a*b + 63*b^2)*x) + ((128*a^2 + 352*a*b + 193*b^2)*Co
sh[c + d*x]*Sinh[c + d*x])/(256*d) - (b*(416*a + 447*b)*Cosh[c + d*x]^3*Sin
h[c + d*x])/(384*d) + (b*(160*a + 513*b)*Cosh[c + d*x]^5*Sinh[c + d*x])/(48
0*d) - (41*b^2*Cosh[c + d*x]^7*Sinh[c + d*x])/(80*d) + (b^2*Cosh[c + d*x]^9
*Sinh[c + d*x])/(10*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-2ax^2+(a+b)x^4\right)^2}{(1-x^2)^6}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh^9(c+dx)\sinh(c+dx)}{10d} + \frac{\text{Subst}\left(\int \frac{-b^2+10(a^2-b^2)x^2-10(3a^2+}{}\right.}{}$$

$$= -\frac{41b^2\cosh^7(c+dx)\sinh(c+dx)}{80d} + \frac{b^2\cosh^9(c+dx)\sinh(c+d}{10d}$$

$$= \frac{b(160a+513b)\cosh^5(c+dx)\sinh(c+dx)}{480d} - \frac{41b^2\cosh^7(c+dx}{80d}$$

$$= -\frac{b(416a+447b)\cosh^3(c+dx)\sinh(c+dx)}{384d} + \frac{b(160a+513b)}{}$$

$$= \frac{(128a^2+352ab+193b^2)\cosh(c+dx)\sinh(c+dx)}{256d} - \frac{b(416a+}{}$$

$$= -\frac{1}{256}\left(128a^2+160ab+63b^2\right)x + \frac{(128a^2+352ab+193b^2)\cosh}{256d}$$

Mathematica [A]
time = 0.27, size = 139, normalized size = 0.86

$$-\frac{15360a^2c+19200abc+7560b^2c+15360a^2dx+19200abdx+7560b^2dx-60(128a^2+240ab+105b^2)\sinh(2(c+dx))+360b(8a+5b)\sinh(4(c+dx))-320ab\sinh(6(c+dx))-450b^2\sinh(6(c+dx))+75b^2\sinh(8(c+dx))-6b^2\sinh(10(c+dx))}{30720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/30720*(15360*a^2*c + 19200*a*b*c + 7560*b^2*c + 15360*a^2*d*x + 19200*a*
b*d*x + 7560*b^2*d*x - 60*(128*a^2 + 240*a*b + 105*b^2)*Sinh[2*(c + d*x)] +
 360*b*(8*a + 5*b)*Sinh[4*(c + d*x)] - 320*a*b*Sinh[6*(c + d*x)] - 450*b^2*
Sinh[6*(c + d*x)] + 75*b^2*Sinh[8*(c + d*x)] - 6*b^2*Sinh[10*(c + d*x)])/d
```

Maple [A]
time = 1.52, size = 130, normalized size = 0.81

method	result
default	$\frac{\left(-\frac{15}{64}b^2-\frac{3}{8}ab\right)\sinh(4dx+4c)}{4d}+\frac{\left(\frac{45}{512}b^2+\frac{1}{16}ab\right)\sinh(6dx+6c)}{6d}+\frac{\left(\frac{105}{256}b^2+\frac{15}{16}ab+\frac{1}{2}a^2\right)\sinh(2dx+2c)}{2d}-\frac{a^2x}{2}-\frac{63b^2x}{256}-\frac{5abx}{8}-$
risch	$-\frac{63b^2x}{256}-\frac{5abx}{8}-\frac{a^2x}{2}+\frac{b^2\mathrm{e}^{10dx+10c}}{10240d}-\frac{5b^2\mathrm{e}^{8dx+8c}}{4096d}+\frac{15b^2\mathrm{e}^{6dx+6c}}{2048d}+\frac{b\,\mathrm{e}^{6dx+6c}a}{192d}-\frac{3\,\mathrm{e}^{4dx+4c}ab}{64d}-\frac{15\,\mathrm{e}^{4dx+4c}b^2}{512d}+\mathrm{e}^{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-15/64*b^2-3/8*a*b)*sinh(4*d*x+4*c)/d+1/6*(45/512*b^2+1/16*a*b)*sinh(6
*d*x+6*c)/d+1/2*(105/256*b^2+15/16*a*b+1/2*a^2)*sinh(2*d*x+2*c)/d-1/2*a^2*x
-63/256*b^2*x-5/8*a*b*x-5/2048*b^2*sinh(8*d*x+8*c)/d+1/5120*b^2*sinh(10*d*x
+10*c)/d
```

Maxima [A]
time = 0.27, size = 260, normalized size = 1.61

$$-\frac{1}{8}\,a^{2}\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{20480}\,b^{2}\left(\frac{\left(25\,e^{(-2\,dx-2\,c)}-150\,e^{(-4\,dx-4\,c)}+600\,e^{(-6\,dx-6\,c)}-2100\,e^{(-8\,dx-8\,c)}-2\right)e^{(10\,dx+10\,c)}}{d}+\frac{5040\,(dx+c)}{d}+\frac{2100\,e^{(-2\,dx-2\,c)}-600\,e^{(-4\,dx-4\,c)}+150\,e^{(-6\,dx-6\,c)}-25\,e^{(-8\,dx-8\,c)}+2\,e^{(-10\,dx-10\,c)}}{d}\right)-\frac{1}{192}\,ab\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/20480*b^2*((25*
e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8
*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x -
 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) +
 2*e^(-10*d*x - 10*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x -
 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e
^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(149) = 298$.
time = 0.43, size = 305, normalized size = 1.89

$$\frac{15\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{9}+30\,\left(6\,b^{2}\cosh\left(dx+c\right)^{3}-5\,b^{2}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{7}+3\,\left(126\,b^{2}\cosh\left(dx+c\right)^{5}-350\,b^{2}\cosh\left(dx+c\right)^{3}+5\,\left(32\,ab+45\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{5}+10\,\left(18\,b^{2}\cosh\left(dx+c\right)^{7}-105\,b^{2}\cosh\left(dx+c\right)^{5}+5\,\left(32\,ab+45\,b^{2}\right)\cosh\left(dx+c\right)^{3}-36\,\left(8\,ab+5\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}-30\,\left(128\,a^{2}+160\,ab+63\,b^{2}\right)dx+15\,\left(b^{2}\cosh\left(dx+c\right)^{9}-10\,b^{2}\cosh\left(dx+c\right)^{7}+\left(32\,ab+45\,b^{2}\right)\cosh\left(dx+c\right)^{5}-24\,\left(8\,ab+5\,b^{2}\right)\cosh\left(dx+c\right)^{3}+2\,\left(128\,a^{2}+240\,ab+105\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/7680*(15*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 30*(6*b^2*cosh(d*x + c)^3 -
5*b^2*cosh(d*x + c))*sinh(d*x + c)^7 + 3*(126*b^2*cosh(d*x + c)^5 - 350*b^2
*cosh(d*x + c)^3 + 5*(32*a*b + 45*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*
(18*b^2*cosh(d*x + c)^7 - 105*b^2*cosh(d*x + c)^5 + 5*(32*a*b + 45*b^2)*cos
h(d*x + c)^3 - 36*(8*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 30*(128*
a^2 + 160*a*b + 63*b^2)*d*x + 15*(b^2*cosh(d*x + c)^9 - 10*b^2*cosh(d*x + c
)^7 + (32*a*b + 45*b^2)*cosh(d*x + c)^5 - 24*(8*a*b + 5*b^2)*cosh(d*x + c)^
3 + 2*(128*a^2 + 240*a*b + 105*b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(155) = 310$.
time = 2.25, size = 484, normalized size = 3.01

$$\begin{cases}\frac{a^{2}x\sinh^{2}\left(c+dx\right)}{2}-\frac{a^{2}x\cosh^{2}\left(c+dx\right)}{2}+\frac{a^{2}\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d}+\frac{5abx\sinh^{6}\left(c+dx\right)}{8}-\frac{15abx\sinh^{4}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{8}+\frac{15abx\sinh^{2}\left(c+dx\right)\cosh^{4}\left(c+dx\right)}{8}-\frac{5abx\cosh^{6}\left(c+dx\right)}{8}+\frac{11ab\sinh^{5}\left(c+dx\right)\cosh\left(c+dx\right)}{8d}-\frac{5ab\sinh^{3}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{3d}+\frac{5ab\sinh\left(c+dx\right)\cosh^{5}\left(c+dx\right)}{8d}+\frac{63b^{2}x\sinh^{10}\left(c+dx\right)}{256}-\frac{315b^{2}x\sinh^{8}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{256}+\frac{315b^{2}x\sinh^{6}\left(c+dx\right)\cosh^{4}\left(c+dx\right)}{128}-\frac{315b^{2}x\sinh^{4}\left(c+dx\right)\cosh^{6}\left(c+dx\right)}{128}+\frac{315b^{2}x\sinh^{2}\left(c+dx\right)\cosh^{8}\left(c+dx\right)}{256}-\frac{63b^{2}x\cosh^{10}\left(c+dx\right)}{256}+\frac{193b^{2}\sinh^{9}\left(c+dx\right)\cosh\left(c+dx\right)}{256d}-\frac{237b^{2}\sinh^{7}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{128d}+\frac{21b^{2}\sinh^{5}\left(c+dx\right)\cosh^{5}\left(c+dx\right)}{10d}-\frac{147b^{2}\sinh^{3}\left(c+dx\right)\cosh^{7}\left(c+dx\right)}{128d}+\frac{63b^{2}\sinh\left(c+dx\right)\cosh^{9}\left(c+dx\right)}{256d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^{4}\left(c\right)\right)^{2}\sinh^{2}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 5*a*b*x*sinh(c + d*x)**6/8 - 15*a*b*x*sinh
(c + d*x)**4*cosh(c + d*x)**2/8 + 15*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**
4/8 - 5*a*b*x*cosh(c + d*x)**6/8 + 11*a*b*sinh(c + d*x)**5*cosh(c + d*x)/(8
*d) - 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3*d) + 5*a*b*sinh(c + d*x)*c
osh(c + d*x)**5/(8*d) + 63*b**2*x*sinh(c + d*x)**10/256 - 315*b**2*x*sinh(c
 + d*x)**8*cosh(c + d*x)**2/256 + 315*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)
**4/128 - 315*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**6/128 + 315*b**2*x*sin
h(c + d*x)**2*cosh(c + d*x)**8/256 - 63*b**2*x*cosh(c + d*x)**10/256 + 193*
b**2*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) - 237*b**2*sinh(c + d*x)**7*cos
h(c + d*x)**3/(128*d) + 21*b**2*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) -
147*b**2*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d) + 63*b**2*sinh(c + d*x)*
cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c)**2, T
rue))
```

Giac [**A**]
time = 0.45, size = 241, normalized size = 1.50

$$-\frac{1}{256}\,(128\,a^2+160\,ab+63\,b^2)x+\frac{b^2e^{(10\,dx+10\,c)}}{10240\,d}-\frac{5\,b^2e^{(8\,dx+8\,c)}}{4096\,d}+\frac{5\,b^2e^{(-8\,dx-8\,c)}}{4096\,d}-\frac{b^2e^{(-10\,dx-10\,c)}}{10240\,d}+\frac{(32\,ab+45\,b^2)e^{(6\,dx+6\,c)}}{6144\,d}-\frac{3\,(8\,ab+5\,b^2)e^{(4\,dx+4\,c)}}{512\,d}+\frac{(128\,a^2+240\,ab+105\,b^2)e^{(2\,dx+2\,c)}}{1024\,d}-\frac{(128\,a^2+240\,ab+105\,b^2)e^{(-2\,dx-2\,c)}}{1024\,d}+\frac{3\,(8\,ab+5\,b^2)e^{(-4\,dx-4\,c)}}{512\,d}-\frac{(32\,ab+45\,b^2)e^{(-6\,dx-6\,c)}}{6144\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/256*(128*a^2 + 160*a*b + 63*b^2)*x + 1/10240*b^2*e^(10*d*x + 10*c)/d - 5
/4096*b^2*e^(8*d*x + 8*c)/d + 5/4096*b^2*e^(-8*d*x - 8*c)/d - 1/10240*b^2*e
^(-10*d*x - 10*c)/d + 1/6144*(32*a*b + 45*b^2)*e^(6*d*x + 6*c)/d - 3/512*(8
*a*b + 5*b^2)*e^(4*d*x + 4*c)/d + 1/1024*(128*a^2 + 240*a*b + 105*b^2)*e^(2
*d*x + 2*c)/d - 1/1024*(128*a^2 + 240*a*b + 105*b^2)*e^(-2*d*x - 2*c)/d + 3
/512*(8*a*b + 5*b^2)*e^(-4*d*x - 4*c)/d - 1/6144*(32*a*b + 45*b^2)*e^(-6*d*
x - 6*c)/d
```

Mupad [**B**]
time = 0.41, size = 149, normalized size = 0.93

$$\frac{960\,a^2\sinh(2\,c+2\,d\,x)+\frac{1575\,b^2\sinh(2\,c+2\,d\,x)}{2}-225\,b^2\sinh(4\,c+4\,d\,x)+\frac{225\,b^2\sinh(6\,c+6\,d\,x)}{4}-\frac{75\,b^2\sinh(8\,c+8\,d\,x)}{8}+\frac{3\,b^2\sinh(10\,c+10\,d\,x)}{4}+1800\,a\,b\sinh(2\,c+2\,d\,x)-360\,a\,b\sinh(4\,c+4\,d\,x)+40\,a\,b\sinh(6\,c+6\,d\,x)-1920\,a^2\,d\,x-945\,b^2\,d\,x-2400\,a\,b\,d\,x}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^4)^2,x)
```

```
[Out] (960*a^2*sinh(2*c + 2*d*x) + (1575*b^2*sinh(2*c + 2*d*x))/2 - 225*b^2*sinh(
4*c + 4*d*x) + (225*b^2*sinh(6*c + 6*d*x))/4 - (75*b^2*sinh(8*c + 8*d*x))/8
 + (3*b^2*sinh(10*c + 10*d*x))/4 + 1800*a*b*sinh(2*c + 2*d*x) - 360*a*b*sin
h(4*c + 4*d*x) + 40*a*b*sinh(6*c + 6*d*x) - 1920*a^2*d*x - 945*b^2*d*x - 24
00*a*b*d*x)/(3840*d)
```

3.198 $\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=92

$$\frac{(a+b)^2\cosh(c+dx)}{d}-\frac{4b(a+b)\cosh^3(c+dx)}{3d}+\frac{2b(a+3b)\cosh^5(c+dx)}{5d}-\frac{4b^2\cosh^7(c+dx)}{7d}+\frac{b^2\cosh^9(c+}{9d}$$

```
[Out] (a+b)^2*cosh(d*x+c)/d-4/3*b*(a+b)*cosh(d*x+c)^3/d+2/5*b*(a+3*b)*cosh(d*x+c)
^5/d-4/7*b^2*cosh(d*x+c)^7/d+1/9*b^2*cosh(d*x+c)^9/d
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3294, 1104}

$$\frac{2b(a+3b)\cosh^5(c+dx)}{5d}-\frac{4b(a+b)\cosh^3(c+dx)}{3d}+\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^9(c+dx)}{9d}-\frac{4b^2\cosh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((a + b)^2*Cosh[c + d*x])/d - (4*b*(a + b)*Cosh[c + d*x]^3)/(3*d) + (2*b*(a
 + 3*b)*Cosh[c + d*x]^5)/(5*d) - (4*b^2*Cosh[c + d*x]^7)/(7*d) + (b^2*Cosh[
c + d*x]^9)/(9*d)
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(a+b-2bx^2+bx^4\right)^2\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)-4ab\left(1+\frac{b}{a}\right)x^2+2ab\left(1+\frac{3b}{a}\right)x^4-4\right.}{d}$$

$$= \frac{(a+b)^2\cosh(c+dx)}{d}-\frac{4b(a+b)\cosh^3(c+dx)}{3d}+\frac{2b(a+3b)\,\text{co}}{5d}$$

Mathematica [A]
time = 0.04, size = 164, normalized size = 1.78

$$\frac{a^2\cosh(c)\cosh(dx)}{d}+\frac{5ab\cosh(c+dx)}{4d}+\frac{63b^2\cosh(c+dx)}{128d}-\frac{5ab\cosh(3(c+dx))}{24d}-\frac{7b^2\cosh(3(c+dx))}{64d}+\frac{ab\cosh(5(c+dx))}{40d}+\frac{9b^2\cosh(5(c+dx))}{320d}-\frac{9b^2\cosh(7(c+dx))}{1792d}+\frac{b^2\cosh(9(c+dx))}{2304d}+\frac{a^2\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (a^2*Cosh[c]*Cosh[d*x])/d + (5*a*b*Cosh[c + d*x])/(4*d) + (63*b^2*Cosh[c +
d*x])/(128*d) - (5*a*b*Cosh[3*(c + d*x)])/(24*d) - (7*b^2*Cosh[3*(c + d*x)]
)/(64*d) + (a*b*Cosh[5*(c + d*x)])/(40*d) + (9*b^2*Cosh[5*(c + d*x)])/(320*
d) - (9*b^2*Cosh[7*(c + d*x)])/(1792*d) + (b^2*Cosh[9*(c + d*x)])/(2304*d)
+ (a^2*Sinh[c]*Sinh[d*x])/d
```

Maple [A]
time = 0.78, size = 107, normalized size = 1.16

method	result
default	$\frac{\left(-\frac{21}{64}b^2-\frac{5}{8}ab\right)\cosh(3dx+3c)}{3d}+\frac{\left(\frac{9}{64}b^2+\frac{1}{8}ab\right)\cosh(5dx+5c)}{5d}+\frac{\left(\frac{63}{128}b^2+\frac{5}{4}ab+a^2\right)\cosh(dx+c)}{d}-\frac{9b^2\cosh(7dx+7c)}{1792d}+\frac{b^2\cosh(9d}{2304d}$
risch	$\frac{b^2e^{9dx+9c}}{4608d}-\frac{9b^2e^{7dx+7c}}{3584d}+\frac{9b^2e^{5dx+5c}}{640d}+\frac{b\,e^{5dx+5c}a}{80d}-\frac{7\,e^{3dx+3c}b^2}{128d}-\frac{5\,e^{3dx+3c}ab}{48d}+\frac{e^{dx+c}a^2}{2d}+\frac{5ab\,e^{dx+c}}{8d}+\frac{63\,e^{dx+c}b^2}{256d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-21/64*b^2-5/8*a*b)/d*cosh(3*d*x+3*c)+1/5*(9/64*b^2+1/8*a*b)/d*cosh(5*
d*x+5*c)+(63/128*b^2+5/4*a*b+a^2)/d*cosh(d*x+c)-9/1792*b^2/d*cosh(7*d*x+7*c
)+1/2304*b^2/d*cosh(9*d*x+9*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(84) = 168.
time = 0.27, size = 226, normalized size = 2.46

$$-\frac{1}{161280}b^2\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\frac{39690\,e^{(-dx-c)}-8820\,e^{(-3\,dx-3\,c)}+2268\,e^{(-5\,dx-5\,c)}-405\,e^{(-7\,dx-7\,c)}+35\,e^{(-9\,dx-9\,c)}}{d}\right)+\frac{1}{240}ab\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{a^2\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/161280*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d
*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x
 - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c
) + 35*e^(-9*d*x - 9*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x
+ 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d +
 3*e^(-5*d*x - 5*c)/d) + a^2*cosh(d*x + c)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(84) = 168$.
time = 0.41, size = 279, normalized size = 3.03

$$\frac{35\,b^2\cosh\left(dx+c\right)^9+315\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8-405\,b^2\cosh\left(dx+c\right)^7+105\,\left(28\,b^2\cosh\left(dx+c\right)^3-27\,b^2\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^6+252\,\left(8\,ab+9\,b^2\right)\cosh\left(dx+c\right)^5+315\,\left(14\,b^2\cosh\left(dx+c\right)^5-45\,b^2\cosh\left(dx+c\right)^3+4\,\left(8\,ab+9\,b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^4-420\,\left(40\,ab+21\,b^2\right)\cosh\left(dx+c\right)^3+315\,\left(4\,b^2\cosh\left(dx+c\right)^7-27\,b^2\cosh\left(dx+c\right)^5+8\,\left(8\,ab+9\,b^2\right)\cosh\left(dx+c\right)^3-4\,\left(40\,ab+21\,b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2+630\,\left(128\,a^2+160\,ab+63\,b^2\right)\cosh\left(dx+c\right)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^2*cosh(d*x + c)^9 + 315*b^2*cosh(d*x + c)*sinh(d*x + c)^8 - 4
05*b^2*cosh(d*x + c)^7 + 105*(28*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c)
)*sinh(d*x + c)^6 + 252*(8*a*b + 9*b^2)*cosh(d*x + c)^5 + 315*(14*b^2*cosh(
d*x + c)^5 - 45*b^2*cosh(d*x + c)^3 + 4*(8*a*b + 9*b^2)*cosh(d*x + c))*sinh
(d*x + c)^4 - 420*(40*a*b + 21*b^2)*cosh(d*x + c)^3 + 315*(4*b^2*cosh(d*x +
 c)^7 - 27*b^2*cosh(d*x + c)^5 + 8*(8*a*b + 9*b^2)*cosh(d*x + c)^3 - 4*(40*
a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 630*(128*a^2 + 160*a*b + 63*
b^2)*cosh(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(83) = 166$.
time = 1.54, size = 204, normalized size = 2.22

$$\begin{cases} \frac{a^2\cosh\left(c+dx\right)}{d}+\frac{2ab\sinh^4\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{8ab\sinh^2\left(c+dx\right)\cosh^3\left(c+dx\right)}{3d}+\frac{16ab\cosh^5\left(c+dx\right)}{15d}+\frac{b^2\sinh^8\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{8b^2\sinh^6\left(c+dx\right)\cosh^3\left(c+dx\right)}{3d}+\frac{16b^2\sinh^4\left(c+dx\right)\cosh^5\left(c+dx\right)}{5d}-\frac{64b^2\sinh^2\left(c+dx\right)\cosh^7\left(c+dx\right)}{35d}+\frac{128b^2\cosh^9\left(c+dx\right)}{315d} & \text{for}\ d\neq 0\\ x\left(a+b\sinh^4\left(c\right)\right)^2\sinh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Piecewise((a**2*cosh(c + d*x)/d + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d -
8*a*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15
*d) + b**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**2*sinh(c + d*x)**6*cosh(
c + d*x)**3/(3*d) + 16*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 64*b*
*2*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**2*cosh(c + d*x)**9/(31
5*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(84) = 168$.

time = 0.48, size = 220, normalized size = 2.39

$$\frac{b^2 e^{(9\,dx+9\,c)}}{4608\,d} - \frac{9\,b^2 e^{(7\,dx+7\,c)}}{3584\,d} - \frac{9\,b^2 e^{(-7\,dx-7\,c)}}{3584\,d} + \frac{b^2 e^{(-9\,dx-9\,c)}}{4608\,d} + \frac{(8\,ab+9\,b^2)e^{(5\,dx+5\,c)}}{640\,d} - \frac{(40\,ab+21\,b^2)e^{(3\,dx+3\,c)}}{384\,d} + \frac{(128\,a^2+160\,ab+63\,b^2)e^{(dx+c)}}{256\,d} + \frac{(128\,a^2+160\,ab+63\,b^2)e^{(-dx-c)}}{256\,d} - \frac{(40\,ab+21\,b^2)e^{(-3\,dx-3\,c)}}{384\,d} + \frac{(8\,ab+9\,b^2)e^{(-5\,dx-5\,c)}}{640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/4608*b^2*e^(9*d*x + 9*c)/d - 9/3584*b^2*e^(7*d*x + 7*c)/d - 9/3584*b^2*e^
(-7*d*x - 7*c)/d + 1/4608*b^2*e^(-9*d*x - 9*c)/d + 1/640*(8*a*b + 9*b^2)*e^
(5*d*x + 5*c)/d - 1/384*(40*a*b + 21*b^2)*e^(3*d*x + 3*c)/d + 1/256*(128*a^
2 + 160*a*b + 63*b^2)*e^(d*x + c)/d + 1/256*(128*a^2 + 160*a*b + 63*b^2)*e^
(-d*x - c)/d - 1/384*(40*a*b + 21*b^2)*e^(-3*d*x - 3*c)/d + 1/640*(8*a*b +
9*b^2)*e^(-5*d*x - 5*c)/d
```

Mupad [B]

time = 0.88, size = 111, normalized size = 1.21

$$\frac{a^2\cosh(c+d\,x) + \frac{2\,a\,b\cosh(c+d\,x)^5}{5} - \frac{4\,a\,b\cosh(c+d\,x)^3}{3} + 2\,a\,b\cosh(c+d\,x) + \frac{b^2\cosh(c+d\,x)^9}{9} - \frac{4\,b^2\cosh(c+d\,x)^7}{7} + \frac{6\,b^2\cosh(c+d\,x)^5}{5} - \frac{4\,b^2\cosh(c+d\,x)^3}{3} + b^2\cosh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^4)^2,x)
```

```
[Out] (a^2*cosh(c + d*x) + b^2*cosh(c + d*x) - (4*b^2*cosh(c + d*x)^3)/3 + (6*b^2
*cosh(c + d*x)^5)/5 - (4*b^2*cosh(c + d*x)^7)/7 + (b^2*cosh(c + d*x)^9)/9 +
 2*a*b*cosh(c + d*x) - (4*a*b*cosh(c + d*x)^3)/3 + (2*a*b*cosh(c + d*x)^5)/
5)/d
```

3.199 $\int \left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=125

$$\frac{1}{128}\left(128a^2+96ab+35b^2\right)x-\frac{b(160a+93b)\cosh(c+dx)\sinh(c+dx)}{128d}+\frac{b(96a+163b)\cosh^3(c+dx)\sinh(c}{192d}$$

```
[Out] 1/128*(128*a^2+96*a*b+35*b^2)*x-1/128*b*(160*a+93*b)*cosh(d*x+c)*sinh(d*x+c
)/d+1/192*b*(96*a+163*b)*cosh(d*x+c)^3*sinh(d*x+c)/d-25/48*b^2*cosh(d*x+c)^
5*sinh(d*x+c)/d+1/8*b^2*cosh(d*x+c)^7*sinh(d*x+c)/d
```

Rubi [A]
time $=0.12$, antiderivative size $=125$, normalized size of antiderivative $=1.00$, number of steps used $=6$, number of rules used $=5$, integrand size $=14$, $\frac{\text{number of rules}}{\text{integrand size}}=0.357$, Rules used $=\{3288, 1171, 1828, 393, 212\}$

$$\frac{1}{128}x\left(128a^2+96ab+35b^2\right)+\frac{b(96a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{b(160a+93b)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{b^2\sinh(c+dx)\cosh^7(c+dx)}{8d}-\frac{25b^2\sinh(c+dx)\cosh^5(c+dx)}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((128*a^2 + 96*a*b + 35*b^2)*x)/128 - (b*(160*a + 93*b)*Cosh[c + d*x]*Sinh[
c + d*x])/(128*d) + (b*(96*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d
) - (25*b^2*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^2*Cosh[c + d*x]^7*Si
nh[c + d*x])/(8*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
```

```
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\sinh^4(c+dx)\right)^2 \, dx &= \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^2}{\left(1-x^2\right)^5} \, dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{b^2\cosh^7(c+dx)\sinh(c+dx)}{8d}-\frac{\text{Subst}\left(\int \frac{-8a^2+b^2+8\left(3a^2+b^2\right)x^2-8(3a-b)(a+b)x^4+8(a}{\left(1-x^2\right)^4}\right.}{8d}\\
&= -\frac{25b^2\cosh^5(c+dx)\sinh(c+dx)}{48d}+\frac{b^2\cosh^7(c+dx)\sinh(c+dx)}{8d}+\frac{\text{Subst}\left(\right.}{}\\
&= \frac{b(96a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d}-\frac{25b^2\cosh^5(c+dx)\sinh(c+dx)}{48d}\\
&= -\frac{b(160a+93b)\cosh(c+dx)\sinh(c+dx)}{128d}+\frac{b(96a+163b)\cosh^3(c+dx)\text{sinl}}{192d}\\
&= \frac{1}{128}\left(128a^2+96ab+35b^2\right)x-\frac{b(160a+93b)\cosh(c+dx)\sinh(c+dx)}{128d}+\frac{b(9}{}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 92, normalized size = 0.74

$$\frac{24\left(128a^2+96ab+35b^2\right)(c+dx)-96b(16a+7b)\sinh(2(c+dx))+24b(8a+7b)\sinh(4(c+dx))-32b^2\sinh(6(c+dx))+3b^2\sinh(8(c+dx))}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (24*(128*a^2 + 96*a*b + 35*b^2)*(c + d*x) - 96*b*(16*a + 7*b)*Sinh[2*(c + d
*x)] + 24*b*(8*a + 7*b)*Sinh[4*(c + d*x)] - 32*b^2*Sinh[6*(c + d*x)] + 3*b^
2*Sinh[8*(c + d*x)])/(3072*d)
```

Maple [A]
time = 1.12, size = 100, normalized size = 0.80

method	result
default	$a^{2} x+\frac{\left(-\frac{7}{16} b^{2}-a b\right) \sinh \left(2 d x+2 c\right)}{2 d}+\frac{\left(\frac{7}{32} b^{2}+\frac{1}{4} a b\right) \sinh \left(4 d x+4 c\right)}{4 d}+\frac{35 b^{2} x}{128}+\frac{3 a b x}{4}-\frac{b^{2} \sinh \left(6 d x+6 c\right)}{96 d}+\frac{b^{2} \sinh \left(8 d x+8 c\right)}{1024 d}$
risch	$\frac{35 b^{2} x}{128}+a^{2} x+\frac{3 a b x}{4}+\frac{b^{2} {\mathrm e}^{8 d x+8 c}}{2048 d}-\frac{b^{2} {\mathrm e}^{6 d x+6 c}}{192 d}+\frac{7 \,{\mathrm e}^{4 d x+4 c} b^{2}}{256 d}+\frac{{\mathrm e}^{4 d x+4 c} a b}{32 d}-\frac{7 \,{\mathrm e}^{2 d x+2 c} b^{2}}{64 d}-\frac{{\mathrm e}^{2 d x+2 c} a b}{4 d}+\frac{7 \,{\mathrm e}^{-2 d x-2 c}}{64 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+1/2*(-7/16*b^2-a*b)*sinh(2*d*x+2*c)/d+1/4*(7/32*b^2+1/4*a*b)*sinh(4*d
*x+4*c)/d+35/128*b^2*x+3/4*a*b*x-1/96*b^2*sinh(6*d*x+6*c)/d+1/1024*b^2*sinh
(8*d*x+8*c)/d
```

Maxima [A]
time = 0.27, size = 183, normalized size = 1.46

$$\frac{1}{32} \, a b {\left(24 \, x + \frac{e^{\left(4 \, d x + 4 \, c\right)}}{d} - \frac{8 \, e^{\left(2 \, d x + 2 \, c\right)}}{d} + \frac{8 \, e^{\left(-2 \, d x - 2 \, c\right)}}{d} - \frac{e^{\left(-4 \, d x - 4 \, c\right)}}{d}\right)} + a^{2} x - \frac{1}{6144} \, b^{2} {\left(\frac{{\left(32 \, e^{\left(-2 \, d x - 2 \, c\right)} - 168 \, e^{\left(-4 \, d x - 4 \, c\right)} + 672 \, e^{\left(-6 \, d x - 6 \, c\right)} - 3\right)} e^{\left(8 \, d x + 8 \, c\right)}}{d} - \frac{1680 \, {\left(d x + c\right)}}{d} - \frac{672 \, e^{\left(-2 \, d x - 2 \, c\right)} - 168 \, e^{\left(-4 \, d x - 4 \, c\right)} + 32 \, e^{\left(-6 \, d x - 6 \, c\right)} - 3 \, e^{\left(-8 \, d x - 8 \, c\right)}}{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + a^2*x - 1/6144*b^2*((32*e^(-2*d*x - 2*c) - 168
*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x
 + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c
) - 3*e^(-8*d*x - 8*c))/d)
```

Fricas [A]
time = 0.41, size = 205, normalized size = 1.64

$$\frac{3 \, b^{2} \cosh\left(d x + c\right) \sinh\left(d x + c\right)^{7} + 3 \, {\left(7 \, b^{2} \cosh\left(d x + c\right)^{3} - 8 \, b^{2} \cosh\left(d x + c\right)\right)} \sinh\left(d x + c\right)^{5} + {\left(21 \, b^{2} \cosh\left(d x + c\right)^{5} - 80 \, b^{2} \cosh\left(d x + c\right)^{3} + 12 \, {\left(8 \, a b + 7 \, b^{2}\right)} \cosh\left(d x + c\right)\right)} \sinh\left(d x + c\right)^{3} + 3 \, {\left(128 \, a^{2} + 96 \, a b + 35 \, b^{2}\right)} d x + 3 \, {\left(b^{2} \cosh\left(d x + c\right)^{7} - 8 \, b^{2} \cosh\left(d x + c\right)^{5} + 4 \, {\left(8 \, a b + 7 \, b^{2}\right)} \cosh\left(d x + c\right)^{3} - 8 \, {\left(16 \, a b + 7 \, b^{2}\right)} \cosh\left(d x + c\right)\right)} \sinh\left(d x + c\right)}{384 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^2*cosh(d*x + c)^3 - 8*b
^2*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^2*cosh(d*x + c)^5 - 80*b^2*cosh(d
```

```
*x + c)^3 + 12*(8*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(128*a^2
+ 96*a*b + 35*b^2)*d*x + 3*(b^2*cosh(d*x + c)^7 - 8*b^2*cosh(d*x + c)^5 + 4
*(8*a*b + 7*b^2)*cosh(d*x + c)^3 - 8*(16*a*b + 7*b^2)*cosh(d*x + c))*sinh(d
*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(116) = 232$.
time $= 1.04$, size $= 332$, normalized size $= 2.66$

$$\begin{cases} a^2x + \frac{3abx\sinh^4(c+dx)}{4} - \frac{3abx\sinh^2(c+dx)\cosh^2(c+dx)}{2} + \frac{3abx\cosh^4(c+dx)}{4} + \frac{5ab\sinh^3(c+dx)\cosh(c+dx)}{4d} - \frac{3ab\sinh(c+dx)\cosh^3(c+dx)}{4d} + \frac{35b^2x\sinh^8(c+dx)}{128} - \frac{35b^2x\sinh^6(c+dx)\cosh^2(c+dx)}{32} + \frac{105b^2x\sinh^4(c+dx)\cosh^4(c+dx)}{64} - \frac{35b^2x\sinh^2(c+dx)\cosh^6(c+dx)}{32} + \frac{35b^2x\cosh^8(c+dx)}{128} + \frac{93b^2\sinh^7(c+dx)\cosh(c+dx)}{128d} - \frac{511b^2\sinh^5(c+dx)\cosh^3(c+dx)}{384d} + \frac{385b^2\sinh^3(c+dx)\cosh^5(c+dx)}{384d} - \frac{35b^2\sinh(c+dx)\cosh^7(c+dx)}{128d} & \text{for } d \neq 0 \\ x\left(a + b\sinh^4(c)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Piecewise((a**2*x + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh(c + d*x)**2*c
osh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c + d*x)**3*cos
h(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d) + 35*b**2*x*s
inh(c + d*x)**8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*
b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sinh(c + d*x)**2*co
sh(c + d*x)**6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**2*sinh(c + d*x)*
*7*cosh(c + d*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*
d) + 385*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**2*sinh(c +
d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2, True))
```

Giac [**A**]
time $= 0.42$, size $= 183$, normalized size $= 1.46$

$$\frac{1}{128}\left(128\,a^2 + 96\,ab + 35\,b^2\right)x + \frac{b^2e^{(8\,dx+8\,c)}}{2048\,d} - \frac{b^2e^{(6\,dx+6\,c)}}{192\,d} + \frac{b^2e^{(-6\,dx-6\,c)}}{192\,d} - \frac{b^2e^{(-8\,dx-8\,c)}}{2048\,d} + \frac{(8\,ab + 7\,b^2)e^{(4\,dx+4\,c)}}{256\,d} - \frac{(16\,ab + 7\,b^2)e^{(2\,dx+2\,c)}}{64\,d} + \frac{(16\,ab + 7\,b^2)e^{(-2\,dx-2\,c)}}{64\,d} - \frac{(8\,ab + 7\,b^2)e^{(-4\,dx-4\,c)}}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/128*(128*a^2 + 96*a*b + 35*b^2)*x + 1/2048*b^2*e^(8*d*x + 8*c)/d - 1/192*
b^2*e^(6*d*x + 6*c)/d + 1/192*b^2*e^(-6*d*x - 6*c)/d - 1/2048*b^2*e^(-8*d*x
 - 8*c)/d + 1/256*(8*a*b + 7*b^2)*e^(4*d*x + 4*c)/d - 1/64*(16*a*b + 7*b^2)
*e^(2*d*x + 2*c)/d + 1/64*(16*a*b + 7*b^2)*e^(-2*d*x - 2*c)/d - 1/256*(8*a*
b + 7*b^2)*e^(-4*d*x - 4*c)/d
```

Mupad [**B**]
time $= 0.29$, size $= 108$, normalized size $= 0.86$

$$\frac{21\,b^2\sinh(4\,c + 4\,d\,x) - 84\,b^2\sinh(2\,c + 2\,d\,x) - 4\,b^2\sinh(6\,c + 6\,d\,x) + \frac{3\,b^2\sinh(8\,c+8\,d\,x)}{8} - 192\,a\,b\sinh(2\,c + 2\,d\,x) + 24\,a\,b\sinh(4\,c + 4\,d\,x) + 384\,a^2\,d\,x + 105\,b^2\,d\,x + 288\,a\,b\,d\,x}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2,x)
```

```
[Out] (21*b^2*sinh(4*c + 4*d*x) - 84*b^2*sinh(2*c + 2*d*x) - 4*b^2*sinh(6*c + 6*d
*x) + (3*b^2*sinh(8*c + 8*d*x))/8 - 192*a*b*sinh(2*c + 2*d*x) + 24*a*b*sinh
(4*c + 4*d*x) + 384*a^2*d*x + 105*b^2*d*x + 288*a*b*d*x)/(384*d)
```

3.200 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=92

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b(2a+3b)\cosh^3(c+dx)}{3d}-\frac{3b^2\cosh^5(c+dx)}{5d}+\frac{b^2\cosh^7}{7}$$

```
[Out] -a^2*arctanh(cosh(d*x+c))/d-b*(2*a+b)*cosh(d*x+c)/d+1/3*b*(2*a+3*b)*cosh(d*
x+c)^3/d-3/5*b^2*cosh(d*x+c)^5/d+1/7*b^2*cosh(d*x+c)^7/d
```

Rubi [A]
time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$,
Rules used = {3294, 1167, 212}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a+3b)\cosh^3(c+dx)}{3d}-\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b^2\cosh^7(c+dx)}{7d}-\frac{3b^2\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -((a^2*ArcTanh[Cosh[c + d*x]])/d) - (b*(2*a + b)*Cosh[c + d*x])/d + (b*(2*a
 + 3*b)*Cosh[c + d*x]^3)/(3*d) - (3*b^2*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[
c + d*x]^7)/(7*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^2}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(b(2a+b)-b(2a+3b)x^2+3b^2x^4-b^2x^6+\frac{a^2}{1-x^2}\right)\,d}{d}\\ &= -\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b(2a+3b)\cosh^3(c+dx)}{3d}-\frac{3b^2\cosh}{5}\\ &= -\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b(2a+3}{}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 146, normalized size = 1.59

$$-\frac{3ab\cosh(c+dx)}{2d}-\frac{35b^2\cosh(c+dx)}{64d}+\frac{ab\cosh(3(c+dx))}{6d}+\frac{7b^2\cosh(3(c+dx))}{64d}-\frac{7b^2\cosh(5(c+dx))}{320d}+\frac{b^2\cosh(7(c+dx))}{448d}-\frac{a^2\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a^2\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-3*a*b*Cosh[c + d*x])/(2*d) - (35*b^2*Cosh[c + d*x])/(64*d) + (a*b*Cosh[3*
(c + d*x)])/(6*d) + (7*b^2*Cosh[3*(c + d*x)])/(64*d) - (7*b^2*Cosh[5*(c + d
*x)])/(320*d) + (b^2*Cosh[7*(c + d*x)])/(448*d) - (a^2*Log[Cosh[c/2 + (d*x)
/2]])/d + (a^2*Log[Sinh[c/2 + (d*x)/2]])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(86) = 172$.
time = 1.32, size = 234, normalized size = 2.54

method	result
risch	$\frac{b^2 e^{7dx+7c}}{896d}-\frac{7b^2 e^{5dx+5c}}{640d}+\frac{e^{3dx+3c}ab}{12d}+\frac{7\,e^{3dx+3c}b^2}{128d}-\frac{3ab\,e^{dx+c}}{4d}-\frac{35\,e^{dx+c}b^2}{128d}-\frac{3\,e^{-dx-c}ab}{4d}-\frac{35\,e^{-dx-c}b^2}{128d}+\frac{e^{-3dx-3}}{12d}$
default	$\frac{b^2\left(\frac{\left(\cosh^7(dx+c)\right)}{7}+\frac{\left(\cosh^5(dx+c)\right)}{5}+\frac{\left(\cosh^3(dx+c)\right)}{3}+\cosh(dx+c)-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)-4b^2\left(\frac{\left(\cosh^5(dx+c)\right)}{5}+\frac{\left(\cosh^3(dx+c)\right)}{3}+\cosh(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/7*cosh(d*x+c)^7+1/5*cosh(d*x+c)^5+1/3*cosh(d*x+c)^3+cosh(d*x+c)
-2*arctanh(exp(d*x+c)))-4*b^2*(1/5*cosh(d*x+c)^5+1/3*cosh(d*x+c)^3+cosh(d*x
+c)-2*arctanh(exp(d*x+c)))+2*a*b*(1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(e
xp(d*x+c)))+6*b^2*(1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x+c)))-4*a
*b*(cosh(d*x+c)-2*arctanh(exp(d*x+c)))-4*b^2*(cosh(d*x+c)-2*arctanh(exp(d*x
```

```
+c)))-2*a^2*arctanh(exp(d*x+c))-4*a*b*arctanh(exp(d*x+c))-2*b^2*arctanh(exp
(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(86) = 172$.
time = 0.27, size = 177, normalized size = 1.92

$$-\frac{1}{4480}b^2\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d}\right)+\frac{1}{12}ab\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/12*a*b*(e^(3*d*x + 3*c)/d
- 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + a^2*log(tanh(1
/2*d*x + 1/2*c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1575 vs. $2(86) = 172$.
time = 0.43, size = 1575, normalized size = 17.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/13440*(15*b^2*cosh(d*x + c)^14 + 210*b^2*cosh(d*x + c)*sinh(d*x + c)^13 +
 15*b^2*sinh(d*x + c)^14 - 147*b^2*cosh(d*x + c)^12 + 21*(65*b^2*cosh(d*x +
 c)^2 - 7*b^2)*sinh(d*x + c)^12 + 84*(65*b^2*cosh(d*x + c)^3 - 21*b^2*cosh(
d*x + c))*sinh(d*x + c)^11 + 35*(32*a*b + 21*b^2)*cosh(d*x + c)^10 + 7*(214
5*b^2*cosh(d*x + c)^4 - 1386*b^2*cosh(d*x + c)^2 + 160*a*b + 105*b^2)*sinh(
d*x + c)^10 + 70*(429*b^2*cosh(d*x + c)^5 - 462*b^2*cosh(d*x + c)^3 + 5*(32
*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 105*(96*a*b + 35*b^2)*cosh(
d*x + c)^8 + 105*(429*b^2*cosh(d*x + c)^6 - 693*b^2*cosh(d*x + c)^4 + 15*(3
2*a*b + 21*b^2)*cosh(d*x + c)^2 - 96*a*b - 35*b^2)*sinh(d*x + c)^8 + 24*(21
45*b^2*cosh(d*x + c)^7 - 4851*b^2*cosh(d*x + c)^5 + 175*(32*a*b + 21*b^2)*c
osh(d*x + c)^3 - 35*(96*a*b + 35*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 105*
(96*a*b + 35*b^2)*cosh(d*x + c)^6 + 21*(2145*b^2*cosh(d*x + c)^8 - 6468*b^2
*cosh(d*x + c)^6 + 350*(32*a*b + 21*b^2)*cosh(d*x + c)^4 - 140*(96*a*b + 35
*b^2)*cosh(d*x + c)^2 - 480*a*b - 175*b^2)*sinh(d*x + c)^6 + 42*(715*b^2*co
sh(d*x + c)^9 - 2772*b^2*cosh(d*x + c)^7 + 210*(32*a*b + 21*b^2)*cosh(d*x +
 c)^5 - 140*(96*a*b + 35*b^2)*cosh(d*x + c)^3 - 15*(96*a*b + 35*b^2)*cosh(d
*x + c))*sinh(d*x + c)^5 + 35*(32*a*b + 21*b^2)*cosh(d*x + c)^4 + 35*(429*b
^2*cosh(d*x + c)^10 - 2079*b^2*cosh(d*x + c)^8 + 210*(32*a*b + 21*b^2)*cosh
(d*x + c)^6 - 210*(96*a*b + 35*b^2)*cosh(d*x + c)^4 - 45*(96*a*b + 35*b^2)*
```

```
cosh(d*x + c)^2 + 32*a*b + 21*b^2)*sinh(d*x + c)^4 - 147*b^2*cosh(d*x + c)^
2 + 140*(39*b^2*cosh(d*x + c)^11 - 231*b^2*cosh(d*x + c)^9 + 30*(32*a*b + 2
1*b^2)*cosh(d*x + c)^7 - 42*(96*a*b + 35*b^2)*cosh(d*x + c)^5 - 15*(96*a*b
+ 35*b^2)*cosh(d*x + c)^3 + (32*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^
3 + 21*(65*b^2*cosh(d*x + c)^12 - 462*b^2*cosh(d*x + c)^10 + 75*(32*a*b + 2
1*b^2)*cosh(d*x + c)^8 - 140*(96*a*b + 35*b^2)*cosh(d*x + c)^6 - 75*(96*a*b
 + 35*b^2)*cosh(d*x + c)^4 + 10*(32*a*b + 21*b^2)*cosh(d*x + c)^2 - 7*b^2)*
sinh(d*x + c)^2 + 15*b^2 - 13440*(a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)
^6*sinh(d*x + c) + 21*a^2*cosh(d*x + c)^5*sinh(d*x + c)^2 + 35*a^2*cosh(d*x
 + c)^4*sinh(d*x + c)^3 + 35*a^2*cosh(d*x + c)^3*sinh(d*x + c)^4 + 21*a^2*c
osh(d*x + c)^2*sinh(d*x + c)^5 + 7*a^2*cosh(d*x + c)*sinh(d*x + c)^6 + a^2*
sinh(d*x + c)^7)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 13440*(a^2*cosh(d
*x + c)^7 + 7*a^2*cosh(d*x + c)^6*sinh(d*x + c) + 21*a^2*cosh(d*x + c)^5*si
nh(d*x + c)^2 + 35*a^2*cosh(d*x + c)^4*sinh(d*x + c)^3 + 35*a^2*cosh(d*x +
c)^3*sinh(d*x + c)^4 + 21*a^2*cosh(d*x + c)^2*sinh(d*x + c)^5 + 7*a^2*cosh(
d*x + c)*sinh(d*x + c)^6 + a^2*sinh(d*x + c)^7)*log(cosh(d*x + c) + sinh(d*
x + c) - 1) + 14*(15*b^2*cosh(d*x + c)^13 - 126*b^2*cosh(d*x + c)^11 + 25*(
32*a*b + 21*b^2)*cosh(d*x + c)^9 - 60*(96*a*b + 35*b^2)*cosh(d*x + c)^7 - 4
5*(96*a*b + 35*b^2)*cosh(d*x + c)^5 + 10*(32*a*b + 21*b^2)*cosh(d*x + c)^3
- 21*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x +
c)^6*sinh(d*x + c) + 21*d*cosh(d*x + c)^5*sinh(d*x + c)^2 + 35*d*cosh(d*x +
 c)^4*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)^3*sinh(d*x + c)^4 + 21*d*cosh(d*
x + c)^2*sinh(d*x + c)^5 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x +
 c)^7)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(86) = 172.
time = 0.46, size = 196, normalized size = 2.13

$$\frac{15\,b^2e^{(7\,dx+7\,c)}-147\,b^2e^{(5\,dx+5\,c)}+1120\,abe^{(3\,dx+3\,c)}+735\,b^2e^{(3\,dx+3\,c)}-10080\,abe^{(dx+c)}-3675\,b^2e^{(dx+c)}-13440\,a^2\log\left(e^{(dx+c)}+1\right)+13440\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right)-\left(10080\,abe^{(6\,dx+6\,c)}+3675\,b^2e^{(6\,dx+6\,c)}-1120\,abe^{(4\,dx+4\,c)}-735\,b^2e^{(4\,dx+4\,c)}+147\,b^2e^{(2\,dx+2\,c)}-15\,b^2\right)e^{(-7\,dx-7\,c)}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*b^2*e^(7*d*x + 7*c) - 147*b^2*e^(5*d*x + 5*c) + 1120*a*b*e^(3*d
*x + 3*c) + 735*b^2*e^(3*d*x + 3*c) - 10080*a*b*e^(d*x + c) - 3675*b^2*e^(d
```

```
*x + c) - 13440*a^2*log(e^(d*x + c) + 1) + 13440*a^2*log(abs(e^(d*x + c) -
1)) - (10080*a*b*e^(6*d*x + 6*c) + 3675*b^2*e^(6*d*x + 6*c) - 1120*a*b*e^(4
*d*x + 4*c) - 735*b^2*e^(4*d*x + 4*c) + 147*b^2*e^(2*d*x + 2*c) - 15*b^2)*e
^(-7*d*x - 7*c))/d
```

Mupad [B]
time = 0.34, size = 198, normalized size = 2.15

$$\frac{b^2\,\mathrm{e}^{-7\,c-7\,d\,x}}{896\,d}-\frac{2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}-\frac{\mathrm{e}^{-c-d\,x}\,\left(35\,b^2+96\,a\,b\right)}{128\,d}-\frac{7\,b^2\,\mathrm{e}^{-5\,c-5\,d\,x}}{640\,d}-\frac{7\,b^2\,\mathrm{e}^{5\,c+5\,d\,x}}{640\,d}-\frac{\mathrm{e}^{c+d\,x}\,\left(35\,b^2+96\,a\,b\right)}{128\,d}+\frac{b^2\,\mathrm{e}^{7\,c+7\,d\,x}}{896\,d}+\frac{b\,\mathrm{e}^{-3\,c-3\,d\,x}\,\left(32\,a+21\,b\right)}{384\,d}+\frac{b\,\mathrm{e}^{3\,c+3\,d\,x}\,\left(32\,a+21\,b\right)}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x),x)
```

```
[Out] (b^2*exp(- 7*c - 7*d*x))/(896*d) - (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2
))/(d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^2)^(1/2) - (exp(- c - d*x)*(96*a*b + 3
5*b^2))/(128*d) - (7*b^2*exp(- 5*c - 5*d*x))/(640*d) - (7*b^2*exp(5*c + 5*d
*x))/(640*d) - (exp(c + d*x)*(96*a*b + 35*b^2))/(128*d) + (b^2*exp(7*c + 7*
d*x))/(896*d) + (b*exp(- 3*c - 3*d*x)*(32*a + 21*b))/(384*d) + (b*exp(3*c +
 3*d*x)*(32*a + 21*b))/(384*d)
```

3.201 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=103

$$-\frac{1}{16}b(16a+5b)x-\frac{a^2\coth(c+dx)}{d}+\frac{b(16a+11b)\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{13b^2\cosh^3(c+dx)\sinh(c+dx)}{24d}$$

```
[Out] -1/16*b*(16*a+5*b)*x-a^2*coth(d*x+c)/d+1/16*b*(16*a+11*b)*cosh(d*x+c)*sinh(
d*x+c)/d-13/24*b^2*cosh(d*x+c)^3*sinh(d*x+c)/d+1/6*b^2*cosh(d*x+c)^5*sinh(d
*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3296, 1273, 1819, 464, 212}

$$-\frac{a^2\coth(c+dx)}{d}+\frac{b(16a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}bx(16a+5b)+\frac{b^2\sinh(c+dx)\cosh^5(c+dx)}{6d}-\frac{13b^2\sinh(c+dx)\cosh^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/16*(b*(16*a + 5*b)*x) - (a^2*Coth[c + d*x])/d + (b*(16*a + 11*b)*Cosh[c
+ d*x]*Sinh[c + d*x])/(16*d) - (13*b^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d
) + (b^2*Cosh[c + d*x]^5*Sinh[c + d*x])/(6*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
```

```
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^2}{x^2\left(1-x^2\right)^4}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b^2\cosh^5(c+dx)\sinh(c+dx)}{6d} - \frac{\text{Subst}\Big(\int \frac{-6a^2+\left(18a^2+b^2\right)x^2-6(3a}{x^2(1-}}{} \\
&= -\frac{13b^2\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b^2\cosh^5(c+dx)\sinh(c+}{6d} \\
&= \frac{b(16a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b^2\cosh^3(c+dx)\,\mathrm{s}}{24d} \\
&= -\frac{a^2\coth(c+dx)}{d} + \frac{b(16a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \\
&= -\frac{1}{16}b(16a+5b)x - \frac{a^2\coth(c+dx)}{d} + \frac{b(16a+11b)\cosh(c+}{16d}
\end{aligned}
$$

Mathematica [A]

time = 0.23, size = 77, normalized size = 0.75

$$\frac{-192a^2\coth(c+dx)+b(-192ac-60bc-192adx-60bdx+(96a+45b)\sinh(2(c+dx))-9b\sinh(4(c+dx))+b\sinh(6(c+dx)))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-192*a^2*Coth[c + d*x] + b*(-192*a*c - 60*b*c - 192*a*d*x - 60*b*d*x + (96
*a + 45*b)*Sinh[2*(c + d*x)] - 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])
)/(192*d)
```

Maple [A]
time = 1.25, size = 168, normalized size = 1.63

method	result
risch	$-abx-\frac{5b^2x}{16}+\frac{b^2\mathrm{e}^{6dx+6c}}{384d}-\frac{3\,\mathrm{e}^{4dx+4c}b^2}{128d}+\frac{\mathrm{e}^{2dx+2c}ab}{4d}+\frac{15\,\mathrm{e}^{2dx+2c}b^2}{128d}-\frac{\mathrm{e}^{-2dx-2c}ab}{4d}-\frac{15\,\mathrm{e}^{-2dx-2c}b^2}{128d}+\frac{3\,\mathrm{e}^{-4dx-4c}b^2}{128d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*b*x-5/16*b^2*x+1/384*b^2/d*exp(6*d*x+6*c)-3/128/d*exp(4*d*x+4*c)*b^2+1/4
/d*exp(2*d*x+2*c)*a*b+15/128/d*exp(2*d*x+2*c)*b^2-1/4/d*exp(-2*d*x-2*c)*a*b
-15/128/d*exp(-2*d*x-2*c)*b^2+3/128/d*exp(-4*d*x-4*c)*b^2-1/384*b^2/d*exp(-
6*d*x-6*c)-2*a^2/d/(exp(2*d*x+2*c)-1)
```

Maxima [A]
time = 0.27, size = 146, normalized size = 1.42

$$-\frac{1}{4}\,ab\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{384}\,b^2\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}}{d}\right)+\frac{2\,a^2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b^2*((9*e^(
-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/
d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 2*a^
2/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(95) = 190.
time = 0.38, size = 217, normalized size = 2.11

$$\frac{b^2\cosh(dx+c)^7+7\,b^2\cosh(dx+c)\sinh(dx+c)^6-10\,b^2\cosh(dx+c)^5+5\left(7\,b^2\cosh(dx+c)^3-10\,b^2\cosh(dx+c)\right)\sinh(dx+c)^4+6\,(16\,ab+9\,b^2)\cosh(dx+c)^3+\left(21\,b^2\cosh(dx+c)^5-100\,b^2\cosh(dx+c)^3+18\,(16\,ab+9\,b^2)\cosh(dx+c)\right)\sinh(dx+c)^2-3\,(128\,a^2+32\,ab+15\,b^2)\cosh(dx+c)-24\,((16\,ab+5\,b^2)dx-16\,a^2)\sinh(dx+c)}{384\,d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(b^2*cosh(d*x + c)^7 + 7*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 10*b^2*c
osh(d*x + c)^5 + 5*(7*b^2*cosh(d*x + c)^3 - 10*b^2*cosh(d*x + c))*sinh(d*x
+ c)^4 + 6*(16*a*b + 9*b^2)*cosh(d*x + c)^3 + (21*b^2*cosh(d*x + c)^5 - 100
*b^2*cosh(d*x + c)^3 + 18*(16*a*b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 -
 3*(128*a^2 + 32*a*b + 15*b^2)*cosh(d*x + c) - 24*((16*a*b + 5*b^2)*d*x - 1
6*a^2)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 179, normalized size = 1.74

$$\frac{b^2e^{(6\,dx+6\,c)} - 9\,b^2e^{(4\,dx+4\,c)} + 96\,abe^{(2\,dx+2\,c)} + 45\,b^2e^{(2\,dx+2\,c)} - 24\,(16\,ab + 5\,b^2)(dx+c) + \left(352\,abe^{(6\,dx+6\,c)} + 110\,b^2e^{(6\,dx+6\,c)} - 96\,abe^{(4\,dx+4\,c)} - 45\,b^2e^{(4\,dx+4\,c)} + 9\,b^2e^{(2\,dx+2\,c)} - b^2\right)e^{(-6\,dx-6\,c)} - \frac{768\,a^2}{e^{(2\,dx+2\,c)}-1}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/384*(b^2*e^(6*d*x + 6*c) - 9*b^2*e^(4*d*x + 4*c) + 96*a*b*e^(2*d*x + 2*c)
 + 45*b^2*e^(2*d*x + 2*c) - 24*(16*a*b + 5*b^2)*(d*x + c) + (352*a*b*e^(6*d
*x + 6*c) + 110*b^2*e^(6*d*x + 6*c) - 96*a*b*e^(4*d*x + 4*c) - 45*b^2*e^(4*
d*x + 4*c) + 9*b^2*e^(2*d*x + 2*c) - b^2)*e^(-6*d*x - 6*c) - 768*a^2/(e^(2*
d*x + 2*c) - 1))/d
```

Mupad [**B**]
time = 0.23, size = 148, normalized size = 1.44

$$\frac{3\,b^2\,\mathrm{e}^{-4\,c-4\,d\,x}}{128\,d}-\frac{2\,a^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,\left(15\,b^2+32\,a\,b\right)}{128\,d}-x\,\left(\frac{5\,b^2}{16}+a\,b\right)-\frac{3\,b^2\,\mathrm{e}^{4\,c+4\,d\,x}}{128\,d}-\frac{b^2\,\mathrm{e}^{-6\,c-6\,d\,x}}{384\,d}+\frac{b^2\,\mathrm{e}^{6\,c+6\,d\,x}}{384\,d}+\frac{b\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(32\,a+15\,b\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^2,x)
```

```
[Out] (3*b^2*exp(- 4*c - 4*d*x))/(128*d) - (2*a^2)/(d*(exp(2*c + 2*d*x) - 1)) - (
exp(- 2*c - 2*d*x)*(32*a*b + 15*b^2))/(128*d) - x*(a*b + (5*b^2)/16) - (3*b
^2*exp(4*c + 4*d*x))/(128*d) - (b^2*exp(- 6*c - 6*d*x))/(384*d) + (b^2*exp(
6*c + 6*d*x))/(384*d) + (b*exp(2*c + 2*d*x)*(32*a + 15*b))/(128*d)
```

3.202 $\int \text{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=92

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b(2a+b)\cosh(c+dx)}{d}-\frac{2b^2\cosh^3(c+dx)}{3d}+\frac{b^2\cosh^5(c+dx)}{5d}-\frac{a^2\coth(c+dx)\text{csch}}{2d}$$

```
[Out] 1/2*a^2*arctanh(cosh(d*x+c))/d+b*(2*a+b)*cosh(d*x+c)/d-2/3*b^2*cosh(d*x+c)^
3/d+1/5*b^2*cosh(d*x+c)^5/d-1/2*a^2*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3294, 1171, 1824, 212}

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b^2\cosh^5(c+dx)}{5d}-\frac{2b^2\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(2*a + b)*Cosh[c + d*x])/d - (2*b^2
*Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^5)/(5*d) - (a^2*Coth[c + d*x]*
Csch[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \text{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^2}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= -\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{-a^2-4ab-2b^2+2b(2a+3b}{1-x^2}\right.}{}\\
&= -\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d} - \frac{\text{Subst}\left(\int \left(-2b(2a+b)+4b\right.\right.}{}\\
&= \frac{b(2a+b)\cosh(c+dx)}{d} - \frac{2b^2\cosh^3(c+dx)}{3d} + \frac{b^2\cosh^5(c+dx}{5d}\\
&= \frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d} + \frac{b(2a+b)\cosh(c+dx)}{d} - \frac{2b^2\cosh}{3}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 144, normalized size = 1.57

$$\frac{2ab\cosh(c)\cosh(dx)}{d}+\frac{5b^2\cosh(c+dx)}{8d}-\frac{5b^2\cosh(3(c+dx))}{48d}+\frac{b^2\cosh(5(c+dx))}{80d}-\frac{a^2\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^2\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{2ab\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (2*a*b*Cosh[c]*Cosh[d*x])/d + (5*b^2*Cosh[c + d*x])/(8*d) - (5*b^2*Cosh[3*(
c + d*x)])/(48*d) + (b^2*Cosh[5*(c + d*x)])/(80*d) - (a^2*Csch[(c + d*x)/2]
^2)/(8*d) - (a^2*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/
(8*d) + (2*a*b*Sinh[c]*Sinh[d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(84) = 168$.
time = 1.52, size = 200, normalized size = 2.17

method	result
risch	$\frac{b^2 e^{5dx+5c}}{160d}-\frac{5\,e^{3dx+3c}b^2}{96d}+\frac{ab\,e^{dx+c}}{d}+\frac{5\,e^{dx+c}b^2}{16d}+\frac{e^{-dx-c}ab}{d}+\frac{5\,e^{-dx-c}b^2}{16d}-\frac{5\,e^{-3dx-3c}b^2}{96d}+\frac{b^2 e^{-5dx-5c}}{160d}-\frac{a^2 e^{dx+c}}{d\left(e^{2d}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/160*b^2/d*exp(5*d*x+5*c)-5/96/d*exp(3*d*x+3*c)*b^2+a*b/d*exp(d*x+c)+5/16/
d*exp(d*x+c)*b^2+1/d*exp(-d*x-c)*a*b+5/16/d*exp(-d*x-c)*b^2-5/96/d*exp(-3*d
*x-3*c)*b^2+1/160*b^2/d*exp(-5*d*x-5*c)-a^2*exp(d*x+c)*(1+exp(2*d*x+2*c))/d
/(exp(2*d*x+2*c)-1)^2+1/2*a^2/d*ln(exp(d*x+c)+1)-1/2*a^2/d*ln(exp(d*x+c)-1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(84) = 168$.
time = 0.28, size = 204, normalized size = 2.22

$$\frac{1}{480}\,b^2\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+ab\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}\,a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/480*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a*b*(
e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^
(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2
*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2272 vs. $2(84) = 168$.
time = 0.51, size = 2272, normalized size = 24.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^2*cosh(d*x + c)^14 + 42*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + 3*b
^2*sinh(d*x + c)^14 - 31*b^2*cosh(d*x + c)^12 + (273*b^2*cosh(d*x + c)^2 -
31*b^2)*sinh(d*x + c)^12 + 12*(91*b^2*cosh(d*x + c)^3 - 31*b^2*cosh(d*x + c
))*sinh(d*x + c)^11 + (480*a*b + 203*b^2)*cosh(d*x + c)^10 + (3003*b^2*cosh
(d*x + c)^4 - 2046*b^2*cosh(d*x + c)^2 + 480*a*b + 203*b^2)*sinh(d*x + c)^1
0 + 2*(3003*b^2*cosh(d*x + c)^5 - 3410*b^2*cosh(d*x + c)^3 + 5*(480*a*b + 2
03*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(
d*x + c)^8 + (9009*b^2*cosh(d*x + c)^6 - 15345*b^2*cosh(d*x + c)^4 + 45*(48
0*a*b + 203*b^2)*cosh(d*x + c)^2 - 480*a^2 - 480*a*b - 175*b^2)*sinh(d*x +
c)^8 + 8*(1287*b^2*cosh(d*x + c)^7 - 3069*b^2*cosh(d*x + c)^5 + 15*(480*a*b
 + 203*b^2)*cosh(d*x + c)^3 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c))*s
```

```
inh(d*x + c)^7 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^6 + (9009*b^2*c
osh(d*x + c)^8 - 28644*b^2*cosh(d*x + c)^6 + 210*(480*a*b + 203*b^2)*cosh(d
*x + c)^4 - 140*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^2 - 480*a^2 - 480*
a*b - 175*b^2)*sinh(d*x + c)^6 + 2*(3003*b^2*cosh(d*x + c)^9 - 12276*b^2*co
sh(d*x + c)^7 + 126*(480*a*b + 203*b^2)*cosh(d*x + c)^5 - 140*(96*a^2 + 96*
a*b + 35*b^2)*cosh(d*x + c)^3 - 15*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^5 + (480*a*b + 203*b^2)*cosh(d*x + c)^4 + (3003*b^2*cosh(d*
x + c)^10 - 15345*b^2*cosh(d*x + c)^8 + 210*(480*a*b + 203*b^2)*cosh(d*x +
c)^6 - 350*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^4 - 75*(96*a^2 + 96*a*b
 + 35*b^2)*cosh(d*x + c)^2 + 480*a*b + 203*b^2)*sinh(d*x + c)^4 - 31*b^2*co
sh(d*x + c)^2 + 4*(273*b^2*cosh(d*x + c)^11 - 1705*b^2*cosh(d*x + c)^9 + 30
*(480*a*b + 203*b^2)*cosh(d*x + c)^7 - 70*(96*a^2 + 96*a*b + 35*b^2)*cosh(d
*x + c)^5 - 25*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^3 + (480*a*b + 203*
b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (273*b^2*cosh(d*x + c)^12 - 2046*b^2*
cosh(d*x + c)^10 + 45*(480*a*b + 203*b^2)*cosh(d*x + c)^8 - 140*(96*a^2 + 9
6*a*b + 35*b^2)*cosh(d*x + c)^6 - 75*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x +
c)^4 + 6*(480*a*b + 203*b^2)*cosh(d*x + c)^2 - 31*b^2)*sinh(d*x + c)^2 + 3*
b^2 + 240*(a^2*cosh(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*
sinh(d*x + c)^9 - 2*a^2*cosh(d*x + c)^7 + 2*(18*a^2*cosh(d*x + c)^2 - a^2)*
sinh(d*x + c)^7 + a^2*cosh(d*x + c)^5 + 14*(6*a^2*cosh(d*x + c)^3 - a^2*cos
h(d*x + c))*sinh(d*x + c)^6 + (126*a^2*cosh(d*x + c)^4 - 42*a^2*cosh(d*x +
c)^2 + a^2)*sinh(d*x + c)^5 + (126*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x +
c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(42*a^2*cosh(d*x + c)^6 - 3
5*a^2*cosh(d*x + c)^4 + 5*a^2*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(18*a^2*
cosh(d*x + c)^7 - 21*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3)*sinh(d*x
+ c)^2 + (9*a^2*cosh(d*x + c)^8 - 14*a^2*cosh(d*x + c)^6 + 5*a^2*cosh(d*x +
 c)^4)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 240*(a^2*cos
h(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 -
2*a^2*cosh(d*x + c)^7 + 2*(18*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^7 +
a^2*cosh(d*x + c)^5 + 14*(6*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d
*x + c)^6 + (126*a^2*cosh(d*x + c)^4 - 42*a^2*cosh(d*x + c)^2 + a^2)*sinh(d
*x + c)^5 + (126*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x + c)^3 + 5*a^2*cosh(
d*x + c))*sinh(d*x + c)^4 + 2*(42*a^2*cosh(d*x + c)^6 - 35*a^2*cosh(d*x + c
)^4 + 5*a^2*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(18*a^2*cosh(d*x + c)^7 -
21*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (9*a^2*co
sh(d*x + c)^8 - 14*a^2*cosh(d*x + c)^6 + 5*a^2*cosh(d*x + c)^4)*sinh(d*x +
c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(21*b^2*cosh(d*x + c)^13 - 1
86*b^2*cosh(d*x + c)^11 + 5*(480*a*b + 203*b^2)*cosh(d*x + c)^9 - 20*(96*a^
2 + 96*a*b + 35*b^2)*cosh(d*x + c)^7 - 15*(96*a^2 + 96*a*b + 35*b^2)*cosh(d
*x + c)^5 + 2*(480*a*b + 203*b^2)*cosh(d*x + c)^3 - 31*b^2*cosh(d*x + c))*s
inh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*si
nh(d*x + c)^9 - 2*d*cosh(d*x + c)^7 + 2*(18*d*cosh(d*x + c)^2 - d)*sinh(d*x
 + c)^7 + 14*(6*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^6 + d*co
sh(d*x + c)^5 + (126*d*cosh(d*x + c)^4 - 42*d*cosh(d*x + c)^2 + d)*sinh(d*x
 + c)^5 + (126*d*cosh(d*x + c)^5 - 70*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c)
```

```
)*sinh(d*x + c)^4 + 2*(42*d*cosh(d*x + c)^6 - 35*d*cosh(d*x + c)^4 + 5*d*co
sh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(18*d*cosh(d*x + c)^7 - 21*d*cosh(d*x +
c)^5 + 5*d*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (9*d*cosh(d*x + c)^8 - 14*d*c
osh(d*x + c)^6 + 5*d*cosh(d*x + c)^4)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(84) = 168.
time = 0.46, size = 182, normalized size = 1.98

$$\frac{3\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5-40\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+480\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)+240\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+120\,a^2\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-120\,a^2\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{480\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*b^2*(e^(d*x + c) + e^(-d*x - c))^5 - 40*b^2*(e^(d*x + c) + e^(-d*x
 - c))^3 + 480*a*b*(e^(d*x + c) + e^(-d*x - c)) + 240*b^2*(e^(d*x + c) + e^
(-d*x - c)) + 120*a^2*log(e^(d*x + c) + e^(-d*x - c) + 2) - 120*a^2*log(e^(
d*x + c) + e^(-d*x - c) - 2) - 480*a^2*(e^(d*x + c) + e^(-d*x - c))/((e^(d*
x + c) + e^(-d*x - c))^2 - 4))/d
```

Mupad [**B**]
time = 0.83, size = 214, normalized size = 2.33

$$\frac{\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}-\frac{5\,b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{96\,d}-\frac{5\,b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{96\,d}+\frac{b^2\,\mathrm{e}^{-5\,c-5\,d\,x}}{160\,d}+\frac{b^2\,\mathrm{e}^{5\,c+5\,d\,x}}{160\,d}+\frac{b\,\mathrm{e}^{-c-d\,x}\,\left(16\,a+5\,b\right)}{16\,d}-\frac{a^2\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}+\frac{b\,\mathrm{e}^{c+d\,x}\,\left(16\,a+5\,b\right)}{16\,d}-\frac{2\,a^2\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^3,x)
```

```
[Out] (atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^
2)^(1/2) - (5*b^2*exp(- 3*c - 3*d*x))/(96*d) - (5*b^2*exp(3*c + 3*d*x))/(96
*d) + (b^2*exp(- 5*c - 5*d*x))/(160*d) + (b^2*exp(5*c + 5*d*x))/(160*d) + (
b*exp(- c - d*x)*(16*a + 5*b))/(16*d) - (a^2*exp(c + d*x))/(d*(exp(2*c + 2*
d*x) - 1)) + (b*exp(c + d*x)*(16*a + 5*b))/(16*d) - (2*a^2*exp(c + d*x))/(d
*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.203 $\int \mathbf{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{1}{8}b(16a+3b)x+\frac{a^2\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}-\frac{5b^2\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^2\cosh^3(c+dx)\sinh(c}{4d}$$

```
[Out] 1/8*b*(16*a+3*b)*x+a^2*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d-5/8*b^2*cosh(d
*x+c)*sinh(d*x+c)/d+1/4*b^2*cosh(d*x+c)^3*sinh(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3296, 1273, 1819, 1275, 213}

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}+\frac{1}{8}bx(16a+3b)+\frac{b^2\sinh(c+dx)\cosh^3(c+dx)}{4d}-\frac{5b^2\sinh(c+dx)\cosh(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (b*(16*a + 3*b)*x)/8 + (a^2*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)
- (5*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]^3*Sinh[c +
 d*x])/(4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
```

```
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^2}{x^4\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{b^2\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{4a^2-12a^2x^2+\left(12a^2+8ab-b^2}{x^4\left(1-x^2\right)^2}\right.}{}\\
&= -\frac{5b^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^2\cosh^3(c+dx)\sinh(c+dx}{4d}\\
&= -\frac{5b^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^2\cosh^3(c+dx)\sinh(c+dx}{4d}\\
&= \frac{a^2\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d} - \frac{5b^2\cosh(c+dx)\sinh(c+}{8d}\\
&= \frac{1}{8}b(16a+3b)x + \frac{a^2\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d} - \frac{5b^2\cos}{}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 68, normalized size = 0.75

$$\frac{-32a^2\coth(c+dx)\left(-2+\operatorname{csch}^2(c+dx)\right)+3b(12bc+64adx+12bdx-8b\sinh(2(c+dx))+b\sinh(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-32*a^2*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + 3*b*(12*b*c + 64*a*d*x + 12
*b*d*x - 8*b*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)]))/(96*d)
```

Maple [A]
time = 1.32, size = 115, normalized size = 1.26

method	result	size
risch	$2abx+\frac{3b^{2}x}{8}+\frac{{\mathrm e}^{4dx+4c}b^{2}}{64d}-\frac{{\mathrm e}^{2dx+2c}b^{2}}{8d}+\frac{{\mathrm e}^{-2dx-2c}b^{2}}{8d}-\frac{{\mathrm e}^{-4dx-4c}b^{2}}{64d}-\frac{4a^{2}\left(3\,{\mathrm e}^{2dx+2c}-1\right)}{3d\left({\mathrm e}^{2dx+2c}-1\right)^{3}}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b*x+3/8*b^2*x+1/64/d*exp(4*d*x+4*c)*b^2-1/8/d*exp(2*d*x+2*c)*b^2+1/8/d*
exp(-2*d*x-2*c)*b^2-1/64/d*exp(-4*d*x-4*c)*b^2-4/3*a^2*(3*exp(2*d*x+2*c)-1)
/d/(exp(2*d*x+2*c)-1)^3
```

Maxima [A]
time = 0.28, size = 165, normalized size = 1.81

$$\frac{1}{64}\,b^{2}\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+2\,abx+\frac{4}{3}\,a^{2}\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}-\frac{1}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 2*a*b*x + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^
(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-
2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(83) = 166$.
time = 0.40, size = 300, normalized size = 3.30

$$\frac{3\,b^{2}\cosh\left(dx+c\right)^{7}+21\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}-33\,b^{2}\cosh\left(dx+c\right)^{5}+15\,\left(7\,b^{2}\cosh\left(dx+c\right)^{3}-11\,b^{2}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}+\left(128\,a^{2}+81\,b^{2}\right)\cosh\left(dx+c\right)^{3}+8\left(3\left(16\,ab+3\,b^{2}\right)dx-16\,a^{2}\right)\sinh\left(dx+c\right)^{3}+3\left(21\,b^{2}\cosh\left(dx+c\right)^{5}-110\,b^{2}\cosh\left(dx+c\right)^{3}+\left(128\,a^{2}+81\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}-3\left(128\,a^{2}+17\,b^{2}\right)\cosh\left(dx+c\right)-24\left(3\left(16\,ab+3\,b^{2}\right)dx-\left(3\left(16\,ab+3\,b^{2}\right)dx-16\,a^{2}\right)\cosh\left(dx+c\right)^{2}-16\,a^{2}\right)\sinh\left(dx+c\right)}{192\left(d\sinh\left(dx+c\right)^{3}+3\left(d\cosh\left(dx+c\right)^{2}-d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(3*b^2*cosh(d*x + c)^7 + 21*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 33*b^
2*cosh(d*x + c)^5 + 15*(7*b^2*cosh(d*x + c)^3 - 11*b^2*cosh(d*x + c))*sinh(
d*x + c)^4 + (128*a^2 + 81*b^2)*cosh(d*x + c)^3 + 8*(3*(16*a*b + 3*b^2)*d*x
```

```
 - 16*a^2)*sinh(d*x + c)^3 + 3*(21*b^2*cosh(d*x + c)^5 - 110*b^2*cosh(d*x +
 c)^3 + (128*a^2 + 81*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(128*a^2 + 17
*b^2)*cosh(d*x + c) - 24*(3*(16*a*b + 3*b^2)*d*x - (3*(16*a*b + 3*b^2)*d*x
- 16*a^2)*cosh(d*x + c)^2 - 16*a^2)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(
d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 142, normalized size = 1.56

$$\frac{3\,b^2e^{(4\,dx+4\,c)} - 24\,b^2e^{(2\,dx+2\,c)} + 24\,(16\,ab + 3\,b^2)(dx + c) - 3\,\left(96\,abe^{(4\,dx+4\,c)} + 18\,b^2e^{(4\,dx+4\,c)} - 8\,b^2e^{(2\,dx+2\,c)} + b^2\right)e^{(-4\,dx-4\,c)} - \frac{256\,(3\,a^2e^{(2\,dx+2\,c)}-a^2)}{(e^{(2\,dx+2\,c)}-1)^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*b^2*e^(4*d*x + 4*c) - 24*b^2*e^(2*d*x + 2*c) + 24*(16*a*b + 3*b^2)
*(d*x + c) - 3*(96*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) - 8*b^2*e^(
2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c) - 256*(3*a^2*e^(2*d*x + 2*c) - a^2)/(e
^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [**B**]
time = 0.17, size = 164, normalized size = 1.80

$$\frac{b\,x\,(16\,a+3\,b)}{8} - \frac{4\,a^2}{3\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)} + \frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d} - \frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d} - \frac{b^2\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,d} + \frac{b^2\,\mathrm{e}^{4\,c+4\,d\,x}}{64\,d} - \frac{8\,a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^4,x)
```

```
[Out] (b*x*(16*a + 3*b))/8 - (4*a^2)/(3*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x)
+ 1)) + (b^2*exp(- 2*c - 2*d*x))/(8*d) - (b^2*exp(2*c + 2*d*x))/(8*d) - (b^
2*exp(- 4*c - 4*d*x))/(64*d) + (b^2*exp(4*c + 4*d*x))/(64*d) - (8*a^2*exp(2
*c + 2*d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*
x) - 1))
```

3.204 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=101

$$-\frac{a(3a+16b)\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}+\frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-$$

```
[Out] -1/8*a*(3*a+16*b)*arctanh(cosh(d*x+c))/d-b^2*cosh(d*x+c)/d+1/3*b^2*cosh(d*x
+c)^3/d+3/8*a^2*coth(d*x+c)*csch(d*x+c)/d-1/4*a^2*coth(d*x+c)*csch(d*x+c)^3
/d
```

Rubi [A]

time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {3294, 1171, 1828, 1167, 212}

$$-\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{a(3a+16b)\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/8*(a*(3*a + 16*b)*ArcTanh[Cosh[c + d*x]])/d - (b^2*Cosh[c + d*x])/d + (b
^2*Cosh[c + d*x]^3)/(3*d) + (3*a^2*Coth[c + d*x]*Csch[c + d*x])/(8*d) - (a^
2*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^2}{\left(1-x^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= -\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} + \frac{\text{Subst}\Big(\int \frac{-(a+2b)(3a+2b)+4b(2a+3}{(1-x^2)}}{}\\
&= \frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} -\\
&= \frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} -\\
&= -\frac{b^2\cosh(c+dx)}{d} + \frac{b^2\cosh^3(c+dx)}{3d} + \frac{3a^2\coth(c+dx)\operatorname{csch}(c}{8d}\\
&= -\frac{a(3a+16b)\tanh^{-1}(\cosh(c+dx))}{8d} - \frac{b^2\cosh(c+dx)}{d} + \frac{b^2\cos}{}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 186, normalized size = 1.84

$$-\frac{3b^2\cosh(c+dx)}{4d}+\frac{b^2\cosh(3(c+dx))}{12d}+\frac{3a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{2ab\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{2ab\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{3a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cosh[3*(c + d*x)])/(12*d) + (3*a^2*Csch
[(c + d*x)/2]^2)/(32*d) - (a^2*Csch[(c + d*x)/2]^4)/(64*d) - (2*a*b*Log[Cos
h[c/2 + (d*x)/2]])/d + (2*a*b*Log[Sinh[c/2 + (d*x)/2]])/d + (3*a^2*Log[Tanh
[(c + d*x)/2]])/(8*d) + (3*a^2*Sech[(c + d*x)/2]^2)/(32*d) + (a^2*Sech[(c +
 d*x)/2]^4)/(64*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(93) = 186$.
time = 1.52, size = 195, normalized size = 1.93

method	result
risch	$\frac{\mathrm{e}^{3dx+3c} b^{2}}{24d}-\frac{3\,\mathrm{e}^{dx+c} b^{2}}{8d}-\frac{3\,\mathrm{e}^{-dx-c} b^{2}}{8d}+\frac{\mathrm{e}^{-3dx-3c} b^{2}}{24d}+\frac{a^{2} \mathrm{e}^{dx+c} \left(3\,\mathrm{e}^{6dx+6c}-11\,\mathrm{e}^{4dx+4c}-11\,\mathrm{e}^{2dx+2c}+3\right)}{4d \left(\mathrm{e}^{2dx+2c}-1\right)^{4}}-\frac{3a^{2} \ln\left(\mathrm{e}^{dx+c}+1\right)}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*b^2-3/8/d*exp(d*x+c)*b^2-3/8/d*exp(-d*x-c)*b^2+1/24/d
*exp(-3*d*x-3*c)*b^2+1/4*a^2*exp(d*x+c)*(3*exp(6*d*x+6*c)-11*exp(4*d*x+4*c)
-11*exp(2*d*x+2*c)+3)/d/(exp(2*d*x+2*c)-1)^4-3/8*a^2/d*ln(exp(d*x+c)+1)-2*a
*b/d*ln(exp(d*x+c)+1)+3/8*a^2/d*ln(exp(d*x+c)-1)+2*a*b/d*ln(exp(d*x+c)-1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(93) = 186$.
time = 0.27, size = 234, normalized size = 2.32

$$\frac{1}{24}\,b^{2}\left(\frac{e^{\left(3\,dx+3\,c\right)}}{d}-\frac{9\,e^{\left(dx+c\right)}}{d}-\frac{9\,e^{\left(-dx-c\right)}}{d}+\frac{e^{\left(-3\,dx-3\,c\right)}}{d}\right)-\frac{1}{8}\,a^{2}\left(\frac{3\,\log\left(e^{\left(-dx-c\right)}+1\right)}{d}-\frac{3\,\log\left(e^{\left(-dx-c\right)}-1\right)}{d}+\frac{2\,\left(3\,e^{\left(-dx-c\right)}-11\,e^{\left(-3\,dx-3\,c\right)}-11\,e^{\left(-5\,dx-5\,c\right)}+3\,e^{\left(-7\,dx-7\,c\right)}\right)}{d\left(4\,e^{\left(-2\,dx-2\,c\right)}-6\,e^{\left(-4\,dx-4\,c\right)}+4\,e^{\left(-6\,dx-6\,c\right)}-e^{\left(-8\,dx-8\,c\right)}-1\right)}\right)-2\,ab\left(\frac{\log\left(e^{\left(-dx-c\right)}+1\right)}{d}-\frac{\log\left(e^{\left(-dx-c\right)}-1\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) - 1/8*a^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/
d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7
*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*
c) - e^(-8*d*x - 8*c) - 1))) - 2*a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x
 - c) - 1)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3356 vs. $2(93) = 186$.
time = 0.44, size = 3356, normalized size = 33.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(b^2*cosh(d*x + c)^14 + 14*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + b^2*si
nh(d*x + c)^14 - 13*b^2*cosh(d*x + c)^12 + 13*(7*b^2*cosh(d*x + c)^2 - b^2)
*sinh(d*x + c)^12 + 52*(7*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d
*x + c)^11 + 3*(6*a^2 + 11*b^2)*cosh(d*x + c)^10 + (1001*b^2*cosh(d*x + c)^
4 - 858*b^2*cosh(d*x + c)^2 + 18*a^2 + 33*b^2)*sinh(d*x + c)^10 + 2*(1001*b
^2*cosh(d*x + c)^5 - 1430*b^2*cosh(d*x + c)^3 + 15*(6*a^2 + 11*b^2)*cosh(d*
x + c))*sinh(d*x + c)^9 - 3*(22*a^2 + 7*b^2)*cosh(d*x + c)^8 + 3*(1001*b^2*
cosh(d*x + c)^6 - 2145*b^2*cosh(d*x + c)^4 + 45*(6*a^2 + 11*b^2)*cosh(d*x +
 c)^2 - 22*a^2 - 7*b^2)*sinh(d*x + c)^8 + 24*(143*b^2*cosh(d*x + c)^7 - 429
*b^2*cosh(d*x + c)^5 + 15*(6*a^2 + 11*b^2)*cosh(d*x + c)^3 - (22*a^2 + 7*b^
2)*cosh(d*x + c))*sinh(d*x + c)^7 - 3*(22*a^2 + 7*b^2)*cosh(d*x + c)^6 + 3*
(1001*b^2*cosh(d*x + c)^8 - 4004*b^2*cosh(d*x + c)^6 + 210*(6*a^2 + 11*b^2)
*cosh(d*x + c)^4 - 28*(22*a^2 + 7*b^2)*cosh(d*x + c)^2 - 22*a^2 - 7*b^2)*si
nh(d*x + c)^6 + 2*(1001*b^2*cosh(d*x + c)^9 - 5148*b^2*cosh(d*x + c)^7 + 37
8*(6*a^2 + 11*b^2)*cosh(d*x + c)^5 - 84*(22*a^2 + 7*b^2)*cosh(d*x + c)^3 -
9*(22*a^2 + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(6*a^2 + 11*b^2)*cosh
(d*x + c)^4 + (1001*b^2*cosh(d*x + c)^10 - 6435*b^2*cosh(d*x + c)^8 + 630*(
6*a^2 + 11*b^2)*cosh(d*x + c)^6 - 210*(22*a^2 + 7*b^2)*cosh(d*x + c)^4 - 45
*(22*a^2 + 7*b^2)*cosh(d*x + c)^2 + 18*a^2 + 33*b^2)*sinh(d*x + c)^4 - 13*b
^2*cosh(d*x + c)^2 + 4*(91*b^2*cosh(d*x + c)^11 - 715*b^2*cosh(d*x + c)^9 +
 90*(6*a^2 + 11*b^2)*cosh(d*x + c)^7 - 42*(22*a^2 + 7*b^2)*cosh(d*x + c)^5
- 15*(22*a^2 + 7*b^2)*cosh(d*x + c)^3 + 3*(6*a^2 + 11*b^2)*cosh(d*x + c))*s
inh(d*x + c)^3 + (91*b^2*cosh(d*x + c)^12 - 858*b^2*cosh(d*x + c)^10 + 135*
(6*a^2 + 11*b^2)*cosh(d*x + c)^8 - 84*(22*a^2 + 7*b^2)*cosh(d*x + c)^6 - 45
*(22*a^2 + 7*b^2)*cosh(d*x + c)^4 + 18*(6*a^2 + 11*b^2)*cosh(d*x + c)^2 - 1
3*b^2)*sinh(d*x + c)^2 + b^2 - 3*((3*a^2 + 16*a*b)*cosh(d*x + c)^11 + 11*(3
*a^2 + 16*a*b)*cosh(d*x + c)*sinh(d*x + c)^10 + (3*a^2 + 16*a*b)*sinh(d*x +
 c)^11 - 4*(3*a^2 + 16*a*b)*cosh(d*x + c)^9 + (55*(3*a^2 + 16*a*b)*cosh(d*x
 + c)^2 - 12*a^2 - 64*a*b)*sinh(d*x + c)^9 + 3*(55*(3*a^2 + 16*a*b)*cosh(d*
x + c)^3 - 12*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^8 + 6*(3*a^2 +
16*a*b)*cosh(d*x + c)^7 + 6*(55*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 - 24*(3*a^
2 + 16*a*b)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b)*sinh(d*x + c)^7 + 42*(11*(3*a
^2 + 16*a*b)*cosh(d*x + c)^5 - 8*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 + (3*a^2
+ 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 - 4*(3*a^2 + 16*a*b)*cosh(d*x + c)
^5 + 2*(231*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 - 252*(3*a^2 + 16*a*b)*cosh(d*
x + c)^4 + 63*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 - 6*a^2 - 32*a*b)*sinh(d*x +
 c)^5 + 2*(165*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 - 252*(3*a^2 + 16*a*b)*cosh
(d*x + c)^5 + 105*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 - 10*(3*a^2 + 16*a*b)*co
sh(d*x + c))*sinh(d*x + c)^4 + (3*a^2 + 16*a*b)*cosh(d*x + c)^3 + (165*(3*a
^2 + 16*a*b)*cosh(d*x + c)^8 - 336*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 + 210*(
3*a^2 + 16*a*b)*cosh(d*x + c)^4 - 40*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 + 3*a
^2 + 16*a*b)*sinh(d*x + c)^3 + (55*(3*a^2 + 16*a*b)*cosh(d*x + c)^9 - 144*(
3*a^2 + 16*a*b)*cosh(d*x + c)^7 + 126*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 - 40
```

```
*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 + 3*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(
d*x + c)^2 + (11*(3*a^2 + 16*a*b)*cosh(d*x + c)^10 - 36*(3*a^2 + 16*a*b)*co
sh(d*x + c)^8 + 42*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 - 20*(3*a^2 + 16*a*b)*c
osh(d*x + c)^4 + 3*(3*a^2 + 16*a*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cos
h(d*x + c) + sinh(d*x + c) + 1) + 3*((3*a^2 + 16*a*b)*cosh(d*x + c)^11 + 11
*(3*a^2 + 16*a*b)*cosh(d*x + c)*sinh(d*x + c)^10 + (3*a^2 + 16*a*b)*sinh(d*
x + c)^11 - 4*(3*a^2 + 16*a*b)*cosh(d*x + c)^9 + (55*(3*a^2 + 16*a*b)*cosh(
d*x + c)^2 - 12*a^2 - 64*a*b)*sinh(d*x + c)^9 + 3*(55*(3*a^2 + 16*a*b)*cosh
(d*x + c)^3 - 12*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^8 + 6*(3*a^2
 + 16*a*b)*cosh(d*x + c)^7 + 6*(55*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 - 24*(3
*a^2 + 16*a*b)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b)*sinh(d*x + c)^7 + 42*(11*(
3*a^2 + 16*a*b)*cosh(d*x + c)^5 - 8*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 + (3*a
^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 - 4*(3*a^2 + 16*a*b)*cosh(d*x +
 c)^5 + 2*(231*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 - 252*(3*a^2 + 16*a*b)*cosh
(d*x + c)^4 + 63*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 - 6*a^2 - 32*a*b)*sinh(d*
x + c)^5 + 2*(165*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 - 252*(3*a^2 + 16*a*b)*c
osh(d*x + c)^5 + 105*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 - 10*(3*a^2 + 16*a*b)
*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^2 + 16*a*b)*cosh(d*x + c)^3 + (165*(
3*a^2 + 16*a*b)*cosh(d*x + c)^8 - 336*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 + 21
0*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 - 40*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 +
3*a^2 + 16*a*b)*sinh(d*x + c)^3 + (55*(3*a^2 + 16*a*b)*cosh(d*x + c)^9 - 14
4*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 + 126*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 -
 40*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 + 3*(3*a^2 + 16*a*b)*cosh(d*x + c))*si
nh(d*x + c)^2 + (11*(3*a^2 + 16*a*b)*cosh(d*x +...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.59, size = 179, normalized size = 1.77

$$\frac{2\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-24\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-3\,(3\,a^2+16\,ab)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)+3\,(3\,a^2+16\,ab)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)+\frac{12\left(3\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-20\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\right)^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 24*b^2*(e^(d*x + c) + e^(-d*x
- c)) - 3*(3*a^2 + 16*a*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) + 3*(3*a^2 +
```

```
 16*a*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) + 12*(3*a^2*(e^(d*x + c) + e^(
-d*x - c))^3 - 20*a^2*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x
 - c))^2 - 4)^2)/d
```

Mupad [B]
time = 0.22, size = 328, normalized size = 3.25

$$\frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{3\,b^2\,\mathrm{e}^{-c-d\,x}}{8\,d}-\frac{3\,b^2\,\mathrm{e}^{c+d\,x}}{8\,d}+\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(3\,a^2\,\sqrt{-d^2}+16\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{9\,a^4+96\,a^3\,b+256\,a^2\,b^2}}\right)\,\sqrt{9\,a^4+96\,a^3\,b+256\,a^2\,b^2}}{4\,\sqrt{-d^2}}-\frac{6\,a^2\,\mathrm{e}^{c+d\,x}}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{4\,a^2\,\mathrm{e}^{c+d\,x}}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}+\frac{3\,a^2\,\mathrm{e}^{c+d\,x}}{4\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{a^2\,\mathrm{e}^{c+d\,x}}{2\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^5,x)
```

```
[Out] (b^2*exp(- 3*c - 3*d*x))/(24*d) - (3*b^2*exp(- c - d*x))/(8*d) - (3*b^2*exp
(c + d*x))/(8*d) + (b^2*exp(3*c + 3*d*x))/(24*d) - (atan((exp(d*x)*exp(c)*(
3*a^2*(-d^2)^(1/2) + 16*a*b*(-d^2)^(1/2)))/(d*(96*a^3*b + 9*a^4 + 256*a^2*b
^2)^(1/2)))*(96*a^3*b + 9*a^4 + 256*a^2*b^2)^(1/2))/(4*(-d^2)^(1/2)) - (6*a
^2*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*
d*x) - 1)) - (4*a^2*exp(c + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*
x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (3*a^2*exp(c + d*x))/(4*
d*(exp(2*c + 2*d*x) - 1)) - (a^2*exp(c + d*x))/(2*d*(exp(4*c + 4*d*x) - 2*e
xp(2*c + 2*d*x) + 1))
```

3.205 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=84

$$-\frac{b^2x}{2}-\frac{a(a+2b)\coth(c+dx)}{d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a^2\coth^5(c+dx)}{5d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

```
[Out] -1/2*b^2*x-a*(a+2*b)*coth(d*x+c)/d+2/3*a^2*coth(d*x+c)^3/d-1/5*a^2*coth(d*x
+c)^5/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3296, 1273, 1816, 213}

$$-\frac{a^2\coth^5(c+dx)}{5d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a(a+2b)\coth(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/2*(b^2*x) - (a*(a + 2*b)*Coth[c + d*x])/d + (2*a^2*Coth[c + d*x]^3)/(3*d
) - (a^2*Coth[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^2}{x^6\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{\text{Subst}\left(\int \frac{-2a^2+6a^2x^2-2a(3a+2b)x^4+}{x^6\left(1-x^2\right)}\right.}{2}\\
&= \frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{\text{Subst}\left(\int \left(-\frac{2a^2}{x^6}+\frac{4a^2}{x^4}-\frac{2a(a+2b)}{x^2}\right.\right.}{2}\\
&= -\frac{a(a+2b)\coth(c+dx)}{d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a^2\coth^5(c+d}{5d}\\
&= -\frac{b^2x}{2}-\frac{a(a+2b)\coth(c+dx)}{d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a^2\coth^5}{5}
\end{aligned}
$$

Mathematica [A]
time = 0.60, size = 67, normalized size = 0.80

$$
\frac{-4a\coth(c+dx)\left(8a+30b-4a\operatorname{csch}^2(c+dx)+3a\operatorname{csch}^4(c+dx)\right)+15b^2(-2(c+dx)+\sinh(2(c+dx)))}{60d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-4*a*Coth[c + d*x]*(8*a + 30*b - 4*a*Csch[c + d*x]^2 + 3*a*Csch[c + d*x]^4
) + 15*b^2*(-2*(c + d*x) + Sinh[2*(c + d*x)]))/(60*d)
```

Maple [A]
time = 1.56, size = 140, normalized size = 1.67

method	result
risch	$-\frac{b^2x}{2}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}b^2}{8d}-\frac{4a\left(15b\,e^{8dx+8c}-60b\,e^{6dx+6c}+40a\,e^{4dx+4c}+90b\,e^{4dx+4c}-20a\,e^{2dx+2c}-60b\,e^{2dx+2c}+4a+15\right.}{15d\left(e^{2dx+2c}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2*x+1/8/d*exp(2*d*x+2*c)*b^2-1/8/d*exp(-2*d*x-2*c)*b^2-4/15*a*(15*b*
exp(8*d*x+8*c)-60*b*exp(6*d*x+6*c)+40*a*exp(4*d*x+4*c)+90*b*exp(4*d*x+4*c)-
20*a*exp(2*d*x+2*c)-60*b*exp(2*d*x+2*c)+4*a+15*b)/d/(exp(2*d*x+2*c)-1)^5
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(76) = 152$.
time = 0.28, size = 267, normalized size = 3.18

$$-\frac{1}{8}\,b^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{16}{15}\,a^2\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1)}-\frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1)}-\frac{1}{d(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1)}\right)+\frac{4\,ab}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 16/15*a^2*(5*e^(-
2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/
(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(76) = 152$.
time = 0.38, size = 457, normalized size = 5.44

$$\frac{15\,b^2\cosh\left(dx+c\right)^7+105\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6-\left(64\,a^2+240\,ab+75\,b^2\right)\cosh\left(dx+c\right)^5-4\left(15\,b^2dx-16\,a^2-60\,ab\right)\sinh\left(dx+c\right)^5+5\left(105\,b^2\cosh\left(dx+c\right)^3-\left(64\,a^2+240\,ab+75\,b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^4+5\left(64\,a^2+144\,ab+27\,b^2\right)\cosh\left(dx+c\right)^3+20\left(15\,b^2dx-2\left(15\,b^2dx-16\,a^2-60\,ab\right)\cosh\left(dx+c\right)^2-16\,a^2-60\,ab\right)\sinh\left(dx+c\right)^3+5\left(63\,b^2\cosh\left(dx+c\right)^5-2\left(64\,a^2+240\,ab+75\,b^2\right)\cosh\left(dx+c\right)^3+3\left(64\,a^2+144\,ab+27\,b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2-5\left(128\,a^2+96\,ab+15\,b^2\right)\cosh\left(dx+c\right)-20\left(\left(15\,b^2dx-16\,a^2-60\,ab\right)\cosh\left(dx+c\right)^4+30\,b^2dx-3\left(15\,b^2dx-16\,a^2-60\,ab\right)\cosh\left(dx+c\right)^2-32\,a^2-120\,ab\right)\sinh\left(dx+c\right)}{120\left(d\sinh\left(dx+c\right)^5+5\left(2\,d\cosh\left(dx+c\right)^2-d\right)\sinh\left(dx+c\right)^3+5\left(d\cosh\left(dx+c\right)^4-3\,d\cosh\left(dx+c\right)^2+2\,d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(15*b^2*cosh(d*x + c)^7 + 105*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - (64
*a^2 + 240*a*b + 75*b^2)*cosh(d*x + c)^5 - 4*(15*b^2*d*x - 16*a^2 - 60*a*b)
*sinh(d*x + c)^5 + 5*(105*b^2*cosh(d*x + c)^3 - (64*a^2 + 240*a*b + 75*b^2)
*cosh(d*x + c))*sinh(d*x + c)^4 + 5*(64*a^2 + 144*a*b + 27*b^2)*cosh(d*x +
c)^3 + 20*(15*b^2*d*x - 2*(15*b^2*d*x - 16*a^2 - 60*a*b)*cosh(d*x + c)^2 -
16*a^2 - 60*a*b)*sinh(d*x + c)^3 + 5*(63*b^2*cosh(d*x + c)^5 - 2*(64*a^2 +
240*a*b + 75*b^2)*cosh(d*x + c)^3 + 3*(64*a^2 + 144*a*b + 27*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^2 - 5*(128*a^2 + 96*a*b + 15*b^2)*cosh(d*x + c) - 20*((
15*b^2*d*x - 16*a^2 - 60*a*b)*cosh(d*x + c)^4 + 30*b^2*d*x - 3*(15*b^2*d*x
- 16*a^2 - 60*a*b)*cosh(d*x + c)^2 - 32*a^2 - 120*a*b)*sinh(d*x + c))/(d*si
nh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x
 + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(76) = 152$.
time = 0.49, size = 166, normalized size = 1.98

$$-\frac{60\,(dx+c)b^2-15\,b^2e^{(2\,dx+2\,c)}-15\,\left(2\,b^2e^{(2\,dx+2\,c)}-b^2\right)e^{(-2\,dx-2\,c)}+\frac{32\,\left(15\,abe^{(8\,dx+8\,c)}-60\,abe^{(6\,dx+6\,c)}+40\,a^2e^{(4\,dx+4\,c)}+90\,abe^{(4\,dx+4\,c)}-20\,a^2e^{(2\,dx+2\,c)}-60\,abe^{(2\,dx+2\,c)}+4\,a^2+15\,ab\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/120*(60*(d*x + c)*b^2 - 15*b^2*e^(2*d*x + 2*c) - 15*(2*b^2*e^(2*d*x + 2*
c) - b^2)*e^(-2*d*x - 2*c) + 32*(15*a*b*e^(8*d*x + 8*c) - 60*a*b*e^(6*d*x +
 6*c) + 40*a^2*e^(4*d*x + 4*c) + 90*a*b*e^(4*d*x + 4*c) - 20*a^2*e^(2*d*x +
 2*c) - 60*a*b*e^(2*d*x + 2*c) + 4*a^2 + 15*a*b)/(e^(2*d*x + 2*c) - 1)^5)/d
```

Mupad [**B**]
time = 0.75, size = 397, normalized size = 4.73

$$\frac{b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}-\frac{\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(4\,a^2+3\,b\,a\right)}{5\,d}-\frac{4\,a\,b}{5\,d}-\frac{12\,a\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{5\,d}+\frac{4\,a\,b\,{\mathrm{e}}^{6\,c+6\,d\,x}}{5\,d}}{6\,{\mathrm{e}}^{4\,c+4\,d\,x}-4\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{b^2\,x}{2}-\frac{\frac{4\,\left(4\,a^2+3\,b\,a\right)}{15\,d}-\frac{8\,a\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{5\,d}+\frac{4\,a\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{b^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}-\frac{\frac{8\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(4\,a^2+3\,b\,a\right)}{5\,d}+\frac{4\,a\,b}{5\,d}-\frac{16\,a\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{5\,d}-\frac{16\,a\,b\,{\mathrm{e}}^{6\,c+6\,d\,x}}{5\,d}+\frac{4\,a\,b\,{\mathrm{e}}^{8\,c+8\,d\,x}}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}-10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}-5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}-1}-\frac{8\,a\,b}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^6,x)
```

```
[Out] (b^2*exp(2*c + 2*d*x))/(8*d) - ((4*exp(2*c + 2*d*x)*(3*a*b + 4*a^2))/(5*d)
- (4*a*b)/(5*d) - (12*a*b*exp(4*c + 4*d*x))/(5*d) + (4*a*b*exp(6*c + 6*d*x)
)/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + ex
p(8*c + 8*d*x) + 1) - (b^2*x)/2 - ((4*(3*a*b + 4*a^2))/(15*d) - (8*a*b*exp(
2*c + 2*d*x))/(5*d) + (4*a*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d*x) -
 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (b^2*exp(- 2*c - 2*d*x))/(8*d
) - ((8*exp(4*c + 4*d*x)*(3*a*b + 4*a^2))/(5*d) + (4*a*b)/(5*d) - (16*a*b*e
xp(2*c + 2*d*x))/(5*d) - (16*a*b*exp(6*c + 6*d*x))/(5*d) + (4*a*b*exp(8*c +
 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*
d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - (8*a*b)/(5*d*(exp(2*c
 + 2*d*x) - 1))
```

3.206 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=111

$$\frac{a(5a+16b)\tanh^{-1}(\cosh(c+dx))}{16d}+\frac{b^2\cosh(c+dx)}{d}-\frac{a(5a+16b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d}+5a^2\coth(c+$$

```
[Out] 1/16*a*(5*a+16*b)*arctanh(cosh(d*x+c))/d+b^2*cosh(d*x+c)/d-1/16*a*(5*a+16*b
)*coth(d*x+c)*csch(d*x+c)/d+5/24*a^2*coth(d*x+c)*csch(d*x+c)^3/d-1/6*a^2*co
th(d*x+c)*csch(d*x+c)^5/d
```

Rubi [A]

time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3294, 1171, 1828, 396, 212}

$$-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}+\frac{a(5a+16b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a(5a+16b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d}+\frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (a*(5*a + 16*b)*ArcTanh[Cosh[c + d*x]])/(16*d) + (b^2*Cosh[c + d*x])/d - (a
*(5*a + 16*b)*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^2*Coth[c + d*x]*Cs
ch[c + d*x]^3)/(24*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \text{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^2}{\left(1-x^2\right)^4}\,dx,x,\cosh(c+dx)\right)}{d}$$

$$= -\frac{a^2\coth(c+dx)\text{csch}^5(c+dx)}{6d} - \text{Subst}\left(\int \frac{-5a^2-12ab-6b^2+6b(2a+}{(1-x^2)}\right.$$

$$= \frac{5a^2\coth(c+dx)\text{csch}^3(c+dx)}{24d} - \frac{a^2\coth(c+dx)\text{csch}^5(c+dx)}{6d}$$

$$= -\frac{a(5a+16b)\coth(c+dx)\text{csch}(c+dx)}{16d} + \frac{5a^2\coth(c+dx)\text{csc}}{24d}$$

$$= \frac{b^2\cosh(c+dx)}{d} - \frac{a(5a+16b)\coth(c+dx)\text{csch}(c+dx)}{16d} + 5a^2$$

$$= \frac{a(5a+16b)\tanh^{-1}(\cosh(c+dx))}{16d} + \frac{b^2\cosh(c+dx)}{d} - a(5a+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(111) = 222.
time = 0.04, size = 240, normalized size = 2.16

$$\frac{b^2\cosh(c)\cosh(dx)}{d} - \frac{5a^2\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{ab\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{4d} + \frac{a^2\text{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a^2\text{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{5a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16d} - \frac{ab\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{5a^2\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{ab\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{4d} - \frac{a^2\text{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a^2\text{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{b^2\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (b^2*Cosh[c]*Cosh[d*x])/d - (5*a^2*Csch[(c + d*x)/2]^2)/(64*d) - (a*b*Csch[
(c + d*x)/2]^2)/(4*d) + (a^2*Csch[(c + d*x)/2]^4)/(64*d) - (a^2*Csch[(c + d
*x)/2]^6)/(384*d) - (5*a^2*Log[Tanh[(c + d*x)/2]])/(16*d) - (a*b*Log[Tanh[(
c + d*x)/2]])/d - (5*a^2*Sech[(c + d*x)/2]^2)/(64*d) - (a*b*Sech[(c + d*x)/
2]^2)/(4*d) - (a^2*Sech[(c + d*x)/2]^4)/(64*d) - (a^2*Sech[(c + d*x)/2]^6)/
(384*d) + (b^2*Sinh[c]*Sinh[d*x])/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(103) = 206$.
time = 1.54, size = 250, normalized size = 2.25

method	result
risch	$\frac{e^{dx+c} b^{2}}{2d}+\frac{e^{-dx-c} b^{2}}{2d}-\frac{a\,e^{dx+c}\left(15a\,e^{10dx+10c}+48b\,e^{10dx+10c}-85a\,e^{8dx+8c}-144b\,e^{8dx+8c}+198a\,e^{6dx+6c}+96b\,e^{6dx+6c}+198a\,e^{4c}\right.}{24d\left(e^{2dx+2c}-1\right)^{6}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*exp(d*x+c)*b^2+1/2/d*exp(-d*x-c)*b^2-1/24*a*exp(d*x+c)*(15*a*exp(10*d
*x+10*c)+48*b*exp(10*d*x+10*c)-85*a*exp(8*d*x+8*c)-144*b*exp(8*d*x+8*c)+198
*a*exp(6*d*x+6*c)+96*b*exp(6*d*x+6*c)+198*a*exp(4*d*x+4*c)+96*b*exp(4*d*x+4
*c)-85*a*exp(2*d*x+2*c)-144*b*exp(2*d*x+2*c)+15*a+48*b)/d/(exp(2*d*x+2*c)-1
)^6-5/16*a^2/d*ln(exp(d*x+c)-1)-a*b/d*ln(exp(d*x+c)-1)+5/16*a^2/d*ln(exp(d*
x+c)+1)+a*b/d*ln(exp(d*x+c)+1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(103) = 206$.
time = 0.27, size = 299, normalized size = 2.69

$$\frac{1}{2}\,b^{2}\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{48}\,a^{2}\left(\frac{15\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15\,e^{(-dx-c)}-85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}+198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9\,dx-9\,c)}+15\,e^{(-11\,dx-11\,c)}\right)}{d(6\,e^{(-2\,dx-2\,c)}-15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}-15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}-e^{(-12\,dx-12\,c)}-1)}\right)+ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/48*a^2*(15*log(e^(-d*x - c) +
1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c
) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*
e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6
*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*
c) - 1))) + a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(
-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1
)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4500 vs. $2(103) = 206$.
time = 0.42, size = 4500, normalized size = 40.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(24*b^2*cosh(d*x + c)^14 + 336*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + 24
*b^2*sinh(d*x + c)^14 - 6*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^12 + 6*(3
64*b^2*cosh(d*x + c)^2 - 5*a^2 - 16*a*b - 20*b^2)*sinh(d*x + c)^12 + 24*(36
4*b^2*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^11 + 2*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^10 + 2*(12012*b^2*c
osh(d*x + c)^4 - 198*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^2 + 85*a^2 + 1
44*a*b + 108*b^2)*sinh(d*x + c)^10 + 4*(12012*b^2*cosh(d*x + c)^5 - 330*(5*
a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^3 + 5*(85*a^2 + 144*a*b + 108*b^2)*cos
h(d*x + c))*sinh(d*x + c)^9 - 12*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^8
 + 6*(12012*b^2*cosh(d*x + c)^6 - 495*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x +
c)^4 + 15*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^2 - 66*a^2 - 32*a*b -
20*b^2)*sinh(d*x + c)^8 + 48*(1716*b^2*cosh(d*x + c)^7 - 99*(5*a^2 + 16*a*b
 + 20*b^2)*cosh(d*x + c)^5 + 5*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^3
 - 2*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 12*(33*a^2
 + 16*a*b + 10*b^2)*cosh(d*x + c)^6 + 12*(6006*b^2*cosh(d*x + c)^8 - 462*(5
*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^6 + 35*(85*a^2 + 144*a*b + 108*b^2)*c
osh(d*x + c)^4 - 28*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^2 - 33*a^2 - 1
6*a*b - 10*b^2)*sinh(d*x + c)^6 + 24*(2002*b^2*cosh(d*x + c)^9 - 198*(5*a^2
 + 16*a*b + 20*b^2)*cosh(d*x + c)^7 + 21*(85*a^2 + 144*a*b + 108*b^2)*cosh(
d*x + c)^5 - 28*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^3 - 3*(33*a^2 + 16
*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(85*a^2 + 144*a*b + 108*b
^2)*cosh(d*x + c)^4 + 2*(12012*b^2*cosh(d*x + c)^10 - 1485*(5*a^2 + 16*a*b
+ 20*b^2)*cosh(d*x + c)^8 + 210*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^
6 - 420*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^4 - 90*(33*a^2 + 16*a*b +
10*b^2)*cosh(d*x + c)^2 + 85*a^2 + 144*a*b + 108*b^2)*sinh(d*x + c)^4 + 8*(
1092*b^2*cosh(d*x + c)^11 - 165*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^9 +
 30*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^7 - 84*(33*a^2 + 16*a*b + 10
*b^2)*cosh(d*x + c)^5 - 30*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^3 + (85
*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(5*a^2 + 16*a*
b + 20*b^2)*cosh(d*x + c)^2 + 6*(364*b^2*cosh(d*x + c)^12 - 66*(5*a^2 + 16*
a*b + 20*b^2)*cosh(d*x + c)^10 + 15*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x +
 c)^8 - 56*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^6 - 30*(33*a^2 + 16*a*b
 + 10*b^2)*cosh(d*x + c)^4 + 2*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^2
 - 5*a^2 - 16*a*b - 20*b^2)*sinh(d*x + c)^2 + 24*b^2 + 3*((5*a^2 + 16*a*b)*
cosh(d*x + c)^13 + 13*(5*a^2 + 16*a*b)*cosh(d*x + c)*sinh(d*x + c)^12 + (5*
a^2 + 16*a*b)*sinh(d*x + c)^13 - 6*(5*a^2 + 16*a*b)*cosh(d*x + c)^11 + 6*(1
```

```
3*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 - 5*a^2 - 16*a*b)*sinh(d*x + c)^11 + 22*
(13*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b)*cosh(d*x + c))*si
nh(d*x + c)^10 + 15*(5*a^2 + 16*a*b)*cosh(d*x + c)^9 + 5*(143*(5*a^2 + 16*a
*b)*cosh(d*x + c)^4 - 66*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 + 15*a^2 + 48*a*b
)*sinh(d*x + c)^9 + 9*(143*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 - 110*(5*a^2 +
16*a*b)*cosh(d*x + c)^3 + 15*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^
8 - 20*(5*a^2 + 16*a*b)*cosh(d*x + c)^7 + 4*(429*(5*a^2 + 16*a*b)*cosh(d*x
+ c)^6 - 495*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 + 135*(5*a^2 + 16*a*b)*cosh(d
*x + c)^2 - 25*a^2 - 80*a*b)*sinh(d*x + c)^7 + 4*(429*(5*a^2 + 16*a*b)*cosh
(d*x + c)^7 - 693*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 + 315*(5*a^2 + 16*a*b)*c
osh(d*x + c)^3 - 35*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 15*(5
*a^2 + 16*a*b)*cosh(d*x + c)^5 + 3*(429*(5*a^2 + 16*a*b)*cosh(d*x + c)^8 -
924*(5*a^2 + 16*a*b)*cosh(d*x + c)^6 + 630*(5*a^2 + 16*a*b)*cosh(d*x + c)^4
 - 140*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 + 25*a^2 + 80*a*b)*sinh(d*x + c)^5
+ 5*(143*(5*a^2 + 16*a*b)*cosh(d*x + c)^9 - 396*(5*a^2 + 16*a*b)*cosh(d*x +
 c)^7 + 378*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 - 140*(5*a^2 + 16*a*b)*cosh(d*
x + c)^3 + 15*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^4 - 6*(5*a^2 +
16*a*b)*cosh(d*x + c)^3 + 2*(143*(5*a^2 + 16*a*b)*cosh(d*x + c)^10 - 495*(5
*a^2 + 16*a*b)*cosh(d*x + c)^8 + 630*(5*a^2 + 16*a*b)*cosh(d*x + c)^6 - 350
*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 + 75*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 - 1
5*a^2 - 48*a*b)*sinh(d*x + c)^3 + 6*(13*(5*a^2 + 16*a*b)*cosh(d*x + c)^11 -
 55*(5*a^2 + 16*a*b)*cosh(d*x + c)^9 + 90*(5*a^2 + 16*a*b)*cosh(d*x + c)^7
- 70*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 + 25*(5*a^2 + 16*a*b)*cosh(d*x + c)^3
 - 3*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^2 + 16*a*b)*cos
h(d*x + c) + (13*(5*a^2 + 16*a*b)*cosh(d*x + c)^12 - 66*(5*a^2 + 16*a*b)*co
sh(d*x + c)^10 + 135*(5*a^2 + 16*a*b)*cosh(d*x + c)^8 - 140*(5*a^2 + 16*a*b
)*cosh(d*x + c)^6 + 75*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 - 18*(5*a^2 + 16*a*
b)*cosh(d*x + c)^2 + 5*a^2 + 16*a*b)*sinh(d*x + c))*log(cosh(d*x + c) + sin
h(d*x + c) + 1) - 3*((5*a^2 + 16*a*b)*cosh(d*x + c)^13 + 13*(5*a^2 + 16*a*b
)*cosh(d*x + c)*sinh(d*x + c)^12 + (5*a^2 + 16*a*b)*sinh(d*x + c)^13 - 6*(5
*a^2 + 16*a*b)*cosh(d*x + c)^11 + 6*(13*(5*a^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(103) = 206.
time = 0.51, size = 243, normalized size = 2.19

$$\frac{48\,b^{2}\left(e^{(dx+c)}+e^{(-dx-c)}\right)+3\,(5\,a^{2}+16\,ab)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\,(5\,a^{2}+16\,ab)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(15\,a^{2}\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{5}+48\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{5}-160\,a^{2}\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{3}-384\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{3}+528\,a^{2}\left(e^{(dx+c)}+e^{(-dx-c)}\right)+768\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{2}-4\right)^{3}}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/96*(48*b^2*(e^(d*x + c) + e^(-d*x - c)) + 3*(5*a^2 + 16*a*b)*log(e^(d*x +
 c) + e^(-d*x - c) + 2) - 3*(5*a^2 + 16*a*b)*log(e^(d*x + c) + e^(-d*x - c)
 - 2) - 4*(15*a^2*(e^(d*x + c) + e^(-d*x - c))^5 + 48*a*b*(e^(d*x + c) + e^
(-d*x - c))^5 - 160*a^2*(e^(d*x + c) + e^(-d*x - c))^3 - 384*a*b*(e^(d*x +
c) + e^(-d*x - c))^3 + 528*a^2*(e^(d*x + c) + e^(-d*x - c)) + 768*a*b*(e^(d
*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^3)/d
```

Mupad [B]

time = 0.82, size = 535, normalized size = 4.82

$$\frac{b^2\,{\mathrm{e}}^{c+d\,x}}{2\,d}-\frac{\frac{8\,{\mathrm{e}}^{5\,c+5\,d\,x}\,\left(4\,a^2+3\,b\,a\right)}{3\,d}+\frac{4\,a\,b\,{\mathrm{e}}^{c+d\,x}}{3\,d}-\frac{16\,a\,b\,{\mathrm{e}}^{3\,c+3\,d\,x}}{3\,d}-\frac{16\,a\,b\,{\mathrm{e}}^{7\,c+7\,d\,x}}{3\,d}+\frac{4\,a\,b\,{\mathrm{e}}^{9\,c+9\,d\,x}}{3\,d}}{15\,{\mathrm{e}}^{4\,c+4\,d\,x}-6\,{\mathrm{e}}^{2\,c+2\,d\,x}-20\,{\mathrm{e}}^{6\,c+6\,d\,x}+15\,{\mathrm{e}}^{8\,c+8\,d\,x}-6\,{\mathrm{e}}^{10\,c+10\,d\,x}+{\mathrm{e}}^{12\,c+12\,d\,x}+1}+\frac{b^2\,{\mathrm{e}}^{-c-d\,x}}{2\,d}+\frac{\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\left(5\,a^2\,\sqrt{-d^2}+16\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{25\,a^4+160\,a^3\,b+256\,a^2\,b^2}}\right)\,\sqrt{25\,a^4+160\,a^3\,b+256\,a^2\,b^2}}{8\,\sqrt{-d^2}}-\frac{a^2\,{\mathrm{e}}^{c+d\,x}}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}-\frac{22\,a^2\,{\mathrm{e}}^{c+d\,x}}{3\,d\,\left(6\,{\mathrm{e}}^{4\,c+4\,d\,x}-4\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1\right)}-\frac{16\,a^2\,{\mathrm{e}}^{c+d\,x}}{3\,d\,\left(5\,{\mathrm{e}}^{2\,c+2\,d\,x}-10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}-5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}-1\right)}-\frac{{\mathrm{e}}^{c+d\,x}\,\left(5\,a^2+16\,b\,a\right)}{8\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{{\mathrm{e}}^{c+d\,x}\,\left(32\,a\,b-5\,a^2\right)}{12\,d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^7,x)
```

```
[Out] (b^2*exp(c + d*x))/(2*d) - ((8*exp(5*c + 5*d*x)*(3*a*b + 4*a^2))/(3*d) + (4
*a*b*exp(c + d*x))/(3*d) - (16*a*b*exp(3*c + 3*d*x))/(3*d) - (16*a*b*exp(7*
c + 7*d*x))/(3*d) + (4*a*b*exp(9*c + 9*d*x))/(3*d))/(15*exp(4*c + 4*d*x) -
6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c
 + 10*d*x) + exp(12*c + 12*d*x) + 1) + (b^2*exp(- c - d*x))/(2*d) + (atan((
exp(d*x)*exp(c)*(5*a^2*(-d^2)^(1/2) + 16*a*b*(-d^2)^(1/2)))/(d*(160*a^3*b +
 25*a^4 + 256*a^2*b^2)^(1/2)))*(160*a^3*b + 25*a^4 + 256*a^2*b^2)^(1/2))/(8
*(-d^2)^(1/2)) - (a^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) - 1)) - (22*a^2*exp(c + d*x))/(3*d*(6*exp(4*c + 4
*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) -
(16*a^2*exp(c + d*x))/(3*d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (exp(c +
d*x)*(16*a*b + 5*a^2))/(8*d*(exp(2*c + 2*d*x) - 1)) - (exp(c + d*x)*(32*a*b
 - 5*a^2))/(12*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.207 $\int \sinh^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=220

$$\frac{(a+b)^3\cosh(c+dx)}{d}-\frac{2(a+b)^2(a+4b)\cosh^3(c+dx)}{3d}+\frac{(a+b)\left(a^2+17ab+28b^2\right)\cosh^5(c+dx)}{5d}-4b(3a^2$$

```
[Out] (a+b)^3*cosh(d*x+c)/d-2/3*(a+b)^2*(a+4*b)*cosh(d*x+c)^3/d+1/5*(a+b)*(a^2+17
*a*b+28*b^2)*cosh(d*x+c)^5/d-4/7*b*(3*a^2+15*a*b+14*b^2)*cosh(d*x+c)^7/d+1/
9*b*(3*a^2+45*a*b+70*b^2)*cosh(d*x+c)^9/d-2/11*b^2*(9*a+28*b)*cosh(d*x+c)^1
1/d+1/13*b^2*(3*a+28*b)*cosh(d*x+c)^13/d-8/15*b^3*cosh(d*x+c)^15/d+1/17*b^3
*cosh(d*x+c)^17/d
```

Rubi [A]
time = 0.16, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3294, 1167}

$$\frac{b(3a^2+45ab+70b^2)\cosh^9(c+dx)}{9d}-\frac{4b(3a^2+15ab+14b^2)\cosh^7(c+dx)}{7d}+\frac{(a+b)\left(a^2+17ab+28b^2\right)\cosh^5(c+dx)}{5d}+\frac{b^2(3a+28b)\cosh^{13}(c+dx)}{13d}-\frac{2b^2(9a+28b)\cosh^{11}(c+dx)}{11d}-\frac{2(a+b)^2(a+4b)\cosh^3(c+dx)}{3d}+\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{b^3\cosh^{17}(c+dx)}{17d}-\frac{8b^3\cosh^{15}(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((a + b)^3*Cosh[c + d*x])/d - (2*(a + b)^2*(a + 4*b)*Cosh[c + d*x]^3)/(3*d)
 + ((a + b)*(a^2 + 17*a*b + 28*b^2)*Cosh[c + d*x]^5)/(5*d) - (4*b*(3*a^2 +
15*a*b + 14*b^2)*Cosh[c + d*x]^7)/(7*d) + (b*(3*a^2 + 45*a*b + 70*b^2)*Cosh
[c + d*x]^9)/(9*d) - (2*b^2*(9*a + 28*b)*Cosh[c + d*x]^11)/(11*d) + (b^2*(3
*a + 28*b)*Cosh[c + d*x]^13)/(13*d) - (8*b^3*Cosh[c + d*x]^15)/(15*d) + (b^
3*Cosh[c + d*x]^17)/(17*d)
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \sinh^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int\left(1-x^2\right)^2\left(a+b-2bx^2+bx^4\right)^3\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int\left((a+b)^3-2(a+b)^2(a+4b)x^2+(a+b)\left(a^2+17ab+\right.\right.\right.}{}$$

$$= \frac{(a+b)^3\cosh(c+dx)}{d} - \frac{2(a+b)^2(a+4b)\cosh^3(c+dx)}{3d} + \frac{(a+}{}$$

Mathematica [**A**]
time = 1.61, size = 288, normalized size = 1.31

$$\frac{1531530(20480a^3+48384a^2b+41184ab^2+12155b^3)\cosh(c+dx)-2042040(2560a^3+8064a^2b+7722ab^2+2431b^3)\cosh(3(c+dx))+627314688a^3\cosh(5(c+dx))+4234374144a^2b\cosh(5(c+dx))+5256210960ab^2\cosh(5(c+dx))+1895421528b^3\cosh(5(c+dx))-756138240a^2b\cosh(7(c+dx))-1501774560ab^2\cosh(7(c+dx))-676936260b^3\cosh(7(c+dx))+65345280a^2b\cosh(9(c+dx))+318558240ab^2\cosh(9(c+dx))+202502300b^3\cosh(9(c+dx))-43439760ab^2\cosh(11(c+dx))-47338200b^3\cosh(11(c+dx))+2827440ab^2\cosh(13(c+dx))+8011080b^3\cosh(13(c+dx))-867867b^3\cosh(15(c+dx))+45045b^3\cosh(17(c+dx))}{50185175040d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (1531530*(20480*a^3 + 48384*a^2*b + 41184*a*b^2 + 12155*b^3)*Cosh[c + d*x]
 - 2042040*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*Cosh[3*(c + d*x)]
 + 627314688*a^3*Cosh[5*(c + d*x)] + 4234374144*a^2*b*Cosh[5*(c + d*x)] + 5
256210960*a*b^2*Cosh[5*(c + d*x)] + 1895421528*b^3*Cosh[5*(c + d*x)] - 7561
38240*a^2*b*Cosh[7*(c + d*x)] - 1501774560*a*b^2*Cosh[7*(c + d*x)] - 676936
260*b^3*Cosh[7*(c + d*x)] + 65345280*a^2*b*Cosh[9*(c + d*x)] + 318558240*a*
b^2*Cosh[9*(c + d*x)] + 202502300*b^3*Cosh[9*(c + d*x)] - 43439760*a*b^2*Co
sh[11*(c + d*x)] - 47338200*b^3*Cosh[11*(c + d*x)] + 2827440*a*b^2*Cosh[13*
(c + d*x)] + 8011080*b^3*Cosh[13*(c + d*x)] - 867867*b^3*Cosh[15*(c + d*x)]
 + 45045*b^3*Cosh[17*(c + d*x)])/(50185175040*d)
```

Maple [**A**]
time = 1.35, size = 259, normalized size = 1.18

method	result
default	$\frac{\left(-\frac{85}{8192}b^3-\frac{39}{4096}a\,b^2\right)\cosh(11dx+11c)}{11d}+\frac{\left(\frac{17}{8192}b^3+\frac{3}{4096}a\,b^2\right)\cosh(13dx+13c)}{13d}+\frac{\left(-\frac{1547}{16384}b^3-\frac{429}{2048}a\,b^2-\frac{27}{256}a^2b\right)\cosh(7dx+7c)}{7d}+$
risch	$\frac{429a\,b^2\mathrm{e}^{5dx+5c}}{8192d}-\frac{1287a\,b^2\mathrm{e}^{3dx+3c}}{8192d}+\frac{b^3\mathrm{e}^{-17dx-17c}}{2228224d}+\frac{189b\,\mathrm{e}^{dx+c}a^2}{256d}+\frac{595b^3\mathrm{e}^{9dx+9c}}{294912d}+\frac{\mathrm{e}^{5dx+5c}a^3}{160d}+\frac{1547\,\mathrm{e}^{5dx+5c}b^3}{81920d}-5\,\mathrm{e}^{3}$

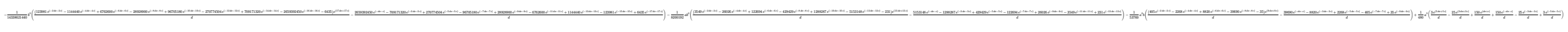
Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*(-85/8192*b^3-39/4096*a*b^2)/d*cosh(11*d*x+11*c)+1/13*(17/8192*b^3+3/4
096*a*b^2)/d*cosh(13*d*x+13*c)+1/7*(-1547/16384*b^3-429/2048*a*b^2-27/256*a
^2*b)/d*cosh(7*d*x+7*c)+1/9*(595/16384*b^3+117/2048*a*b^2+3/256*a^2*b)/d*co
sh(9*d*x+9*c)+1/3*(-2431/8192*b^3-3861/4096*a*b^2-63/64*a^2*b-5/16*a^3)/d*c
```

```
osh(3*d*x+3*c)+1/5*(1547/8192*b^3+2145/4096*a*b^2+27/64*a^2*b+1/16*a^3)/d*c
osh(5*d*x+5*c)+(12155/32768*b^3+1287/1024*a*b^2+189/128*a^2*b+5/8*a^3)/d*co
sh(d*x+c)-17/983040*b^3/d*cosh(15*d*x+15*c)+1/1114112*b^3/d*cosh(17*d*x+17*
c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(204) = 408$.
time = 0.27, size = 600, normalized size = 2.73



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/14338621440*b^3*((123981*e^(-2*d*x - 2*c) - 1144440*e^(-4*d*x - 4*c) + 6
762600*e^(-6*d*x - 6*c) - 28928900*e^(-8*d*x - 8*c) + 96705180*e^(-10*d*x -
 10*c) - 270774504*e^(-12*d*x - 12*c) + 709171320*e^(-14*d*x - 14*c) - 2659
392450*e^(-16*d*x - 16*c) - 6435)*e^(17*d*x + 17*c)/d - (2659392450*e^(-d*x
 - c) - 709171320*e^(-3*d*x - 3*c) + 270774504*e^(-5*d*x - 5*c) - 96705180*
e^(-7*d*x - 7*c) + 28928900*e^(-9*d*x - 9*c) - 6762600*e^(-11*d*x - 11*c) +
 1144440*e^(-13*d*x - 13*c) - 123981*e^(-15*d*x - 15*c) + 6435*e^(-17*d*x -
 17*c))/d) - 1/8200192*a*b^2*((3549*e^(-2*d*x - 2*c) - 26026*e^(-4*d*x - 4*
c) + 122694*e^(-6*d*x - 6*c) - 429429*e^(-8*d*x - 8*c) + 1288287*e^(-10*d*x
 - 10*c) - 5153148*e^(-12*d*x - 12*c) - 231)*e^(13*d*x + 13*c)/d - (5153148
*e^(-d*x - c) - 1288287*e^(-3*d*x - 3*c) + 429429*e^(-5*d*x - 5*c) - 122694
*e^(-7*d*x - 7*c) + 26026*e^(-9*d*x - 9*c) - 3549*e^(-11*d*x - 11*c) + 231*
e^(-13*d*x - 13*c))/d) - 1/53760*a^2*b*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*
d*x - 4*c) + 8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x
+ 9*c)/d - (39690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5
*c) - 405*e^(-7*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) + 1/480*a^3*(3*e^(5*d*
x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d
- 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(204) = 408$.
time = 0.37, size = 1030, normalized size = 4.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/50185175040*(45045*b^3*cosh(d*x + c)^17 + 765765*b^3*cosh(d*x + c)*sinh(d
*x + c)^16 - 867867*b^3*cosh(d*x + c)^15 + 765765*(40*b^3*cosh(d*x + c)^3 -
 17*b^3*cosh(d*x + c))*sinh(d*x + c)^14 + 471240*(6*a*b^2 + 17*b^3)*cosh(d*
x + c)^13 + 255255*(1092*b^3*cosh(d*x + c)^5 - 1547*b^3*cosh(d*x + c)^3 + 2
```

```
4*(6*a*b^2 + 17*b^3)*cosh(d*x + c))*sinh(d*x + c)^12 - 556920*(78*a*b^2 + 8
5*b^3)*cosh(d*x + c)^11 + 153153*(5720*b^3*cosh(d*x + c)^7 - 17017*b^3*cosh
(d*x + c)^5 + 880*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^3 - 40*(78*a*b^2 + 85*b^
3)*cosh(d*x + c))*sinh(d*x + c)^10 + 340340*(192*a^2*b + 936*a*b^2 + 595*b^
3)*cosh(d*x + c)^9 + 765765*(1430*b^3*cosh(d*x + c)^9 - 7293*b^3*cosh(d*x +
 c)^7 + 792*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^5 - 120*(78*a*b^2 + 85*b^3)*co
sh(d*x + c)^3 + 4*(192*a^2*b + 936*a*b^2 + 595*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^8 - 437580*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*cosh(d*x + c)^7 + 255
255*(2184*b^3*cosh(d*x + c)^11 - 17017*b^3*cosh(d*x + c)^9 + 3168*(6*a*b^2
+ 17*b^3)*cosh(d*x + c)^7 - 1008*(78*a*b^2 + 85*b^3)*cosh(d*x + c)^5 + 112*
(192*a^2*b + 936*a*b^2 + 595*b^3)*cosh(d*x + c)^3 - 12*(1728*a^2*b + 3432*a
*b^2 + 1547*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 1225224*(512*a^3 + 3456*a
^2*b + 4290*a*b^2 + 1547*b^3)*cosh(d*x + c)^5 + 765765*(140*b^3*cosh(d*x +
c)^13 - 1547*b^3*cosh(d*x + c)^11 + 440*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^9
- 240*(78*a*b^2 + 85*b^3)*cosh(d*x + c)^7 + 56*(192*a^2*b + 936*a*b^2 + 595
*b^3)*cosh(d*x + c)^5 - 20*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*cosh(d*x +
c)^3 + 8*(512*a^3 + 3456*a^2*b + 4290*a*b^2 + 1547*b^3)*cosh(d*x + c))*sinh
(d*x + c)^4 - 2042040*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*cosh(
d*x + c)^3 + 765765*(8*b^3*cosh(d*x + c)^15 - 119*b^3*cosh(d*x + c)^13 + 48
*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^11 - 40*(78*a*b^2 + 85*b^3)*cosh(d*x + c)
^9 + 16*(192*a^2*b + 936*a*b^2 + 595*b^3)*cosh(d*x + c)^7 - 12*(1728*a^2*b
+ 3432*a*b^2 + 1547*b^3)*cosh(d*x + c)^5 + 16*(512*a^3 + 3456*a^2*b + 4290*
a*b^2 + 1547*b^3)*cosh(d*x + c)^3 - 8*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 +
 2431*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 1531530*(20480*a^3 + 48384*a^2*
b + 41184*a*b^2 + 12155*b^3)*cosh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(204) = 408$.
time = 17.34, size = 592, normalized size = 2.69

$$\begin{cases} \frac{a^3\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4a^3\sinh^2(c+dx)\cosh^3(c+dx)}{3d} + \frac{8a^3\cosh^5(c+dx)}{15d} + \frac{3a^2b\sinh^8(c+dx)\cosh(c+dx)}{d} - \frac{8a^2b\sinh^6(c+dx)\cosh^3(c+dx)}{d} + \frac{48a^2b\sinh^4(c+dx)\cosh^5(c+dx)}{5d} - \frac{192a^2b\sinh^2(c+dx)\cosh^7(c+dx)}{35d} + \frac{128a^2b\cosh^9(c+dx)}{105d} + \frac{3ab^2\sinh^{12}(c+dx)\cosh(c+dx)}{d} - \frac{12ab^2\sinh^{10}(c+dx)\cosh^3(c+dx)}{d} + \frac{24ab^2\sinh^8(c+dx)\cosh^5(c+dx)}{d} - \frac{192ab^2\sinh^6(c+dx)\cosh^7(c+dx)}{7d} + \frac{128ab^2\sinh^4(c+dx)\cosh^9(c+dx)}{7d} - \frac{512ab^2\sinh^2(c+dx)\cosh^{11}(c+dx)}{77d} + \frac{1024ab^2\cosh^{13}(c+dx)}{1001d} + \frac{b^3\sinh^{16}(c+dx)\cosh(c+dx)}{d} - \frac{16b^3\sinh^{14}(c+dx)\cosh^3(c+dx)}{3d} + \frac{224b^3\sinh^{12}(c+dx)\cosh^5(c+dx)}{15d} - \frac{128b^3\sinh^{10}(c+dx)\cosh^7(c+dx)}{5d} + \frac{256b^3\sinh^8(c+dx)\cosh^9(c+dx)}{9d} - \frac{2048b^3\sinh^6(c+dx)\cosh^{11}(c+dx)}{99d} + \frac{4096b^3\sinh^4(c+dx)\cosh^{13}(c+dx)}{429d} - \frac{16384b^3\sinh^2(c+dx)\cosh^{15}(c+dx)}{6435d} + \frac{32768b^3\cosh^{17}(c+dx)}{109395d} & \text{for}\: d \neq 0 \\ x\left(a + b\sinh^4(c)\right)^3\sinh^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*a**3*sinh(c + d*x)**2*
cosh(c + d*x)**3/(3*d) + 8*a**3*cosh(c + d*x)**5/(15*d) + 3*a**2*b*sinh(c +
 d*x)**8*cosh(c + d*x)/d - 8*a**2*b*sinh(c + d*x)**6*cosh(c + d*x)**3/d + 4
8*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 192*a**2*b*sinh(c + d*x)
**2*cosh(c + d*x)**7/(35*d) + 128*a**2*b*cosh(c + d*x)**9/(105*d) + 3*a*b**
2*sinh(c + d*x)**12*cosh(c + d*x)/d - 12*a*b**2*sinh(c + d*x)**10*cosh(c +
d*x)**3/d + 24*a*b**2*sinh(c + d*x)**8*cosh(c + d*x)**5/d - 192*a*b**2*sinh
(c + d*x)**6*cosh(c + d*x)**7/(7*d) + 128*a*b**2*sinh(c + d*x)**4*cosh(c +
d*x)**9/(7*d) - 512*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**11/(77*d) + 1024
*a*b**2*cosh(c + d*x)**13/(1001*d) + b**3*sinh(c + d*x)**16*cosh(c + d*x)/d
```

```
 - 16*b**3*sinh(c + d*x)**14*cosh(c + d*x)**3/(3*d) + 224*b**3*sinh(c + d*x
)**12*cosh(c + d*x)**5/(15*d) - 128*b**3*sinh(c + d*x)**10*cosh(c + d*x)**7
/(5*d) + 256*b**3*sinh(c + d*x)**8*cosh(c + d*x)**9/(9*d) - 2048*b**3*sinh(
c + d*x)**6*cosh(c + d*x)**11/(99*d) + 4096*b**3*sinh(c + d*x)**4*cosh(c +
d*x)**13/(429*d) - 16384*b**3*sinh(c + d*x)**2*cosh(c + d*x)**15/(6435*d) +
 32768*b**3*cosh(c + d*x)**17/(109395*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*
*3*sinh(c)**5, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(204) = 408.
time = 0.54, size = 520, normalized size = 2.36

$$\frac{b^{3}e^{(17\,dx+17\,c)}}{2228224\,d}-\frac{17\,b^{3}e^{(15\,dx+15\,c)}}{1966080\,d}-\frac{17\,b^{3}e^{(-15\,dx-15\,c)}}{1966080\,d}+\frac{b^{3}e^{(-17\,dx-17\,c)}}{2228224\,d}+\frac{(6\,ab^{2}+17\,b^{3})e^{(13\,dx+13\,c)}}{212992\,d}-\frac{(78\,ab^{2}+85\,b^{3})e^{(11\,dx+11\,c)}}{180224\,d}+\frac{(192\,a^{2}b+936\,ab^{2}+595\,b^{3})e^{(9\,dx+9\,c)}}{294912\,d}-\frac{(1728\,a^{2}b+3432\,ab^{2}+1547\,b^{3})e^{(7\,dx+7\,c)}}{229376\,d}+\frac{(512\,a^{3}+3456\,a^{2}b+4290\,ab^{2}+1547\,b^{3})e^{(5\,dx+5\,c)}}{81920\,d}-\frac{(2560\,a^{3}+8064\,a^{2}b+7722\,ab^{2}+2431\,b^{3})e^{(3\,dx+3\,c)}}{49152\,d}+\frac{(20480\,a^{3}+48384\,a^{2}b+41184\,ab^{2}+12155\,b^{3})e^{(dx+c)}}{65536\,d}+\frac{(20480\,a^{3}+48384\,a^{2}b+41184\,ab^{2}+12155\,b^{3})e^{(-dx-c)}}{65536\,d}-\frac{(2560\,a^{3}+8064\,a^{2}b+7722\,ab^{2}+2431\,b^{3})e^{(-3\,dx-3\,c)}}{49152\,d}+\frac{(512\,a^{3}+3456\,a^{2}b+4290\,ab^{2}+1547\,b^{3})e^{(-5\,dx-5\,c)}}{81920\,d}-\frac{(1728\,a^{2}b+3432\,ab^{2}+1547\,b^{3})e^{(-7\,dx-7\,c)}}{229376\,d}+\frac{(192\,a^{2}b+936\,ab^{2}+595\,b^{3})e^{(-9\,dx-9\,c)}}{294912\,d}-\frac{(78\,ab^{2}+85\,b^{3})e^{(-11\,dx-11\,c)}}{180224\,d}+\frac{(6\,ab^{2}+17\,b^{3})e^{(-13\,dx-13\,c)}}{212992\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2228224*b^3*e^(17*d*x + 17*c)/d - 17/1966080*b^3*e^(15*d*x + 15*c)/d - 17
/1966080*b^3*e^(-15*d*x - 15*c)/d + 1/2228224*b^3*e^(-17*d*x - 17*c)/d + 1/
212992*(6*a*b^2 + 17*b^3)*e^(13*d*x + 13*c)/d - 1/180224*(78*a*b^2 + 85*b^3
)*e^(11*d*x + 11*c)/d + 1/294912*(192*a^2*b + 936*a*b^2 + 595*b^3)*e^(9*d*x
 + 9*c)/d - 1/229376*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*e^(7*d*x + 7*c)/d
 + 1/81920*(512*a^3 + 3456*a^2*b + 4290*a*b^2 + 1547*b^3)*e^(5*d*x + 5*c)/d
 - 1/49152*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*e^(3*d*x + 3*c)/
d + 1/65536*(20480*a^3 + 48384*a^2*b + 41184*a*b^2 + 12155*b^3)*e^(d*x + c)
/d + 1/65536*(20480*a^3 + 48384*a^2*b + 41184*a*b^2 + 12155*b^3)*e^(-d*x -
c)/d - 1/49152*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*e^(-3*d*x -
3*c)/d + 1/81920*(512*a^3 + 3456*a^2*b + 4290*a*b^2 + 1547*b^3)*e^(-5*d*x -
 5*c)/d - 1/229376*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*e^(-7*d*x - 7*c)/d
+ 1/294912*(192*a^2*b + 936*a*b^2 + 595*b^3)*e^(-9*d*x - 9*c)/d - 1/180224*
(78*a*b^2 + 85*b^3)*e^(-11*d*x - 11*c)/d + 1/212992*(6*a*b^2 + 17*b^3)*e^(-
13*d*x - 13*c)/d
```

Mupad [**B**]
time = 1.76, size = 319, normalized size = 1.45

$$\frac{\frac{a^{3}\cosh(c+d\,x)^{5}}{5}-\frac{2\,a^{3}\cosh(c+d\,x)^{3}}{3}+a^{3}\cosh(c+d\,x)+\frac{a^{2}\,b\cosh(c+d\,x)^{9}}{3}-\frac{12\,a^{2}\,b\cosh(c+d\,x)^{7}}{7}+\frac{18\,a^{2}\,b\cosh(c+d\,x)^{5}}{5}-4\,a^{2}\,b\cosh(c+d\,x)^{3}+3\,a^{2}\,b\cosh(c+d\,x)+\frac{3\,a\,b^{2}\cosh(c+d\,x)^{13}}{13}-\frac{18\,a\,b^{2}\cosh(c+d\,x)^{11}}{11}+5\,a\,b^{2}\cosh(c+d\,x)^{9}-\frac{60\,a\,b^{2}\cosh(c+d\,x)^{7}}{7}+9\,a\,b^{2}\cosh(c+d\,x)^{5}-6\,a\,b^{2}\cosh(c+d\,x)^{3}+3\,a\,b^{2}\cosh(c+d\,x)+\frac{b^{3}\cosh(c+d\,x)^{17}}{17}-\frac{8\,b^{3}\cosh(c+d\,x)^{15}}{15}+\frac{28\,b^{3}\cosh(c+d\,x)^{13}}{13}-\frac{56\,b^{3}\cosh(c+d\,x)^{11}}{11}+\frac{70\,b^{3}\cosh(c+d\,x)^{9}}{9}-8\,b^{3}\cosh(c+d\,x)^{7}+\frac{28\,b^{3}\cosh(c+d\,x)^{5}}{5}-\frac{8\,b^{3}\cosh(c+d\,x)^{3}}{3}+b^{3}\cosh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^5*(a + b*sinh(c + d*x)^4)^3,x)
```

```
[Out] (a^3*cosh(c + d*x) + b^3*cosh(c + d*x) - (2*a^3*cosh(c + d*x)^3)/3 + (a^3*c
osh(c + d*x)^5)/5 - (8*b^3*cosh(c + d*x)^3)/3 + (28*b^3*cosh(c + d*x)^5)/5
- 8*b^3*cosh(c + d*x)^7 + (70*b^3*cosh(c + d*x)^9)/9 - (56*b^3*cosh(c + d*x
)^11)/11 + (28*b^3*cosh(c + d*x)^13)/13 - (8*b^3*cosh(c + d*x)^15)/15 + (b^
3*cosh(c + d*x)^17)/17 - 6*a*b^2*cosh(c + d*x)^3 - 4*a^2*b*cosh(c + d*x)^3
```

```
+ 9*a*b^2*cosh(c + d*x)^5 + (18*a^2*b*cosh(c + d*x)^5)/5 - (60*a*b^2*cosh(c
 + d*x)^7)/7 - (12*a^2*b*cosh(c + d*x)^7)/7 + 5*a*b^2*cosh(c + d*x)^9 + (a^
2*b*cosh(c + d*x)^9)/3 - (18*a*b^2*cosh(c + d*x)^11)/11 + (3*a*b^2*cosh(c +
 d*x)^13)/13 + 3*a*b^2*cosh(c + d*x) + 3*a^2*b*cosh(c + d*x))/d
```

3.208 $\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

<u>**Optimal**</u>. Leaf size=183

$$-\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{(a+b)^2(a+7b)\cosh^3(c+dx)}{3d}-\frac{3b(a+b)(3a+7b)\cosh^5(c+dx)}{5d}+\frac{b(3a^2+30ab+}{}$$

```
[Out] -(a+b)^3*cosh(d*x+c)/d+1/3*(a+b)^2*(a+7*b)*cosh(d*x+c)^3/d-3/5*b*(a+b)*(3*a
+7*b)*cosh(d*x+c)^5/d+1/7*b*(3*a^2+30*a*b+35*b^2)*cosh(d*x+c)^7/d-5/9*b^2*(
3*a+7*b)*cosh(d*x+c)^9/d+3/11*b^2*(a+7*b)*cosh(d*x+c)^11/d-7/13*b^3*cosh(d*
x+c)^13/d+1/15*b^3*cosh(d*x+c)^15/d
```

<u>**Rubi**</u> [A]
time = 0.13, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3294, 1167}

$$\frac{b(3a^2+30ab+35b^2)\cosh^7(c+dx)}{7d}+\frac{3b^2(a+7b)\cosh^{11}(c+dx)}{11d}-\frac{5b^2(3a+7b)\cosh^9(c+dx)}{9d}-\frac{3b(a+b)(3a+7b)\cosh^5(c+dx)}{5d}+\frac{(a+b)^2(a+7b)\cosh^3(c+dx)}{3d}-\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{b^3\cosh^{15}(c+dx)}{15d}-\frac{7b^3\cosh^{13}(c+dx)}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(((a + b)^3*Cosh[c + d*x])/d) + ((a + b)^2*(a + 7*b)*Cosh[c + d*x]^3)/(3*d
) - (3*b*(a + b)*(3*a + 7*b)*Cosh[c + d*x]^5)/(5*d) + (b*(3*a^2 + 30*a*b +
35*b^2)*Cosh[c + d*x]^7)/(7*d) - (5*b^2*(3*a + 7*b)*Cosh[c + d*x]^9)/(9*d)
+ (3*b^2*(a + 7*b)*Cosh[c + d*x]^11)/(11*d) - (7*b^3*Cosh[c + d*x]^13)/(13*
d) + (b^3*Cosh[c + d*x]^15)/(15*d)
```

<u>Rule 1167</u>

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

<u>Rule 3294</u>

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

<u>Rubi steps</u>

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+b-2bx^2+bx^4\right)^3\,dx,x,\cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left((a+b)^3-(a+b)^2(a+7b)x^2+3b(a+b)(3a+7b)x\right.\right.}{}$$

$$= -\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{(a+b)^2(a+7b)\cosh^3(c+dx)}{3d}-\frac{3b(}{}$$

Mathematica [A]
time = 1.74, size = 185, normalized size = 1.01

$$\frac{-135135(4096a^3+8960a^2b+7392ab^2+2145b^3)\cosh(c+dx)+15015(4096a^3+16128a^2b+15840ab^2+5005b^3)\cosh(3(c+dx))+b(-27027(1792a^2+2640ab+1001b^2)\cosh(5(c+dx))+19305(256a^2+880ab+455b^2)\cosh(7(c+dx))-7b(715(528a+455b)\cosh(9(c+dx))-1755(16a+35b)\cosh(11(c+dx))+7425b\cosh(13(c+dx))-429b\cosh(15(c+dx))))}{738017280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-135135*(4096*a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*Cosh[c + d*x] + 15
015*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5005*b^3)*Cosh[3*(c + d*x)] + b
*(-27027*(1792*a^2 + 2640*a*b + 1001*b^2)*Cosh[5*(c + d*x)] + 19305*(256*a^
2 + 880*a*b + 455*b^2)*Cosh[7*(c + d*x)] - 7*b*(715*(528*a + 455*b)*Cosh[9*
(c + d*x)] - 1755*(16*a + 35*b)*Cosh[11*(c + d*x)] + 7425*b*Cosh[13*(c + d*
x)] - 429*b*Cosh[15*(c + d*x)])))/(738017280*d)
```

Maple [A]
time = 1.13, size = 222, normalized size = 1.21

method	result
default	$\frac{\left(-\frac{455}{16384}b^3-\frac{33}{1024}a\,b^2\right)\cosh(9dx+9c)}{9d}+\frac{\left(\frac{105}{16384}b^3+\frac{3}{1024}a\,b^2\right)\cosh(11dx+11c)}{11d}+\frac{\left(-\frac{3003}{16384}b^3-\frac{495}{1024}a\,b^2-\frac{21}{64}a^2b\right)\cosh(5dx+5c)}{5d}+$
risch	$-\frac{99a\,b^2e^{5dx+5c}}{2048d}+\frac{165a\,b^2e^{3dx+3c}}{1024d}-\frac{105b\,e^{dx+c}a^2}{128d}-\frac{455b^3e^{9dx+9c}}{294912d}-\frac{3003\,e^{5dx+5c}b^3}{163840d}+\frac{e^{3dx+3c}a^3}{24d}+\frac{5005\,e^{3dx+3c}b^3}{98304d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(-455/16384*b^3-33/1024*a*b^2)/d*cosh(9*d*x+9*c)+1/11*(105/16384*b^3+3/
1024*a*b^2)/d*cosh(11*d*x+11*c)+1/5*(-3003/16384*b^3-495/1024*a*b^2-21/64*a
^2*b)/d*cosh(5*d*x+5*c)+1/7*(1365/16384*b^3+165/1024*a*b^2+3/64*a^2*b)/d*co
sh(7*d*x+7*c)+(-6435/16384*b^3-693/512*a*b^2-105/64*a^2*b-3/4*a^3)/d*cosh(d
*x+c)+1/3*(5005/16384*b^3+495/512*a*b^2+63/64*a^2*b+1/4*a^3)/d*cosh(3*d*x+3
*c)-15/212992*b^3/d*cosh(13*d*x+13*c)+1/245760*b^3/d*cosh(15*d*x+15*c)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(169) = 338$.
time = 0.29, size = 501, normalized size = 2.74

$$-\frac{1}{210862080}\,b^{3}\left(\frac{\left(7425\,e^{(-2\,dx-2\,c)}-61425\,e^{(-4\,dx-4\,c)}+325325\,e^{(-6\,dx-6\,c)}-1254825\,e^{(-8\,dx-8\,c)}+3864861\,e^{(-10\,dx-10\,c)}-10735725\,e^{(-12\,dx-12\,c)}+41409225\,e^{(-14\,dx-14\,c)}-429\right)e^{(15\,dx+15\,c)}}{d}+\frac{41409225\,e^{(-dx-c)}-10735725\,e^{(-3\,dx-3\,c)}+3864861\,e^{(-5\,dx-5\,c)}-1254825\,e^{(-7\,dx-7\,c)}+325325\,e^{(-9\,dx-9\,c)}-61425\,e^{(-11\,dx-11\,c)}+7425\,e^{(-13\,dx-13\,c)}-429\,e^{(-15\,dx-15\,c)}}{d}\right)-\frac{1}{473088}\,ab^{2}\left(\frac{\left(847\,e^{(-2\,dx-2\,c)}-5445\,e^{(-4\,dx-4\,c)}+22869\,e^{(-6\,dx-6\,c)}-76230\,e^{(-8\,dx-8\,c)}+320166\,e^{(-10\,dx-10\,c)}-63\right)e^{(11\,dx+11\,c)}}{d}+\frac{320166\,e^{(-dx-c)}-76230\,e^{(-3\,dx-3\,c)}+22869\,e^{(-5\,dx-5\,c)}-5445\,e^{(-7\,dx-7\,c)}+847\,e^{(-9\,dx-9\,c)}-63\,e^{(-11\,dx-11\,c)}}{d}\right)-\frac{3}{4480}\,a^{2}b\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d}\right)+\frac{1}{24}\,a^{3}\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/210862080*b^3*((7425*e^(-2*d*x - 2*c) - 61425*e^(-4*d*x - 4*c) + 325325*
e^(-6*d*x - 6*c) - 1254825*e^(-8*d*x - 8*c) + 3864861*e^(-10*d*x - 10*c) -
10735725*e^(-12*d*x - 12*c) + 41409225*e^(-14*d*x - 14*c) - 429)*e^(15*d*x
+ 15*c)/d + (41409225*e^(-d*x - c) - 10735725*e^(-3*d*x - 3*c) + 3864861*e^
(-5*d*x - 5*c) - 1254825*e^(-7*d*x - 7*c) + 325325*e^(-9*d*x - 9*c) - 61425
*e^(-11*d*x - 11*c) + 7425*e^(-13*d*x - 13*c) - 429*e^(-15*d*x - 15*c))/d)
- 1/473088*a*b^2*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(
-6*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^
(11*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e
^(-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*
d*x - 11*c))/d) - 3/4480*a^2*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c)
 + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*
e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/24*a^3*
(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/
d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(169) = 338$.
time = 0.54, size = 795, normalized size = 4.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/738017280*(3003*b^3*cosh(d*x + c)^15 + 45045*b^3*cosh(d*x + c)*sinh(d*x +
 c)^14 - 51975*b^3*cosh(d*x + c)^13 + 15015*(91*b^3*cosh(d*x + c)^3 - 45*b^
3*cosh(d*x + c))*sinh(d*x + c)^12 + 12285*(16*a*b^2 + 35*b^3)*cosh(d*x + c)
^11 + 9009*(1001*b^3*cosh(d*x + c)^5 - 1650*b^3*cosh(d*x + c)^3 + 15*(16*a*
b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 - 5005*(528*a*b^2 + 455*b^3)*
cosh(d*x + c)^9 + 45045*(429*b^3*cosh(d*x + c)^7 - 1485*b^3*cosh(d*x + c)^5
 + 45*(16*a*b^2 + 35*b^3)*cosh(d*x + c)^3 - (528*a*b^2 + 455*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^8 + 19305*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x +
c)^7 + 15015*(1001*b^3*cosh(d*x + c)^9 - 5940*b^3*cosh(d*x + c)^7 + 378*(16
*a*b^2 + 35*b^3)*cosh(d*x + c)^5 - 28*(528*a*b^2 + 455*b^3)*cosh(d*x + c)^3
 + 9*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 270
```

```
27*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(d*x + c)^5 + 45045*(91*b^3*cos
h(d*x + c)^11 - 825*b^3*cosh(d*x + c)^9 + 90*(16*a*b^2 + 35*b^3)*cosh(d*x +
 c)^7 - 14*(528*a*b^2 + 455*b^3)*cosh(d*x + c)^5 + 15*(256*a^2*b + 880*a*b^
2 + 455*b^3)*cosh(d*x + c)^3 - 3*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(
d*x + c))*sinh(d*x + c)^4 + 15015*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5
005*b^3)*cosh(d*x + c)^3 + 45045*(7*b^3*cosh(d*x + c)^13 - 90*b^3*cosh(d*x
+ c)^11 + 15*(16*a*b^2 + 35*b^3)*cosh(d*x + c)^9 - 4*(528*a*b^2 + 455*b^3)*
cosh(d*x + c)^7 + 9*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x + c)^5 - 6*(
1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 + (4096*a^3 + 16128*a^2
*b + 15840*a*b^2 + 5005*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 135135*(4096*
a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*cosh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(167) = 334$.
time = 9.95, size = 484, normalized size = 2.64

$$\begin{cases} \frac{a^{3}\sinh^{2}(c+dx)\cosh(c+dx)}{d} - \frac{2a^{3}\cosh^{3}(c+dx)}{3d} + \frac{3a^{2}b\sinh^{6}(c+dx)\cosh(c+dx)}{d} - \frac{6a^{2}b\sinh^{4}(c+dx)\cosh^{3}(c+dx)}{d} + \frac{24a^{2}b\sinh^{2}(c+dx)\cosh^{5}(c+dx)}{5d} - \frac{48a^{2}b\cosh^{7}(c+dx)}{35d} + \frac{3ab^{2}\sinh^{10}(c+dx)\cosh(c+dx)}{d} - \frac{10ab^{2}\sinh^{8}(c+dx)\cosh^{3}(c+dx)}{d} + \frac{16ab^{2}\sinh^{6}(c+dx)\cosh^{5}(c+dx)}{d} - \frac{96ab^{2}\sinh^{4}(c+dx)\cosh^{7}(c+dx)}{7d} + \frac{128ab^{2}\sinh^{2}(c+dx)\cosh^{9}(c+dx)}{21d} - \frac{256ab^{2}\cosh^{11}(c+dx)}{231d} + \frac{b^{3}\sinh^{14}(c+dx)\cosh(c+dx)}{d} - \frac{14b^{3}\sinh^{12}(c+dx)\cosh^{3}(c+dx)}{3d} + \frac{56b^{3}\sinh^{10}(c+dx)\cosh^{5}(c+dx)}{5d} - \frac{16b^{3}\sinh^{8}(c+dx)\cosh^{7}(c+dx)}{d} + \frac{128b^{3}\sinh^{6}(c+dx)\cosh^{9}(c+dx)}{9d} - \frac{256b^{3}\sinh^{4}(c+dx)\cosh^{11}(c+dx)}{33d} + \frac{1024b^{3}\sinh^{2}(c+dx)\cosh^{13}(c+dx)}{429d} - \frac{2048b^{3}\cosh^{15}(c+dx)}{6435d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^{4}(c)\right)^{3}\sinh^{3}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**3*cosh(c + d*x)**3/
(3*d) + 3*a**2*b*sinh(c + d*x)**6*cosh(c + d*x)/d - 6*a**2*b*sinh(c + d*x)*
*4*cosh(c + d*x)**3/d + 24*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) -
 48*a**2*b*cosh(c + d*x)**7/(35*d) + 3*a*b**2*sinh(c + d*x)**10*cosh(c + d*
x)/d - 10*a*b**2*sinh(c + d*x)**8*cosh(c + d*x)**3/d + 16*a*b**2*sinh(c + d
*x)**6*cosh(c + d*x)**5/d - 96*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**7/(7*
d) + 128*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**9/(21*d) - 256*a*b**2*cosh(
c + d*x)**11/(231*d) + b**3*sinh(c + d*x)**14*cosh(c + d*x)/d - 14*b**3*sin
h(c + d*x)**12*cosh(c + d*x)**3/(3*d) + 56*b**3*sinh(c + d*x)**10*cosh(c +
d*x)**5/(5*d) - 16*b**3*sinh(c + d*x)**8*cosh(c + d*x)**7/d + 128*b**3*sinh
(c + d*x)**6*cosh(c + d*x)**9/(9*d) - 256*b**3*sinh(c + d*x)**4*cosh(c + d*
x)**11/(33*d) + 1024*b**3*sinh(c + d*x)**2*cosh(c + d*x)**13/(429*d) - 2048
*b**3*cosh(c + d*x)**15/(6435*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**3*sinh(
c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(169) = 338$.
time = 0.52, size = 446, normalized size = 2.44

$$\frac{b^{3}e^{(15\,dx+15\,c)}}{491520\,d} - \frac{15\,b^{3}e^{(13\,dx+13\,c)}}{425984\,d} - \frac{15\,b^{3}e^{(-13\,dx-13\,c)}}{425984\,d} + \frac{b^{3}e^{(-15\,dx-15\,c)}}{491520\,d} + \frac{3\,(16\,ab^{2}+35\,b^{3})e^{(11\,dx+11\,c)}}{360448\,d} - \frac{(528\,ab^{2}+455\,b^{3})e^{(9\,dx+9\,c)}}{294912\,d} + \frac{3\,(256\,a^{2}b+880\,ab^{2}+455\,b^{3})e^{(7\,dx+7\,c)}}{229376\,d} - \frac{3\,(1792\,a^{2}b+2640\,ab^{2}+1001\,b^{3})e^{(5\,dx+5\,c)}}{163840\,d} + \frac{(4096\,a^{3}+16128\,a^{2}b+15840\,ab^{2}+5005\,b^{3})e^{(3\,dx+3\,c)}}{98304\,d} - \frac{3\,(4096\,a^{3}+8960\,a^{2}b+7392\,ab^{2}+2145\,b^{3})e^{(dx+c)}}{32768\,d} - \frac{3\,(4096\,a^{3}+8960\,a^{2}b+7392\,ab^{2}+2145\,b^{3})e^{(-dx-c)}}{32768\,d} + \frac{(4096\,a^{3}+16128\,a^{2}b+15840\,ab^{2}+5005\,b^{3})e^{(-3\,dx-3\,c)}}{98304\,d} - \frac{3\,(1792\,a^{2}b+2640\,ab^{2}+1001\,b^{3})e^{(-5\,dx-5\,c)}}{163840\,d} + \frac{3\,(256\,a^{2}b+880\,ab^{2}+455\,b^{3})e^{(-7\,dx-7\,c)}}{229376\,d} - \frac{(528\,ab^{2}+455\,b^{3})e^{(-9\,dx-9\,c)}}{294912\,d} + \frac{3\,(16\,ab^{2}+35\,b^{3})e^{(-11\,dx-11\,c)}}{360448\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/491520*b^3*e^(15*d*x + 15*c)/d - 15/425984*b^3*e^(13*d*x + 13*c)/d - 15/4
25984*b^3*e^(-13*d*x - 13*c)/d + 1/491520*b^3*e^(-15*d*x - 15*c)/d + 3/3604
48*(16*a*b^2 + 35*b^3)*e^(11*d*x + 11*c)/d - 1/294912*(528*a*b^2 + 455*b^3)
*e^(9*d*x + 9*c)/d + 3/229376*(256*a^2*b + 880*a*b^2 + 455*b^3)*e^(7*d*x +
7*c)/d - 3/163840*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*e^(5*d*x + 5*c)/d +
1/98304*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5005*b^3)*e^(3*d*x + 3*c)/d
 - 3/32768*(4096*a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*e^(d*x + c)/d -
3/32768*(4096*a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*e^(-d*x - c)/d + 1/
98304*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5005*b^3)*e^(-3*d*x - 3*c)/d
- 3/163840*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*e^(-5*d*x - 5*c)/d + 3/2293
76*(256*a^2*b + 880*a*b^2 + 455*b^3)*e^(-7*d*x - 7*c)/d - 1/294912*(528*a*b
^2 + 455*b^3)*e^(-9*d*x - 9*c)/d + 3/360448*(16*a*b^2 + 35*b^3)*e^(-11*d*x
- 11*c)/d
```

Mupad [**B**]
time = 1.33, size = 266, normalized size = 1.45

$$-\frac{-\frac{a^3\cosh(c+d\,x)^3}{3}+a^3\cosh(c+d\,x)-\frac{3\,a^2\,b\cosh(c+d\,x)^7}{7}+\frac{9\,a^2\,b\cosh(c+d\,x)^5}{5}-3\,a^2\,b\cosh(c+d\,x)^3+3\,a^2\,b\cosh(c+d\,x)-\frac{3\,a\,b^2\cosh(c+d\,x)^{11}}{11}+\frac{5\,a\,b^2\cosh(c+d\,x)^9}{3}-\frac{30\,a\,b^2\cosh(c+d\,x)^7}{7}+6\,a\,b^2\cosh(c+d\,x)^5-5\,a\,b^2\cosh(c+d\,x)^3+3\,a\,b^2\cosh(c+d\,x)-\frac{b^3\cosh(c+d\,x)^{15}}{15}+\frac{7\,b^3\cosh(c+d\,x)^{13}}{13}-\frac{21\,b^3\cosh(c+d\,x)^{11}}{11}+\frac{35\,b^3\cosh(c+d\,x)^9}{9}-5\,b^3\cosh(c+d\,x)^7+\frac{21\,b^3\cosh(c+d\,x)^5}{5}-\frac{7\,b^3\cosh(c+d\,x)^3}{3}+b^3\cosh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^4)^3,x)
```

```
[Out] -(a^3*cosh(c + d*x) + b^3*cosh(c + d*x) - (a^3*cosh(c + d*x)^3)/3 - (7*b^3*
cosh(c + d*x)^3)/3 + (21*b^3*cosh(c + d*x)^5)/5 - 5*b^3*cosh(c + d*x)^7 + (
35*b^3*cosh(c + d*x)^9)/9 - (21*b^3*cosh(c + d*x)^11)/11 + (7*b^3*cosh(c +
d*x)^13)/13 - (b^3*cosh(c + d*x)^15)/15 - 5*a*b^2*cosh(c + d*x)^3 - 3*a^2*b
*cosh(c + d*x)^3 + 6*a*b^2*cosh(c + d*x)^5 + (9*a^2*b*cosh(c + d*x)^5)/5 -
(30*a*b^2*cosh(c + d*x)^7)/7 - (3*a^2*b*cosh(c + d*x)^7)/7 + (5*a*b^2*cosh(
c + d*x)^9)/3 - (3*a*b^2*cosh(c + d*x)^11)/11 + 3*a*b^2*cosh(c + d*x) + 3*a
^2*b*cosh(c + d*x))/d
```

3.209 $\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=143

$$\frac{(a+b)^3\cosh(c+dx)}{d}-\frac{2b(a+b)^2\cosh^3(c+dx)}{d}+\frac{3b(a+b)(a+5b)\cosh^5(c+dx)}{5d}-\frac{4b^2(3a+5b)\cosh^7(c+d}{7d}$$

```
[Out] (a+b)^3*cosh(d*x+c)/d-2*b*(a+b)^2*cosh(d*x+c)^3/d+3/5*b*(a+b)*(a+5*b)*cosh(
d*x+c)^5/d-4/7*b^2*(3*a+5*b)*cosh(d*x+c)^7/d+1/3*b^2*(a+5*b)*cosh(d*x+c)^9/
d-6/11*b^3*cosh(d*x+c)^11/d+1/13*b^3*cosh(d*x+c)^13/d
```

Rubi [A]
time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3294, 1104}

$$\frac{b^2(a+5b)\cosh^9(c+dx)}{3d}-\frac{4b^2(3a+5b)\cosh^7(c+dx)}{7d}+\frac{3b(a+b)(a+5b)\cosh^5(c+dx)}{5d}-\frac{2b(a+b)^2\cosh^3(c+dx)}{d}+\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{b^3\cosh^{13}(c+dx)}{13d}-\frac{6b^3\cosh^{11}(c+dx)}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((a + b)^3*Cosh[c + d*x])/d - (2*b*(a + b)^2*Cosh[c + d*x]^3)/d + (3*b*(a +
 b)*(a + 5*b)*Cosh[c + d*x]^5)/(5*d) - (4*b^2*(3*a + 5*b)*Cosh[c + d*x]^7)/
(7*d) + (b^2*(a + 5*b)*Cosh[c + d*x]^9)/(3*d) - (6*b^3*Cosh[c + d*x]^11)/(1
1*d) + (b^3*Cosh[c + d*x]^13)/(13*d)
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int\left(a+b-2bx^2+bx^4\right)^3\,dx,x,\cosh(c+dx)\right)}{d}$$

$$= \text{Subst}\Big(\int \Big(a^3\Big(1+\frac{b(3a^2+3ab+b^2)}{a^3}\Big)-6b(a+b)^2x^2+12b^2(a+b)\ ($$

$$= \frac{(a+b)^3\cosh(c+dx)}{d}-\frac{2b(a+b)^2\cosh^3(c+dx)}{d}+\frac{3b(a+b)(}{}$$

Mathematica [A]
time = 0.74, size = 157, normalized size = 1.10

$$\frac{60060(1024a^3+1920a^2b+1512ab^2+429b^3)\cosh(c+dx)-15015b(1280a^2+1344ab+429b^2)\cosh(3(c+dx))+3003b(768a^2+1728ab+715b^2)\cosh(5(c+dx))-4290b^2(216a+143b)\cosh(7(c+dx))+10010b^2(8a+13b)\cosh(9(c+dx))-17745b^3\cosh(11(c+dx))+1155b^3\cosh(13(c+dx))}{61501440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (60060*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*Cosh[c + d*x] - 15015
*b*(1280*a^2 + 1344*a*b + 429*b^2)*Cosh[3*(c + d*x)] + 3003*b*(768*a^2 + 17
28*a*b + 715*b^2)*Cosh[5*(c + d*x)] - 4290*b^2*(216*a + 143*b)*Cosh[7*(c +
d*x)] + 10010*b^2*(8*a + 13*b)*Cosh[9*(c + d*x)] - 17745*b^3*Cosh[11*(c + d
*x)] + 1155*b^3*Cosh[13*(c + d*x)])/(61501440*d)
```

Maple [A]
time = 0.96, size = 183, normalized size = 1.28

method	result
default	$\frac{\left(-\frac{143}{2048}b^3-\frac{27}{256}a\,b^2\right)\cosh(7dx+7c)}{7d}+\frac{\left(\frac{39}{2048}b^3+\frac{3}{256}a\,b^2\right)\cosh(9dx+9c)}{9d}+\frac{\left(-\frac{1287}{4096}b^3-\frac{63}{64}a\,b^2-\frac{15}{16}a^2b\right)\cosh(3dx+3c)}{3d}+$ $\big(\frac{715}{4096}b^3-$
risch	$\frac{27a\,b^2\mathrm{e}^{5dx+5c}}{640d}-\frac{21a\,b^2\mathrm{e}^{3dx+3c}}{128d}+\frac{15b\,\mathrm{e}^{dx+c}a^2}{16d}+\frac{13b^3\mathrm{e}^{9dx+9c}}{12288d}+\frac{143\,\mathrm{e}^{5dx+5c}b^3}{8192d}-\frac{429\,\mathrm{e}^{3dx+3c}b^3}{8192d}+\frac{b^3\mathrm{e}^{13dx+13c}}{106496d}-$ $\frac{13b^3\mathrm{e}^{1}}{90}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(-143/2048*b^3-27/256*a*b^2)/d*cosh(7*d*x+7*c)+1/9*(39/2048*b^3+3/256*a
*b^2)/d*cosh(9*d*x+9*c)+1/3*(-1287/4096*b^3-63/64*a*b^2-15/16*a^2*b)/d*cosh
(3*d*x+3*c)+1/5*(715/4096*b^3+27/64*a*b^2+3/16*a^2*b)/d*cosh(5*d*x+5*c)+(42
9/1024*b^3+189/128*a*b^2+15/8*a^2*b+a^3)/d*cosh(d*x+c)-13/45056*b^3/d*cosh(
11*d*x+11*c)+1/53248*b^3/d*cosh(13*d*x+13*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(133) = 266.
time = 0.27, size = 399, normalized size = 2.79

$$-\frac{1}{24600576}b^3\left(\frac{\left(3549e^{(-2dx-2c)}-26026e^{(-4dx-4c)}+122694e^{(-6dx-6c)}-429429e^{(-8dx-8c)}+1288287e^{(-10dx-10c)}-5153148e^{(-12dx-12c)}-231\right)e^{(13dx+13c)}}{d}-\frac{5153148e^{(-dx-c)}-1288287e^{(-3dx-3c)}+429429e^{(-5dx-5c)}-122694e^{(-7dx-7c)}+26026e^{(-9dx-9c)}-3549e^{(-11dx-11c)}+231e^{(-13dx-13c)}}{d}\right)-\frac{1}{53760}ab^2\left(\frac{\left(405e^{(-2dx-2c)}-2268e^{(-4dx-4c)}+8820e^{(-6dx-6c)}-39690e^{(-8dx-8c)}-35\right)e^{(9dx+9c)}}{d}-\frac{39690e^{(-dx-c)}-8820e^{(-3dx-3c)}+2268e^{(-5dx-5c)}-405e^{(-7dx-7c)}+35e^{(-9dx-9c)}}{d}\right)+\frac{1}{160}a^2b\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)+\frac{a^3\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/24600576*b^3*((3549*e^(-2*d*x - 2*c) - 26026*e^(-4*d*x - 4*c) + 122694*e
^(-6*d*x - 6*c) - 429429*e^(-8*d*x - 8*c) + 1288287*e^(-10*d*x - 10*c) - 51
53148*e^(-12*d*x - 12*c) - 231)*e^(13*d*x + 13*c)/d - (5153148*e^(-d*x - c)
 - 1288287*e^(-3*d*x - 3*c) + 429429*e^(-5*d*x - 5*c) - 122694*e^(-7*d*x -
7*c) + 26026*e^(-9*d*x - 9*c) - 3549*e^(-11*d*x - 11*c) + 231*e^(-13*d*x -
13*c))/d) - 1/53760*a*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) +
8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (3
9690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(
-7*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) + 1/160*a^2*b*(3*e^(5*d*x + 5*c)/d
- 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*
d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^3*cosh(d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(133) = 266$.
time = 0.39, size = 594, normalized size = 4.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/61501440*(1155*b^3*cosh(d*x + c)^13 + 15015*b^3*cosh(d*x + c)*sinh(d*x +
c)^12 - 17745*b^3*cosh(d*x + c)^11 + 15015*(22*b^3*cosh(d*x + c)^3 - 13*b^3
*cosh(d*x + c))*sinh(d*x + c)^10 + 10010*(8*a*b^2 + 13*b^3)*cosh(d*x + c)^9
 + 45045*(33*b^3*cosh(d*x + c)^5 - 65*b^3*cosh(d*x + c)^3 + 2*(8*a*b^2 + 13
*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 - 4290*(216*a*b^2 + 143*b^3)*cosh(d*x
+ c)^7 + 30030*(66*b^3*cosh(d*x + c)^7 - 273*b^3*cosh(d*x + c)^5 + 28*(8*a*
b^2 + 13*b^3)*cosh(d*x + c)^3 - (216*a*b^2 + 143*b^3)*cosh(d*x + c))*sinh(d
*x + c)^6 + 3003*(768*a^2*b + 1728*a*b^2 + 715*b^3)*cosh(d*x + c)^5 + 15015
*(55*b^3*cosh(d*x + c)^9 - 390*b^3*cosh(d*x + c)^7 + 84*(8*a*b^2 + 13*b^3)*
cosh(d*x + c)^5 - 10*(216*a*b^2 + 143*b^3)*cosh(d*x + c)^3 + (768*a^2*b + 1
728*a*b^2 + 715*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 15015*(1280*a^2*b + 1
344*a*b^2 + 429*b^3)*cosh(d*x + c)^3 + 15015*(6*b^3*cosh(d*x + c)^11 - 65*b
^3*cosh(d*x + c)^9 + 24*(8*a*b^2 + 13*b^3)*cosh(d*x + c)^7 - 6*(216*a*b^2 +
 143*b^3)*cosh(d*x + c)^5 + 2*(768*a^2*b + 1728*a*b^2 + 715*b^3)*cosh(d*x +
 c)^3 - 3*(1280*a^2*b + 1344*a*b^2 + 429*b^3)*cosh(d*x + c))*sinh(d*x + c)^
2 + 60060*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*cosh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(131) = 262$.
time = 5.43, size = 377, normalized size = 2.64

$$\begin{cases} \frac{a^{3} \cosh{\left(c+dx \right)}}{d} + \frac{3a^{2}b \sinh^{4}{\left(c+dx \right)} \cosh{\left(c+dx \right)}}{d} - \frac{4a^{2}b \sinh^{2}{\left(c+dx \right)} \cosh^{3}{\left(c+dx \right)}}{d} + \frac{8a^{2}b \cosh^{5}{\left(c+dx \right)}}{5d} + \frac{3ab^{2} \sinh^{8}{\left(c+dx \right)} \cosh{\left(c+dx \right)}}{d} - \frac{8ab^{2} \sinh^{6}{\left(c+dx \right)} \cosh^{3}{\left(c+dx \right)}}{d} + \frac{48ab^{2} \sinh^{4}{\left(c+dx \right)} \cosh^{5}{\left(c+dx \right)}}{5d} - \frac{192ab^{2} \sinh^{2}{\left(c+dx \right)} \cosh^{7}{\left(c+dx \right)}}{35d} + \frac{128ab^{2} \cosh^{9}{\left(c+dx \right)}}{105d} + \frac{b^{3} \sinh^{12}{\left(c+dx \right)} \cosh{\left(c+dx \right)}}{d} - \frac{4b^{3} \sinh^{10}{\left(c+dx \right)} \cosh^{3}{\left(c+dx \right)}}{d} + \frac{8b^{3} \sinh^{8}{\left(c+dx \right)} \cosh^{5}{\left(c+dx \right)}}{d} - \frac{64b^{3} \sinh^{6}{\left(c+dx \right)} \cosh^{7}{\left(c+dx \right)}}{7d} + \frac{128b^{3} \sinh^{4}{\left(c+dx \right)} \cosh^{9}{\left(c+dx \right)}}{21d} - \frac{512b^{3} \sinh^{2}{\left(c+dx \right)} \cosh^{11}{\left(c+dx \right)}}{231d} + \frac{1024b^{3} \cosh^{13}{\left(c+dx \right)}}{3003d} & \text{for}\: d \neq 0 \\ x \left(a + b \sinh^{4}{\left(c \right)}\right)^{3} \sinh{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Piecewise((a**3*cosh(c + d*x)/d + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d
 - 4*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5
/(5*d) + 3*a*b**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*a*b**2*sinh(c + d*x)
**6*cosh(c + d*x)**3/d + 48*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d)
- 192*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*a*b**2*cosh(c +
 d*x)**9/(105*d) + b**3*sinh(c + d*x)**12*cosh(c + d*x)/d - 4*b**3*sinh(c +
 d*x)**10*cosh(c + d*x)**3/d + 8*b**3*sinh(c + d*x)**8*cosh(c + d*x)**5/d -
 64*b**3*sinh(c + d*x)**6*cosh(c + d*x)**7/(7*d) + 128*b**3*sinh(c + d*x)**
4*cosh(c + d*x)**9/(21*d) - 512*b**3*sinh(c + d*x)**2*cosh(c + d*x)**11/(23
1*d) + 1024*b**3*cosh(c + d*x)**13/(3003*d), Ne(d, 0)), (x*(a + b*sinh(c)**
4)**3*sinh(c), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(133) = 266.
time = 0.51, size = 372, normalized size = 2.60

$$\frac{b^{3}e^{(13\,dx+13\,c)}}{106496\,d}-\frac{13\,b^{3}e^{(11\,dx+11\,c)}}{90112\,d}-\frac{13\,b^{3}e^{(-11\,dx-11\,c)}}{90112\,d}+\frac{b^{3}e^{(-13\,dx-13\,c)}}{106496\,d}+\frac{(8\,ab^{2}+13\,b^{3})e^{(9\,dx+9\,c)}}{12288\,d}-\frac{(216\,ab^{2}+143\,b^{3})e^{(7\,dx+7\,c)}}{28672\,d}+\frac{(768\,a^{2}b+1728\,ab^{2}+715\,b^{3})e^{(5\,dx+5\,c)}}{40960\,d}-\frac{(1280\,a^{2}b+1344\,ab^{2}+429\,b^{3})e^{(3\,dx+3\,c)}}{8192\,d}+\frac{(1024\,a^{3}+1920\,a^{2}b+1512\,ab^{2}+429\,b^{3})e^{(dx+c)}}{2048\,d}+\frac{(1024\,a^{3}+1920\,a^{2}b+1512\,ab^{2}+429\,b^{3})e^{(-dx-c)}}{2048\,d}-\frac{(1280\,a^{2}b+1344\,ab^{2}+429\,b^{3})e^{(-3\,dx-3\,c)}}{8192\,d}+\frac{(768\,a^{2}b+1728\,ab^{2}+715\,b^{3})e^{(-5\,dx-5\,c)}}{40960\,d}-\frac{(216\,ab^{2}+143\,b^{3})e^{(-7\,dx-7\,c)}}{28672\,d}+\frac{(8\,ab^{2}+13\,b^{3})e^{(-9\,dx-9\,c)}}{12288\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/106496*b^3*e^(13*d*x + 13*c)/d - 13/90112*b^3*e^(11*d*x + 11*c)/d - 13/90
112*b^3*e^(-11*d*x - 11*c)/d + 1/106496*b^3*e^(-13*d*x - 13*c)/d + 1/12288*
(8*a*b^2 + 13*b^3)*e^(9*d*x + 9*c)/d - 1/28672*(216*a*b^2 + 143*b^3)*e^(7*d
*x + 7*c)/d + 1/40960*(768*a^2*b + 1728*a*b^2 + 715*b^3)*e^(5*d*x + 5*c)/d
- 1/8192*(1280*a^2*b + 1344*a*b^2 + 429*b^3)*e^(3*d*x + 3*c)/d + 1/2048*(10
24*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*e^(d*x + c)/d + 1/2048*(1024*a^
3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*e^(-d*x - c)/d - 1/8192*(1280*a^2*b
+ 1344*a*b^2 + 429*b^3)*e^(-3*d*x - 3*c)/d + 1/40960*(768*a^2*b + 1728*a*b^
2 + 715*b^3)*e^(-5*d*x - 5*c)/d - 1/28672*(216*a*b^2 + 143*b^3)*e^(-7*d*x -
 7*c)/d + 1/12288*(8*a*b^2 + 13*b^3)*e^(-9*d*x - 9*c)/d
```

Mupad [**B**]
time = 1.07, size = 211, normalized size = 1.48

$$\frac{a^3\cosh(c+d\,x)+\frac{3\,a^2\,b\cosh(c+d\,x)^5}{5}-2\,a^2\,b\cosh(c+d\,x)^3+3\,a^2\,b\cosh(c+d\,x)+\frac{a\,b^2\cosh(c+d\,x)^9}{3}-\frac{12\,a\,b^2\cosh(c+d\,x)^7}{7}+\frac{18\,a\,b^2\cosh(c+d\,x)^5}{5}-4\,a\,b^2\cosh(c+d\,x)^3+3\,a\,b^2\cosh(c+d\,x)+\frac{b^3\cosh(c+d\,x)^{13}}{13}-\frac{6\,b^3\cosh(c+d\,x)^{11}}{11}+\frac{5\,b^3\cosh(c+d\,x)^9}{3}-\frac{20\,b^3\cosh(c+d\,x)^7}{7}+3\,b^3\cosh(c+d\,x)^5-2\,b^3\cosh(c+d\,x)^3+b^3\cosh(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^4)^3,x)
```

```
[Out] (a^3*cosh(c + d*x) + b^3*cosh(c + d*x) - 2*b^3*cosh(c + d*x)^3 + 3*b^3*cosh
(c + d*x)^5 - (20*b^3*cosh(c + d*x)^7)/7 + (5*b^3*cosh(c + d*x)^9)/3 - (6*b
```

```
^3*cosh(c + d*x)^11)/11 + (b^3*cosh(c + d*x)^13)/13 - 4*a*b^2*cosh(c + d*x)
^3 - 2*a^2*b*cosh(c + d*x)^3 + (18*a*b^2*cosh(c + d*x)^5)/5 + (3*a^2*b*cosh
(c + d*x)^5)/5 - (12*a*b^2*cosh(c + d*x)^7)/7 + (a*b^2*cosh(c + d*x)^9)/3 +
 3*a*b^2*cosh(c + d*x) + 3*a^2*b*cosh(c + d*x))/d
```

3.210 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx$

Optimal. Leaf size=158

$$-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(3a^2+3ab+b^2)\cosh(c+dx)}{d}+\frac{b(3a^2+9ab+5b^2)\cosh^3(c+dx)}{3d}-b^2(9a+10$$

```
[Out] -a^3*arctanh(cosh(d*x+c))/d-b*(3*a^2+3*a*b+b^2)*cosh(d*x+c)/d+1/3*b*(3*a^2+
9*a*b+5*b^2)*cosh(d*x+c)^3/d-1/5*b^2*(9*a+10*b)*cosh(d*x+c)^5/d+1/7*b^2*(3*
a+10*b)*cosh(d*x+c)^7/d-5/9*b^3*cosh(d*x+c)^9/d+1/11*b^3*cosh(d*x+c)^11/d
```

Rubi [A]
time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3294, 1167, 212}

$$-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(3a^2+9ab+5b^2)\cosh^3(c+dx)}{3d}-\frac{b(3a^2+3ab+b^2)\cosh(c+dx)}{d}+\frac{b^2(3a+10b)\cosh^7(c+dx)}{7d}-\frac{b^2(9a+10b)\cosh^5(c+dx)}{5d}+\frac{b^3\cosh^{11}(c+dx)}{11d}-\frac{5b^3\cosh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((a^3*ArcTanh[Cosh[c + d*x]])/d) - (b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x])
/d + (b*(3*a^2 + 9*a*b + 5*b^2)*Cosh[c + d*x]^3)/(3*d) - (b^2*(9*a + 10*b)*
Cosh[c + d*x]^5)/(5*d) + (b^2*(3*a + 10*b)*Cosh[c + d*x]^7)/(7*d) - (5*b^3*
Cosh[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]^11)/(11*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{1-x^2}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\operatorname{Subst}\left(\int \left(b(3a^2+3ab+b^2)-b(3a^2+9ab+5b^2)\,x^2+b^2(9a+\right.\right.}{}\\ &= -\frac{b(3a^2+3ab+b^2)\cosh(c+dx)}{d}+\frac{b(3a^2+9ab+5b^2)\cosh^3(c+}{3d}\\ &= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(3a^2+3ab+b^2)\cosh(c+dx)}{d}+\end{aligned}$$

Mathematica [A]
time = 0.28, size = 139, normalized size = 0.88

$$\frac{-20790b(384a^2+280ab+77b^2)\cosh(c+dx)+6930b(8a+5b)(16a+11b)\cosh(3(c+dx))-2079b^2(112a+55b)\cosh(5(c+dx))+495b^2(48a+55b)\cosh(7(c+dx))-4235b^3\cosh(9(c+dx))+315b^3\cosh(11(c+dx))+3548160a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{3548160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-20790*b*(384*a^2 + 280*a*b + 77*b^2)*Cosh[c + d*x] + 6930*b*(8*a + 5*b)*(
16*a + 11*b)*Cosh[3*(c + d*x)] - 2079*b^2*(112*a + 55*b)*Cosh[5*(c + d*x)]
+ 495*b^2*(48*a + 55*b)*Cosh[7*(c + d*x)] - 4235*b^3*Cosh[9*(c + d*x)] + 31
5*b^3*Cosh[11*(c + d*x)] + 3548160*a^3*Log[Tanh[(c + d*x)/2]])/(3548160*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(148) = 296$.
time = 1.35, size = 533, normalized size = 3.37

method	result
risch	$-\frac{21a\,b^2e^{5dx+5c}}{640d}+\frac{21a\,b^2e^{3dx+3c}}{128d}+\frac{a^3\ln(e^{dx+c}-1)}{d}-\frac{a^3\ln(e^{dx+c}+1)}{d}-\frac{9b\,e^{dx+c}a^2}{8d}-\frac{11b^3e^{9dx+9c}}{18432d}-\frac{33\,e^{5dx+5c}b^3}{2048d}+55$
default	$\frac{-6a\,b^2\operatorname{arctanh}(e^{dx+c})-6a^2b\operatorname{arctanh}(e^{dx+c})-2a^3\operatorname{arctanh}(e^{dx+c})-2b^3\operatorname{arctanh}(e^{dx+c})+3a\,b^2\left(\frac{(\cosh^7(dx+c))}{7}+\frac{(\cosh^5(dx+c))}{5}+(\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-6*a*b^2*arctanh(exp(d*x+c))-6*a^2*b*arctanh(exp(d*x+c))-2*a^3*arctanh
(exp(d*x+c))-2*b^3*arctanh(exp(d*x+c))+3*a*b^2*(1/7*cosh(d*x+c)^7+1/5*cosh(
d*x+c)^5+1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x+c)))-12*a*b^2*(1/5
```

```
*cosh(d*x+c)^5+1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x+c)))+3*a^2*b
*(1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x+c)))+18*a*b^2*(1/3*cosh(d
*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x+c)))-6*a^2*b*(cosh(d*x+c)-2*arctanh(e
xp(d*x+c)))-12*a*b^2*(cosh(d*x+c)-2*arctanh(exp(d*x+c)))-6*b^3*(cosh(d*x+c)
-2*arctanh(exp(d*x+c)))+b^3*(1/11*cosh(d*x+c)^11+1/9*cosh(d*x+c)^9+1/7*cosh
(d*x+c)^7+1/5*cosh(d*x+c)^5+1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x
+c)))-6*b^3*(1/9*cosh(d*x+c)^9+1/7*cosh(d*x+c)^7+1/5*cosh(d*x+c)^5+1/3*cosh
(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x+c)))+15*b^3*(1/7*cosh(d*x+c)^7+1/5*
cosh(d*x+c)^5+1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x+c)))-20*b^3*(
1/5*cosh(d*x+c)^5+1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x+c)))+15*b
^3*(1/3*cosh(d*x+c)^3+cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(148) = 296$.
time = 0.28, size = 327, normalized size = 2.07

$$-\frac{1}{1419264}\,b^{3}\left(\frac{\left(847\,e^{\left(-2\,dx-2\,c\right)}-5445\,e^{\left(-4\,dx-4\,c\right)}+22869\,e^{\left(-6\,dx-6\,c\right)}-76230\,e^{\left(-8\,dx-8\,c\right)}+320166\,e^{\left(-10\,dx-10\,c\right)}-63\right)e^{\left(11\,dx+11\,c\right)}}{d}+\frac{320166\,e^{\left(-dx-c\right)}-76230\,e^{\left(-3\,dx-3\,c\right)}+22869\,e^{\left(-5\,dx-5\,c\right)}-5445\,e^{\left(-7\,dx-7\,c\right)}+847\,e^{\left(-9\,dx-9\,c\right)}-63\,e^{\left(-11\,dx-11\,c\right)}}{d}\right)-\frac{3}{4480}\,ab^{2}\left(\frac{\left(49\,e^{\left(-2\,dx-2\,c\right)}-245\,e^{\left(-4\,dx-4\,c\right)}+1225\,e^{\left(-6\,dx-6\,c\right)}-5\right)e^{\left(7\,dx+7\,c\right)}}{d}+\frac{1225\,e^{\left(-dx-c\right)}-245\,e^{\left(-3\,dx-3\,c\right)}+49\,e^{\left(-5\,dx-5\,c\right)}-5\,e^{\left(-7\,dx-7\,c\right)}}{d}\right)+\frac{1}{8}\,a^{2}b\left(\frac{e^{\left(3\,dx+3\,c\right)}}{d}-\frac{9\,e^{\left(dx+c\right)}}{d}-\frac{9\,e^{\left(-dx-c\right)}}{d}+\frac{e^{\left(-3\,dx-3\,c\right)}}{d}\right)+\frac{a^{3}\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/1419264*b^3*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-6
*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^(1
1*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e^(
-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*d*
x - 11*c))/d) - 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) +
 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^
(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/8*a^2*b*(
e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d
) + a^3*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3824 vs. $2(148) = 296$.
time = 0.43, size = 3824, normalized size = 24.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/7096320*(315*b^3*cosh(d*x + c)^22 + 6930*b^3*cosh(d*x + c)*sinh(d*x + c)^
21 + 315*b^3*sinh(d*x + c)^22 - 4235*b^3*cosh(d*x + c)^20 + 385*(189*b^3*co
sh(d*x + c)^2 - 11*b^3)*sinh(d*x + c)^20 + 7700*(63*b^3*cosh(d*x + c)^3 - 1
1*b^3*cosh(d*x + c))*sinh(d*x + c)^19 + 495*(48*a*b^2 + 55*b^3)*cosh(d*x +
c)^18 + 55*(41895*b^3*cosh(d*x + c)^4 - 14630*b^3*cosh(d*x + c)^2 + 432*a*b
^2 + 495*b^3)*sinh(d*x + c)^18 + 330*(25137*b^3*cosh(d*x + c)^5 - 14630*b^3
```

*cosh(d*x + c)^3 + 27*(48*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 -
 2079*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^16 + 33*(712215*b^3*cosh(d*x + c)^
6 - 621775*b^3*cosh(d*x + c)^4 - 7056*a*b^2 - 3465*b^3 + 2295*(48*a*b^2 + 5
5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 528*(101745*b^3*cosh(d*x + c)^7
 - 124355*b^3*cosh(d*x + c)^5 + 765*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^3 - 63
*(112*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 + 6930*(128*a^2*b + 1
68*a*b^2 + 55*b^3)*cosh(d*x + c)^14 + 330*(305235*b^3*cosh(d*x + c)^8 - 497
420*b^3*cosh(d*x + c)^6 + 4590*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^4 + 2688*a
^2*b + 3528*a*b^2 + 1155*b^3 - 756*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^14 + 4620*(33915*b^3*cosh(d*x + c)^9 - 71060*b^3*cosh(d*x + c)^
7 + 918*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^5 - 252*(112*a*b^2 + 55*b^3)*cosh
(d*x + c)^3 + 21*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^13 - 20790*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^12 + 2310*(88
179*b^3*cosh(d*x + c)^10 - 230945*b^3*cosh(d*x + c)^8 + 3978*(48*a*b^2 + 55
*b^3)*cosh(d*x + c)^6 - 1638*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^4 - 3456*a^
2*b - 2520*a*b^2 - 693*b^3 + 273*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^12 + 8*(27776385*b^3*cosh(d*x + c)^11 - 88913825*b^3*
cosh(d*x + c)^9 + 1969110*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^7 - 1135134*(11
2*a*b^2 + 55*b^3)*cosh(d*x + c)^5 + 315315*(128*a^2*b + 168*a*b^2 + 55*b^3)
*cosh(d*x + c)^3 - 31185*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c))*si
nh(d*x + c)^11 - 20790*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^10 +
22*(9258795*b^3*cosh(d*x + c)^12 - 35565530*b^3*cosh(d*x + c)^10 + 984555*(
48*a*b^2 + 55*b^3)*cosh(d*x + c)^8 - 756756*(112*a*b^2 + 55*b^3)*cosh(d*x +
 c)^6 + 315315*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^4 - 362880*a^
2*b - 264600*a*b^2 - 72765*b^3 - 62370*(384*a^2*b + 280*a*b^2 + 77*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^10 + 220*(712215*b^3*cosh(d*x + c)^13 - 3233230
*b^3*cosh(d*x + c)^11 + 109395*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^9 - 108108
*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^7 + 63063*(128*a^2*b + 168*a*b^2 + 55*b
^3)*cosh(d*x + c)^5 - 20790*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^
3 - 945*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 6
930*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^8 + 330*(305235*b^3*cosh
(d*x + c)^14 - 1616615*b^3*cosh(d*x + c)^12 + 65637*(48*a*b^2 + 55*b^3)*cos
h(d*x + c)^10 - 81081*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^8 + 63063*(128*a^2
*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^6 - 31185*(384*a^2*b + 280*a*b^2 + 7
7*b^3)*cosh(d*x + c)^4 + 2688*a^2*b + 3528*a*b^2 + 1155*b^3 - 2835*(384*a^2
*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 2640*(20349*b^3
*cosh(d*x + c)^15 - 124355*b^3*cosh(d*x + c)^13 + 5967*(48*a*b^2 + 55*b^3)*
cosh(d*x + c)^11 - 9009*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^9 + 9009*(128*a^
2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^7 - 6237*(384*a^2*b + 280*a*b^2 + 7
7*b^3)*cosh(d*x + c)^5 - 945*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)
^3 + 21*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 2
079*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^6 + 231*(101745*b^3*cosh(d*x + c)^16
 - 710600*b^3*cosh(d*x + c)^14 + 39780*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^12
 - 72072*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^10 + 90090*(128*a^2*b + 168*a*b
^2 + 55*b^3)*cosh(d*x + c)^8 - 83160*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(

```
d*x + c)^6 - 18900*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^4 - 1008*
a*b^2 - 495*b^3 + 840*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^6 + 462*(17955*b^3*cosh(d*x + c)^17 - 142120*b^3*cosh(d*x + c)^1
5 + 9180*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^13 - 19656*(112*a*b^2 + 55*b^3)*
cosh(d*x + c)^11 + 30030*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^9 -
 35640*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^7 - 11340*(384*a^2*b
+ 280*a*b^2 + 77*b^3)*cosh(d*x + c)^5 + 840*(128*a^2*b + 168*a*b^2 + 55*b^3
)*cosh(d*x + c)^3 - 27*(112*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^5
- 4235*b^3*cosh(d*x + c)^2 + 495*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^4 + 165*
(13965*b^3*cosh(d*x + c)^18 - 124355*b^3*cosh(d*x + c)^16 + 9180*(48*a*b^2
+ 55*b^3)*cosh(d*x + c)^14 - 22932*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^12 +
42042*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^10 - 62370*(384*a^2*b
+ 280*a*b^2 + 77*b^3)*cosh(d*x + c)^8 - 26460*(384*a^2*b + 280*a*b^2 + 77*b
^3)*cosh(d*x + c)^6 + 2940*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^4
 + 144*a*b^2 + 165*b^3 - 189*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + 660*(735*b^3*cosh(d*x + c)^19 - 7315*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(148) = 296.
time = 0.54, size = 377, normalized size = 2.39

$$\frac{315\,b^3e^{(11\,dx+11\,c)} - 4235\,b^3e^{(9\,dx+9\,c)} + 23760\,ab^2e^{(7\,dx+7\,c)} + 27225\,b^3e^{(7\,dx+7\,c)} - 232848\,ab^2e^{(5\,dx+5\,c)} - 114345\,b^3e^{(5\,dx+5\,c)} + 887040\,a^2be^{(3\,dx+3\,c)} + 1164240\,ab^2e^{(3\,dx+3\,c)} + 381150\,b^3e^{(3\,dx+3\,c)} - 7983360\,a^2be^{(dx+c)} - 5821200\,ab^2e^{(dx+c)} - 1600830\,b^3e^{(dx+c)} - 7096320\,a^3\log\left(e^{(dx+c)}+1\right) + 7096320\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right) - \left(7983360\,a^2be^{(10\,dx+10\,c)} + 5821200\,ab^2e^{(10\,dx+10\,c)} + 1600830\,b^3e^{(10\,dx+10\,c)} - 887040\,a^2be^{(8\,dx+8\,c)} - 1164240\,ab^2e^{(8\,dx+8\,c)} - 381150\,b^3e^{(8\,dx+8\,c)} + 232848\,ab^2e^{(6\,dx+6\,c)} + 114345\,b^3e^{(6\,dx+6\,c)} - 23760\,ab^2e^{(4\,dx+4\,c)} - 27225\,b^3e^{(4\,dx+4\,c)} + 4235\,b^3e^{(2\,dx+2\,c)} - 315\,b^3\right)e^{(-11\,dx-11\,c)}}{7096320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/7096320*(315*b^3*e^(11*d*x + 11*c) - 4235*b^3*e^(9*d*x + 9*c) + 23760*a*b
^2*e^(7*d*x + 7*c) + 27225*b^3*e^(7*d*x + 7*c) - 232848*a*b^2*e^(5*d*x + 5*
c) - 114345*b^3*e^(5*d*x + 5*c) + 887040*a^2*b*e^(3*d*x + 3*c) + 1164240*a*
b^2*e^(3*d*x + 3*c) + 381150*b^3*e^(3*d*x + 3*c) - 7983360*a^2*b*e^(d*x + c
) - 5821200*a*b^2*e^(d*x + c) - 1600830*b^3*e^(d*x + c) - 7096320*a^3*log(e
^(d*x + c) + 1) + 7096320*a^3*log(abs(e^(d*x + c) - 1)) - (7983360*a^2*b*e^
(10*d*x + 10*c) + 5821200*a*b^2*e^(10*d*x + 10*c) + 1600830*b^3*e^(10*d*x +
 10*c) - 887040*a^2*b*e^(8*d*x + 8*c) - 1164240*a*b^2*e^(8*d*x + 8*c) - 381
150*b^3*e^(8*d*x + 8*c) + 232848*a*b^2*e^(6*d*x + 6*c) + 114345*b^3*e^(6*d*
```

```
x + 6*c) - 23760*a*b^2*e^(4*d*x + 4*c) - 27225*b^3*e^(4*d*x + 4*c) + 4235*b
^3*e^(2*d*x + 2*c) - 315*b^3)*e^(-11*d*x - 11*c))/d
```

Mupad [B]
time = 0.65, size = 326, normalized size = 2.06

$$\frac{e^{-3c-3dx}\left(128a^2b+168ab^2+55b^3\right)}{1024d}-\frac{2\,\mathrm{atan}\left(\frac{a^3e^{dx}e^c\sqrt{-d^2}}{d\sqrt{a^6}}\right)\sqrt{a^6}}{\sqrt{-d^2}}+\frac{e^{3c+3dx}\left(128a^2b+168ab^2+55b^3\right)}{1024d}-\frac{11b^3e^{-9c-9dx}}{18432d}-\frac{11b^3e^{9c+9dx}}{18432d}+\frac{b^3e^{-11c-11dx}}{22528d}+\frac{b^3e^{11c+11dx}}{22528d}-\frac{3be^{-c-dx}\left(384a^2+280ab+77b^2\right)}{1024d}+\frac{b^2e^{-7c-7dx}\left(48a+55b\right)}{14336d}+\frac{b^2e^{7c+7dx}\left(48a+55b\right)}{14336d}-\frac{3b^2e^{-5c-5dx}\left(112a+55b\right)}{10240d}-\frac{3b^2e^{5c+5dx}\left(112a+55b\right)}{10240d}-\frac{3be^{c+dx}\left(384a^2+280ab+77b^2\right)}{1024d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x),x)
```

```
[Out] (exp(- 3*c - 3*d*x)*(168*a*b^2 + 128*a^2*b + 55*b^3))/(1024*d) - (2*atan((a
^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/2)
 + (exp(3*c + 3*d*x)*(168*a*b^2 + 128*a^2*b + 55*b^3))/(1024*d) - (11*b^3*e
xp(- 9*c - 9*d*x))/(18432*d) - (11*b^3*exp(9*c + 9*d*x))/(18432*d) + (b^3*e
xp(- 11*c - 11*d*x))/(22528*d) + (b^3*exp(11*c + 11*d*x))/(22528*d) - (3*b*
exp(- c - d*x)*(280*a*b + 384*a^2 + 77*b^2))/(1024*d) + (b^2*exp(- 7*c - 7*
d*x)*(48*a + 55*b))/(14336*d) + (b^2*exp(7*c + 7*d*x)*(48*a + 55*b))/(14336
*d) - (3*b^2*exp(- 5*c - 5*d*x)*(112*a + 55*b))/(10240*d) - (3*b^2*exp(5*c
+ 5*d*x)*(112*a + 55*b))/(10240*d) - (3*b*exp(c + d*x)*(280*a*b + 384*a^2 +
 77*b^2))/(1024*d)
```

3.211 $\int \mathrm{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=148

$$\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b(3a^2+3ab+b^2)\cosh(c+dx)}{d}-\frac{2b^2(3a+2b)\cosh^3(c+dx)}{3d}+\frac{3b^2(a+2b)\cosh^5(c}{5d}$$

```
[Out] 1/2*a^3*arctanh(cosh(d*x+c))/d+b*(3*a^2+3*a*b+b^2)*cosh(d*x+c)/d-2/3*b^2*(3
*a+2*b)*cosh(d*x+c)^3/d+3/5*b^2*(a+2*b)*cosh(d*x+c)^5/d-4/7*b^3*cosh(d*x+c)
^7/d+1/9*b^3*cosh(d*x+c)^9/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3294, 1171, 1824, 212}

$$\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\mathrm{csch}(c+dx)}{2d}+\frac{b(3a^2+3ab+b^2)\cosh(c+dx)}{d}+\frac{3b^2(a+2b)\cosh^5(c+dx)}{5d}-\frac{2b^2(3a+2b)\cosh^3(c+dx)}{3d}+\frac{b^3\cosh^9(c+dx)}{9d}-\frac{4b^3\cosh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x]
)/d - (2*b^2*(3*a + 2*b)*Cosh[c + d*x]^3)/(3*d) + (3*b^2*(a + 2*b)*Cosh[c +
 d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(9*d
) - (a^3*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-a^3-6a^2b-6ab^2-2b^3+2b(3}{}\right.}{}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \left(-2b(3a^2+3ab+b^2\right.\right.}{}$$

$$= \frac{b(3a^2+3ab+b^2)\cosh(c+dx)}{d} - \frac{2b^2(3a+2b)\cosh^3(c+dx)}{3d} +$$

$$= \frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d} + \frac{b(3a^2+3ab+b^2)\cosh(c+dx)}{d} - $$

Mathematica [A]
time = 0.28, size = 155, normalized size = 1.05

$$\frac{1890b(128a^2+80ab+21b^2)\cosh(c+dx) - 1260b^2(20a+7b)\cosh(3(c+dx)) + 3024ab^2\cosh(5(c+dx)) + 2268b^3\cosh(5(c+dx)) - 405b^3\cosh(7(c+dx)) + 35b^3\cosh(9(c+dx)) - 10080a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) - 40320a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - 10080a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (1890*b*(128*a^2 + 80*a*b + 21*b^2)*Cosh[c + d*x] - 1260*b^2*(20*a + 7*b)*C
osh[3*(c + d*x)] + 3024*a*b^2*Cosh[5*(c + d*x)] + 2268*b^3*Cosh[5*(c + d*x)
] - 405*b^3*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(c + d*x)] - 10080*a^3*Csch[(
c + d*x)/2]^2 - 40320*a^3*Log[Tanh[(c + d*x)/2]] - 10080*a^3*Sech[(c + d*x)
/2]^2)/(80640*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(136) = 272.
time = 1.48, size = 379, normalized size = 2.56

method	result

risch	$\frac{b^3 e^{9dx+9c}}{4608d} - \frac{9b^3 e^{7dx+7c}}{3584d} + \frac{3a\, b^2 e^{5dx+5c}}{160d} + \frac{9\, e^{5dx+5c} b^3}{640d} - \frac{5a\, b^2 e^{3dx+3c}}{32d} - \frac{7\, e^{3dx+3c} b^3}{128d} + \frac{3b\, e^{dx+c} a^2}{2d} + \frac{15a\, e^{dx+c} b^2}{16d} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4608*b^3/d*exp(9*d*x+9*c)-9/3584*b^3/d*exp(7*d*x+7*c)+3/160*a*b^2/d*exp(5
*d*x+5*c)+9/640/d*exp(5*d*x+5*c)*b^3-5/32*a*b^2/d*exp(3*d*x+3*c)-7/128/d*ex
p(3*d*x+3*c)*b^3+3/2*b/d*exp(d*x+c)*a^2+15/16*a/d*exp(d*x+c)*b^2+63/256*b^3
/d*exp(d*x+c)+3/2/d*exp(-d*x-c)*a^2*b+15/16*a/d*exp(-d*x-c)*b^2+63/256*b^3/
d*exp(-d*x-c)-5/32*a*b^2/d*exp(-3*d*x-3*c)-7/128*b^3/d*exp(-3*d*x-3*c)+3/16
0*a*b^2/d*exp(-5*d*x-5*c)+9/640/d*exp(-5*d*x-5*c)*b^3-9/3584*b^3/d*exp(-7*d
*x-7*c)+1/4608*b^3/d*exp(-9*d*x-9*c)-a^3*exp(d*x+c)*(1+exp(2*d*x+2*c))/d/(e
xp(2*d*x+2*c)-1)^2+1/2*a^3/d*ln(exp(d*x+c)+1)-1/2*a^3/d*ln(exp(d*x+c)-1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(136) = 272$.
time = 0.29, size = 334, normalized size = 2.26

$$-\frac{1}{161280}\, b^3 \left(\frac{\left(405\, e^{(-2\, dx-2\, c)} - 2268\, e^{(-4\, dx-4\, c)} + 8820\, e^{(-6\, dx-6\, c)} - 39690\, e^{(-8\, dx-8\, c)} - 35\right) e^{(9\, dx+9\, c)}}{d} - \frac{39690\, e^{(-dx-c)} - 8820\, e^{(-3\, dx-3\, c)} + 2268\, e^{(-5\, dx-5\, c)} - 405\, e^{(-7\, dx-7\, c)} + 35\, e^{(-9\, dx-9\, c)}}{d}\right) + \frac{1}{160}\, ab^2 \left(\frac{3\, e^{(5\, dx+5\, c)}}{d} - \frac{25\, e^{(3\, dx+3\, c)}}{d} + \frac{150\, e^{(dx+c)}}{d} + \frac{150\, e^{(-dx-c)}}{d} - \frac{25\, e^{(-3\, dx-3\, c)}}{d} + \frac{3\, e^{(-5\, dx-5\, c)}}{d}\right) + \frac{3}{2}\, a^2 b \left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right) + \frac{1}{2}\, a^3 \left(\frac{\log\left(e^{(-dx-c)} + 1\right)}{d} - \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{2\left(e^{(-dx-c)} + e^{(-3\, dx-3\, c)}\right)}{d\left(2\, e^{(-2\, dx-2\, c)} - e^{(-4\, dx-4\, c)} - 1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d
*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x
 - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c
) + 35*e^(-9*d*x - 9*c))/d) + 1/160*a*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*
x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d
 + 3*e^(-5*d*x - 5*c)/d) + 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2
*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) +
 e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4895 vs. $2(136) = 272$.
time = 0.42, size = 4895, normalized size = 33.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/161280*(35*b^3*cosh(d*x + c)^22 + 770*b^3*cosh(d*x + c)*sinh(d*x + c)^21
+ 35*b^3*sinh(d*x + c)^22 - 475*b^3*cosh(d*x + c)^20 + 5*(1617*b^3*cosh(d*x
```

```
 + c)^2 - 95*b^3)*sinh(d*x + c)^20 + 100*(539*b^3*cosh(d*x + c)^3 - 95*b^3*
cosh(d*x + c))*sinh(d*x + c)^19 + (3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^18
+ (256025*b^3*cosh(d*x + c)^4 - 90250*b^3*cosh(d*x + c)^2 + 3024*a*b^2 + 31
13*b^3)*sinh(d*x + c)^18 + 6*(153615*b^3*cosh(d*x + c)^5 - 90250*b^3*cosh(d
*x + c)^3 + 3*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - 9*(
3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^16 + 3*(870485*b^3*cosh(d*x + c)^6 - 7
67125*b^3*cosh(d*x + c)^4 - 10416*a*b^2 - 4587*b^3 + 51*(3024*a*b^2 + 3113*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 48*(124355*b^3*cosh(d*x + c)^7 - 1
53425*b^3*cosh(d*x + c)^5 + 17*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^3 - 3*
(3472*a*b^2 + 1529*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 + 126*(1920*a^2*b +
 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^14 + 6*(1865325*b^3*cosh(d*x + c)^8 -
3068500*b^3*cosh(d*x + c)^6 + 510*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^4 +
 40320*a^2*b + 34104*a*b^2 + 9933*b^3 - 180*(3472*a*b^2 + 1529*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^14 + 4*(4352425*b^3*cosh(d*x + c)^9 - 9205500*b^3*c
osh(d*x + c)^7 + 2142*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^5 - 1260*(3472*
a*b^2 + 1529*b^3)*cosh(d*x + c)^3 + 441*(1920*a^2*b + 1624*a*b^2 + 473*b^3)
*cosh(d*x + c))*sinh(d*x + c)^13 - 630*(256*a^3 + 384*a^2*b + 280*a*b^2 + 7
7*b^3)*cosh(d*x + c)^12 + 2*(11316305*b^3*cosh(d*x + c)^10 - 29917875*b^3*c
osh(d*x + c)^8 + 9282*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^6 - 8190*(3472*
a*b^2 + 1529*b^3)*cosh(d*x + c)^4 - 80640*a^3 - 120960*a^2*b - 88200*a*b^2
- 24255*b^3 + 5733*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^12 + 8*(3086265*b^3*cosh(d*x + c)^11 - 9972625*b^3*cosh(d*x + c)
^9 + 3978*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^7 - 4914*(3472*a*b^2 + 1529
*b^3)*cosh(d*x + c)^5 + 5733*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x +
 c)^3 - 945*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c))*sinh(
d*x + c)^11 - 630*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^
10 + 2*(11316305*b^3*cosh(d*x + c)^12 - 43879550*b^3*cosh(d*x + c)^10 + 218
79*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^8 - 36036*(3472*a*b^2 + 1529*b^3)*
cosh(d*x + c)^6 + 63063*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^4
 - 80640*a^3 - 120960*a^2*b - 88200*a*b^2 - 24255*b^3 - 20790*(256*a^3 + 38
4*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(435242
5*b^3*cosh(d*x + c)^13 - 19945250*b^3*cosh(d*x + c)^11 + 12155*(3024*a*b^2
+ 3113*b^3)*cosh(d*x + c)^9 - 25740*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^7
 + 63063*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^5 - 34650*(256*a
^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^3 - 1575*(256*a^3 + 384*
a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 126*(1920*a^2*
b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^8 + 6*(1865325*b^3*cosh(d*x + c)^14
 - 9972625*b^3*cosh(d*x + c)^12 + 7293*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c
)^10 - 19305*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^8 + 63063*(1920*a^2*b +
1624*a*b^2 + 473*b^3)*cosh(d*x + c)^6 - 51975*(256*a^3 + 384*a^2*b + 280*a*
b^2 + 77*b^3)*cosh(d*x + c)^4 + 40320*a^2*b + 34104*a*b^2 + 9933*b^3 - 4725
*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
8 + 48*(124355*b^3*cosh(d*x + c)^15 - 767125*b^3*cosh(d*x + c)^13 + 663*(30
24*a*b^2 + 3113*b^3)*cosh(d*x + c)^11 - 2145*(3472*a*b^2 + 1529*b^3)*cosh(d
*x + c)^9 + 9009*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^7 - 1039
```

```
5*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^5 - 1575*(256*a^
3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^3 + 21*(1920*a^2*b + 1624
*a*b^2 + 473*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 9*(3472*a*b^2 + 1529*b^3
)*cosh(d*x + c)^6 + 3*(870485*b^3*cosh(d*x + c)^16 - 6137000*b^3*cosh(d*x +
 c)^14 + 6188*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^12 - 24024*(3472*a*b^2
+ 1529*b^3)*cosh(d*x + c)^10 + 126126*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*c
osh(d*x + c)^8 - 194040*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x
 + c)^6 - 44100*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^4
- 10416*a*b^2 - 4587*b^3 + 1176*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^6 + 6*(153615*b^3*cosh(d*x + c)^17 - 1227400*b^3*co
sh(d*x + c)^15 + 1428*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^13 - 6552*(3472
*a*b^2 + 1529*b^3)*cosh(d*x + c)^11 + 42042*(1920*a^2*b + 1624*a*b^2 + 473*
b^3)*cosh(d*x + c)^9 - 83160*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cos
h(d*x + c)^7 - 26460*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x +
c)^5 + 1176*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^3 - 9*(3472*a
*b^2 + 1529*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 475*b^3*cosh(d*x + c)^2 +
 (3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^4 + (256025*b^3*cosh(d*x + c)^18 - 2
301375*b^3*cosh(d*x + c)^16 + 3060*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^14
 - 16380*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^12 + 126126*(1920*a^2*b + 16
24*a*b^2 + 473*b^3)*cosh(d*x + c)^10 - 311850*(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(136) = 272.
time = 0.58, size = 300, normalized size = 2.03

$$\frac{35\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^9-720\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^7+3024\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+6048\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5-40320\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-26880\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+241920\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+241920\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+80640\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+40320\,a^3\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-40320\,a^3\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{161280\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/161280*(35*b^3*(e^(d*x + c) + e^(-d*x - c))^9 - 720*b^3*(e^(d*x + c) + e^
(-d*x - c))^7 + 3024*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 + 6048*b^3*(e^(d*
x + c) + e^(-d*x - c))^5 - 40320*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 268
80*b^3*(e^(d*x + c) + e^(-d*x - c))^3 + 241920*a^2*b*(e^(d*x + c) + e^(-d*x
 - c)) + 241920*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 80640*b^3*(e^(d*x + c)
 + e^(-d*x - c)) + 40320*a^3*log(e^(d*x + c) + e^(-d*x - c) + 2) - 40320*a^
```

```
3*log(e^(d*x + c) + e^(-d*x - c) - 2) - 161280*a^3*(e^(d*x + c) + e^(-d*x -
 c))/((e^(d*x + c) + e^(-d*x - c))^2 - 4))/d
```

Mupad [B]
time = 1.19, size = 326, normalized size = 2.20

$$\frac{\operatorname{atan}\left(\frac{a^3\,e^{d\,x}\,e^c\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}}-\frac{9\,b^3\,e^{-7\,c-7\,d\,x}}{3584\,d}-\frac{9\,b^3\,e^{7\,c+7\,d\,x}}{3584\,d}+\frac{b^3\,e^{-9\,c-9\,d\,x}}{4608\,d}+\frac{b^3\,e^{9\,c+9\,d\,x}}{4608\,d}+\frac{3\,b\,e^{-c-d\,x}\,\left(128\,a^2+80\,a\,b+21\,b^2\right)}{256\,d}+\frac{3\,b^2\,e^{-5\,c-5\,d\,x}\,\left(4\,a+3\,b\right)}{640\,d}+\frac{3\,b^2\,e^{5\,c+5\,d\,x}\,\left(4\,a+3\,b\right)}{640\,d}-\frac{b^2\,e^{-3\,c-3\,d\,x}\,\left(20\,a+7\,b\right)}{128\,d}-\frac{b^2\,e^{3\,c+3\,d\,x}\,\left(20\,a+7\,b\right)}{128\,d}+\frac{3\,b\,e^{c+d\,x}\,\left(128\,a^2+80\,a\,b+21\,b^2\right)}{256\,d}-\frac{a^3\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^3\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^3,x)
```

```
[Out] (atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^
2)^(1/2) - (9*b^3*exp(- 7*c - 7*d*x))/(3584*d) - (9*b^3*exp(7*c + 7*d*x))/(
3584*d) + (b^3*exp(- 9*c - 9*d*x))/(4608*d) + (b^3*exp(9*c + 9*d*x))/(4608*
d) + (3*b*exp(- c - d*x)*(80*a*b + 128*a^2 + 21*b^2))/(256*d) + (3*b^2*exp(
- 5*c - 5*d*x)*(4*a + 3*b))/(640*d) + (3*b^2*exp(5*c + 5*d*x)*(4*a + 3*b))/
(640*d) - (b^2*exp(- 3*c - 3*d*x)*(20*a + 7*b))/(128*d) - (b^2*exp(3*c + 3*
d*x)*(20*a + 7*b))/(128*d) + (3*b*exp(c + d*x)*(80*a*b + 128*a^2 + 21*b^2))
/(256*d) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(c + d
*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.212 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=142

$$-\frac{3a^2(a+8b)\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{b^2(3a+b)\cosh(c+dx)}{d}+\frac{b^2(a+b)\cosh^3(c+dx)}{d}-\frac{3b^3\cosh^5(c+dx)}{5d}$$

```
[Out] -3/8*a^2*(a+8*b)*arctanh(cosh(d*x+c))/d-b^2*(3*a+b)*cosh(d*x+c)/d+b^2*(a+b)
*cosh(d*x+c)^3/d-3/5*b^3*cosh(d*x+c)^5/d+1/7*b^3*cosh(d*x+c)^7/d+3/8*a^3*co
th(d*x+c)*csch(d*x+c)/d-1/4*a^3*coth(d*x+c)*csch(d*x+c)^3/d
```

Rubi [A]
time = 0.19, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3294, 1171, 1828, 1824, 212}

$$-\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{3a^2(a+8b)\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{b^2(a+b)\cosh^3(c+dx)}{d}-\frac{b^2(3a+b)\cosh(c+dx)}{d}+\frac{b^3\cosh^7(c+dx)}{7d}-\frac{3b^3\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-3*a^2*(a + 8*b)*ArcTanh[Cosh[c + d*x]])/(8*d) - (b^2*(3*a + b)*Cosh[c + d
*x])/d + (b^2*(a + b)*Cosh[c + d*x]^3)/d - (3*b^3*Cosh[c + d*x]^5)/(5*d) +
(b^3*Cosh[c + d*x]^7)/(7*d) + (3*a^3*Coth[c + d*x]*Csch[c + d*x])/(8*d) - (
a^3*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} + \frac{\text{Subst}\Big(\int \frac{-3a^3-12a^2b-12ab^2-4b^3+}{}}{}$$

$$= \frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} -$$

$$= \frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} -$$

$$= -\frac{b^2(3a+b)\cosh(c+dx)}{d} + \frac{b^2(a+b)\cosh^3(c+dx)}{d} - \frac{3b^3\cosh}{5}$$

$$= -\frac{3a^2(a+8b)\tanh^{-1}(\cosh(c+dx))}{8d} - \frac{b^2(3a+b)\cosh(c+dx)}{d}$$

Mathematica [A]
time = 0.29, size = 173, normalized size = 1.22

$$\frac{-35b^2(144a+35b)\cosh(c+dx)+35b^2(16a+7b)\cosh(3(c+dx))-49b^3\cosh(5(c+dx))+5b^3\cosh(7(c+dx))+210a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)-35a^3\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)+840a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+6720a^2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+210a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+35a^3\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-35*b^2*(144*a + 35*b)*Cosh[c + d*x] + 35*b^2*(16*a + 7*b)*Cosh[3*(c + d*x
)] - 49*b^3*Cosh[5*(c + d*x)] + 5*b^3*Cosh[7*(c + d*x)] + 210*a^3*Csch[(c +
 d*x)/2]^2 - 35*a^3*Csch[(c + d*x)/2]^4 + 840*a^3*Log[Tanh[(c + d*x)/2]] +
6720*a^2*b*Log[Tanh[(c + d*x)/2]] + 210*a^3*Sech[(c + d*x)/2]^2 + 35*a^3*Se
ch[(c + d*x)/2]^4)/(2240*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(132) = 264$.
time = 1.55, size = 336, normalized size = 2.37

method	result
risch	$\frac{b^3 e^{7dx+7c}}{896d} - \frac{7\, e^{5dx+5c} b^3}{640d} + \frac{7\, e^{3dx+3c} b^3}{128d} + \frac{a\, b^2 e^{3dx+3c}}{8d} - \frac{9a\, e^{dx+c} b^2}{8d} - \frac{35 b^3 e^{dx+c}}{128d} - \frac{9a\, e^{-dx-c} b^2}{8d} - \frac{35 b^3 e^{-dx-c}}{128d} + 7b^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/896*b^3/d*exp(7*d*x+7*c)-7/640/d*exp(5*d*x+5*c)*b^3+7/128/d*exp(3*d*x+3*c
)*b^3+1/8*a*b^2/d*exp(3*d*x+3*c)-9/8*a/d*exp(d*x+c)*b^2-35/128*b^3/d*exp(d*
x+c)-9/8*a/d*exp(-d*x-c)*b^2-35/128*b^3/d*exp(-d*x-c)+7/128*b^3/d*exp(-3*d*
x-3*c)+1/8*a*b^2/d*exp(-3*d*x-3*c)-7/640/d*exp(-5*d*x-5*c)*b^3+1/896*b^3/d*
exp(-7*d*x-7*c)+1/4*a^3*exp(d*x+c)*(3*exp(6*d*x+6*c)-11*exp(4*d*x+4*c)-11*e
xp(2*d*x+2*c)+3)/d/(exp(2*d*x+2*c)-1)^4+3/8*a^3/d*ln(exp(d*x+c)-1)+3*a^2*b/
d*ln(exp(d*x+c)-1)-3/8*a^3/d*ln(exp(d*x+c)+1)-3*a^2*b/d*ln(exp(d*x+c)+1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(132) = 264$.
time = 0.28, size = 340, normalized size = 2.39

$$-\frac{1}{4480}\, b^3 \left(\frac{\left(49\, e^{(-2\,dx-2\,c)} - 245\, e^{(-4\,dx-4\,c)} + 1225\, e^{(-6\,dx-6\,c)} - 5\right) e^{(7\,dx+7\,c)}}{d} + \frac{1225\, e^{(-dx-c)} - 245\, e^{(-3\,dx-3\,c)} + 49\, e^{(-5\,dx-5\,c)} - 5\, e^{(-7\,dx-7\,c)}}{d}\right) + \frac{1}{8}\, ab^2 \left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\, e^{(dx+c)}}{d} - \frac{9\, e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right) - \frac{1}{8}\, a^3 \left(\frac{3\, \log\left(e^{(-dx-c)} + 1\right)}{d} - \frac{3\, \log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{2\left(3\, e^{(-dx-c)} - 11\, e^{(-3\,dx-3\,c)} - 11\, e^{(-5\,dx-5\,c)} + 3\, e^{(-7\,dx-7\,c)}\right)}{d(4\, e^{(-2\,dx-2\,c)} - 6\, e^{(-4\,dx-4\,c)} + 4\, e^{(-6\,dx-6\,c)} - e^{(-8\,dx-8\,c)} - 1)}\right) - 3\, a^2 b \left(\frac{\log\left(e^{(-dx-c)} + 1\right)}{d} - \frac{\log\left(e^{(-dx-c)} - 1\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/8*a*b^2*(e^(3*d*x + 3*c)/d
 - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) - 1/8*a^3*(3*lo
g(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*
e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*
x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))
) - 3*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6441 vs. $2(132) = 264$.
time = 0.46, size = 6441, normalized size = 45.36

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] 1/4480*(5*b^3*cosh(d*x + c)^22 + 110*b^3*cosh(d*x + c)*sinh(d*x + c)^21 + 5
*b^3*sinh(d*x + c)^22 - 69*b^3*cosh(d*x + c)^20 + 3*(385*b^3*cosh(d*x + c)^
2 - 23*b^3)*sinh(d*x + c)^20 + 20*(385*b^3*cosh(d*x + c)^3 - 69*b^3*cosh(d*
x + c))*sinh(d*x + c)^19 + (560*a*b^2 + 471*b^3)*cosh(d*x + c)^18 + (36575*
b^3*cosh(d*x + c)^4 - 13110*b^3*cosh(d*x + c)^2 + 560*a*b^2 + 471*b^3)*sinh
(d*x + c)^18 + 18*(7315*b^3*cosh(d*x + c)^5 - 4370*b^3*cosh(d*x + c)^3 + (5
60*a*b^2 + 471*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - (7280*a*b^2 + 2519*b^
3)*cosh(d*x + c)^16 + (373065*b^3*cosh(d*x + c)^6 - 334305*b^3*cosh(d*x + c
)^4 - 7280*a*b^2 - 2519*b^3 + 153*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^16 + 16*(53295*b^3*cosh(d*x + c)^7 - 66861*b^3*cosh(d*x + c)^5
+ 51*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^3 - (7280*a*b^2 + 2519*b^3)*cosh(d
*x + c))*sinh(d*x + c)^15 + 6*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c
)^14 + 6*(266475*b^3*cosh(d*x + c)^8 - 445740*b^3*cosh(d*x + c)^6 + 510*(56
0*a*b^2 + 471*b^3)*cosh(d*x + c)^4 + 560*a^3 + 3080*a*b^2 + 891*b^3 - 20*(7
280*a*b^2 + 2519*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^14 + 4*(621775*b^3*cos
h(d*x + c)^9 - 1337220*b^3*cosh(d*x + c)^7 + 2142*(560*a*b^2 + 471*b^3)*cos
h(d*x + c)^5 - 140*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^3 + 21*(560*a^3 +
3080*a*b^2 + 891*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 - 14*(880*a^3 + 840*a
*b^2 + 231*b^3)*cosh(d*x + c)^12 + 2*(1616615*b^3*cosh(d*x + c)^10 - 434596
5*b^3*cosh(d*x + c)^8 + 9282*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^6 - 910*(7
280*a*b^2 + 2519*b^3)*cosh(d*x + c)^4 - 6160*a^3 - 5880*a*b^2 - 1617*b^3 +
273*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 24
*(146965*b^3*cosh(d*x + c)^11 - 482885*b^3*cosh(d*x + c)^9 + 1326*(560*a*b^
2 + 471*b^3)*cosh(d*x + c)^7 - 182*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^5
+ 91*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^3 - 7*(880*a^3 + 840*a*
b^2 + 231*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 14*(880*a^3 + 840*a*b^2 +
231*b^3)*cosh(d*x + c)^10 + 2*(1616615*b^3*cosh(d*x + c)^12 - 6374082*b^3*c
osh(d*x + c)^10 + 21879*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^8 - 4004*(7280*
a*b^2 + 2519*b^3)*cosh(d*x + c)^6 + 3003*(560*a^3 + 3080*a*b^2 + 891*b^3)*c
osh(d*x + c)^4 - 6160*a^3 - 5880*a*b^2 - 1617*b^3 - 462*(880*a^3 + 840*a*b^
2 + 231*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(621775*b^3*cosh(d*x + c
)^13 - 2897310*b^3*cosh(d*x + c)^11 + 12155*(560*a*b^2 + 471*b^3)*cosh(d*x
+ c)^9 - 2860*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^7 + 3003*(560*a^3 + 308
0*a*b^2 + 891*b^3)*cosh(d*x + c)^5 - 770*(880*a^3 + 840*a*b^2 + 231*b^3)*co
sh(d*x + c)^3 - 35*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^9 + 6*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^8 + 6*(266475*b^3
*cosh(d*x + c)^14 - 1448655*b^3*cosh(d*x + c)^12 + 7293*(560*a*b^2 + 471*b^
3)*cosh(d*x + c)^10 - 2145*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^8 + 3003*(
560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^6 - 1155*(880*a^3 + 840*a*b^2
 + 231*b^3)*cosh(d*x + c)^4 + 560*a^3 + 3080*a*b^2 + 891*b^3 - 105*(880*a^3
 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(53295*b^3*co
```

```
sh(d*x + c)^15 - 334305*b^3*cosh(d*x + c)^13 + 1989*(560*a*b^2 + 471*b^3)*c
osh(d*x + c)^11 - 715*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^9 + 1287*(560*a
^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^7 - 693*(880*a^3 + 840*a*b^2 + 231
*b^3)*cosh(d*x + c)^5 - 105*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^3
 + 3*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - (728
0*a*b^2 + 2519*b^3)*cosh(d*x + c)^6 + (373065*b^3*cosh(d*x + c)^16 - 267444
0*b^3*cosh(d*x + c)^14 + 18564*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^12 - 800
8*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^10 + 18018*(560*a^3 + 3080*a*b^2 +
891*b^3)*cosh(d*x + c)^8 - 12936*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x +
 c)^6 - 2940*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^4 - 7280*a*b^2 -
 2519*b^3 + 168*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 6*(21945*b^3*cosh(d*x + c)^17 - 178296*b^3*cosh(d*x + c)^15 + 1428
*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^13 - 728*(7280*a*b^2 + 2519*b^3)*cosh(
d*x + c)^11 + 2002*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^9 - 1848*
(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^7 - 588*(880*a^3 + 840*a*b^2
+ 231*b^3)*cosh(d*x + c)^5 + 56*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x +
 c)^3 - (7280*a*b^2 + 2519*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 69*b^3*cos
h(d*x + c)^2 + (560*a*b^2 + 471*b^3)*cosh(d*x + c)^4 + (36575*b^3*cosh(d*x
+ c)^18 - 334305*b^3*cosh(d*x + c)^16 + 3060*(560*a*b^2 + 471*b^3)*cosh(d*x
 + c)^14 - 1820*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^12 + 6006*(560*a^3 +
3080*a*b^2 + 891*b^3)*cosh(d*x + c)^10 - 6930*(880*a^3 + 840*a*b^2 + 231*b^
3)*cosh(d*x + c)^8 - 2940*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^6 +
 420*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^4 + 560*a*b^2 + 471*b^3
 - 15*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(1925*b^
3*cosh(d*x + c)^19 - 19665*b^3*cosh(d*x + c)^17 + 204*(560*a*b^2 + 471*b^3)
*cosh(d*x + c)^15 - 140*(7280*a*b^2 + 2519*b^3)...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(132) = 264.
time = 0.62, size = 271, normalized size = 1.91

$$\frac{5\,b^{3}\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{7}-84\,b^{3}\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{5}+560\,ab^{2}\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{3}+560\,b^{3}\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{3}-6720\,ab^{2}\left(e^{(dx+c)}+e^{(-dx-c)}\right)-2240\,b^{3}\left(e^{(dx+c)}+e^{(-dx-c)}\right)-840\,\left(a^{3}+8\,a^{2}b\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)+840\,\left(a^{3}+8\,a^{2}b\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)+\frac{1120\,\left(3\,a^{3}\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{3}-20\,a^{3}\left(e^{(dx+c)}+e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{2}-4\right)^{2}}}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*b^3*(e^(d*x + c) + e^(-d*x - c))^7 - 84*b^3*(e^(d*x + c) + e^(-d*
x - c))^5 + 560*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 + 560*b^3*(e^(d*x + c)
 + e^(-d*x - c))^3 - 6720*a*b^2*(e^(d*x + c) + e^(-d*x - c)) - 2240*b^3*(e^
(d*x + c) + e^(-d*x - c)) - 840*(a^3 + 8*a^2*b)*log(e^(d*x + c) + e^(-d*x -
 c) + 2) + 840*(a^3 + 8*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) + 1120*(
3*a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 20*a^3*(e^(d*x + c) + e^(-d*x - c)))
/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^2)/d
```

Mupad [B]
time = 1.15, size = 421, normalized size = 2.96

$$\frac{b^3\,\mathrm{e}^{-7\,c-7\,d\,x}}{896\,d}-\frac{7\,b^3\,\mathrm{e}^{-5\,c-5\,d\,x}}{640\,d}-\frac{7\,b^3\,\mathrm{e}^{5\,c+5\,d\,x}}{640\,d}-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\left(a^3\,\sqrt{-d^2}+8\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+16\,a^5\,b+64\,a^4\,b^2}}\right)\,\sqrt{a^6+16\,a^5\,b+64\,a^4\,b^2}}{4\,\sqrt{-d^2}}+\frac{b^3\,\mathrm{e}^{7\,c+7\,d\,x}}{896\,d}-\frac{6\,a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{b^2\,\mathrm{e}^{c+d\,x}\,\left(144\,a+35\,b\right)}{128\,d}-\frac{4\,a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}+\frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}\,\left(16\,a+7\,b\right)}{128\,d}+\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}\,\left(16\,a+7\,b\right)}{128\,d}-\frac{b^2\,\mathrm{e}^{-c-d\,x}\,\left(144\,a+35\,b\right)}{128\,d}+\frac{3\,a^3\,\mathrm{e}^{c+d\,x}}{4\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{a^3\,\mathrm{e}^{c+d\,x}}{2\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^5,x)
```

```
[Out] (b^3*exp(- 7*c - 7*d*x))/(896*d) - (7*b^3*exp(- 5*c - 5*d*x))/(640*d) - (7*
b^3*exp(5*c + 5*d*x))/(640*d) - (3*atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2)
+ 8*a^2*b*(-d^2)^(1/2)))/(d*(16*a^5*b + a^6 + 64*a^4*b^2)^(1/2)))*(16*a^5*b
 + a^6 + 64*a^4*b^2)^(1/2))/(4*(-d^2)^(1/2)) + (b^3*exp(7*c + 7*d*x))/(896*
d) - (6*a^3*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp
(6*c + 6*d*x) - 1)) - (b^2*exp(c + d*x)*(144*a + 35*b))/(128*d) - (4*a^3*ex
p(c + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x
) + exp(8*c + 8*d*x) + 1)) + (b^2*exp(- 3*c - 3*d*x)*(16*a + 7*b))/(128*d)
+ (b^2*exp(3*c + 3*d*x)*(16*a + 7*b))/(128*d) - (b^2*exp(- c - d*x)*(144*a
+ 35*b))/(128*d) + (3*a^3*exp(c + d*x))/(4*d*(exp(2*c + 2*d*x) - 1)) - (a^3
*exp(c + d*x))/(2*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.213 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=156

$$\frac{a^2(5a+24b)\tanh^{-1}(\cosh(c+dx))}{16d}+\frac{b^2(3a+b)\cosh(c+dx)}{d}-\frac{2b^3\cosh^3(c+dx)}{3d}+\frac{b^3\cosh^5(c+dx)}{5d}-a^2(5a$$

```
[Out] 1/16*a^2*(5*a+24*b)*arctanh(cosh(d*x+c))/d+b^2*(3*a+b)*cosh(d*x+c)/d-2/3*b^
3*cosh(d*x+c)^3/d+1/5*b^3*cosh(d*x+c)^5/d-1/16*a^2*(5*a+24*b)*coth(d*x+c)*c
sch(d*x+c)/d+5/24*a^3*coth(d*x+c)*csch(d*x+c)^3/d-1/6*a^3*coth(d*x+c)*csch(
d*x+c)^5/d
```

Rubi [A]

time = 0.21, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3294, 1171, 1828, 1824, 212}

$$-\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}+\frac{a^2(5a+24b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^2(5a+24b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d}+\frac{b^2(3a+b)\cosh(c+dx)}{d}+\frac{b^3\cosh^5(c+dx)}{5d}-\frac{2b^3\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (a^2*(5*a + 24*b)*ArcTanh[Cosh[c + d*x]])/(16*d) + (b^2*(3*a + b)*Cosh[c +
d*x])/d - (2*b^3*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d) - (a^
2*(5*a + 24*b)*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^3*Coth[c + d*x]*C
sch[c + d*x]^3)/(24*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^4}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= -\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d} - \frac{\operatorname{Subst}\left(\int \frac{-5a^3-18a^2b-18ab^2-6b^3+}{}\right.}{}\\
&= \frac{5a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d} - \frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}\\
&= -\frac{a^2(5a+24b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d} + \frac{5a^3\coth(c+dx)\operatorname{csc}}{24d}\\
&= -\frac{a^2(5a+24b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d} + \frac{5a^3\coth(c+dx)\operatorname{csc}}{24d}\\
&= \frac{b^2(3a+b)\cosh(c+dx)}{d} - \frac{2b^3\cosh^3(c+dx)}{3d} + \frac{b^3\cosh^5(c+dx}{5d}\\
&= \frac{a^2(5a+24b)\tanh^{-1}(\cosh(c+dx))}{16d} + \frac{b^2(3a+b)\cosh(c+dx)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.52, size = 223, normalized size = 1.43

$$
-\frac{-240b^2(24a+5b)\cosh(c+dx)+200b^3\cosh(3(c+dx))-24b^3\cosh(5(c+dx))+150a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+720a^2b\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)-30a^3\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)+5a^3\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)+600a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+2880a^2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+150a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+720a^2b\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+30a^3\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)+5a^3\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{1920d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/1920*(-240*b^2*(24*a + 5*b)*Cosh[c + d*x] + 200*b^3*Cosh[3*(c + d*x)] -
24*b^3*Cosh[5*(c + d*x)] + 150*a^3*Csch[(c + d*x)/2]^2 + 720*a^2*b*Csch[(c
+ d*x)/2]^2 - 30*a^3*Csch[(c + d*x)/2]^4 + 5*a^3*Csch[(c + d*x)/2]^6 + 600*
a^3*Log[Tanh[(c + d*x)/2]] + 2880*a^2*b*Log[Tanh[(c + d*x)/2]] + 150*a^3*Se
ch[(c + d*x)/2]^2 + 720*a^2*b*Sech[(c + d*x)/2]^2 + 30*a^3*Sech[(c + d*x)/2
]^4 + 5*a^3*Sech[(c + d*x)/2]^6)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(144) = 288$.
time = 1.52, size = 358, normalized size = 2.29

method	result
risch	$\frac{\mathrm{e}^{5dx+5c}b^{3}}{160d}-\frac{5\,\mathrm{e}^{3dx+3c}b^{3}}{96d}+\frac{3a\,\mathrm{e}^{dx+c}b^{2}}{2d}+\frac{5b^{3}\mathrm{e}^{dx+c}}{16d}+\frac{3a\,\mathrm{e}^{-dx-c}b^{2}}{2d}+\frac{5b^{3}\mathrm{e}^{-dx-c}}{16d}-\frac{5b^{3}\mathrm{e}^{-3dx-3c}}{96d}+\frac{\mathrm{e}^{-5dx-5c}b^{3}}{160d}-\ a^{2}\mathrm{e}^{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/160/d*exp(5*d*x+5*c)*b^3-5/96/d*exp(3*d*x+3*c)*b^3+3/2*a/d*exp(d*x+c)*b^2
+5/16*b^3/d*exp(d*x+c)+3/2*a/d*exp(-d*x-c)*b^2+5/16*b^3/d*exp(-d*x-c)-5/96*
b^3/d*exp(-3*d*x-3*c)+1/160/d*exp(-5*d*x-5*c)*b^3-1/24*a^2*exp(d*x+c)*(15*a
*exp(10*d*x+10*c)+72*b*exp(10*d*x+10*c)-85*a*exp(8*d*x+8*c)-216*b*exp(8*d*x
+8*c)+198*a*exp(6*d*x+6*c)+144*b*exp(6*d*x+6*c)+198*a*exp(4*d*x+4*c)+144*b*
exp(4*d*x+4*c)-85*a*exp(2*d*x+2*c)-216*b*exp(2*d*x+2*c)+15*a+72*b)/d/(exp(2
*d*x+2*c)-1)^6-5/16*a^3/d*ln(exp(d*x+c)-1)-3/2*a^2*b/d*ln(exp(d*x+c)-1)+5/1
6*a^3/d*ln(exp(d*x+c)+1)+3/2*a^2*b/d*ln(exp(d*x+c)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(144) = 288$.
time = 0.29, size = 390, normalized size = 2.50

$$\frac{1}{480}\,b^{3}\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{3}{2}\,ab^{2}\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{48}\,a^{3}\left(\frac{15\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15\,e^{(-dx-c)}-85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}+198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9\,dx-9\,c)}+15\,e^{(-11\,dx-11\,c)}\right)}{d(6\,e^{(-2\,dx-2\,c)}-15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}-15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}-e^{(-12\,dx-12\,c)}-1)}\right)+\frac{3}{2}\,a^{2}b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/480*b^3*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 3/2*a
*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/48*a^3*(15*log(e^(-d*x - c) + 1)/
d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c) +
 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(
-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c)
- 1))) + 3/2*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(
```

```
e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c)
- 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8547 vs. $2(144) = 288$.
time = 0.49, size = 8547, normalized size = 54.79

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^3*cosh(d*x + c)^22 + 66*b^3*cosh(d*x + c)*sinh(d*x + c)^21 + 3*b
^3*sinh(d*x + c)^22 - 43*b^3*cosh(d*x + c)^20 + (693*b^3*cosh(d*x + c)^2 -
43*b^3)*sinh(d*x + c)^20 + 20*(231*b^3*cosh(d*x + c)^3 - 43*b^3*cosh(d*x +
c))*sinh(d*x + c)^19 + 15*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^18 + 5*(4389*b^
3*cosh(d*x + c)^4 - 1634*b^3*cosh(d*x + c)^2 + 144*a*b^2 + 69*b^3)*sinh(d*x
 + c)^18 + 6*(13167*b^3*cosh(d*x + c)^5 - 8170*b^3*cosh(d*x + c)^3 + 45*(48
*a*b^2 + 23*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - 15*(20*a^3 + 96*a^2*b +
240*a*b^2 + 79*b^3)*cosh(d*x + c)^16 + 3*(74613*b^3*cosh(d*x + c)^6 - 69445
*b^3*cosh(d*x + c)^4 - 100*a^3 - 480*a^2*b - 1200*a*b^2 - 395*b^3 + 765*(48
*a*b^2 + 23*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 48*(10659*b^3*cosh(d*x
 + c)^7 - 13889*b^3*cosh(d*x + c)^5 + 255*(48*a*b^2 + 23*b^3)*cosh(d*x + c)
^3 - 5*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c))*sinh(d*x + c
)^15 + 10*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^14 + 10
*(95931*b^3*cosh(d*x + c)^8 - 166668*b^3*cosh(d*x + c)^6 + 4590*(48*a*b^2 +
 23*b^3)*cosh(d*x + c)^4 + 170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3 - 180*
(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^14
+ 20*(74613*b^3*cosh(d*x + c)^9 - 166668*b^3*cosh(d*x + c)^7 + 6426*(48*a*b
^2 + 23*b^3)*cosh(d*x + c)^5 - 420*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)
*cosh(d*x + c)^3 + 7*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^13 - 90*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x
 + c)^12 + 2*(969969*b^3*cosh(d*x + c)^10 - 2708355*b^3*cosh(d*x + c)^8 + 1
39230*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^6 - 13650*(20*a^3 + 96*a^2*b + 240*
a*b^2 + 79*b^3)*cosh(d*x + c)^4 - 1980*a^3 - 1440*a^2*b - 1800*a*b^2 - 495*
b^3 + 455*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^12 + 8*(264537*b^3*cosh(d*x + c)^11 - 902785*b^3*cosh(d*x + c)^9
+ 59670*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^7 - 8190*(20*a^3 + 96*a^2*b + 240
*a*b^2 + 79*b^3)*cosh(d*x + c)^5 + 455*(170*a^3 + 432*a^2*b + 648*a*b^2 + 1
87*b^3)*cosh(d*x + c)^3 - 135*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(
d*x + c))*sinh(d*x + c)^11 - 90*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cos
h(d*x + c)^10 + 2*(969969*b^3*cosh(d*x + c)^12 - 3972254*b^3*cosh(d*x + c)^
10 + 328185*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^8 - 60060*(20*a^3 + 96*a^2*b
+ 240*a*b^2 + 79*b^3)*cosh(d*x + c)^6 + 5005*(170*a^3 + 432*a^2*b + 648*a*b
^2 + 187*b^3)*cosh(d*x + c)^4 - 1980*a^3 - 1440*a^2*b - 1800*a*b^2 - 495*b^
```

```
3 - 2970*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^10 + 20*(74613*b^3*cosh(d*x + c)^13 - 361114*b^3*cosh(d*x + c)^11 + 36
465*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^9 - 8580*(20*a^3 + 96*a^2*b + 240*a*b
^2 + 79*b^3)*cosh(d*x + c)^7 + 1001*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*
b^3)*cosh(d*x + c)^5 - 990*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x
 + c)^3 - 45*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*
x + c)^9 + 10*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^8 +
 10*(95931*b^3*cosh(d*x + c)^14 - 541671*b^3*cosh(d*x + c)^12 + 65637*(48*a
*b^2 + 23*b^3)*cosh(d*x + c)^10 - 19305*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79
*b^3)*cosh(d*x + c)^8 + 3003*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*co
sh(d*x + c)^6 - 4455*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^
4 + 170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3 - 405*(44*a^3 + 32*a^2*b + 40
*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(31977*b^3*cosh(d*x
+ c)^15 - 208335*b^3*cosh(d*x + c)^13 + 29835*(48*a*b^2 + 23*b^3)*cosh(d*x
+ c)^11 - 10725*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^9 +
2145*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^7 - 4455*(44
*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^5 - 675*(44*a^3 + 32*a^2
*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 5*(170*a^3 + 432*a^2*b + 648*a*b^
2 + 187*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 15*(20*a^3 + 96*a^2*b + 240*a
*b^2 + 79*b^3)*cosh(d*x + c)^6 + (223839*b^3*cosh(d*x + c)^16 - 1666680*b^3
*cosh(d*x + c)^14 + 278460*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^12 - 120120*(2
0*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^10 + 30030*(170*a^3 +
432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^8 - 83160*(44*a^3 + 32*a^2*b
 + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^6 - 18900*(44*a^3 + 32*a^2*b + 40*a*b^2
 + 11*b^3)*cosh(d*x + c)^4 - 300*a^3 - 1440*a^2*b - 3600*a*b^2 - 1185*b^3 +
 280*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 2*(39501*b^3*cosh(d*x + c)^17 - 333336*b^3*cosh(d*x + c)^15 + 6426
0*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^13 - 32760*(20*a^3 + 96*a^2*b + 240*a*b
^2 + 79*b^3)*cosh(d*x + c)^11 + 10010*(170*a^3 + 432*a^2*b + 648*a*b^2 + 18
7*b^3)*cosh(d*x + c)^9 - 35640*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh
(d*x + c)^7 - 11340*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^5
 + 280*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^3 - 45*(20
*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 43*b
^3*cosh(d*x + c)^2 + 15*(48*a*b^2 + 23*b^3)*cos...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(144) = 288.

time = 0.59, size = 321, normalized size = 2.06

$$\frac{3\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5-40\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+720\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+240\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+15\,(5\,a^3+24\,a^2b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\,(5\,a^3+24\,a^2b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{20\left(15\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+72\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5-160\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-576\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+528\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+1152\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\right)^3}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/480*(3*b^3*(e^(d*x + c) + e^(-d*x - c))^5 - 40*b^3*(e^(d*x + c) + e^(-d*x
 - c))^3 + 720*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 240*b^3*(e^(d*x + c) +
e^(-d*x - c)) + 15*(5*a^3 + 24*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) -
 15*(5*a^3 + 24*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 20*(15*a^3*(e^
(d*x + c) + e^(-d*x - c))^5 + 72*a^2*b*(e^(d*x + c) + e^(-d*x - c))^5 - 160
*a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 576*a^2*b*(e^(d*x + c) + e^(-d*x - c)
)^3 + 528*a^3*(e^(d*x + c) + e^(-d*x - c)) + 1152*a^2*b*(e^(d*x + c) + e^(-
d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^3)/d
```

Mupad [B]
time = 1.14, size = 633, normalized size = 4.06

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(5\,a^3\,\sqrt{-d^2}+24\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{25\,a^6+240\,a^5\,b+576\,a^4\,b^2}}\right)\,\sqrt{25\,a^6+240\,a^5\,b+576\,a^4\,b^2}}{8\,\sqrt{-d^2}}-\frac{\frac{4\,\mathrm{e}^{5\,c+5\,d\,x}\,\left(8\,a^3+9\,b\,a^2\right)}{3\,d}-\frac{8\,a^2\,b\,\mathrm{e}^{3\,c+3\,d\,x}}{d}-\frac{8\,a^2\,b\,\mathrm{e}^{7\,c+7\,d\,x}}{d}+\frac{2\,a^2\,b\,\mathrm{e}^{9\,c+9\,d\,x}}{d}+\frac{2\,a^2\,b\,\mathrm{e}^{c+d\,x}}{d}}{15\,\mathrm{e}^{4\,c+4\,d\,x}-6\,\mathrm{e}^{2\,c+2\,d\,x}-20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}-6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1}-\frac{5\,b^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{96\,d}-\frac{5\,b^3\,\mathrm{e}^{3\,c+3\,d\,x}}{96\,d}+\frac{b^3\,\mathrm{e}^{-5\,c-5\,d\,x}}{160\,d}+\frac{b^3\,\mathrm{e}^{5\,c+5\,d\,x}}{160\,d}-\frac{\mathrm{e}^{c+d\,x}\,\left(5\,a^3+24\,b\,a^2\right)}{8\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{a^3\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}+\frac{b^2\,\mathrm{e}^{c+d\,x}\,\left(24\,a+5\,b\right)}{16\,d}-\frac{\mathrm{e}^{c+d\,x}\,\left(48\,a^2\,b-5\,a^3\right)}{12\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{22\,a^3\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}-\frac{16\,a^3\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1\right)}+\frac{b^2\,\mathrm{e}^{-c-d\,x}\,\left(24\,a+5\,b\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^7,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(5*a^3*(-d^2)^(1/2) + 24*a^2*b*(-d^2)^(1/2)))/(d*(24
0*a^5*b + 25*a^6 + 576*a^4*b^2)^(1/2)))*(240*a^5*b + 25*a^6 + 576*a^4*b^2)^
(1/2))/(8*(-d^2)^(1/2)) - ((4*exp(5*c + 5*d*x)*(9*a^2*b + 8*a^3))/(3*d) - (
8*a^2*b*exp(3*c + 3*d*x))/d - (8*a^2*b*exp(7*c + 7*d*x))/d + (2*a^2*b*exp(9
*c + 9*d*x))/d + (2*a^2*b*exp(c + d*x))/d)/(15*exp(4*c + 4*d*x) - 6*exp(2*c
 + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x
) + exp(12*c + 12*d*x) + 1) - (5*b^3*exp(- 3*c - 3*d*x))/(96*d) - (5*b^3*ex
p(3*c + 3*d*x))/(96*d) + (b^3*exp(- 5*c - 5*d*x))/(160*d) + (b^3*exp(5*c +
5*d*x))/(160*d) - (exp(c + d*x)*(24*a^2*b + 5*a^3))/(8*d*(exp(2*c + 2*d*x)
- 1)) - (a^3*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) +
exp(6*c + 6*d*x) - 1)) + (b^2*exp(c + d*x)*(24*a + 5*b))/(16*d) - (exp(c +
d*x)*(48*a^2*b - 5*a^3))/(12*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
 - (22*a^3*exp(c + d*x))/(3*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*
exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (16*a^3*exp(c + d*x))/(3*d*(5*e
xp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8
*d*x) + exp(10*c + 10*d*x) - 1)) + (b^2*exp(- c - d*x)*(24*a + 5*b))/(16*d)
```

3.214 $\int \operatorname{csch}^9(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 dx$

Optimal. Leaf size=171

$$-\frac{a(35a^2+144ab+384b^2)\tanh^{-1}(\cosh(c+dx))}{128d}-\frac{b^3\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}+\frac{a^2(35a+144b)\operatorname{coth}(}{12}$$

```
[Out] -1/128*a*(35*a^2+144*a*b+384*b^2)*arctanh(cosh(d*x+c))/d-b^3*cosh(d*x+c)/d+
1/3*b^3*cosh(d*x+c)^3/d+1/128*a^2*(35*a+144*b)*coth(d*x+c)*csch(d*x+c)/d-1/
192*a^2*(35*a+144*b)*coth(d*x+c)*csch(d*x+c)^3/d+7/48*a^3*coth(d*x+c)*csch(
d*x+c)^5/d-1/8*a^3*coth(d*x+c)*csch(d*x+c)^7/d
```

Rubi [A]
time = 0.23, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$,
Rules used = {3294, 1171, 1828, 1167, 212}

$$-\frac{a^3\coth(c+dx)\operatorname{csch}^7(c+dx)}{8d}+\frac{7a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{48d}-\frac{a(35a^2+144ab+384b^2)\tanh^{-1}(\cosh(c+dx))}{128d}-\frac{a^2(35a+144b)\coth(c+dx)\operatorname{csch}^3(c+dx)}{192d}+\frac{a^2(35a+144b)\coth(c+dx)\operatorname{csch}(c+dx)}{128d}+\frac{b^3\cosh^3(c+dx)}{3d}-\frac{b^3\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^9*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/128*(a*(35*a^2 + 144*a*b + 384*b^2)*ArcTanh[Cosh[c + d*x]])/d - (b^3*Cos
h[c + d*x])/d + (b^3*Cosh[c + d*x]^3)/(3*d) + (a^2*(35*a + 144*b)*Coth[c +
d*x]*Csch[c + d*x])/(128*d) - (a^2*(35*a + 144*b)*Coth[c + d*x]*Csch[c + d*
x]^3)/(192*d) + (7*a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(48*d) - (a^3*Coth[c
+ d*x]*Csch[c + d*x]^7)/(8*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
```

```
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^9(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= -\frac{\operatorname{Subst}\Big(\int\frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^5}\,dx,x,\cosh(c+dx)\Big)}{d}\\
&= -\frac{a^3\coth(c+dx)\operatorname{csch}^7(c+dx)}{8d}+\frac{\operatorname{Subst}\Big(\int\frac{-(a+2b)\left(7a^2+10ab+4b^2\right.}{}}{}\\
&= \frac{7a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{48d}-\frac{a^3\coth(c+dx)\operatorname{csch}^7(c+dx}{8d}\\
&= -\frac{a^2(35a+144b)\coth(c+dx)\operatorname{csch}^3(c+dx)}{192d}+\frac{7a^3\coth(c+d}{4}\\
&= \frac{a^2(35a+144b)\coth(c+dx)\operatorname{csch}(c+dx)}{128d}-\frac{a^2(35a+144b)\,\mathrm{co}}{}\\
&= \frac{a^2(35a+144b)\coth(c+dx)\operatorname{csch}(c+dx)}{128d}-\frac{a^2(35a+144b)\,\mathrm{co}}{}\\
&= -\frac{b^3\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}+\frac{a^2(35a+144b)\coth(c}{128}\\
&= -\frac{a\left(35a^2+144ab+384b^2\right)\tanh^{-1}(\cosh(c+dx))}{128d}-\frac{b^3\cosh(c}{d}
\end{aligned}
$$

Mathematica [A]
time = 1.27, size = 219, normalized size = 1.28

$$\frac{-4608b^3\cosh(c+dx)+512b^3\cosh(3(c+dx))+a\left(12a(35a+144b)\operatorname{csch}^2\left(\tfrac{1}{2}(c+dx)\right)-18a(5a+16b)\operatorname{csch}^4\left(\tfrac{1}{2}(c+dx)\right)+20a^2\operatorname{csch}^6\left(\tfrac{1}{2}(c+dx)\right)-3a^2\operatorname{csch}^8\left(\tfrac{1}{2}(c+dx)\right)+48\left(35a^2+144ab+384b^2\right)\log\left(\tanh\left(\tfrac{1}{2}(c+dx)\right)\right)+12a(35a+144b)\operatorname{sech}^2\left(\tfrac{1}{2}(c+dx)\right)+18a(5a+16b)\operatorname{sech}^4\left(\tfrac{1}{2}(c+dx)\right)+20a^2\operatorname{sech}^6\left(\tfrac{1}{2}(c+dx)\right)+3a^2\operatorname{sech}^8\left(\tfrac{1}{2}(c+dx)\right)\right)}{6144d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^9*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-4608*b^3*Cosh[c + d*x] + 512*b^3*Cosh[3*(c + d*x)] + a*(12*a*(35*a + 144*
b)*Csch[(c + d*x)/2]^2 - 18*a*(5*a + 16*b)*Csch[(c + d*x)/2]^4 + 20*a^2*Csc
h[(c + d*x)/2]^6 - 3*a^2*Csch[(c + d*x)/2]^8 + 48*(35*a^2 + 144*a*b + 384*b
^2)*Log[Tanh[(c + d*x)/2]] + 12*a*(35*a + 144*b)*Sech[(c + d*x)/2]^2 + 18*a
*(5*a + 16*b)*Sech[(c + d*x)/2]^4 + 20*a^2*Sech[(c + d*x)/2]^6 + 3*a^2*Sech
[(c + d*x)/2]^8))/(6144*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(159) = 318$.
time = 1.52, size = 375, normalized size = 2.19

method	result

risch	$\frac{e^{3dx+3c}b^3}{24d} - \frac{3b^3 e^{dx+c}}{8d} - \frac{3b^3 e^{-dx-c}}{8d} + \frac{b^3 e^{-3dx-3c}}{24d} + \frac{a^2 e^{dx+c}(105a\, e^{14dx+14c} + 432b\, e^{14dx+14c} - 805a\, e^{12dx+12c} - 3312b\, e^{12dx+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*b^3-3/8*b^3/d*exp(d*x+c)-3/8*b^3/d*exp(-d*x-c)+1/24*b
^3/d*exp(-3*d*x-3*c)+1/192*a^2*exp(d*x+c)*(105*a*exp(14*d*x+14*c)+432*b*exp
(14*d*x+14*c)-805*a*exp(12*d*x+12*c)-3312*b*exp(12*d*x+12*c)+2681*a*exp(10*
d*x+10*c)+7344*b*exp(10*d*x+10*c)-5053*a*exp(8*d*x+8*c)-4464*b*exp(8*d*x+8*
c)-5053*a*exp(6*d*x+6*c)-4464*b*exp(6*d*x+6*c)+2681*a*exp(4*d*x+4*c)+7344*b
*exp(4*d*x+4*c)-805*a*exp(2*d*x+2*c)-3312*b*exp(2*d*x+2*c)+105*a+432*b)/d/(
exp(2*d*x+2*c)-1)^8+35/128*a^3/d*ln(exp(d*x+c)-1)+9/8*a^2*b/d*ln(exp(d*x+c)
-1)+3*a/d*ln(exp(d*x+c)-1)*b^2-35/128*a^3/d*ln(exp(d*x+c)+1)-9/8*a^2*b/d*ln
(exp(d*x+c)+1)-3*a/d*ln(exp(d*x+c)+1)*b^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(159) = 318$.
time = 0.30, size = 463, normalized size = 2.71

$$\frac{1}{24}b^3\left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d}\right) - \frac{1}{384}a^3\left(\frac{105\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{105\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(105e^{(-dx-c)} - 805e^{(-3dx-3c)} + 2681e^{(-5dx-5c)} - 5053e^{(-7dx-7c)} - 5053e^{(-9dx-9c)} + 2681e^{(-11dx-11c)} - 805e^{(-13dx-13c)} + 105e^{(-15dx-15c)}\right)}{d(8e^{(-2dx-2c)} - 28e^{(-4dx-4c)} + 56e^{(-6dx-6c)} - 70e^{(-8dx-8c)} + 56e^{(-10dx-10c)} - 28e^{(-12dx-12c)} + 8e^{(-14dx-14c)} - e^{(-16dx-16c)} - 1)}\right) - \frac{3}{8}a^2b\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{3\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(3e^{(-dx-c)} - 11e^{(-3dx-3c)} - 11e^{(-5dx-5c)} + 3e^{(-7dx-7c)}\right)}{d(4e^{(-2dx-2c)} - 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} - e^{(-8dx-8c)} - 1)}\right) - 3ab^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) - 1/384*a^3*(105*log(e^(-d*x - c) + 1)/d - 105*log(e^(-d*x - c)
 - 1)/d + 2*(105*e^(-d*x - c) - 805*e^(-3*d*x - 3*c) + 2681*e^(-5*d*x - 5*c
) - 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9*c) + 2681*e^(-11*d*x - 11*c)
 - 805*e^(-13*d*x - 13*c) + 105*e^(-15*d*x - 15*c))/(d*(8*e^(-2*d*x - 2*c)
- 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) - 70*e^(-8*d*x - 8*c) + 56*e^(-
10*d*x - 10*c) - 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) - e^(-16*d*x
- 16*c) - 1))) - 3/8*a^2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c)
- 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*
e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
 - 6*c) - e^(-8*d*x - 8*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) + 1)/d - log(
e^(-d*x - c) - 1)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10848 vs. $2(159) = 318$.
time = 0.48, size = 10848, normalized size = 63.44

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(16*b^3*cosh(d*x + c)^22 + 352*b^3*cosh(d*x + c)*sinh(d*x + c)^21 + 1
6*b^3*sinh(d*x + c)^22 - 272*b^3*cosh(d*x + c)^20 + 16*(231*b^3*cosh(d*x +
c)^2 - 17*b^3)*sinh(d*x + c)^20 + 320*(77*b^3*cosh(d*x + c)^3 - 17*b^3*cosh
(d*x + c))*sinh(d*x + c)^19 + 2*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x +
c)^18 + 2*(58520*b^3*cosh(d*x + c)^4 - 25840*b^3*cosh(d*x + c)^2 + 105*a^3
+ 432*a^2*b + 728*b^3)*sinh(d*x + c)^18 + 12*(35112*b^3*cosh(d*x + c)^5 - 2
5840*b^3*cosh(d*x + c)^3 + 3*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c))
*sinh(d*x + c)^17 - 2*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^16 +
2*(596904*b^3*cosh(d*x + c)^6 - 658920*b^3*cosh(d*x + c)^4 - 805*a^3 - 3312
*a^2*b - 1880*b^3 + 153*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^16 + 32*(85272*b^3*cosh(d*x + c)^7 - 131784*b^3*cosh(d*x + c)^5
 + 51*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^3 - (805*a^3 + 3312*a^2
*b + 1880*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 + 2*(2681*a^3 + 7344*a^2*b +
 2512*b^3)*cosh(d*x + c)^ ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**9*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(159) = 318.
time = 0.60, size = 335, normalized size = 1.96

$$\frac{32\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-384\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)-3\left(35\,a^3+144\,a^2b+384\,ab^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)+3\left(35\,a^3+144\,a^2b+384\,ab^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)+\frac{4\left(105\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^7+432\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^7-1540\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5-6336\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+8176\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+29952\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-17856\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)-46080\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\right)^4}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/768*(32*b^3*(e^(d*x + c) + e^(-d*x - c))^3 - 384*b^3*(e^(d*x + c) + e^(-d
*x - c)) - 3*(35*a^3 + 144*a^2*b + 384*a*b^2)*log(e^(d*x + c) + e^(-d*x - c
) + 2) + 3*(35*a^3 + 144*a^2*b + 384*a*b^2)*log(e^(d*x + c) + e^(-d*x - c)
- 2) + 4*(105*a^3*(e^(d*x + c) + e^(-d*x - c))^7 + 432*a^2*b*(e^(d*x + c) +
 e^(-d*x - c))^7 - 1540*a^3*(e^(d*x + c) + e^(-d*x - c))^5 - 6336*a^2*b*(e^
(d*x + c) + e^(-d*x - c))^5 + 8176*a^3*(e^(d*x + c) + e^(-d*x - c))^3 + 299
52*a^2*b*(e^(d*x + c) + e^(-d*x - c))^3 - 17856*a^3*(e^(d*x + c) + e^(-d*x
- c)) - 46080*a^2*b*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x -
 c))^2 - 4)^4)/d
```

Mupad [B]
time = 1.12, size = 759, normalized size = 4.44

$$\frac{b^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{3\,b^3\,\mathrm{e}^{-c-d\,x}}{8\,d}-\frac{3\,b^3\,\mathrm{e}^{c+d\,x}}{8\,d}+\frac{b^3\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(35\,a^3\,\sqrt{-d^2}+384\,a\,b^2\,\sqrt{-d^2}+144\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{1225\,a^6+10080\,a^5\,b+47616\,a^4\,b^2+110592\,a^3\,b^3+147456\,a^2\,b^4}}\right)\,\sqrt{1225\,a^6+10080\,a^5\,b+47616\,a^4\,b^2+110592\,a^3\,b^3+147456\,a^2\,b^4}}{64\,\sqrt{-d^2}}+\frac{\mathrm{e}^{c+d\,x}\,\left(35\,a^3+144\,b\,a^2\right)}{64\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{\mathrm{e}^{c+d\,x}\,\left(a^3+48\,b\,a^2\right)}{4\,d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}-\frac{\mathrm{e}^{c+d\,x}\,\left(35\,a^3+144\,b\,a^2\right)}{96\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{\mathrm{e}^{c+d\,x}\,\left(432\,a^2\,b-7\,a^3\right)}{24\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{170\,a^3\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1\right)}-\frac{404\,a^3\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(15\,\mathrm{e}^{4\,c+4\,d\,x}-6\,\mathrm{e}^{2\,c+2\,d\,x}-20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}-6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)}-\frac{112\,a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(7\,\mathrm{e}^{2\,c+2\,d\,x}-21\,\mathrm{e}^{4\,c+4\,d\,x}+35\,\mathrm{e}^{6\,c+6\,d\,x}-35\,\mathrm{e}^{8\,c+8\,d\,x}+21\,\mathrm{e}^{10\,c+10\,d\,x}-7\,\mathrm{e}^{12\,c+12\,d\,x}+\mathrm{e}^{14\,c+14\,d\,x}-1\right)}-\frac{32\,a^3\,\mathrm{e}^{c+d\,x}}{d\,\left(28\,\mathrm{e}^{4\,c+4\,d\,x}-8\,\mathrm{e}^{2\,c+2\,d\,x}-56\,\mathrm{e}^{6\,c+6\,d\,x}+70\,\mathrm{e}^{8\,c+8\,d\,x}-56\,\mathrm{e}^{10\,c+10\,d\,x}+28\,\mathrm{e}^{12\,c+12\,d\,x}-8\,\mathrm{e}^{14\,c+14\,d\,x}+\mathrm{e}^{16\,c+16\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^9,x)
```

```
[Out] (b^3*exp(- 3*c - 3*d*x))/(24*d) - (3*b^3*exp(- c - d*x))/(8*d) - (3*b^3*exp
(c + d*x))/(8*d) + (b^3*exp(3*c + 3*d*x))/(24*d) - (atan((exp(d*x)*exp(c)*(
35*a^3*(-d^2)^(1/2) + 384*a*b^2*(-d^2)^(1/2) + 144*a^2*b*(-d^2)^(1/2)))/(d*
(10080*a^5*b + 1225*a^6 + 147456*a^2*b^4 + 110592*a^3*b^3 + 47616*a^4*b^2)^
(1/2)))*(10080*a^5*b + 1225*a^6 + 147456*a^2*b^4 + 110592*a^3*b^3 + 47616*a
^4*b^2)^(1/2))/(64*(-d^2)^(1/2)) + (exp(c + d*x)*(144*a^2*b + 35*a^3))/(64*
d*(exp(2*c + 2*d*x) - 1)) - (exp(c + d*x)*(48*a^2*b + a^3))/(4*d*(6*exp(4*c
 + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)
) - (exp(c + d*x)*(144*a^2*b + 35*a^3))/(96*d*(exp(4*c + 4*d*x) - 2*exp(2*c
 + 2*d*x) + 1)) - (exp(c + d*x)*(432*a^2*b - 7*a^3))/(24*d*(3*exp(2*c + 2*d
*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (170*a^3*exp(c + d*x))/
(3*d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*ex
p(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (404*a^3*exp(c + d*x))/(3*d*(15
*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c +
 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (112*a^3*exp(c
+ d*x))/(d*(7*exp(2*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x)
- 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(
14*c + 14*d*x) - 1)) - (32*a^3*exp(c + d*x))/(d*(28*exp(4*c + 4*d*x) - 8*ex
p(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(10*c +
10*d*x) + 28*exp(12*c + 12*d*x) - 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x)
 + 1))
```

3.215 $\int \operatorname{csch}^{11}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=189

$$\frac{3a(21a^2+80ab+128b^2)\tanh^{-1}(\cosh(c+dx))}{256d}+\frac{b^3\cosh(c+dx)}{d}-\frac{3a(21a^2+80ab+128b^2)\coth(c+dx)\mathrm{csc}}{256d}$$

```
[Out] 3/256*a*(21*a^2+80*a*b+128*b^2)*arctanh(cosh(d*x+c))/d+b^3*cosh(d*x+c)/d-3/
256*a*(21*a^2+80*a*b+128*b^2)*coth(d*x+c)*csch(d*x+c)/d+1/128*a^2*(21*a+80*
b)*coth(d*x+c)*csch(d*x+c)^3/d-1/160*a^2*(21*a+80*b)*coth(d*x+c)*csch(d*x+c
)^5/d+9/80*a^3*coth(d*x+c)*csch(d*x+c)^7/d-1/10*a^3*coth(d*x+c)*csch(d*x+c)
^9/d
```

Rubi [A]
time = 0.26, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3294, 1171, 1828, 396, 212}

$$-\frac{a^3\coth(c+dx)\operatorname{csch}^9(c+dx)}{10d}+\frac{9a^3\coth(c+dx)\operatorname{csch}^7(c+dx)}{80d}+\frac{3a(21a^2+80ab+128b^2)\tanh^{-1}(\cosh(c+dx))}{256d}-\frac{3a(21a^2+80ab+128b^2)\coth(c+dx)\operatorname{csch}(c+dx)}{256d}-\frac{a^2(21a+80b)\coth(c+dx)\operatorname{csch}^5(c+dx)}{160d}+\frac{a^2(21a+80b)\coth(c+dx)\operatorname{csch}^3(c+dx)}{128d}+\frac{b^3\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^11*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (3*a*(21*a^2 + 80*a*b + 128*b^2)*ArcTanh[Cosh[c + d*x]])/(256*d) + (b^3*Cos
h[c + d*x])/d - (3*a*(21*a^2 + 80*a*b + 128*b^2)*Coth[c + d*x]*Csch[c + d*x
])/(256*d) + (a^2*(21*a + 80*b)*Coth[c + d*x]*Csch[c + d*x]^3)/(128*d) - (a
^2*(21*a + 80*b)*Coth[c + d*x]*Csch[c + d*x]^5)/(160*d) + (9*a^3*Coth[c + d
*x]*Csch[c + d*x]^7)/(80*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^9)/(10*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \text{csch}^{11}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^6}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{a^3\coth(c+dx)\text{csch}^9(c+dx)}{10d} - \frac{\text{Subst}\left(\int \frac{-9a^3-30a^2b-30ab^2-1}{}\right.}{} \\
&= \frac{9a^3\coth(c+dx)\text{csch}^7(c+dx)}{80d} - \frac{a^3\coth(c+dx)\text{csch}^9(c+d}{10d} \\
&= -\frac{a^2(21a+80b)\coth(c+dx)\text{csch}^5(c+dx)}{160d} + \frac{9a^3\coth(c+d}{8} \\
&= \frac{a^2(21a+80b)\coth(c+dx)\text{csch}^3(c+dx)}{128d} - \frac{a^2(21a+80b)\,\text{co}}{} \\
&= -\frac{3a(21a^2+80ab+128b^2)\coth(c+dx)\text{csch}(c+dx)}{256d} + \frac{a^2(21}{} \\
&= \frac{b^3\cosh(c+dx)}{d} - \frac{3a(21a^2+80ab+128b^2)\coth(c+dx)\text{csch}}{256d} \\
&= \frac{3a(21a^2+80ab+128b^2)\tanh^{-1}(\cosh(c+dx))}{256d} + \frac{b^3\cosh(c+}{d}
\end{aligned}
$$

Mathematica [A]
time = 1.72, size = 265, normalized size = 1.40

$$\frac{b^3\cosh(c+dx)}{d} - \frac{a\left(60(21a^2+80ab+128b^2)\,\text{csch}^2\left(\frac{1}{2}(c+dx)\right) - 40a(7a+24b)\text{csch}^4\left(\frac{1}{2}(c+dx)\right) + 10a(7a+16b)\text{csch}^6\left(\frac{1}{2}(c+dx)\right) - 15a^2\text{csch}^8\left(\frac{1}{2}(c+dx)\right) + 2a^2\text{csch}^{10}\left(\frac{1}{2}(c+dx)\right) + 240(21a^2+80ab+128b^2)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + 60(21a^2+80ab+128b^2)\,\text{sech}^2\left(\frac{1}{2}(c+dx)\right) + 40a(7a+24b)\text{sech}^4\left(\frac{1}{2}(c+dx)\right) + 10a(7a+16b)\text{sech}^6\left(\frac{1}{2}(c+dx)\right) + 15a^2\text{sech}^8\left(\frac{1}{2}(c+dx)\right) + 2a^2\text{sech}^{10}\left(\frac{1}{2}(c+dx)\right)\right)}{20480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^11*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (b^3*Cosh[c + d*x])/d - (a*(60*(21*a^2 + 80*a*b + 128*b^2)*Csch[(c + d*x)/2
]^2 - 40*a*(7*a + 24*b)*Csch[(c + d*x)/2]^4 + 10*a*(7*a + 16*b)*Csch[(c + d
*x)/2]^6 - 15*a^2*Csch[(c + d*x)/2]^8 + 2*a^2*Csch[(c + d*x)/2]^10 + 240*(2
1*a^2 + 80*a*b + 128*b^2)*Log[Tanh[(c + d*x)/2]] + 60*(21*a^2 + 80*a*b + 12
8*b^2)*Sech[(c + d*x)/2]^2 + 40*a*(7*a + 24*b)*Sech[(c + d*x)/2]^4 + 10*a*(
7*a + 16*b)*Sech[(c + d*x)/2]^6 + 15*a^2*Sech[(c + d*x)/2]^8 + 2*a^2*Sech[(
c + d*x)/2]^10))/(20480*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(177) = 354$.
time = 1.54, size = 548, normalized size = 2.90

method	result
risch	$\frac{b^3 e^{dx+c}}{2d}+\frac{b^3 e^{-dx-c}}{2d}-\frac{a\,e^{dx+c}\left(1200ab+315a^2-11600ab\,e^{2dx+2c}+1920b^2+53280ab\,e^{10dx+10c}-93120ab\,e^{6dx+6c}+53280ab\,e^{8dx+8}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^3/d*exp(d*x+c)+1/2*b^3/d*exp(-d*x-c)-1/640*a*exp(d*x+c)*(1200*a*b+315
*a^2-11600*a*b*exp(2*d*x+2*c)+1920*b^2+53280*a*b*exp(10*d*x+10*c)-93120*a*b
*exp(6*d*x+6*c)+53280*a*b*exp(8*d*x+8*c)+13188*a^2*exp(4*d*x+4*c)+1200*a*b*
exp(18*d*x+18*c)-11600*a*b*exp(16*d*x+16*c)+50240*a*b*exp(14*d*x+14*c)-9312
0*a*b*exp(12*d*x+12*c)+50240*a*b*exp(4*d*x+4*c)+315*a^2*exp(18*d*x+18*c)+19
20*b^2*exp(18*d*x+18*c)-3045*a^2*exp(16*d*x+16*c)-13440*b^2*exp(16*d*x+16*c
)+13188*a^2*exp(14*d*x+14*c)+38400*b^2*exp(14*d*x+14*c)-33660*a^2*exp(12*d*
x+12*c)-33660*a^2*exp(6*d*x+6*c)+26880*b^2*exp(8*d*x+8*c)-13440*b^2*exp(2*d
*x+2*c)-53760*b^2*exp(6*d*x+6*c)+38400*b^2*exp(4*d*x+4*c)-3045*a^2*exp(2*d*
x+2*c)-53760*b^2*exp(12*d*x+12*c)+55970*a^2*exp(10*d*x+10*c)+26880*b^2*exp(
10*d*x+10*c)+55970*a^2*exp(8*d*x+8*c))/d/(exp(2*d*x+2*c)-1)^10-63/256*a^3/d
*ln(exp(d*x+c)-1)-15/16*a^2*b/d*ln(exp(d*x+c)-1)-3/2*a/d*ln(exp(d*x+c)-1)*b
^2+63/256*a^3/d*ln(exp(d*x+c)+1)+15/16*a^2*b/d*ln(exp(d*x+c)+1)+3/2*a/d*ln(
exp(d*x+c)+1)*b^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(177) = 354$.
time = 0.29, size = 573, normalized size = 3.03

$$\frac{1}{2}b^3\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{1280}a^3\left(\frac{315\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{315\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(315e^{(-dx-c)}-3045e^{(-3dx-3c)}+13188e^{(-5dx-5c)}-33660e^{(-7dx-7c)}+55970e^{(-9dx-9c)}+55970e^{(-11dx-11c)}-33660e^{(-13dx-13c)}+13188e^{(-15dx-15c)}-3045e^{(-17dx-17c)}+315e^{(-19dx-19c)}\right)}{d(10e^{(-2dx-2c)}-45e^{(-4dx-4c)}+120e^{(-6dx-6c)}-210e^{(-8dx-8c)}+252e^{(-10dx-10c)}-210e^{(-12dx-12c)}+120e^{(-14dx-14c)}-45e^{(-16dx-16c)}+10e^{(-18dx-18c)}-e^{(-20dx-20c)}-1)}\right)+\frac{1}{16}a^2b\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15e^{(-dx-c)}-85e^{(-3dx-3c)}+198e^{(-5dx-5c)}+198e^{(-7dx-7c)}-85e^{(-9dx-9c)}+15e^{(-11dx-11c)}\right)}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(-10dx-10c)}-e^{(-12dx-12c)}-1)}\right)+\frac{3}{2}ab^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3dx-3c)}\right)}{d(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/1280*a^3*(315*log(e^(-d*x - c)
 + 1)/d - 315*log(e^(-d*x - c) - 1)/d + 2*(315*e^(-d*x - c) - 3045*e^(-3*d*
x - 3*c) + 13188*e^(-5*d*x - 5*c) - 33660*e^(-7*d*x - 7*c) + 55970*e^(-9*d*
x - 9*c) + 55970*e^(-11*d*x - 11*c) - 33660*e^(-13*d*x - 13*c) + 13188*e^(-
15*d*x - 15*c) - 3045*e^(-17*d*x - 17*c) + 315*e^(-19*d*x - 19*c))/(d*(10*e
^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) + 120*e^(-6*d*x - 6*c) - 210*e^(-8*d*
x - 8*c) + 252*e^(-10*d*x - 10*c) - 210*e^(-12*d*x - 12*c) + 120*e^(-14*d*x
 - 14*c) - 45*e^(-16*d*x - 16*c) + 10*e^(-18*d*x - 18*c) - e^(-20*d*x - 20*
c) - 1))) + 1/16*a^2*b*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) -
1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 19
8*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(
-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x -
8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 3/2*a*b^2*(log(e^
```

```
(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x -
 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 13503 vs. $2(177) = 354$.
time = 0.48, size = 13503, normalized size = 71.44

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/1280*(640*b^3*cosh(d*x + c)^22 + 14080*b^3*cosh(d*x + c)*sinh(d*x + c)^21
 + 640*b^3*sinh(d*x + c)^22 - 30*(21*a^3 + 80*a^2*b + 128*a*b^2 + 192*b^3)*
cosh(d*x + c)^20 + 30*(4928*b^3*cosh(d*x + c)^2 - 21*a^3 - 80*a^2*b - 128*a
*b^2 - 192*b^3)*sinh(d*x + c)^20 + 200*(4928*b^3*cosh(d*x + c)^3 - 3*(21*a^
3 + 80*a^2*b + 128*a*b^2 + 192*b^3)*cosh(d*x + c))*sinh(d*x + c)^19 + 10*(6
09*a^3 + 2320*a^2*b + 2688*a*b^2 + 2240*b^3)*cosh(d*x + c)^18 + 10*(468160*
b^3*cosh(d*x + c)^4 + 609*a^3 + 2320*a^2*b + 2688*a*b^2 + 2240*b^3 - 570*(2
1*a^3 + 80*a^2*b + 128*a*b^2 + 192*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^18 +
 180*(93632*b^3*cosh(d*x + c)^5 - 190*(21*a^3 + 80*a^2*b + 128*a*b^2 + 192*
b^3)*cosh(d*x + c)^3 + (609*a^3 + 2320*a^2*b + 2688*a*b^2 + 2240*b^3)*cosh(
d*x + c))*sinh(d*x + c)^17 - 8*(3297*a^3 + 12560*a^2*b + 9600*a*b^2 + 6000*
b^3)*cosh(d*x + c)^16 + 2*(23876160*b^3*cosh(d*x + c)^6 - 72675*(21*a^3 + 8
0*a^2*b + 128*a*b^2 + 192*b^3)*cosh(d*x + c)^4 - 13188*a^3 - 50240*a^2*b -
38400*a*b^2 - 24000*b^3 + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**11*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(177) = 354$.
time = 0.64, size = 477, normalized size = 2.52

$$\frac{1280\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+15\,(21\,a^3+80\,a^2b+128\,ab^2)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\,(21\,a^3+80\,a^2b+128\,ab^2)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(315\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^9+1200\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^9+1920\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^9-5880\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^7-22400\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^7-30720\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^7+43008\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+163840\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+184320\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5-151680\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-542720\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-491520\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+247040\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+675840\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+491520\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\right)^5}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2560*(1280*b^3*(e^(d*x + c) + e^(-d*x - c)) + 15*(21*a^3 + 80*a^2*b + 128
*a*b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 15*(21*a^3 + 80*a^2*b + 128*a
*b^2)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 4*(315*a^3*(e^(d*x + c) + e^(-d
*x - c))^9 + 1200*a^2*b*(e^(d*x + c) + e^(-d*x - c))^9 + 1920*a*b^2*(e^(d*x
 + c) + e^(-d*x - c))^9 - 5880*a^3*(e^(d*x + c) + e^(-d*x - c))^7 - 22400*a
^2*b*(e^(d*x + c) + e^(-d*x - c))^7 - 30720*a*b^2*(e^(d*x + c) + e^(-d*x -
c))^7 + 43008*a^3*(e^(d*x + c) + e^(-d*x - c))^5 + 163840*a^2*b*(e^(d*x + c
) + e^(-d*x - c))^5 + 184320*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 - 151680*
a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 542720*a^2*b*(e^(d*x + c) + e^(-d*x -
c))^3 - 491520*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 + 247040*a^3*(e^(d*x +
c) + e^(-d*x - c)) + 675840*a^2*b*(e^(d*x + c) + e^(-d*x - c)) + 491520*a*b
^2*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^5)/d
```

Mupad [B]
time = 1.11, size = 1194, normalized size = 6.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^11,x)
```

```
[Out] (b^3*exp(c + d*x))/(2*d) - ((24*exp(5*c + 5*d*x)*(7*a*b^2 + 4*a^2*b))/(5*d)
 - (48*exp(7*c + 7*d*x)*(7*a*b^2 + 8*a^2*b))/(5*d) - (48*exp(11*c + 11*d*x)
*(7*a*b^2 + 8*a^2*b))/(5*d) + (24*exp(13*c + 13*d*x)*(7*a*b^2 + 4*a^2*b))/(
5*d) + (4*exp(9*c + 9*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(5*d) - (48*a
*b^2*exp(3*c + 3*d*x))/(5*d) - (48*a*b^2*exp(15*c + 15*d*x))/(5*d) + (6*a*b
^2*exp(17*c + 17*d*x))/(5*d) + (6*a*b^2*exp(c + d*x))/(5*d))/(45*exp(4*c +
4*d*x) - 10*exp(2*c + 2*d*x) - 120*exp(6*c + 6*d*x) + 210*exp(8*c + 8*d*x)
- 252*exp(10*c + 10*d*x) + 210*exp(12*c + 12*d*x) - 120*exp(14*c + 14*d*x)
+ 45*exp(16*c + 16*d*x) - 10*exp(18*c + 18*d*x) + exp(20*c + 20*d*x) + 1) +
 (b^3*exp(- c - d*x))/(2*d) + (3*atan((exp(d*x)*exp(c)*(21*a^3*(-d^2)^(1/2)
 + 128*a*b^2*(-d^2)^(1/2) + 80*a^2*b*(-d^2)^(1/2)))/(d*(3360*a^5*b + 441*a^
6 + 16384*a^2*b^4 + 20480*a^3*b^3 + 11776*a^4*b^2)^(1/2)))*(3360*a^5*b + 44
1*a^6 + 16384*a^2*b^4 + 20480*a^3*b^3 + 11776*a^4*b^2)^(1/2))/(128*(-d^2)^(
1/2)) - (exp(c + d*x)*(208*a^2*b + a^3))/(5*d*(5*exp(2*c + 2*d*x) - 10*exp(
4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x
) - 1)) - (exp(c + d*x)*(80*a^2*b + 21*a^3))/(80*d*(3*exp(2*c + 2*d*x) - 3*
exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (3*exp(c + d*x)*(464*a^2*b - 3*
a^3))/(40*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) +
 exp(8*c + 8*d*x) + 1)) - (3*exp(c + d*x)*(128*a*b^2 + 80*a^2*b + 21*a^3))/
(128*d*(exp(2*c + 2*d*x) - 1)) - (1032*a^3*exp(c + d*x))/(5*d*(7*exp(2*c +
2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(8*c + 8*d*x) +
21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) - 1)) + (
exp(c + d*x)*(400*a^2*b - 1536*a*b^2 + 105*a^3))/(320*d*(exp(4*c + 4*d*x) -
 2*exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(32*a^2*b + 209*a^3))/(5*d*(15*
```

```
exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c +
8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (176*a^3*exp(c +
 d*x))/(d*(28*exp(4*c + 4*d*x) - 8*exp(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) +
 70*exp(8*c + 8*d*x) - 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) - 8*ex
p(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) - (256*a^3*exp(c + d*x))/(5*d*(
9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) - 126*exp(8*
c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) + 36*exp(14*c +
 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1))
```

3.216 $\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=220

$$-\frac{(231a^3+840a^2b+1152ab^2+1024b^3)\tanh^{-1}(\cosh(c+dx))}{1024d}+\frac{3a(77a^2+280ab+384b^2)\coth(c+dx)\operatorname{csch}(}{1024d}$$

```
[Out] -1/1024*(231*a^3+840*a^2*b+1152*a*b^2+1024*b^3)*arctanh(cosh(d*x+c))/d+3/10
24*a*(77*a^2+280*a*b+384*b^2)*coth(d*x+c)*csch(d*x+c)/d-1/512*a*(77*a^2+280
*a*b+384*b^2)*coth(d*x+c)*csch(d*x+c)^3/d+7/640*a^2*(11*a+40*b)*coth(d*x+c)
*csch(d*x+c)^5/d-3/320*a^2*(11*a+40*b)*coth(d*x+c)*csch(d*x+c)^7/d+11/120*a
^3*coth(d*x+c)*csch(d*x+c)^9/d-1/12*a^3*coth(d*x+c)*csch(d*x+c)^11/d
```

Rubi [A]
time = 0.28, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3294, 1171, 1828, 393, 212}

$$-\frac{a^3\coth(c+dx)\operatorname{csch}^{11}(c+dx)}{12d}+\frac{11a^3\coth(c+dx)\operatorname{csch}^9(c+dx)}{120d}-\frac{a(77a^2+280ab+384b^2)\coth(c+dx)\operatorname{csch}^3(c+dx)}{512d}+\frac{3a(77a^2+280ab+384b^2)\coth(c+dx)\operatorname{csch}(c+dx)}{1024d}-\frac{3a^2(11a+40b)\coth(c+dx)\operatorname{csch}^7(c+dx)}{320d}+\frac{7a^2(11a+40b)\coth(c+dx)\operatorname{csch}^5(c+dx)}{640d}-\frac{(231a^3+840a^2b+1152ab^2+1024b^3)\tanh^{-1}(\cosh(c+dx))}{1024d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^13*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/1024*((231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*ArcTanh[Cosh[c + d*x
]])/d + (3*a*(77*a^2 + 280*a*b + 384*b^2)*Coth[c + d*x]*Csch[c + d*x])/(102
4*d) - (a*(77*a^2 + 280*a*b + 384*b^2)*Coth[c + d*x]*Csch[c + d*x]^3)/(512*
d) + (7*a^2*(11*a + 40*b)*Coth[c + d*x]*Csch[c + d*x]^5)/(640*d) - (3*a^2*(
11*a + 40*b)*Coth[c + d*x]*Csch[c + d*x]^7)/(320*d) + (11*a^3*Coth[c + d*x]
*Csch[c + d*x]^9)/(120*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^11)/(12*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{(1-x^2)^7}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= -\frac{a^3\coth(c+dx)\operatorname{csch}^{11}(c+dx)}{12d}+\frac{\operatorname{Subst}\left(\int \frac{-11a^3-36a^2b-36ab^2-1}{}\right.}{}\\
&= \frac{11a^3\coth(c+dx)\operatorname{csch}^9(c+dx)}{120d}-\frac{a^3\coth(c+dx)\operatorname{csch}^{11}(c+d}{12d}\\
&= -\frac{3a^2(11a+40b)\coth(c+dx)\operatorname{csch}^7(c+dx)}{320d}+\frac{11a^3\coth(c+d}{12}\\
&= \frac{7a^2(11a+40b)\coth(c+dx)\operatorname{csch}^5(c+dx)}{640d}-\frac{3a^2(11a+40b)\,\mathrm{co}}{}\\
&= -\frac{a(77a^2+280ab+384b^2)\coth(c+dx)\operatorname{csch}^3(c+dx)}{512d}+\frac{7a^2(1}{}\\
&= \frac{3a(77a^2+280ab+384b^2)\coth(c+dx)\operatorname{csch}(c+dx)}{1024d}-\frac{a(77a^2}{}\\
&= -\frac{(231a^3+840a^2b+1152ab^2+1024b^3)\tanh^{-1}(\cosh(c+dx))}{1024d}
\end{aligned}
$$

Mathematica [A]
time = 1.49, size = 246, normalized size = 1.12

$$\frac{-30a(76555a^2+75816ab+45696b^2)\coth(c+dx)\operatorname{csch}^{11}(c+dx)+2a(750629a^2+2074200ab+1422720b^2)\cosh(3(c+dx))\operatorname{csch}^{12}(c+dx)-9a(77099a^2+280360ab+246400b^2)\cosh(5(c+dx))\operatorname{csch}^{12}(c+dx)+63a(3421a^2+12440ab+14720b^2)\cosh(7(c+dx))\operatorname{csch}^{12}(c+dx)-525a(77a^2+280ab+384b^2)\cosh(9(c+dx))\operatorname{csch}^{12}(c+dx)+45a(77a^2+280ab+384b^2)\cosh(11(c+dx))\operatorname{csch}^{12}(c+dx)+15360(231a^3+840a^2b+1152ab^2+1024b^3)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{15728640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^13*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-30*a*(76555*a^2 + 75816*a*b + 45696*b^2)*Coth[c + d*x]*Csch[c + d*x]^11 +
 2*a*(750629*a^2 + 2074200*a*b + 1422720*b^2)*Cosh[3*(c + d*x)]*Csch[c + d*
x]^12 - 9*a*(77099*a^2 + 280360*a*b + 246400*b^2)*Cosh[5*(c + d*x)]*Csch[c
+ d*x]^12 + 63*a*(3421*a^2 + 12440*a*b + 14720*b^2)*Cosh[7*(c + d*x)]*Csch[
c + d*x]^12 - 525*a*(77*a^2 + 280*a*b + 384*b^2)*Cosh[9*(c + d*x)]*Csch[c +
 d*x]^12 + 45*a*(77*a^2 + 280*a*b + 384*b^2)*Cosh[11*(c + d*x)]*Csch[c + d*
x]^12 + 15360*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*Log[Tanh[(c + d
*x)/2]])/(15728640*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(206) = 412$.
time = 1.50, size = 632, normalized size = 2.87

method	result
risch	$a\,\mathrm{e}^{dx+c}\big(12600ab+3465a^2-147000ab\,\mathrm{e}^{2dx+2c}+17280b^2-2274480ab\,\mathrm{e}^{10dx+10c}-2523240ab\,\mathrm{e}^{6dx+6c}+4148400ab\,\mathrm{e}^{8dx+8c}+215523a^2\mathrm{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7680*a*exp(d*x+c)*(12600*a*b+3465*a^2-147000*a*b*exp(2*d*x+2*c)+17280*b^2
-2274480*a*b*exp(10*d*x+10*c)-2523240*a*b*exp(6*d*x+6*c)+4148400*a*b*exp(8*
d*x+8*c)+215523*a^2*exp(4*d*x+4*c)+783720*a*b*exp(18*d*x+18*c)+12600*a*b*ex
p(22*d*x+22*c)-2523240*a*b*exp(16*d*x+16*c)+4148400*a*b*exp(14*d*x+14*c)-22
74480*a*b*exp(12*d*x+12*c)-147000*a*b*exp(20*d*x+20*c)+783720*a*b*exp(4*d*x
+4*c)-40425*a^2*exp(20*d*x+20*c)-201600*b^2*exp(20*d*x+20*c)+215523*a^2*exp
(18*d*x+18*c)+927360*b^2*exp(18*d*x+18*c)-693891*a^2*exp(16*d*x+16*c)-22176
00*b^2*exp(16*d*x+16*c)+1501258*a^2*exp(14*d*x+14*c)+2845440*b^2*exp(14*d*x
+14*c)-2296650*a^2*exp(12*d*x+12*c)+3465*a^2*exp(22*d*x+22*c)+17280*b^2*exp
(22*d*x+22*c)-693891*a^2*exp(6*d*x+6*c)+2845440*b^2*exp(8*d*x+8*c)-201600*b
^2*exp(2*d*x+2*c)-2217600*b^2*exp(6*d*x+6*c)+927360*b^2*exp(4*d*x+4*c)-4042
5*a^2*exp(2*d*x+2*c)-1370880*b^2*exp(12*d*x+12*c)-2296650*a^2*exp(10*d*x+10
*c)-1370880*b^2*exp(10*d*x+10*c)+1501258*a^2*exp(8*d*x+8*c))/d/(exp(2*d*x+2
*c)-1)^12-231/1024*a^3/d*ln(exp(d*x+c)+1)-105/128*a^2*b/d*ln(exp(d*x+c)+1)-
9/8*a/d*ln(exp(d*x+c)+1)*b^2-1/d*ln(exp(d*x+c)+1)*b^3+231/1024*a^3/d*ln(exp
(d*x+c)-1)+105/128*a^2*b/d*ln(exp(d*x+c)-1)+9/8*a/d*ln(exp(d*x+c)-1)*b^2+1/
d*ln(exp(d*x+c)-1)*b^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(206) = 412$.
time = 0.30, size = 720, normalized size = 3.27

$$-\frac{1}{15360}a^3\left(\frac{3465\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3465\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3465e^{(-dx-c)}-40425e^{(-3dx-3c)}+215523e^{(-5dx-5c)}-693891e^{(-7dx-7c)}+1501258e^{(-9dx-9c)}-2296650e^{(-11dx-11c)}-2296650e^{(-13dx-13c)}+1501258e^{(-15dx-15c)}-693891e^{(-17dx-17c)}+215523e^{(-19dx-19c)}-40425e^{(-21dx-21c)}+3465e^{(-23dx-23c)}\right)}{d\left(12e^{(-2dx-2c)}-66e^{(-4dx-4c)}+220e^{(-6dx-6c)}-495e^{(-8dx-8c)}+792e^{(-10dx-10c)}-924e^{(-12dx-12c)}+792e^{(-14dx-14c)}-495e^{(-16dx-16c)}+220e^{(-18dx-18c)}-66e^{(-20dx-20c)}+12e^{(-22dx-22c)}-e^{(-24dx-24c)}-1\right)}\right)-\frac{1}{128}a^2b\left(\frac{105\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{105\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(105e^{(-dx-c)}-805e^{(-3dx-3c)}+2681e^{(-5dx-5c)}-5053e^{(-7dx-7c)}-5053e^{(-9dx-9c)}+2681e^{(-11dx-11c)}-805e^{(-13dx-13c)}+105e^{(-15dx-15c)}\right)}{d\left(8e^{(-2dx-2c)}-28e^{(-4dx-4c)}+56e^{(-6dx-6c)}-70e^{(-8dx-8c)}+56e^{(-10dx-10c)}-28e^{(-12dx-12c)}+8e^{(-14dx-14c)}-e^{(-16dx-16c)}-1\right)}\right)-\frac{3}{8}ab^2\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3e^{(-dx-c)}-11e^{(-3dx-3c)}-11e^{(-5dx-5c)}+3e^{(-7dx-7c)}\right)}{d\left(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8dx-8c)}-1\right)}\right)-b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/15360*a^3*(3465*log(e^(-d*x - c) + 1)/d - 3465*log(e^(-d*x - c) - 1)/d +
 2*(3465*e^(-d*x - c) - 40425*e^(-3*d*x - 3*c) + 215523*e^(-5*d*x - 5*c) -
693891*e^(-7*d*x - 7*c) + 1501258*e^(-9*d*x - 9*c) - 2296650*e^(-11*d*x - 1
1*c) - 2296650*e^(-13*d*x - 13*c) + 1501258*e^(-15*d*x - 15*c) - 693891*e^(
-17*d*x - 17*c) + 215523*e^(-19*d*x - 19*c) - 40425*e^(-21*d*x - 21*c) + 34
65*e^(-23*d*x - 23*c))/(d*(12*e^(-2*d*x - 2*c) - 66*e^(-4*d*x - 4*c) + 220*
e^(-6*d*x - 6*c) - 495*e^(-8*d*x - 8*c) + 792*e^(-10*d*x - 10*c) - 924*e^(-
12*d*x - 12*c) + 792*e^(-14*d*x - 14*c) - 495*e^(-16*d*x - 16*c) + 220*e^(-
18*d*x - 18*c) - 66*e^(-20*d*x - 20*c) + 12*e^(-22*d*x - 22*c) - e^(-24*d*x
 - 24*c) - 1))) - 1/128*a^2*b*(105*log(e^(-d*x - c) + 1)/d - 105*log(e^(-d*
```

```
x - c) - 1)/d + 2*(105*e^(-d*x - c) - 805*e^(-3*d*x - 3*c) + 2681*e^(-5*d*x
 - 5*c) - 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9*c) + 2681*e^(-11*d*x -
 11*c) - 805*e^(-13*d*x - 13*c) + 105*e^(-15*d*x - 15*c))/(d*(8*e^(-2*d*x -
 2*c) - 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) - 70*e^(-8*d*x - 8*c) + 5
6*e^(-10*d*x - 10*c) - 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) - e^(-1
6*d*x - 16*c) - 1))) - 3/8*a*b^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x
 - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c
) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(
-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) - b^3*(log(e^(-d*x - c) + 1)/d - lo
g(e^(-d*x - c) - 1)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 17811 vs. $2(206) = 412$.
time = 0.53, size = 17811, normalized size = 80.96

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/15360*(90*(77*a^3 + 280*a^2*b + 384*a*b^2)*cosh(d*x + c)^23 + 2070*(77*a^
3 + 280*a^2*b + 384*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^22 + 90*(77*a^3 + 28
0*a^2*b + 384*a*b^2)*sinh(d*x + c)^23 - 1050*(77*a^3 + 280*a^2*b + 384*a*b^
2)*cosh(d*x + c)^21 - 30*(2695*a^3 + 9800*a^2*b + 13440*a*b^2 - 759*(77*a^3
 + 280*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^21 + 630*(253*(77*
a^3 + 280*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 35*(77*a^3 + 280*a^2*b + 384
*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^20 + 126*(3421*a^3 + 12440*a^2*b + 147
20*a*b^2)*cosh(d*x + c)^19 + 126*(6325*(77*a^3 + 280*a^2*b + 384*a*b^2)*cos
h(d*x + c)^4 + 3421*a^3 + 12440*a^2*b + 14720*a*b^2 - 1750*(77*a^3 + 280*a^
2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^19 + 798*(3795*(77*a^3 + 28
0*a^2*b + 384*a*b^2)*cosh(d*x + c)^5 - 1750*(77*a^3 + 280*a^2*b + 384*a*b^2
)*cosh(d*x + c)^3 + 3*(3421*a^3 + 12440*a^2*b + 14720*a*b^2)*cosh(d*x + c))
*sinh(d*x + c)^18 - 18*(77099*a^3 + 280360*a^2*b + 246400*a*b^2)*cosh(d*x +
 c)^17 + 18*(504735*(77*a ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**13*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(206) = 412.
time = 0.62, size = 537, normalized size = 2.44

$$-\frac{15\,(231\,a^{3}+840\,a^{2}b+1152\,ab^{2}+1024\,b^{3})\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\,(231\,a^{3}+840\,a^{2}b+1152\,ab^{2}+1024\,b^{3})\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(3465\,a^{3}(e^{(dx+c)}+e^{(-dx-c)})^{11}+12600\,a^{2}b(e^{(dx+c)}+e^{(-dx-c)})^{11}+17280\,ab^{2}(e^{(dx+c)}+e^{(-dx-c)})^{11}-78540\,a^{3}(e^{(dx+c)}+e^{(-dx-c)})^{9}-285600\,a^{2}b(e^{(dx+c)}+e^{(-dx-c)})^{9}-391680\,ab^{2}(e^{(dx+c)}+e^{(-dx-c)})^{9}+731808\,a^{3}(e^{(dx+c)}+e^{(-dx-c)})^{7}+2661120\,a^{2}b(e^{(dx+c)}+e^{(-dx-c)})^{7}+3502080\,ab^{2}(e^{(dx+c)}+e^{(-dx-c)})^{7}-3560832\,a^{3}(e^{(dx+c)}+e^{(-dx-c)})^{5}-12948480\,a^{2}b(e^{(dx+c)}+e^{(-dx-c)})^{5}-15482880\,ab^{2}(e^{(dx+c)}+e^{(-dx-c)})^{5}+9391360\,a^{3}(e^{(dx+c)}+e^{(-dx-c)})^{3}+32839680\,a^{2}b(e^{(dx+c)}+e^{(-dx-c)})^{3}+33914880\,ab^{2}(e^{(dx+c)}+e^{(-dx-c)})^{3}-12180480\,a^{3}(e^{(dx+c)}+e^{(-dx-c)})-34283520\,a^{2}b(e^{(dx+c)}+e^{(-dx-c)})-29491200\,ab^{2}(e^{(dx+c)}+e^{(-dx-c)})\right)}{\left((e^{(dx+c)}+e^{(-dx-c)})^{2}-4\right)^{6}}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/30720*(15*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*log(e^(d*x + c)
+ e^(-d*x - c) + 2) - 15*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*log(
e^(d*x + c) + e^(-d*x - c) - 2) - 4*(3465*a^3*(e^(d*x + c) + e^(-d*x - c))^
11 + 12600*a^2*b*(e^(d*x + c) + e^(-d*x - c))^11 + 17280*a*b^2*(e^(d*x + c)
 + e^(-d*x - c))^11 - 78540*a^3*(e^(d*x + c) + e^(-d*x - c))^9 - 285600*a^2
*b*(e^(d*x + c) + e^(-d*x - c))^9 - 391680*a*b^2*(e^(d*x + c) + e^(-d*x - c
))^9 + 731808*a^3*(e^(d*x + c) + e^(-d*x - c))^7 + 2661120*a^2*b*(e^(d*x +
c) + e^(-d*x - c))^7 + 3502080*a*b^2*(e^(d*x + c) + e^(-d*x - c))^7 - 35608
32*a^3*(e^(d*x + c) + e^(-d*x - c))^5 - 12948480*a^2*b*(e^(d*x + c) + e^(-d
*x - c))^5 - 15482880*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 + 9391360*a^3*(e
^(d*x + c) + e^(-d*x - c))^3 + 32839680*a^2*b*(e^(d*x + c) + e^(-d*x - c))^
3 + 33914880*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12180480*a^3*(e^(d*x +
c) + e^(-d*x - c)) - 34283520*a^2*b*(e^(d*x + c) + e^(-d*x - c)) - 29491200
*a*b^2*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^6
)/d
```

Mupad [**B**]
time = 1.10, size = 1314, normalized size = 5.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^13,x)
```

```
[Out] (3*exp(c + d*x)*(384*a*b^2 + 280*a^2*b + 77*a^3))/(512*d*(exp(2*c + 2*d*x)
- 1)) - (5632*a^3*exp(c + d*x))/(3*d*(11*exp(2*c + 2*d*x) - 55*exp(4*c + 4*
d*x) + 165*exp(6*c + 6*d*x) - 330*exp(8*c + 8*d*x) + 462*exp(10*c + 10*d*x)
 - 462*exp(12*c + 12*d*x) + 330*exp(14*c + 14*d*x) - 165*exp(16*c + 16*d*x)
 + 55*exp(18*c + 18*d*x) - 11*exp(20*c + 20*d*x) + exp(22*c + 22*d*x) - 1))
 - (1024*a^3*exp(c + d*x))/(3*d*(66*exp(4*c + 4*d*x) - 12*exp(2*c + 2*d*x)
- 220*exp(6*c + 6*d*x) + 495*exp(8*c + 8*d*x) - 792*exp(10*c + 10*d*x) + 92
4*exp(12*c + 12*d*x) - 792*exp(14*c + 14*d*x) + 495*exp(16*c + 16*d*x) - 22
0*exp(18*c + 18*d*x) + 66*exp(20*c + 20*d*x) - 12*exp(22*c + 22*d*x) + exp(
24*c + 24*d*x) + 1)) - (exp(c + d*x)*(2424*a^2*b + a^3))/(6*d*(15*exp(4*c +
 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) -
6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (atan((exp(d*x)*exp(c)*(2
```

```
31*a^3*(-d^2)^(1/2) + 1024*b^3*(-d^2)^(1/2) + 1152*a*b^2*(-d^2)^(1/2) + 840
*a^2*b*(-d^2)^(1/2)))/(d*(2359296*a*b^5 + 388080*a^5*b + 53361*a^6 + 104857
6*b^6 + 3047424*a^2*b^4 + 2408448*a^3*b^3 + 1237824*a^4*b^2)^(1/2)))*(23592
96*a*b^5 + 388080*a^5*b + 53361*a^6 + 1048576*b^6 + 3047424*a^2*b^4 + 24084
48*a^3*b^3 + 1237824*a^4*b^2)^(1/2))/(512*(-d^2)^(1/2)) - (exp(c + d*x)*(10
200*a^2*b - 11*a^3))/(60*d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (exp(c +
d*x)*(384*a*b^2 + 280*a^2*b + 77*a^3))/(256*d*(exp(4*c + 4*d*x) - 2*exp(2*c
 + 2*d*x) + 1)) + (exp(c + d*x)*(280*a^2*b - 5760*a*b^2 + 77*a^3))/(320*d*(
3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (42*exp(
c + d*x)*(8*a^2*b + 15*a^3))/(d*(7*exp(2*c + 2*d*x) - 21*exp(4*c + 4*d*x) +
 35*exp(6*c + 6*d*x) - 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) - 7*exp(
12*c + 12*d*x) + exp(14*c + 14*d*x) - 1)) - (23488*a^3*exp(c + d*x))/(5*d*(
9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) - 126*exp(8*
c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) + 36*exp(14*c +
 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1)) - (3*exp(c + d*x
)*(640*a*b^2 + 40*a^2*b + 11*a^3))/(160*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c +
 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (4*exp(c + d*x)*(12
0*a^2*b + 3361*a^3))/(5*d*(28*exp(4*c + 4*d*x) - 8*exp(2*c + 2*d*x) - 56*ex
p(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(10*c + 10*d*x) + 28*exp(12*c
+ 12*d*x) - 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) - (20864*a^3*ex
p(c + d*x))/(5*d*(45*exp(4*c + 4*d*x) - 10*exp(2*c + 2*d*x) - 120*exp(6*c +
 6*d*x) + 210*exp(8*c + 8*d*x) - 252*exp(10*c + 10*d*x) + 210*exp(12*c + 12
*d*x) - 120*exp(14*c + 14*d*x) + 45*exp(16*c + 16*d*x) - 10*exp(18*c + 18*d
*x) + exp(20*c + 20*d*x) + 1))
```

3.217 $\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=255

$$-\frac{(1024a^3+1920a^2b+1512ab^2+429b^3)\,x}{2048}+\frac{(1024a^3+4224a^2b+4632ab^2+1619b^3)\cosh(c+dx)\sinh(c+}{2048d}$$

```
[Out] -1/2048*(1024*a^3+1920*a^2*b+1512*a*b^2+429*b^3)*x+1/2048*(1024*a^3+4224*a^
2*b+4632*a*b^2+1619*b^3)*cosh(d*x+c)*sinh(d*x+c)/d-1/3072*b*(4992*a^2+10728
*a*b+5549*b^2)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/3840*b*(1920*a^2+12312*a*b+105
79*b^2)*cosh(d*x+c)^5*sinh(d*x+c)/d-1/4480*b^2*(6888*a+11821*b)*cosh(d*x+c)
^7*sinh(d*x+c)/d+1/1680*b^2*(504*a+2593*b)*cosh(d*x+c)^9*sinh(d*x+c)/d-85/1
68*b^3*cosh(d*x+c)^11*sinh(d*x+c)/d+1/14*b^3*cosh(d*x+c)^13*sinh(d*x+c)/d
```

Rubi [A]

time = 0.40, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3296, 1271, 1828, 1171, 393, 212}

$$\frac{b(1920a^2+12312ab+10579b^2)\sinh(c+dx)\cosh^5(c+dx)}{3840d}-\frac{b(4992a^2+10728ab+5549b^2)\sinh(c+dx)\cosh^3(c+dx)}{3072d}+\frac{(1024a^3+4224a^2b+4632ab^2+1619b^3)\sinh(c+dx)\cosh(c+dx)}{2048d}-\frac{x(1024a^3+1920a^2b+1512ab^2+429b^3)}{2048}+\frac{b^2(504a+2593b)\sinh(c+dx)\cosh^9(c+dx)}{1680d}-\frac{b^2(6888a+11821b)\sinh(c+dx)\cosh^7(c+dx)}{4480d}+\frac{b^3\sinh(c+dx)\cosh^{13}(c+dx)}{14d}-\frac{85b^3\sinh(c+dx)\cosh^{11}(c+dx)}{168d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/2048*((1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*x) + ((1024*a^3 + 4
224*a^2*b + 4632*a*b^2 + 1619*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(2048*d) -
(b*(4992*a^2 + 10728*a*b + 5549*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(3072*d
) + (b*(1920*a^2 + 12312*a*b + 10579*b^2)*Cosh[c + d*x]^5*Sinh[c + d*x])/(3
840*d) - (b^2*(6888*a + 11821*b)*Cosh[c + d*x]^7*Sinh[c + d*x])/(4480*d) +
(b^2*(504*a + 2593*b)*Cosh[c + d*x]^9*Sinh[c + d*x])/(1680*d) - (85*b^3*Cos
h[c + d*x]^11*Sinh[c + d*x])/(168*d) + (b^3*Cosh[c + d*x]^13*Sinh[c + d*x])
/(14*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-2ax^2+(a+b)x^4\right)^3}{\left(1-x^2\right)^8}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh^{13}(c+dx)\sinh(c+dx)}{14d} + \text{Subst}\Big(\int \frac{-b^3+14\left(a^3-b^3\right)x^2-14(5a}{}$$

$$= -\frac{85b^3\cosh^{11}(c+dx)\sinh(c+dx)}{168d} + \frac{b^3\cosh^{13}(c+dx)\sinh(c}{14d}$$

$$= \frac{b^2(504a+2593b)\cosh^9(c+dx)\sinh(c+dx)}{1680d} - \frac{85b^3\cosh^{11}(c}{}$$

$$= -\frac{b^2(6888a+11821b)\cosh^7(c+dx)\sinh(c+dx)}{4480d} + \frac{b^2(504a+}{}$$

$$= \frac{b(1920a^2+12312ab+10579b^2)\cosh^5(c+dx)\sinh(c+dx)}{3840d} -$$

$$= -\frac{b(4992a^2+10728ab+5549b^2)\cosh^3(c+dx)\sinh(c+dx)}{3072d} +$$

$$= \frac{(1024a^3+4224a^2b+4632ab^2+1619b^3)\cosh(c+dx)\sinh(c+}{2048d}$$

$$= -\frac{(1024a^3+1920a^2b+1512ab^2+429b^3)\,x}{2048} + \frac{(1024a^3+4224a}{}$$

Mathematica [A]
time = 0.51, size = 189, normalized size = 0.74

$$\frac{-840(1024a^3+1920a^2b+1512ab^2+429b^3)(c+dx)+105(4096a^3+11520a^2b+10080ab^2+3003b^3)\sinh(2(c+dx))-105b(2304a^2+2880ab+1001b^2)\sinh(4(c+dx))+35b(768a^2+2160ab+1001b^2)\sinh(6(c+dx))-105b^2(120a+91b)\sinh(8(c+dx))+21b^2(48a+91b)\sinh(10(c+dx))-245b^3\sinh(12(c+dx))+15b^3\sinh(14(c+dx))}{1720320d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-840*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*(c + d*x) + 105*(4096*
a^3 + 11520*a^2*b + 10080*a*b^2 + 3003*b^3)*Sinh[2*(c + d*x)] - 105*b*(2304
*a^2 + 2880*a*b + 1001*b^2)*Sinh[4*(c + d*x)] + 35*b*(768*a^2 + 2160*a*b +
1001*b^2)*Sinh[6*(c + d*x)] - 105*b^2*(120*a + 91*b)*Sinh[8*(c + d*x)] + 21
*b^2*(48*a + 91*b)*Sinh[10*(c + d*x)] - 245*b^3*Sinh[12*(c + d*x)] + 15*b^3
*Sinh[14*(c + d*x)])/(1720320*d)
```

Maple [A]
time = 1.89, size = 215, normalized size = 0.84

method	result
default	$\frac{\left(-\frac{91}{2048}b^{3}-\frac{15}{256}a\,b^{2}\right)\sinh(8dx+8c)}{8d}+\frac{\left(\frac{91}{8192}b^{3}+\frac{3}{512}a\,b^{2}\right)\sinh(10dx+10c)}{10d}+\frac{\left(-\frac{1001}{4096}b^{3}-\frac{45}{64}a\,b^{2}-\frac{9}{16}a^{2}b\right)\sinh(4dx+4c)}{4d}+\frac{\left(\frac{1001}{8192}b^{3}+\right.}{}$
risch	$-\frac{15a^{2}bx}{16}+\frac{3b^{2}\mathrm{e}^{10dx+10c}a}{10240d}-\frac{15b^{2}\mathrm{e}^{8dx+8c}a}{4096d}+\frac{45b\,\mathrm{e}^{2dx+2c}a^{2}}{128d}+\frac{b^{3}\mathrm{e}^{14dx+14c}}{229376d}-\frac{7b^{3}\mathrm{e}^{12dx+12c}}{98304d}+\frac{7b^{3}\mathrm{e}^{-12dx-12c}}{98304d}-\frac{b^{3}\mathrm{e}^{-14dx}}{22937}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-91/2048*b^3-15/256*a*b^2)*sinh(8*d*x+8*c)/d+1/10*(91/8192*b^3+3/512*a
*b^2)*sinh(10*d*x+10*c)/d+1/4*(-1001/4096*b^3-45/64*a*b^2-9/16*a^2*b)*sinh(
4*d*x+4*c)/d+1/6*(1001/8192*b^3+135/512*a*b^2+3/32*a^2*b)*sinh(6*d*x+6*c)/d
+1/2*(3003/8192*b^3+315/256*a*b^2+45/32*a^2*b+1/2*a^3)*sinh(2*d*x+2*c)/d-1/
2*a^3*x-429/2048*b^3*x-189/256*a*b^2*x-15/16*a^2*b*x-7/49152*b^3*sinh(12*d*
x+12*c)/d+1/114688*b^3*sinh(14*d*x+14*c)/d
```

Maxima [**A**]
time = 0.28, size = 442, normalized size = 1.73

$$-\frac{1}{8}a^{3}\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{3440640}b^{3}\left(\frac{\left(245e^{(-2dx-2c)}-1911e^{(-4dx-4c)}+9555e^{(-6dx-6c)}-35035e^{(-8dx-8c)}+105105e^{(-10dx-10c)}-315315e^{(-12dx-12c)}-15\right)e^{(14dx+14c)}}{d}+\frac{720720(dx+c)}{d}+\frac{315315e^{(-2dx-2c)}-105105e^{(-4dx-4c)}+35035e^{(-6dx-6c)}-9555e^{(-8dx-8c)}+1911e^{(-10dx-10c)}-245e^{(-12dx-12c)}+15e^{(-14dx-14c)}}{d}\right)-\frac{3}{20480}ab^{2}\left(\frac{\left(25e^{(-2dx-2c)}-150e^{(-4dx-4c)}+600e^{(-6dx-6c)}-2100e^{(-8dx-8c)}-2\right)e^{(10dx+10c)}}{d}+\frac{5040(dx+c)}{d}+\frac{2100e^{(-2dx-2c)}-600e^{(-4dx-4c)}+150e^{(-6dx-6c)}-25e^{(-8dx-8c)}+2e^{(-10dx-10c)}}{d}\right)-\frac{1}{128}a^{2}b\left(\frac{\left(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}+\frac{120(dx+c)}{d}+\frac{45e^{(-2dx-2c)}-9e^{(-4dx-4c)}+e^{(-6dx-6c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/3440640*b^3*((2
45*e^(-2*d*x - 2*c) - 1911*e^(-4*d*x - 4*c) + 9555*e^(-6*d*x - 6*c) - 35035
*e^(-8*d*x - 8*c) + 105105*e^(-10*d*x - 10*c) - 315315*e^(-12*d*x - 12*c) -
 15)*e^(14*d*x + 14*c)/d + 720720*(d*x + c)/d + (315315*e^(-2*d*x - 2*c) -
105105*e^(-4*d*x - 4*c) + 35035*e^(-6*d*x - 6*c) - 9555*e^(-8*d*x - 8*c) +
1911*e^(-10*d*x - 10*c) - 245*e^(-12*d*x - 12*c) + 15*e^(-14*d*x - 14*c))/d
) - 3/20480*a*b^2*((25*e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*
d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c
)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c)
- 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 1/128*a^2*b*((9*e^(-2*d*
x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (
45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 627 vs. 2(239) = 478.
time = 0.39, size = 627, normalized size = 2.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/860160*(105*b^3*cosh(d*x + c)*sinh(d*x + c)^13 + 210*(13*b^3*cosh(d*x + c
)^3 - 7*b^3*cosh(d*x + c))*sinh(d*x + c)^11 + 35*(429*b^3*cosh(d*x + c)^5 -
 770*b^3*cosh(d*x + c)^3 + 3*(48*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh(d*x +
c)^9 + 60*(429*b^3*cosh(d*x + c)^7 - 1617*b^3*cosh(d*x + c)^5 + 21*(48*a*b^
2 + 91*b^3)*cosh(d*x + c)^3 - 7*(120*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh(d*
x + c)^7 + 21*(715*b^3*cosh(d*x + c)^9 - 4620*b^3*cosh(d*x + c)^7 + 126*(48
*a*b^2 + 91*b^3)*cosh(d*x + c)^5 - 140*(120*a*b^2 + 91*b^3)*cosh(d*x + c)^3
 + 5*(768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 7
0*(39*b^3*cosh(d*x + c)^11 - 385*b^3*cosh(d*x + c)^9 + 18*(48*a*b^2 + 91*b^
3)*cosh(d*x + c)^7 - 42*(120*a*b^2 + 91*b^3)*cosh(d*x + c)^5 + 5*(768*a^2*b
 + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 - 3*(2304*a^2*b + 2880*a*b^2 + 10
01*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 420*(1024*a^3 + 1920*a^2*b + 1512*
a*b^2 + 429*b^3)*d*x + 105*(b^3*cosh(d*x + c)^13 - 14*b^3*cosh(d*x + c)^11
+ (48*a*b^2 + 91*b^3)*cosh(d*x + c)^9 - 4*(120*a*b^2 + 91*b^3)*cosh(d*x + c
)^7 + (768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c)^5 - 2*(2304*a^2*b +
 2880*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 + (4096*a^3 + 11520*a^2*b + 10080*a
*b^2 + 3003*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(250) = 500$.
time = 7.78, size = 877, normalized size = 3.44

$$\begin{cases} [illegible] & \text{for } d \neq 0 \\ x\left(a + b\sinh^4(c)\right)^3 \sinh^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 15*a**2*b*x*sinh(c + d*x)**6/16 - 45*a**2*
b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 45*a**2*b*x*sinh(c + d*x)**2*cos
h(c + d*x)**4/16 - 15*a**2*b*x*cosh(c + d*x)**6/16 + 33*a**2*b*sinh(c + d*x
)**5*cosh(c + d*x)/(16*d) - 5*a**2*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d
) + 15*a**2*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + 189*a*b**2*x*sinh(c +
 d*x)**10/256 - 945*a*b**2*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 945*a*
b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 945*a*b**2*x*sinh(c + d*x)**
4*cosh(c + d*x)**6/128 + 945*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256
 - 189*a*b**2*x*cosh(c + d*x)**10/256 + 579*a*b**2*sinh(c + d*x)**9*cosh(c
+ d*x)/(256*d) - 711*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + 63*
a*b**2*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) - 441*a*b**2*sinh(c + d*x)*
*3*cosh(c + d*x)**7/(128*d) + 189*a*b**2*sinh(c + d*x)*cosh(c + d*x)**9/(25
6*d) + 429*b**3*x*sinh(c + d*x)**14/2048 - 3003*b**3*x*sinh(c + d*x)**12*co
sh(c + d*x)**2/2048 + 9009*b**3*x*sinh(c + d*x)**10*cosh(c + d*x)**4/2048 -
 15015*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**6/2048 + 15015*b**3*x*sinh(c
+ d*x)**6*cosh(c + d*x)**8/2048 - 9009*b**3*x*sinh(c + d*x)**4*cosh(c + d*x
)**10/2048 + 3003*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**12/2048 - 429*b**3
```

```
*x*cosh(c + d*x)**14/2048 + 1619*b**3*sinh(c + d*x)**13*cosh(c + d*x)/(2048
*d) - 4511*b**3*sinh(c + d*x)**11*cosh(c + d*x)**3/(1536*d) + 171457*b**3*s
inh(c + d*x)**9*cosh(c + d*x)**5/(30720*d) - 429*b**3*sinh(c + d*x)**7*cosh
(c + d*x)**7/(70*d) + 40469*b**3*sinh(c + d*x)**5*cosh(c + d*x)**9/(10240*d
) - 715*b**3*sinh(c + d*x)**3*cosh(c + d*x)**11/(512*d) + 429*b**3*sinh(c +
 d*x)*cosh(c + d*x)**13/(2048*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**3*sinh(
c)**2, True))
```

Giac [A]
time = 0.57, size = 401, normalized size = 1.57

$$\frac{b^3e^{(14\,dx+14\,c)}}{229376\,d}-\frac{7\,b^3e^{(12\,dx+12\,c)}}{98304\,d}+\frac{7\,b^3e^{(-12\,dx-12\,c)}}{98304\,d}-\frac{b^3e^{(-14\,dx-14\,c)}}{229376\,d}-\frac{1}{2048}\,(1024\,a^3+1920\,a^2b+1512\,ab^2+429\,b^3)x+\frac{(48\,ab^2+91\,b^3)e^{(10\,dx+10\,c)}}{163840\,d}-\frac{(120\,ab^2+91\,b^3)e^{(8\,dx+8\,c)}}{32768\,d}+\frac{(768\,a^2b+2160\,ab^2+1001\,b^3)e^{(6\,dx+6\,c)}}{98304\,d}-\frac{(2304\,a^2b+2880\,ab^2+1001\,b^3)e^{(4\,dx+4\,c)}}{32768\,d}+\frac{(4096\,a^3+11520\,a^2b+10080\,ab^2+3003\,b^3)e^{(2\,dx+2\,c)}}{32768\,d}-\frac{(4096\,a^3+11520\,a^2b+10080\,ab^2+3003\,b^3)e^{(-2\,dx-2\,c)}}{32768\,d}+\frac{(2304\,a^2b+2880\,ab^2+1001\,b^3)e^{(-4\,dx-4\,c)}}{32768\,d}-\frac{(768\,a^2b+2160\,ab^2+1001\,b^3)e^{(-6\,dx-6\,c)}}{98304\,d}+\frac{(120\,ab^2+91\,b^3)e^{(-8\,dx-8\,c)}}{32768\,d}-\frac{(48\,ab^2+91\,b^3)e^{(-10\,dx-10\,c)}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/229376*b^3*e^(14*d*x + 14*c)/d - 7/98304*b^3*e^(12*d*x + 12*c)/d + 7/9830
4*b^3*e^(-12*d*x - 12*c)/d - 1/229376*b^3*e^(-14*d*x - 14*c)/d - 1/2048*(10
24*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*x + 1/163840*(48*a*b^2 + 91*b^3
)*e^(10*d*x + 10*c)/d - 1/32768*(120*a*b^2 + 91*b^3)*e^(8*d*x + 8*c)/d + 1/
98304*(768*a^2*b + 2160*a*b^2 + 1001*b^3)*e^(6*d*x + 6*c)/d - 1/32768*(2304
*a^2*b + 2880*a*b^2 + 1001*b^3)*e^(4*d*x + 4*c)/d + 1/32768*(4096*a^3 + 115
20*a^2*b + 10080*a*b^2 + 3003*b^3)*e^(2*d*x + 2*c)/d - 1/32768*(4096*a^3 +
11520*a^2*b + 10080*a*b^2 + 3003*b^3)*e^(-2*d*x - 2*c)/d + 1/32768*(2304*a^
2*b + 2880*a*b^2 + 1001*b^3)*e^(-4*d*x - 4*c)/d - 1/98304*(768*a^2*b + 2160
*a*b^2 + 1001*b^3)*e^(-6*d*x - 6*c)/d + 1/32768*(120*a*b^2 + 91*b^3)*e^(-8*
d*x - 8*c)/d - 1/163840*(48*a*b^2 + 91*b^3)*e^(-10*d*x - 10*c)/d
```

Mupad [B]
time = 0.79, size = 393, normalized size = 1.54

$$\frac{\mathrm{e}^{6\,c+6\,d\,x}\,\left(768\,a^2\,b+2160\,a\,b^2+1001\,b^3\right)}{98304\,d}-\frac{\mathrm{e}^{-6\,c-6\,d\,x}\,\left(768\,a^2\,b+2160\,a\,b^2+1001\,b^3\right)}{98304\,d}-x\,\left(\frac{a^3}{2}+\frac{15\,a^2\,b}{16}+\frac{189\,a\,b^2}{256}+\frac{429\,b^3}{2048}\right)+\frac{\mathrm{e}^{-4\,c-4\,d\,x}\,\left(2304\,a^2\,b+2880\,a\,b^2+1001\,b^3\right)}{32768\,d}-\frac{\mathrm{e}^{4\,c+4\,d\,x}\,\left(2304\,a^2\,b+2880\,a\,b^2+1001\,b^3\right)}{32768\,d}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,\left(4096\,a^3+11520\,a^2\,b+10080\,a\,b^2+3003\,b^3\right)}{32768\,d}+\frac{\mathrm{e}^{2\,c+2\,d\,x}\,\left(4096\,a^3+11520\,a^2\,b+10080\,a\,b^2+3003\,b^3\right)}{32768\,d}+\frac{7\,b^3\,\mathrm{e}^{-12\,c-12\,d\,x}}{98304\,d}-\frac{7\,b^3\,\mathrm{e}^{12\,c+12\,d\,x}}{98304\,d}-\frac{b^3\,\mathrm{e}^{-14\,c-14\,d\,x}}{229376\,d}+\frac{b^3\,\mathrm{e}^{14\,c+14\,d\,x}}{229376\,d}-\frac{b^2\,\mathrm{e}^{-10\,c-10\,d\,x}\,\left(48\,a+91\,b\right)}{163840\,d}+\frac{b^2\,\mathrm{e}^{10\,c+10\,d\,x}\,\left(48\,a+91\,b\right)}{163840\,d}+\frac{b^2\,\mathrm{e}^{-8\,c-8\,d\,x}\,\left(120\,a+91\,b\right)}{32768\,d}-\frac{b^2\,\mathrm{e}^{8\,c+8\,d\,x}\,\left(120\,a+91\,b\right)}{32768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^4)^3,x)
```

```
[Out] (exp(6*c + 6*d*x)*(2160*a*b^2 + 768*a^2*b + 1001*b^3))/(98304*d) - (exp(- 6
*c - 6*d*x)*(2160*a*b^2 + 768*a^2*b + 1001*b^3))/(98304*d) - x*((189*a*b^2)
/256 + (15*a^2*b)/16 + a^3/2 + (429*b^3)/2048) + (exp(- 4*c - 4*d*x)*(2880*
a*b^2 + 2304*a^2*b + 1001*b^3))/(32768*d) - (exp(4*c + 4*d*x)*(2880*a*b^2 +
 2304*a^2*b + 1001*b^3))/(32768*d) - (exp(- 2*c - 2*d*x)*(10080*a*b^2 + 115
20*a^2*b + 4096*a^3 + 3003*b^3))/(32768*d) + (exp(2*c + 2*d*x)*(10080*a*b^2
 + 11520*a^2*b + 4096*a^3 + 3003*b^3))/(32768*d) + (7*b^3*exp(- 12*c - 12*d
*x))/(98304*d) - (7*b^3*exp(12*c + 12*d*x))/(98304*d) - (b^3*exp(- 14*c - 1
4*d*x))/(229376*d) + (b^3*exp(14*c + 14*d*x))/(229376*d) - (b^2*exp(- 10*c
- 10*d*x)*(48*a + 91*b))/(163840*d) + (b^2*exp(10*c + 10*d*x)*(48*a + 91*b)
)/(163840*d) + (b^2*exp(- 8*c - 8*d*x)*(120*a + 91*b))/(32768*d) - (b^2*exp
(8*c + 8*d*x)*(120*a + 91*b))/(32768*d)
```

3.218 $\int \left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=211

$$\frac{(1024a^3+1152a^2b+840ab^2+231b^3)\,x}{1024}-\frac{b(1920a^2+2232ab+793b^2)\cosh(c+dx)\sinh(c+dx)}{1024d}+\frac{b(1152a^2}{}$$

```
[Out] 1/1024*(1024*a^3+1152*a^2*b+840*a*b^2+231*b^3)*x-1/1024*b*(1920*a^2+2232*a*
b+793*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/1536*b*(1152*a^2+3912*a*b+2279*b^2)*
cosh(d*x+c)^3*sinh(d*x+c)/d-1/1920*b^2*(3000*a+3481*b)*cosh(d*x+c)^5*sinh(d
*x+c)/d+3/320*b^2*(40*a+139*b)*cosh(d*x+c)^7*sinh(d*x+c)/d-61/120*b^3*cosh(
d*x+c)^9*sinh(d*x+c)/d+1/12*b^3*cosh(d*x+c)^11*sinh(d*x+c)/d
```

Rubi [A]
time = 0.28, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3288, 1171, 1828, 393, 212}

$$\frac{b(1152a^2+3912ab+2279b^2)\sinh(c+dx)\cosh^3(c+dx)}{1536d}-\frac{b(1920a^2+2232ab+793b^2)\sinh(c+dx)\cosh(c+dx)}{1024d}+\frac{x(1024a^3+1152a^2b+840ab^2+231b^3)}{1024}+\frac{3b^2(40a+139b)\sinh(c+dx)\cosh^7(c+dx)}{320d}-\frac{b^2(3000a+3481b)\sinh(c+dx)\cosh^5(c+dx)}{1920d}+\frac{b^3\sinh(c+dx)\cosh^{11}(c+dx)}{12d}-\frac{61b^3\sinh(c+dx)\cosh^9(c+dx)}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*x)/1024 - (b*(1920*a^2 + 223
2*a*b + 793*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(1024*d) + (b*(1152*a^2 + 391
2*a*b + 2279*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(1536*d) - (b^2*(3000*a +
3481*b)*Cosh[c + d*x]^5*Sinh[c + d*x])/(1920*d) + (3*b^2*(40*a + 139*b)*Cos
h[c + d*x]^7*Sinh[c + d*x])/(320*d) - (61*b^3*Cosh[c + d*x]^9*Sinh[c + d*x]
)/(120*d) + (b^3*Cosh[c + d*x]^11*Sinh[c + d*x])/(12*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{\left(1-x^2\right)^7}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh^{11}(c+dx)\sinh(c+dx)}{12d} - \frac{\text{Subst}\left(\int \frac{-12a^3+b^3+12\left(5a^3+b^3\right)x^2-12\left(10a^3+3a^2b-\right.}{}\right.}{}$$

$$= -\frac{61b^3\cosh^9(c+dx)\sinh(c+dx)}{120d} + \frac{b^3\cosh^{11}(c+dx)\sinh(c+dx)}{12d} + \frac{\text{Subst}}{}$$

$$= \frac{3b^2(40a+139b)\cosh^7(c+dx)\sinh(c+dx)}{320d} - \frac{61b^3\cosh^9(c+dx)\sinh(c+d}{120d}$$

$$= -\frac{b^2(3000a+3481b)\cosh^5(c+dx)\sinh(c+dx)}{1920d} + \frac{3b^2(40a+139b)\cosh^7(c}{320d}$$

$$= \frac{b\left(1152a^2+3912ab+2279b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{1536d} - \frac{b^2(3000a+3481}{}$$

$$= -\frac{b\left(1920a^2+2232ab+793b^2\right)\cosh(c+dx)\sinh(c+dx)}{1024d} + \frac{b(1152a^2+3912}{}$$

$$= \frac{\left(1024a^3+1152a^2b+840ab^2+231b^3\right)x}{1024} - \frac{b(1920a^2+2232ab+793b^2)\cosh}{1024d}$$

Mathematica [A]
time = 0.31, size = 156, normalized size = 0.74

$$\frac{120\left(1024a^3+1152a^2b+840ab^2+231b^3\right)(c+dx)-720b\left(128a^2+112ab+33b^2\right)\sinh(2(c+dx))+45b\left(256a^2+448ab+165b^2\right)\sinh(4(c+dx))-40b^2(96a+55b)\sinh(6(c+dx))+45b^2(8a+11b)\sinh(8(c+dx))-72b^3\sinh(10(c+dx))+5b^3\sinh(12(c+dx))}{122880d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (120*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*(c + d*x) - 720*b*(128*a
^2 + 112*a*b + 33*b^2)*Sinh[2*(c + d*x)] + 45*b*(256*a^2 + 448*a*b + 165*b^
2)*Sinh[4*(c + d*x)] - 40*b^2*(96*a + 55*b)*Sinh[6*(c + d*x)] + 45*b^2*(8*a
 + 11*b)*Sinh[8*(c + d*x)] - 72*b^3*Sinh[10*(c + d*x)] + 5*b^3*Sinh[12*(c +
 d*x)])/(122880*d)
```

Maple [A]
time = 1.48, size = 177, normalized size = 0.84

method	result
default	$a^3x + \frac{\left(-\frac{55}{512}b^3-\frac{3}{16}a\,b^2\right)\sinh(6dx+6c)}{6d} + \frac{\left(\frac{33}{1024}b^3+\frac{3}{128}a\,b^2\right)\sinh(8dx+8c)}{8d} + \frac{\left(-\frac{99}{256}b^3-\frac{21}{16}a\,b^2-\frac{3}{2}a^2b\right)\sinh(2dx+2c)}{2d} +$

risch	$\frac{9a^2bx}{8}+\frac{3b^2\mathrm{e}^{8dx+8c}a}{2048d}-\frac{3b\,\mathrm{e}^{2dx+2c}a^2}{8d}+\frac{b^3\mathrm{e}^{12dx+12c}}{49152d}-\frac{b^3\mathrm{e}^{-12dx-12c}}{49152d}+\frac{99b^3\mathrm{e}^{-2dx-2c}}{1024d}-\frac{495b^3\mathrm{e}^{-4dx-4c}}{16384d}+\frac{231b^3x}{1024}+21$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+1/6*(-55/512*b^3-3/16*a*b^2)*sinh(6*d*x+6*c)/d+1/8*(33/1024*b^3+3/128
*a*b^2)*sinh(8*d*x+8*c)/d+1/2*(-99/256*b^3-21/16*a*b^2-3/2*a^2*b)*sinh(2*d*
x+2*c)/d+1/4*(495/2048*b^3+21/32*a*b^2+3/8*a^2*b)*sinh(4*d*x+4*c)/d+231/102
4*b^3*x+105/128*a*b^2*x+9/8*a^2*b*x-3/5120*b^3*sinh(10*d*x+10*c)/d+1/24576*
b^3*sinh(12*d*x+12*c)/d
```

Maxima [A]
time = 0.27, size = 344, normalized size = 1.63

$$\frac{3}{64}a^2b\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+a^3x-\frac{1}{245760}b^3\left(\frac{\left(72e^{(-2dx-2c)}-495e^{(-4dx-4c)}+2200e^{(-6dx-6c)}-7425e^{(-8dx-8c)}+23760e^{(-10dx-10c)}-5\right)e^{(12dx+12c)}}{d}-\frac{55440(dx+c)}{d}-\frac{23760e^{(-2dx-2c)}-7425e^{(-4dx-4c)}+2200e^{(-6dx-6c)}-495e^{(-8dx-8c)}+72e^{(-10dx-10c)}-5e^{(-12dx-12c)}}{d}\right)-\frac{1}{2048}ab^2\left(\frac{\left(32e^{(-2dx-2c)}-168e^{(-4dx-4c)}+672e^{(-6dx-6c)}-3\right)e^{(8dx+8c)}}{d}-\frac{1680(dx+c)}{d}-\frac{672e^{(-2dx-2c)}-168e^{(-4dx-4c)}+32e^{(-6dx-6c)}-3e^{(-8dx-8c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) + a^3*x - 1/245760*b^3*((72*e^(-2*d*x - 2*c) -
 495*e^(-4*d*x - 4*c) + 2200*e^(-6*d*x - 6*c) - 7425*e^(-8*d*x - 8*c) + 237
60*e^(-10*d*x - 10*c) - 5)*e^(12*d*x + 12*c)/d - 55440*(d*x + c)/d - (23760
*e^(-2*d*x - 2*c) - 7425*e^(-4*d*x - 4*c) + 2200*e^(-6*d*x - 6*c) - 495*e^(
-8*d*x - 8*c) + 72*e^(-10*d*x - 10*c) - 5*e^(-12*d*x - 12*c))/d) - 1/2048*a
*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) -
3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4
*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(197) = 394$.
time = 0.38, size = 461, normalized size = 2.18

$$\frac{15b^3\cosh(dx+c)\sinh(dx+c)^{11}+5\left(55b^3\cosh(dx+c)^3-36b^3\cosh(dx+c)\right)\sinh(dx+c)^9+90\left(11b^3\cosh(dx+c)^5-24b^3\cosh(dx+c)^3+(8ab^2+11b^3)\cosh(dx+c)\right)\sinh(dx+c)^7+6\left(165b^3\cosh(dx+c)^7-756b^3\cosh(dx+c)^5+105(8ab^2+11b^3)\cosh(dx+c)^3-10(96ab^2+55b^3)\cosh(dx+c)\right)\sinh(dx+c)^5+5\left(55b^3\cosh(dx+c)^9-432b^3\cosh(dx+c)^7+126(8ab^2+11b^3)\cosh(dx+c)^5-40(96ab^2+55b^3)\cosh(dx+c)^3+9(256a^2b+448ab^2+165b^3)\cosh(dx+c)\right)\sinh(dx+c)^3+30(1024a^3+1152a^2b+840ab^2+231b^3)dx+15\left(b^3\cosh(dx+c)^{11}-12b^3\cosh(dx+c)^9+6(8ab^2+11b^3)\cosh(dx+c)^7-4(96ab^2+55b^3)\cosh(dx+c)^5+3(256a^2b+448ab^2+165b^3)\cosh(dx+c)^3-24(128a^2b+112ab^2+33b^3)\cosh(dx+c)\right)\sinh(dx+c)}{30720d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/30720*(15*b^3*cosh(d*x + c)*sinh(d*x + c)^11 + 5*(55*b^3*cosh(d*x + c)^3
- 36*b^3*cosh(d*x + c))*sinh(d*x + c)^9 + 90*(11*b^3*cosh(d*x + c)^5 - 24*b
^3*cosh(d*x + c)^3 + (8*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 6*
(165*b^3*cosh(d*x + c)^7 - 756*b^3*cosh(d*x + c)^5 + 105*(8*a*b^2 + 11*b^3)
*cosh(d*x + c)^3 - 10*(96*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 +
5*(55*b^3*cosh(d*x + c)^9 - 432*b^3*cosh(d*x + c)^7 + 126*(8*a*b^2 + 11*b^3
```

```
)*cosh(d*x + c)^5 - 40*(96*a*b^2 + 55*b^3)*cosh(d*x + c)^3 + 9*(256*a^2*b +
 448*a*b^2 + 165*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(1024*a^3 + 1152*
a^2*b + 840*a*b^2 + 231*b^3)*d*x + 15*(b^3*cosh(d*x + c)^11 - 12*b^3*cosh(d
*x + c)^9 + 6*(8*a*b^2 + 11*b^3)*cosh(d*x + c)^7 - 4*(96*a*b^2 + 55*b^3)*co
sh(d*x + c)^5 + 3*(256*a^2*b + 448*a*b^2 + 165*b^3)*cosh(d*x + c)^3 - 24*(1
28*a^2*b + 112*a*b^2 + 33*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(202) = 404$.
time = 4.39, size = 666, normalized size = 3.16

$$\begin{cases} a^3 x + \frac{9a^2bx\sinh^4(c+dx)}{8} - \frac{9a^2bx\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{9a^2bx\cosh^4(c+dx)}{8} + \frac{15a^2b\sinh^3(c+dx)\cosh(c+dx)}{8d} - \frac{9a^2b\sinh(c+dx)\cosh^3(c+dx)}{8d} + \frac{105ab^2x\sinh^8(c+dx)}{128} - \frac{105ab^2x\sinh^6(c+dx)\cosh^2(c+dx)}{32} + \frac{315ab^2x\sinh^4(c+dx)\cosh^4(c+dx)}{64} - \frac{105ab^2x\sinh^2(c+dx)\cosh^6(c+dx)}{32} + \frac{105ab^2x\cosh^8(c+dx)}{128} + \frac{279ab^2\sinh^7(c+dx)\cosh(c+dx)}{128d} - \frac{511ab^2\sinh^5(c+dx)\cosh^3(c+dx)}{128d} + \frac{385ab^2\sinh^3(c+dx)\cosh^5(c+dx)}{128d} - \frac{105ab^2\sinh(c+dx)\cosh^7(c+dx)}{128d} + \frac{231b^3x\sinh^{12}(c+dx)}{1024} - \frac{693b^3x\sinh^{10}(c+dx)\cosh^2(c+dx)}{512} + \frac{3465b^3x\sinh^8(c+dx)\cosh^4(c+dx)}{1024} - \frac{1155b^3x\sinh^6(c+dx)\cosh^6(c+dx)}{256} + \frac{3465b^3x\sinh^4(c+dx)\cosh^8(c+dx)}{1024} - \frac{693b^3x\sinh^2(c+dx)\cosh^{10}(c+dx)}{512} + \frac{231b^3x\cosh^{12}(c+dx)}{1024} + \frac{793b^3\sinh^{11}(c+dx)\cosh(c+dx)}{1024d} - \frac{7337b^3\sinh^9(c+dx)\cosh^3(c+dx)}{3072d} + \frac{9273b^3\sinh^7(c+dx)\cosh^5(c+dx)}{2560d} - \frac{7623b^3\sinh^5(c+dx)\cosh^7(c+dx)}{2560d} + \frac{1309b^3\sinh^3(c+dx)\cosh^9(c+dx)}{1024d} - \frac{231b^3\sinh(c+dx)\cosh^{11}(c+dx)}{1024d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^4(c)\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Piecewise((a**3*x + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b*x*sinh(c + d*x
)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 15*a**2*b*sinh(c
+ d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*
d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)**6*cosh
(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 105*a
*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c + d*x)**
8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a*b**2*sinh
(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)**3*cosh(c
+ d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + 231
*b**3*x*sinh(c + d*x)**12/1024 - 693*b**3*x*sinh(c + d*x)**10*cosh(c + d*x)
**2/512 + 3465*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**4/1024 - 1155*b**3*x*
sinh(c + d*x)**6*cosh(c + d*x)**6/256 + 3465*b**3*x*sinh(c + d*x)**4*cosh(c
 + d*x)**8/1024 - 693*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**10/512 + 231*b
**3*x*cosh(c + d*x)**12/1024 + 793*b**3*sinh(c + d*x)**11*cosh(c + d*x)/(10
24*d) - 7337*b**3*sinh(c + d*x)**9*cosh(c + d*x)**3/(3072*d) + 9273*b**3*si
nh(c + d*x)**7*cosh(c + d*x)**5/(2560*d) - 7623*b**3*sinh(c + d*x)**5*cosh(
c + d*x)**7/(2560*d) + 1309*b**3*sinh(c + d*x)**3*cosh(c + d*x)**9/(1024*d)
 - 231*b**3*sinh(c + d*x)*cosh(c + d*x)**11/(1024*d), Ne(d, 0)), (x*(a + b*
sinh(c)**4)**3, True))
```

Giac [**A**]
time = 0.42, size = 327, normalized size = 1.55

$$\frac{b^3e^{(12\,dx+12\,c)}}{49152\,d} - \frac{3\,b^3e^{(10\,dx+10\,c)}}{10240\,d} + \frac{3\,b^3e^{(-10\,dx-10\,c)}}{10240\,d} - \frac{b^3e^{(-12\,dx-12\,c)}}{49152\,d} + \frac{1}{1024}\left(1024\,a^3 + 1152\,a^2b + 840\,ab^2 + 231\,b^3\right)x + \frac{3\,(8\,ab^2 + 11\,b^3)e^{(8\,dx+8\,c)}}{16384\,d} - \frac{(96\,ab^2 + 55\,b^3)e^{(6\,dx+6\,c)}}{6144\,d} + \frac{3\,(256\,a^2b + 448\,ab^2 + 165\,b^3)e^{(4\,dx+4\,c)}}{16384\,d} - \frac{3\,(128\,a^2b + 112\,ab^2 + 33\,b^3)e^{(2\,dx+2\,c)}}{1024\,d} + \frac{3\,(128\,a^2b + 112\,ab^2 + 33\,b^3)e^{(-2\,dx-2\,c)}}{1024\,d} - \frac{3\,(256\,a^2b + 448\,ab^2 + 165\,b^3)e^{(-4\,dx-4\,c)}}{16384\,d} + \frac{(96\,ab^2 + 55\,b^3)e^{(-6\,dx-6\,c)}}{6144\,d} - \frac{3\,(8\,ab^2 + 11\,b^3)e^{(-8\,dx-8\,c)}}{16384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/49152*b^3*e^(12*d*x + 12*c)/d - 3/10240*b^3*e^(10*d*x + 10*c)/d + 3/10240
*b^3*e^(-10*d*x - 10*c)/d - 1/49152*b^3*e^(-12*d*x - 12*c)/d + 1/1024*(1024
```

```
*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*x + 3/16384*(8*a*b^2 + 11*b^3)*e^(
8*d*x + 8*c)/d - 1/6144*(96*a*b^2 + 55*b^3)*e^(6*d*x + 6*c)/d + 3/16384*(25
6*a^2*b + 448*a*b^2 + 165*b^3)*e^(4*d*x + 4*c)/d - 3/1024*(128*a^2*b + 112*
a*b^2 + 33*b^3)*e^(2*d*x + 2*c)/d + 3/1024*(128*a^2*b + 112*a*b^2 + 33*b^3)
*e^(-2*d*x - 2*c)/d - 3/16384*(256*a^2*b + 448*a*b^2 + 165*b^3)*e^(-4*d*x -
 4*c)/d + 1/6144*(96*a*b^2 + 55*b^3)*e^(-6*d*x - 6*c)/d - 3/16384*(8*a*b^2
+ 11*b^3)*e^(-8*d*x - 8*c)/d
```

Mupad [B]
time = 0.56, size = 210, normalized size = 1.00

$$\frac{\frac{7425\,b^3\,\sinh\left(4\,c+4\,d\,x\right)}{8}-2970\,b^3\,\sinh\left(2\,c+2\,d\,x\right)-275\,b^3\,\sinh\left(6\,c+6\,d\,x\right)+\frac{495\,b^3\,\sinh\left(8\,c+8\,d\,x\right)}{8}-9\,b^3\,\sinh\left(10\,c+10\,d\,x\right)+\frac{5\,b^3\,\sinh\left(12\,c+12\,d\,x\right)}{8}-10080\,a\,b^2\,\sinh\left(2\,c+2\,d\,x\right)-11520\,a^2\,b\,\sinh\left(2\,c+2\,d\,x\right)+2520\,a\,b^2\,\sinh\left(4\,c+4\,d\,x\right)+1440\,a^2\,b\,\sinh\left(4\,c+4\,d\,x\right)-480\,a\,b^2\,\sinh\left(6\,c+6\,d\,x\right)+45\,a\,b^2\,\sinh\left(8\,c+8\,d\,x\right)+15360\,a^3\,d\,x+3465\,b^3\,d\,x+12600\,a\,b^2\,d\,x+17280\,a^2\,b\,d\,x}{15360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3,x)
```

```
[Out] ((7425*b^3*sinh(4*c + 4*d*x))/8 - 2970*b^3*sinh(2*c + 2*d*x) - 275*b^3*sinh
(6*c + 6*d*x) + (495*b^3*sinh(8*c + 8*d*x))/8 - 9*b^3*sinh(10*c + 10*d*x) +
 (5*b^3*sinh(12*c + 12*d*x))/8 - 10080*a*b^2*sinh(2*c + 2*d*x) - 11520*a^2*
b*sinh(2*c + 2*d*x) + 2520*a*b^2*sinh(4*c + 4*d*x) + 1440*a^2*b*sinh(4*c +
4*d*x) - 480*a*b^2*sinh(6*c + 6*d*x) + 45*a*b^2*sinh(8*c + 8*d*x) + 15360*a
^3*d*x + 3465*b^3*d*x + 12600*a*b^2*d*x + 17280*a^2*b*d*x)/(15360*d)
```

3.219 $\int \mathbf{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=181

$$-\frac{3}{256}b\left(128a^2+80ab+21b^2\right)x-\frac{a^3\coth(c+dx)}{d}+\frac{b(384a^2+528ab+193b^2)\cosh(c+dx)\sinh(c+dx)}{256d}-b^2$$

```
[Out] -3/256*b*(128*a^2+80*a*b+21*b^2)*x-a^3*coth(d*x+c)/d+1/256*b*(384*a^2+528*a
*b+193*b^2)*cosh(d*x+c)*sinh(d*x+c)/d-1/128*b^2*(208*a+149*b)*cosh(d*x+c)^3
*sinh(d*x+c)/d+1/160*b^2*(80*a+171*b)*cosh(d*x+c)^5*sinh(d*x+c)/d-41/80*b^3
*cosh(d*x+c)^7*sinh(d*x+c)/d+1/10*b^3*cosh(d*x+c)^9*sinh(d*x+c)/d
```

Rubi [A]
time = 0.31, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3296, 1273, 1819, 464, 212}

$$-\frac{a^3\coth(c+dx)}{d}+\frac{b(384a^2+528ab+193b^2)\sinh(c+dx)\cosh(c+dx)}{256d}-\frac{3}{256}bx\left(128a^2+80ab+21b^2\right)+\frac{b^2(80a+171b)\sinh(c+dx)\cosh^5(c+dx)}{160d}-\frac{b^2(208a+149b)\sinh(c+dx)\cosh^3(c+dx)}{128d}+\frac{b^3\sinh(c+dx)\cosh^9(c+dx)}{10d}-\frac{41b^3\sinh(c+dx)\cosh^7(c+dx)}{80d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-3*b*(128*a^2 + 80*a*b + 21*b^2)*x)/256 - (a^3*Coth[c + d*x])/d + (b*(384*
a^2 + 528*a*b + 193*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) - (b^2*(208*a
 + 149*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*d) + (b^2*(80*a + 171*b)*Cosh
[c + d*x]^5*Sinh[c + d*x])/(160*d) - (41*b^3*Cosh[c + d*x]^7*Sinh[c + d*x])
/(80*d) + (b^3*Cosh[c + d*x]^9*Sinh[c + d*x])/(10*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
```

```
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^2(1-x^2)^6}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{b^3\cosh^9(c+dx)\sinh(c+dx)}{10d}-\frac{\operatorname{Subst}\left(\int\frac{-10a^3+(50a^3+b^3)x^2-10(}{}\right.}{}$$

$$=-\frac{41b^3\cosh^7(c+dx)\sinh(c+dx)}{80d}+\frac{b^3\cosh^9(c+dx)\sinh(c+}{10d}$$

$$=\frac{b^2(80a+171b)\cosh^5(c+dx)\sinh(c+dx)}{160d}-\frac{41b^3\cosh^7(c+d}{80}$$

$$=-\frac{b^2(208a+149b)\cosh^3(c+dx)\sinh(c+dx)}{128d}+\frac{b^2(80a+171}{}$$

$$=\frac{b(384a^2+528ab+193b^2)\cosh(c+dx)\sinh(c+dx)}{256d}-\frac{b^2(208}{}$$

$$=-\frac{a^3\coth(c+dx)}{d}+\frac{b(384a^2+528ab+193b^2)\cosh(c+dx)\mathrm{si}}{256d}$$

$$=-\frac{3}{256}b\left(128a^2+80ab+21b^2\right)x-\frac{a^3\coth(c+dx)}{d}+\frac{b(384a^2}{}$$

Mathematica [A]
time = 0.57, size = 134, normalized size = 0.74

$$\frac{-120b(128a^2+80ab+21b^2)(c+dx)-10240a^3\coth(c+dx)+60b(128a^2+120ab+35b^2)\sinh(2(c+dx))-120b^2(12a+5b)\sinh(4(c+dx))+10b^2(16a+15b)\sinh(6(c+dx))-25b^3\sinh(8(c+dx))+2b^3\sinh(10(c+dx))}{10240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-120*b*(128*a^2 + 80*a*b + 21*b^2)*(c + d*x) - 10240*a^3*Coth[c + d*x] + 6
0*b*(128*a^2 + 120*a*b + 35*b^2)*Sinh[2*(c + d*x)] - 120*b^2*(12*a + 5*b)*S
inh[4*(c + d*x)] + 10*b^2*(16*a + 15*b)*Sinh[6*(c + d*x)] - 25*b^3*Sinh[8*(
c + d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(10240*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(169)=338$.
time = 1.25, size = 357, normalized size = 1.97

method	result
risch	$-\frac{3a^2bx}{2}-\frac{15ab^2x}{16}-\frac{63b^3x}{256}+\frac{b^3e^{10dx+10c}}{10240d}-\frac{5b^3e^{8dx+8c}}{4096d}+\frac{15b^3e^{6dx+6c}}{2048d}+\frac{b^2e^{6dx+6c}a}{128d}-\frac{9b^2e^{4dx+4c}a}{128d}-\frac{15b^3e^{4dx+4c}}{512d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*a^2*b*x-15/16*a*b^2*x-63/256*b^3*x+1/10240*b^3/d*exp(10*d*x+10*c)-5/40
96*b^3/d*exp(8*d*x+8*c)+15/2048*b^3/d*exp(6*d*x+6*c)+1/128*b^2/d*exp(6*d*x+
6*c)*a-9/128*b^2/d*exp(4*d*x+4*c)*a-15/512*b^3/d*exp(4*d*x+4*c)+3/8*b/d*exp
(2*d*x+2*c)*a^2+45/128/d*exp(2*d*x+2*c)*a*b^2+105/1024*b^3/d*exp(2*d*x+2*c)
-3/8*b/d*exp(-2*d*x-2*c)*a^2-45/128/d*exp(-2*d*x-2*c)*a*b^2-105/1024*b^3/d*
exp(-2*d*x-2*c)+9/128*b^2/d*exp(-4*d*x-4*c)*a+15/512*b^3/d*exp(-4*d*x-4*c)-
15/2048*b^3/d*exp(-6*d*x-6*c)-1/128*b^2/d*exp(-6*d*x-6*c)*a+5/4096*b^3/d*ex
p(-8*d*x-8*c)-1/10240*b^3/d*exp(-10*d*x-10*c)-2*a^3/d/(exp(2*d*x+2*c)-1)
```

Maxima [**A**]
time = 0.29, size = 284, normalized size = 1.57

$$-\frac{3}{8}\,a^{2}b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{20480}\,b^{3}\left(\frac{\left(25\,e^{(-2\,dx-2\,c)}-150\,e^{(-4\,dx-4\,c)}+600\,e^{(-6\,dx-6\,c)}-2100\,e^{(-8\,dx-8\,c)}-2\right)e^{(10\,dx+10\,c)}}{d}+\frac{5040\,(dx+c)}{d}+\frac{2100\,e^{(-2\,dx-2\,c)}-600\,e^{(-4\,dx-4\,c)}+150\,e^{(-6\,dx-6\,c)}-25\,e^{(-8\,dx-8\,c)}+2\,e^{(-10\,dx-10\,c)}}{d}\right)-\frac{1}{128}\,ab^{2}\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}}{d}\right)+\frac{2\,a^{3}}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/20480*b^3*((2
5*e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(
-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x
 - 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c)
 + 2*e^(-10*d*x - 10*c))/d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d
*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) -
 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1
))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(169) = 338$.
time = 0.38, size = 474, normalized size = 2.62

$$\frac{2\,b^{3}\cosh\left(dx+c\right)^{11}+22\,b^{3}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{10}-27\,b^{3}\cosh\left(dx+c\right)^{9}+3\left(110\,b^{3}\cosh\left(dx+c\right)^{3}-81\,b^{3}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{8}+5\left(32\,ab^{2}+35\,b^{3}\right)\cosh\left(dx+c\right)^{7}+7\left(132\,b^{3}\cosh\left(dx+c\right)^{5}-324\,b^{3}\cosh\left(dx+c\right)^{3}+5\left(32\,ab^{2}+35\,b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{6}-50\left(32\,ab^{2}+15\,b^{3}\right)\cosh\left(dx+c\right)^{5}+\left(660\,b^{3}\cosh\left(dx+c\right)^{7}-3402\,b^{3}\cosh\left(dx+c\right)^{5}+175\left(32\,ab^{2}+35\,b^{3}\right)\cosh\left(dx+c\right)^{3}-250\left(32\,ab^{2}+15\,b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}+60\left(128\,a^{2}b+144\,ab^{2}+45\,b^{3}\right)\cosh\left(dx+c\right)^{3}+\left(110\,b^{3}\cosh\left(dx+c\right)^{9}-972\,b^{3}\cosh\left(dx+c\right)^{7}+105\left(32\,ab^{2}+35\,b^{3}\right)\cosh\left(dx+c\right)^{5}-500\left(32\,ab^{2}+15\,b^{3}\right)\cosh\left(dx+c\right)^{3}+180\left(128\,a^{2}b+144\,ab^{2}+45\,b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}-20\left(1024\,a^{3}+384\,a^{2}b+360\,ab^{2}+105\,b^{3}\right)\cosh\left(dx+c\right)+80\left(256\,a^{3}-3\left(128\,a^{2}b+80\,ab^{2}+21\,b^{3}\right)dx\right)\sinh\left(dx+c\right)}{20480\,d\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/20480*(2*b^3*cosh(d*x + c)^11 + 22*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 2
7*b^3*cosh(d*x + c)^9 + 3*(110*b^3*cosh(d*x + c)^3 - 81*b^3*cosh(d*x + c))*
sinh(d*x + c)^8 + 5*(32*a*b^2 + 35*b^3)*cosh(d*x + c)^7 + 7*(132*b^3*cosh(d
*x + c)^5 - 324*b^3*cosh(d*x + c)^3 + 5*(32*a*b^2 + 35*b^3)*cosh(d*x + c))*
sinh(d*x + c)^6 - 50*(32*a*b^2 + 15*b^3)*cosh(d*x + c)^5 + (660*b^3*cosh(d*
```

```
x + c)^7 - 3402*b^3*cosh(d*x + c)^5 + 175*(32*a*b^2 + 35*b^3)*cosh(d*x + c)
^3 - 250*(32*a*b^2 + 15*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 60*(128*a^2*b
 + 144*a*b^2 + 45*b^3)*cosh(d*x + c)^3 + (110*b^3*cosh(d*x + c)^9 - 972*b^3
*cosh(d*x + c)^7 + 105*(32*a*b^2 + 35*b^3)*cosh(d*x + c)^5 - 500*(32*a*b^2
+ 15*b^3)*cosh(d*x + c)^3 + 180*(128*a^2*b + 144*a*b^2 + 45*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^2 - 20*(1024*a^3 + 384*a^2*b + 360*a*b^2 + 105*b^3)*cosh
(d*x + c) + 80*(256*a^3 - 3*(128*a^2*b + 80*a*b^2 + 21*b^3)*d*x)*sinh(d*x +
 c))/(d*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(169) = 338.
time = 0.56, size = 355, normalized size = 1.96

$$\frac{2\,b^{3}e^{\left(10\,dx+10\,c\right)}-25\,b^{3}e^{\left(8\,dx+8\,c\right)}+160\,ab^{2}e^{\left(6\,dx+6\,c\right)}+150\,b^{3}e^{\left(6\,dx+6\,c\right)}-1440\,ab^{2}e^{\left(4\,dx+4\,c\right)}-600\,b^{3}e^{\left(4\,dx+4\,c\right)}+7680\,a^{2}be^{\left(2\,dx+2\,c\right)}+7200\,ab^{2}e^{\left(2\,dx+2\,c\right)}+2100\,b^{3}e^{\left(2\,dx+2\,c\right)}-240\,\left(128\,a^{2}b+80\,ab^{2}+21\,b^{3}\right)\left(dx+c\right)-\frac{40960\,a^{3}}{e^{\left(2\,dx+2\,c\right)}-1}+\left(35072\,a^{2}be^{\left(10\,dx+10\,c\right)}+21920\,ab^{2}e^{\left(10\,dx+10\,c\right)}+5754\,b^{3}e^{\left(10\,dx+10\,c\right)}-7680\,a^{2}be^{\left(8\,dx+8\,c\right)}-7200\,ab^{2}e^{\left(8\,dx+8\,c\right)}-2100\,b^{3}e^{\left(8\,dx+8\,c\right)}+1440\,ab^{2}e^{\left(6\,dx+6\,c\right)}+600\,b^{3}e^{\left(6\,dx+6\,c\right)}-160\,ab^{2}e^{\left(4\,dx+4\,c\right)}-150\,b^{3}e^{\left(4\,dx+4\,c\right)}+25\,b^{3}e^{\left(2\,dx+2\,c\right)}-2\,b^{3}\right)e^{\left(-10\,dx-10\,c\right)}}{20480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/20480*(2*b^3*e^(10*d*x + 10*c) - 25*b^3*e^(8*d*x + 8*c) + 160*a*b^2*e^(6*
d*x + 6*c) + 150*b^3*e^(6*d*x + 6*c) - 1440*a*b^2*e^(4*d*x + 4*c) - 600*b^3
*e^(4*d*x + 4*c) + 7680*a^2*b*e^(2*d*x + 2*c) + 7200*a*b^2*e^(2*d*x + 2*c)
+ 2100*b^3*e^(2*d*x + 2*c) - 240*(128*a^2*b + 80*a*b^2 + 21*b^3)*(d*x + c)
- 40960*a^3/(e^(2*d*x + 2*c) - 1) + (35072*a^2*b*e^(10*d*x + 10*c) + 21920*
a*b^2*e^(10*d*x + 10*c) + 5754*b^3*e^(10*d*x + 10*c) - 7680*a^2*b*e^(8*d*x
+ 8*c) - 7200*a*b^2*e^(8*d*x + 8*c) - 2100*b^3*e^(8*d*x + 8*c) + 1440*a*b^2
*e^(6*d*x + 6*c) + 600*b^3*e^(6*d*x + 6*c) - 160*a*b^2*e^(4*d*x + 4*c) - 15
0*b^3*e^(4*d*x + 4*c) + 25*b^3*e^(2*d*x + 2*c) - 2*b^3)*e^(-10*d*x - 10*c))
/d
```

Mupad [**B**]
time = 1.09, size = 265, normalized size = 1.46

$$\frac{5\,b^3\,{\mathrm{e}}^{-8\,c-8\,d\,x}}{4096\,d}-\frac{2\,a^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{5\,b^3\,{\mathrm{e}}^{8\,c+8\,d\,x}}{4096\,d}-\frac{b^3\,{\mathrm{e}}^{-10\,c-10\,d\,x}}{10240\,d}+\frac{b^3\,{\mathrm{e}}^{10\,c+10\,d\,x}}{10240\,d}-\frac{3\,b\,x\,\left(128\,a^2+80\,a\,b+21\,b^2\right)}{256}-\frac{3\,b\,{\mathrm{e}}^{-2\,c-2\,d\,x}\,\left(128\,a^2+120\,a\,b+35\,b^2\right)}{1024\,d}+\frac{3\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(128\,a^2+120\,a\,b+35\,b^2\right)}{1024\,d}+\frac{3\,b^2\,{\mathrm{e}}^{-4\,c-4\,d\,x}\,\left(12\,a+5\,b\right)}{512\,d}-\frac{3\,b^2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(12\,a+5\,b\right)}{512\,d}-\frac{b^2\,{\mathrm{e}}^{-6\,c-6\,d\,x}\,\left(16\,a+15\,b\right)}{2048\,d}+\frac{b^2\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(16\,a+15\,b\right)}{2048\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^2,x)
```

```
[Out] (5*b^3*exp(- 8*c - 8*d*x))/(4096*d) - (2*a^3)/(d*(exp(2*c + 2*d*x) - 1)) -
(5*b^3*exp(8*c + 8*d*x))/(4096*d) - (b^3*exp(- 10*c - 10*d*x))/(10240*d) +
(b^3*exp(10*c + 10*d*x))/(10240*d) - (3*b*x*(80*a*b + 128*a^2 + 21*b^2))/25
6 - (3*b*exp(- 2*c - 2*d*x)*(120*a*b + 128*a^2 + 35*b^2))/(1024*d) + (3*b*e
xp(2*c + 2*d*x)*(120*a*b + 128*a^2 + 35*b^2))/(1024*d) + (3*b^2*exp(- 4*c -
 4*d*x)*(12*a + 5*b))/(512*d) - (3*b^2*exp(4*c + 4*d*x)*(12*a + 5*b))/(512*
d) - (b^2*exp(- 6*c - 6*d*x)*(16*a + 15*b))/(2048*d) + (b^2*exp(6*c + 6*d*x
)*(16*a + 15*b))/(2048*d)
```

3.220 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=161

$$\frac{1}{128}b\left(384a^2+144ab+35b^2\right)x+\frac{a^3\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}-\frac{3b^2(80a+31b)\cosh(c+dx)\sinh(c+$$

```
[Out] 1/128*b*(384*a^2+144*a*b+35*b^2)*x+a^3*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/
d-3/128*b^2*(80*a+31*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/192*b^2*(144*a+163*b)*c
osh(d*x+c)^3*sinh(d*x+c)/d-25/48*b^3*cosh(d*x+c)^5*sinh(d*x+c)/d+1/8*b^3*co
sh(d*x+c)^7*sinh(d*x+c)/d
```

Rubi [A]
time = 0.27, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3296, 1273, 1819, 1275, 213}

$$-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}+\frac{1}{128}bx\left(384a^2+144ab+35b^2\right)+\frac{b^2(144a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{3b^2(80a+31b)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{b^3\sinh(c+dx)\cosh^7(c+dx)}{8d}-\frac{25b^3\sinh(c+dx)\cosh^5(c+dx)}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (b*(384*a^2 + 144*a*b + 35*b^2)*x)/128 + (a^3*Coth[c + d*x])/d - (a^3*Coth[
c + d*x]^3)/(3*d) - (3*b^2*(80*a + 31*b)*Cosh[c + d*x]*Sinh[c + d*x])/(128*
d) + (b^2*(144*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) - (25*b^3*
Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^3*Cosh[c + d*x]^7*Sinh[c + d*x])
/(8*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^4\left(1-x^2\right)^5}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh^7(c+dx)\sinh(c+dx)}{8d} + \frac{\text{Subst}\left(\int \frac{8a^3-40a^3x^2+\left(80a^3+24a^2}{}\right.}{}$$

$$= -\frac{25b^3\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b^3\cosh^7(c+dx)\sinh(c+}{8d}$$

$$= \frac{b^2(144a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d} - \frac{25b^3\cosh^5(c+}{4}$$

$$= -\frac{3b^2(80a+31b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b^2(144a+163b}{}$$

$$= -\frac{3b^2(80a+31b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b^2(144a+163b}{}$$

$$= \frac{a^3\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} - \frac{3b^2(80a+31b)\cosh(c+}{128d}$$

$$= \frac{1}{128}b\left(384a^2+144ab+35b^2\right)x + \frac{a^3\coth(c+dx)}{d} - \frac{a^3\coth^3(}{3d}$$

Mathematica [A]
time = 0.55, size = 131, normalized size = 0.81

$$\frac{-1024a^3\coth(c+dx)\left(-2+\operatorname{csch}^2(c+dx)\right)+b(9216a^2c+3456abc+840b^2c+9216a^2dx+3456abdx+840b^2dx-96b(24a+7b)\sinh(2(c+dx))+24b(12a+7b)\sinh(4(c+dx))-32b^2\sinh(6(c+dx))+3b^2\sinh(8(c+dx)))}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-1024*a^3*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + b*(9216*a^2*c + 3456*a*b*
c + 840*b^2*c + 9216*a^2*d*x + 3456*a*b*d*x + 840*b^2*d*x - 96*b*(24*a + 7*
b)*Sinh[2*(c + d*x)] + 24*b*(12*a + 7*b)*Sinh[4*(c + d*x)] - 32*b^2*Sinh[6*
(c + d*x)] + 3*b^2*Sinh[8*(c + d*x)]))/(3072*d)
```

Maple [A]
time = 1.42, size = 264, normalized size = 1.64

method	result
risch	$3a^2bx+\frac{9ab^2x}{8}+\frac{35b^3x}{128}+\frac{b^3e^{8dx+8c}}{2048d}-\frac{b^3e^{6dx+6c}}{192d}+\frac{3b^2e^{4dx+4c}a}{64d}+\frac{7b^3e^{4dx+4c}}{256d}-\frac{3e^{2dx+2c}ab^2}{8d}-\frac{7b^3e^{2dx+2c}}{64d}+3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a^2*b*x+9/8*a*b^2*x+35/128*b^3*x+1/2048*b^3/d*exp(8*d*x+8*c)-1/192*b^3/d*
exp(6*d*x+6*c)+3/64*b^2/d*exp(4*d*x+4*c)*a+7/256*b^3/d*exp(4*d*x+4*c)-3/8/d
*exp(2*d*x+2*c)*a*b^2-7/64*b^3/d*exp(2*d*x+2*c)+3/8/d*exp(-2*d*x-2*c)*a*b^2
+7/64*b^3/d*exp(-2*d*x-2*c)-3/64*b^2/d*exp(-4*d*x-4*c)*a-7/256*b^3/d*exp(-4
*d*x-4*c)+1/192*b^3/d*exp(-6*d*x-6*c)-1/2048*b^3/d*exp(-8*d*x-8*c)-4/3*a^3*
(3*exp(2*d*x+2*c)-1)/d/(exp(2*d*x+2*c)-1)^3
```

Maxima [**A**]
time = 0.29, size = 282, normalized size = 1.75

$$\frac{3}{64}\,ab^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+3\,a^2bx-\frac{1}{6144}\,b^3\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+672\,e^{(-6\,dx-6\,c)}-3\right)e^{(8\,dx+8\,c)}}{d}-\frac{1680\,(dx+c)}{d}-\frac{672\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+32\,e^{(-6\,dx-6\,c)}-3\,e^{(-8\,dx-8\,c)}}{d}\right)+\frac{4}{3}\,a^3\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) + 3*a^2*b*x - 1/6144*b^3*((32*e^(-2*d*x - 2*c)
 - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 168
0*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x
 - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*
d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*
x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(149) = 298.
time = 0.39, size = 567, normalized size = 3.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/6144*(3*b^3*cosh(d*x + c)^11 + 33*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 41
*b^3*cosh(d*x + c)^9 + 9*(55*b^3*cosh(d*x + c)^3 - 41*b^3*cosh(d*x + c))*si
nh(d*x + c)^8 + 3*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^7 + 21*(66*b^3*cosh(d*x
 + c)^5 - 164*b^3*cosh(d*x + c)^3 + (96*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh
(d*x + c)^6 - 3*(1056*a*b^2 + 425*b^3)*cosh(d*x + c)^5 + 3*(330*b^3*cosh(d*
x + c)^7 - 1722*b^3*cosh(d*x + c)^5 + 35*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^
3 - 5*(1056*a*b^2 + 425*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 8*(512*a^3 +
972*a*b^2 + 319*b^3)*cosh(d*x + c)^3 - 16*(256*a^3 - 3*(384*a^2*b + 144*a*b
^2 + 35*b^3)*d*x)*sinh(d*x + c)^3 + 3*(55*b^3*cosh(d*x + c)^9 - 492*b^3*cos
h(d*x + c)^7 + 21*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^5 - 10*(1056*a*b^2 + 42
5*b^3)*cosh(d*x + c)^3 + 8*(512*a^3 + 972*a*b^2 + 319*b^3)*cosh(d*x + c))*s
inh(d*x + c)^2 - 24*(512*a^3 + 204*a*b^2 + 63*b^3)*cosh(d*x + c) + 48*(256*
```

```
a^3 - 3*(384*a^2*b + 144*a*b^2 + 35*b^3)*d*x - (256*a^3 - 3*(384*a^2*b + 14
4*a*b^2 + 35*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^3 +
 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.59, size = 285, normalized size = 1.77

$$\frac{3\,b^3e^{(8\,dx+8\,c)}-32\,b^3e^{(6\,dx+6\,c)}+288\,ab^2e^{(4\,dx+4\,c)}+168\,b^3e^{(4\,dx+4\,c)}-2304\,ab^2e^{(2\,dx+2\,c)}-672\,b^3e^{(2\,dx+2\,c)}+48\,(384\,a^2b+144\,ab^2+35\,b^3)(dx+c)-\left(19200\,a^2be^{(8\,dx+8\,c)}+7200\,ab^2e^{(8\,dx+8\,c)}+1750\,b^3e^{(8\,dx+8\,c)}-2304\,ab^2e^{(6\,dx+6\,c)}-672\,b^3e^{(6\,dx+6\,c)}+288\,ab^2e^{(4\,dx+4\,c)}+168\,b^3e^{(4\,dx+4\,c)}-32\,b^3e^{(2\,dx+2\,c)}+3\,b^3\right)e^{(-8\,dx-8\,c)}-\frac{8192\,(3\,a^3e^{(2\,dx+2\,c)}-a^3)}{(e^{(2\,dx+2\,c)}-1)^3}}{6144\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/6144*(3*b^3*e^(8*d*x + 8*c) - 32*b^3*e^(6*d*x + 6*c) + 288*a*b^2*e^(4*d*x
 + 4*c) + 168*b^3*e^(4*d*x + 4*c) - 2304*a*b^2*e^(2*d*x + 2*c) - 672*b^3*e^
(2*d*x + 2*c) + 48*(384*a^2*b + 144*a*b^2 + 35*b^3)*(d*x + c) - (19200*a^2*
b*e^(8*d*x + 8*c) + 7200*a*b^2*e^(8*d*x + 8*c) + 1750*b^3*e^(8*d*x + 8*c) -
 2304*a*b^2*e^(6*d*x + 6*c) - 672*b^3*e^(6*d*x + 6*c) + 288*a*b^2*e^(4*d*x
+ 4*c) + 168*b^3*e^(4*d*x + 4*c) - 32*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-8*d*
x - 8*c) - 8192*(3*a^3*e^(2*d*x + 2*c) - a^3)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [**B**]
time = 1.07, size = 269, normalized size = 1.67

$$x\left(3\,a^2\,b+\frac{9\,a\,b^2}{8}+\frac{35\,b^3}{128}\right)-\frac{4\,a^3}{3\,d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{b^3\,{\mathrm{e}}^{-6\,c-6\,d\,x}}{192\,d}-\frac{b^3\,{\mathrm{e}}^{6\,c+6\,d\,x}}{192\,d}-\frac{b^3\,{\mathrm{e}}^{-8\,c-8\,d\,x}}{2048\,d}+\frac{b^3\,{\mathrm{e}}^{8\,c+8\,d\,x}}{2048\,d}-\frac{8\,a^3\,{\mathrm{e}}^{2\,c+2\,d\,x}}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}-\frac{b^2\,{\mathrm{e}}^{-4\,c-4\,d\,x}\,\left(12\,a+7\,b\right)}{256\,d}+\frac{b^2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(12\,a+7\,b\right)}{256\,d}+\frac{b^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}\,\left(24\,a+7\,b\right)}{64\,d}-\frac{b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(24\,a+7\,b\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^4,x)
```

```
[Out] x*((9*a*b^2)/8 + 3*a^2*b + (35*b^3)/128) - (4*a^3)/(3*d*(exp(4*c + 4*d*x) -
 2*exp(2*c + 2*d*x) + 1)) + (b^3*exp(- 6*c - 6*d*x))/(192*d) - (b^3*exp(6*c
 + 6*d*x))/(192*d) - (b^3*exp(- 8*c - 8*d*x))/(2048*d) + (b^3*exp(8*c + 8*d
*x))/(2048*d) - (8*a^3*exp(2*c + 2*d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4
*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (b^2*exp(- 4*c - 4*d*x)*(12*a + 7*b)
)/(256*d) + (b^2*exp(4*c + 4*d*x)*(12*a + 7*b))/(256*d) + (b^2*exp(- 2*c -
2*d*x)*(24*a + 7*b))/(64*d) - (b^2*exp(2*c + 2*d*x)*(24*a + 7*b))/(64*d)
```

3.221 $\int \text{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=148

$$-\frac{1}{16}b^2(24a+5b)x-\frac{a^2(a+3b)\coth(c+dx)}{d}+\frac{2a^3\coth^3(c+dx)}{3d}-\frac{a^3\coth^5(c+dx)}{5d}+\frac{b^2(24a+11b)\cosh(c+d}{16d}$$

```
[Out] -1/16*b^2*(24*a+5*b)*x-a^2*(a+3*b)*coth(d*x+c)/d+2/3*a^3*coth(d*x+c)^3/d-1/
5*a^3*coth(d*x+c)^5/d+1/16*b^2*(24*a+11*b)*cosh(d*x+c)*sinh(d*x+c)/d-13/24*
b^3*cosh(d*x+c)^3*sinh(d*x+c)/d+1/6*b^3*cosh(d*x+c)^5*sinh(d*x+c)/d
```

Rubi [A]
time = 0.26, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3296, 1273, 1819, 1816, 213}

$$-\frac{a^3\coth^5(c+dx)}{5d}+\frac{2a^3\coth^3(c+dx)}{3d}-\frac{a^2(a+3b)\coth(c+dx)}{d}+\frac{b^2(24a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}b^2x(24a+5b)+\frac{b^3\sinh(c+dx)\cosh^5(c+dx)}{6d}-\frac{13b^3\sinh(c+dx)\cosh^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/16*(b^2*(24*a + 5*b)*x) - (a^2*(a + 3*b)*Coth[c + d*x])/d + (2*a^3*Coth[
c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d) + (b^2*(24*a + 11*b)*Cosh[c
 + d*x]*Sinh[c + d*x])/(16*d) - (13*b^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*
d) + (b^3*Cosh[c + d*x]^5*Sinh[c + d*x])/(6*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int\frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^6\left(1-x^2\right)^4}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{b^3\cosh^5(c+dx)\sinh(c+dx)}{6d}-\frac{\text{Subst}\left(\int\frac{-6a^3+30a^3x^2-6a^2(10a+3}{}\right.}{}$$

$$=-\frac{13b^3\cosh^3(c+dx)\sinh(c+dx)}{24d}+\frac{b^3\cosh^5(c+dx)\sinh(c+}{6d}$$

$$=\frac{b^2(24a+11b)\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{13b^3\cosh^3(c+dx)}{24d}$$

$$=\frac{b^2(24a+11b)\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{13b^3\cosh^3(c+dx)}{24d}$$

$$=-\frac{a^2(a+3b)\coth(c+dx)}{d}+\frac{2a^3\coth^3(c+dx)}{3d}-\frac{a^3\coth^5(c+}{5d}$$

$$=-\frac{1}{16}b^2(24a+5b)x-\frac{a^2(a+3b)\coth(c+dx)}{d}+\frac{2a^3\coth^3(c+}{3d}$$

Mathematica [A]

time = 0.81, size = 110, normalized size = 0.74

$$\frac{-64a^2\coth(c+dx)\left(8a+45b-4a\operatorname{csch}^2(c+dx)+3a\operatorname{csch}^4(c+dx)\right)+5b^2(-288ac-60bc-288adx-60bdx+9(16a+5b)\sinh(2(c+dx))-9b\sinh(4(c+dx))+b\sinh(6(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-64*a^2*Coth[c + d*x]*(8*a + 45*b - 4*a*Csch[c + d*x]^2 + 3*a*Csch[c + d*x
]^4) + 5*b^2*(-288*a*c - 60*b*c - 288*a*d*x - 60*b*d*x + 9*(16*a + 5*b)*Sin
h[2*(c + d*x)] - 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)]))/(960*d)
```

Maple [A]
time = 1.55, size = 253, normalized size = 1.71

method	result
risch	$-\frac{3a\,b^{2}x}{2}-\frac{5b^{3}x}{16}+\frac{b^{3}\mathrm{e}^{6dx+6c}}{384d}-\frac{3b^{3}\mathrm{e}^{4dx+4c}}{128d}+\frac{3\,\mathrm{e}^{2dx+2c}a\,b^{2}}{8d}+\frac{15b^{3}\mathrm{e}^{2dx+2c}}{128d}-\frac{3\,\mathrm{e}^{-2dx-2c}a\,b^{2}}{8d}-\frac{15b^{3}\mathrm{e}^{-2dx-2c}}{128d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*a*b^2*x-5/16*b^3*x+1/384*b^3/d*exp(6*d*x+6*c)-3/128*b^3/d*exp(4*d*x+4*
c)+3/8/d*exp(2*d*x+2*c)*a*b^2+15/128*b^3/d*exp(2*d*x+2*c)-3/8/d*exp(-2*d*x-
2*c)*a*b^2-15/128*b^3/d*exp(-2*d*x-2*c)+3/128*b^3/d*exp(-4*d*x-4*c)-1/384*b
^3/d*exp(-6*d*x-6*c)-2/15*a^2*(45*b*exp(8*d*x+8*c)-180*b*exp(6*d*x+6*c)+80*
a*exp(4*d*x+4*c)+270*b*exp(4*d*x+4*c)-40*a*exp(2*d*x+2*c)-180*b*exp(2*d*x+2
*c)+8*a+45*b)/d/(exp(2*d*x+2*c)-1)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(136) = 272.
time = 0.28, size = 359, normalized size = 2.43

$$-\frac{3}{8}\,ab^{2}\left(4\,x-\frac{e^{\left(2\,dx+2\,c\right)}}{d}+\frac{e^{\left(-2\,dx-2\,c\right)}}{d}\right)-\frac{1}{384}\,b^{3}\left(\frac{\left(9\,e^{\left(-2\,dx-2\,c\right)}-45\,e^{\left(-4\,dx-4\,c\right)}-1\right)e^{\left(6\,dx+6\,c\right)}}{d}+\frac{120\,\left(dx+c\right)}{d}+\frac{45\,e^{\left(-2\,dx-2\,c\right)}-9\,e^{\left(-4\,dx-4\,c\right)}+e^{\left(-6\,dx-6\,c\right)}}{d}\right)-\frac{16}{15}\,a^{3}\left(\frac{5\,e^{\left(-2\,dx-2\,c\right)}}{d\left(5\,e^{\left(-2\,dx-2\,c\right)}-10\,e^{\left(-4\,dx-4\,c\right)}+10\,e^{\left(-6\,dx-6\,c\right)}-5\,e^{\left(-8\,dx-8\,c\right)}+e^{\left(-10\,dx-10\,c\right)}-1\right)}-\frac{10\,e^{\left(-4\,dx-4\,c\right)}}{d\left(5\,e^{\left(-2\,dx-2\,c\right)}-10\,e^{\left(-4\,dx-4\,c\right)}+10\,e^{\left(-6\,dx-6\,c\right)}-5\,e^{\left(-8\,dx-8\,c\right)}+e^{\left(-10\,dx-10\,c\right)}-1\right)}-\frac{1}{d\left(5\,e^{\left(-2\,dx-2\,c\right)}-10\,e^{\left(-4\,dx-4\,c\right)}+10\,e^{\left(-6\,dx-6\,c\right)}-5\,e^{\left(-8\,dx-8\,c\right)}+e^{\left(-10\,dx-10\,c\right)}-1\right)}\right)+\frac{6\,a^{2}b}{d\left(e^{\left(-2\,dx-2\,c\right)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b^3*((9*e
^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c
)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) - 16
/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x
- 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) - 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 768 vs. 2(136) = 272.
time = 0.38, size = 768, normalized size = 5.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/1920*(5*b^3*cosh(d*x + c)^11 + 55*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 70
*b^3*cosh(d*x + c)^9 + 15*(55*b^3*cosh(d*x + c)^3 - 42*b^3*cosh(d*x + c))*s
inh(d*x + c)^8 + 20*(36*a*b^2 + 25*b^3)*cosh(d*x + c)^7 + 70*(33*b^3*cosh(d
*x + c)^5 - 84*b^3*cosh(d*x + c)^3 + 2*(36*a*b^2 + 25*b^3)*cosh(d*x + c))*s
inh(d*x + c)^6 - (1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 1625*b^3)*cosh(d*x +
 c)^5 + 8*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*sinh(d*x + c)^5
 + 5*(330*b^3*cosh(d*x + c)^7 - 1764*b^3*cosh(d*x + c)^5 + 140*(36*a*b^2 +
25*b^3)*cosh(d*x + c)^3 - (1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 1625*b^3)*c
osh(d*x + c))*sinh(d*x + c)^4 + 20*(256*a^3 + 864*a^2*b + 324*a*b^2 + 125*b
^3)*cosh(d*x + c)^3 - 40*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x -
 2*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^3 + 5*(55*b^3*cosh(d*x + c)^9 - 504*b^3*cosh(d*x + c)^7 + 84*(36*a
*b^2 + 25*b^3)*cosh(d*x + c)^5 - 2*(1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 16
25*b^3)*cosh(d*x + c)^3 + 12*(256*a^3 + 864*a^2*b + 324*a*b^2 + 125*b^3)*co
sh(d*x + c))*sinh(d*x + c)^2 - 10*(1024*a^3 + 1152*a^2*b + 360*a*b^2 + 131*
b^3)*cosh(d*x + c) + 40*((128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*
cosh(d*x + c)^4 + 256*a^3 + 1440*a^2*b - 30*(24*a*b^2 + 5*b^3)*d*x - 3*(128
*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c
))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*
cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(136) = 272.
time = 0.57, size = 286, normalized size = 1.93

$$\frac{5\,b^3e^{(6\,dx+6\,c)} - 45\,b^3e^{(4\,dx+4\,c)} + 720\,ab^2e^{(2\,dx+2\,c)} + 225\,b^3e^{(2\,dx+2\,c)} - 120\,(24\,ab^2 + 5\,b^3)(dx+c) + 5\,\left(528\,ab^2e^{(6\,dx+6\,c)} + 110\,b^3e^{(6\,dx+6\,c)} - 144\,ab^2e^{(4\,dx+4\,c)} - 45\,b^3e^{(4\,dx+4\,c)} + 9\,b^3e^{(2\,dx+2\,c)} - b^3\right)e^{(-6\,dx-6\,c)} - \frac{256\,(45\,a^2be^{(8\,dx+8\,c)} - 180\,a^2be^{(6\,dx+6\,c)} + 80\,a^3e^{(4\,dx+4\,c)} + 270\,a^2be^{(4\,dx+4\,c)} - 40\,a^3e^{(2\,dx+2\,c)} - 180\,a^2be^{(2\,dx+2\,c)} + 8\,a^3 + 45\,a^2b)}{\left(e^{(2\,dx+2\,c)} - 1\right)^5}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*b^3*e^(6*d*x + 6*c) - 45*b^3*e^(4*d*x + 4*c) + 720*a*b^2*e^(2*d*x
 + 2*c) + 225*b^3*e^(2*d*x + 2*c) - 120*(24*a*b^2 + 5*b^3)*(d*x + c) + 5*(5
```

```
28*a*b^2*e^(6*d*x + 6*c) + 110*b^3*e^(6*d*x + 6*c) - 144*a*b^2*e^(4*d*x + 4
*c) - 45*b^3*e^(4*d*x + 4*c) + 9*b^3*e^(2*d*x + 2*c) - b^3)*e^(-6*d*x - 6*c
) - 256*(45*a^2*b*e^(8*d*x + 8*c) - 180*a^2*b*e^(6*d*x + 6*c) + 80*a^3*e^(4
*d*x + 4*c) + 270*a^2*b*e^(4*d*x + 4*c) - 40*a^3*e^(2*d*x + 2*c) - 180*a^2*
b*e^(2*d*x + 2*c) + 8*a^3 + 45*a^2*b)/(e^(2*d*x + 2*c) - 1)^5)/d
```

Mupad [B]
time = 1.03, size = 511, normalized size = 3.45

$$\frac{\frac{6a^2b}{5d}-\frac{2e^{2c+2dx}\left(8a^3+9ba^2\right)}{5d}+\frac{18a^2be^{4c+4dx}}{5d}-\frac{6a^2be^{6c+6dx}}{5d}}{6e^{4c+4dx}-4e^{2c+2dx}-4e^{6c+6dx}+e^{8c+8dx}+1}-\frac{\frac{2\left(8a^3+9ba^2\right)}{15d}-\frac{12a^2be^{2c+2dx}}{5d}+\frac{6a^2be^{4c+4dx}}{5d}}{3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1}-\frac{\frac{6a^2b}{5d}+\frac{4e^{4c+4dx}\left(8a^3+9ba^2\right)}{5d}-\frac{24a^2be^{2c+2dx}}{5d}-\frac{24a^2be^{6c+6dx}}{5d}+\frac{6a^2be^{8c+8dx}}{5d}}{5e^{2c+2dx}-10e^{4c+4dx}+10e^{6c+6dx}-5e^{8c+8dx}+e^{10c+10dx}-1}-\frac{b^2x\left(24a+5b\right)}{16}+\frac{3b^3e^{-4c-4dx}}{128d}-\frac{3b^3e^{4c+4dx}}{128d}-\frac{b^3e^{-6c-6dx}}{384d}+\frac{b^3e^{6c+6dx}}{384d}-\frac{3b^2e^{-2c-2dx}\left(16a+5b\right)}{128d}+\frac{3b^2e^{2c+2dx}\left(16a+5b\right)}{128d}-\frac{12a^2b}{5d\left(e^{2c+2dx}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^6,x)
```

```
[Out] ((6*a^2*b)/(5*d) - (2*exp(2*c + 2*d*x)*(9*a^2*b + 8*a^3))/(5*d) + (18*a^2*b
*exp(4*c + 4*d*x))/(5*d) - (6*a^2*b*exp(6*c + 6*d*x))/(5*d))/(6*exp(4*c + 4
*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (
(2*(9*a^2*b + 8*a^3))/(15*d) - (12*a^2*b*exp(2*c + 2*d*x))/(5*d) + (6*a^2*b
*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*
c + 6*d*x) - 1) - ((6*a^2*b)/(5*d) + (4*exp(4*c + 4*d*x)*(9*a^2*b + 8*a^3))
/(5*d) - (24*a^2*b*exp(2*c + 2*d*x))/(5*d) - (24*a^2*b*exp(6*c + 6*d*x))/(5
*d) + (6*a^2*b*exp(8*c + 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c +
4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)
 - (b^2*x*(24*a + 5*b))/16 + (3*b^3*exp(- 4*c - 4*d*x))/(128*d) - (3*b^3*ex
p(4*c + 4*d*x))/(128*d) - (b^3*exp(- 6*c - 6*d*x))/(384*d) + (b^3*exp(6*c +
 6*d*x))/(384*d) - (3*b^2*exp(- 2*c - 2*d*x)*(16*a + 5*b))/(128*d) + (3*b^2
*exp(2*c + 2*d*x)*(16*a + 5*b))/(128*d) - (12*a^2*b)/(5*d*(exp(2*c + 2*d*x)
 - 1))
```

3.222 $\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=133

$$\frac{3}{8}b^2(8a+b)x+\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^2(a+b)\coth^3(c+dx)}{d}+\frac{3a^3\coth^5(c+dx)}{5d}-\frac{a^3\coth^7(c+dx)}{7d}-5b^3$$

```
[Out] 3/8*b^2*(8*a+b)*x+a^2*(a+3*b)*coth(d*x+c)/d-a^2*(a+b)*coth(d*x+c)^3/d+3/5*a
^3*coth(d*x+c)^5/d-1/7*a^3*coth(d*x+c)^7/d-5/8*b^3*cosh(d*x+c)*sinh(d*x+c)/
d+1/4*b^3*cosh(d*x+c)^3*sinh(d*x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3296, 1273, 1819, 1816, 213}

$$-\frac{a^3\coth^7(c+dx)}{7d}+\frac{3a^3\coth^5(c+dx)}{5d}-\frac{a^2(a+b)\coth^3(c+dx)}{d}+\frac{a^2(a+3b)\coth(c+dx)}{d}+\frac{3}{8}b^2x(8a+b)+\frac{b^3\sinh(c+dx)\cosh^3(c+dx)}{4d}-\frac{5b^3\sinh(c+dx)\cosh(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^8*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (3*b^2*(8*a + b)*x)/8 + (a^2*(a + 3*b)*Coth[c + d*x])/d - (a^2*(a + b)*Coth
[c + d*x]^3)/d + (3*a^3*Coth[c + d*x]^5)/(5*d) - (a^3*Coth[c + d*x]^7)/(7*d
) - (5*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^3*Cosh[c + d*x]^3*Sinh[c
 + d*x])/(4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^8\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{\text{Subst}\left(\int \frac{4a^3-20a^3x^2+4a^2(10a+3b)x}{}\right.}{}\\
&= -\frac{5b^3\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^3\cosh^3(c+dx)\sinh(c+dx}{4d}\\
&= -\frac{5b^3\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^3\cosh^3(c+dx)\sinh(c+dx}{4d}\\
&= \frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^2(a+b)\coth^3(c+dx)}{d}+\frac{3a^3\coth^5}{5a}\\
&= \frac{3}{8}b^2(8a+b)x+\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^2(a+b)\coth^3(c+}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.55, size = 106, normalized size = 0.80

$$\frac{-32a^2\coth(c+dx)\left(-2(8a+35b)+(8a+35b)\operatorname{csch}^2(c+dx)-6a\operatorname{csch}^4(c+dx)+5a\operatorname{csch}^6(c+dx)\right)+35b^2(12(8a+b)(c+dx)-8b\sinh(2(c+dx))+b\sinh(4(c+dx)))}{1120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^8*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-32*a^2*Coth[c + d*x]*(-2*(8*a + 35*b) + (8*a + 35*b)*Csch[c + d*x]^2 - 6*
a*Csch[c + d*x]^4 + 5*a*Csch[c + d*x]^6) + 35*b^2*(12*(8*a + b)*(c + d*x) -
 8*b*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)]))/(1120*d)
```

Maple [**A**]
time = 1.54, size = 207, normalized size = 1.56

method	result
risch	$3a\,b^2x+\frac{3b^3x}{8}+\frac{b^3\mathrm{e}^{4dx+4c}}{64d}-\frac{b^3\mathrm{e}^{2dx+2c}}{8d}+\frac{b^3\mathrm{e}^{-2dx-2c}}{8d}-\frac{b^3\mathrm{e}^{-4dx-4c}}{64d}-\frac{4a^2\left(105b\,\mathrm{e}^{10dx+10c}-455b\,\mathrm{e}^{8dx+8c}+280a\,\mathrm{e}^{6dx+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a*b^2*x+3/8*b^3*x+1/64*b^3/d*exp(4*d*x+4*c)-1/8*b^3/d*exp(2*d*x+2*c)+1/8*
b^3/d*exp(-2*d*x-2*c)-1/64*b^3/d*exp(-4*d*x-4*c)-4/35*a^2*(105*b*exp(10*d*x
+10*c)-455*b*exp(8*d*x+8*c)+280*a*exp(6*d*x+6*c)+770*b*exp(6*d*x+6*c)-168*a
*exp(4*d*x+4*c)-630*b*exp(4*d*x+4*c)+56*a*exp(2*d*x+2*c)+245*b*exp(2*d*x+2*
c)-8*a-35*b)/d/(exp(2*d*x+2*c)-1)^7
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(123) = 246.
time = 0.28, size = 537, normalized size = 4.04

$$\frac{1}{64}b^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+3ab^2x+\frac{32}{35}a^3\left(\frac{7e^{(-2dx-2c)}}{d(7e^{(-2dx-2c)}-21e^{(-4dx-4c)}+35e^{(-6dx-6c)}-35e^{(-8dx-8c)}+21e^{(-10dx-10c)}-7e^{(-12dx-12c)}+e^{(-14dx-14c)}-1)}-\frac{21e^{(-4dx-4c)}}{d(7e^{(-2dx-2c)}-21e^{(-4dx-4c)}+35e^{(-6dx-6c)}-35e^{(-8dx-8c)}+21e^{(-10dx-10c)}-7e^{(-12dx-12c)}+e^{(-14dx-14c)}-1)}+\frac{35e^{(-6dx-6c)}}{d(7e^{(-2dx-2c)}-21e^{(-4dx-4c)}+35e^{(-6dx-6c)}-35e^{(-8dx-8c)}+21e^{(-10dx-10c)}-7e^{(-12dx-12c)}+e^{(-14dx-14c)}-1)}-\frac{1}{d(7e^{(-2dx-2c)}-21e^{(-4dx-4c)}+35e^{(-6dx-6c)}-35e^{(-8dx-8c)}+21e^{(-10dx-10c)}-7e^{(-12dx-12c)}+e^{(-14dx-14c)}-1)}\right)+4a^2b\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/64*b^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3*a*b^2*x + 32/35*a^3*(7*e^(-2*d*x - 2*c)/(d*(
7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d
*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*
c) - 1)) - 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^
(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(
-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x -
8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) -
1)) - 1/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c)
- 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-
14*d*x - 14*c) - 1))) + 4*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c)
- 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) -
3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(123) = 246$.
time = 0.42, size = 928, normalized size = 6.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(35*b^3*cosh(d*x + c)^11 + 385*b^3*cosh(d*x + c)*sinh(d*x + c)^10 -
525*b^3*cosh(d*x + c)^9 + 525*(11*b^3*cosh(d*x + c)^3 - 9*b^3*cosh(d*x + c)
)*sinh(d*x + c)^8 + (1024*a^3 + 4480*a^2*b + 2695*b^3)*cosh(d*x + c)^7 - 8*
(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*sinh(d*x + c)^7 + 7*(2310*b
^3*cosh(d*x + c)^5 - 6300*b^3*cosh(d*x + c)^3 + (1024*a^3 + 4480*a^2*b + 26
95*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 7*(1024*a^3 + 4480*a^2*b + 975*b^3
)*cosh(d*x + c)^5 + 56*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x - 3*(
128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x +
c)^5 + 35*(330*b^3*cosh(d*x + c)^7 - 1890*b^3*cosh(d*x + c)^5 + (1024*a^3 +
 4480*a^2*b + 2695*b^3)*cosh(d*x + c)^3 - (1024*a^3 + 4480*a^2*b + 975*b^3)
*cosh(d*x + c))*sinh(d*x + c)^4 + 42*(512*a^3 + 1600*a^2*b + 215*b^3)*cosh(
d*x + c)^3 - 56*(5*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x
 + c)^4 + 384*a^3 + 1680*a^2*b - 315*(8*a*b^2 + b^3)*d*x - 10*(128*a^3 + 56
0*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 7*(27
5*b^3*cosh(d*x + c)^9 - 2700*b^3*cosh(d*x + c)^7 + 3*(1024*a^3 + 4480*a^2*b
 + 2695*b^3)*cosh(d*x + c)^5 - 10*(1024*a^3 + 4480*a^2*b + 975*b^3)*cosh(d*
x + c)^3 + 18*(512*a^3 + 1600*a^2*b + 215*b^3)*cosh(d*x + c))*sinh(d*x + c)
^2 - 70*(512*a^3 + 576*a^2*b + 63*b^3)*cosh(d*x + c) - 56*((128*a^3 + 560*a
^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 5*(128*a^3 + 560*a^2*b -
105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 640*a^3 - 2800*a^2*b + 525*(8*a*
b^2 + b^3)*d*x + 9*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x
 + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^7 + 7*(3*d*cosh(d*x + c)^2 - d)*si
nh(d*x + c)^5 + 7*(5*d*cosh(d*x + c)^4 - 10*d*cosh(d*x + c)^2 + 3*d)*sinh(d
*x + c)^3 + 7*(d*cosh(d*x + c)^6 - 5*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^
2 - 5*d)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**8*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(123) = 246.

time = 0.58, size = 253, normalized size = 1.90

$$\frac{35\,b^{3}e^{(4\,dx+4\,c)}-280\,b^{3}e^{(2\,dx+2\,c)}+840\,(8\,ab^{2}+b^{3})(dx+c)-35\,\left(144\,ab^{2}e^{(4\,dx+4\,c)}+18\,b^{3}e^{(4\,dx+4\,c)}-8\,b^{3}e^{(2\,dx+2\,c)}+b^{3}\right)e^{(-4\,dx-4\,c)}-\frac{256\,(105\,a^{2}be^{(10\,dx+10\,c)}-455\,a^{2}be^{(8\,dx+8\,c)}+280\,a^{3}e^{(6\,dx+6\,c)}+770\,a^{2}be^{(6\,dx+6\,c)}-168\,a^{3}e^{(4\,dx+4\,c)}-630\,a^{2}be^{(4\,dx+4\,c)}+56\,a^{3}e^{(2\,dx+2\,c)}+245\,a^{2}be^{(2\,dx+2\,c)}-8\,a^{3}-35\,a^{2}b)}{(e^{(2\,dx+2\,c)}-1)^{7}}}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2240*(35*b^3*e^(4*d*x + 4*c) - 280*b^3*e^(2*d*x + 2*c) + 840*(8*a*b^2 + b
^3)*(d*x + c) - 35*(144*a*b^2*e^(4*d*x + 4*c) + 18*b^3*e^(4*d*x + 4*c) - 8*
b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x - 4*c) - 256*(105*a^2*b*e^(10*d*x + 10
*c) - 455*a^2*b*e^(8*d*x + 8*c) + 280*a^3*e^(6*d*x + 6*c) + 770*a^2*b*e^(6*
d*x + 6*c) - 168*a^3*e^(4*d*x + 4*c) - 630*a^2*b*e^(4*d*x + 4*c) + 56*a^3*e
^(2*d*x + 2*c) + 245*a^2*b*e^(2*d*x + 2*c) - 8*a^3 - 35*a^2*b)/(e^(2*d*x +
2*c) - 1)^7)/d
```

Mupad [**B**]

time = 1.03, size = 749, normalized size = 5.63

$$\frac{\frac{32\,a^2\,b}{35\,d}-\frac{16\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(8\,a^3+9\,b\,a^2\right)}{35\,d}+\frac{192\,a^2\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{35\,d}-\frac{16\,a^2\,b\,{\mathrm{e}}^{6\,c+6\,d\,x}}{7\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}-10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}-5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}-1}-\frac{\frac{16\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(8\,a^3+9\,b\,a^2\right)}{7\,d}+\frac{24\,a^2\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{7\,d}-\frac{96\,a^2\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{7\,d}-\frac{96\,a^2\,b\,{\mathrm{e}}^{8\,c+8\,d\,x}}{7\,d}+\frac{24\,a^2\,b\,{\mathrm{e}}^{10\,c+10\,d\,x}}{7\,d}}{7\,{\mathrm{e}}^{2\,c+2\,d\,x}-21\,{\mathrm{e}}^{4\,c+4\,d\,x}+35\,{\mathrm{e}}^{6\,c+6\,d\,x}-35\,{\mathrm{e}}^{8\,c+8\,d\,x}+21\,{\mathrm{e}}^{10\,c+10\,d\,x}-7\,{\mathrm{e}}^{12\,c+12\,d\,x}+{\mathrm{e}}^{14\,c+14\,d\,x}-1}-\frac{\frac{4\,a^2\,b}{7\,d}+\frac{8\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(8\,a^3+9\,b\,a^2\right)}{7\,d}-\frac{32\,a^2\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{7\,d}-\frac{64\,a^2\,b\,{\mathrm{e}}^{6\,c+6\,d\,x}}{7\,d}+\frac{20\,a^2\,b\,{\mathrm{e}}^{8\,c+8\,d\,x}}{7\,d}}{15\,{\mathrm{e}}^{4\,c+4\,d\,x}-6\,{\mathrm{e}}^{2\,c+2\,d\,x}-20\,{\mathrm{e}}^{6\,c+6\,d\,x}+15\,{\mathrm{e}}^{8\,c+8\,d\,x}-6\,{\mathrm{e}}^{10\,c+10\,d\,x}+{\mathrm{e}}^{12\,c+12\,d\,x}+1}+\frac{\frac{32\,a^2\,b}{35\,d}-\frac{8\,a^2\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{7\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{\frac{4\,\left(8\,a^3+9\,b\,a^2\right)}{35\,d}-\frac{96\,a^2\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{35\,d}+\frac{12\,a^2\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{7\,d}}{6\,{\mathrm{e}}^{4\,c+4\,d\,x}-4\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}+\frac{3\,b^2\,x\,\left(8\,a+b\right)}{8}+\frac{b^3\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}-\frac{b^3\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}-\frac{b^3\,{\mathrm{e}}^{-4\,c-4\,d\,x}}{64\,d}+\frac{b^3\,{\mathrm{e}}^{4\,c+4\,d\,x}}{64\,d}-\frac{4\,a^2\,b}{7\,d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^8,x)
```

```
[Out] ((32*a^2*b)/(35*d) - (16*exp(2*c + 2*d*x)*(9*a^2*b + 8*a^3))/(35*d) + (192*
a^2*b*exp(4*c + 4*d*x))/(35*d) - (16*a^2*b*exp(6*c + 6*d*x))/(7*d))/(5*exp(
2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*
x) + exp(10*c + 10*d*x) - 1) - ((16*exp(6*c + 6*d*x)*(9*a^2*b + 8*a^3))/(7*
d) + (24*a^2*b*exp(2*c + 2*d*x))/(7*d) - (96*a^2*b*exp(4*c + 4*d*x))/(7*d)
- (96*a^2*b*exp(8*c + 8*d*x))/(7*d) + (24*a^2*b*exp(10*c + 10*d*x))/(7*d))/
(7*exp(2*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(8*
c + 8*d*x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 14*d
*x) - 1) - ((4*a^2*b)/(7*d) + (8*exp(4*c + 4*d*x)*(9*a^2*b + 8*a^3))/(7*d)
- (32*a^2*b*exp(2*c + 2*d*x))/(7*d) - (64*a^2*b*exp(6*c + 6*d*x))/(7*d) + (
20*a^2*b*exp(8*c + 8*d*x))/(7*d))/(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x)
 - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(1
2*c + 12*d*x) + 1) + ((32*a^2*b)/(35*d) - (8*a^2*b*exp(2*c + 2*d*x))/(7*d))
/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((4*(9*
a^2*b + 8*a^3))/(35*d) - (96*a^2*b*exp(2*c + 2*d*x))/(35*d) + (12*a^2*b*exp
(4*c + 4*d*x))/(7*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c
+ 6*d*x) + exp(8*c + 8*d*x) + 1) + (3*b^2*x*(8*a + b))/8 + (b^3*exp(- 2*c -
 2*d*x))/(8*d) - (b^3*exp(2*c + 2*d*x))/(8*d) - (b^3*exp(- 4*c - 4*d*x))/(6
4*d) + (b^3*exp(4*c + 4*d*x))/(64*d) - (4*a^2*b)/(7*d*(exp(4*c + 4*d*x) - 2
*exp(2*c + 2*d*x) + 1))
```

3.223 $\int \operatorname{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=140

$$-\frac{b^3x}{2}-\frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d}+\frac{2a^2(2a+3b)\coth^3(c+dx)}{3d}-\frac{3a^2(2a+b)\coth^5(c+dx)}{5d}+\frac{4a^3\coth^7(c}{7d}$$

```
[Out] -1/2*b^3*x-a*(a^2+3*a*b+3*b^2)*coth(d*x+c)/d+2/3*a^2*(2*a+3*b)*coth(d*x+c)^
3/d-3/5*a^2*(2*a+b)*coth(d*x+c)^5/d+4/7*a^3*coth(d*x+c)^7/d-1/9*a^3*coth(d*
x+c)^9/d+1/2*b^3*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3296, 1273, 1816, 213}

$$-\frac{a^3\coth^9(c+dx)}{9d}+\frac{4a^3\coth^7(c+dx)}{7d}-\frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d}-\frac{3a^2(2a+b)\coth^5(c+dx)}{5d}+\frac{2a^2(2a+3b)\coth^3(c+dx)}{3d}+\frac{b^3\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^10*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/2*(b^3*x) - (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d + (2*a^2*(2*a + 3*
b)*Coth[c + d*x]^3)/(3*d) - (3*a^2*(2*a + b)*Coth[c + d*x]^5)/(5*d) + (4*a^
3*Coth[c + d*x]^7)/(7*d) - (a^3*Coth[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]
*Sinh[c + d*x])/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^{10}\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-2a^3+10a^3x^2-2a^2(10a+3}{}\right.}{}$$

$$= \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \left(-\frac{2a^3}{x^{10}}+\frac{8a^3}{x^8}-\frac{6a^2(2}{x^6}\right.\right.}{}$$

$$= -\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d} + \frac{2a^2(2a+3b)\coth^3(c+d}{3d}$$

$$= -\frac{b^3x}{2} - \frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d} + \frac{2a^2(2a+3b)\coth^3}{3d}$$

Mathematica [A]
time = 0.45, size = 115, normalized size = 0.82

$$\frac{-4a\coth(c+dx)\left(128a^2+504ab+945b^2-4a(16a+63b)\operatorname{csch}^2(c+dx)+3a(16a+63b)\operatorname{csch}^4(c+dx)-40a^2\operatorname{csch}^6(c+dx)+35a^2\operatorname{csch}^8(c+dx)\right)+315b^3(-2(c+dx)+\sinh(2(c+dx)))}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^10*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (-4*a*Coth[c + d*x]*(128*a^2 + 504*a*b + 945*b^2 - 4*a*(16*a + 63*b)*Csch[c
 + d*x]^2 + 3*a*(16*a + 63*b)*Csch[c + d*x]^4 - 40*a^2*Csch[c + d*x]^6 + 35
*a^2*Csch[c + d*x]^8) + 315*b^3*(-2*(c + d*x) + Sinh[2*(c + d*x)]))/(1260*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(128) = 256$.
time = 1.57, size = 322, normalized size = 2.30

method	result
risch	$-\frac{b^3x}{2}+\frac{b^3\mathrm{e}^{2dx+2c}}{8d}-\frac{b^3\mathrm{e}^{-2dx-2c}}{8d}-\frac{2a\left(945b^2\mathrm{e}^{16dx+16c}-7560b^2\mathrm{e}^{14dx+14c}+5040ab\,\mathrm{e}^{12dx+12c}+26460b^2\mathrm{e}^{12dx+12c}-22680ab\,\mathrm{e}^{10d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^3*x+1/8*b^3/d*exp(2*d*x+2*c)-1/8*b^3/d*exp(-2*d*x-2*c)-2/315*a*(945*
b^2*exp(16*d*x+16*c)-7560*b^2*exp(14*d*x+14*c)+5040*a*b*exp(12*d*x+12*c)+26
460*b^2*exp(12*d*x+12*c)-22680*a*b*exp(10*d*x+10*c)-52920*b^2*exp(10*d*x+10
*c)+16128*a^2*exp(8*d*x+8*c)+40824*a*b*exp(8*d*x+8*c)+66150*b^2*exp(8*d*x+8
*c)-10752*a^2*exp(6*d*x+6*c)-37296*a*b*exp(6*d*x+6*c)-52920*b^2*exp(6*d*x+6
*c)+4608*a^2*exp(4*d*x+4*c)+18144*a*b*exp(4*d*x+4*c)+26460*b^2*exp(4*d*x+4*
c)-1152*a^2*exp(2*d*x+2*c)-4536*a*b*exp(2*d*x+2*c)-7560*b^2*exp(2*d*x+2*c)+
128*a^2+504*a*b+945*b^2)/d/(exp(2*d*x+2*c)-1)^9
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(128) = 256$.
time = 0.27, size = 842, normalized size = 6.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 256/315*a^3*(9*e^
(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x
- 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12
*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1
)) - 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*
e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-1
2*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x -
 18*c) - 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x -
4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c)
- 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^
(-18*d*x - 18*c) - 1)) - 126*e^(-8*d*x - 8*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e
^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d
*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x -
 16*c) + e^(-18*d*x - 18*c) - 1)) - 1/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x
 - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*
c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) +
 e^(-18*d*x - 18*c) - 1))) - 16/5*a^2*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x
 - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*
```

```
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))) + 6*a*b^2/(d*(e^(-
2*d*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1314 vs. $2(128) = 256$.
time = 0.40, size = 1314, normalized size = 9.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/2520*(315*b^3*cosh(d*x + c)^11 + 3465*b^3*cosh(d*x + c)*sinh(d*x + c)^10
- (1024*a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x + c)^9 - 4*(315*
b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*sinh(d*x + c)^9 + 9*(5775*b^3*
cosh(d*x + c)^3 - (1024*a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^8 + 9*(1024*a^3 + 4032*a^2*b + 5880*a*b^2 + 1225*b^3)*c
osh(d*x + c)^7 + 36*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2 - 4*(3
15*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^2)*sinh(d*x +
 c)^7 + 21*(6930*b^3*cosh(d*x + c)^5 - 4*(1024*a^3 + 4032*a^2*b + 7560*a*b^
2 + 2835*b^3)*cosh(d*x + c)^3 + 3*(1024*a^3 + 4032*a^2*b + 5880*a*b^2 + 122
5*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 9*(4096*a^3 + 16128*a^2*b + 16800*a
*b^2 + 2625*b^3)*cosh(d*x + c)^5 - 36*(1260*b^3*d*x + 14*(315*b^3*d*x - 256
*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^4 - 1024*a^3 - 4032*a^2*b - 7
560*a*b^2 - 21*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x +
 c)^2)*sinh(d*x + c)^5 + 9*(11550*b^3*cosh(d*x + c)^7 - 14*(1024*a^3 + 4032
*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x + c)^5 + 35*(1024*a^3 + 4032*a^2*b
 + 5880*a*b^2 + 1225*b^3)*cosh(d*x + c)^3 - 5*(4096*a^3 + 16128*a^2*b + 168
00*a*b^2 + 2625*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 42*(2048*a^3 + 6144*a
^2*b + 5040*a*b^2 + 675*b^3)*cosh(d*x + c)^3 - 12*(28*(315*b^3*d*x - 256*a^
3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^6 - 8820*b^3*d*x - 105*(315*b^3*
d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^4 + 7168*a^3 + 28224
*a^2*b + 52920*a*b^2 + 120*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 9*(1925*b^3*cosh(d*x + c)^9 - 4*(1024*
a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x + c)^7 + 21*(1024*a^3 +
4032*a^2*b + 5880*a*b^2 + 1225*b^3)*cosh(d*x + c)^5 - 10*(4096*a^3 + 16128*
a^2*b + 16800*a*b^2 + 2625*b^3)*cosh(d*x + c)^3 + 14*(2048*a^3 + 6144*a^2*b
 + 5040*a*b^2 + 675*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 126*(1024*a^3 + 1
152*a^2*b + 840*a*b^2 + 105*b^3)*cosh(d*x + c) - 36*((315*b^3*d*x - 256*a^3
 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^8 - 7*(315*b^3*d*x - 256*a^3 - 10
08*a^2*b - 1890*a*b^2)*cosh(d*x + c)^6 + 4410*b^3*d*x + 20*(315*b^3*d*x - 2
56*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^4 - 3584*a^3 - 14112*a^2*b
- 26460*a*b^2 - 28*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d
```

```
*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^9 + 9*(4*d*cosh(d*x + c)^2 - d)*
sinh(d*x + c)^7 + 9*(14*d*cosh(d*x + c)^4 - 21*d*cosh(d*x + c)^2 + 4*d)*sin
h(d*x + c)^5 + 3*(28*d*cosh(d*x + c)^6 - 105*d*cosh(d*x + c)^4 + 120*d*cosh
(d*x + c)^2 - 28*d)*sinh(d*x + c)^3 + 9*(d*cosh(d*x + c)^8 - 7*d*cosh(d*x +
 c)^6 + 20*d*cosh(d*x + c)^4 - 28*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**10*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(128) = 256.
time = 0.59, size = 360, normalized size = 2.57

$$-\frac{1260\,(dx+c)b^3 - 315\,b^3e^{(2\,dx+2\,c)} - 315\,(2\,b^3e^{(2\,dx+2\,c)} - b^3)e^{(-2\,dx-2\,c)} + \frac{16\,(945\,ab^2e^{(16\,dx+16\,c)} - 7560\,ab^2e^{(14\,dx+14\,c)} + 5040\,a^2be^{(12\,dx+12\,c)} + 26460\,ab^2e^{(12\,dx+12\,c)} - 22680\,a^2be^{(10\,dx+10\,c)} - 52920\,ab^2e^{(10\,dx+10\,c)} + 16128\,a^3e^{(8\,dx+8\,c)} + 40824\,a^2be^{(8\,dx+8\,c)} + 66150\,ab^2e^{(8\,dx+8\,c)} - 10752\,a^3e^{(6\,dx+6\,c)} - 37296\,a^2be^{(6\,dx+6\,c)} - 52920\,ab^2e^{(6\,dx+6\,c)} + 4608\,a^3e^{(4\,dx+4\,c)} + 18144\,a^2be^{(4\,dx+4\,c)} + 26460\,ab^2e^{(4\,dx+4\,c)} - 1152\,a^3e^{(2\,dx+2\,c)} - 4536\,a^2be^{(2\,dx+2\,c)} - 7560\,ab^2e^{(2\,dx+2\,c)} + 128\,a^3 + 504\,a^2b + 945\,ab^2)}{(e^{(2\,dx+2\,c)} - 1)^9}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/2520*(1260*(d*x + c)*b^3 - 315*b^3*e^(2*d*x + 2*c) - 315*(2*b^3*e^(2*d*x
 + 2*c) - b^3)*e^(-2*d*x - 2*c) + 16*(945*a*b^2*e^(16*d*x + 16*c) - 7560*a*
b^2*e^(14*d*x + 14*c) + 5040*a^2*b*e^(12*d*x + 12*c) + 26460*a*b^2*e^(12*d*
x + 12*c) - 22680*a^2*b*e^(10*d*x + 10*c) - 52920*a*b^2*e^(10*d*x + 10*c) +
 16128*a^3*e^(8*d*x + 8*c) + 40824*a^2*b*e^(8*d*x + 8*c) + 66150*a*b^2*e^(8
*d*x + 8*c) - 10752*a^3*e^(6*d*x + 6*c) - 37296*a^2*b*e^(6*d*x + 6*c) - 529
20*a*b^2*e^(6*d*x + 6*c) + 4608*a^3*e^(4*d*x + 4*c) + 18144*a^2*b*e^(4*d*x
+ 4*c) + 26460*a*b^2*e^(4*d*x + 4*c) - 1152*a^3*e^(2*d*x + 2*c) - 4536*a^2*
b*e^(2*d*x + 2*c) - 7560*a*b^2*e^(2*d*x + 2*c) + 128*a^3 + 504*a^2*b + 945*
a*b^2)/(e^(2*d*x + 2*c) - 1)^9)/d
```

Mupad [**B**]
time = 1.08, size = 1500, normalized size = 10.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^10,x)
```

```
[Out] ((2*a*b^2)/(3*d) - (2*exp(2*c + 2*d*x)*(7*a*b^2 + 4*a^2*b))/(3*d) + (2*exp(
4*c + 4*d*x)*(7*a*b^2 + 8*a^2*b))/d + (10*exp(8*c + 8*d*x)*(7*a*b^2 + 8*a^2
```

```
*b))/(3*d) - (2*exp(10*c + 10*d*x)*(7*a*b^2 + 4*a^2*b))/d - (2*exp(6*c + 6*
d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(9*d) + (14*a*b^2*exp(12*c + 12*d*x
))/(3*d) - (2*a*b^2*exp(14*c + 14*d*x))/(3*d))/(28*exp(4*c + 4*d*x) - 8*exp
(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(10*c + 1
0*d*x) + 28*exp(12*c + 12*d*x) - 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x)
+ 1) - ((2*(105*a*b^2 + 144*a^2*b + 128*a^3))/(315*d) - (8*exp(2*c + 2*d*x)
*(7*a*b^2 + 8*a^2*b))/(21*d) + (4*exp(4*c + 4*d*x)*(7*a*b^2 + 4*a^2*b))/(7*
d) - (8*a*b^2*exp(6*c + 6*d*x))/(3*d) + (2*a*b^2*exp(8*c + 8*d*x))/(3*d))/(
5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c
+ 8*d*x) + exp(10*c + 10*d*x) - 1) - ((2*(7*a*b^2 + 4*a^2*b))/(21*d) - (4*a
*b^2*exp(2*c + 2*d*x))/(3*d) + (2*a*b^2*exp(4*c + 4*d*x))/(3*d))/(3*exp(2*c
 + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((2*a*b^2)/(3*d) +
 (8*exp(4*c + 4*d*x)*(7*a*b^2 + 4*a^2*b))/(3*d) - (16*exp(6*c + 6*d*x)*(7*a
*b^2 + 8*a^2*b))/(3*d) - (16*exp(10*c + 10*d*x)*(7*a*b^2 + 8*a^2*b))/(3*d)
+ (8*exp(12*c + 12*d*x)*(7*a*b^2 + 4*a^2*b))/(3*d) + (4*exp(8*c + 8*d*x)*(1
05*a*b^2 + 144*a^2*b + 128*a^3))/(9*d) - (16*a*b^2*exp(2*c + 2*d*x))/(3*d)
- (16*a*b^2*exp(14*c + 14*d*x))/(3*d) + (2*a*b^2*exp(16*c + 16*d*x))/(3*d))
/(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) - 126*exp(
8*c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) + 36*exp(14*c
 + 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1) + ((2*(7*a*b^2
+ 8*a^2*b))/(21*d) + (20*exp(4*c + 4*d*x)*(7*a*b^2 + 8*a^2*b))/(21*d) - (20
*exp(6*c + 6*d*x)*(7*a*b^2 + 4*a^2*b))/(21*d) - (2*exp(2*c + 2*d*x)*(105*a*
b^2 + 144*a^2*b + 128*a^3))/(63*d) + (10*a*b^2*exp(8*c + 8*d*x))/(3*d) - (2
*a*b^2*exp(10*c + 10*d*x))/(3*d))/(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x)
 - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(1
2*c + 12*d*x) + 1) + ((2*(7*a*b^2 + 8*a^2*b))/(21*d) - (2*exp(2*c + 2*d*x)*
(7*a*b^2 + 4*a^2*b))/(7*d) + (2*a*b^2*exp(4*c + 4*d*x))/d - (2*a*b^2*exp(6*
c + 6*d*x))/(3*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6
*d*x) + exp(8*c + 8*d*x) + 1) - (b^3*x)/2 - ((2*(7*a*b^2 + 4*a^2*b))/(21*d)
 - (4*exp(2*c + 2*d*x)*(7*a*b^2 + 8*a^2*b))/(7*d) - (40*exp(6*c + 6*d*x)*(7
*a*b^2 + 8*a^2*b))/(21*d) + (10*exp(8*c + 8*d*x)*(7*a*b^2 + 4*a^2*b))/(7*d)
 + (2*exp(4*c + 4*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(21*d) - (4*a*b^2
*exp(10*c + 10*d*x))/d + (2*a*b^2*exp(12*c + 12*d*x))/(3*d))/(7*exp(2*c + 2
*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(8*c + 8*d*x) + 2
1*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) - 1) - (b^
3*exp(- 2*c - 2*d*x))/(8*d) + (b^3*exp(2*c + 2*d*x))/(8*d) - (4*a*b^2)/(3*d
*(exp(2*c + 2*d*x) - 1))
```

3.224 $\int \mathbf{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=147

$$b^3x+\frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d}-\frac{a(5a^2+9ab+3b^2)\coth^3(c+dx)}{3d}+\frac{a^2(10a+9b)\coth^5(c+dx)}{5d}-a^2(10$$

```
[Out] b^3*x+a*(a^2+3*a*b+3*b^2)*coth(d*x+c)/d-1/3*a*(5*a^2+9*a*b+3*b^2)*coth(d*x+
c)^3/d+1/5*a^2*(10*a+9*b)*coth(d*x+c)^5/d-1/7*a^2*(10*a+3*b)*coth(d*x+c)^7/
d+5/9*a^3*coth(d*x+c)^9/d-1/11*a^3*coth(d*x+c)^11/d
```

Rubi [A]
time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3296, 1275, 213}

$$-\frac{a^3\coth^{11}(c+dx)}{11d}+\frac{5a^3\coth^9(c+dx)}{9d}-\frac{a(5a^2+9ab+3b^2)\coth^3(c+dx)}{3d}+\frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d}-\frac{a^2(10a+3b)\coth^7(c+dx)}{7d}+\frac{a^2(10a+9b)\coth^5(c+dx)}{5d}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^12*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] b^3*x + (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d - (a*(5*a^2 + 9*a*b + 3*b
^2)*Coth[c + d*x]^3)/(3*d) + (a^2*(10*a + 9*b)*Coth[c + d*x]^5)/(5*d) - (a^
2*(10*a + 3*b)*Coth[c + d*x]^7)/(7*d) + (5*a^3*Coth[c + d*x]^9)/(9*d) - (a^
3*Coth[c + d*x]^11)/(11*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^{12}(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{12}} - \frac{5a^3}{x^{10}} + \frac{a^2(10a+3b)}{x^8} - \frac{a^2(10a+9b)}{x^6} + \frac{a(5a^2+9ab+3b^2)}{x^4}\right.\right.}{d}$$

$$= \frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d} - \frac{a(5a^2+9ab+3b^2)\coth^3(c}{3d}$$

$$= b^3x + \frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d} - \frac{a(5a^2+9ab+3b^2)\,\mathrm{c}}{3d}$$

Mathematica [A]
time = 6.09, size = 239, normalized size = 1.63

$$\frac{b^3(c+dx)}{d} + \frac{2(640a^3\cosh(c+dx)+2376a^2b\cosh(c+dx)+3465ab^2\cosh(c+dx))\operatorname{csch}(c+dx)}{3465d} + \frac{(-640a^3\cosh(c+dx)-2376a^2b\cosh(c+dx)-3465ab^2\cosh(c+dx))\operatorname{csch}^3(c+dx)}{3465d} + \frac{2(80a^3\cosh(c+dx)+297a^2b\cosh(c+dx))\operatorname{csch}^5(c+dx)}{1155d} + \frac{(-80a^3\cosh(c+dx)-297a^2b\cosh(c+dx))\operatorname{csch}^7(c+dx)}{693d} + \frac{10a^3\coth(c+dx)\operatorname{csch}^8(c+dx)}{99d} - \frac{a^3\coth(c+dx)\operatorname{csch}^{10}(c+dx)}{11d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^12*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (b^3*(c + d*x))/d + (2*(640*a^3*Cosh[c + d*x] + 2376*a^2*b*Cosh[c + d*x] +
3465*a*b^2*Cosh[c + d*x])*Csch[c + d*x])/(3465*d) + ((-640*a^3*Cosh[c + d*x
] - 2376*a^2*b*Cosh[c + d*x] - 3465*a*b^2*Cosh[c + d*x])*Csch[c + d*x]^3)/(
3465*d) + (2*(80*a^3*Cosh[c + d*x] + 297*a^2*b*Cosh[c + d*x])*Csch[c + d*x]
^5)/(1155*d) + ((-80*a^3*Cosh[c + d*x] - 297*a^2*b*Cosh[c + d*x])*Csch[c +
d*x]^7)/(693*d) + (10*a^3*Coth[c + d*x]*Csch[c + d*x]^8)/(99*d) - (a^3*Coth
[c + d*x]*Csch[c + d*x]^10)/(11*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(137) = 274$.
time = 1.54, size = 328, normalized size = 2.23

method	result
risch	$b^3x - \frac{4a\left(10395b^2\mathrm{e}^{18dx+18c} - 86625b^2\mathrm{e}^{16dx+16c} + 83160ab\,\mathrm{e}^{14dx+14c} + 318780b^2\mathrm{e}^{14dx+14c} - 382536ab\,\mathrm{e}^{12dx+12c} - 679140b^2\mathrm{e}^{12dx+1}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*x-4/3465*a*(10395*b^2*exp(18*d*x+18*c)-86625*b^2*exp(16*d*x+16*c)+83160
*a*b*exp(14*d*x+14*c)+318780*b^2*exp(14*d*x+14*c)-382536*a*b*exp(12*d*x+12*
c)-679140*b^2*exp(12*d*x+12*c)+295680*a^2*exp(10*d*x+10*c)+715176*a*b*exp(1
```

```
0*d*x+10*c)+921690*b^2*exp(10*d*x+10*c)-211200*a^2*exp(8*d*x+8*c)-700920*a*
b*exp(8*d*x+8*c)-824670*b^2*exp(8*d*x+8*c)+105600*a^2*exp(6*d*x+6*c)+392040
*a*b*exp(6*d*x+6*c)+485100*b^2*exp(6*d*x+6*c)-35200*a^2*exp(4*d*x+4*c)-1306
80*a*b*exp(4*d*x+4*c)-180180*b^2*exp(4*d*x+4*c)+7040*a^2*exp(2*d*x+2*c)+261
36*a*b*exp(2*d*x+2*c)+38115*b^2*exp(2*d*x+2*c)-640*a^2-2376*a*b-3465*b^2)/d
/(exp(2*d*x+2*c)-1)^11
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1291 vs. $2(137) = 274$.
time = 0.29, size = 1291, normalized size = 8.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] b^3*x + 512/693*a^3*(11*e^(-2*d*x - 2*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4
*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x
- 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x
- 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c
) - 1)) - 55*e^(-4*d*x - 4*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c)
 + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 4
62*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 5
5*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) + 1
65*e^(-6*d*x - 6*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(
-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*
d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d
*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 330*e^(-8*d
*x - 8*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6
*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c
) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c)
 - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) + 462*e^(-10*d*x - 10*c
)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 33
0*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*
e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^
(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 1/(d*(11*e^(-2*d*x - 2*c) - 5
5*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-
10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-
16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x
 - 22*c) - 1))) + 96/35*a^2*b*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) -
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10
*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) - 21*e^(-4*d
*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) - 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^
```

```
(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
- 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) - 1/(d*(7*e^(-2*d
*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c)
 + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)))
 + 4*a*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
+ e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1607 vs. $2(137) = 274$.
time = 0.39, size = 1607, normalized size = 10.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(2*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^11 + 22*(640*a^
3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^10 + (3465*b^3*d*x
 - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*sinh(d*x + c)^11 - 22*(640*a^3 + 237
6*a^2*b + 3465*a*b^2)*cosh(d*x + c)^9 - 11*(3465*b^3*d*x - 1280*a^3 - 4752*
a^2*b - 6930*a*b^2 - 5*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*
cosh(d*x + c)^2)*sinh(d*x + c)^9 + 66*(5*(640*a^3 + 2376*a^2*b + 3465*a*b^2
)*cosh(d*x + c)^3 - 3*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c))*si
nh(d*x + c)^8 + 110*(640*a^3 + 2376*a^2*b + 3087*a*b^2)*cosh(d*x + c)^7 + 1
1*(17325*b^3*d*x + 30*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*c
osh(d*x + c)^4 - 6400*a^3 - 23760*a^2*b - 34650*a*b^2 - 36*(3465*b^3*d*x -
1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 154*
(6*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^5 - 12*(640*a^3 + 2376
*a^2*b + 3465*a*b^2)*cosh(d*x + c)^3 + 5*(640*a^3 + 2376*a^2*b + 3087*a*b^2
)*cosh(d*x + c))*sinh(d*x + c)^6 - 330*(640*a^3 + 2376*a^2*b + 2415*a*b^2)*
cosh(d*x + c)^5 + 33*(14*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2
)*cosh(d*x + c)^6 - 17325*b^3*d*x - 42*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*
b - 6930*a*b^2)*cosh(d*x + c)^4 + 6400*a^3 + 23760*a^2*b + 34650*a*b^2 + 35
*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)*sinh(
d*x + c)^5 + 22*(30*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^7 - 1
26*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^5 + 175*(640*a^3 + 237
6*a^2*b + 3087*a*b^2)*cosh(d*x + c)^3 - 75*(640*a^3 + 2376*a^2*b + 2415*a*b
^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 660*(640*a^3 + 1872*a^2*b + 1533*a*b^2
)*cosh(d*x + c)^3 + 11*(15*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b
^2)*cosh(d*x + c)^8 - 84*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2
)*cosh(d*x + c)^6 + 103950*b^3*d*x + 175*(3465*b^3*d*x - 1280*a^3 - 4752*a^
2*b - 6930*a*b^2)*cosh(d*x + c)^4 - 38400*a^3 - 142560*a^2*b - 207900*a*b^2
 - 150*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 + 22*(5*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^
```

```
9 - 36*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^7 + 105*(640*a^3 +
 2376*a^2*b + 3087*a*b^2)*cosh(d*x + c)^5 - 150*(640*a^3 + 2376*a^2*b + 241
5*a*b^2)*cosh(d*x + c)^3 + 90*(640*a^3 + 1872*a^2*b + 1533*a*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^2 - 4620*(128*a^3 + 144*a^2*b + 105*a*b^2)*cosh(d*x + c
) + 11*((3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^1
0 - 9*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^8 +
 35*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^6 - 1
45530*b^3*d*x - 75*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh
(d*x + c)^4 + 53760*a^3 + 199584*a^2*b + 291060*a*b^2 + 90*(3465*b^3*d*x -
1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh
(d*x + c)^11 + 11*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^9 + 11*(30*d*cosh
(d*x + c)^4 - 36*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^7 + 33*(14*d*cosh(d
*x + c)^6 - 42*d*cosh(d*x + c)^4 + 35*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c
)^5 + 11*(15*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 175*d*cosh(d*x + c)
^4 - 150*d*cosh(d*x + c)^2 + 30*d)*sinh(d*x + c)^3 + 11*(d*cosh(d*x + c)^10
 - 9*d*cosh(d*x + c)^8 + 35*d*cosh(d*x + c)^6 - 75*d*cosh(d*x + c)^4 + 90*d
*cosh(d*x + c)^2 - 42*d)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**12*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(137) = 274.
time = 0.59, size = 359, normalized size = 2.44

$$\frac{3465\,(dx+c)b^3-\frac{4\,\left(10395\,ab^2e^{(18\,dx+18\,c)}-86625\,ab^2e^{(16\,dx+16\,c)}+83160\,a^2be^{(14\,dx+14\,c)}+318780\,ab^2e^{(14\,dx+14\,c)}-382536\,a^2be^{(12\,dx+12\,c)}-679140\,ab^2e^{(12\,dx+12\,c)}+295680\,a^3e^{(10\,dx+10\,c)}+715176\,a^2be^{(10\,dx+10\,c)}+921690\,ab^2e^{(10\,dx+10\,c)}-211200\,a^3e^{(8\,dx+8\,c)}-700920\,a^2be^{(8\,dx+8\,c)}-824670\,ab^2e^{(8\,dx+8\,c)}+105600\,a^3e^{(6\,dx+6\,c)}+392040\,a^2be^{(6\,dx+6\,c)}+485100\,ab^2e^{(6\,dx+6\,c)}-35200\,a^3e^{(4\,dx+4\,c)}-130680\,a^2be^{(4\,dx+4\,c)}-180180\,ab^2e^{(4\,dx+4\,c)}+7040\,a^3e^{(2\,dx+2\,c)}+26136\,a^2be^{(2\,dx+2\,c)}+38115\,ab^2e^{(2\,dx+2\,c)}-640\,a^3-2376\,a^2b-3465\,ab^2\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^{11}}}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/3465*(3465*(d*x + c)*b^3 - 4*(10395*a*b^2*e^(18*d*x + 18*c) - 86625*a*b^2
*e^(16*d*x + 16*c) + 83160*a^2*b*e^(14*d*x + 14*c) + 318780*a*b^2*e^(14*d*x
 + 14*c) - 382536*a^2*b*e^(12*d*x + 12*c) - 679140*a*b^2*e^(12*d*x + 12*c)
+ 295680*a^3*e^(10*d*x + 10*c) + 715176*a^2*b*e^(10*d*x + 10*c) + 921690*a*
b^2*e^(10*d*x + 10*c) - 211200*a^3*e^(8*d*x + 8*c) - 700920*a^2*b*e^(8*d*x
+ 8*c) - 824670*a*b^2*e^(8*d*x + 8*c) + 105600*a^3*e^(6*d*x + 6*c) + 392040
*a^2*b*e^(6*d*x + 6*c) + 485100*a*b^2*e^(6*d*x + 6*c) - 35200*a^3*e^(4*d*x
+ 4*c) - 130680*a^2*b*e^(4*d*x + 4*c) - 180180*a*b^2*e^(4*d*x + 4*c) + 7040
```

```
*a^3*e^(2*d*x + 2*c) + 26136*a^2*b*e^(2*d*x + 2*c) + 38115*a*b^2*e^(2*d*x +
 2*c) - 640*a^3 - 2376*a^2*b - 3465*a*b^2)/(e^(2*d*x + 2*c) - 1)^11)/d
```

Mupad [B]
time = 0.98, size = 1955, normalized size = 13.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^12,x)

[Out] ((64*a*b^2)/(165*d) - (32*exp(2*c + 2*d*x)*(7*a*b^2 + 4*a^2*b))/(55*d) + (1
28*exp(4*c + 4*d*x)*(7*a*b^2 + 8*a^2*b))/(55*d) + (64*exp(8*c + 8*d*x)*(7*a
*b^2 + 8*a^2*b))/(11*d) - (224*exp(10*c + 10*d*x)*(7*a*b^2 + 4*a^2*b))/(55*
d) - (32*exp(6*c + 6*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(99*d) + (1792
*a*b^2*exp(12*c + 12*d*x))/(165*d) - (96*a*b^2*exp(14*c + 14*d*x))/(55*d))/
(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) - 126*exp(8
*c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) + 36*exp(14*c
+ 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1) - ((4*(105*a*b^2
 + 144*a^2*b + 128*a^3))/(693*d) - (32*exp(2*c + 2*d*x)*(7*a*b^2 + 8*a^2*b)
)/(77*d) + (8*exp(4*c + 4*d*x)*(7*a*b^2 + 4*a^2*b))/(11*d) - (128*a*b^2*exp
(6*c + 6*d*x))/(33*d) + (12*a*b^2*exp(8*c + 8*d*x))/(11*d))/(15*exp(4*c + 4
*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*
exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - ((4*(7*a*b^2 + 4*a^2*b))/(55
*d) - (32*exp(2*c + 2*d*x)*(7*a*b^2 + 8*a^2*b))/(55*d) - (32*exp(6*c + 6*d*
x)*(7*a*b^2 + 8*a^2*b))/(11*d) + (28*exp(8*c + 8*d*x)*(7*a*b^2 + 4*a^2*b))/
(11*d) + (4*exp(4*c + 4*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(33*d) - (4
48*a*b^2*exp(10*c + 10*d*x))/(55*d) + (84*a*b^2*exp(12*c + 12*d*x))/(55*d))
/(28*exp(4*c + 4*d*x) - 8*exp(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) + 70*exp(8
*c + 8*d*x) - 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) - 8*exp(14*c +
14*d*x) + exp(16*c + 16*d*x) + 1) - ((4*(7*a*b^2 + 4*a^2*b))/(55*d) - (64*a
*b^2*exp(2*c + 2*d*x))/(55*d) + (36*a*b^2*exp(4*c + 4*d*x))/(55*d))/(6*exp(
4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) +
 1) - ((96*exp(6*c + 6*d*x)*(7*a*b^2 + 4*a^2*b))/(11*d) - (192*exp(8*c + 8*
d*x)*(7*a*b^2 + 8*a^2*b))/(11*d) - (192*exp(12*c + 12*d*x)*(7*a*b^2 + 8*a^2
*b))/(11*d) + (96*exp(14*c + 14*d*x)*(7*a*b^2 + 4*a^2*b))/(11*d) + (16*exp(
10*c + 10*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(11*d) + (24*a*b^2*exp(2*
c + 2*d*x))/(11*d) - (192*a*b^2*exp(4*c + 4*d*x))/(11*d) - (192*a*b^2*exp(1
6*c + 16*d*x))/(11*d) + (24*a*b^2*exp(18*c + 18*d*x))/(11*d))/(11*exp(2*c +
 2*d*x) - 55*exp(4*c + 4*d*x) + 165*exp(6*c + 6*d*x) - 330*exp(8*c + 8*d*x)
 + 462*exp(10*c + 10*d*x) - 462*exp(12*c + 12*d*x) + 330*exp(14*c + 14*d*x)
 - 165*exp(16*c + 16*d*x) + 55*exp(18*c + 18*d*x) - 11*exp(20*c + 20*d*x) +
 exp(22*c + 22*d*x) - 1) - ((12*a*b^2)/(55*d) + (144*exp(4*c + 4*d*x)*(7*a*
b^2 + 4*a^2*b))/(55*d) - (384*exp(6*c + 6*d*x)*(7*a*b^2 + 8*a^2*b))/(55*d)
- (576*exp(10*c + 10*d*x)*(7*a*b^2 + 8*a^2*b))/(55*d) + (336*exp(12*c + 12*
```

```
d*x)*(7*a*b^2 + 4*a^2*b))/(55*d) + (8*exp(8*c + 8*d*x)*(105*a*b^2 + 144*a^2
*b + 128*a^3))/(11*d) - (192*a*b^2*exp(2*c + 2*d*x))/(55*d) - (768*a*b^2*ex
p(14*c + 14*d*x))/(55*d) + (108*a*b^2*exp(16*c + 16*d*x))/(55*d))/(45*exp(4
*c + 4*d*x) - 10*exp(2*c + 2*d*x) - 120*exp(6*c + 6*d*x) + 210*exp(8*c + 8*
d*x) - 252*exp(10*c + 10*d*x) + 210*exp(12*c + 12*d*x) - 120*exp(14*c + 14*
d*x) + 45*exp(16*c + 16*d*x) - 10*exp(18*c + 18*d*x) + exp(20*c + 20*d*x) +
 1) + b^3*x + ((32*(7*a*b^2 + 8*a^2*b))/(385*d) + (96*exp(4*c + 4*d*x)*(7*a
*b^2 + 8*a^2*b))/(77*d) - (16*exp(6*c + 6*d*x)*(7*a*b^2 + 4*a^2*b))/(11*d)
- (8*exp(2*c + 2*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(231*d) + (64*a*b^
2*exp(8*c + 8*d*x))/(11*d) - (72*a*b^2*exp(10*c + 10*d*x))/(55*d))/(7*exp(2
*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(8*c + 8*d*
x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) - 1)
 + ((32*(7*a*b^2 + 8*a^2*b))/(385*d) - (16*exp(2*c + 2*d*x)*(7*a*b^2 + 4*a^
2*b))/(55*d) + (128*a*b^2*exp(4*c + 4*d*x))/(55*d) - (48*a*b^2*exp(6*c + 6*
d*x))/(55*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*
x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) + ((64*a*b^2)/(165*d) - (
24*a*b^2*exp(2*c + 2*d*x))/(55*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x)
 + exp(6*c + 6*d*x) - 1) - (12*a*b^2)/(55*d*(exp(4*c + 4*d*x) - 2*exp(2*c +
 2*d*x) + 1))
```

3.225 $\int \operatorname{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=144

$$-\frac{(a+b)^3\coth(c+dx)}{d}+\frac{2a(a+b)^2\coth^3(c+dx)}{d}-\frac{3a(a+b)(5a+b)\coth^5(c+dx)}{5d}+\frac{4a^2(5a+3b)\coth^7(c}{7d}$$

```
[Out] -(a+b)^3*coth(d*x+c)/d+2*a*(a+b)^2*coth(d*x+c)^3/d-3/5*a*(a+b)*(5*a+b)*coth
(d*x+c)^5/d+4/7*a^2*(5*a+3*b)*coth(d*x+c)^7/d-1/3*a^2*(5*a+b)*coth(d*x+c)^9
/d+6/11*a^3*coth(d*x+c)^11/d-1/13*a^3*coth(d*x+c)^13/d
```

Rubi [A]
time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3296, 1122}

$$-\frac{a^3\coth^{13}(c+dx)}{13d}+\frac{6a^3\coth^{11}(c+dx)}{11d}-\frac{a^2(5a+b)\coth^9(c+dx)}{3d}+\frac{4a^2(5a+3b)\coth^7(c+dx)}{7d}-\frac{3a(a+b)(5a+b)\coth^5(c+dx)}{5d}+\frac{2a(a+b)^2\coth^3(c+dx)}{d}-\frac{(a+b)^3\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^14*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(((a + b)^3*Coth[c + d*x])/d) + (2*a*(a + b)^2*Coth[c + d*x]^3)/d - (3*a*(
a + b)*(5*a + b)*Coth[c + d*x]^5)/(5*d) + (4*a^2*(5*a + 3*b)*Coth[c + d*x]^
7)/(7*d) - (a^2*(5*a + b)*Coth[c + d*x]^9)/(3*d) + (6*a^3*Coth[c + d*x]^11)
/(11*d) - (a^3*Coth[c + d*x]^13)/(13*d)
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^{14}}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\operatorname{Subst}\Big(\int\Big(\frac{a^3}{x^{14}}-\frac{6a^3}{x^{12}}+\frac{3a^2(5a+b)}{x^{10}}-\frac{4a^2(5a+3b)}{x^8}+\frac{3a(a+b)(5a+b)}{x^6}-6a}{d}$$

$$=-\frac{(a+b)^3\coth(c+dx)}{d}+\frac{2a(a+b)^2\coth^3(c+dx)}{d}-\frac{3a(a+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 350 vs. 2(144) = 288.
time = 2.21, size = 350, normalized size = 2.43

$$-\frac{(8580(1024a^3+1152a^2b+840ab^2+231b^3)\cosh(c+dx)-6435(1024a^3+2944a^2b+2408ab^2+693b^3)\cosh(3(c+dx))+3660800a^3\cosh(5(c+dx))+13087360a^2b\cosh(5(c+dx))+13093080ab^2\cosh(5(c+dx))+4129125b^3\cosh(5(c+dx))-1464320a^3\cosh(7(c+dx))-5234944a^2b\cosh(7(c+dx))-6390384ab^2\cosh(7(c+dx))-2312310b^3\cosh(7(c+dx))+399360a^3\cosh(9(c+dx))+1427712a^2b\cosh(9(c+dx))+1873872ab^2\cosh(9(c+dx))+810810b^3\cosh(9(c+dx))-66560a^3\cosh(11(c+dx))-237952a^2b\cosh(11(c+dx))-312312ab^2\cosh(11(c+dx))-165165b^3\cosh(11(c+dx))+5120a^3\cosh(13(c+dx))+18304a^2b\cosh(13(c+dx))+24024ab^2\cosh(13(c+dx))+15015b^3\cosh(13(c+dx)))\operatorname{csch}^{13}(c+dx)}{61501440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^14*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/61501440*((8580*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*Cosh[c + d
*x] - 6435*(1024*a^3 + 2944*a^2*b + 2408*a*b^2 + 693*b^3)*Cosh[3*(c + d*x)]
 + 3660800*a^3*Cosh[5*(c + d*x)] + 13087360*a^2*b*Cosh[5*(c + d*x)] + 13093
080*a*b^2*Cosh[5*(c + d*x)] + 4129125*b^3*Cosh[5*(c + d*x)] - 1464320*a^3*C
osh[7*(c + d*x)] - 5234944*a^2*b*Cosh[7*(c + d*x)] - 6390384*a*b^2*Cosh[7*(
c + d*x)] - 2312310*b^3*Cosh[7*(c + d*x)] + 399360*a^3*Cosh[9*(c + d*x)] +
1427712*a^2*b*Cosh[9*(c + d*x)] + 1873872*a*b^2*Cosh[9*(c + d*x)] + 810810*
b^3*Cosh[9*(c + d*x)] - 66560*a^3*Cosh[11*(c + d*x)] - 237952*a^2*b*Cosh[11
*(c + d*x)] - 312312*a*b^2*Cosh[11*(c + d*x)] - 165165*b^3*Cosh[11*(c + d*x
)] + 5120*a^3*Cosh[13*(c + d*x)] + 18304*a^2*b*Cosh[13*(c + d*x)] + 24024*a
*b^2*Cosh[13*(c + d*x)] + 15015*b^3*Cosh[13*(c + d*x)])*Csch[c + d*x]^13)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(134) = 268.
time = 1.50, size = 564, normalized size = 3.92

method	result
risch	$-\frac{2\left(24024ab^2-17008992ab^2e^{14dx+14c}+20646912a^2be^{12dx+12c}+24216192ab^2e^{12dx+12c}-21250944a^2be^{10dx+10c}-23207184ab^2e^{10d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15015*(24024*a*b^2-17008992*a*b^2*exp(14*d*x+14*c)+20646912*a^2*b*exp(12
*d*x+12*c)+24216192*a*b^2*exp(12*d*x+12*c)-21250944*a^2*b*exp(10*d*x+10*c)-
```

```
23207184*a*b^2*exp(10*d*x+10*c)-2042040*a*b^2*exp(18*d*x+18*c)+2306304*a^2*
b*exp(16*d*x+16*c)+7711704*a*b^2*exp(16*d*x+16*c)-10762752*a^2*b*exp(14*d*x
+14*c)+1427712*a^2*b*exp(4*d*x+4*c)-237952*a^2*b*exp(2*d*x+2*c)+18304*a^2*b
+5120*a^3+15015*b^3+15135120*a*b^2*exp(8*d*x+8*c)-6630624*a*b^2*exp(6*d*x+6
*c)+1873872*a*b^2*exp(4*d*x+4*c)+240240*a*b^2*exp(20*d*x+20*c)+13087360*a^2
*b*exp(8*d*x+8*c)-312312*a*b^2*exp(2*d*x+2*c)-5234944*a^2*b*exp(6*d*x+6*c)+
15015*b^3*exp(24*d*x+24*c)-3303300*b^3*exp(18*d*x+18*c)+7432425*b^3*exp(16*
d*x+16*c)-11891880*b^3*exp(14*d*x+14*c)-66560*a^3*exp(2*d*x+2*c)+8785920*a^
3*exp(12*d*x+12*c)+13873860*b^3*exp(12*d*x+12*c)-6589440*a^3*exp(10*d*x+10*
c)-180180*b^3*exp(22*d*x+22*c)+990990*b^3*exp(20*d*x+20*c)-180180*b^3*exp(2
*d*x+2*c)+7432425*b^3*exp(8*d*x+8*c)+399360*a^3*exp(4*d*x+4*c)+990990*b^3*e
xp(4*d*x+4*c)-11891880*b^3*exp(10*d*x+10*c)+3660800*a^3*exp(8*d*x+8*c)-1464
320*a^3*exp(6*d*x+6*c)-3303300*b^3*exp(6*d*x+6*c))/d/(exp(2*d*x+2*c)-1)^13
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1916 vs. $2(134) = 268$.
time = 0.30, size = 1916, normalized size = 13.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -2048/3003*a^3*(13*e^(-2*d*x - 2*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x
- 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10
*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x -
 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x -
22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 78*e^(-4*d*x - 4
*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) -
715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) +
1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c)
- 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) +
e^(-26*d*x - 26*c) - 1)) + 286*e^(-6*d*x - 6*c)/(d*(13*e^(-2*d*x - 2*c) - 7
8*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(
-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*
e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e
^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 715*
e^(-8*d*x - 8*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*
d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d
*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18
*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d
*x - 24*c) + e^(-26*d*x - 26*c) - 1)) + 1287*e^(-10*d*x - 10*c)/(d*(13*e^(-
2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x -
 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x
 - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*
```

```
x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26
*c) - 1)) - 1716*e^(-12*d*x - 12*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x
 - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10
*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x -
 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x -
22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 1/(d*(13*e^(-2*d
*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*
c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x -
14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x -
 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c)
 - 1))) - 256/105*a^2*b*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(
-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x
 - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 1
6*c) + e^(-18*d*x - 18*c) - 1)) - 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c
) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*
e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-
16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2
*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8
*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c
) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 126*e^(-8*d*x - 8*c)/
(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^
(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14
*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 1/(d*(9*e^
(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x
 - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 1
4*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1))) - 16/5*a*b^2*(5*e^(
-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)
/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) - 1))) + 2*b^3/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2323 vs. 2(134) = 268.
time = 0.40, size = 2323, normalized size = 16.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -4/15015*((2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^1
2 - 48*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 +
 (2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*sinh(d*x + c)^12 - 52*(6
```

```
40*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^10 - 2*(16640*a^
3 + 59488*a^2*b + 78078*a*b^2 + 90090*b^3 - 33*(2560*a^3 + 9152*a^2*b + 120
12*a*b^2 + 15015*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 - 40*(22*(640*a^3 +
 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)^3 - 13*(640*a^3 + 2288*a^2*b + 3003
*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 78*(2560*a^3 + 9152*a^2*b + 13552*
a*b^2 + 12705*b^3)*cosh(d*x + c)^8 + 3*(165*(2560*a^3 + 9152*a^2*b + 12012*
a*b^2 + 15015*b^3)*cosh(d*x + c)^4 + 66560*a^3 + 237952*a^2*b + 352352*a*b^
2 + 330330*b^3 - 780*(640*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^8 - 96*(33*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh
(d*x + c)^5 - 65*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)^3 + 52*(
320*a^3 + 1144*a^2*b + 1309*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 572*(12
80*a^3 + 4576*a^2*b + 7581*a*b^2 + 5775*b^3)*cosh(d*x + c)^6 + 4*(231*(2560
*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^6 - 2730*(640*a^
3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^4 - 183040*a^3 - 6543
68*a^2*b - 1084083*a*b^2 - 825825*b^3 + 546*(2560*a^3 + 9152*a^2*b + 13552*
a*b^2 + 12705*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 24*(132*(640*a^3 + 22
88*a^2*b + 3003*a*b^2)*cosh(d*x + c)^7 - 546*(640*a^3 + 2288*a^2*b + 3003*a
*b^2)*cosh(d*x + c)^5 + 1456*(320*a^3 + 1144*a^2*b + 1309*a*b^2)*cosh(d*x +
 c)^3 - 143*(1280*a^3 + 4576*a^2*b + 4011*a*b^2)*cosh(d*x + c))*sinh(d*x +
c)^5 + 143*(12800*a^3 + 53824*a^2*b + 79884*a*b^2 + 51975*b^3)*cosh(d*x + c
)^4 + (495*(2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^
8 - 10920*(640*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^6 +
5460*(2560*a^3 + 9152*a^2*b + 13552*a*b^2 + 12705*b^3)*cosh(d*x + c)^4 + 18
30400*a^3 + 7696832*a^2*b + 11423412*a*b^2 + 7432425*b^3 - 8580*(1280*a^3 +
 4576*a^2*b + 7581*a*b^2 + 5775*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 16*
(55*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)^9 - 390*(640*a^3 + 22
88*a^2*b + 3003*a*b^2)*cosh(d*x + c)^7 + 2184*(320*a^3 + 1144*a^2*b + 1309*
a*b^2)*cosh(d*x + c)^5 - 715*(1280*a^3 + 4576*a^2*b + 4011*a*b^2)*cosh(d*x
+ c)^3 + 143*(3200*a^3 + 9424*a^2*b + 6489*a*b^2)*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4392960*a^3 + 10323456*a^2*b + 12108096*a*b^2 + 6936930*b^3 - 3432*
(960*a^3 + 4664*a^2*b + 5859*a*b^2 + 3465*b^3)*cosh(d*x + c)^2 + 6*(11*(256
0*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^10 - 390*(640*a
^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^8 + 364*(2560*a^3 +
9152*a^2*b + 13552*a*b^2 + 12705*b^3)*cosh(d*x + c)^6 - 1430*(1280*a^3 + 45
76*a^2*b + 7581*a*b^2 + 5775*b^3)*cosh(d*x + c)^4 - 549120*a^3 - 2667808*a^
2*b - 3351348*a*b^2 - 1981980*b^3 + 143*(12800*a^3 + 53824*a^2*b + 79884*a*
b^2 + 51975*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 8*(6*(640*a^3 + 2288*a^
2*b + 3003*a*b^2)*cosh(d*x + c)^11 - 65*(640*a^3 + 2288*a^2*b + 3003*a*b^2)
*cosh(d*x + c)^9 + 624*(320*a^3 + 1144*a^2*b + 1309*a*b^2)*cosh(d*x + c)^7
- 429*(1280*a^3 + 4576*a^2*b + 4011*a*b^2)*cosh(d*x + c)^5 + 286*(3200*a^3
+ 9424*a^2*b + 6489*a*b^2)*cosh(d*x + c)^3 - 858*(960*a^3 + 1528*a^2*b + 90
3*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^14 + 14*d*cosh(d*x
+ c)*sinh(d*x + c)^13 + d*sinh(d*x + c)^14 - 14*d*cosh(d*x + c)^12 + 7*(13*
d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^12 + 4*(91*d*cosh(d*x + c)^3 - 36*d*
cosh(d*x + c))*sinh(d*x + c)^11 + 91*d*cosh(d*x + c)^10 + 7*(143*d*cosh(d*x
```

```
 + c)^4 - 132*d*cosh(d*x + c)^2 + 13*d)*sinh(d*x + c)^10 + 2*(1001*d*cosh(d
*x + c)^5 - 1320*d*cosh(d*x + c)^3 + 325*d*cosh(d*x + c))*sinh(d*x + c)^9 -
 364*d*cosh(d*x + c)^8 + 7*(429*d*cosh(d*x + c)^6 - 990*d*cosh(d*x + c)^4 +
 585*d*cosh(d*x + c)^2 - 52*d)*sinh(d*x + c)^8 + 8*(429*d*cosh(d*x + c)^7 -
 1188*d*cosh(d*x + c)^5 + 975*d*cosh(d*x + c)^3 - 208*d*cosh(d*x + c))*sinh
(d*x + c)^7 + 1001*d*cosh(d*x + c)^6 + 7*(429*d*cosh(d*x + c)^8 - 1848*d*co
sh(d*x + c)^6 + 2730*d*cosh(d*x + c)^4 - 1456*d*cosh(d*x + c)^2 + 143*d)*si
nh(d*x + c)^6 + 2*(1001*d*cosh(d*x + c)^9 - 4752*d*cosh(d*x + c)^7 + 8190*d
*cosh(d*x + c)^5 - 5824*d*cosh(d*x + c)^3 + 1287*d*cosh(d*x + c))*sinh(d*x
+ c)^5 - 2002*d*cosh(d*x + c)^4 + 7*(143*d*cosh(d*x + c)^10 - 990*d*cosh(d*
x + c)^8 + 2730*d*cosh(d*x + c)^6 - 3640*d*cosh(d*x + c)^4 + 2145*d*cosh(d*
x + c)^2 - 286*d)*sinh(d*x + c)^4 + 4*(91*d*cosh(d*x + c)^11 - 660*d*cosh(d
*x + c)^9 + 1950*d*cosh(d*x + c)^7 - 2912*d*cosh(d*x + c)^5 + 2145*d*cosh(d
*x + c)^3 - 572*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3003*d*cosh(d*x + c)^2 +
 7*(13*d*cosh(d*x + c)^12 - 132*d*cosh(d*x + c)^10 + 585*d*cosh(d*x + c)^8
- 1456*d*cosh(d*x + c)^6 + 2145*d*cosh(d*x + c)^4 - 1716*d*cosh(d*x + c)^2
+ 429*d)*sinh(d*x + c)^2 + 2*(7*d*cosh(d*x + c)^13 - 72*d*cosh(d*x + c)^11
+ 325*d*cosh(d*x + c)^9 - 832*d*cosh(d*x + c)^7 + 1287*d*cosh(d*x + c)^5 -
1144*d*cosh(d*x + c)^3 + 429*d*cosh(d*x + c))*s...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**14*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(134) = 268.
time = 0.60, size = 563, normalized size = 3.91

$$-\frac{2 \, {\left(15015 \, b^{3} e^{\left(24 \, d x + 24 \, c\right)} - 180180 \, b^{3} e^{\left(22 \, d x + 22 \, c\right)} + 240240 \, a b^{2} e^{\left(20 \, d x + 20 \, c\right)} + 990990 \, b^{3} e^{\left(20 \, d x + 20 \, c\right)} - 2042040 \, a b^{2} e^{\left(18 \, d x + 18 \, c\right)} - 3303300 \, b^{3} e^{\left(18 \, d x + 18 \, c\right)} + 2306304 \, a^{2} b e^{\left(16 \, d x + 16 \, c\right)} + 7711704 \, a b^{2} e^{\left(16 \, d x + 16 \, c\right)} + 7432425 \, b^{3} e^{\left(16 \, d x + 16 \, c\right)} - 10762752 \, a^{2} b e^{\left(14 \, d x + 14 \, c\right)} - 17008992 \, a b^{2} e^{\left(14 \, d x + 14 \, c\right)} - 11891880 \, b^{3} e^{\left(14 \, d x + 14 \, c\right)} + 8785920 \, a^{3} e^{\left(12 \, d x + 12 \, c\right)} + 20646912 \, a^{2} b e^{\left(12 \, d x + 12 \, c\right)} + 24216192 \, a b^{2} e^{\left(12 \, d x + 12 \, c\right)} + 13873860 \, b^{3} e^{\left(12 \, d x + 12 \, c\right)} - 6589440 \, a^{3} e^{\left(10 \, d x + 10 \, c\right)} - 21250944 \, a^{2} b e^{\left(10 \, d x + 10 \, c\right)} - 23207184 \, a b^{2} e^{\left(10 \, d x + 10 \, c\right)} - 11891880 \, b^{3} e^{\left(10 \, d x + 10 \, c\right)} + 3660800 \, a^{3} e^{\left(8 \, d x + 8 \, c\right)} + 13087360 \, a^{2} b e^{\left(8 \, d x + 8 \, c\right)} + 15135120 \, a b^{2} e^{\left(8 \, d x + 8 \, c\right)} + 7432425 \, b^{3} e^{\left(8 \, d x + 8 \, c\right)} - 1464320 \, a^{3} e^{\left(6 \, d x + 6 \, c\right)} - 5234944 \, a^{2} b e^{\left(6 \, d x + 6 \, c\right)} - 6630624 \, a b^{2} e^{\left(6 \, d x + 6 \, c\right)} - 3303300 \, b^{3} e^{\left(6 \, d x + 6 \, c\right)} + 399360 \, a^{3} e^{\left(4 \, d x + 4 \, c\right)} + 1427712 \, a^{2} b e^{\left(4 \, d x + 4 \, c\right)} + 1873872 \, a b^{2} e^{\left(4 \, d x + 4 \, c\right)} + 990990 \, b^{3} e^{\left(4 \, d x + 4 \, c\right)} - 66560 \, a^{3} e^{\left(2 \, d x + 2 \, c\right)} - 237952 \, a^{2} b e^{\left(2 \, d x + 2 \, c\right)} - 312312 \, a b^{2} e^{\left(2 \, d x + 2 \, c\right)} - 180180 \, b^{3} e^{\left(2 \, d x + 2 \, c\right)} + 5120 \, a^{3} + 18304 \, a^{2} b + 24024 \, a b^{2} + 15015 \, b^{3}\right)}}{15015 \, d {\left(e^{\left(2 \, d x + 2 \, c\right)} - 1\right)}^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -2/15015*(15015*b^3*e^(24*d*x + 24*c) - 180180*b^3*e^(22*d*x + 22*c) + 2402
40*a*b^2*e^(20*d*x + 20*c) + 990990*b^3*e^(20*d*x + 20*c) - 2042040*a*b^2*e
^(18*d*x + 18*c) - 3303300*b^3*e^(18*d*x + 18*c) + 2306304*a^2*b*e^(16*d*x
+ 16*c) + 7711704*a*b^2*e^(16*d*x + 16*c) + 7432425*b^3*e^(16*d*x + 16*c) -
 10762752*a^2*b*e^(14*d*x + 14*c) - 17008992*a*b^2*e^(14*d*x + 14*c) - 1189
1880*b^3*e^(14*d*x + 14*c) + 8785920*a^3*e^(12*d*x + 12*c) + 20646912*a^2*b
*e^(12*d*x + 12*c) + 24216192*a*b^2*e^(12*d*x + 12*c) + 13873860*b^3*e^(12*
```

```
d*x + 12*c) - 6589440*a^3*e^(10*d*x + 10*c) - 21250944*a^2*b*e^(10*d*x + 10
*c) - 23207184*a*b^2*e^(10*d*x + 10*c) - 11891880*b^3*e^(10*d*x + 10*c) + 3
660800*a^3*e^(8*d*x + 8*c) + 13087360*a^2*b*e^(8*d*x + 8*c) + 15135120*a*b^
2*e^(8*d*x + 8*c) + 7432425*b^3*e^(8*d*x + 8*c) - 1464320*a^3*e^(6*d*x + 6*
c) - 5234944*a^2*b*e^(6*d*x + 6*c) - 6630624*a*b^2*e^(6*d*x + 6*c) - 330330
0*b^3*e^(6*d*x + 6*c) + 399360*a^3*e^(4*d*x + 4*c) + 1427712*a^2*b*e^(4*d*x
 + 4*c) + 1873872*a*b^2*e^(4*d*x + 4*c) + 990990*b^3*e^(4*d*x + 4*c) - 6656
0*a^3*e^(2*d*x + 2*c) - 237952*a^2*b*e^(2*d*x + 2*c) - 312312*a*b^2*e^(2*d*
x + 2*c) - 180180*b^3*e^(2*d*x + 2*c) + 5120*a^3 + 18304*a^2*b + 24024*a*b^
2 + 15015*b^3)/(d*(e^(2*d*x + 2*c) - 1)^13)
```

Mupad [B]
time = 1.14, size = 2500, normalized size = 17.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^14,x)
```

```
[Out] ((6*b^3*exp(4*c + 4*d*x))/(13*d) - (2*b^3*exp(6*c + 6*d*x))/(13*d) + (2*b^2
*(96*a + 55*b))/(715*d) - (6*b^2*exp(2*c + 2*d*x)*(8*a + 11*b))/(143*d))/(6
*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d
*x) + 1) - ((2*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(3003*d) - (1
2*b^3*exp(10*c + 10*d*x))/(13*d) + (2*b^3*exp(12*c + 12*d*x))/(13*d) - (4*b
*exp(2*c + 2*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(143*d) + (2*b*exp(4*c + 4*
d*x)*(448*a*b + 256*a^2 + 165*b^2))/(143*d) + (30*b^2*exp(8*c + 8*d*x)*(8*a
 + 11*b))/(143*d) - (8*b^2*exp(6*c + 6*d*x)*(96*a + 55*b))/(143*d))/(7*exp(
2*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(8*c + 8*d
*x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) - 1
) - ((8*exp(6*c + 6*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(14
3*d) - (18*b^3*exp(16*c + 16*d*x))/(13*d) + (2*b^3*exp(18*c + 18*d*x))/(13*
d) - (2*b^2*(96*a + 55*b))/(715*d) - (24*b*exp(4*c + 4*d*x)*(112*a*b + 128*
a^2 + 33*b^2))/(143*d) - (84*b*exp(8*c + 8*d*x)*(112*a*b + 128*a^2 + 33*b^2
))/(143*d) + (6*b*exp(2*c + 2*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(715*d) +
 (84*b*exp(10*c + 10*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(715*d) + (72*b^2*
exp(14*c + 14*d*x)*(8*a + 11*b))/(143*d) - (168*b^2*exp(12*c + 12*d*x)*(96*
a + 55*b))/(715*d))/(45*exp(4*c + 4*d*x) - 10*exp(2*c + 2*d*x) - 120*exp(6*
c + 6*d*x) + 210*exp(8*c + 8*d*x) - 252*exp(10*c + 10*d*x) + 210*exp(12*c +
 12*d*x) - 120*exp(14*c + 14*d*x) + 45*exp(16*c + 16*d*x) - 10*exp(18*c + 1
8*d*x) + exp(20*c + 20*d*x) + 1) - ((2*b*(448*a*b + 256*a^2 + 165*b^2))/(21
45*d) - (8*b^3*exp(6*c + 6*d*x))/(13*d) + (2*b^3*exp(8*c + 8*d*x))/(13*d) +
 (12*b^2*exp(4*c + 4*d*x)*(8*a + 11*b))/(143*d) - (8*b^2*exp(2*c + 2*d*x)*(
96*a + 55*b))/(715*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6
*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) + ((2*b*(112*a*b
 + 128*a^2 + 33*b^2))/(429*d) - (2*exp(2*c + 2*d*x)*(840*a*b^2 + 1152*a^2*b
```

```
 + 1024*a^3 + 231*b^3))/(429*d) + (14*b^3*exp(12*c + 12*d*x))/(13*d) - (2*b
^3*exp(14*c + 14*d*x))/(13*d) + (14*b*exp(4*c + 4*d*x)*(112*a*b + 128*a^2 +
 33*b^2))/(143*d) - (14*b*exp(6*c + 6*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(
429*d) - (42*b^2*exp(10*c + 10*d*x)*(8*a + 11*b))/(143*d) + (14*b^2*exp(8*c
 + 8*d*x)*(96*a + 55*b))/(143*d))/(28*exp(4*c + 4*d*x) - 8*exp(2*c + 2*d*x)
 - 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(10*c + 10*d*x) + 28*e
xp(12*c + 12*d*x) - 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1) - ((2*b^
3)/(13*d) + (8*exp(12*c + 12*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*
b^3))/(13*d) - (24*b^3*exp(2*c + 2*d*x))/(13*d) - (24*b^3*exp(22*c + 22*d*x
))/(13*d) + (2*b^3*exp(24*c + 24*d*x))/(13*d) - (48*b*exp(10*c + 10*d*x)*(1
12*a*b + 128*a^2 + 33*b^2))/(13*d) - (48*b*exp(14*c + 14*d*x)*(112*a*b + 12
8*a^2 + 33*b^2))/(13*d) + (6*b*exp(8*c + 8*d*x)*(448*a*b + 256*a^2 + 165*b^
2))/(13*d) + (6*b*exp(16*c + 16*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(13*d)
+ (12*b^2*exp(4*c + 4*d*x)*(8*a + 11*b))/(13*d) + (12*b^2*exp(20*c + 20*d*x
)*(8*a + 11*b))/(13*d) - (8*b^2*exp(6*c + 6*d*x)*(96*a + 55*b))/(13*d) - (8
*b^2*exp(18*c + 18*d*x)*(96*a + 55*b))/(13*d))/(13*exp(2*c + 2*d*x) - 78*ex
p(4*c + 4*d*x) + 286*exp(6*c + 6*d*x) - 715*exp(8*c + 8*d*x) + 1287*exp(10*
c + 10*d*x) - 1716*exp(12*c + 12*d*x) + 1716*exp(14*c + 14*d*x) - 1287*exp(
16*c + 16*d*x) + 715*exp(18*c + 18*d*x) - 286*exp(20*c + 20*d*x) + 78*exp(2
2*c + 22*d*x) - 13*exp(24*c + 24*d*x) + exp(26*c + 26*d*x) - 1) - ((2*b^3*e
xp(4*c + 4*d*x))/(13*d) - (4*b^3*exp(2*c + 2*d*x))/(13*d) + (2*b^2*(8*a + 1
1*b))/(143*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x)
- 1) + ((2*b*(112*a*b + 128*a^2 + 33*b^2))/(429*d) + (10*b^3*exp(8*c + 8*d*
x))/(13*d) - (2*b^3*exp(10*c + 10*d*x))/(13*d) - (2*b*exp(2*c + 2*d*x)*(448
*a*b + 256*a^2 + 165*b^2))/(429*d) - (20*b^2*exp(6*c + 6*d*x)*(8*a + 11*b))
/(143*d) + (4*b^2*exp(4*c + 4*d*x)*(96*a + 55*b))/(143*d))/(15*exp(4*c + 4*
d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*e
xp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - ((2*b*(448*a*b + 256*a^2 + 16
5*b^2))/(2145*d) + (8*exp(4*c + 4*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 +
 231*b^3))/(429*d) - (16*b^3*exp(14*c + 14*d*x))/(13*d) + (2*b^3*exp(16*c +
 16*d*x))/(13*d) - (16*b*exp(2*c + 2*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(42
9*d) - (112*b*exp(6*c + 6*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(429*d) + (28*
b*exp(8*c + 8*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(429*d) + (56*b^2*exp(12*
c + 12*d*x)*(8*a + 11*b))/(143*d) - (112*b^2*exp(10*c + 10*d*x)*(96*a + 55*
b))/(715*d))/(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x
) - 126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) +
 36*exp(14*c + 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1) + (
(2*b^3)/(13*d) - (4*exp(10*c + 10*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 +
 231*b^3))/(13*d) + (22*b^3*exp(20*c + 20*d*x))/(13*d) - (2*b^3*exp(22*c +
22*d*x))/(13*d) + (20*b*exp(8*c + 8*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(13*
d) + (28*b*exp(12*c + 12*d*x)*(112*a*b + 128*a^...
```

3.226 $\int \operatorname{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=182

$$\frac{(a+b)^3\coth(c+dx)}{d}-\frac{(a+b)^2(7a+b)\coth^3(c+dx)}{3d}+\frac{3a(a+b)(7a+3b)\coth^5(c+dx)}{5d}-\frac{a(35a^2+30ab+$$

```
[Out] (a+b)^3*coth(d*x+c)/d-1/3*(a+b)^2*(7*a+b)*coth(d*x+c)^3/d+3/5*a*(a+b)*(7*a+
3*b)*coth(d*x+c)^5/d-1/7*a*(35*a^2+30*a*b+3*b^2)*coth(d*x+c)^7/d+5/9*a^2*(7
*a+3*b)*coth(d*x+c)^9/d-3/11*a^2*(7*a+b)*coth(d*x+c)^11/d+7/13*a^3*coth(d*x
+c)^13/d-1/15*a^3*coth(d*x+c)^15/d
```

Rubi [A]

time = 0.11, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3296, 1275}

$$-\frac{a^3\coth^{15}(c+dx)}{15d}+\frac{7a^3\coth^{13}(c+dx)}{13d}-\frac{a(35a^2+30ab+3b^2)\coth^7(c+dx)}{7d}-\frac{3a^2(7a+b)\coth^{11}(c+dx)}{11d}+\frac{5a^2(7a+3b)\coth^9(c+dx)}{9d}+\frac{3a(a+b)(7a+3b)\coth^5(c+dx)}{5d}-\frac{(a+b)^2(7a+b)\coth^3(c+dx)}{3d}+\frac{(a+b)^3\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^16*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((a + b)^3*Coth[c + d*x])/d - ((a + b)^2*(7*a + b)*Coth[c + d*x]^3)/(3*d) +
 (3*a*(a + b)*(7*a + 3*b)*Coth[c + d*x]^5)/(5*d) - (a*(35*a^2 + 30*a*b + 3*
b^2)*Coth[c + d*x]^7)/(7*d) + (5*a^2*(7*a + 3*b)*Coth[c + d*x]^9)/(9*d) - (
3*a^2*(7*a + b)*Coth[c + d*x]^11)/(11*d) + (7*a^3*Coth[c + d*x]^13)/(13*d)
- (a^3*Coth[c + d*x]^15)/(15*d)
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)\left(a-2ax^2+(a+b)x^4\right)^3}{x^{16}}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{16}}-\frac{7a^3}{x^{14}}+\frac{3a^2(7a+b)}{x^{12}}-\frac{5a^2(7a+3b)}{x^{10}}+\frac{a(35a^2+30ab+3b^2)}{x^8}+\right.\right.}{d}$$

$$=\frac{(a+b)^3\coth(c+dx)}{d}-\frac{(a+b)^2(7a+b)\coth^3(c+dx)}{3d}+\frac{3a(a}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 404 vs. 2(182) = 364.
time = 3.12, size = 404, normalized size = 2.22

$$-\frac{(45045(1024a^3+1152a^2b+840ab^2+231b^3)\cosh(c+dx)-5005(7168a^3+20352a^2b+16632ab^2+4785b^3)\cosh(3(c+dx))+21525504a^3\cosh(5(c+dx))+74954880a^2b\cosh(5(c+dx))+74162088ab^2\cosh(5(c+dx))+23288265b^3\cosh(5(c+dx))-9784320a^3\cosh(7(c+dx))-34070400a^2b\cosh(7(c+dx))-39999960ab^2\cosh(7(c+dx))-14189175b^3\cosh(7(c+dx))+3261440a^3\cosh(9(c+dx))+11356800a^2b\cosh(9(c+dx))+14054040ab^2\cosh(9(c+dx))+5720715b^3\cosh(9(c+dx))-752640a^3\cosh(11(c+dx))-2620800a^2b\cosh(11(c+dx))-3243240ab^2\cosh(11(c+dx))-1486485b^3\cosh(11(c+dx))+107520a^3\cosh(13(c+dx))+374400a^2b\cosh(13(c+dx))+463320ab^2\cosh(13(c+dx))+225225b^3\cosh(13(c+dx))-7168a^3\cosh(15(c+dx))-24960a^2b\cosh(15(c+dx))-30888ab^2\cosh(15(c+dx))-15015b^3\cosh(15(c+dx)))\operatorname{csch}^{15}(c+dx)}{369008640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^16*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/369008640*((45045*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*Cosh[c +
 d*x] - 5005*(7168*a^3 + 20352*a^2*b + 16632*a*b^2 + 4785*b^3)*Cosh[3*(c +
d*x)] + 21525504*a^3*Cosh[5*(c + d*x)] + 74954880*a^2*b*Cosh[5*(c + d*x)] +
 74162088*a*b^2*Cosh[5*(c + d*x)] + 23288265*b^3*Cosh[5*(c + d*x)] - 978432
0*a^3*Cosh[7*(c + d*x)] - 34070400*a^2*b*Cosh[7*(c + d*x)] - 39999960*a*b^2
*Cosh[7*(c + d*x)] - 14189175*b^3*Cosh[7*(c + d*x)] + 3261440*a^3*Cosh[9*(c
 + d*x)] + 11356800*a^2*b*Cosh[9*(c + d*x)] + 14054040*a*b^2*Cosh[9*(c + d*
x)] + 5720715*b^3*Cosh[9*(c + d*x)] - 752640*a^3*Cosh[11*(c + d*x)] - 26208
00*a^2*b*Cosh[11*(c + d*x)] - 3243240*a*b^2*Cosh[11*(c + d*x)] - 1486485*b^
3*Cosh[11*(c + d*x)] + 107520*a^3*Cosh[13*(c + d*x)] + 374400*a^2*b*Cosh[13
*(c + d*x)] + 463320*a*b^2*Cosh[13*(c + d*x)] + 225225*b^3*Cosh[13*(c + d*x
)] - 7168*a^3*Cosh[15*(c + d*x)] - 24960*a^2*b*Cosh[15*(c + d*x)] - 30888*a
*b^2*Cosh[15*(c + d*x)] - 15015*b^3*Cosh[15*(c + d*x)])*Csch[c + d*x]^15)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(168) = 336.
time = 1.61, size = 622, normalized size = 3.42

method	result
risch	$-\frac{4(-30888ab^2+118301040ab^2e^{14dx+14c}-113393280a^2be^{12dx+12c}-118918800ab^2e^{12dx+12c}+74954880a^2be^{10dx+10c}+83459376ab}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/45045*(-30888*a*b^2+118301040*a*b^2*exp(14*d*x+14*c)-113393280*a^2*b*exp
(12*d*x+12*c)-118918800*a*b^2*exp(12*d*x+12*c)+74954880*a^2*b*exp(10*d*x+10
*c)+83459376*a*b^2*exp(10*d*x+10*c)+35675640*a*b^2*exp(18*d*x+18*c)-5436288
0*a^2*b*exp(16*d*x+16*c)-80463240*a*b^2*exp(16*d*x+16*c)+106254720*a^2*b*ex
p(14*d*x+14*c)-2620800*a^2*b*exp(4*d*x+4*c)+374400*a^2*b*exp(2*d*x+2*c)-249
60*a^2*b-7168*a^3-15015*b^3-41081040*a*b^2*exp(8*d*x+8*c)+14054040*a*b^2*ex
p(6*d*x+6*c)-3243240*a*b^2*exp(4*d*x+4*c)-9297288*a*b^2*exp(20*d*x+20*c)-34
070400*a^2*b*exp(8*d*x+8*c)+463320*a*b^2*exp(2*d*x+2*c)+11356800*a^2*b*exp(
6*d*x+6*c)-555555*b^3*exp(24*d*x+24*c)+1081080*a*b^2*exp(22*d*x+22*c)+11531
520*a^2*b*exp(18*d*x+18*c)+25600575*b^3*exp(18*d*x+18*c)-43108065*b^3*exp(1
6*d*x+16*c)+53513460*b^3*exp(14*d*x+14*c)+107520*a^3*exp(2*d*x+2*c)-3587584
0*a^3*exp(12*d*x+12*c)-49549500*b^3*exp(12*d*x+12*c)+21525504*a^3*exp(10*d*
x+10*c)+3153150*b^3*exp(22*d*x+22*c)-10900890*b^3*exp(20*d*x+20*c)+45045*b^
3*exp(26*d*x+26*c)+46126080*a^3*exp(14*d*x+14*c)+225225*b^3*exp(2*d*x+2*c)-
17342325*b^3*exp(8*d*x+8*c)-752640*a^3*exp(4*d*x+4*c)-1531530*b^3*exp(4*d*x
+4*c)+34189155*b^3*exp(10*d*x+10*c)-9784320*a^3*exp(8*d*x+8*c)+3261440*a^3*
exp(6*d*x+6*c)+6276270*b^3*exp(6*d*x+6*c))/d/(exp(2*d*x+2*c)-1)^15
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2731 vs. $2(168) = 336$.
time = 0.30, size = 2731, normalized size = 15.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 4096/6435*a^3*(15*e^(-2*d*x - 2*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x
- 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 1
0*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x
- 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d
*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*
x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 105*e^(-4*d*x - 4*c)/(d*(15*e^(-2*d*
x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8
*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x -
 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*
x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d
*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) + 455*e^(-6*d
*x - 6*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x -
6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x -
12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x
 - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d
*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x -
30*c) - 1)) - 1365*e^(-8*d*x - 8*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x
 - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x -
```

```
10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x
 - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*
d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d
*x - 28*c) + e^(-30*d*x - 30*c) - 1)) + 3003*e^(-10*d*x - 10*c)/(d*(15*e^(-
2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x
 - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d
*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-2
0*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-
26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 5005*e^
(-12*d*x - 12*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6
*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12
*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(
-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e
^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30
*d*x - 30*c) - 1)) + 6435*e^(-14*d*x - 14*c)/(d*(15*e^(-2*d*x - 2*c) - 105*
e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-
10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e
^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 136
5*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15
*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 1/(d*(15*e^(-2*d*x - 2*c)
 - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 300
3*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) -
6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c)
 + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c
) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1))) + 512/231*a^2*b*(11*e
^(-2*d*x - 2*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d
*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x
- 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x -
 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 55*e^(-4*d*x -
4*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) -
 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 3
30*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11
*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) + 165*e^(-6*d*x - 6*c)/(d*(1
1*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8
*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*
d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*
x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 330*e^(-8*d*x - 8*c)/(d*(11*e^(-2*d*
x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c
) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c
) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c)
+ e^(-22*d*x - 22*c) - 1)) + 462*e^(-10*d*x - 10*c)/(d*(11*e^(-2*d*x - 2*c)
 - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*
e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*
e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22
*d*x - 22*c) - 1)) - 1/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*
```

```
e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-
12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-1
8*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1))) + 96/35*a
*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c)...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2967 vs. $2(168) = 336$.
time = 0.39, size = 2967, normalized size = 16.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 8/45045*((3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^1
3 + 13*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)*sin
h(d*x + c)^12 - 2*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*sinh(d*x
 + c)^13 - 15*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x +
 c)^11 + 6*(8960*a^3 + 31200*a^2*b + 38610*a*b^2 + 65065*b^3 - 26*(1792*a^3
 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 +
 11*(26*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^3
- 15*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c))*sinh
(d*x + c)^10 + 210*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x
 + c)^9 - 10*(143*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x
 + c)^4 + 37632*a^3 + 131040*a^2*b + 216216*a*b^2 + 234234*b^3 - 165*(1792*
a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9
 + 9*(143*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^
5 - 275*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c)^3
+ 210*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x + c))*sinh(d
*x + c)^8 - 182*(8960*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*cosh(d*x
 + c)^7 - 4*(858*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x
+ c)^6 - 2475*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c
)^4 - 407680*a^3 - 1419600*a^2*b - 2918916*a*b^2 - 2147145*b^3 + 3780*(896*
a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7
+ 2*(858*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^7
 - 3465*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c)^5
+ 8820*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x + c)^3 - 63
7*(8960*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*cosh(d*x + c))*sinh(d*
x + c)^6 + 1365*(3584*a^3 + 8256*a^2*b + 1980*a*b^2 - 3025*b^3)*cosh(d*x +
c)^5 - 6*(429*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c
)^8 - 2310*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^6
 + 8820*(896*a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x + c)^4 + 81
5360*a^3 + 3800160*a^2*b + 6396390*a*b^2 + 3578575*b^3 - 1274*(4480*a^3 + 1
5600*a^2*b + 32076*a*b^2 + 23595*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 5*
(143*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^9 - 9
```

```
90*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c)^7 + 529
2*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x + c)^5 - 1274*(8
960*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*cosh(d*x + c)^3 + 1365*(35
84*a^3 + 8256*a^2*b + 1980*a*b^2 - 3025*b^3)*cosh(d*x + c))*sinh(d*x + c)^4
 - 429*(25088*a^3 + 24000*a^2*b + 3492*a*b^2 - 10395*b^3)*cosh(d*x + c)^3 -
 2*(286*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c)^10 -
 2475*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^8 + 17
640*(896*a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x + c)^6 - 6370*(
4480*a^3 + 15600*a^2*b + 32076*a*b^2 + 23595*b^3)*cosh(d*x + c)^4 - 5381376
*a^3 - 32329440*a^2*b - 40980654*a*b^2 - 19324305*b^3 + 13650*(1792*a^3 + 8
352*a^2*b + 14058*a*b^2 + 7865*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 3*(2
6*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^11 - 275
*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c)^9 + 2520*
(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x + c)^7 - 1274*(896
0*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*cosh(d*x + c)^5 + 4550*(3584
*a^3 + 8256*a^2*b + 1980*a*b^2 - 3025*b^3)*cosh(d*x + c)^3 - 429*(25088*a^3
 + 24000*a^2*b + 3492*a*b^2 - 10395*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 2
860*(1792*a^3 - 1248*a^2*b - 108*a*b^2 + 693*b^3)*cosh(d*x + c) - 2*(13*(17
92*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c)^12 - 165*(1792*
a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^10 + 1890*(896*a^3
 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x + c)^8 - 1274*(4480*a^3 + 1
5600*a^2*b + 32076*a*b^2 + 23595*b^3)*cosh(d*x + c)^6 + 6825*(1792*a^3 + 83
52*a^2*b + 14058*a*b^2 + 7865*b^3)*cosh(d*x + c)^4 + 20500480*a^3 + 5491200
0*a^2*b + 59304960*a*b^2 + 25765740*b^3 - 1287*(12544*a^3 + 75360*a^2*b + 9
5526*a*b^2 + 45045*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^17
 + 17*d*cosh(d*x + c)*sinh(d*x + c)^16 + d*sinh(d*x + c)^17 - 15*d*cosh(d*x
 + c)^15 + (136*d*cosh(d*x + c)^2 - 15*d)*sinh(d*x + c)^15 + 5*(136*d*cosh(
d*x + c)^3 - 45*d*cosh(d*x + c))*sinh(d*x + c)^14 + 104*d*cosh(d*x + c)^13
+ (2380*d*cosh(d*x + c)^4 - 1575*d*cosh(d*x + c)^2 + 106*d)*sinh(d*x + c)^1
3 + 13*(476*d*cosh(d*x + c)^5 - 525*d*cosh(d*x + c)^3 + 104*d*cosh(d*x + c)
)*sinh(d*x + c)^12 - 440*d*cosh(d*x + c)^11 + (12376*d*cosh(d*x + c)^6 - 20
475*d*cosh(d*x + c)^4 + 8268*d*cosh(d*x + c)^2 - 470*d)*sinh(d*x + c)^11 +
11*(1768*d*cosh(d*x + c)^7 - 4095*d*cosh(d*x + c)^5 + 2704*d*cosh(d*x + c)^
3 - 440*d*cosh(d*x + c))*sinh(d*x + c)^10 + 1260*d*cosh(d*x + c)^9 + 5*(486
2*d*cosh(d*x + c)^8 - 15015*d*cosh(d*x + c)^6 + 15158*d*cosh(d*x + c)^4 - 5
170*d*cosh(d*x + c)^2 + 294*d)*sinh(d*x + c)^9 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**16*(a+b*sinh(d*x+c)**4)**3,x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(168) = 336.
time = 0.63, size = 621, normalized size = 3.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -4/45045*(45045*b^3*e^(26*d*x + 26*c) - 555555*b^3*e^(24*d*x + 24*c) + 1081
080*a*b^2*e^(22*d*x + 22*c) + 3153150*b^3*e^(22*d*x + 22*c) - 9297288*a*b^2
*e^(20*d*x + 20*c) - 10900890*b^3*e^(20*d*x + 20*c) + 11531520*a^2*b*e^(18*
d*x + 18*c) + 35675640*a*b^2*e^(18*d*x + 18*c) + 25600575*b^3*e^(18*d*x + 1
8*c) - 54362880*a^2*b*e^(16*d*x + 16*c) - 80463240*a*b^2*e^(16*d*x + 16*c)
- 43108065*b^3*e^(16*d*x + 16*c) + 46126080*a^3*e^(14*d*x + 14*c) + 1062547
20*a^2*b*e^(14*d*x + 14*c) + 118301040*a*b^2*e^(14*d*x + 14*c) + 53513460*b
^3*e^(14*d*x + 14*c) - 35875840*a^3*e^(12*d*x + 12*c) - 113393280*a^2*b*e^(
12*d*x + 12*c) - 118918800*a*b^2*e^(12*d*x + 12*c) - 49549500*b^3*e^(12*d*x
 + 12*c) + 21525504*a^3*e^(10*d*x + 10*c) + 74954880*a^2*b*e^(10*d*x + 10*c
) + 83459376*a*b^2*e^(10*d*x + 10*c) + 34189155*b^3*e^(10*d*x + 10*c) - 978
4320*a^3*e^(8*d*x + 8*c) - 34070400*a^2*b*e^(8*d*x + 8*c) - 41081040*a*b^2*
e^(8*d*x + 8*c) - 17342325*b^3*e^(8*d*x + 8*c) + 3261440*a^3*e^(6*d*x + 6*c
) + 11356800*a^2*b*e^(6*d*x + 6*c) + 14054040*a*b^2*e^(6*d*x + 6*c) + 62762
70*b^3*e^(6*d*x + 6*c) - 752640*a^3*e^(4*d*x + 4*c) - 2620800*a^2*b*e^(4*d*
x + 4*c) - 3243240*a*b^2*e^(4*d*x + 4*c) - 1531530*b^3*e^(4*d*x + 4*c) + 10
7520*a^3*e^(2*d*x + 2*c) + 374400*a^2*b*e^(2*d*x + 2*c) + 463320*a*b^2*e^(2
*d*x + 2*c) + 225225*b^3*e^(2*d*x + 2*c) - 7168*a^3 - 24960*a^2*b - 30888*a
*b^2 - 15015*b^3)/(d*(e^(2*d*x + 2*c) - 1)^15)
```

Mupad [B]
time = 1.22, size = 2500, normalized size = 13.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^16,x)
```

```
[Out] ((32*b^3)/(455*d) - (8*b^3*exp(2*c + 2*d*x))/(105*d))/(3*exp(2*c + 2*d*x) -
 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((8*exp(8*c + 8*d*x)*(840*a*b
^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(39*d) - (352*b^3*exp(18*c + 18*d*x)
)/(91*d) + (44*b^3*exp(20*c + 20*d*x))/(105*d) + (4*b^2*(8*a + 11*b))/(455*
d) - (64*b*exp(6*c + 6*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(91*d) - (128*b*e
xp(10*c + 10*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(65*d) + (4*b*exp(4*c + 4*d
*x)*(448*a*b + 256*a^2 + 165*b^2))/(91*d) + (24*b*exp(12*c + 12*d*x)*(448*a
*b + 256*a^2 + 165*b^2))/(65*d) + (132*b^2*exp(16*c + 16*d*x)*(8*a + 11*b))
```

```
/(91*d) - (32*b^2*exp(2*c + 2*d*x)*(96*a + 55*b))/(1365*d) - (64*b^2*exp(14
*c + 14*d*x)*(96*a + 55*b))/(91*d))/(66*exp(4*c + 4*d*x) - 12*exp(2*c + 2*d
*x) - 220*exp(6*c + 6*d*x) + 495*exp(8*c + 8*d*x) - 792*exp(10*c + 10*d*x)
+ 924*exp(12*c + 12*d*x) - 792*exp(14*c + 14*d*x) + 495*exp(16*c + 16*d*x)
- 220*exp(18*c + 18*d*x) + 66*exp(20*c + 20*d*x) - 12*exp(22*c + 22*d*x) +
exp(24*c + 24*d*x) + 1) + ((192*b^3*exp(4*c + 4*d*x))/(455*d) - (16*b^3*exp
(6*c + 6*d*x))/(105*d) + (32*b^2*(96*a + 55*b))/(15015*d) - (16*b^2*exp(2*c
 + 2*d*x)*(8*a + 11*b))/(455*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x)
+ 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((4*
(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(6435*d) - (96*b^3*exp(10*c
+ 10*d*x))/(65*d) + (4*b^3*exp(12*c + 12*d*x))/(15*d) - (64*b*exp(2*c + 2*d
*x)*(112*a*b + 128*a^2 + 33*b^2))/(2145*d) + (12*b*exp(4*c + 4*d*x)*(448*a*
b + 256*a^2 + 165*b^2))/(715*d) + (4*b^2*exp(8*c + 8*d*x)*(8*a + 11*b))/(13
*d) - (32*b^2*exp(6*c + 6*d*x)*(96*a + 55*b))/(429*d))/(28*exp(4*c + 4*d*x)
 - 8*exp(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(
10*c + 10*d*x) + 28*exp(12*c + 12*d*x) - 8*exp(14*c + 14*d*x) + exp(16*c +
16*d*x) + 1) - ((32*exp(6*c + 6*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 2
31*b^3))/(429*d) - (288*b^3*exp(16*c + 16*d*x))/(91*d) + (8*b^3*exp(18*c +
18*d*x))/(21*d) - (32*b^2*(96*a + 55*b))/(15015*d) - (192*b*exp(4*c + 4*d*x
)*(112*a*b + 128*a^2 + 33*b^2))/(1001*d) - (128*b*exp(8*c + 8*d*x)*(112*a*b
 + 128*a^2 + 33*b^2))/(143*d) + (8*b*exp(2*c + 2*d*x)*(448*a*b + 256*a^2 +
165*b^2))/(1001*d) + (144*b*exp(10*c + 10*d*x)*(448*a*b + 256*a^2 + 165*b^2
))/(715*d) + (96*b^2*exp(14*c + 14*d*x)*(8*a + 11*b))/(91*d) - (64*b^2*exp(
12*c + 12*d*x)*(96*a + 55*b))/(143*d))/(11*exp(2*c + 2*d*x) - 55*exp(4*c +
4*d*x) + 165*exp(6*c + 6*d*x) - 330*exp(8*c + 8*d*x) + 462*exp(10*c + 10*d*
x) - 462*exp(12*c + 12*d*x) + 330*exp(14*c + 14*d*x) - 165*exp(16*c + 16*d*
x) + 55*exp(18*c + 18*d*x) - 11*exp(20*c + 20*d*x) + exp(22*c + 22*d*x) - 1
) - ((32*exp(14*c + 14*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/
(15*d) + (8*b^3*exp(2*c + 2*d*x))/(15*d) - (32*b^3*exp(4*c + 4*d*x))/(5*d)
- (32*b^3*exp(24*c + 24*d*x))/(5*d) + (8*b^3*exp(26*c + 26*d*x))/(15*d) - (
64*b*exp(12*c + 12*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(5*d) - (64*b*exp(16*
c + 16*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(5*d) + (8*b*exp(10*c + 10*d*x)*(
448*a*b + 256*a^2 + 165*b^2))/(5*d) + (8*b*exp(18*c + 18*d*x)*(448*a*b + 25
6*a^2 + 165*b^2))/(5*d) + (16*b^2*exp(6*c + 6*d*x)*(8*a + 11*b))/(5*d) + (1
6*b^2*exp(22*c + 22*d*x)*(8*a + 11*b))/(5*d) - (32*b^2*exp(8*c + 8*d*x)*(96
*a + 55*b))/(15*d) - (32*b^2*exp(20*c + 20*d*x)*(96*a + 55*b))/(15*d))/(15*
exp(2*c + 2*d*x) - 105*exp(4*c + 4*d*x) + 455*exp(6*c + 6*d*x) - 1365*exp(8
*c + 8*d*x) + 3003*exp(10*c + 10*d*x) - 5005*exp(12*c + 12*d*x) + 6435*exp(
14*c + 14*d*x) - 6435*exp(16*c + 16*d*x) + 5005*exp(18*c + 18*d*x) - 3003*e
xp(20*c + 20*d*x) + 1365*exp(22*c + 22*d*x) - 455*exp(24*c + 24*d*x) + 105*
exp(26*c + 26*d*x) - 15*exp(28*c + 28*d*x) + exp(30*c + 30*d*x) - 1) - ((4*
b*(448*a*b + 256*a^2 + 165*b^2))/(5005*d) - (64*b^3*exp(6*c + 6*d*x))/(91*d
) + (4*b^3*exp(8*c + 8*d*x))/(21*d) + (8*b^2*exp(4*c + 4*d*x)*(8*a + 11*b))
/(91*d) - (32*b^2*exp(2*c + 2*d*x)*(96*a + 55*b))/(3003*d))/(15*exp(4*c + 4
*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*
```

```
exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) + ((64*b*(112*a*b + 128*a^2 +
33*b^2))/(15015*d) - (32*exp(2*c + 2*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^
3 + 231*b^3))/(6435*d) + (128*b^3*exp(12*c + 12*d*x))/(65*d) - (32*b^3*exp(
14*c + 14*d*x))/(105*d) + (256*b*exp(4*c + 4*d*x)*(112*a*b + 128*a^2 + 33*b
^2))/(2145*d) - (32*b*exp(6*c + 6*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(715*
d) - (32*b^2*exp(10*c + 10*d*x)*(8*a + 11*b))/(65*d) + (64*b^2*exp(8*c + 8*
d*x)*(96*a + 55*b))/(429*d))/(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84
*exp(6*c + 6*d*x) - 126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(
12*c + 12*d*x) + 36*exp(14*c + 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c +
18*d*x) - 1) - ((4*b^3)/(105*d) + (16*exp(12*c + 12*d*x)*(840*a*b^2 + 1152*
a^2*b + 1024*a^3 + 231*b^3))/(15*d) - (32*b^3*exp(2*c + 2*d*x))/(35*d) - (1
92*b^3*exp(22*c + 22*d*x))/(35*d) + (52*b^3*exp(24*c + 24*d*x))/(105*d) - (
192*b*exp(10*c + 10*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(35*d) - (256*b*exp(
14*c + 14*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(3...
```

3.227 $\int \operatorname{csch}^{18}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=221

$$-\frac{(a+b)^3\coth(c+dx)}{d}+\frac{2(a+b)^2(4a+b)\coth^3(c+dx)}{3d}-\frac{(a+b)\left(28a^2+17ab+b^2\right)\coth^5(c+dx)}{5d}+\frac{4a(14a$$

```
[Out] -(a+b)^3*coth(d*x+c)/d+2/3*(a+b)^2*(4*a+b)*coth(d*x+c)^3/d-1/5*(a+b)*(28*a^
2+17*a*b+b^2)*coth(d*x+c)^5/d+4/7*a*(14*a^2+15*a*b+3*b^2)*coth(d*x+c)^7/d-1
/9*a*(70*a^2+45*a*b+3*b^2)*coth(d*x+c)^9/d+2/11*a^2*(28*a+9*b)*coth(d*x+c)^
11/d-1/13*a^2*(28*a+3*b)*coth(d*x+c)^13/d+8/15*a^3*coth(d*x+c)^15/d-1/17*a^
3*coth(d*x+c)^17/d
```

Rubi [A]
time = 0.14, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$,
Rules used = {3296, 1275}

$$-\frac{a^3\coth^{17}(c+dx)}{17d}+\frac{8a^3\coth^{15}(c+dx)}{15d}-\frac{a\left(70a^2+45ab+3b^2\right)\coth^9(c+dx)}{9d}+\frac{4a\left(14a^2+15ab+3b^2\right)\coth^7(c+dx)}{7d}-\frac{(a+b)\left(28a^2+17ab+b^2\right)\coth^5(c+dx)}{5d}-\frac{a^2(28a+3b)\coth^{13}(c+dx)}{13d}+\frac{2a^2(28a+9b)\coth^{11}(c+dx)}{11d}+\frac{2(a+b)^2(4a+b)\coth^3(c+dx)}{3d}-\frac{(a+b)^3\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^18*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(((a + b)^3*Coth[c + d*x])/d) + (2*(a + b)^2*(4*a + b)*Coth[c + d*x]^3)/(3
*d) - ((a + b)*(28*a^2 + 17*a*b + b^2)*Coth[c + d*x]^5)/(5*d) + (4*a*(14*a^
2 + 15*a*b + 3*b^2)*Coth[c + d*x]^7)/(7*d) - (a*(70*a^2 + 45*a*b + 3*b^2)*C
oth[c + d*x]^9)/(9*d) + (2*a^2*(28*a + 9*b)*Coth[c + d*x]^11)/(11*d) - (a^2
*(28*a + 3*b)*Coth[c + d*x]^13)/(13*d) + (8*a^3*Coth[c + d*x]^15)/(15*d) -
(a^3*Coth[c + d*x]^17)/(17*d)
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^{18}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2(a-2ax^2+(a+b)x^4)^3}{x^{18}}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{18}} - \frac{8a^3}{x^{16}} + \frac{a^2(28a+3b)}{x^{14}} - \frac{2a^2(28a+9b)}{x^{12}} + \frac{a(70a^2+45ab+3b^{}}{x^{10}}\right.\right.}{}$$

$$= -\frac{(a+b)^3\coth(c+dx)}{d} + \frac{2(a+b)^2(4a+b)\coth^3(c+dx)}{3d} -$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 458 vs. $2(221) = 442$.

time = 4.63, size = 458, normalized size = 2.07

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^18*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/6273146880*((680680*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*Cosh[c
 + d*x] - 272272*(2048*a^3 + 5760*a^2*b + 4704*a*b^2 + 1353*b^3)*Cosh[3*(c
+ d*x)] + 354844672*a^3*Cosh[5*(c + d*x)] + 1211857920*a^2*b*Cosh[5*(c + d*
x)] + 1189284096*a*b^2*Cosh[5*(c + d*x)] + 372263892*b^3*Cosh[5*(c + d*x)]
- 177422336*a^3*Cosh[7*(c + d*x)] - 605928960*a^2*b*Cosh[7*(c + d*x)] - 692
659968*a*b^2*Cosh[7*(c + d*x)] - 242288046*b^3*Cosh[7*(c + d*x)] + 68239360
*a^3*Cosh[9*(c + d*x)] + 233049600*a^2*b*Cosh[9*(c + d*x)] + 277717440*a*b^
2*Cosh[9*(c + d*x)] + 108738630*b^3*Cosh[9*(c + d*x)] - 19496960*a^3*Cosh[1
1*(c + d*x)] - 66585600*a^2*b*Cosh[11*(c + d*x)] - 79347840*a*b^2*Cosh[11*(
c + d*x)] - 33693660*b^3*Cosh[11*(c + d*x)] + 3899392*a^3*Cosh[13*(c + d*x)
] + 13317120*a^2*b*Cosh[13*(c + d*x)] + 15869568*a*b^2*Cosh[13*(c + d*x)] +
 6942936*b^3*Cosh[13*(c + d*x)] - 487424*a^3*Cosh[15*(c + d*x)] - 1664640*a
^2*b*Cosh[15*(c + d*x)] - 1983696*a*b^2*Cosh[15*(c + d*x)] - 867867*b^3*Cos
h[15*(c + d*x)] + 28672*a^3*Cosh[17*(c + d*x)] + 97920*a^2*b*Cosh[17*(c + d
*x)] + 116688*a*b^2*Cosh[17*(c + d*x)] + 51051*b^3*Cosh[17*(c + d*x)])*Csch
[c + d*x]^17)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(205) = 410$.

time = 1.64, size = 680, normalized size = 3.08

method	result
risch	$-\frac{16(697016320a^3e^{16dx+16c}+116688ab^2-1775057856ab^2e^{14dx+14c}+1211857920a^2be^{12dx+12c}+1316707392ab^2e^{12dx+12c}-605928}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -16/765765*(697016320*a^3*exp(16*d*x+16*c)+116688*a*b^2-1775057856*a*b^2*ex
p(14*d*x+14*c)+1211857920*a^2*b*exp(12*d*x+12*c)+1316707392*a*b^2*exp(12*d*
x+12*c)-605928960*a^2*b*exp(10*d*x+10*c)-707362656*a*b^2*exp(10*d*x+10*c)-1
132457040*a*b^2*exp(18*d*x+18*c)+1582289280*a^2*b*exp(16*d*x+16*c)+17042282
40*a*b^2*exp(16*d*x+16*c)-1736317440*a^2*b*exp(14*d*x+14*c)+13317120*a^2*b*
exp(4*d*x+4*c)-1664640*a^2*b*exp(2*d*x+2*c)+97920*a^2*b+28672*a^3+168030720
*a^2*b*exp(20*d*x+20*c)+51051*b^3+277717440*a*b^2*exp(8*d*x+8*c)-79347840*a
*b^2*exp(6*d*x+6*c)+15869568*a*b^2*exp(4*d*x+4*c)+494290368*a*b^2*exp(20*d*
x+20*c)+233049600*a^2*b*exp(8*d*x+8*c)-1983696*a*b^2*exp(2*d*x+2*c)-6658560
0*a^2*b*exp(6*d*x+6*c)+36807771*b^3*exp(24*d*x+24*c)-127423296*a*b^2*exp(22
*d*x+22*c)-798145920*a^2*b*exp(18*d*x+18*c)-541906365*b^3*exp(18*d*x+18*c)+
699143445*b^3*exp(16*d*x+16*c)-680611932*b^3*exp(14*d*x+14*c)-487424*a^3*ex
p(2*d*x+2*c)+354844672*a^3*exp(12*d*x+12*c)+502035534*b^3*exp(12*d*x+12*c)-
177422336*a^3*exp(10*d*x+10*c)-129771642*b^3*exp(22*d*x+22*c)+312227916*b^3
*exp(20*d*x+20*c)+14702688*a*b^2*exp(24*d*x+24*c)-6381375*b^3*exp(26*d*x+26
*c)-557613056*a^3*exp(14*d*x+14*c)-867867*b^3*exp(2*d*x+2*c)+115120005*b^3*
exp(8*d*x+8*c)+3899392*a^3*exp(4*d*x+4*c)+6942936*b^3*exp(4*d*x+4*c)+510510
*b^3*exp(28*d*x+28*c)-279095817*b^3*exp(10*d*x+10*c)+68239360*a^3*exp(8*d*x
+8*c)-19496960*a^3*exp(6*d*x+6*c)-34204170*b^3*exp(6*d*x+6*c))/d/(exp(2*d*x
+2*c)-1)^17
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3719 vs. $2(205) = 410$.
time = 0.30, size = 3719, normalized size = 16.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -65536/109395*a^3*(17*e^(-2*d*x - 2*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*
d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x
 - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-
16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 1237
6*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) -
680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e
^(-34*d*x - 34*c) - 1)) - 136*e^(-4*d*x - 4*c)/(d*(17*e^(-2*d*x - 2*c) - 13
6*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^
(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24
310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c
```

```
) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x -
26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 3
2*c) + e^(-34*d*x - 34*c) - 1)) + 680*e^(-6*d*x - 6*c)/(d*(17*e^(-2*d*x - 2
*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) +
 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14
*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*
x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-2
6*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32
*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) - 2380*e^(-8*d*x - 8*c)/(d*(17*e^(-
2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x
 - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14
*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*
e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2
380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) -
17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) + 6188*e^(-10*d*x - 10*c)/
(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 238
0*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 1
9448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*
c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x
- 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x
 - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) - 12376*e^(-12*
d*x - 12*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x
- 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x
 - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-
18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188
*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 13
6*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) + 1
9448*e^(-14*d*x - 14*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 68
0*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 1237
6*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c)
+ 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x -
22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x
- 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*
c) - 1)) - 24310*e^(-16*d*x - 16*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x
 - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x -
10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*
d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e
^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680
*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-
34*d*x - 34*c) - 1)) - 1/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 6
80*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 123
76*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c)
 + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x -
 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x
 - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34
```

```
*c) - 1))) - 2048/1001*a^2*b*(13*e^(-2*d*x - 2*c)/(d*(13*e^(-2*d*x - 2*c) -
 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e
^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 128
7*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78
*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 78
*e^(-4*d*x - 4*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6
*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*
d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-1
8*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*
d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) + 286*e^(-6*d*x - 6*c)/(d*(13*e^(-2*
d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8
*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3585 vs. $2(205) = 410$.
time = 0.38, size = 3585, normalized size = 16.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -16/765765*((28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x
+ c)^14 - 14*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*cosh(d*x
 + c)*sinh(d*x + c)^13 + (28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b
^3)*sinh(d*x + c)^14 - 34*(14336*a^3 + 48960*a^2*b + 58344*a*b^2 + 213213*b
^3)*cosh(d*x + c)^12 - (487424*a^3 + 1664640*a^2*b + 1983696*a*b^2 + 724924
2*b^3 - 91*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^12 - 4*(91*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 4
59459*b^3)*cosh(d*x + c)^3 - 204*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 81
081*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + 17*(229376*a^3 + 783360*a^2*b +
1798368*a*b^2 + 2573571*b^3)*cosh(d*x + c)^10 + (1001*(28672*a^3 + 97920*a^
2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x + c)^4 + 3899392*a^3 + 13317120*a
^2*b + 30572256*a*b^2 + 43750707*b^3 - 2244*(14336*a^3 + 48960*a^2*b + 5834
4*a*b^2 + 213213*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 - 2*(1001*(28672*a^
3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*cosh(d*x + c)^5 - 7480*(7168*a
^3 + 24480*a^2*b + 29172*a*b^2 - 81081*b^3)*cosh(d*x + c)^3 + 85*(229376*a^
3 + 783360*a^2*b + 68640*a*b^2 - 1756755*b^3)*cosh(d*x + c))*sinh(d*x + c)^
9 - 68*(286720*a^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*cosh(d*x +
 c)^8 + (3003*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*
x + c)^6 - 16830*(14336*a^3 + 48960*a^2*b + 58344*a*b^2 + 213213*b^3)*cosh(
d*x + c)^4 - 19496960*a^3 - 66585600*a^2*b - 206771136*a*b^2 - 163975812*b^
3 + 765*(229376*a^3 + 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^8 - 8*(429*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 -
459459*b^3)*cosh(d*x + c)^7 - 6732*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 -
```

```
81081*b^3)*cosh(d*x + c)^5 + 255*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 -
 1756755*b^3)*cosh(d*x + c)^3 - 272*(71680*a^3 + 244800*a^2*b - 176748*a*b^
2 - 351351*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 17*(4014080*a^3 + 23592960
*a^2*b + 45412224*a*b^2 + 25138113*b^3)*cosh(d*x + c)^6 + (3003*(28672*a^3
+ 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x + c)^8 - 31416*(14336*a
^3 + 48960*a^2*b + 58344*a*b^2 + 213213*b^3)*cosh(d*x + c)^6 + 3570*(229376
*a^3 + 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*cosh(d*x + c)^4 + 682393
60*a^3 + 401080320*a^2*b + 772007808*a*b^2 + 427347921*b^3 - 1904*(286720*a
^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 - 2*(1001*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*cosh
(d*x + c)^9 - 26928*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 81081*b^3)*cosh
(d*x + c)^7 + 2142*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 - 1756755*b^3)*
cosh(d*x + c)^5 - 7616*(71680*a^3 + 244800*a^2*b - 176748*a*b^2 - 351351*b^
3)*cosh(d*x + c)^3 + 51*(4014080*a^3 + 3824640*a^2*b - 12739584*a*b^2 - 115
94583*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 442*(401408*a^3 + 3176640*a^2*b
 + 4162488*a*b^2 + 1857471*b^3)*cosh(d*x + c)^4 + (1001*(28672*a^3 + 97920*
a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x + c)^10 - 16830*(14336*a^3 + 48
960*a^2*b + 58344*a*b^2 + 213213*b^3)*cosh(d*x + c)^8 + 3570*(229376*a^3 +
783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*cosh(d*x + c)^6 - 4760*(286720*
a^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*cosh(d*x + c)^4 - 1774223
36*a^3 - 1404074880*a^2*b - 1839819696*a*b^2 - 821002182*b^3 + 255*(4014080
*a^3 + 23592960*a^2*b + 45412224*a*b^2 + 25138113*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 - 4*(91*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*
cosh(d*x + c)^11 - 3740*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 81081*b^3)*
cosh(d*x + c)^9 + 510*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 - 1756755*b^
3)*cosh(d*x + c)^7 - 3808*(71680*a^3 + 244800*a^2*b - 176748*a*b^2 - 351351
*b^3)*cosh(d*x + c)^5 + 85*(4014080*a^3 + 3824640*a^2*b - 12739584*a*b^2 -
11594583*b^3)*cosh(d*x + c)^3 - 884*(200704*a^3 - 217440*a^2*b - 480876*a*b
^2 - 297297*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 557613056*a^3 - 173631744
0*a^2*b - 1775057856*a*b^2 - 680611932*b^3 + 221*(4759552*a^3 + 12643200*a^
2*b + 13669392*a*b^2 + 5435199*b^3)*cosh(d*x + c)^2 + (91*(28672*a^3 + 9792
0*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x + c)^12 - 2244*(14336*a^3 + 4
8960*a^2*b + 58344*a*b^2 + 213213*b^3)*cosh(d*x + c)^10 + 765*(229376*a^3 +
 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*cosh(d*x + c)^8 - 1904*(286720
*a^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*cosh(d*x + c)^6 + 255*(4
014080*a^3 + 23592960*a^2*b + 45412224*a*b^2 + 25138113*b^3)*cosh(d*x + c)^
4 + 1051860992*a^3 + 2794147200*a^2*b + 3020935632*a*b^2 + 1201178979*b^3 -
 2652*(401408*a^3 + 3176640*a^2*b + 4162488*a*b^2 + 1857471*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^2 - 2*(7*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459
459*b^3)*cosh(d*x + c)^13 - 408*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 810
81*b^3)*cosh(d*x + c)^11 + 85*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 - 17
56755*b^3)*cosh(d*x + c)^9 - 1088*(71680*a^3 + 244800*a^2*b - 176748*a*b^2
- 351351*b^3)*cosh(d*x + c)^7 + 51*(4014080*a^3 + 3824640*a^2*b - 12739584*
a*b^2 - 11594583*b^3)*cosh(d*x + c)^5 - 1768*(2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**18*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 679 vs. 2(205) = 410.
time = 0.64, size = 679, normalized size = 3.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -16/765765*(510510*b^3*e^(28*d*x + 28*c) - 6381375*b^3*e^(26*d*x + 26*c) +
14702688*a*b^2*e^(24*d*x + 24*c) + 36807771*b^3*e^(24*d*x + 24*c) - 1274232
96*a*b^2*e^(22*d*x + 22*c) - 129771642*b^3*e^(22*d*x + 22*c) + 168030720*a^
2*b*e^(20*d*x + 20*c) + 494290368*a*b^2*e^(20*d*x + 20*c) + 312227916*b^3*e
^(20*d*x + 20*c) - 798145920*a^2*b*e^(18*d*x + 18*c) - 1132457040*a*b^2*e^(
18*d*x + 18*c) - 541906365*b^3*e^(18*d*x + 18*c) + 697016320*a^3*e^(16*d*x
+ 16*c) + 1582289280*a^2*b*e^(16*d*x + 16*c) + 1704228240*a*b^2*e^(16*d*x +
 16*c) + 699143445*b^3*e^(16*d*x + 16*c) - 557613056*a^3*e^(14*d*x + 14*c)
- 1736317440*a^2*b*e^(14*d*x + 14*c) - 1775057856*a*b^2*e^(14*d*x + 14*c) -
 680611932*b^3*e^(14*d*x + 14*c) + 354844672*a^3*e^(12*d*x + 12*c) + 121185
7920*a^2*b*e^(12*d*x + 12*c) + 1316707392*a*b^2*e^(12*d*x + 12*c) + 5020355
34*b^3*e^(12*d*x + 12*c) - 177422336*a^3*e^(10*d*x + 10*c) - 605928960*a^2*
b*e^(10*d*x + 10*c) - 707362656*a*b^2*e^(10*d*x + 10*c) - 279095817*b^3*e^(
10*d*x + 10*c) + 68239360*a^3*e^(8*d*x + 8*c) + 233049600*a^2*b*e^(8*d*x +
8*c) + 277717440*a*b^2*e^(8*d*x + 8*c) + 115120005*b^3*e^(8*d*x + 8*c) - 19
496960*a^3*e^(6*d*x + 6*c) - 66585600*a^2*b*e^(6*d*x + 6*c) - 79347840*a*b^
2*e^(6*d*x + 6*c) - 34204170*b^3*e^(6*d*x + 6*c) + 3899392*a^3*e^(4*d*x + 4
*c) + 13317120*a^2*b*e^(4*d*x + 4*c) + 15869568*a*b^2*e^(4*d*x + 4*c) + 694
2936*b^3*e^(4*d*x + 4*c) - 487424*a^3*e^(2*d*x + 2*c) - 1664640*a^2*b*e^(2*
d*x + 2*c) - 1983696*a*b^2*e^(2*d*x + 2*c) - 867867*b^3*e^(2*d*x + 2*c) + 2
8672*a^3 + 97920*a^2*b + 116688*a*b^2 + 51051*b^3)/(d*(e^(2*d*x + 2*c) - 1)
^17)
```

Mupad [**B**]
time = 1.31, size = 2500, normalized size = 11.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^18,x)
```

```
[Out] ((24*b^3)/(595*d) - (64*exp(10*c + 10*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a
^3 + 231*b^3))/(85*d) + (6864*b^3*exp(20*c + 20*d*x))/(595*d) - (104*b^3*ex
p(22*c + 22*d*x))/(85*d) + (48*b*exp(8*c + 8*d*x)*(112*a*b + 128*a^2 + 33*b
^2))/(17*d) + (576*b*exp(12*c + 12*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(85*d
) - (24*b*exp(6*c + 6*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(119*d) - (144*b*
exp(14*c + 14*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(119*d) - (48*b^2*exp(2*c
 + 2*d*x)*(8*a + 11*b))/(595*d) - (528*b^2*exp(18*c + 18*d*x)*(8*a + 11*b))
/(119*d) + (16*b^2*exp(4*c + 4*d*x)*(96*a + 55*b))/(119*d) + (264*b^2*exp(1
6*c + 16*d*x)*(96*a + 55*b))/(119*d))/(91*exp(4*c + 4*d*x) - 14*exp(2*c + 2
*d*x) - 364*exp(6*c + 6*d*x) + 1001*exp(8*c + 8*d*x) - 2002*exp(10*c + 10*d
*x) + 3003*exp(12*c + 12*d*x) - 3432*exp(14*c + 14*d*x) + 3003*exp(16*c + 1
6*d*x) - 2002*exp(18*c + 18*d*x) + 1001*exp(20*c + 20*d*x) - 364*exp(22*c +
 22*d*x) + 91*exp(24*c + 24*d*x) - 14*exp(26*c + 26*d*x) + exp(28*c + 28*d*
x) + 1) + ((24*b^3)/(595*d) - (4*b^3*exp(2*c + 2*d*x))/(85*d))/(6*exp(4*c +
 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) -
 ((64*exp(8*c + 8*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(221*
d) - (1056*b^3*exp(18*c + 18*d*x))/(119*d) + (88*b^3*exp(20*c + 20*d*x))/(8
5*d) + (48*b^2*(8*a + 11*b))/(7735*d) - (192*b*exp(6*c + 6*d*x)*(112*a*b +
128*a^2 + 33*b^2))/(221*d) - (3456*b*exp(10*c + 10*d*x)*(112*a*b + 128*a^2
+ 33*b^2))/(1105*d) + (72*b*exp(4*c + 4*d*x)*(448*a*b + 256*a^2 + 165*b^2))
/(1547*d) + (144*b*exp(12*c + 12*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(221*d
) + (4752*b^2*exp(16*c + 16*d*x)*(8*a + 11*b))/(1547*d) - (32*b^2*exp(2*c +
 2*d*x)*(96*a + 55*b))/(1547*d) - (2112*b^2*exp(14*c + 14*d*x)*(96*a + 55*b
))/(1547*d))/(13*exp(2*c + 2*d*x) - 78*exp(4*c + 4*d*x) + 286*exp(6*c + 6*d
*x) - 715*exp(8*c + 8*d*x) + 1287*exp(10*c + 10*d*x) - 1716*exp(12*c + 12*d
*x) + 1716*exp(14*c + 14*d*x) - 1287*exp(16*c + 16*d*x) + 715*exp(18*c + 18
*d*x) - 286*exp(20*c + 20*d*x) + 78*exp(22*c + 22*d*x) - 13*exp(24*c + 24*d
*x) + exp(26*c + 26*d*x) - 1) + ((48*b^3*exp(4*c + 4*d*x))/(119*d) - (8*b^3
*exp(6*c + 6*d*x))/(51*d) + (8*b^2*(96*a + 55*b))/(4641*d) - (48*b^2*exp(2*
c + 2*d*x)*(8*a + 11*b))/(1547*d))/(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x
) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(
12*c + 12*d*x) + 1) - ((64*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(
109395*d) - (192*b^3*exp(10*c + 10*d*x))/(85*d) + (112*b^3*exp(12*c + 12*d*
x))/(255*d) - (384*b*exp(2*c + 2*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(12155*
d) + (48*b*exp(4*c + 4*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(2431*d) + (96*b
^2*exp(8*c + 8*d*x)*(8*a + 11*b))/(221*d) - (64*b^2*exp(6*c + 6*d*x)*(96*a
+ 55*b))/(663*d))/(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c +
6*d*x) - 126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d
*x) + 36*exp(14*c + 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1
) - ((64*exp(6*c + 6*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(6
63*d) - (792*b^3*exp(16*c + 16*d*x))/(119*d) + (44*b^3*exp(18*c + 18*d*x))/
```

```
(51*d) - (8*b^2*(96*a + 55*b))/(4641*d) - (48*b*exp(4*c + 4*d*x)*(112*a*b +
 128*a^2 + 33*b^2))/(221*d) - (288*b*exp(8*c + 8*d*x)*(112*a*b + 128*a^2 +
33*b^2))/(221*d) + (12*b*exp(2*c + 2*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(1
547*d) + (72*b*exp(10*c + 10*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(221*d) +
(3168*b^2*exp(14*c + 14*d*x)*(8*a + 11*b))/(1547*d) - (176*b^2*exp(12*c + 1
2*d*x)*(96*a + 55*b))/(221*d))/(66*exp(4*c + 4*d*x) - 12*exp(2*c + 2*d*x) -
 220*exp(6*c + 6*d*x) + 495*exp(8*c + 8*d*x) - 792*exp(10*c + 10*d*x) + 924
*exp(12*c + 12*d*x) - 792*exp(14*c + 14*d*x) + 495*exp(16*c + 16*d*x) - 220
*exp(18*c + 18*d*x) + 66*exp(20*c + 20*d*x) - 12*exp(22*c + 22*d*x) + exp(2
4*c + 24*d*x) + 1) - ((64*exp(14*c + 14*d*x)*(840*a*b^2 + 1152*a^2*b + 1024
*a^3 + 231*b^3))/(17*d) + (4*b^3*exp(2*c + 2*d*x))/(17*d) - (72*b^3*exp(4*c
 + 4*d*x))/(17*d) - (312*b^3*exp(24*c + 24*d*x))/(17*d) + (28*b^3*exp(26*c
+ 26*d*x))/(17*d) - (336*b*exp(12*c + 12*d*x)*(112*a*b + 128*a^2 + 33*b^2))
/(17*d) - (432*b*exp(16*c + 16*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(17*d) +
(36*b*exp(10*c + 10*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(17*d) + (60*b*exp(
18*c + 18*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(17*d) + (48*b^2*exp(6*c + 6*
d*x)*(8*a + 11*b))/(17*d) + (144*b^2*exp(22*c + 22*d*x)*(8*a + 11*b))/(17*d
) - (40*b^2*exp(8*c + 8*d*x)*(96*a + 55*b))/(17*d) - (88*b^2*exp(20*c + 20*
d*x)*(96*a + 55*b))/(17*d))/(120*exp(4*c + 4*d*x) - 16*exp(2*c + 2*d*x) - 5
60*exp(6*c + 6*d*x) + 1820*exp(8*c + 8*d*x) - 4368*exp(10*c + 10*d*x) + 800
8*exp(12*c + 12*d*x) - 11440*exp(14*c + 14*d*x) + 12870*exp(16*c + 16*d*x)
- 11440*exp(18*c + 18*d*x) + 8008*exp(20*c + 20*d*x) - 4368*exp(22*c + 22*d
*x) + 1820*exp(24*c + 24*d*x) - 560*exp(26*c + 26*d*x) + 120*exp(28*c + 28*
d*x) - 16*exp(30*c + 30*d*x) + exp(32*c + 32*d*x) + 1) - ((12*b*(448*a*b +
256*a^2 + 165*b^2))/(17017*d) - (96*b^3*exp(6*c + 6*d*x))/(119*d) + (4*b^3*
exp(8*c + 8*d*x))/(17*d) + (144*b^2*exp(4*c + 4...
```

3.228 $\int \operatorname{csch}^{20}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=248

$$\frac{(a+b)^3\coth(c+dx)}{d}-\frac{(a+b)^2(3a+b)\coth^3(c+dx)}{d}+\frac{3(a+b)\left(12a^2+9ab+b^2\right)\coth^5(c+dx)}{5d}-\frac{(84a^3+$$

```
[Out] (a+b)^3*coth(d*x+c)/d-(a+b)^2*(3*a+b)*coth(d*x+c)^3/d+3/5*(a+b)*(12*a^2+9*a
*b+b^2)*coth(d*x+c)^5/d-1/7*(84*a^3+105*a^2*b+30*a*b^2+b^3)*coth(d*x+c)^7/d
+1/3*a*(42*a^2+35*a*b+5*b^2)*coth(d*x+c)^9/d-3/11*a*(42*a^2+21*a*b+b^2)*cot
h(d*x+c)^11/d+21/13*a^2*(4*a+b)*coth(d*x+c)^13/d-1/5*a^2*(12*a+b)*coth(d*x+
c)^15/d+9/17*a^3*coth(d*x+c)^17/d-1/19*a^3*coth(d*x+c)^19/d
```

Rubi [A]
time = 0.16, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{3296, 1275\}$

$$-\frac{a^3\coth^{19}(c+dx)}{19d}+\frac{9a^3\coth^{17}(c+dx)}{17d}-\frac{3a\left(42a^2+21ab+b^2\right)\coth^{11}(c+dx)}{11d}+\frac{a\left(42a^2+35ab+5b^2\right)\coth^9(c+dx)}{3d}+\frac{3(a+b)\left(12a^2+9ab+b^2\right)\coth^5(c+dx)}{5d}-\frac{a^2(12a+b)\coth^{15}(c+dx)}{5d}+\frac{21a^2(4a+b)\coth^{13}(c+dx)}{13d}-\frac{\left(84a^3+105a^2b+30ab^2+b^3\right)\coth^7(c+dx)}{7d}-\frac{(a+b)^2(3a+b)\coth^3(c+dx)}{d}+\frac{(a+b)^3\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^20*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((a + b)^3*Coth[c + d*x])/d - ((a + b)^2*(3*a + b)*Coth[c + d*x]^3)/d + (3*
(a + b)*(12*a^2 + 9*a*b + b^2)*Coth[c + d*x]^5)/(5*d) - ((84*a^3 + 105*a^2*
b + 30*a*b^2 + b^3)*Coth[c + d*x]^7)/(7*d) + (a*(42*a^2 + 35*a*b + 5*b^2)*C
oth[c + d*x]^9)/(3*d) - (3*a*(42*a^2 + 21*a*b + b^2)*Coth[c + d*x]^11)/(11*
d) + (21*a^2*(4*a + b)*Coth[c + d*x]^13)/(13*d) - (a^2*(12*a + b)*Coth[c +
d*x]^15)/(5*d) + (9*a^3*Coth[c + d*x]^17)/(17*d) - (a^3*Coth[c + d*x]^19)/(
19*d)
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^{20}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^3\left(a-2ax^2+(a+b)x^4\right)^3}{x^{20}}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{20}} - \frac{9a^3}{x^{18}} + \frac{3a^2(12a+b)}{x^{16}} - \frac{21a^2(4a+b)}{x^{14}} + \frac{3a\left(42a^2+21ab+b^2\right)}{x^{12}}\right.\right.}{}$$

$$= \frac{(a+b)^3\coth(c+dx)}{d} - \frac{(a+b)^2(3a+b)\coth^3(c+dx)}{d} + \frac{3(a}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 512 vs. 2(248) = 496.
time = 5.99, size = 512, normalized size = 2.06

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^20*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/79459860480*((7759752*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*Cosh
[c + d*x] - 2116296*(3072*a^3 + 8576*a^2*b + 7000*a*b^2 + 2013*b^3)*Cosh[3*
(c + d*x)] + 4334174208*a^3*Cosh[5*(c + d*x)] + 14582690304*a^2*b*Cosh[5*(c
 + d*x)] + 14221509120*a*b^2*Cosh[5*(c + d*x)] + 4440518082*b^3*Cosh[5*(c +
 d*x)] - 2333786112*a^3*Cosh[7*(c + d*x)] - 7852217856*a^2*b*Cosh[7*(c + d*
x)] - 8803791360*a*b^2*Cosh[7*(c + d*x)] - 3047642598*b^3*Cosh[7*(c + d*x)]
 + 1000194048*a^3*Cosh[9*(c + d*x)] + 3365236224*a^2*b*Cosh[9*(c + d*x)] +
3906077760*a*b^2*Cosh[9*(c + d*x)] + 1489040982*b^3*Cosh[9*(c + d*x)] - 333
398016*a^3*Cosh[11*(c + d*x)] - 1121745408*a^2*b*Cosh[11*(c + d*x)] - 13020
25920*a*b^2*Cosh[11*(c + d*x)] - 527386002*b^3*Cosh[11*(c + d*x)] + 8334950
4*a^3*Cosh[13*(c + d*x)] + 280436352*a^2*b*Cosh[13*(c + d*x)] + 325506480*a
*b^2*Cosh[13*(c + d*x)] + 134271423*b^3*Cosh[13*(c + d*x)] - 14708736*a^3*C
osh[15*(c + d*x)] - 49488768*a^2*b*Cosh[15*(c + d*x)] - 57442320*a*b^2*Cosh
[15*(c + d*x)] - 23694957*b^3*Cosh[15*(c + d*x)] + 1634304*a^3*Cosh[17*(c +
 d*x)] + 5498752*a^2*b*Cosh[17*(c + d*x)] + 6382480*a*b^2*Cosh[17*(c + d*x)
] + 2632773*b^3*Cosh[17*(c + d*x)] - 86016*a^3*Cosh[19*(c + d*x)] - 289408*
a^2*b*Cosh[19*(c + d*x)] - 335920*a*b^2*Cosh[19*(c + d*x)] - 138567*b^3*Cos
h[19*(c + d*x)])*Csch[c + d*x]^19)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 737 vs. 2(232) = 464.
time = 1.68, size = 738, normalized size = 2.98

method	result
risch	$-\frac{32\left(-6501261312a^{3}e^{16dx+16c}-335920a\,b^{2}+15573923040a\,b^{2}e^{14dx+14c}-7852217856a^{2}b\,e^{12dx+12c}-8958986400a\,b^{2}e^{12dx+12c}+33\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -32/4849845*(-6501261312*a^3*exp(16*d*x+16*c)-335920*a*b^2+15573923040*a*b^
2*exp(14*d*x+14*c)-7852217856*a^2*b*exp(12*d*x+12*c)-8958986400*a*b^2*exp(1
2*d*x+12*c)+3365236224*a^2*b*exp(10*d*x+10*c)+3906077760*a*b^2*exp(10*d*x+1
0*c)+18774904720*a*b^2*exp(18*d*x+18*c)-20011694976*a^2*b*exp(16*d*x+16*c)-
20101788720*a*b^2*exp(16*d*x+16*c)+14582690304*a^2*b*exp(14*d*x+14*c)-49488
768*a^2*b*exp(4*d*x+4*c)+5498752*a^2*b*exp(2*d*x+2*c)-289408*a^2*b-86016*a^
3+1862340480*a^2*b*exp(22*d*x+22*c)-8897848960*a^2*b*exp(20*d*x+20*c)-13856
7*b^3-1302025920*a*b^2*exp(8*d*x+8*c)+325506480*a*b^2*exp(6*d*x+6*c)-574423
20*a*b^2*exp(4*d*x+4*c)-12256713040*a*b^2*exp(20*d*x+20*c)-1121745408*a^2*b
*exp(8*d*x+8*c)+6382480*a*b^2*exp(2*d*x+2*c)+280436352*a^2*b*exp(6*d*x+6*c)
+155195040*a*b^2*exp(26*d*x+26*c)-1270797957*b^3*exp(24*d*x+24*c)+528771672
0*a*b^2*exp(22*d*x+22*c)+17837083264*a^2*b*exp(18*d*x+18*c)+7296522519*b^3*
exp(18*d*x+18*c)-7366637421*b^3*exp(16*d*x+16*c)+5711316039*b^3*exp(14*d*x+
14*c)+1634304*a^3*exp(2*d*x+2*c)-2333786112*a^3*exp(12*d*x+12*c)-3403621221
*b^3*exp(12*d*x+12*c)+1000194048*a^3*exp(10*d*x+10*c)+3106533573*b^3*exp(22
*d*x+22*c)-5504019807*b^3*exp(20*d*x+20*c)-1352413920*a*b^2*exp(24*d*x+24*c
)+355978623*b^3*exp(26*d*x+26*c)+4334174208*a^3*exp(14*d*x+14*c)+2632773*b^
3*exp(2*d*x+2*c)-532235847*b^3*exp(8*d*x+8*c)-14708736*a^3*exp(4*d*x+4*c)-2
3694957*b^3*exp(4*d*x+4*c)+4849845*b^3*exp(30*d*x+30*c)-61108047*b^3*exp(28
*d*x+28*c)+7945986048*a^3*exp(18*d*x+18*c)+1550149029*b^3*exp(10*d*x+10*c)-
333398016*a^3*exp(8*d*x+8*c)+83349504*a^3*exp(6*d*x+6*c)+134271423*b^3*exp(
6*d*x+6*c))/d/(exp(2*d*x+2*c)-1)^19
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4883 vs. $2(232) = 464$.
time = 0.33, size = 4883, normalized size = 19.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 131072/230945*a^3*(19*e^(-2*d*x - 2*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*
d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*
x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(
-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 755
82*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c)
```

```
 - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32
*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) -
 1)) - 171*e^(-4*d*x - 4*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c)
+ 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) -
 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 1
6*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d
*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^
(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e
^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) + 969*
e^(-6*d*x - 6*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6
*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-
12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 9237
8*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c)
- 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x -
28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x -
 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) - 3876*e^(-8*d*x
- 8*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c
) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 1
2*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d
*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^
(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 38
76*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 1
9*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) + 11628*e^(-10*d*x - 10*c)/
(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 387
6*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) +
50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18
*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*
x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-
30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-3
6*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) - 27132*e^(-12*d*x - 12*c)/(d*(19*
e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8
*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e
^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 9
2378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*
c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x
- 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x -
 36*c) + e^(-38*d*x - 38*c) - 1)) + 50388*e^(-14*d*x - 14*c)/(d*(19*e^(-2*d
*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x -
8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d
*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^
(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27
132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c)
 - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c)
+ e^(-38*d*x - 38*c) - 1)) - 75582*e^(-16*d*x - 16*c)/(d*(19*e^(-2*d*x - 2*
c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) +
```

```
11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14
*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*
x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(
-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*
e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-3
8*d*x - 38*c) - 1)) + 92378*e^(-18*d*x - 18*c)/(d*(19*e^(-2*d*x - 2*c) - 17
1*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e
^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 7
5582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*
c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x
 - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*
d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x -
 38*c) - 1)) - 1/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6
*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-
12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 9237
8*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c)
- 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x -...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4259 vs. $2(232) = 464$.
time = 0.56, size = 4259, normalized size = 17.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 64/4849845*((43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*
x + c)^15 + 15*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh
(d*x + c)*sinh(d*x + c)^14 - 2*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 124
7103*b^3)*sinh(d*x + c)^15 - 19*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 -
1538823*b^3)*cosh(d*x + c)^13 + 2*(408576*a^3 + 1374688*a^2*b + 1595620*a*b
^2 + 15935205*b^3 - 105*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 13*(35*(43008*a^3 + 144704*a^2*b + 1
67960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^3 - 19*(43008*a^3 + 144704*a^2*b +
 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c))*sinh(d*x + c)^12 + 57*(129024*a
^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^11 - 6*(455*(
21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x + c)^4 + 1225
728*a^3 + 4124064*a^2*b + 17719780*a*b^2 + 31639465*b^3 - 494*(21504*a^3 +
72352*a^2*b + 83980*a*b^2 + 838695*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 +
 11*(273*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x +
 c)^5 - 494*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*
x + c)^3 + 57*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh
(d*x + c))*sinh(d*x + c)^10 - 969*(43008*a^3 + 144704*a^2*b - 529880*a*b^2
- 586443*b^3)*cosh(d*x + c)^9 - 2*(5005*(21504*a^3 + 72352*a^2*b + 83980*a*
```

```
b^2 + 1247103*b^3)*cosh(d*x + c)^6 - 13585*(21504*a^3 + 72352*a^2*b + 83980
*a*b^2 + 838695*b^3)*cosh(d*x + c)^4 - 20837376*a^3 - 70109088*a^2*b - 4194
80100*a*b^2 - 351267345*b^3 + 3135*(64512*a^3 + 217056*a^2*b + 932620*a*b^2
 + 1665235*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 9*(715*(43008*a^3 + 1447
04*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^7 - 2717*(43008*a^3 +
144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^5 + 1045*(129024*a
^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^3 - 969*(4300
8*a^3 + 144704*a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^8 + 6783*(24576*a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d*x
 + c)^7 - 6*(2145*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cos
h(d*x + c)^8 - 10868*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)*c
osh(d*x + c)^6 + 6270*(64512*a^3 + 217056*a^2*b + 932620*a*b^2 + 1665235*b^
3)*cosh(d*x + c)^4 + 27783168*a^3 + 248673824*a^2*b + 549145220*a*b^2 + 303
230785*b^3 - 11628*(21504*a^3 + 72352*a^2*b + 432900*a*b^2 + 362505*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^7 + (5005*(43008*a^3 + 144704*a^2*b + 167960*a
*b^2 - 2355639*b^3)*cosh(d*x + c)^9 - 32604*(43008*a^3 + 144704*a^2*b + 167
960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^7 + 26334*(129024*a^3 + 434112*a^2*b
 - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^5 - 81396*(43008*a^3 + 144704*
a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x + c)^3 + 47481*(24576*a^3 - 545
92*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 323*
(1548288*a^3 - 8564416*a^2*b - 12926680*a*b^2 - 6120543*b^3)*cosh(d*x + c)^
5 - 2*(3003*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x
+ c)^10 - 24453*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)*cosh(d
*x + c)^8 + 26334*(64512*a^3 + 217056*a^2*b + 932620*a*b^2 + 1665235*b^3)*c
osh(d*x + c)^6 - 122094*(21504*a^3 + 72352*a^2*b + 432900*a*b^2 + 362505*b^
3)*cosh(d*x + c)^4 - 250048512*a^3 - 3065771296*a^2*b - 4040697700*a*b^2 -
1763542209*b^3 + 20349*(86016*a^3 + 769888*a^2*b + 1700140*a*b^2 + 938795*b
^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + (1365*(43008*a^3 + 144704*a^2*b + 16
7960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^11 - 13585*(43008*a^3 + 144704*a^2*
b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^9 + 18810*(129024*a^3 + 43411
2*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^7 - 122094*(43008*a^3 +
 144704*a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x + c)^5 + 237405*(24576*
a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d*x + c)^3 - 1615*(1548
288*a^3 - 8564416*a^2*b - 12926680*a*b^2 - 6120543*b^3)*cosh(d*x + c))*sinh
(d*x + c)^4 - 323*(8687616*a^3 + 15456448*a^2*b + 15194920*a*b^2 + 6026163*
b^3)*cosh(d*x + c)^3 - 2*(455*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247
103*b^3)*cosh(d*x + c)^12 - 5434*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 8
38695*b^3)*cosh(d*x + c)^10 + 9405*(64512*a^3 + 217056*a^2*b + 932620*a*b^2
 + 1665235*b^3)*cosh(d*x + c)^8 - 81396*(21504*a^3 + 72352*a^2*b + 432900*a
*b^2 + 362505*b^3)*cosh(d*x + c)^6 + 33915*(86016*a^3 + 769888*a^2*b + 1700
140*a*b^2 + 938795*b^3)*cosh(d*x + c)^4 + 2569943040*a^3 + 6422325280*a^2*b
 + 6933472780*a*b^2 + 2675035935*b^3 - 3230*(774144*a^3 + 9491552*a^2*b + 1
2509900*a*b^2 + 5459883*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (105*(43008
*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^13 - 1482*(
43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^11 + 3
```

```
135*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^
9 - 34884*(43008*a^3 + 144704*a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x +
 c)^7 + 142443*(24576*a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d
*x + c)^5 - 3230*(1548288*a^3 - 8564416*a^2*b -...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**20*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 737 vs. 2(232) = 464.
time = 0.65, size = 737, normalized size = 2.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -32/4849845*(4849845*b^3*e^(30*d*x + 30*c) - 61108047*b^3*e^(28*d*x + 28*c)
 + 155195040*a*b^2*e^(26*d*x + 26*c) + 355978623*b^3*e^(26*d*x + 26*c) - 13
52413920*a*b^2*e^(24*d*x + 24*c) - 1270797957*b^3*e^(24*d*x + 24*c) + 18623
40480*a^2*b*e^(22*d*x + 22*c) + 5287716720*a*b^2*e^(22*d*x + 22*c) + 310653
3573*b^3*e^(22*d*x + 22*c) - 8897848960*a^2*b*e^(20*d*x + 20*c) - 122567130
40*a*b^2*e^(20*d*x + 20*c) - 5504019807*b^3*e^(20*d*x + 20*c) + 7945986048*
a^3*e^(18*d*x + 18*c) + 17837083264*a^2*b*e^(18*d*x + 18*c) + 18774904720*a
*b^2*e^(18*d*x + 18*c) + 7296522519*b^3*e^(18*d*x + 18*c) - 6501261312*a^3*
e^(16*d*x + 16*c) - 20011694976*a^2*b*e^(16*d*x + 16*c) - 20101788720*a*b^2
*e^(16*d*x + 16*c) - 7366637421*b^3*e^(16*d*x + 16*c) + 4334174208*a^3*e^(1
4*d*x + 14*c) + 14582690304*a^2*b*e^(14*d*x + 14*c) + 15573923040*a*b^2*e^(
14*d*x + 14*c) + 5711316039*b^3*e^(14*d*x + 14*c) - 2333786112*a^3*e^(12*d*
x + 12*c) - 7852217856*a^2*b*e^(12*d*x + 12*c) - 8958986400*a*b^2*e^(12*d*x
 + 12*c) - 3403621221*b^3*e^(12*d*x + 12*c) + 1000194048*a^3*e^(10*d*x + 10
*c) + 3365236224*a^2*b*e^(10*d*x + 10*c) + 3906077760*a*b^2*e^(10*d*x + 10*
c) + 1550149029*b^3*e^(10*d*x + 10*c) - 333398016*a^3*e^(8*d*x + 8*c) - 112
1745408*a^2*b*e^(8*d*x + 8*c) - 1302025920*a*b^2*e^(8*d*x + 8*c) - 53223584
7*b^3*e^(8*d*x + 8*c) + 83349504*a^3*e^(6*d*x + 6*c) + 280436352*a^2*b*e^(6
*d*x + 6*c) + 325506480*a*b^2*e^(6*d*x + 6*c) + 134271423*b^3*e^(6*d*x + 6*
c) - 14708736*a^3*e^(4*d*x + 4*c) - 49488768*a^2*b*e^(4*d*x + 4*c) - 574423
20*a*b^2*e^(4*d*x + 4*c) - 23694957*b^3*e^(4*d*x + 4*c) + 1634304*a^3*e^(2*
d*x + 2*c) + 5498752*a^2*b*e^(2*d*x + 2*c) + 6382480*a*b^2*e^(2*d*x + 2*c)
```

```
+ 2632773*b^3*e^(2*d*x + 2*c) - 86016*a^3 - 289408*a^2*b - 335920*a*b^2 - 1
38567*b^3)/(d*(e^(2*d*x + 2*c) - 1)^19)
```

Mupad [**B**]
time = 1.42, size = 2500, normalized size = 10.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^20,x)
```

```
[Out] ((512*b*(112*a*b + 128*a^2 + 33*b^2))/(138567*d) + (1792*b^3*exp(8*c + 8*d*
x))/(969*d) - (448*b^3*exp(10*c + 10*d*x))/(969*d) - (64*b*exp(2*c + 2*d*x)
*(448*a*b + 256*a^2 + 165*b^2))/(12597*d) - (256*b^2*exp(6*c + 6*d*x)*(8*a
+ 11*b))/(969*d) + (512*b^2*exp(4*c + 4*d*x)*(96*a + 55*b))/(12597*d))/(9*e
xp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) - 126*exp(8*c +
 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) + 36*exp(14*c + 14
*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1) - ((8*b*(448*a*b + 2
56*a^2 + 165*b^2))/(12597*d) + (128*exp(4*c + 4*d*x)*(840*a*b^2 + 1152*a^2*
b + 1024*a^3 + 231*b^3))/(4199*d) - (2816*b^3*exp(14*c + 14*d*x))/(323*d) +
 (440*b^3*exp(16*c + 16*d*x))/(323*d) - (512*b*exp(2*c + 2*d*x)*(112*a*b +
128*a^2 + 33*b^2))/(12597*d) - (2560*b*exp(6*c + 6*d*x)*(112*a*b + 128*a^2
+ 33*b^2))/(4199*d) + (880*b*exp(8*c + 8*d*x)*(448*a*b + 256*a^2 + 165*b^2)
)/(4199*d) + (704*b^2*exp(12*c + 12*d*x)*(8*a + 11*b))/(323*d) - (2816*b^2*
exp(10*c + 10*d*x)*(96*a + 55*b))/(4199*d))/(66*exp(4*c + 4*d*x) - 12*exp(2
*c + 2*d*x) - 220*exp(6*c + 6*d*x) + 495*exp(8*c + 8*d*x) - 792*exp(10*c +
10*d*x) + 924*exp(12*c + 12*d*x) - 792*exp(14*c + 14*d*x) + 495*exp(16*c +
16*d*x) - 220*exp(18*c + 18*d*x) + 66*exp(20*c + 20*d*x) - 12*exp(22*c + 22
*d*x) + exp(24*c + 24*d*x) + 1) - ((512*exp(18*c + 18*d*x)*(840*a*b^2 + 115
2*a^2*b + 1024*a^3 + 231*b^3))/(19*d) + (128*b^3*exp(6*c + 6*d*x))/(19*d) -
 (1536*b^3*exp(8*c + 8*d*x))/(19*d) - (1536*b^3*exp(28*c + 28*d*x))/(19*d)
+ (128*b^3*exp(30*c + 30*d*x))/(19*d) - (3072*b*exp(16*c + 16*d*x)*(112*a*b
 + 128*a^2 + 33*b^2))/(19*d) - (3072*b*exp(20*c + 20*d*x)*(112*a*b + 128*a^
2 + 33*b^2))/(19*d) + (384*b*exp(14*c + 14*d*x)*(448*a*b + 256*a^2 + 165*b^
2))/(19*d) + (384*b*exp(22*c + 22*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(19*d
) + (768*b^2*exp(10*c + 10*d*x)*(8*a + 11*b))/(19*d) + (768*b^2*exp(26*c +
26*d*x)*(8*a + 11*b))/(19*d) - (512*b^2*exp(12*c + 12*d*x)*(96*a + 55*b))/(
19*d) - (512*b^2*exp(24*c + 24*d*x)*(96*a + 55*b))/(19*d))/(19*exp(2*c + 2*
d*x) - 171*exp(4*c + 4*d*x) + 969*exp(6*c + 6*d*x) - 3876*exp(8*c + 8*d*x)
+ 11628*exp(10*c + 10*d*x) - 27132*exp(12*c + 12*d*x) + 50388*exp(14*c + 14
*d*x) - 75582*exp(16*c + 16*d*x) + 92378*exp(18*c + 18*d*x) - 92378*exp(20*
c + 20*d*x) + 75582*exp(22*c + 22*d*x) - 50388*exp(24*c + 24*d*x) + 27132*e
xp(26*c + 26*d*x) - 11628*exp(28*c + 28*d*x) + 3876*exp(30*c + 30*d*x) - 96
9*exp(32*c + 32*d*x) + 171*exp(34*c + 34*d*x) - 19*exp(36*c + 36*d*x) + exp
(38*c + 38*d*x) - 1) + ((128*b^3)/(4845*d) - (1792*exp(10*c + 10*d*x)*(840*
```

```
a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(1615*d) + (128128*b^3*exp(20*c +
 20*d*x))/(4845*d) - (2912*b^3*exp(22*c + 22*d*x))/(969*d) + (3584*b*exp(8*
c + 8*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(969*d) + (3584*b*exp(12*c + 12*d*
x)*(112*a*b + 128*a^2 + 33*b^2))/(323*d) - (224*b*exp(6*c + 6*d*x)*(448*a*b
 + 256*a^2 + 165*b^2))/(969*d) - (704*b*exp(14*c + 14*d*x)*(448*a*b + 256*a
^2 + 165*b^2))/(323*d) - (64*b^2*exp(2*c + 2*d*x)*(8*a + 11*b))/(969*d) - (
9152*b^2*exp(18*c + 18*d*x)*(8*a + 11*b))/(969*d) + (128*b^2*exp(4*c + 4*d*
x)*(96*a + 55*b))/(969*d) + (1408*b^2*exp(16*c + 16*d*x)*(96*a + 55*b))/(32
3*d))/(15*exp(2*c + 2*d*x) - 105*exp(4*c + 4*d*x) + 455*exp(6*c + 6*d*x) -
1365*exp(8*c + 8*d*x) + 3003*exp(10*c + 10*d*x) - 5005*exp(12*c + 12*d*x) +
 6435*exp(14*c + 14*d*x) - 6435*exp(16*c + 16*d*x) + 5005*exp(18*c + 18*d*x
) - 3003*exp(20*c + 20*d*x) + 1365*exp(22*c + 22*d*x) - 455*exp(24*c + 24*d
*x) + 105*exp(26*c + 26*d*x) - 15*exp(28*c + 28*d*x) + exp(30*c + 30*d*x) -
 1) + ((128*b^3)/(4845*d) - (32*b^3*exp(2*c + 2*d*x))/(969*d))/(5*exp(2*c +
 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) +
exp(10*c + 10*d*x) - 1) - ((128*exp(8*c + 8*d*x)*(840*a*b^2 + 1152*a^2*b +
1024*a^3 + 231*b^3))/(323*d) - (18304*b^3*exp(18*c + 18*d*x))/(969*d) + (22
88*b^3*exp(20*c + 20*d*x))/(969*d) + (32*b^2*(8*a + 11*b))/(6783*d) - (1024
*b*exp(6*c + 6*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(969*d) - (1536*b*exp(10*
c + 10*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(323*d) + (16*b*exp(4*c + 4*d*x)*
(448*a*b + 256*a^2 + 165*b^2))/(323*d) + (352*b*exp(12*c + 12*d*x)*(448*a*b
 + 256*a^2 + 165*b^2))/(323*d) + (13728*b^2*exp(16*c + 16*d*x)*(8*a + 11*b)
)/(2261*d) - (128*b^2*exp(2*c + 2*d*x)*(96*a + 55*b))/(6783*d) - (5632*b^2*
exp(14*c + 14*d*x)*(96*a + 55*b))/(2261*d))/(91*exp(4*c + 4*d*x) - 14*exp(2
*c + 2*d*x) - 364*exp(6*c + 6*d*x) + 1001*exp(8*c + 8*d*x) - 2002*exp(10*c
+ 10*d*x) + 3003*exp(12*c + 12*d*x) - 3432*exp(14*c + 14*d*x) + 3003*exp(16
*c + 16*d*x) - 2002*exp(18*c + 18*d*x) + 1001*exp(20*c + 20*d*x) - 364*exp(
22*c + 22*d*x) + 91*exp(24*c + 24*d*x) - 14*exp(26*c + 26*d*x) + exp(28*c +
 28*d*x) + 1) + ((128*b^3*exp(4*c + 4*d*x))/(323*d) - (160*b^3*exp(6*c + 6*
d*x))/(969*d) + (128*b^2*(96*a + 55*b))/(88179*d) - (64*b^2*exp(2*c + 2*d*x
)*(8*a + 11*b))/(2261*d))/(7*exp(2*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*ex
p(6*c + 6*d*x) - 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c +
 12*d*x) + exp(14*c + 14*d*x) - 1) - ((128*(840...
```

3.229 $\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{a\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{7/4}d}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{7/4}d}+\frac{\cosh(c+dx)}{bd}-\frac{\cosh^3(c+dx)}{3bd}$$

```
[Out] cosh(d*x+c)/b/d-1/3*cosh(d*x+c)^3/b/d-1/2*a*arctan(b^(1/4)*cosh(d*x+c)/(a^(
1/2)-b^(1/2))^(1/2))/b^(7/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*a*arctanh(b^(1/4
)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/b^(7/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3294, 1184, 1180, 211, 214}

$$-\frac{a\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cosh^3(c+dx)}{3bd}+\frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/2*(a*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(Sqrt[Sqrt
[a] - Sqrt[b]]*b^(7/4)*d) + (a*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a]
 + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(7/4)*d) + Cosh[c + d*x]/(b*d) -
 Cosh[c + d*x]^3/(3*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{x^2}{b}+\frac{a-ax^2}{b\left(a-b+2bx^2-bx^4\right)}\right)\,dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)}{bd}-\frac{\cosh^3(c+dx)}{3bd}-\frac{\text{Subst}\left(\int \frac{a-ax^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{bd} \\
&= \frac{\cosh(c+dx)}{bd}-\frac{\cosh^3(c+dx)}{3bd}+\frac{a\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx\right.}{2bd} \\
&= -\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{7/4}d}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{7/4}d}+\frac{\cosh(c+dx)}{bd}-
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.33, size = 390, normalized size = 2.64

$$
\frac{18\cosh(c+dx)-2\cosh(3(c+dx))-3a\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-c-dx-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)+3c\#1^2+3dx\#1^2+6\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2-3c\#1^4-3dx\#1^4-6\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4+c\#1^6+dx\#1^6+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{24bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (18*Cosh[c + d*x] - 2*Cosh[3*(c + d*x)] - 3*a*RootSum[b - 4*b*#1^2 - 16*a*#
1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2]
 - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 3*c*#
1^2 + 3*d*x*#1^2 + 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
 d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 3*c*#1^4 - 3*d*x*#1^4 - 6*Log[-C
osh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x
)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*
x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#
1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(24*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(110) = 220$.
time = 1.69, size = 263, normalized size = 1.78

method	result
risch	$-\frac{e^{3dx+3c}}{24bd}+\frac{3\,e^{dx+c}}{8bd}+\frac{3\,e^{-dx-c}}{8bd}-\frac{e^{-3dx-3c}}{24bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(256a\,b^{7}d^{4}-256b^{8}d^{4}\right)\_Z^{4}+32a^{2}b^{4}d^{2}\_Z^{2}-a^{4}\right)}\right. -$
derivativedivides	$-\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-$
default	$-\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/b
/(tanh(1/2*d*x+1/2*c)+1)+1/3/b/(tanh(1/2*d*x+1/2*c)-1)^3+1/2/b/(tanh(1/2*d*
x+1/2*c)-1)^2-1/2/b/(tanh(1/2*d*x+1/2*c)-1)+8*a^2/b*(-1/16*(a*b)^(1/2)/a/b/
(-(a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(
1/2)+2*a)/(-(a*b)^(1/2)*a-a*b)^(1/2))-1/16*(a*b)^(1/2)/a/b/((a*b)^(1/2)*a-a
*b)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(
1/2)*a-a*b)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/24*(e^(6*d*x + 6*c) - 9*e^(4*d*x + 4*c) - 9*e^(2*d*x + 2*c) + 1)*e^(-3*d
*x - 3*c)/(b*d) - 1/128*integrate(256*(a*e^(7*d*x + 7*c) - 3*a*e^(5*d*x + 5
*c) + 3*a*e^(3*d*x + 3*c) - a*e^(d*x + c))/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(
6*d*x + 6*c) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*
c))*e^(4*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. 2(110) = 220.
time = 0.45, size = 1617, normalized size = 10.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/24*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6
+ 3*(5*cosh(d*x + c)^2 - 3)*sinh(d*x + c)^4 - 9*cosh(d*x + c)^4 + 4*(5*cosh
(d*x + c)^3 - 9*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 - 18*
cosh(d*x + c)^2 - 3)*sinh(d*x + c)^2 - 6*(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(
d*x + c)^2*sinh(d*x + c) + 3*b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d
*x + c)^3)*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4
)) + a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c
)*sinh(d*x + c) + a^3*sinh(d*x + c)^2 + a^3 + 2*(a^2*b^2*d*cosh(d*x + c) +
a^2*b^2*d*sinh(d*x + c) - ((a*b^5 - b^6)*d^3*cosh(d*x + c) + (a*b^5 - b^6)*
d^3*sinh(d*x + c))*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3
 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)
*d^2))) + 6*(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*
b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d*x + c)^3)*sqrt(-((a*b^3 - b^
4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2)
)*log(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*
x + c)^2 + a^3 - 2*(a^2*b^2*d*cosh(d*x + c) + a^2*b^2*d*sinh(d*x + c) - ((a
*b^5 - b^6)*d^3*cosh(d*x + c) + (a*b^5 - b^6)*d^3*sinh(d*x + c))*sqrt(a^5/(
(a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^
7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) - 6*(b*d*cosh(d*x + c
)^3 + 3*b*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*b*d*cosh(d*x + c)*sinh(d*x +
c)^2 + b*d*sinh(d*x + c)^3)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*
a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cosh(d*x + c)^2 + 2*
```

```
a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2 + a^3 + 2*(a^2*b^2*d*
cosh(d*x + c) + a^2*b^2*d*sinh(d*x + c) + ((a*b^5 - b^6)*d^3*cosh(d*x + c)
+ (a*b^5 - b^6)*d^3*sinh(d*x + c))*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)
))*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)
/((a*b^3 - b^4)*d^2))) + 6*(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(d*x + c)^2*sin
h(d*x + c) + 3*b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d*x + c)^3)*sqr
t(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b
^3 - b^4)*d^2))*log(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c)
 + a^3*sinh(d*x + c)^2 + a^3 - 2*(a^2*b^2*d*cosh(d*x + c) + a^2*b^2*d*sinh(
d*x + c) + ((a*b^5 - b^6)*d^3*cosh(d*x + c) + (a*b^5 - b^6)*d^3*sinh(d*x +
c))*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(((a*b^3 - b^4)*d^2*sqrt
(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))) - 9*cosh
(d*x + c)^2 + 6*(cosh(d*x + c)^5 - 6*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sin
h(d*x + c) + 1)/(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(d*x + c)^2*sinh(d*x + c)
+ 3*b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 753 vs. 2(110) = 220.
time = 0.59, size = 753, normalized size = 5.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

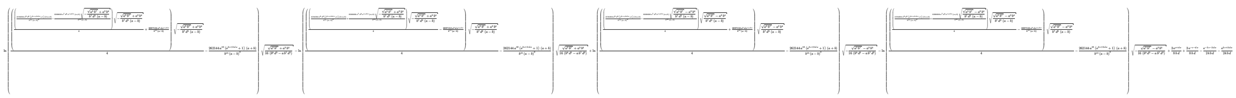
```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/24*(12*((4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 5*sqrt(a*b)*sqrt(-b^
2 + sqrt(a*b)*b)*a*b)*b^2 + (4*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^2 + 5*sqrt(-b
^2 + sqrt(a*b)*b)*a*b^3)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sq
rt(-(b^4 - sqrt(b^8 + (a*b^3 - b^4)*b^4))/b^4))/(4*a^2*b^5 + a*b^6 - 5*b^7)
 - 6*(4*sqrt(a*b)*sqrt(b^2 + sqrt(a*b)*b)*a*b^4 - 3*sqrt(a*b)*sqrt(b^2 + sq
rt(a*b)*b)*b^5 + (4*sqrt(b^2 + sqrt(a*b)*b)*a^2*b - 3*sqrt(b^2 + sqrt(a*b)*
b)*a*b^2 + 4*sqrt(a*b)*sqrt(b^2 + sqrt(a*b)*b)*a^2 - 3*sqrt(a*b)*sqrt(b^2 +
 sqrt(a*b)*b)*a*b)*b^2*abs(b) - (4*sqrt(b^2 + sqrt(a*b)*b)*a^2*b^2 - 3*sqrt
(b^2 + sqrt(a*b)*b)*a*b^3 + 4*sqrt(a*b)*sqrt(b^2 + sqrt(a*b)*b)*a*b^2 - 3*s
qrt(a*b)*sqrt(b^2 + sqrt(a*b)*b)*b^3)*b^2 - (4*sqrt(b^2 + sqrt(a*b)*b)*a^2*
b^3 - 3*sqrt(b^2 + sqrt(a*b)*b)*a*b^4)*abs(b))*log(2*sqrt((b^4 + sqrt(b^8 +
```

```
 (a*b^3 - b^4)*b^4))/b^4) + e^(d*x + c) + e^(-d*x - c))/(4*a^2*b^6 - 7*a*b^
7 + 3*b^8) - 6*((4*sqrt(a*b)*sqrt(b^2 + sqrt(a*b)*b)*a^2 - 3*sqrt(a*b)*sqrt
(b^2 + sqrt(a*b)*b)*a*b)*b^2 + (4*sqrt(b^2 + sqrt(a*b)*b)*a^2*b^2 - 3*sqrt(
b^2 + sqrt(a*b)*b)*a*b^3)*abs(b))*log(abs(-2*sqrt((b^4 + sqrt(b^8 + (a*b^3
- b^4)*b^4))/b^4) + e^(d*x + c) + e^(-d*x - c)))/(4*a^2*b^5 - 7*a*b^6 + 3*b
^7) + (b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*b^2*(e^(d*x + c) + e^(-d*x -
 c)))/b^3)/d
```

Mupad [B]

time = 9.80, size = 1124, normalized size = 7.59



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log((((((4194304*a^8*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^11*(a - b)^2)
 - (8388608*a^7*d^3*exp(c + d*x)*(a + b)*(-((a^5*b^7)^(1/2) + a^2*b^4)/(b^7
*d^2*(a - b)))^(1/2))/(b^10*(a - b)))*(-((a^5*b^7)^(1/2) + a^2*b^4)/(b^7*d^
2*(a - b)))^(1/2))/4 + (2097152*a^9*d*exp(c + d*x))/(b^13*(a - b)))*(-((a^5
*b^7)^(1/2) + a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (262144*a^10*(exp(2*c
+ 2*d*x) + 1)*(a + b))/(b^15*(a - b)^2))*(((a^5*b^7)^(1/2) + a^2*b^4)/(16*(
b^8*d^2 - a*b^7*d^2)))^(1/2) - log((((((4194304*a^8*d^2*(exp(2*c + 2*d*x) +
 1)*(3*a + b))/(b^11*(a - b)^2) + (8388608*a^7*d^3*exp(c + d*x)*(a + b)*(-(
(a^5*b^7)^(1/2) + a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/(b^10*(a - b)))*(-((a^
5*b^7)^(1/2) + a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (2097152*a^9*d*exp(c
+ d*x))/(b^13*(a - b)))*(-((a^5*b^7)^(1/2) + a^2*b^4)/(b^7*d^2*(a - b)))^(1
/2))/4 - (262144*a^10*(exp(2*c + 2*d*x) + 1)*(a + b))/(b^15*(a - b)^2))*(((
a^5*b^7)^(1/2) + a^2*b^4)/(16*(b^8*d^2 - a*b^7*d^2)))^(1/2) + log((((((4194
304*a^8*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^11*(a - b)^2) - (8388608*a
^7*d^3*exp(c + d*x)*(a + b)*(((a^5*b^7)^(1/2) - a^2*b^4)/(b^7*d^2*(a - b)))
^(1/2))/(b^10*(a - b)))*(((a^5*b^7)^(1/2) - a^2*b^4)/(b^7*d^2*(a - b)))^(1/
2))/4 + (2097152*a^9*d*exp(c + d*x))/(b^13*(a - b)))*(((a^5*b^7)^(1/2) - a^
2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (262144*a^10*(exp(2*c + 2*d*x) + 1)*(a
 + b))/(b^15*(a - b)^2))*(-((a^5*b^7)^(1/2) - a^2*b^4)/(16*(b^8*d^2 - a*b^7
*d^2)))^(1/2) - log((((((4194304*a^8*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/
(b^11*(a - b)^2) + (8388608*a^7*d^3*exp(c + d*x)*(a + b)*(((a^5*b^7)^(1/2)
- a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/(b^10*(a - b)))*(((a^5*b^7)^(1/2) - a^
2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (2097152*a^9*d*exp(c + d*x))/(b^13*(a
- b)))*(((a^5*b^7)^(1/2) - a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (262144*a
^10*(exp(2*c + 2*d*x) + 1)*(a + b))/(b^15*(a - b)^2))*(-((a^5*b^7)^(1/2) -
a^2*b^4)/(16*(b^8*d^2 - a*b^7*d^2)))^(1/2) + (3*exp(c + d*x))/(8*b*d) + (3*
exp(- c - d*x))/(8*b*d) - exp(- 3*c - 3*d*x)/(24*b*d) - exp(3*c + 3*d*x)/(2
4*b*d)
```

3.230 $\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{5/4}d}+\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{5/4}d}-\frac{\cosh(c+dx)}{bd}$$

```
[Out] -cosh(d*x+c)/b/d+1/2*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*a^
(1/2)/b^(5/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*arctanh(b^(1/4)*cosh(d*x+c)/(a^
(1/2)+b^(1/2))^(1/2))*a^(1/2)/b^(5/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3294, 1184, 1107, 211, 214}

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (Sqrt[a]*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[S
qrt[a] - Sqrt[b]]*b^(5/4)*d) + (Sqrt[a]*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqr
t[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)*d) - Cosh[c + d*x
]/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
```

```
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{a}{b(a-b+2bx^2-bx^4)}\right)dx, x, \cosh(c+dx)\right)}{d}\\
&= -\frac{\cosh(c+dx)}{bd}+\frac{a\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{bd}\\
&= -\frac{\cosh(c+dx)}{bd}-\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\,\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2\sqrt{b}\,d}+\sqrt{a}\,\text{Su}\\
&= \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{5/4}d}+\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{5/4}d}-\frac{\cosh(c+d}{bd}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.20, size = 235, normalized size = 1.69

$$-\frac{2\cosh(c+dx)+a\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-c\#1-dx\#1-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1+c\#1^3+dx\#1^3+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^3}{-b-8a\#1^2+3b\#1^2-3b\#1^4+b\#1^6}\&\right]}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/2*(2*Cosh[c + d*x] + a*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b
*#1^6 + b*#1^8 & , (-(c*#1) - d*x*#1 - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1 + c*#1^3 + d*x*#
1^3 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*#1^6)
 & ])/(b*d)
```

Maple [A]

time = 1.62, size = 174, normalized size = 1.25

method	result
risch	$-\frac{e^{dx+c}}{2bd}-\frac{e^{-dx-c}}{2bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(256a\,b^5d^4-256b^6d^4\right)\_Z^4+32a\,d^2\_Z^2b^3-a^2\right)}\_R\ln\left(e^{2dx+2c}+\left(\left(\frac{128b}{a}\right.\right.\right.\right.$
derivativedivides	$\frac{\frac{2a^2\left(\frac{\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}-2a}{4\sqrt{\sqrt{ab}\,a-ab}}\right)}{4a\sqrt{\sqrt{ab}\,a-ab}}-\frac{\arctan\left(\frac{-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}+2a}{4\sqrt{-\sqrt{ab}\,a-ab}}\right)}{4a\sqrt{-\sqrt{ab}\,a-ab}}\right)}{b}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ }{b\left(\mathrm{ta}\right.}}{d}$
default	$\frac{\frac{2a^2\left(\frac{\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}-2a}{4\sqrt{\sqrt{ab}\,a-ab}}\right)}{4a\sqrt{\sqrt{ab}\,a-ab}}-\frac{\arctan\left(\frac{-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}+2a}{4\sqrt{-\sqrt{ab}\,a-ab}}\right)}{4a\sqrt{-\sqrt{ab}\,a-ab}}\right)}{b}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ }{b\left(\mathrm{ta}\right.}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2/b*(1/4/a/((a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+
1/2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2))-1/4/a/(-(a*b)^(1/2)*
a-a*b)^(1/2)*arctan(1/4*(-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(a
*b)^(1/2)*a-a*b)^(1/2)))-1/b/(tanh(1/2*d*x+1/2*c)+1)+1/b/(tanh(1/2*d*x+1/2*
c)-1))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/32*integrate(256*(a*e^(5*
d*x + 5*c) - a*e^(3*d*x + 3*c))/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6*c
) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4*d*
x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1247 vs. $2(99) = 198$.
time = 0.43, size = 1247, normalized size = 8.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*((b*d*cosh(d*x + c) + b*d*sinh(d*x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(
a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))*log(a^2*cosh
(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2
 + 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) - ((a*b^4 - b^5)*d^3*co
sh(d*x + c) + (a*b^4 - b^5)*d^3*sinh(d*x + c))*sqrt(a^3/((a^2*b^5 - 2*a*b^6
 + b^7)*d^4)))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)
*d^4)) + a)/((a*b^2 - b^3)*d^2))) - (b*d*cosh(d*x + c) + b*d*sinh(d*x + c))
*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((
a*b^2 - b^3)*d^2))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x +
 c) + a^2*sinh(d*x + c)^2 + a^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d
*x + c) - ((a*b^4 - b^5)*d^3*cosh(d*x + c) + (a*b^4 - b^5)*d^3*sinh(d*x + c
))*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)))*sqrt(-((a*b^2 - b^3)*d^2*sqrt
(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))) + (b*d*cos
h(d*x + c) + b*d*sinh(d*x + c))*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5
- 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(a^2*cosh(d*x + c)^2 +
2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2 + 2*(a^2*b*d*
cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a*b^4 - b^5)*d^3*cosh(d*x + c) +
(a*b^4 - b^5)*d^3*sinh(d*x + c))*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)))
*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a
*b^2 - b^3)*d^2))) - (b*d*cosh(d*x + c) + b*d*sinh(d*x + c))*sqrt(((a*b^2 -
 b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2
))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d
*x + c)^2 + a^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a*b^
4 - b^5)*d^3*cosh(d*x + c) + (a*b^4 - b^5)*d^3*sinh(d*x + c))*sqrt(a^3/((a^
2*b^5 - 2*a*b^6 + b^7)*d^4)))*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 -
2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))) - 2*cosh(d*x + c)^2 - 4*cos
h(d*x + c)*sinh(d*x + c) - 2*sinh(d*x + c)^2 - 2)/(b*d*cosh(d*x + c) + b*d*
sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(99) = 198.
time = 0.58, size = 525, normalized size = 3.78

$$\frac{\frac{2\left(4\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,ab^2+5\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,b^3+\left(4\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b+5\sqrt{-b^2+\sqrt{ab}\,b}\,ab^2\right)|b|\right)\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{b^2-\sqrt{b^4+(ab-b^2)b^2}}{b^2}}}\right)}{4a^2b^4+ab^5-5b^6}-\frac{\left(4\sqrt{ab}\sqrt{b^2+\sqrt{ab}\,b}\,ab^2-3\sqrt{ab}\sqrt{b^2+\sqrt{ab}\,b}\,b^3-\left(4\sqrt{b^2+\sqrt{ab}\,b}\,a^2b-3\sqrt{b^2+\sqrt{ab}\,b}\,ab^2\right)|b|\right)\log\left(2\sqrt{\frac{b^2+\sqrt{b^4+(ab-b^2)b^2}}{b^2}}+e^{(dx+c)}+e^{(-dx-c)}\right)}{4a^2b^4-7ab^5+3b^6}+\frac{\left(4\sqrt{ab}\sqrt{b^2+\sqrt{ab}\,b}\,ab^2-3\sqrt{ab}\sqrt{b^2+\sqrt{ab}\,b}\,b^3-\left(4\sqrt{b^2+\sqrt{ab}\,b}\,a^2b-3\sqrt{b^2+\sqrt{ab}\,b}\,ab^2\right)|b|\right)\log\left(\left|-2\sqrt{\frac{b^2+\sqrt{b^4+(ab-b^2)b^2}}{b^2}}+e^{(dx+c)}+e^{(-dx-c)}\right|\right)}{4a^2b^4-7ab^5+3b^6}-\frac{2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/4*(2*(4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^2 + 5*sqrt(a*b)*sqrt(-b^2
+ sqrt(a*b)*b)*b^3 + (4*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b + 5*sqrt(-b^2 + sqrt
(a*b)*b)*a*b^2)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(b^2
- sqrt(b^4 + (a*b - b^2)*b^2))/b^2))/(4*a^2*b^4 + a*b^5 - 5*b^6) - (4*sqrt(
a*b)*sqrt(b^2 + sqrt(a*b)*b)*a*b^2 - 3*sqrt(a*b)*sqrt(b^2 + sqrt(a*b)*b)*b^
3 - (4*sqrt(b^2 + sqrt(a*b)*b)*a^2*b - 3*sqrt(b^2 + sqrt(a*b)*b)*a*b^2)*abs
(b))*log(2*sqrt((b^2 + sqrt(b^4 + (a*b - b^2)*b^2))/b^2) + e^(d*x + c) + e^
(-d*x - c))/(4*a^2*b^4 - 7*a*b^5 + 3*b^6) + (4*sqrt(a*b)*sqrt(b^2 + sqrt(a*
b)*b)*a*b^2 - 3*sqrt(a*b)*sqrt(b^2 + sqrt(a*b)*b)*b^3 - (4*sqrt(b^2 + sqrt(
a*b)*b)*a^2*b - 3*sqrt(b^2 + sqrt(a*b)*b)*a*b^2)*abs(b))*log(abs(-2*sqrt((b
^2 + sqrt(b^4 + (a*b - b^2)*b^2))/b^2) + e^(d*x + c) + e^(-d*x - c)))/(4*a^
2*b^4 - 7*a*b^5 + 3*b^6) - 2*(e^(d*x + c) + e^(-d*x - c))/b)/d
```

Mupad [**B**]
time = 7.85, size = 1046, normalized size = 7.53

$$\ln\left(\frac{\left(\frac{\left(\frac{4194304\,a^6d^2\left(e^{2c+2dx}+1\right)(3a+b)}{b^9(a-b)^2}+\frac{16777216\,a^6d^3e^{c+dx}\sqrt{-\frac{\sqrt{a^3b^5}+ab^3}{b^5d^2(a-b)}}}{b^8(a-b)}\right)\sqrt{-\frac{\sqrt{a^3b^5}+ab^3}{b^5d^2(a-b)}}}{4}-\frac{2097152\,a^7d\,e^{c+dx}}{b^{11}(a-b)}\right)\sqrt{-\frac{\sqrt{a^3b^5}+ab^3}{b^5d^2(a-b)}}}{4}-\frac{262144\,a^7\left(e^{2c+2dx}+1\right)(a+b)}{b^{12}(a-b)^2}\right)\sqrt{\frac{\sqrt{a^3b^5}+ab^3}{16\left(b^6d^2-ab^5d^2\right)}}-\ln\left(\frac{\left(\frac{\left(\frac{4194304\,a^6d^2\left(e^{2c+2dx}+1\right)(3a+b)}{b^9(a-b)^2}-\frac{16777216\,a^6d^3e^{c+dx}\sqrt{-\frac{\sqrt{a^3b^5}+ab^3}{b^5d^2(a-b)}}}{b^8(a-b)}\right)\sqrt{-\frac{\sqrt{a^3b^5}+ab^3}{b^5d^2(a-b)}}}{4}+\frac{2097152\,a^7d\,e^{c+dx}}{b^{11}(a-b)}\right)\sqrt{-\frac{\sqrt{a^3b^5}+ab^3}{b^5d^2(a-b)}}}{4}-\frac{262144\,a^7\left(e^{2c+2dx}+1\right)(a+b)}{b^{12}(a-b)^2}\right)\sqrt{\frac{\sqrt{a^3b^5}+ab^3}{16\left(b^6d^2-ab^5d^2\right)}}-\ln\left(\frac{\left(\frac{\left(\frac{4194304\,a^6d^2\left(e^{2c+2dx}+1\right)(3a+b)}{b^9(a-b)^2}-\frac{16777216\,a^6d^3e^{c+dx}\sqrt{\frac{\sqrt{a^3b^5}-ab^3}{b^5d^2(a-b)}}}{b^8(a-b)}\right)\sqrt{\frac{\sqrt{a^3b^5}-ab^3}{b^5d^2(a-b)}}}{4}+\frac{2097152\,a^7d\,e^{c+dx}}{b^{11}(a-b)}\right)\sqrt{\frac{\sqrt{a^3b^5}-ab^3}{b^5d^2(a-b)}}}{4}-\frac{262144\,a^7\left(e^{2c+2dx}+1\right)(a+b)}{b^{12}(a-b)^2}\right)\sqrt{-\frac{\sqrt{a^3b^5}-ab^3}{16\left(b^6d^2-ab^5d^2\right)}}+\ln\left(\frac{\left(\frac{\left(\frac{4194304\,a^6d^2\left(e^{2c+2dx}+1\right)(3a+b)}{b^9(a-b)^2}+\frac{16777216\,a^6d^3e^{c+dx}\sqrt{\frac{\sqrt{a^3b^5}-ab^3}{b^5d^2(a-b)}}}{b^8(a-b)}\right)\sqrt{\frac{\sqrt{a^3b^5}-ab^3}{b^5d^2(a-b)}}}{4}-\frac{2097152\,a^7d\,e^{c+dx}}{b^{11}(a-b)}\right)\sqrt{\frac{\sqrt{a^3b^5}-ab^3}{b^5d^2(a-b)}}}{4}-\frac{262144\,a^7\left(e^{2c+2dx}+1\right)(a+b)}{b^{12}(a-b)^2}\right)\sqrt{-\frac{\sqrt{a^3b^5}-ab^3}{16\left(b^6d^2-ab^5d^2\right)}}-\frac{e^{c+dx}}{2bd}-\frac{e^{-c-dx}}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log((((((4194304*a^6*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^9*(a - b)^2)
+ (16777216*a^6*d^3*exp(c + d*x)*(-((a^3*b^5)^(1/2) + a*b^3)/(b^5*d^2*(a -
b)))^(1/2))/(b^8*(a - b)))*(-((a^3*b^5)^(1/2) + a*b^3)/(b^5*d^2*(a - b)))^(
1/2))/4 - (2097152*a^7*d*exp(c + d*x))/(b^11*(a - b)))*(-((a^3*b^5)^(1/2) +
 a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 - (262144*a^7*(exp(2*c + 2*d*x) + 1)*(a
```

```
 + b))/(b^12*(a - b)^2))*(((a^3*b^5)^(1/2) + a*b^3)/(16*(b^6*d^2 - a*b^5*d^
2)))^(1/2) - log((((((4194304*a^6*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^
9*(a - b)^2) - (16777216*a^6*d^3*exp(c + d*x)*(-((a^3*b^5)^(1/2) + a*b^3)/(
b^5*d^2*(a - b)))^(1/2))/(b^8*(a - b)))*(-((a^3*b^5)^(1/2) + a*b^3)/(b^5*d^
2*(a - b)))^(1/2))/4 + (2097152*a^7*d*exp(c + d*x))/(b^11*(a - b)))*(-((a^3
*b^5)^(1/2) + a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 - (262144*a^7*(exp(2*c + 2
*d*x) + 1)*(a + b))/(b^12*(a - b)^2))*(((a^3*b^5)^(1/2) + a*b^3)/(16*(b^6*d
^2 - a*b^5*d^2)))^(1/2) - log((((((4194304*a^6*d^2*(exp(2*c + 2*d*x) + 1)*(
3*a + b))/(b^9*(a - b)^2) - (16777216*a^6*d^3*exp(c + d*x)*(((a^3*b^5)^(1/2
) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/(b^8*(a - b)))*(((a^3*b^5)^(1/2) - a*b
^3)/(b^5*d^2*(a - b)))^(1/2))/4 + (2097152*a^7*d*exp(c + d*x))/(b^11*(a - b
)))*(((a^3*b^5)^(1/2) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 - (262144*a^7*(e
xp(2*c + 2*d*x) + 1)*(a + b))/(b^12*(a - b)^2))*(-((a^3*b^5)^(1/2) - a*b^3)
/(16*(b^6*d^2 - a*b^5*d^2)))^(1/2) + log((((((4194304*a^6*d^2*(exp(2*c + 2*
d*x) + 1)*(3*a + b))/(b^9*(a - b)^2) + (16777216*a^6*d^3*exp(c + d*x)*(((a^
3*b^5)^(1/2) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/(b^8*(a - b)))*(((a^3*b^5)^
(1/2) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 - (2097152*a^7*d*exp(c + d*x))/(
b^11*(a - b)))*(((a^3*b^5)^(1/2) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 - (26
2144*a^7*(exp(2*c + 2*d*x) + 1)*(a + b))/(b^12*(a - b)^2))*(-((a^3*b^5)^(1/
2) - a*b^3)/(16*(b^6*d^2 - a*b^5*d^2)))^(1/2) - exp(c + d*x)/(2*b*d) - exp(
- c - d*x)/(2*b*d)
```

3.231 $\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=115

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/4}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/4}d}$$

```
[Out] -1/2*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(3/4)/d/(a^(1/2)
-b^(1/2))^(1/2)+1/2*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/b^
(3/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3294, 1180, 211, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/2*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(Sqrt[Sqrt[a]
- Sqrt[b]]*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt
[b]]]/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{a-b+2bx^2-bx^4}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx,x,\cosh(c+dx)\right)}{2d}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\,dx,x\right.}{2d}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/4}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/4}d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 365, normalized size = 3.17

$$-\frac{\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-c-dx-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)+3c\#1^2+3dx\#1^2+6\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2-3c\#1^4-3dx\#1^4-6\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4+c\#1^6+dx\#1^6+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/8*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-
c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*
#1 - Sinh[(c + d*x)/2]*#1] + 3*c*#1^2 + 3*d*x*#1^2 + 6*Log[-Cosh[(c + d*x)/
2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2
- 3*c*#1^4 - 3*d*x*#1^4 - 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Co
sh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log
[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ]/d
```

Maple [**A**]
time = 1.41, size = 148, normalized size = 1.29

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(-1+\left(256a\,b^3d^4-256b^4d^4\right)\_Z^4+32b^2d^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2dx+2c}+\left(\left(128a\,b^2d^3-128b^3d^3\right)\_R^3\right.\right.$
derivativedivides	$\frac{8a\left(-\frac{\sqrt{ab}\,\arctan\left(\frac{-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}+2a}{4\sqrt{-\sqrt{ab}\,a-ab}}\right)}{16ab\sqrt{-\sqrt{ab}\,a-ab}}-\frac{\sqrt{ab}\,\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}-2a}{4\sqrt{\sqrt{ab}\,a-ab}}\right)}{16ab\sqrt{\sqrt{ab}\,a-ab}}\right)}{d}$
default	$\frac{8a\left(-\frac{\sqrt{ab}\,\arctan\left(\frac{-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}+2a}{4\sqrt{-\sqrt{ab}\,a-ab}}\right)}{16ab\sqrt{-\sqrt{ab}\,a-ab}}-\frac{\sqrt{ab}\,\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}-2a}{4\sqrt{\sqrt{ab}\,a-ab}}\right)}{16ab\sqrt{\sqrt{ab}\,a-ab}}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 8/d*a*(-1/16*(a*b)^(1/2)/a/b/(-(a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(-2*a*ta
nh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(a*b)^(1/2)*a-a*b)^(1/2))-1/16*(a*
b)^(1/2)/a/b/((a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^
2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sinh(d*x + c)^3/(b*sinh(d*x + c)^4 - a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 975 vs. 2(79) = 158.
time = 0.41, size = 975, normalized size = 8.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a
*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x + c) - ((a*b^2 - b^3)*d^3*
cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4
 + b^5)*d^4)))*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4
)) + 1)/((a*b - b^2)*d^2)) + 1) - 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b
^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(b*d*cosh(d*x + c) + b*d*s
inh(d*x + c) - ((a*b^2 - b^3)*d^3*cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*
x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)))*sqrt(-((a*b - b^2)*d^2*sqr
t(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2)) + 1) + 1/4*sqr
t(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2
)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x + c) + ((a*b^2 - b^3)*d^3*cosh(d*x
 + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*
d^4)))*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/(
(a*b - b^2)*d^2)) + 1) - 1/4*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b
^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x +
c) + ((a*b^2 - b^3)*d^3*cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sq
rt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)))*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b
^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2)) + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(79) = 158.
time = 0.54, size = 314, normalized size = 2.73

$$-\frac{\frac{\left(4\sqrt{-b^2-\sqrt{ab}\,b}\,ab+5\sqrt{-b^2-\sqrt{ab}\,b}\,b^2-4\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,a-5\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,b\right)|b|\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{b+\sqrt{(a-b)b+b^2}}{b}}}\right)}{4a^2b^3+ab^4-5b^5}+\frac{\left(4\sqrt{-b^2+\sqrt{ab}\,b}\,ab+5\sqrt{-b^2+\sqrt{ab}\,b}\,b^2+4\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,a+5\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,b\right)|b|\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{b-\sqrt{(a-b)b+b^2}}{b}}}\right)}{4a^2b^3+ab^4-5b^5}}{2d}$$

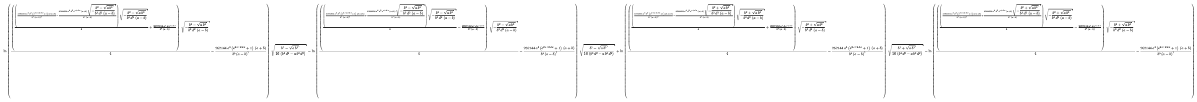
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*((4*sqrt(-b^2 - sqrt(a*b)*b)*a*b + 5*sqrt(-b^2 - sqrt(a*b)*b)*b^2 - 4*
sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a - 5*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)
```

```
*b)*abs(b)*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(b + sqrt((a - b)*
b + b^2))/b))/(4*a^2*b^3 + a*b^4 - 5*b^5) + (4*sqrt(-b^2 + sqrt(a*b)*b)*a*b
 + 5*sqrt(-b^2 + sqrt(a*b)*b)*b^2 + 4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a
+ 5*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b)*abs(b)*arctan(1/2*(e^(d*x + c) +
e^(-d*x - c))/sqrt(-(b - sqrt((a - b)*b + b^2))/b))/(4*a^2*b^3 + a*b^4 - 5*
b^5))/d
```

Mupad [B]
time = 6.16, size = 975, normalized size = 8.48



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log((((((4194304*a^4*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^7*(a - b)^2)
- (8388608*a^4*d^3*exp(c + d*x)*(a + b)*(-(b^2 - (a*b^3)^(1/2))/(b^3*d^2*(a
 - b)))^(1/2))/(b^7*(a - b)))*(-(b^2 - (a*b^3)^(1/2))/(b^3*d^2*(a - b)))^(1
/2))/4 + (2097152*a^4*d*exp(c + d*x))/(b^8*(a - b)))*(-(b^2 - (a*b^3)^(1/2)
)/(b^3*d^2*(a - b)))^(1/2))/4 - (262144*a^4*(exp(2*c + 2*d*x) + 1)*(a + b))
/(b^9*(a - b)^2))*((b^2 - (a*b^3)^(1/2))/(16*(b^4*d^2 - a*b^3*d^2)))^(1/2)
- log((((((4194304*a^4*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^7*(a - b)^2
) + (8388608*a^4*d^3*exp(c + d*x)*(a + b)*(-(b^2 - (a*b^3)^(1/2))/(b^3*d^2*
(a - b)))^(1/2))/(b^7*(a - b)))*(-(b^2 - (a*b^3)^(1/2))/(b^3*d^2*(a - b)))^
(1/2))/4 - (2097152*a^4*d*exp(c + d*x))/(b^8*(a - b)))*(-(b^2 - (a*b^3)^(1/
2))/(b^3*d^2*(a - b)))^(1/2))/4 - (262144*a^4*(exp(2*c + 2*d*x) + 1)*(a + b
))/(b^9*(a - b)^2))*((b^2 - (a*b^3)^(1/2))/(16*(b^4*d^2 - a*b^3*d^2)))^(1/2
) + log((((((4194304*a^4*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^7*(a - b)
^2) - (8388608*a^4*d^3*exp(c + d*x)*(a + b)*(-(b^2 + (a*b^3)^(1/2))/(b^3*d^
2*(a - b)))^(1/2))/(b^7*(a - b)))*(-(b^2 + (a*b^3)^(1/2))/(b^3*d^2*(a - b))
)^(1/2))/4 + (2097152*a^4*d*exp(c + d*x))/(b^8*(a - b)))*(-(b^2 + (a*b^3)^(
1/2))/(b^3*d^2*(a - b)))^(1/2))/4 - (262144*a^4*(exp(2*c + 2*d*x) + 1)*(a +
 b))/(b^9*(a - b)^2))*((b^2 + (a*b^3)^(1/2))/(16*(b^4*d^2 - a*b^3*d^2)))^(1
/2) - log((((((4194304*a^4*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^7*(a -
b)^2) + (8388608*a^4*d^3*exp(c + d*x)*(a + b)*(-(b^2 + (a*b^3)^(1/2))/(b^3*
d^2*(a - b)))^(1/2))/(b^7*(a - b)))*(-(b^2 + (a*b^3)^(1/2))/(b^3*d^2*(a - b
)))^(1/2))/4 - (2097152*a^4*d*exp(c + d*x))/(b^8*(a - b)))*(-(b^2 + (a*b^3)
^(1/2))/(b^3*d^2*(a - b)))^(1/2))/4 - (262144*a^4*(exp(2*c + 2*d*x) + 1)*(a
 + b))/(b^9*(a - b)^2))*((b^2 + (a*b^3)^(1/2))/(16*(b^4*d^2 - a*b^3*d^2)))^
(1/2)
```

3.232 $\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt[4]{b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt[4]{b}\,d}$$

```
[Out] 1/2*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(1/4)/d/a^(1/2)/(
a^(1/2)-b^(1/2))^(1/2)+1/2*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1
/2))/b^(1/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3294, 1107, 211, 214}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\,d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\,d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqr
t[a] - Sqrt[b]]*b^(1/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
```

```
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2\sqrt{a}\,d} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-}\right.}{2\sqrt{}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt[4]{b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt[4]{b}\,d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.12, size = 221, normalized size = 1.77

$$
-\frac{\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-c\#1-dx\#1-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1+c\#1^3+dx\#1^3+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^3}{-b-8a\#1^2+3b\#1^2-3b\#1^4+b\#1^6}\&\right]}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/2*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-
(c*#1) - d*x*#1 - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1 + c*#1^3 + d*x*#1^3 + 2*Log[-Cosh[(c
+ d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1
]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*#1^6) & ]/d
```

Maple [A]
time = 1.71, size = 132, normalized size = 1.06

method	result
risch	$\sum_{\_R=\operatorname{RootOf}\left(-1+\left(256b\,d^4a^3-256a^2b^2d^4\right)\_Z^4+32a\,d^2\_Z^2b\right)} \_R\ln\left(e^{2dx+2c}+\left(\left(128a^2b\,d^3-128a\,b^2d^3\right)\right.\right.$
derivativedivides	$\frac{2a\left(\frac{\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}-2a}{4\sqrt{\sqrt{ab}\,a-ab}}\right)}{4a\sqrt{\sqrt{ab}\,a-ab}}-\frac{\arctan\left(\frac{-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}+2a}{4\sqrt{-\sqrt{ab}\,a-ab}}\right)}{4a\sqrt{-\sqrt{ab}\,a-ab}}\right)}{d}$
default	$\frac{2a\left(\frac{\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}-2a}{4\sqrt{\sqrt{ab}\,a-ab}}\right)}{4a\sqrt{\sqrt{ab}\,a-ab}}-\frac{\arctan\left(\frac{-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}+2a}{4\sqrt{-\sqrt{ab}\,a-ab}}\right)}{4a\sqrt{-\sqrt{ab}\,a-ab}}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*a*(1/4/a/((a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^
2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2))-1/4/a/(-(a*b)^(1/2)*a-a*b)^
(1/2)*arctan(1/4*(-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(a*b)^(1/
2)*a-a*b)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sinh(d*x + c)/(b*sinh(d*x + c)^4 - a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 979 vs. 2(85) = 170.
time = 0.40, size = 979, normalized size = 7.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/(
(a^2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
```

```
h(d*x + c)^2 + 2*(a*d*cosh(d*x + c) + a*d*sinh(d*x + c) - ((a^2*b - a*b^2)*
d^3*cosh(d*x + c) + (a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a
^2*b^2 + a*b^3)*d^4)))*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 +
a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2)) + 1) - 1/4*sqrt(-((a^2 - a*b)*d^2*sqrt
(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(a*d*cosh(d*x
+ c) + a*d*sinh(d*x + c) - ((a^2*b - a*b^2)*d^3*cosh(d*x + c) + (a^2*b - a*
b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)))*sqrt(-((
a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*
d^2)) + 1) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*
d^4)) - 1)/((a^2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*
x + c) + sinh(d*x + c)^2 + 2*(a*d*cosh(d*x + c) + a*d*sinh(d*x + c) + ((a^2
*b - a*b^2)*d^3*cosh(d*x + c) + (a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/(
(a^3*b - 2*a^2*b^2 + a*b^3)*d^4)))*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2
*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2)) + 1) - 1/4*sqrt(((a^2 - a*b
)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2))*log
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(a*d
*cosh(d*x + c) + a*d*sinh(d*x + c) + ((a^2*b - a*b^2)*d^3*cosh(d*x + c) + (
a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4))
)*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^
2 - a*b)*d^2)) + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(85) = 170.
time = 0.54, size = 332, normalized size = 2.66

$$\frac{\frac{\left(4\sqrt{-b^2-\sqrt{ab}\,b}\,a^2b+5\sqrt{-b^2-\sqrt{ab}\,b}\,ab^2+4\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,ab+5\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,b^2\right)|b|\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{b+\sqrt{(a-b)b+b^2}}{b}}}\right)}{4a^3b^3+a^2b^4-5ab^5}+\frac{\left(4\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b+5\sqrt{-b^2+\sqrt{ab}\,b}\,ab^2+4\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,ab+5\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,b^2\right)|b|\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{b-\sqrt{(a-b)b+b^2}}{b}}}\right)}{4a^3b^3+a^2b^4-5ab^5}}{2d}$$

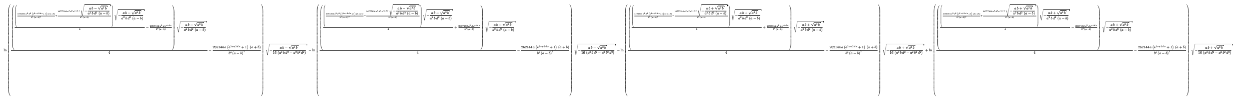
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/2*((4*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b + 5*sqrt(-b^2 - sqrt(a*b)*b)*a*b^2 +
 4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b + 5*sqrt(a*b)*sqrt(-b^2 - sqrt(a*
b)*b)*b^2)*abs(b)*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(b + sqrt((
```

```
a - b)*b + b^2))/b))/(4*a^3*b^3 + a^2*b^4 - 5*a*b^5) + (4*sqrt(-b^2 + sqrt(
a*b)*b)*a^2*b + 5*sqrt(-b^2 + sqrt(a*b)*b)*a*b^2 + 4*sqrt(a*b)*sqrt(-b^2 +
sqrt(a*b)*b)*a*b + 5*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^2)*abs(b)*arctan(
1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(b - sqrt((a - b)*b + b^2))/b))/(4*a
^3*b^3 + a^2*b^4 - 5*a*b^5))/d
```

Mupad [B]
time = 8.15, size = 1007, normalized size = 8.06



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log((((((4194304*a^2*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^5*(a - b)^2)
+ (16777216*a^3*d^3*exp(c + d*x)*(-(a*b - (a^3*b)^(1/2))/(a^2*b*d^2*(a - b)
))^(1/2))/(b^5*(a - b)))*(-(a*b - (a^3*b)^(1/2))/(a^2*b*d^2*(a - b)))^(1/2)
)/4 - (2097152*a^2*d*exp(c + d*x))/(b^6*(a - b)))*(-(a*b - (a^3*b)^(1/2))/(
a^2*b*d^2*(a - b)))^(1/2))/4 - (262144*a*(exp(2*c + 2*d*x) + 1)*(a + b))/(b
^6*(a - b)^2))*(-(a*b - (a^3*b)^(1/2))/(16*(a^3*b*d^2 - a^2*b^2*d^2)))^(1/2
) - log((((((4194304*a^2*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^5*(a - b)
^2) - (16777216*a^3*d^3*exp(c + d*x)*(-(a*b - (a^3*b)^(1/2))/(a^2*b*d^2*(a
- b)))^(1/2))/(b^5*(a - b)))*(-(a*b - (a^3*b)^(1/2))/(a^2*b*d^2*(a - b)))^(
1/2))/4 + (2097152*a^2*d*exp(c + d*x))/(b^6*(a - b)))*(-(a*b - (a^3*b)^(1/2
))/(a^2*b*d^2*(a - b)))^(1/2))/4 - (262144*a*(exp(2*c + 2*d*x) + 1)*(a + b)
)/(b^6*(a - b)^2))*(-(a*b - (a^3*b)^(1/2))/(16*(a^3*b*d^2 - a^2*b^2*d^2)))^
(1/2) - log((((((4194304*a^2*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^5*(a
- b)^2) - (16777216*a^3*d^3*exp(c + d*x)*(-(a*b + (a^3*b)^(1/2))/(a^2*b*d^2
*(a - b)))^(1/2))/(b^5*(a - b)))*(-(a*b + (a^3*b)^(1/2))/(a^2*b*d^2*(a - b)
))^(1/2))/4 + (2097152*a^2*d*exp(c + d*x))/(b^6*(a - b)))*(-(a*b + (a^3*b)^
(1/2))/(a^2*b*d^2*(a - b)))^(1/2))/4 - (262144*a*(exp(2*c + 2*d*x) + 1)*(a
+ b))/(b^6*(a - b)^2))*(-(a*b + (a^3*b)^(1/2))/(16*(a^3*b*d^2 - a^2*b^2*d^2
)))^(1/2) + log((((((4194304*a^2*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^5
*(a - b)^2) + (16777216*a^3*d^3*exp(c + d*x)*(-(a*b + (a^3*b)^(1/2))/(a^2*b
*d^2*(a - b)))^(1/2))/(b^5*(a - b)))*(-(a*b + (a^3*b)^(1/2))/(a^2*b*d^2*(a
- b)))^(1/2))/4 - (2097152*a^2*d*exp(c + d*x))/(b^6*(a - b)))*(-(a*b + (a^3
*b)^(1/2))/(a^2*b*d^2*(a - b)))^(1/2))/4 - (262144*a*(exp(2*c + 2*d*x) + 1)
*(a + b))/(b^6*(a - b)^2))*(-(a*b + (a^3*b)^(1/2))/(16*(a^3*b*d^2 - a^2*b^2
*d^2)))^(1/2)
```

3.233 $\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt[4]{b}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

```
[Out] -arctanh(cosh(d*x+c))/a/d-1/2*b^(1/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b
^(1/2))^(1/2))/a/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cosh
(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3294, 1184, 213, 1180, 211, 214}

$$-\frac{\sqrt[4]{b}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/2*(b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(a*S
qrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cosh[c + d*x]]/(a*d) + (b^(1/4)*ArcTanh
[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a*Sqrt[Sqrt[a] + Sqrt
[b]]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a(-1+x^2)}+\frac{b-bx^2}{a(a-b+2bx^2-bx^4)}\right)\,dx, x, \cosh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx\right.}{ad}\\
&= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{b\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2ad} + \frac{b}{}\\
&= -\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt{\sqrt{}}}\right.}{2a\sqrt{\sqrt{a}+}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.18, size = 385, normalized size = 2.83

$$\frac{8\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-b\mathrm{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-c-dx-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)+3c\#1^2+3dx\#1^2+6\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2-3c\#1^4-3dx\#1^4-6\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4+c\#1^6+dx\#1^6+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{8ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (8*Log[Tanh[(c + d*x)/2]] - b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 -
 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 3*c*#1^2 + 3*d*x*#1^2
 + 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Si
nh[(c + d*x)/2]*#1]*#1^2 - 3*c*#1^4 - 3*d*x*#1^4 - 6*Log[-Cosh[(c + d*x)/2]
 - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 +
c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c
+ d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 -
 3*b*#1^5 + b*#1^7) & ])/(8*a*d)
```

Maple [A]
time = 1.98, size = 164, normalized size = 1.21

method	result
risch	$\frac{\ln(e^{dx+c}-1)}{da}-\frac{\ln(e^{dx+c}+1)}{da}+2\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4096a^5d^4-4096a^4b\,d^4\right)\_Z^4+128a^2b\,d^2\_Z^2-b\right)}\_R\ln\left(e^{2dx+2}\right.\right.$
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+8b\left(-\frac{\sqrt{ab}\arctan\left(\frac{-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}+2a}{4\sqrt{-\sqrt{ab}\,a-ab}}\right)}{16ab\sqrt{-\sqrt{ab}\,a-ab}}-\frac{\sqrt{ab}\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}}{4\sqrt{\sqrt{ab}\,a-ab}}\right)}{16ab\sqrt{\sqrt{ab}\,a-ab}}\right.}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+8b\left(-\frac{\sqrt{ab}\arctan\left(\frac{-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}+2a}{4\sqrt{-\sqrt{ab}\,a-ab}}\right)}{16ab\sqrt{-\sqrt{ab}\,a-ab}}-\frac{\sqrt{ab}\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}}{4\sqrt{\sqrt{ab}\,a-ab}}\right)}{16ab\sqrt{\sqrt{ab}\,a-ab}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(tanh(1/2*d*x+1/2*c))+8*b*(-1/16*(a*b)^(1/2)/a/b/(-(a*b)^(1/2)*a
-a*b)^(1/2)*arctan(1/4*(-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(a*
b)^(1/2)*a-a*b)^(1/2))-1/16*(a*b)^(1/2)/a/b/((a*b)^(1/2)*a-a*b)^(1/2)*arcta
n(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/
2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

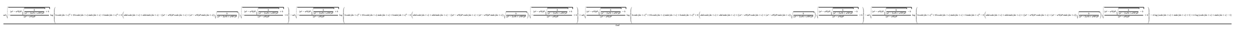
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(7*d*x + 7*c) - 3*b*e^(5*d*x + 5*c) + 3*b*e^(3*d*x + 3*c
) - b*e^(d*x + c))/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c) - 4*a*b*e^(
2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1067 vs. 2(100) = 200.
time = 0.43, size = 1067, normalized size = 7.85



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) +
 b)/((a^3 - a^2*b)*d^2))*log(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2 + 2*(a*b*d*cosh(d*x + c) + a*b*d*sinh(d*x + c) -
((a^4 - a^3*b)*d^3*cosh(d*x + c) + (a^4 - a^3*b)*d^3*sinh(d*x + c))*sqrt(b/
((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2
*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2)) + b) - a*d*sqrt(-((a^3 -
a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2)
)*log(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)
^2 - 2*(a*b*d*cosh(d*x + c) + a*b*d*sinh(d*x + c) - ((a^4 - a^3*b)*d^3*cosh
(d*x + c) + (a^4 - a^3*b)*d^3*sinh(d*x + c))*sqrt(b/((a^5 - 2*a^4*b + a^3*b
^2)*d^4)))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4))
 + b)/((a^3 - a^2*b)*d^2)) + b) + a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5
- 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b*cosh(d*x + c)^2
+ 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*(a*b*d*cosh(d*x +
 c) + a*b*d*sinh(d*x + c) + ((a^4 - a^3*b)*d^3*cosh(d*x + c) + (a^4 - a^3*b
)*d^3*sinh(d*x + c))*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*sqrt(((a^3 -
a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)
) + b) - a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)
) - b)/((a^3 - a^2*b)*d^2))*log(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(
d*x + c) + b*sinh(d*x + c)^2 - 2*(a*b*d*cosh(d*x + c) + a*b*d*sinh(d*x + c)
 + ((a^4 - a^3*b)*d^3*cosh(d*x + c) + (a^4 - a^3*b)*d^3*sinh(d*x + c))*sqrt
(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 -
 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)) + b) - 4*log(cosh(d*x +
 c) + sinh(d*x + c) + 1) + 4*log(cosh(d*x + c) + sinh(d*x + c) - 1))/(a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(100) = 200.
time = 0.47, size = 419, normalized size = 3.08

$$\frac{\frac{\left(\left(4\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,a+5\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,b\right)|a||b|-\left(4\sqrt{-b^2-\sqrt{ab}\,b}\,a^2b+5\sqrt{-b^2-\sqrt{ab}\,b}\,ab^2\right)|b|\right)\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{ab+\sqrt{a^2b^2+(a^2-ab)ab}}{ab}}}\right)}{4a^4b^2+a^3b^3-5a^2b^4}-\frac{\left(\left(4\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,a+5\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,b\right)|a||b|+\left(4\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b+5\sqrt{-b^2+\sqrt{ab}\,b}\,ab^2\right)|b|\right)\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{ab-\sqrt{a^2b^2+(a^2-ab)ab}}{ab}}}\right)}{4a^4b^2+a^3b^3-5a^2b^4}-\frac{\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{a}+\frac{\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/2*(((4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a + 5*sqrt(a*b)*sqrt(-b^2 - sqr
t(a*b)*b)*b)*abs(a)*abs(b) - (4*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b + 5*sqrt(-b^
2 - sqrt(a*b)*b)*a*b^2)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqr
t(-(a*b + sqrt(a^2*b^2 + (a^2 - a*b)*a*b))/(a*b)))/(4*a^4*b^2 + a^3*b^3 - 5
*a^2*b^4) - ((4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a + 5*sqrt(a*b)*sqrt(-b^
2 + sqrt(a*b)*b)*b)*abs(a)*abs(b) + (4*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b + 5*s
qrt(-b^2 + sqrt(a*b)*b)*a*b^2)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x -
c))/sqrt(-(a*b - sqrt(a^2*b^2 + (a^2 - a*b)*a*b))/(a*b)))/(4*a^4*b^2 + a^3*
b^3 - 5*a^2*b^4) - log(e^(d*x + c) + e^(-d*x - c) + 2)/a + log(e^(d*x + c)
+ e^(-d*x - c) - 2)/a)/d
```

Mupad [**B**]
time = 9.19, size = 1243, normalized size = 9.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)),x)
```

```
[Out] log((((((4294967296*a*d^2*(exp(2*c + 2*d*x) + 1)*(26*a*b - 49*a^2 + 15*b^2)
)/(b^6*(a - b)^3) + (8589934592*a^2*d^3*exp(c + d*x)*(3*a*b + 16*a^2 - 15*b
^2)*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/(b^7*(a - b)^2))*(-
(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 - (2147483648*d*exp(c +
 d*x)*(17*a - 15*b))/(b^6*(a - b)^2))*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a
 - b)))^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(3*a*b + 16*a^2 - 15*b
```

```
^2))/(a*b^6*(a - b)^3))*(-(a^2*b + (a^5*b)^(1/2))/(16*(a^5*d^2 - a^4*b*d^2)
))^(1/2) - log((((((4294967296*a*d^2*(exp(2*c + 2*d*x) + 1)*(26*a*b - 49*a^
2 + 15*b^2))/(b^6*(a - b)^3) - (8589934592*a^2*d^3*exp(c + d*x)*(3*a*b + 16
*a^2 - 15*b^2)*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/(b^7*(a
- b)^2))*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 + (214748364
8*d*exp(c + d*x)*(17*a - 15*b))/(b^6*(a - b)^2))*(-(a^2*b + (a^5*b)^(1/2))/
(a^4*d^2*(a - b)))^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(3*a*b + 16
*a^2 - 15*b^2))/(a*b^6*(a - b)^3))*(-(a^2*b + (a^5*b)^(1/2))/(16*(a^5*d^2 -
 a^4*b*d^2)))^(1/2) - (2*atan((exp(d*x)*exp(c)*(65536*a^2*(-a^2*d^2)^(1/2)
+ 50625*b^2*(-a^2*d^2)^(1/2) - 115200*a*b*(-a^2*d^2)^(1/2)))/(65536*a^3*d +
 50625*a*b^2*d - 115200*a^2*b*d)))/(-a^2*d^2)^(1/2) - log((((((4294967296*a
*d^2*(exp(2*c + 2*d*x) + 1)*(26*a*b - 49*a^2 + 15*b^2))/(b^6*(a - b)^3) - (
8589934592*a^2*d^3*exp(c + d*x)*(3*a*b + 16*a^2 - 15*b^2)*(-(a^2*b - (a^5*b
)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/(b^7*(a - b)^2))*(-(a^2*b - (a^5*b)^(1/2
))/(a^4*d^2*(a - b)))^(1/2))/4 + (2147483648*d*exp(c + d*x)*(17*a - 15*b))/
(b^6*(a - b)^2))*(-(a^2*b - (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 + (2
68435456*(exp(2*c + 2*d*x) + 1)*(3*a*b + 16*a^2 - 15*b^2))/(a*b^6*(a - b)^3
))*(-(a^2*b - (a^5*b)^(1/2))/(16*(a^5*d^2 - a^4*b*d^2)))^(1/2) + log((((((4
294967296*a*d^2*(exp(2*c + 2*d*x) + 1)*(26*a*b - 49*a^2 + 15*b^2))/(b^6*(a
- b)^3) + (8589934592*a^2*d^3*exp(c + d*x)*(3*a*b + 16*a^2 - 15*b^2)*(-(a^2
*b - (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/(b^7*(a - b)^2))*(-(a^2*b - (
a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 - (2147483648*d*exp(c + d*x)*(17*
a - 15*b))/(b^6*(a - b)^2))*(-(a^2*b - (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1
/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(3*a*b + 16*a^2 - 15*b^2))/(a*b^
6*(a - b)^3))*(-(a^2*b - (a^5*b)^(1/2))/(16*(a^5*d^2 - a^4*b*d^2)))^(1/2)
```

3.234 $\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=184

$$\frac{b^{3/4}\operatorname{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{\tanh^{-1}(\cosh(c+dx))}{2ad}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}\,d}+\frac{1}{4ad(1-\cosh(c+}$$

```
[Out] 1/2*arctanh(cosh(d*x+c))/a/d+1/4/a/d/(1-cosh(d*x+c))-1/4/a/d/(1+cosh(d*x+c)
)+1/2*b^(3/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^(3/2)/d
/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^(3/4)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b
^(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3294, 1184, 213, 1107, 211, 214}

$$\frac{b^{3/4}\operatorname{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{1}{4ad(1-\cosh(c+dx))}-\frac{1}{4ad(\cosh(c+dx)+1)}+\frac{\tanh^{-1}(\cosh(c+dx))}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^(3/2
)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + ArcTanh[Cosh[c + d*x]]/(2*a*d) + (b^(3/4)*Ar
cTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqr
t[a] + Sqrt[b]]*d) + 1/(4*a*d*(1 - Cosh[c + d*x])) - 1/(4*a*d*(1 + Cosh[c +
 d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+2bx^2-bx^4)}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{4a(-1+x)^2}+\frac{1}{4a(1+x)^2}-\frac{1}{2a(-1+x^2)}+\frac{b}{a(a-b+2bx^2-bx^4)}\right)\,dx, x, \cosh(c+dx}{d}\\
&= \frac{1}{4ad(1-\cosh(c+dx))}-\frac{1}{4ad(1+\cosh(c+dx))}-\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(\right.}{2ad}\\
&= \frac{\tanh^{-1}(\cosh(c+dx))}{2ad}+\frac{1}{4ad(1-\cosh(c+dx))}-\frac{1}{4ad(1+\cosh(c+dx))}-\frac{b^3}{}\\
&= \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{\tanh^{-1}(\cosh(c+dx))}{2ad}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\cos}{\sqrt{\sqrt{a}}}\right.}{2a^{3/2}\sqrt{\sqrt{a}+}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.28, size = 265, normalized size = 1.44

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+4\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+4b\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-c\#1-dx\#1-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1+c\#1^3+dx\#1^3+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^3}{-b-8a\#1^2+3b\#1^2-3b\#1^4+b\#1^6}\&\right]+\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/8*(Csch[(c + d*x)/2]^2 + 4*Log[Tanh[(c + d*x)/2]] + 4*b*RootSum[b - 4*b*
#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(c*#1) - d*x*#1 - 2*L
og[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
+ d*x)/2]*#1]*#1 + c*#1^3 + d*x*#1^3 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^3)/(-b - 8*a*#1^
2 + 3*b*#1^2 - 3*b*#1^4 + b*#1^6) & ] + Sech[(c + d*x)/2]^2)/(a*d)
```

Maple [A]
time = 2.16, size = 181, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}+2b\left(\frac{\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}-2a}{4\sqrt{\sqrt{ab}\,a-ab}}\right)}{4a\sqrt{\sqrt{ab}\,a-ab}}-\frac{\arctan\left(\frac{-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}+2a}{4\sqrt{-\sqrt{ab}\,a-ab}}\right)}{4a\sqrt{-\sqrt{ab}\,a-ab}}\right)-\frac{1}{8a\tanh\left(\frac{d}{2}\right.}}{d}$
default	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}+2b\left(\frac{\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}-2a}{4\sqrt{\sqrt{ab}\,a-ab}}\right)}{4a\sqrt{\sqrt{ab}\,a-ab}}-\frac{\arctan\left(\frac{-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{ab}+2a}{4\sqrt{-\sqrt{ab}\,a-ab}}\right)}{4a\sqrt{-\sqrt{ab}\,a-ab}}\right)-\frac{1}{8a\tanh\left(\frac{d}{2}\right.}}{d}$
risch	$-\frac{e^{dx+c}\left(1+e^{2dx+2c}\right)}{da\left(e^{2dx+2c}-1\right)^2}-\frac{\ln\left(e^{dx+c}-1\right)}{2da}+\frac{\ln\left(e^{dx+c}+1\right)}{2da}+8\left(\sum_{\_R=\text{RootOf}\left(\left(1048576a^7d^4-1048576a^6b\,d^4\right)\_Z^4+204\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a+2*b*(1/4/a/((a*b)^(1/2)*a-a*b)^(1/2)*arcta
n(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/
2))-1/4/a/(-(a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(-2*a*tanh(1/2*d*x+1/2*c)^2
+4*(a*b)^(1/2)+2*a)/(-(a*b)^(1/2)*a-a*b)^(1/2)))-1/8/a/tanh(1/2*d*x+1/2*c)^
2-1/2/a*ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*
c) + a*d) + 1/2*log((e^(d*x + c) + 1)*e^(-c))/(a*d) - 1/2*log((e^(d*x + c)
- 1)*e^(-c))/(a*d) - 8*integrate((b*e^(5*d*x + 5*c) - b*e^(3*d*x + 3*c))/(a
*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c) - 4*a*b*e^(2*d*x + 2*c) + a*b -
2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1954 vs. $2(136) = 272$.
time = 0.44, size = 1954, normalized size = 10.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*cosh(d*x + c)^3 + 12*cosh(d*x + c)*sinh(d*x + c)^2 + 4*sinh(d*x + c
)^3 - (a*d*cosh(d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh
(d*x + c)^4 - 2*a*d*cosh(d*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(
d*x + c)^2 + a*d + 4*(a*d*cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c
))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2
)/((a^4 - a^3*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d
*x + c) + b^2*sinh(d*x + c)^2 + b^2 + 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*si
nh(d*x + c) - ((a^5 - a^4*b)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*sinh(d*x
 + c))*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)))*sqrt(-((a^4 - a^3*b)*d^2*
sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))) + (a
*d*cosh(d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh(d*x + c
)^4 - 2*a*d*cosh(d*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)
^2 + a*d + 4*(a*d*cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(
-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4
- a^3*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c)
+ b^2*sinh(d*x + c)^2 + b^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x +
 c) - ((a^5 - a^4*b)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*sinh(d*x + c))*s
qrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3
/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))) - (a*d*cosh(
d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh(d*x + c)^4 - 2*
a*d*cosh(d*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^2 + a*d
 + 4*(a*d*cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 -
a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*
d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sin
h(d*x + c)^2 + b^2 + 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a
^5 - a^4*b)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*sinh(d*x + c))*sqrt(b^3/(
(a^7 - 2*a^6*b + a^5*b^2)*d^4)))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2
```

```
*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) + (a*d*cosh(d*x + c)^4
 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh(d*x + c)^4 - 2*a*d*cosh(d
*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^2 + a*d + 4*(a*d*
cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 - a^3*b)*d^2
*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))*log(
b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)
^2 + b^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a^5 - a^4*b
)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*sinh(d*x + c))*sqrt(b^3/((a^7 - 2*a
^6*b + a^5*b^2)*d^4)))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a
^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) - 2*(cosh(d*x + c)^4 + 4*cosh(d*
x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d
*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*
x + c) + 1)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*(cosh(d*x + c)^4 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1
)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))
*sinh(d*x + c) + 1)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*cosh(d*x
+ c)^2 + 1)*sinh(d*x + c) + 4*cosh(d*x + c))/(a*d*cosh(d*x + c)^4 + 4*a*d*c
osh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh(d*x + c)^4 - 2*a*d*cosh(d*x + c)^2
+ 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^2 + a*d + 4*(a*d*cosh(d*x +
 c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(136) = 272.
time = 0.51, size = 467, normalized size = 2.54

$$\frac{\frac{2\left(\left(4\sqrt{-b^2-\sqrt{ab}\,b}\,ab+5\sqrt{-b^2-\sqrt{ab}\,b}\,b^2\right)|a||b|-\left(4\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,ab+5\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,b^2\right)|b|\right)\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{ab+\sqrt{a^2b^2+(a^2-ab)ab}}{ab}}}\right)}{(4a^3b^2+a^2b^3-5ab^4)|a|}+\frac{2\left(\left(4\sqrt{-b^2+\sqrt{ab}\,b}\,ab+5\sqrt{-b^2+\sqrt{ab}\,b}\,b^2\right)|a||b|+\left(4\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,ab+5\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,b^2\right)|b|\right)\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{ab-\sqrt{a^2b^2+(a^2-ab)ab}}{ab}}}\right)}{(4a^3b^2+a^2b^3-5ab^4)|a|}+\frac{\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{a}-\frac{\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{a}-\frac{4\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\right)a}}{4d}$$

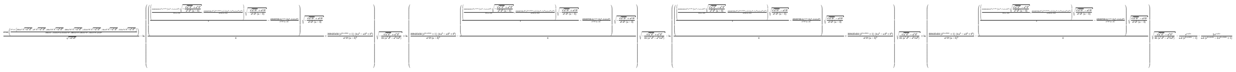
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/4*(2*((4*sqrt(-b^2 - sqrt(a*b)*b)*a*b + 5*sqrt(-b^2 - sqrt(a*b)*b)*b^2)*a
bs(a)*abs(b) - (4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b + 5*sqrt(a*b)*sqrt
(-b^2 - sqrt(a*b)*b)*b^2)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/s
qrt(-(a*b + sqrt(a^2*b^2 + (a^2 - a*b)*a*b))/(a*b)))/((4*a^3*b^2 + a^2*b^3
- 5*a*b^4)*abs(a)) + 2*((4*sqrt(-b^2 + sqrt(a*b)*b)*a*b + 5*sqrt(-b^2 + sqr
```

```
t(a*b)*b)*b^2)*abs(a)*abs(b) + (4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b +
5*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^2)*abs(b))*arctan(1/2*(e^(d*x + c) +
 e^(-d*x - c))/sqrt(-(a*b - sqrt(a^2*b^2 + (a^2 - a*b)*a*b))/(a*b)))/((4*a^
3*b^2 + a^2*b^3 - 5*a*b^4)*abs(a)) + log(e^(d*x + c) + e^(-d*x - c) + 2)/a
- log(e^(d*x + c) + e^(-d*x - c) - 2)/a - 4*(e^(d*x + c) + e^(-d*x - c))/((
(e^(d*x + c) + e^(-d*x - c))^2 - 4)*a))/d
```

Mupad [B]
time = 12.21, size = 1517, normalized size = 8.24



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a - b*sinh(c + d*x)^4)),x)
```

```
[Out] atan((exp(d*x)*exp(c)*(256*a^6*(-a^2*d^2)^(1/2) + b^6*(-a^2*d^2)^(1/2) + 96
*a^2*b^4*(-a^2*d^2)^(1/2) - 288*a^3*b^3*(-a^2*d^2)^(1/2) + 512*a^4*b^2*(-a^
2*d^2)^(1/2) - 16*a*b^5*(-a^2*d^2)^(1/2) - 512*a^5*b*(-a^2*d^2)^(1/2)))/(25
6*a^7*d - 16*a^2*b^5*d + 96*a^3*b^4*d - 288*a^4*b^3*d + 512*a^5*b^2*d + a*b
^6*d - 512*a^6*b*d))/(-a^2*d^2)^(1/2) - log((((((8589934592*d^3*exp(c + d*x
)*(8*a^2 - 7*a*b + 3*b^2)*(-((a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a - b)))^
(1/2))/(b^5*(a - b)^2) - (4294967296*d^2*(exp(2*c + 2*d*x) + 1)*(2*a*b^2 -
7*a^2*b + 12*a^3 + b^3))/(a^2*b^4*(a - b)^3))*(-((a^7*b^3)^(1/2) + a^3*b^2)
/(a^6*d^2*(a - b)))^(1/2))/4 - (4294967296*d*exp(c + d*x)*(2*a^2 - 2*a*b +
b^2))/(a^3*b^4*(a - b)^2))*(-((a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a - b)))
^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(4*a^3 - a*b^2 + b^3))/(a^5*b
^3*(a - b)^3))*(-((a^7*b^3)^(1/2) + a^3*b^2)/(16*(a^7*d^2 - a^6*b*d^2)))^(1
/2) + log((268435456*(exp(2*c + 2*d*x) + 1)*(4*a^3 - a*b^2 + b^3))/(a^5*b^3
*(a - b)^3) - (((((8589934592*d^3*exp(c + d*x)*(8*a^2 - 7*a*b + 3*b^2)*(-((
a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/(b^5*(a - b)^2) + (4294
967296*d^2*(exp(2*c + 2*d*x) + 1)*(2*a*b^2 - 7*a^2*b + 12*a^3 + b^3))/(a^2*
b^4*(a - b)^3))*(-((a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/4 -
 (4294967296*d*exp(c + d*x)*(2*a^2 - 2*a*b + b^2))/(a^3*b^4*(a - b)^2))*(-(
(a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/4)*(-((a^7*b^3)^(1/2)
+ a^3*b^2)/(16*(a^7*d^2 - a^6*b*d^2)))^(1/2) - log((((((8589934592*d^3*exp(
c + d*x)*(8*a^2 - 7*a*b + 3*b^2)*(((a^7*b^3)^(1/2) - a^3*b^2)/(a^6*d^2*(a -
 b)))^(1/2))/(b^5*(a - b)^2) - (4294967296*d^2*(exp(2*c + 2*d*x) + 1)*(2*a*
b^2 - 7*a^2*b + 12*a^3 + b^3))/(a^2*b^4*(a - b)^3))*(((a^7*b^3)^(1/2) - a^3
*b^2)/(a^6*d^2*(a - b)))^(1/2))/4 - (4294967296*d*exp(c + d*x)*(2*a^2 - 2*a
*b + b^2))/(a^3*b^4*(a - b)^2))*(((a^7*b^3)^(1/2) - a^3*b^2)/(a^6*d^2*(a -
b)))^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(4*a^3 - a*b^2 + b^3))/(a
^5*b^3*(a - b)^3))*(((a^7*b^3)^(1/2) - a^3*b^2)/(16*(a^7*d^2 - a^6*b*d^2)))
^(1/2) + log((268435456*(exp(2*c + 2*d*x) + 1)*(4*a^3 - a*b^2 + b^3))/(a^5*
b^3*(a - b)^3) - (((((8589934592*d^3*exp(c + d*x)*(8*a^2 - 7*a*b + 3*b^2)*(
((a^7*b^3)^(1/2) - a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/(b^5*(a - b)^2) + (42
```

```
94967296*d^2*(exp(2*c + 2*d*x) + 1)*(2*a*b^2 - 7*a^2*b + 12*a^3 + b^3))/(a^
2*b^4*(a - b)^3))*(((a^7*b^3)^(1/2) - a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/4
- (4294967296*d*exp(c + d*x)*(2*a^2 - 2*a*b + b^2))/(a^3*b^4*(a - b)^2))*((
(a^7*b^3)^(1/2) - a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/4)*(((a^7*b^3)^(1/2) -
 a^3*b^2)/(16*(a^7*d^2 - a^6*b*d^2)))^(1/2) - exp(c + d*x)/(a*d*(exp(2*c +
2*d*x) - 1)) - (2*exp(c + d*x))/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x)
 + 1))
```

3.235 $\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{x}{2b}-\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/2}d}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/2}d}-\frac{1}{4bd(1-\tanh(c+}$$

```
[Out] 1/2*x/b-1/2*a^(3/4)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/b^
(3/2)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*a^(3/4)*arctanh((a^(1/2)+b^(1/2))^(1/2)
*tanh(d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)+b^(1/2))^(1/2)-1/4/b/d/(1-tanh(d*x
+c))+1/4/b/d/(1+tanh(d*x+c))
```

Rubi [A]

time = 0.18, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3296, 1301, 213, 1144, 214}

$$-\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{1}{4bd(1-\tanh(c+dx))}+\frac{1}{4bd(\tanh(c+dx)+1)}+\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] x/(2*b) - (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]
)/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/2)*d) + (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] +
Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/2)*d) - 1
/(4*b*d*(1 - Tanh[c + d*x])) + 1/(4*b*d*(1 + Tanh[c + d*x]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
```

```
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2(a-2ax^2+(a-b)x^4)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{4b(-1+x)^2}-\frac{1}{4b(1+x)^2}-\frac{1}{2b(-1+x^2)}+\frac{ax^2}{b(a-2ax^2+(a-b)x^4)}\right)\,dx, x, \tanh(c+d}{d}$$

$$= -\frac{1}{4bd(1-\tanh(c+dx))}+\frac{1}{4bd(1+\tanh(c+dx))}-\frac{\text{Subst}(\int \frac{1}{-1+x^2}\,dx, x, \tanh(}{2bd}$$

$$= \frac{x}{2b}-\frac{1}{4bd(1-\tanh(c+dx))}+\frac{1}{4bd(1+\tanh(c+dx))}+\frac{\left(a\left(\sqrt{a}+\sqrt{b}\right)\right)\text{Subs}}{}$$

$$= \frac{x}{2b}-\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/2}d}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/2}d}$$

Mathematica [A]

time = 0.66, size = 158, normalized size = 0.90

$$\frac{2\sqrt{b}\,(c+dx)+\frac{2a\mathrm{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\,\sqrt{b}}}+\frac{2a\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\,\sqrt{b}}}-\sqrt{b}\,\sinh(2(c+dx))}{4b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (2*Sqrt[b]*(c + d*x) + (2*a*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt
[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (2*a*ArcTanh[((Sqrt[a
] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sq
rt[b]] - Sqrt[b]*Sinh[2*(c + d*x)])/(4*b^(3/2)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.69, size = 206, normalized size = 1.18

method	result
risch	$\frac{x}{2b}-\frac{e^{2dx+2c}}{8bd}+\frac{e^{-2dx-2c}}{8bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(256a\,b^{6}d^{4}-256b^{7}d^{4}\right)\_Z^{4}-32a^{2}b^{3}d^{2}\_Z^{2}+a^{3}\right)}\_R\ln\left(e^{2dx+2c}\right.\right.$
derivativedivides	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b}-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_Z^{6}+(6a-16b)\_Z^{4}-4a\_Z^{2}\right.}\right.}{b}}{d}$
default	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b}-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_Z^{6}+(6a-16b)\_Z^{4}-4a\_Z^{2}\right.}\right.}{b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/b/(tanh(1/2*d*x+1/2*c)+1)+1/2/b*ln
(tanh(1/2*d*x+1/2*c)+1)-a/b*sum((_R^4-_R^2)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^
3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_
Z^4-4*a*_Z^2+a))-1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/b/(tanh(1/2*d*x+1/2*c)
-1)-1/2/b*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

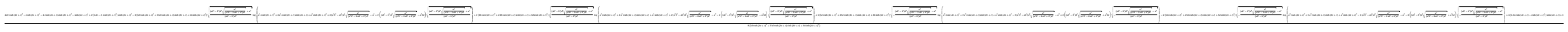
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/8*(4*d*x*e^(2*d*x + 2*c) - e^(4*d*x + 4*c) + 1)*e^(-2*d*x - 2*c)/(b*d) -
1/64*integrate(256*(a*e^(6*d*x + 6*c) - 2*a*e^(4*d*x + 4*c) + a*e^(2*d*x +
2*c))/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6*c) - 4*b^2*e^(2*d*x + 2*c)
+ b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1441 vs. $2(127) = 254$.
time = 0.44, size = 1441, normalized size = 8.23



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*d*x*cosh(d*x + c)^2 - cosh(d*x + c)^4 - 4*cosh(d*x + c)*sinh(d*x + c
)^3 - sinh(d*x + c)^4 + 2*(2*d*x - 3*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2*(
b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)
^2)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2
)/((a*b^3 - b^4)*d^2))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d
*x + c) + a^2*sinh(d*x + c)^2 + 2*(a^2*b^2 - a*b^3)*d^2*sqrt(a^3/((a^2*b^5
- 2*a*b^6 + b^7)*d^4)) - a^2 + 2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*
a*b^6 + b^7)*d^4)) - a^2*b*d)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 -
2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) + 2*(b*d*cosh(d*x + c)^2
+ 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2)*sqrt(((a*b^3 - b
^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2
))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d
*x + c)^2 + 2*(a^2*b^2 - a*b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4
)) - a^2 - 2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) -
 a^2*b*d)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))
 + a^2)/((a*b^3 - b^4)*d^2))) + 2*(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c
)*sinh(d*x + c) + b*d*sinh(d*x + c)^2)*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((
a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))*log(a^2*cosh(d*x
 + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 - 2*(a^2*
b^2 - a*b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2 + 2*((a*b^
4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2*b*d)*sqrt(-((a
*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 -
b^4)*d^2))) - 2*(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) +
b*d*sinh(d*x + c)^2)*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6
+ b^7)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))*log(a^2*cosh(d*x + c)^2 + 2*a^2*co
sh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 - 2*(a^2*b^2 - a*b^3)*d^2*s
qrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2 - 2*((a*b^4 - b^5)*d^3*sqrt(
a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2*b*d)*sqrt(-((a*b^3 - b^4)*d^2*sq
```

```
rt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))) + 4*(2
*d*x*cosh(d*x + c) - cosh(d*x + c)^3)*sinh(d*x + c) + 1)/(b*d*cosh(d*x + c)
^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Not invertible Error: Bad Argumen
t Valu
```

Mupad [**B**]
time = 11.24, size = 2191, normalized size = 12.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log((((((4194304*a^6*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*
b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c
 + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(b^12*(a - b)^2) - (16777216*a^
6*d^3*(((a^3*b^7)^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2)*(40*a*b^2 - 35*
b^3 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*exp(2*
c + 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^11*(a - b)))*(((a^3*b^7)^(1/2)
 + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (2097152*a^7*d*(256*a^2*b - 256*a
*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*x) + 6*b^3*exp(2*c + 2*d*x) + 756*a*b
^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*c + 2*d*x)))/(b^14*(a - b)))*(((a^3*b
^7)^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (524288*a^8*(185*a*b^2 -
 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(2*c
```

```
+ 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*d*x)))/(b^15
*(a - b)^2))*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*(b^7*d^2 - a*b^6*d^2)))^(1/2
) - log((((((4194304*a^6*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*
a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp
(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(b^12*(a - b)^2) + (1677721
6*a^6*d^3*(((a^3*b^7)^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2)*(40*a*b^2 -
 35*b^3 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*ex
p(2*c + 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^11*(a - b)))*(((a^3*b^7)^(
1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 + (2097152*a^7*d*(256*a^2*b - 2
56*a*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*x) + 6*b^3*exp(2*c + 2*d*x) + 756
*a*b^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*c + 2*d*x)))/(b^14*(a - b)))*(((a
^3*b^7)^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (524288*a^8*(185*a*b
^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(
2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*d*x)))/(
b^15*(a - b)^2))*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*(b^7*d^2 - a*b^6*d^2)))^
(1/2) + log((((((4194304*a^6*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 +
930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b
*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(b^12*(a - b)^2) - (167
77216*a^6*d^3*(-((a^3*b^7)^(1/2) - a^2*b^3)/(b^6*d^2*(a - b)))^(1/2)*(40*a*
b^2 - 35*b^3 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b
^2*exp(2*c + 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^11*(a - b)))*(-((a^3*
b^7)^(1/2) - a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (2097152*a^7*d*(256*a^2
*b - 256*a*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*x) + 6*b^3*exp(2*c + 2*d*x)
 + 756*a*b^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*c + 2*d*x)))/(b^14*(a - b))
)*(-((a^3*b^7)^(1/2) - a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (524288*a^8*(
185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b
^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*d
*x)))/(b^15*(a - b)^2))*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*(b^7*d^2 - a*b^6*d
^2)))^(1/2) - log((((((4194304*a^6*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 -
b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768
*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(b^12*(a - b)^2)
+ (16777216*a^6*d^3*(-((a^3*b^7)^(1/2) - a^2*b^3)/(b^6*d^2*(a - b)))^(1/2)*
(40*a*b^2 - 35*b^3 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 3
26*a*b^2*exp(2*c + 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^11*(a - b)))*(-
((a^3*b^7)^(1/2) - a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 + (2097152*a^7*d*(2
56*a^2*b - 256*a*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*x) + 6*b^3*exp(2*c +
2*d*x) + 756*a*b^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*c + 2*d*x)))/(b^14*(a
 - b)))*(-((a^3*b^7)^(1/2) - a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (524288
*a^8*(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x)
- 35*b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c
 + 2*d*x)))/(b^15*(a - b)^2))*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*(b^7*d^2 - a
*b^6*d^2)))^(1/2) + x/(2*b) + exp(- 2*c - 2*d*x)/(8*b*d) - exp(2*c + 2*d*x)
/(8*b*d)
```

3.236 $\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=127

$$-\frac{x}{b}+\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,bd}+\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,bd}$$

```
[Out] -x/b+1/2*a^(1/4)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/b/d/(
a^(1/2)-b^(1/2))^(1/2)+1/2*a^(1/4)*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x
+c)/a^(1/4))/b/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3296, 1301, 213, 1180, 214}

$$\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -(x/b) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])
/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b*d) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]
]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a-2ax^2+(a-b)x^4)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{b(-1+x^2)}+\frac{a(1-x^2)}{b(a-2ax^2+(a-b)x^4)}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+dx)\right)}{bd}+\frac{a\text{Subst}\Big(\int \frac{1-x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+d}{bd}\\
&= -\frac{x}{b}-\frac{\left(\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)\right)\text{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2bd} -\\
&= -\frac{x}{b}+\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,bd}+\frac{\sqrt[4]{a}\tanh^{-1}\Big(\frac{\sqrt{\sqrt{a}+\sqrt{b}}}{\sqrt[4]{a}}}{2\sqrt{\sqrt{a}+\sqrt{b}}\,bd}
\end{aligned}$$

Mathematica [A]

time = 0.33, size = 143, normalized size = 1.13

$$\frac{-2(c+dx)-\frac{\sqrt{a}\operatorname{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (-2*(c + d*x) - (Sqrt[a]*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a
 + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[a]*ArcTanh[((Sqrt[
a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*S
qrt[b]])/(2*b*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.52, size = 139, normalized size = 1.09

method	result
risch	$-\frac{x}{b}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(256a\,b^4d^4-256b^5d^4\right)\_Z^4-32a\,b^2d^2\_Z^2+a\right)}\_R\ln\left(\mathrm{e}^{2dx+2c}+\left(-128a\,b^2d^3+128\right.\right.\right.$
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(\_R^6-3\_R^4+3\_R^2-1\right)\ln(\tanh(}{\_R^7a-3\_R^5a+3\_R^3a-8\_R}\right.}{4b}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(\_R^6-3\_R^4+3\_R^2-1\right)\ln(\tanh(}{\_R^7a-3\_R^5a+3\_R^3a-8\_R}\right.}{4b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/4*a/b*sum((_R^6-3*_R^4+3*_R^2-1)/(_R^
7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a
*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))+1/b*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -16*a*integrate(e^(4*d*x + 4*c)/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6*c
) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4*d*
x)), x) - x/b
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1009 vs. $2(91) = 182$.
time = 0.42, size = 1009, normalized size = 7.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(b*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a
)/((a*b^2 - b^3)*d^2))*log(2*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b
^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - b*d)*sq
rt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a)/((a*b^2
- b^3)*d^2)) - 1) - b*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 +
b^5)*d^4)) + a)/((a*b^2 - b^3)*d^2))*log(2*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3
 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 - 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d
^4)) - b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))
 + a)/((a*b^2 - b^3)*d^2)) - 1) - b*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b
^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-2*(a*b - b^2)*d^2*
sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 -
2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a
*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2)) - 1) + b*sqrt(-((a*b^2 - b^3)*d
^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-2
*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*((a*b^2 - b^3)*d^3*sq
rt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(
a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2)) - 1) + 4*x)/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 13, normalized size = 0.10

$$-\frac{dx+c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -(d*x + c)/(b*d)
```

Mupad [B]
time = 11.05, size = 1861, normalized size = 14.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log((((((524288*a^3*d^2*(31*a*b^2 - 128*a^2*b + 128*a^3 - b^3 + 256*a^3*exp
(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*
b*exp(2*c + 2*d*x)))/(b^8*(a - b)) + (1048576*a^3*d^3*((a*b^2 - (a*b^5)^(1/
2))/(b^4*d^2*(a - b)))^(1/2)*(45*a*b^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3
*exp(2*c + 2*d*x) - 50*a*b^2*exp(2*c + 2*d*x) + 48*a^2*b*exp(2*c + 2*d*x)))
/(b^7*(a - b)))*((a*b^2 - (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2))/4 + (262
144*a^4*d*(72*a*b - 64*a^2 - 9*b^2 + 256*a^2*exp(2*c + 2*d*x) + 31*b^2*exp(
2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d*x)))/(b^9*(a - b)))*((a*b^2 - (a*b^5)^
(1/2))/(b^4*d^2*(a - b)))^(1/2))/4 + (32768*a^4*(128*a*b - 128*a^2 - 15*b^2
 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2*exp(2*c + 2*d*x) - 304*a*b*exp(2*c + 2
*d*x)))/(b^10*(a - b)))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(b^5*d^2 - a*b^4*d^2)
))^(1/2) - log((((((524288*a^3*d^2*(31*a*b^2 - 128*a^2*b + 128*a^3 - b^3 +
256*a^3*exp(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x)
 - 240*a^2*b*exp(2*c + 2*d*x)))/(b^8*(a - b)) - (1048576*a^3*d^3*((a*b^2 -
(a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2)*(45*a*b^2 - 104*a^2*b + 64*a^3 - 3*
b^3 + 4*b^3*exp(2*c + 2*d*x) - 50*a*b^2*exp(2*c + 2*d*x) + 48*a^2*b*exp(2*c
 + 2*d*x)))/(b^7*(a - b)))*((a*b^2 - (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2
))/4 - (262144*a^4*d*(72*a*b - 64*a^2 - 9*b^2 + 256*a^2*exp(2*c + 2*d*x) +
31*b^2*exp(2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d*x)))/(b^9*(a - b)))*((a*b^2
 - (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2))/4 + (32768*a^4*(128*a*b - 128*a
^2 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2*exp(2*c + 2*d*x) - 304*a*b*
exp(2*c + 2*d*x)))/(b^10*(a - b)))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(b^5*d^2 -
 a*b^4*d^2)))^(1/2) - x/b - log((((((524288*a^3*d^2*(31*a*b^2 - 128*a^2*b +
 128*a^3 - b^3 + 256*a^3*exp(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x) + 21*a*b^2
*exp(2*c + 2*d*x) - 240*a^2*b*exp(2*c + 2*d*x)))/(b^8*(a - b)) - (1048576*a
^3*d^3*((a*b^2 + (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2)*(45*a*b^2 - 104*a^
```

```
2*b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c + 2*d*x) - 50*a*b^2*exp(2*c + 2*d*x) +
 48*a^2*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*((a*b^2 + (a*b^5)^(1/2))/(b^4*d
^2*(a - b)))^(1/2))/4 - (262144*a^4*d*(72*a*b - 64*a^2 - 9*b^2 + 256*a^2*ex
p(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d*x)))/(b^9*
(a - b)))*((a*b^2 + (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2))/4 + (32768*a^4
*(128*a*b - 128*a^2 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2*exp(2*c +
2*d*x) - 304*a*b*exp(2*c + 2*d*x)))/(b^10*(a - b)))*(-(a*b^2 + (a*b^5)^(1/2
))/(16*(b^5*d^2 - a*b^4*d^2)))^(1/2) + log((((((524288*a^3*d^2*(31*a*b^2 -
128*a^2*b + 128*a^3 - b^3 + 256*a^3*exp(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x)
 + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*b*exp(2*c + 2*d*x)))/(b^8*(a - b)) +
 (1048576*a^3*d^3*((a*b^2 + (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2)*(45*a*b
^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c + 2*d*x) - 50*a*b^2*exp(2*c
 + 2*d*x) + 48*a^2*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*((a*b^2 + (a*b^5)^(1
/2))/(b^4*d^2*(a - b)))^(1/2))/4 + (262144*a^4*d*(72*a*b - 64*a^2 - 9*b^2 +
 256*a^2*exp(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d
*x)))/(b^9*(a - b)))*((a*b^2 + (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2))/4 +
 (32768*a^4*(128*a*b - 128*a^2 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2
*exp(2*c + 2*d*x) - 304*a*b*exp(2*c + 2*d*x)))/(b^10*(a - b)))*(-(a*b^2 + (
a*b^5)^(1/2))/(16*(b^5*d^2 - a*b^4*d^2)))^(1/2)
```

3.237 $\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{b}\,d}$$

```
[Out] -1/2*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(1/4)/d/b^(1/2)
/(a^(1/2)-b^(1/2))^(1/2)+1/2*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^
(1/4))/a^(1/4)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3296, 1144, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\,d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\,d\sqrt{\sqrt{a}-\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(a^(1/4)*Sqrt
[Sqrt[a] - Sqrt[b]]*Sqrt[b]*d) + ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c +
d*x])/a^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\text{Subst}\left(\int \frac{1}{-a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2d} + \left(1+\frac{\sqrt{a}}{\sqrt{b}}\right)\text{S}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\,\sqrt{b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\,\sqrt{b}\,d}$$

Mathematica [**A**]
time = 0.26, size = 127, normalized size = 1.02

$$\frac{\frac{\text{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}}{2\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/Sqr
t[-a + Sqrt[a]*Sqrt[b]] + ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[
a + Sqrt[a]*Sqrt[b]]]/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[b]*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.78, size = 94, normalized size = 0.75

method	result

derivativedivides	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(\_R^4-\_R^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}}{d}$
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(\_R^4-\_R^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}}{d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(256a^2b^2d^4-256a\,b^3d^4)\_Z^4-32a\,d^2\_Z^2b\right)}\_R\ln\left(\mathrm{e}^{2dx+2c}+(-128a^2b\,d^3+128a\,b^2d^3)\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*sum((_R^4-_R^2)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d
*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^4 - a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 975 vs. 2(85) = 170.
time = 0.41, size = 975, normalized size = 7.80

$$-\frac{1}{4}\sqrt{\frac{(ab-b^2)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+1}{(ab-b^2)d^2}}\log\left(2(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2+2\left((a^2b-ab^2)d^3\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}-ad\right)\sqrt{\frac{(ab-b^2)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+1}{(ab-b^2)d^2}}-1\right)+\frac{1}{4}\sqrt{\frac{(ab-b^2)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+1}{(ab-b^2)d^2}}\log\left(2(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-2\left((a^2b-ab^2)d^3\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}-ad\right)\sqrt{\frac{(ab-b^2)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+1}{(ab-b^2)d^2}}-1\right)+\frac{1}{4}\sqrt{-\frac{(ab-b^2)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}-1}{(ab-b^2)d^2}}\log\left(-2(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2+2\left((a^2b-ab^2)d^3\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+ad\right)\sqrt{-\frac{(ab-b^2)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}-1}{(ab-b^2)d^2}}-1\right)-\frac{1}{4}\sqrt{-\frac{(ab-b^2)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}-1}{(ab-b^2)d^2}}\log\left(-2(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-2\left((a^2b-ab^2)d^3\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+ad\right)\sqrt{-\frac{(ab-b^2)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}-1}{(ab-b^2)d^2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/(
(a*b - b^2)*d^2))*log(2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)
*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
+ 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - a*d)*s
qrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a*b -
 b^2)*d^2)) - 1) + 1/4*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a
*b^3)*d^4)) + 1)/((a*b - b^2)*d^2))*log(2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b -
2*a^2*b^2 + a*b^3)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2 - 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b
```

```
^3)*d^4)) - a*d)*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*
d^4)) + 1)/((a*b - b^2)*d^2)) - 1) + 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^
3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2))*log(-2*(a^2 - a*b)*d
^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x
 + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3
*b - 2*a^2*b^2 + a*b^3)*d^4)) + a*d)*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b
- 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2)) - 1) - 1/4*sqrt(-((a*b -
 b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2))
*log(-2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*((a^2*b - a*
b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + a*d)*sqrt(-((a*b - b^2
)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2)) - 1
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 0
```

Mupad [**B**]
time = 12.90, size = 1859, normalized size = 14.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log((((((262144*a^2*d^2*(102*a*b - 128*a^2 - 22*b^2 - 272*a^2*exp(2*c + 2*d
*x) + 19*b^2*exp(2*c + 2*d*x) + 189*a*b*exp(2*c + 2*d*x)))/(b^6*(a - b)) -
(131072*a^2*d^3*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2)*(119*a*b^
2 - 136*a^2*b + b^3 - 1024*a^3*exp(2*c + 2*d*x) + 9*b^3*exp(2*c + 2*d*x) -
```

```
809*a*b^2*exp(2*c + 2*d*x) + 1808*a^2*b*exp(2*c + 2*d*x)))/(b^6*(a - b)))*(
(a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 + (32768*a*d*(120*a^2*b
 - 129*a*b^2 + b^3 - 1024*a^3*exp(2*c + 2*d*x) - b^3*exp(2*c + 2*d*x) + 201
*a*b^2*exp(2*c + 2*d*x) + 816*a^2*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*((a*b
 + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 - (16384*a*(106*a*b - 128*a
^2 - 2*b^2 + 240*a^2*exp(2*c + 2*d*x) + 3*b^2*exp(2*c + 2*d*x) - 275*a*b*ex
p(2*c + 2*d*x)))/(b^7*(a - b)))*(-(a*b + (a*b^3)^(1/2))/(16*(a*b^3*d^2 - a^
2*b^2*d^2)))^(1/2) - log((((((262144*a^2*d^2*(102*a*b - 128*a^2 - 22*b^2 -
272*a^2*exp(2*c + 2*d*x) + 19*b^2*exp(2*c + 2*d*x) + 189*a*b*exp(2*c + 2*d*
x)))/(b^6*(a - b)) + (131072*a^2*d^3*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a -
 b)))^(1/2)*(119*a*b^2 - 136*a^2*b + b^3 - 1024*a^3*exp(2*c + 2*d*x) + 9*b^
3*exp(2*c + 2*d*x) - 809*a*b^2*exp(2*c + 2*d*x) + 1808*a^2*b*exp(2*c + 2*d*
x)))/(b^6*(a - b)))*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 -
(32768*a*d*(120*a^2*b - 129*a*b^2 + b^3 - 1024*a^3*exp(2*c + 2*d*x) - b^3*e
xp(2*c + 2*d*x) + 201*a*b^2*exp(2*c + 2*d*x) + 816*a^2*b*exp(2*c + 2*d*x)))
/(b^7*(a - b)))*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 - (163
84*a*(106*a*b - 128*a^2 - 2*b^2 + 240*a^2*exp(2*c + 2*d*x) + 3*b^2*exp(2*c
+ 2*d*x) - 275*a*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*(-(a*b + (a*b^3)^(1/2)
)/(16*(a*b^3*d^2 - a^2*b^2*d^2)))^(1/2) + log((((((262144*a^2*d^2*(102*a*b
- 128*a^2 - 22*b^2 - 272*a^2*exp(2*c + 2*d*x) + 19*b^2*exp(2*c + 2*d*x) + 1
89*a*b*exp(2*c + 2*d*x)))/(b^6*(a - b)) - (131072*a^2*d^3*((a*b - (a*b^3)^(
1/2))/(a*b^2*d^2*(a - b)))^(1/2)*(119*a*b^2 - 136*a^2*b + b^3 - 1024*a^3*ex
p(2*c + 2*d*x) + 9*b^3*exp(2*c + 2*d*x) - 809*a*b^2*exp(2*c + 2*d*x) + 1808
*a^2*b*exp(2*c + 2*d*x)))/(b^6*(a - b)))*((a*b - (a*b^3)^(1/2))/(a*b^2*d^2*
(a - b)))^(1/2))/4 + (32768*a*d*(120*a^2*b - 129*a*b^2 + b^3 - 1024*a^3*exp
(2*c + 2*d*x) - b^3*exp(2*c + 2*d*x) + 201*a*b^2*exp(2*c + 2*d*x) + 816*a^2
*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*((a*b - (a*b^3)^(1/2))/(a*b^2*d^2*(a -
 b)))^(1/2))/4 - (16384*a*(106*a*b - 128*a^2 - 2*b^2 + 240*a^2*exp(2*c + 2*
d*x) + 3*b^2*exp(2*c + 2*d*x) - 275*a*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*(
-(a*b - (a*b^3)^(1/2))/(16*(a*b^3*d^2 - a^2*b^2*d^2)))^(1/2) - log((((((262
144*a^2*d^2*(102*a*b - 128*a^2 - 22*b^2 - 272*a^2*exp(2*c + 2*d*x) + 19*b^2
*exp(2*c + 2*d*x) + 189*a*b*exp(2*c + 2*d*x)))/(b^6*(a - b)) + (131072*a^2*
d^3*((a*b - (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2)*(119*a*b^2 - 136*a^2*
b + b^3 - 1024*a^3*exp(2*c + 2*d*x) + 9*b^3*exp(2*c + 2*d*x) - 809*a*b^2*ex
p(2*c + 2*d*x) + 1808*a^2*b*exp(2*c + 2*d*x)))/(b^6*(a - b)))*((a*b - (a*b^
3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 - (32768*a*d*(120*a^2*b - 129*a*b^2
 + b^3 - 1024*a^3*exp(2*c + 2*d*x) - b^3*exp(2*c + 2*d*x) + 201*a*b^2*exp(2
*c + 2*d*x) + 816*a^2*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*((a*b - (a*b^3)^(
1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 - (16384*a*(106*a*b - 128*a^2 - 2*b^2 +
 240*a^2*exp(2*c + 2*d*x) + 3*b^2*exp(2*c + 2*d*x) - 275*a*b*exp(2*c + 2*d*
x)))/(b^7*(a - b)))*(-(a*b - (a*b^3)^(1/2))/(16*(a*b^3*d^2 - a^2*b^2*d^2)))
^(1/2)
```

3.238 $\int \frac{1}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

```
[Out] 1/2*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)
-b^(1/2))^(1/2)+1/2*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^
(3/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3288, 1180, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sinh[c + d*x]^4)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sq
rt[a] - Sqrt[b]]*d) + ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/
4)]/(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{-a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2d} - \left(1+\frac{\sqrt{b}}{\sqrt{a}}\right.$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

Mathematica [A]
time = 0.17, size = 128, normalized size = 1.11

$$\frac{-\frac{\text{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sinh[c + d*x]^4)^(-1),x]
```

```
[Out] (-(ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/S
qrt[-a + Sqrt[a]*Sqrt[b]]) + ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sq
rt[a + Sqrt[a]*Sqrt[b]]]/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.67, size = 102, normalized size = 0.89

method	result

derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_Z^{6}+(6a-16b)\_Z^{4}-4a\_Z^{2}+a\right)}\frac{\left(-\_R^{6}+3\_R^{4}-3\_R^{2}+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{7}a-3\_R^{5}a+3\_R^{3}a-8\_R^{3}b-\_Ra}}{4d}$
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_Z^{6}+(6a-16b)\_Z^{4}-4a\_Z^{2}+a\right)}\frac{\left(-\_R^{6}+3\_R^{4}-3\_R^{2}+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{7}a-3\_R^{5}a+3\_R^{3}a-8\_R^{3}b-\_Ra}}{4d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(256a^{4}d^{4}-256b\,d^{4}a^{3})\_Z^{4}-32a^{2}d^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{2dx+2c}+\left(-\frac{128d^{3}a^{4}}{b}+128a^{3}d^{3}\right)\_R^{3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*sum((-_R^6+3*_R^4-3*_R^2+1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*
ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z
^2+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sinh(d*x + c)^4 - a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(79) = 158$.
time = 0.42, size = 975, normalized size = 8.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a
^2 - a*b)*d^2))*log(2*(a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d
^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c
)^2 + 2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*b*d)
*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a^2 -
 a*b)*d^2)) - b) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b
^2)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(2*(a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*
a^4*b + a^3*b^2)*d^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c
```

```
) + b*sinh(d*x + c)^2 - 2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b
^2)*d^4)) - a*b*d)*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*
d^4)) + 1)/((a^2 - a*b)*d^2)) - b) + 1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^
5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(-2*(a^3 - a^2*b)*d
^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*((a^4 - a^3*b)*d^3*sqrt(b/((a
^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5
- 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2)) - b) - 1/4*sqrt(-((a^2 -
 a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))*l
og(-2*(a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - 2*((a^4 -
a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*sqrt(-((a^2 - a
*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2)) - b
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 1, normalized size = 0.01

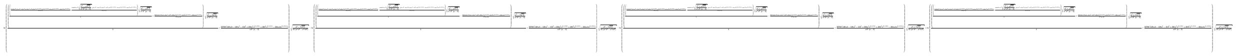
$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 0
```

Mupad [**B**]
time = 10.10, size = 1787, normalized size = 15.54



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log((((((524288*d^2*(31*a*b^2 - 128*a^2*b + 128*a^3 - b^3 + 256*a^3*exp(2*c
 + 2*d*x) + b^3*exp(2*c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*b*ex
p(2*c + 2*d*x)))/(b^5*(a - b)) + (1048576*a*d^3*((a^2 - (a^3*b)^(1/2))/(a^3
```

```
*d^2*(a - b)))^(1/2)*(45*a*b^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c
 + 2*d*x) - 50*a*b^2*exp(2*c + 2*d*x) + 48*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a
 - b)))*((a^2 - (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2))/4 + (262144*d*(72*
a*b - 64*a^2 - 9*b^2 + 256*a^2*exp(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) -
 288*a*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*((a^2 - (a^3*b)^(1/2))/(a^3*d^2*
(a - b)))^(1/2))/4 + (32768*(128*a*b - 128*a^2 - 15*b^2 + 256*a^2*exp(2*c +
 2*d*x) + 29*b^2*exp(2*c + 2*d*x) - 304*a*b*exp(2*c + 2*d*x)))/(a*b^5*(a -
b)))*((a^2 - (a^3*b)^(1/2))/(16*(a^4*d^2 - a^3*b*d^2)))^(1/2) - log(((((52
4288*d^2*(31*a*b^2 - 128*a^2*b + 128*a^3 - b^3 + 256*a^3*exp(2*c + 2*d*x) +
 b^3*exp(2*c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*b*exp(2*c + 2*d
*x)))/(b^5*(a - b)) - (1048576*a*d^3*((a^2 - (a^3*b)^(1/2))/(a^3*d^2*(a - b
)))^(1/2)*(45*a*b^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c + 2*d*x) -
 50*a*b^2*exp(2*c + 2*d*x) + 48*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*((a
^2 - (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2))/4 - (262144*d*(72*a*b - 64*a^
2 - 9*b^2 + 256*a^2*exp(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) - 288*a*b*ex
p(2*c + 2*d*x)))/(b^5*(a - b)))*((a^2 - (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(
1/2))/4 + (32768*(128*a*b - 128*a^2 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 2
9*b^2*exp(2*c + 2*d*x) - 304*a*b*exp(2*c + 2*d*x)))/(a*b^5*(a - b)))*((a^2
- (a^3*b)^(1/2))/(16*(a^4*d^2 - a^3*b*d^2)))^(1/2) - log(((((524288*d^2*(3
1*a*b^2 - 128*a^2*b + 128*a^3 - b^3 + 256*a^3*exp(2*c + 2*d*x) + b^3*exp(2*
c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*b*exp(2*c + 2*d*x)))/(b^5*
(a - b)) - (1048576*a*d^3*((a^2 + (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2)*(
45*a*b^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c + 2*d*x) - 50*a*b^2*e
xp(2*c + 2*d*x) + 48*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*((a^2 + (a^3*b
)^(1/2))/(a^3*d^2*(a - b)))^(1/2))/4 - (262144*d*(72*a*b - 64*a^2 - 9*b^2 +
 256*a^2*exp(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d
*x)))/(b^5*(a - b)))*((a^2 + (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2))/4 + (
32768*(128*a*b - 128*a^2 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2*exp(2
*c + 2*d*x) - 304*a*b*exp(2*c + 2*d*x)))/(a*b^5*(a - b)))*((a^2 + (a^3*b)^(
1/2))/(16*(a^4*d^2 - a^3*b*d^2)))^(1/2) + log(((((524288*d^2*(31*a*b^2 - 1
28*a^2*b + 128*a^3 - b^3 + 256*a^3*exp(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x)
+ 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a - b)) +
(1048576*a*d^3*((a^2 + (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2)*(45*a*b^2 -
104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c + 2*d*x) - 50*a*b^2*exp(2*c + 2*
d*x) + 48*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*((a^2 + (a^3*b)^(1/2))/(a
^3*d^2*(a - b)))^(1/2))/4 + (262144*d*(72*a*b - 64*a^2 - 9*b^2 + 256*a^2*ex
p(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d*x)))/(b^5*
(a - b)))*((a^2 + (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2))/4 + (32768*(128*
a*b - 128*a^2 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2*exp(2*c + 2*d*x)
 - 304*a*b*exp(2*c + 2*d*x)))/(a*b^5*(a - b)))*((a^2 + (a^3*b)^(1/2))/(16*(
a^4*d^2 - a^3*b*d^2)))^(1/2)
```

3.239 $\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=139

$$-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{\coth(c+dx)}{ad}$$

```
[Out] -coth(d*x+c)/a/d-1/2*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*b
^(1/2)/a^(5/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*arctanh((a^(1/2)+b^(1/2))^(1/2
)*tanh(d*x+c)/a^(1/4))*b^(1/2)/a^(5/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3296, 1301, 1144, 214}

$$-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/2*(Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(a^
(5/4)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + (Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]
]*Tanh[c + d*x])/a^(1/4)])/(2*a^(5/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - Coth[c +
 d*x]/(a*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^2(a-2ax^2+(a-b)x^4)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{bx^2}{a(a-2ax^2+(a-b)x^4)}\right)\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{\coth(c+dx)}{ad}+\frac{b\operatorname{Subst}\left(\int \frac{x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{ad} \\
&= -\frac{\coth(c+dx)}{ad}+\frac{\left(\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}\right)\operatorname{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tanh\right.}{2ad} \\
&= -\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+}{\sqrt[4]{a}}\right.}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}
\end{aligned}$$

Mathematica [A]
time = 0.59, size = 143, normalized size = 1.03

$$\frac{\frac{\sqrt{b}\operatorname{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-2\coth(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] ((Sqrt[b]*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt
[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]*ArcTanh[((Sqrt[a] + Sqrt[b])*T
anh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - 2*Cot
h[c + d*x])/(2*a*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.97, size = 130, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{b\Bigg(\sum_{\_\_R=\mathrm{RootOf}\left(a\_\_Z^{8}-4a\_\_Z^{6}+(6a-16b)\_\_Z^{4}-4a\_\_Z^{2}+a\right)}\frac{\left(\_\_R^{4}-\_\_R^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_\_R\right)}{\_\_R^{7}a-3\_\_R^{5}a+3\_\_R^{3}a-8\_\_R^{3}b-\_\_Ra}}{a}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{b\Bigg(\sum_{\_\_R=\mathrm{RootOf}\left(a\_\_Z^{8}-4a\_\_Z^{6}+(6a-16b)\_\_Z^{4}-4a\_\_Z^{2}+a\right)}\frac{\left(\_\_R^{4}-\_\_R^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_\_R\right)}{\_\_R^{7}a-3\_\_R^{5}a+3\_\_R^{3}a-8\_\_R^{3}b-\_\_Ra}}{a}}{d}$
risch	$-\frac{2}{da\left(\mathrm{e}^{2dx+2c}-1\right)}+4\Bigg(\sum_{\_\_R=\mathrm{RootOf}\left(\left(65536a^{6}d^{4}-65536a^{5}b\,d^{4}\right)\_\_Z^{4}-512a^{3}d^{2}\_\_Z^{2}b+b^{2}\right)}\_\_R\ln\Big(\mathrm{e}^{2dx+2c}+\Big($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-1/a*b*sum((_R^4-_R^2)/(_R^7*a-3*_R^5*a+3*_R
^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6
*a-16*b)*_Z^4-4*a*_Z^2+a))-1/2/a/tanh(1/2*d*x+1/2*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(6*d*x + 6*c) - 2*b*e^(4*
d*x + 4*c) + b*e^(2*d*x + 2*c))/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c
) - 4*a*b*e^(2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*
x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. $2(99) = 198$.
time = 0.43, size = 1305, normalized size = 9.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d
*x + c)^2 - a*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2
)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x
+ c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + 2*(a^4 - a^3*b)*d^2*sqrt(b^3/((a
^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 + 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 -
 2*a^6*b + a^5*b^2)*d^4)) - a^2*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7
 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))) - (a*d*cosh(d*x + c)
^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)*sqrt(((
a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2
*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2
*sinh(d*x + c)^2 + 2*(a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*
d^4)) - b^2 - 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)
) - a^2*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^
4)) + b)/((a^3 - a^2*b)*d^2))) - (a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)
*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b
^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(
d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - 2*(a
^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 + 2*((a^5 -
 a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + a^2*b*d)*sqrt(-((a^
3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b
)*d^2))) + (a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*s
inh(d*x + c)^2 - a*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a
^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cos
h(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - 2*(a^4 - a^3*b)*d^2*sqrt(b
^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 - 2*((a^5 - a^4*b)*d^3*sqrt(b^3/(
(a^7 - 2*a^6*b + a^5*b^2)*d^4)) + a^2*b*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^
3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))) + 8)/(a*d*cos
h(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*
d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 21, normalized size = 0.15

$$-\frac{2}{ad(e^{(2\,dx+2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -2/(a*d*(e^(2*d*x + 2*c) - 1))
```

Mupad [**B**]
time = 11.29, size = 2128, normalized size = 15.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)),x)
```

```
[Out] log((((((4194304*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2
+ b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2
*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(a^2*b^4*(a - b)^2) - (16777216*d^3
*(((a^5*b^3)^(1/2) + a^3*b)/(a^5*d^2*(a - b)))^(1/2)*(40*a*b^2 - 35*b^3 + 5
12*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*exp(2*c + 2*d
*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*(((a^5*b^3)^(1/2) + a^3*b
)/(a^5*d^2*(a - b)))^(1/2))/4 - (2097152*d*(256*a^2*b - 256*a*b^2 - 5*b^3 -
 1024*a^3*exp(2*c + 2*d*x) + 6*b^3*exp(2*c + 2*d*x) + 756*a*b^2*exp(2*c + 2
*d*x) + 256*a^2*b*exp(2*c + 2*d*x)))/(a^3*b^4*(a - b)))*(((a^5*b^3)^(1/2) +
 a^3*b)/(a^5*d^2*(a - b)))^(1/2))/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*
a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*
b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*d*x)))/(a^4*b^3*(a - b)^2))*(
((a^5*b^3)^(1/2) + a^3*b)/(16*(a^6*d^2 - a^5*b*d^2)))^(1/2) - log((((((4194
304*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c
 + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2*d*x) - 1392*
a^2*b^2*exp(2*c + 2*d*x)))/(a^2*b^4*(a - b)^2) + (16777216*d^3*(((a^5*b^3)^
(1/2) + a^3*b)/(a^5*d^2*(a - b)))^(1/2)*(40*a*b^2 - 35*b^3 + 512*a^3*exp(2*
c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*exp(2*c + 2*d*x) - 896*a^2
*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*(((a^5*b^3)^(1/2) + a^3*b)/(a^5*d^2*(a
 - b)))^(1/2))/4 + (2097152*d*(256*a^2*b - 256*a*b^2 - 5*b^3 - 1024*a^3*exp
(2*c + 2*d*x) + 6*b^3*exp(2*c + 2*d*x) + 756*a*b^2*exp(2*c + 2*d*x) + 256*a
^2*b*exp(2*c + 2*d*x)))/(a^3*b^4*(a - b)))*(((a^5*b^3)^(1/2) + a^3*b)/(a^5*
d^2*(a - b)))^(1/2))/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3
```

```
- 1024*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c +
 2*d*x) + 2048*a^2*b*exp(2*c + 2*d*x)))/(a^4*b^3*(a - b)^2))*(((a^5*b^3)^(1
/2) + a^3*b)/(16*(a^6*d^2 - a^5*b*d^2)))^(1/2) + log((((((4194304*d^2*(512*
a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 6
27*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2
*c + 2*d*x)))/(a^2*b^4*(a - b)^2) - (16777216*d^3*(-((a^5*b^3)^(1/2) - a^3*
b)/(a^5*d^2*(a - b)))^(1/2)*(40*a*b^2 - 35*b^3 + 512*a^3*exp(2*c + 2*d*x) +
 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*exp(2*c + 2*d*x) - 896*a^2*b*exp(2*c +
 2*d*x)))/(b^5*(a - b)))*(-((a^5*b^3)^(1/2) - a^3*b)/(a^5*d^2*(a - b)))^(1/
2))/4 - (2097152*d*(256*a^2*b - 256*a*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*
x) + 6*b^3*exp(2*c + 2*d*x) + 756*a*b^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*
c + 2*d*x)))/(a^3*b^4*(a - b)))*(-((a^5*b^3)^(1/2) - a^3*b)/(a^5*d^2*(a - b
)))^(1/2))/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3
*exp(2*c + 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) +
2048*a^2*b*exp(2*c + 2*d*x)))/(a^4*b^3*(a - b)^2))*(-((a^5*b^3)^(1/2) - a^3
*b)/(16*(a^6*d^2 - a^5*b*d^2)))^(1/2) - log((((((4194304*d^2*(512*a^4 - 118
4*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*
exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*
x)))/(a^2*b^4*(a - b)^2) + (16777216*d^3*(-((a^5*b^3)^(1/2) - a^3*b)/(a^5*d
^2*(a - b)))^(1/2)*(40*a*b^2 - 35*b^3 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*e
xp(2*c + 2*d*x) + 326*a*b^2*exp(2*c + 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))
/(b^5*(a - b)))*(-((a^5*b^3)^(1/2) - a^3*b)/(a^5*d^2*(a - b)))^(1/2))/4 + (
2097152*d*(256*a^2*b - 256*a*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*x) + 6*b^
3*exp(2*c + 2*d*x) + 756*a*b^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*c + 2*d*x
)))/(a^3*b^4*(a - b)))*(-((a^5*b^3)^(1/2) - a^3*b)/(a^5*d^2*(a - b)))^(1/2)
)/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c
+ 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*
b*exp(2*c + 2*d*x)))/(a^4*b^3*(a - b)^2))*(-((a^5*b^3)^(1/2) - a^3*b)/(16*(
a^6*d^2 - a^5*b*d^2)))^(1/2) - 2/(a*d*(exp(2*c + 2*d*x) - 1))
```

3.240 $\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}+\frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}$$

```
[Out] coth(d*x+c)/a/d-1/3*coth(d*x+c)^3/a/d+1/2*b*arctanh((a^(1/2)-b^(1/2))^(1/2)
*tanh(d*x+c)/a^(1/4))/a^(7/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b*arctanh((a^(1
/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(7/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3296, 1301, 1180, 214}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\coth^3(c+dx)}{3ad}+\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (b*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqr
t[Sqrt[a] - Sqrt[b]]*d) + (b*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x]
)/a^(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) + Coth[c + d*x]/(a*d) - C
oth[c + d*x]^3/(3*a*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{x^4(a-2ax^2+(a-b)x^4)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{1}{ax^2}+\frac{b-bx^2}{a(a-2ax^2+(a-b)x^4)}\right)dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}+\frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{ad}\\
&= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{\left(\left(\sqrt{a}+\sqrt{b}\right)b\right)\operatorname{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x}\right.}{2a^{3/2}d}\\
&= \frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}+
\end{aligned}$$

Mathematica [A]
time = 1.71, size = 165, normalized size = 1.11

$$\frac{-\frac{3b\operatorname{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{3b\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}+4\sqrt{a}\coth(c+dx)-2\sqrt{a}\coth(c+dx)\operatorname{csch}^2(c+dx)}{6a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] ((-3*b*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]
]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (3*b*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c +
 d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + 4*Sqrt[a]*Co
th[c + d*x] - 2*Sqrt[a]*Coth[c + d*x]*Csch[c + d*x]^2)/(6*a^(3/2)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.07, size = 168, normalized size = 1.14

method	result
derivativedivides	$\frac{-\frac{\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(\_R^6-3\_R^4+3\_R^2\right.}{\_R^7a-3\_R^5a+3\_}\right.}{4a}}{d}$
default	$\frac{-\frac{\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(\_R^6-3\_R^4+3\_R^2\right.}{\_R^7a-3\_R^5a+3\_}\right.}{4a}}{d}$
risch	$-\frac{4\left(3\,\mathrm{e}^{2dx+2c}-1\right)}{3ad\left(\mathrm{e}^{2dx+2c}-1\right)^3}+16\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16777216a^8d^4-16777216a^7b\,d^4\right)\_Z^4-8192a^4d^2\_Z^2b^2+b^4\right)}\_R\ln\left(\mathrm{e}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/a*(1/3*tanh(1/2*d*x+1/2*c)^3-3*tanh(1/2*d*x+1/2*c))-1/4/a*b*sum((
_R^6-3*_R^4+3*_R^2-1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*
d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))-1/24/
a/tanh(1/2*d*x+1/2*c)^3+3/8/a/tanh(1/2*d*x+1/2*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -16*b*integrate(e^(4*d*x + 4*c)/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c
) - 4*a*b*e^(2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*
x)), x) - 4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*e^(6*d*x + 6*c) - 3*a*d*e^(4*d*x
 + 4*c) + 3*a*d*e^(2*d*x + 2*c) - a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2206 vs. $2(110) = 220$.

time = 0.44, size = 2206, normalized size = 14.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*d*si
nh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*cosh(d*x + c)^2 - a*d)*sin
h(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cosh(d*x + c)^3 - 3*a*d*cos
h(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c)^4 - 6*a*d*cosh(d*x + c
)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x + c)^5 - 2*a*d*cosh(d*x
+ c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5
/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(b^4*cosh(
d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x + c) + b^4*sinh(d*x + c)^2 - b^4
+ 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + 2*(a^
2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt
(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4
- a^3*b)*d^2))) - 3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*sinh(d*x + c
)^5 + a*d*sinh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*cosh(d*x + c)^
2 - a*d)*sinh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cosh(d*x + c)^3
 - 3*a*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c)^4 - 6*a*d*
cosh(d*x + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x + c)^5 - 2*a
*d*cosh(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 - a^3*b)*
d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*l
og(b^4*cosh(d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x + c) + b^4*sinh(d*x +
 c)^2 - b^4 + 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d
^4)) - 2*(a^2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)
*d^4)))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) +
 b^2)/((a^4 - a^3*b)*d^2))) + 3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*
sinh(d*x + c)^5 + a*d*sinh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*co
sh(d*x + c)^2 - a*d)*sinh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cos
h(d*x + c)^3 - 3*a*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c
)^4 - 6*a*d*cosh(d*x + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x
+ c)^5 - 2*a*d*cosh(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((
a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^2)/((a^4 - a
^3*b)*d^2))*log(b^4*cosh(d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x + c) + b
^4*sinh(d*x + c)^2 - b^4 - 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9 - 2*a^8*b
 + a^7*b^2)*d^4)) + 2*(a^2*b^3*d + (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8
*b + a^7*b^2)*d^4)))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^
7*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) - 3*(a*d*cosh(d*x + c)^6 + 6*a*d*
cosh(d*x + c)*sinh(d*x + c)^5 + a*d*sinh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4
 + 3*(5*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2
+ 4*(5*a*d*cosh(d*x + c)^3 - 3*a*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*
d*cosh(d*x + c)^4 - 6*a*d*cosh(d*x + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*
```

```
(a*d*cosh(d*x + c)^5 - 2*a*d*cosh(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) -
b^2)/((a^4 - a^3*b)*d^2))*log(b^4*cosh(d*x + c)^2 + 2*b^4*cosh(d*x + c)*sin
h(d*x + c) + b^4*sinh(d*x + c)^2 - b^4 - 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/(
(a^9 - 2*a^8*b + a^7*b^2)*d^4)) - 2*(a^2*b^3*d + (a^7 - a^6*b)*d^3*sqrt(b^5
/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9
- 2*a^8*b + a^7*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) - 48*cosh(d*x + c)^
2 - 96*cosh(d*x + c)*sinh(d*x + c) - 48*sinh(d*x + c)^2 + 16)/(a*d*cosh(d*x
 + c)^6 + 6*a*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*d*sinh(d*x + c)^6 - 3*a*d
*cosh(d*x + c)^4 + 3*(5*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^4 + 3*a*d*
cosh(d*x + c)^2 + 4*(5*a*d*cosh(d*x + c)^3 - 3*a*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(5*a*d*cosh(d*x + c)^4 - 6*a*d*cosh(d*x + c)^2 + a*d)*sinh(d*x +
 c)^2 - a*d + 6*(a*d*cosh(d*x + c)^5 - 2*a*d*cosh(d*x + c)^3 + a*d*cosh(d*x
 + c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 34, normalized size = 0.23

$$-\frac{4\left(3\,e^{(2\,dx+2\,c)}-1\right)}{3\,ad(e^{(2\,dx+2\,c)}-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*(e^(2*d*x + 2*c) - 1)^3)
```

Mupad [**B**]
time = 12.33, size = 2178, normalized size = 14.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a - b*sinh(c + d*x)^4)),x)
```

```
[Out] log((((((4194304*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2
+ b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2
```

```
*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(a^4*b^2*(a - b)^2) + (8388608*d^3*
(((a^7*b^5)^(1/2) + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2)*(181*a*b^2 - 432*a^2*
b + 256*a^3 + 5*b^3 - 512*a^3*exp(2*c + 2*d*x) - 6*b^3*exp(2*c + 2*d*x) - 6
22*a*b^2*exp(2*c + 2*d*x) + 1152*a^2*b*exp(2*c + 2*d*x)))/(a^2*b^3*(a - b))
)*(((a^7*b^5)^(1/2) + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2))/4 + (2097152*d*(17
6*a*b - 256*a^2 + 75*b^2 + 1536*a^2*exp(2*c + 2*d*x) - 134*b^2*exp(2*c + 2*
d*x) - 1408*a*b*exp(2*c + 2*d*x)))/(a^5*b*(a - b)))*(((a^7*b^5)^(1/2) + a^4
*b^2)/(a^7*d^2*(a - b)))^(1/2))/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*a^
3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*b^
2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*d*x)))/(a^7*(a - b)^2))*(((a^7*
b^5)^(1/2) + a^4*b^2)/(16*(a^8*d^2 - a^7*b*d^2)))^(1/2) - log(((((4194304*
d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2
*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*
b^2*exp(2*c + 2*d*x)))/(a^4*b^2*(a - b)^2) - (8388608*d^3*(((a^7*b^5)^(1/2)
 + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2)*(181*a*b^2 - 432*a^2*b + 256*a^3 + 5*b
^3 - 512*a^3*exp(2*c + 2*d*x) - 6*b^3*exp(2*c + 2*d*x) - 622*a*b^2*exp(2*c
+ 2*d*x) + 1152*a^2*b*exp(2*c + 2*d*x)))/(a^2*b^3*(a - b)))*(((a^7*b^5)^(1/
2) + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2))/4 - (2097152*d*(176*a*b - 256*a^2 +
 75*b^2 + 1536*a^2*exp(2*c + 2*d*x) - 134*b^2*exp(2*c + 2*d*x) - 1408*a*b*e
xp(2*c + 2*d*x)))/(a^5*b*(a - b)))*(((a^7*b^5)^(1/2) + a^4*b^2)/(a^7*d^2*(a
 - b)))^(1/2))/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024
*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x
) + 2048*a^2*b*exp(2*c + 2*d*x)))/(a^7*(a - b)^2))*(((a^7*b^5)^(1/2) + a^4*
b^2)/(16*(a^8*d^2 - a^7*b*d^2)))^(1/2) - log(((((4194304*d^2*(512*a^4 - 11
84*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3
*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d
*x)))/(a^4*b^2*(a - b)^2) - (8388608*d^3*(-((a^7*b^5)^(1/2) - a^4*b^2)/(a^7
*d^2*(a - b)))^(1/2)*(181*a*b^2 - 432*a^2*b + 256*a^3 + 5*b^3 - 512*a^3*exp
(2*c + 2*d*x) - 6*b^3*exp(2*c + 2*d*x) - 622*a*b^2*exp(2*c + 2*d*x) + 1152*
a^2*b*exp(2*c + 2*d*x)))/(a^2*b^3*(a - b)))*(-((a^7*b^5)^(1/2) - a^4*b^2)/(
a^7*d^2*(a - b)))^(1/2))/4 - (2097152*d*(176*a*b - 256*a^2 + 75*b^2 + 1536*
a^2*exp(2*c + 2*d*x) - 134*b^2*exp(2*c + 2*d*x) - 1408*a*b*exp(2*c + 2*d*x)
))/(a^5*b*(a - b)))*(-((a^7*b^5)^(1/2) - a^4*b^2)/(a^7*d^2*(a - b)))^(1/2))
/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c +
 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b
*exp(2*c + 2*d*x)))/(a^7*(a - b)^2))*(-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^8
*d^2 - a^7*b*d^2)))^(1/2) + log(((((4194304*d^2*(512*a^4 - 1184*a^3*b - 25
3*a*b^3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*
d*x) + 768*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(a^4*b^
2*(a - b)^2) + (8388608*d^3*(-((a^7*b^5)^(1/2) - a^4*b^2)/(a^7*d^2*(a - b))
)^(1/2)*(181*a*b^2 - 432*a^2*b + 256*a^3 + 5*b^3 - 512*a^3*exp(2*c + 2*d*x)
 - 6*b^3*exp(2*c + 2*d*x) - 622*a*b^2*exp(2*c + 2*d*x) + 1152*a^2*b*exp(2*c
 + 2*d*x)))/(a^2*b^3*(a - b)))*(-((a^7*b^5)^(1/2) - a^4*b^2)/(a^7*d^2*(a -
b)))^(1/2))/4 + (2097152*d*(176*a*b - 256*a^2 + 75*b^2 + 1536*a^2*exp(2*c +
 2*d*x) - 134*b^2*exp(2*c + 2*d*x) - 1408*a*b*exp(2*c + 2*d*x)))/(a^5*b*(a
```

```
- b)))*(-((a^7*b^5)^(1/2) - a^4*b^2)/(a^7*d^2*(a - b)))^(1/2))/4 - (524288*
(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*
b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*
d*x)))/(a^7*(a - b)^2))*(-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^8*d^2 - a^7*b*
d^2)))^(1/2) - 4/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - 8/(3*a
*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
```

3.241 $\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=235

$$-\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{9/4}d}-\frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{9/4}d}+\frac{\cosh(c}{b^2}$$

```
[Out] cosh(d*x+c)/b^2/d+1/4*a*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/(a-b)/b^2/d/(a-b+
2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)-1/8*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-
b^(1/2))^(1/2))*a^(1/2)*(5*a^(1/2)-6*b^(1/2))/b^(9/4)/d/(a^(1/2)-b^(1/2))^(
3/2)-1/8*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*a^(1/2)*(5*a^
(1/2)+6*b^(1/2))/b^(9/4)/d/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A]
time = 0.37, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3294, 1219, 1690, 1180, 211, 214}

$$-\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b^2d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}+\frac{\cosh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/8*(Sqrt[a]*(5*Sqrt[a] - 6*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[S
qrt[a] - Sqrt[b]]])/((Sqrt[a] - Sqrt[b])^(3/2)*b^(9/4)*d) - (Sqrt[a]*(5*Sqr
t[a] + 6*Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])
/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(9/4)*d) + Cosh[c + d*x]/(b^2*d) + (a*Cosh[
c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(4*(a - b)*b^2*d*(a - b + 2*b*Cosh[c
+ d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{(a-b+2bx^2-bx^4)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\Big(\int \frac{2a\left(a+\frac{a^2}{b}\right.}{}}{}$$

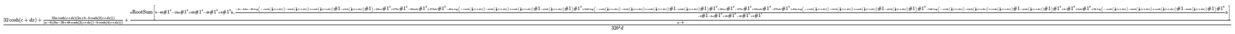
$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\Big(\int \Big(-\frac{8a(a-}{b}}{}$$

$$= \frac{\cosh(c+dx)}{b^2d} + \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{S}}{}$$

$$= \frac{\cosh(c+dx)}{b^2d} + \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \frac{\Big(}{}$$

$$= -\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{9/4}d} - \frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tanh}{8\left(\sqrt{a}+\sqrt{b}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.70, size = 615, normalized size = 2.62



Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (32*Cosh[c + d*x] + (32*a*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/((
a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (a*Root
Sum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(b*c) - b
*d*x - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#
1 - Sinh[(c + d*x)/2]*#1] - 20*a*c*#1^2 + 27*b*c*#1^2 - 20*a*d*x*#1^2 + 27*
b*d*x*#1^2 - 40*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*
x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 54*b*Log[-Cosh[(c + d*x)/2] - Sinh[
(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 20*a*c*#
1^4 - 27*b*c*#1^4 + 20*a*d*x*#1^4 - 27*b*d*x*#1^4 + 40*a*Log[-Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^
```

```
4 - 54*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
- Sinh[(c + d*x)/2]*#1]*#1^4 + b*c*#1^6 + b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(32*
b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(185) = 370$.

time = 5.93, size = 373, normalized size = 1.59

method	result
derivativedivides	$-\frac{2a\left(\frac{\frac{(-2b+a)\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a-4b}-\frac{(3a-8b)\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a-b)}+\frac{(3a+2b)\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a-4b}-\frac{a}{4(a-b)}}{a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+a\left(-\frac{\left(\sqrt{a\right.}}{}\right.\right.}{}$
default	$-\frac{2a\left(\frac{\frac{(-2b+a)\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a-4b}-\frac{(3a-8b)\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a-b)}+\frac{(3a+2b)\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a-4b}-\frac{a}{4(a-b)}}{a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+a\left(-\frac{\left(\sqrt{a\right.}}{}\right.\right.}{}$
risch	$\frac{e^{dx+c}}{2b^{2}d}+\frac{e^{-dx-c}}{2b^{2}d}+\frac{a\,e^{dx+c}\left(-b\,e^{6dx+6c}+4a\,e^{4dx+4c}+b\,e^{4dx+4c}+4a\,e^{2dx+2c}+b\,e^{2dx+2c}-b\right)}{2b^{2}(a-b)d\left(-b\,e^{8dx+8c}+4b\,e^{6dx+6c}+16a\,e^{4dx+4c}-6b\,e^{4dx+4c}+4b\,e^{2dx+2c}-b\right)}+\left(_{\_R=\mathrm{RootC}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a/b^2*((1/4*(-2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^6-1/4*(3*a-8*b)/(a-b
)*tanh(1/2*d*x+1/2*c)^4+1/4*(3*a+2*b)/(a-b)*tanh(1/2*d*x+1/2*c)^2-1/4*a/(a-
b))/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2
*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)+1/4/(a-b)*a*(
-1/4*((a*b)^(1/2)+5*a-6*b)/a/(-(a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(-2*a*ta
nh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(a*b)^(1/2)*a-a*b)^(1/2))+1/4*(-(a
*b)^(1/2)+5*a-6*b)/a/((a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x
```

```
+1/2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2))))-1/b^2/(tanh(1/2*d
*x+1/2*c)-1)+1/b^2/(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b - b^2 + (a*b*e^(10*c) - b^2*e^(10*c))*e^(10*d*x) - (2*a*b*e^(8*c)
- 3*b^2*e^(8*c))*e^(8*d*x) - (20*a^2*e^(6*c) - 17*a*b*e^(6*c) + 2*b^2*e^(6*
c))*e^(6*d*x) - (20*a^2*e^(4*c) - 17*a*b*e^(4*c) + 2*b^2*e^(4*c))*e^(4*d*x)
 - (2*a*b*e^(2*c) - 3*b^2*e^(2*c))*e^(2*d*x))/((a*b^3*d*e^(9*c) - b^4*d*e^(
9*c))*e^(9*d*x) - 4*(a*b^3*d*e^(7*c) - b^4*d*e^(7*c))*e^(7*d*x) - 2*(8*a^2*
b^2*d*e^(5*c) - 11*a*b^3*d*e^(5*c) + 3*b^4*d*e^(5*c))*e^(5*d*x) - 4*(a*b^3*
d*e^(3*c) - b^4*d*e^(3*c))*e^(3*d*x) + (a*b^3*d*e^c - b^4*d*e^c)*e^(d*x)) +
 1/512*integrate(256*(a*b*e^(7*d*x + 7*c) - a*b*e^(d*x + c) + (20*a^2*e^(5*
c) - 27*a*b*e^(5*c))*e^(5*d*x) - (20*a^2*e^(3*c) - 27*a*b*e^(3*c))*e^(3*d*x
))/(a*b^3 - b^4 + (a*b^3*e^(8*c) - b^4*e^(8*c))*e^(8*d*x) - 4*(a*b^3*e^(6*c
) - b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b^2*e^(4*c) - 11*a*b^3*e^(4*c) + 3*b^
4*e^(4*c))*e^(4*d*x) - 4*(a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7664 vs. $2(187) = 374$.
time = 0.58, size = 7664, normalized size = 32.61

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*(a*b - b^2)*cosh(d*x + c)^10 + 80*(a*b - b^2)*cosh(d*x + c)*sinh(d*
x + c)^9 + 8*(a*b - b^2)*sinh(d*x + c)^10 - 8*(2*a*b - 3*b^2)*cosh(d*x + c)
^8 + 8*(45*(a*b - b^2)*cosh(d*x + c)^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^8 + 6
4*(15*(a*b - b^2)*cosh(d*x + c)^3 - (2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^7 - 8*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^6 + 8*(210*(a*b - b^2)*
cosh(d*x + c)^4 - 28*(2*a*b - 3*b^2)*cosh(d*x + c)^2 - 20*a^2 + 17*a*b - 2*
b^2)*sinh(d*x + c)^6 + 16*(126*(a*b - b^2)*cosh(d*x + c)^5 - 28*(2*a*b - 3*
b^2)*cosh(d*x + c)^3 - 3*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^5 - 8*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^4 + 8*(210*(a*b - b^2)*c
osh(d*x + c)^6 - 70*(2*a*b - 3*b^2)*cosh(d*x + c)^4 - 15*(20*a^2 - 17*a*b +
 2*b^2)*cosh(d*x + c)^2 - 20*a^2 + 17*a*b - 2*b^2)*sinh(d*x + c)^4 + 32*(30
*(a*b - b^2)*cosh(d*x + c)^7 - 14*(2*a*b - 3*b^2)*cosh(d*x + c)^5 - 5*(20*a
^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^3 - (20*a^2 - 17*a*b + 2*b^2)*cosh(d*x +
```

```
 c))*sinh(d*x + c)^3 - 8*(2*a*b - 3*b^2)*cosh(d*x + c)^2 + 8*(45*(a*b - b^2
)*cosh(d*x + c)^8 - 28*(2*a*b - 3*b^2)*cosh(d*x + c)^6 - 15*(20*a^2 - 17*a*
b + 2*b^2)*cosh(d*x + c)^4 - 6*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^2 -
2*a*b + 3*b^2)*sinh(d*x + c)^2 + ((a*b^3 - b^4)*d*cosh(d*x + c)^9 + 9*(a*b^
3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a*b^3 - b^4)*d*sinh(d*x + c)^9
- 4*(a*b^3 - b^4)*d*cosh(d*x + c)^7 + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^2
- (a*b^3 - b^4)*d)*sinh(d*x + c)^7 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos
h(d*x + c)^5 + 28*(3*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh
(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 42*(a*
b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x +
 c)^5 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^3 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x
+ c)^5 - 70*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b
^4)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a*b^3 - b^4)*d*cosh(d*x + c)^
6 - 35*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d
*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*cosh(
d*x + c) + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^7 - 21*(a*b^3 - b^4)*d*cosh(d
*x + c)^5 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 -
 b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*(a*b^3 - b^4)*d*cosh(d*x + c)^8
 - 28*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 10*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d
*cosh(d*x + c)^4 - 12*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + (a*b^3 - b^4)*d)*si
nh(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7
 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a
^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4))
+ 15*a^3 - 47*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2)
)*log(-625*a^5 + 2625*a^4*b - 3684*a^3*b^2 + 1728*a^2*b^3 - (625*a^5 - 2625
*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cosh(d*x + c)^2 - 2*(625*a^5 - 2625*a
^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c) - (625*a^5
- 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*sinh(d*x + c)^2 + 2*((125*a^5*b
^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d*cosh(d*x + c) + (125*a^5*b^
2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d*sinh(d*x + c) - 2*((2*a^4*b^
7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*cosh(d*x + c) + (2*a^4
*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sinh(d*x + c))*sqrt
((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*
b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^1
5)*d^4)))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 -
3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*
b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 1
5*a^3 - 47*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2)))
- ((a*b^3 - b^4)*d*cosh(d*x + c)^9 + 9*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d
*x + c)^8 + (a*b^3 - b^4)*d*sinh(d*x + c)^9 - 4*(a*b^3 - b^4)*d*cosh(d*x +
c)^7 + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c
)^7 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^5 + 28*(3*(a*b^3 - b
^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*
(63*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 42*(a*b^3 - b^4)*d*cosh(d*x + c)^2 -
(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^5 - 4*(a*b^3 - b^4)*d*cosh(
```

```
d*x + c)^3 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 70*(a*b^3 - b^4)*d*cos
h(d*x + c)^3 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x +
 c)^4 + 4*(21*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 35*(a*b^3 - b^4)*d*cosh(d*x
 + c)^4 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4
)*d)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*cosh(d*x + c) + 4*(9*(a*b^3 - b^4)*d
*cosh(d*x + c)^7 - 21*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 5*(8*a^2*b^2 - 11*a
*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x
 + c)^2 + (9*(a*b^3 - b^4)*d*cosh(d*x + c)^8 - 28*(a*b^3 - b^4)*d*cosh(d*x
+ c)^6 - 10*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*co...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**9/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. $2(187) = 374$.
time = 0.88, size = 1082, normalized size = 4.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*(((4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 5*sqrt(a*b)*sqrt(-b^2 +
sqrt(a*b)*b)*a*b)*(a*b^2 - b^3)^2*abs(b) + (20*sqrt(-b^2 + sqrt(a*b)*b)*a^4
*b^2 - 23*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^3 - 32*sqrt(-b^2 + sqrt(a*b)*b)*a^
2*b^4 + 35*sqrt(-b^2 + sqrt(a*b)*b)*a*b^5)*abs(-a*b^2 + b^3)*abs(b) - (20*s
qrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^4 - 39*sqrt(a*b)*sqrt(-b^2 + sqrt(a
*b)*b)*a^3*b^5 - 12*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^6 + 61*sqrt(a*
b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^7 - 30*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b
^8)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a*b^3 - b^4 + sq
rt((a^2*b^2 - 2*a*b^3 + b^4)*(a*b^3 - b^4) + (a*b^3 - b^4)^2))/(a*b^3 - b^4
)))/((4*a^4*b^6 - 7*a^3*b^7 - 3*a^2*b^8 + 11*a*b^9 - 5*b^10)*abs(-a*b^2 + b
^3)) - ((4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2 + 5*sqrt(a*b)*sqrt(-b^2 -
 sqrt(a*b)*b)*a*b)*(a*b^2 - b^3)^2*abs(b) - (20*sqrt(-b^2 - sqrt(a*b)*b)*a^
4*b^2 - 23*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^3 - 32*sqrt(-b^2 - sqrt(a*b)*b)*a
^2*b^4 + 35*sqrt(-b^2 - sqrt(a*b)*b)*a*b^5)*abs(-a*b^2 + b^3)*abs(b) - (20*
sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^4 - 39*sqrt(a*b)*sqrt(-b^2 - sqrt(
a*b)*b)*a^3*b^5 - 12*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^6 + 61*sqrt(a
*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^7 - 30*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*
```

```
b^8)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a*b^3 - b^4 - s
qrt((a^2*b^2 - 2*a*b^3 + b^4)*(a*b^3 - b^4) + (a*b^3 - b^4)^2))/(a*b^3 - b^
4)))/((4*a^4*b^6 - 7*a^3*b^7 - 3*a^2*b^8 + 11*a*b^9 - 5*b^10)*abs(-a*b^2 +
b^3)) - 4*(a*b*(e^(d*x + c) + e^(-d*x - c))^3 - 4*a^2*(e^(d*x + c) + e^(-d*
x - c)) - 4*a*b*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x -
c))^4 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)*(a*b^2 - b^3)) -
4*(e^(d*x + c) + e^(-d*x - c))/b^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^9}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^9/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^9/(a - b*sinh(c + d*x)^4)^2, x)
```

3.242 $\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

<u>**Optimal**</u>. Leaf size=210

$$\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{7/4}d}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{7/4}d}-\frac{a\cosh(}{4(a-b)bd\,(a-}$$

```
[Out] -1/4*a*cosh(d*x+c)*(2-cosh(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cosh(d*x+c)^2-b*cos
h(d*x+c)^4)+1/8*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a^(1
/2)-4*b^(1/2))/b^(7/4)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctanh(b^(1/4)*cosh(d
*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(3*a^(1/2)+4*b^(1/2))/b^(7/4)/d/(a^(1/2)+b^(
1/2))^(3/2)
```

<u>**Rubi**</u> [A]
time = 0.27, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3294, 1219, 1180, 211, 214}

$$\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4bd(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((3*Sqrt[a] - 4*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(7/4)*d) - ((3*Sqrt[a] + 4*Sqrt[b])*A
rcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*(Sqrt[a] + Sqrt
[b])^(3/2)*b^(7/4)*d) - (a*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*(a - b)*
b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 214</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

<u>Rule 1180</u>

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \frac{\text{Subst}\Big(\int \frac{4a(a-2}{a-}}{}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\Big(3a-\sqrt{a}\sqrt{b}}{}$$

$$= \frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{7/4}d} - \frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\Bigg(\frac{\sqrt[4]{}}{\sqrt{}}}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.49, size = 737, normalized size = 3.51

$$-\frac{-\frac{16a(-5\cosh(c+dx)+\cosh(3(c+dx)))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))}+\mathrm{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{3ac-4bc+3adx-4bdx+6a\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)-8b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)-5ac\#1^2+12bc\#1^2-5adx\#1^2+12bdx\#1^2-10a\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2+24b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2+5ac\#1^4-12bc\#1^4+5adx\#1^4-12bdx\#1^4+10a\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4-24b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4-3ac\#1^6+4bc\#1^6-3adx\#1^6+4bdx\#1^6-6a\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^6+8b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{32(a-b)bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/32*((-16*a*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh
[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6
*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (3*a*c - 4*b*c + 3*a*d*x - 4*b*d*x + 6*a*Lo
g[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
 d*x)/2]*#1] - 8*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 5*a*c*#1^2 + 12*b*c*#1^2 - 5*a*d*x*#1^2
 + 12*b*d*x*#1^2 - 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 24*b*Log[-Cosh[(c + d*x)/2] -
 Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 5*
a*c*#1^4 - 12*b*c*#1^4 + 5*a*d*x*#1^4 - 12*b*d*x*#1^4 + 10*a*Log[-Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]
*#1^4 - 24*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]
*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 3*a*c*#1^6 + 4*b*c*#1^6 - 3*a*d*x*#1^6 +
 4*b*d*x*#1^6 - 6*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 + 8*b*Log[-Cosh[(c + d*x)/2] - Sinh
[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1)
 - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/((a - b)*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(164) = 328$.
time = 5.59, size = 434, normalized size = 2.07

method	result
derivativedivides	$\frac{128a^2\left(\frac{-\frac{\left(ab-\sqrt{ab}\,a+2\sqrt{ab}\,b\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2(a-b)}-\frac{\sqrt{ab}+b}{2a(a-b)}}{\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4\sqrt{ab}\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+1}+\frac{\left(3\sqrt{ab}\,a-4\sqrt{ab}\,b-ab\right)\arctan\left(\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\right.\right.}{4\sqrt{\sqrt{a}}}\right)}{4a(a-b)\sqrt{\sqrt{ab}\,a-ab}}\right.}{256a\,b^2}$

default	$\frac{128a^2\left(\frac{\frac{-\frac{\left(ab-\sqrt{ab}\,a+2\sqrt{ab}\,b\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2(a-b)}-\frac{\sqrt{ab}+b}{2a(a-b)}}{\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4\sqrt{ab}\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+1}+\frac{\left(3\sqrt{ab}\,a-4\sqrt{ab}\,b-ab\right)\arctan\left(\frac{2a\left(\tanh^2\left(\frac{d}{2}\right.\right.}{4\sqrt{\sqrt{}}}\right.}{4a(a-b)\sqrt{\sqrt{ab}\,a-a}}}{256a\,b^2}\right.}{}$
risch	$\frac{a\,\mathrm{e}^{dx+c}\left(\mathrm{e}^{6dx+6c}-5\,\mathrm{e}^{4dx+4c}-5\,\mathrm{e}^{2dx+2c}+1\right)}{2bd(a-b)\left(-b\,\mathrm{e}^{8dx+8c}+4b\,\mathrm{e}^{6dx+6c}+16a\,\mathrm{e}^{4dx+4c}-6b\,\mathrm{e}^{4dx+4c}+4b\,\mathrm{e}^{2dx+2c}-b\right)}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(65536a^3b^7d^4-19660\right.\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 128/d*a^2*(1/256/a/b^2*((-1/2*(a*b-(a*b)^(1/2)*a+2*(a*b)^(1/2)*b)/a^2/(a-b)
*tanh(1/2*d*x+1/2*c)^2-1/2*((a*b)^(1/2)+b)/a/(a-b))/(tanh(1/2*d*x+1/2*c)^4-
2*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2+1)+1/4*(3*(a*
b)^(1/2)*a-4*(a*b)^(1/2)*b-a*b)/a/(a-b)/((a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/
4*(2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2)))
-1/256/a/b^2*((1/2*((a*b)^(1/2)*a-2*(a*b)^(1/2)*b+a*b)/a^2/(a-b)*tanh(1/2*d
*x+1/2*c)^2+1/2*(b-(a*b)^(1/2))/a/(a-b))/(tanh(1/2*d*x+1/2*c)^4-4*(a*b)^(1/
2)/a*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)^2+1)-1/4*(3*(a*b)^(1/2)*a-
4*(a*b)^(1/2)*b+a*b)/a/(a-b)/(-(a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(-2*a*ta
nh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(a*b)^(1/2)*a-a*b)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*e^(7*d*x + 7*c) - 5*a*e^(5*d*x + 5*c) - 5*a*e^(3*d*x + 3*c) + a*e^(
d*x + c))/(a*b^2*d - b^3*d + (a*b^2*d*e^(8*c) - b^3*d*e^(8*c))*e^(8*d*x) -
4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*d*e^(4*c) - 11*a
*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))*e^(4*d*x) - 4*(a*b^2*d*e^(2*c) - b^3*d*e^
(2*c))*e^(2*d*x)) + 1/128*integrate(64*((3*a*e^(7*c) - 4*b*e^(7*c))*e^(7*d*
x) - (5*a*e^(5*c) - 12*b*e^(5*c))*e^(5*d*x) + (5*a*e^(3*c) - 12*b*e^(3*c))*
e^(3*d*x) - (3*a*e^c - 4*b*e^c)*e^(d*x))/(a*b^2 - b^3 + (a*b^2*e^(8*c) - b^
3*e^(8*c))*e^(8*d*x) - 4*(a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^2
*b*e^(4*c) - 11*a*b^2*e^(4*c) + 3*b^3*e^(4*c))*e^(4*d*x) - 4*(a*b^2*e^(2*c)
 - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6266 vs. $2(161) = 322$.
time = 0.56, size = 6266, normalized size = 29.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
[Out] -1/16*(8*a*cosh(d*x + c)^7 + 56*a*cosh(d*x + c)*sinh(d*x + c)^6 + 8*a*sinh(
d*x + c)^7 - 40*a*cosh(d*x + c)^5 + 8*(21*a*cosh(d*x + c)^2 - 5*a)*sinh(d*x
 + c)^5 + 40*(7*a*cosh(d*x + c)^3 - 5*a*cosh(d*x + c))*sinh(d*x + c)^4 - 40
*a*cosh(d*x + c)^3 + 40*(7*a*cosh(d*x + c)^4 - 10*a*cosh(d*x + c)^2 - a)*si
nh(d*x + c)^3 + 8*(21*a*cosh(d*x + c)^5 - 50*a*cosh(d*x + c)^3 - 15*a*cosh(
d*x + c))*sinh(d*x + c)^2 - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b
^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(
a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*
b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x +
 c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 -
b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 -
4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 -
10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2
- b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2
 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*
cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 +
 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*s
qrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b +
1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9
 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16
*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(-81*a^3 + 405*a^2*b
- 680*a*b^2 + 384*b^3 - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*cosh(d*x
 + c)^2 - 2*(81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*cosh(d*x + c)*sinh(d
*x + c) - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*sinh(d*x + c)^2 + 2*(2
*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*cosh(d*x + c) + 2*(9*a^3*b^
2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*sinh(d*x + c) - ((3*a^4*b^5 - 14*a^3*
b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*cosh(d*x + c) + (3*a^4*b^5 - 14*a^
3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*sinh(d*x + c))*sqrt((81*a^5 - 52
2*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 +
15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)))*sqrt(-((a^
3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*
b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3
*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/((a
^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cosh(d*x + c)
```

```
^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh
(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh
(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*
b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 -
b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)
^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*s
inh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*c
osh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 +
 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x +
c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*
d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*
((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a
^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a
^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5
*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*
a^2 - 15*a*b + 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(-81
*a^3 + 405*a^2*b - 680*a*b^2 + 384*b^3 - (81*a^3 - 405*a^2*b + 680*a*b^2 -
384*b^3)*cosh(d*x + c)^2 - 2*(81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*cos
h(d*x + c)*sinh(d*x + c) - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*sinh(
d*x + c)^2 - 2*(2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*cosh(d*x +
 c) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*sinh(d*x + c) - ((3*
a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*cosh(d*x + c) + (
3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*sinh(d*x + c))*
sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b
^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*
d^4)))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*
a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15
*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*
a*b + 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) - ((a*b^2 - b^3
)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1009 vs. $2(161) = 322$.
time = 0.73, size = 1009, normalized size = 4.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*(((12*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 - sqrt(a*b)*sqrt(-b^2 + s
qrt(a*b)*b)*a*b - 20*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^2)*(a*b - b^2)^2*
abs(b) - 2*(4*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^2 - 7*sqrt(-b^2 + sqrt(a*b)*b)
*a^2*b^3 - 7*sqrt(-b^2 + sqrt(a*b)*b)*a*b^4 + 10*sqrt(-b^2 + sqrt(a*b)*b)*b
^5)*abs(-a*b + b^2)*abs(b) - (4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^3
- 3*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^4 - 6*sqrt(a*b)*sqrt(-b^2 + sq
rt(a*b)*b)*a*b^5 + 5*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^6)*abs(b))*arctan
(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a*b^2 - b^3 + sqrt((a^2*b - 2*a*b^
2 + b^3)*(a*b^2 - b^3) + (a*b^2 - b^3)^2))/(a*b^2 - b^3)))/((4*a^4*b^5 - 7*
a^3*b^6 - 3*a^2*b^7 + 11*a*b^8 - 5*b^9)*abs(-a*b + b^2)) - ((12*sqrt(a*b)*s
qrt(-b^2 - sqrt(a*b)*b)*a^2 - sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b - 20*s
qrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^2)*(a*b - b^2)^2*abs(b) + 2*(4*sqrt(-b^
2 - sqrt(a*b)*b)*a^3*b^2 - 7*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^3 - 7*sqrt(-b^2
 - sqrt(a*b)*b)*a*b^4 + 10*sqrt(-b^2 - sqrt(a*b)*b)*b^5)*abs(-a*b + b^2)*ab
s(b) - (4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^3 - 3*sqrt(a*b)*sqrt(-b^
2 - sqrt(a*b)*b)*a^2*b^4 - 6*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^5 + 5*s
qrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^6)*abs(b))*arctan(1/2*(e^(d*x + c) + e^
(-d*x - c))/sqrt(-(a*b^2 - b^3 - sqrt((a^2*b - 2*a*b^2 + b^3)*(a*b^2 - b^3)
 + (a*b^2 - b^3)^2))/(a*b^2 - b^3)))/((4*a^4*b^5 - 7*a^3*b^6 - 3*a^2*b^7 +
11*a*b^8 - 5*b^9)*abs(-a*b + b^2)) + 4*(a*(e^(d*x + c) + e^(-d*x - c))^3 -
8*a*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^4 - 8*b*
(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)*(a*b - b^2)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^7}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4)^2, x)
```

3.243 $\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=217

$$-\frac{\left(\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{5/4}d}-\frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{5/4}d}+\frac{\cosh(c}{4(a-b)bd\,(a-}$$

```
[Out] 1/4*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cosh(d*x+c)^2-b*co
sh(d*x+c)^4)-1/8*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(a^(1/
2)-2*b^(1/2))/b^(5/4)/d/a^(1/2)/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctanh(b^(1/4)
*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(a^(1/2)+2*b^(1/2))/b^(5/4)/d/a^(1/2)
/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A]

time = 0.22, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3294, 1219, 1180, 211, 214}

$$-\frac{\left(\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\,b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\,b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4bd(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/8*((Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - S
qrt[b]]])/(Sqrt[a]*(Sqrt[a] - Sqrt[b])^(3/2)*b^(5/4)*d) - ((Sqrt[a] + 2*Sqr
t[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*Sqrt[a]*
(Sqrt[a] + Sqrt[b])^(3/2)*b^(5/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c + d
*x]^2))/(4*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a-b+2bx^2-bx^4)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2a(a-3b)}{a-b+2bx}\right.}{8}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \frac{\left(\sqrt{a}-2\sqrt{b}\right)\text{S}}{}$$

$$= -\frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{5/4}d} - \frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt{\sqrt{}}}\right.}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.41, size = 597, normalized size = 2.75

$$\frac{\frac{32\cosh(c+dx)(2a+b-b\cosh(2(c+dx)))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))} + \text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-bc-bdx-2b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)-4ac\#1^2+11bc\#1^2-4adx\#1^2+11bdx\#1^2-8a\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2+22b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2+4ac\#1^4-11bc\#1^4+4adx\#1^4-11bdx\#1^4+8a\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4-22b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4+bc\#1^6+bdx\#1^6+2b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{32(a-b)bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((32*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2
*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b
*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(b*c) - b*d*x - 2*b*Log[-Cosh[(c + d*x)/2]
- Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 4*a*c*
#1^2 + 11*b*c*#1^2 - 4*a*d*x*#1^2 + 11*b*d*x*#1^2 - 8*a*Log[-Cosh[(c + d*x)
/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2
 + 22*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1]*#1^2 + 4*a*c*#1^4 - 11*b*c*#1^4 + 4*a*d*x*#1^4 - 11*
b*d*x*#1^4 + 8*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x
)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 22*b*Log[-Cosh[(c + d*x)/2] - Sinh[(
c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + b*c*#1^6
+ b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*
b*#1^5 + b*#1^7) & ])/(32*(a - b)*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(167) = 334$.
time = 5.13, size = 347, normalized size = 1.60

method	result
derivativedivides	$\frac{\frac{-\frac{(-2b+a)\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b(a-b)}+\frac{(3a-8b)\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b(a-b)}-\frac{(3a+2b)\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b(a-b)}+\frac{a}{2b(a-b)}}{a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{a\left(-\frac{\left(-\sqrt{ab}-a+\right.}{}\right.}{}}{d}$
default	$\frac{\frac{-\frac{(-2b+a)\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b(a-b)}+\frac{(3a-8b)\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b(a-b)}-\frac{(3a+2b)\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b(a-b)}+\frac{a}{2b(a-b)}}{a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{a\left(-\frac{\left(-\sqrt{ab}-a+\right.}{}\right.}{}}{d}$
risch	$\frac{e^{dx+c}\left(-b\,e^{6dx+6c}+4a\,e^{4dx+4c}+b\,e^{4dx+4c}+4a\,e^{2dx+2c}+b\,e^{2dx+2c}-b\right)}{2b(a-b)d\left(-b\,e^{8dx+8c}+4b\,e^{6dx+6c}+16a\,e^{4dx+4c}-6b\,e^{4dx+4c}+4b\,e^{2dx+2c}-b\right)}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(65536a^{5}b^{5}d^{4}-196608a\right.\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(32*(-1/64*(-2*b+a)/b/(a-b)*tanh(1/2*d*x+1/2*c)^6+1/64*(3*a-8*b)/b/(a-b
)*tanh(1/2*d*x+1/2*c)^4-1/64*(3*a+2*b)/b/(a-b)*tanh(1/2*d*x+1/2*c)^2+1/64*a
/b/(a-b))/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d
*x+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)+1/2/b/(
a-b)*a*(-1/4*(-(a*b)^(1/2)-a+2*b)/a/(-(a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(
-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(a*b)^(1/2)*a-a*b)^(1/2))+1
/4*((a*b)^(1/2)-a+2*b)/a/((a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(2*a*tanh(1/2
*d*x+1/2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*e^(5*c) + b*e^(5*c))*e^(5*d*x) + (4*a*e^(3*c) + b*e^(3*c))*e^(3*
d*x) - b*e^(7*d*x + 7*c) - b*e^(d*x + c))/(a*b^2*d - b^3*d + (a*b^2*d*e^(8*
c) - b^3*d*e^(8*c))*e^(8*d*x) - 4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c))*e^(6*d*
x) - 2*(8*a^2*b*d*e^(4*c) - 11*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))*e^(4*d*x)
 - 4*(a*b^2*d*e^(2*c) - b^3*d*e^(2*c))*e^(2*d*x)) + 1/32*integrate(16*((4*a
*e^(5*c) - 11*b*e^(5*c))*e^(5*d*x) - (4*a*e^(3*c) - 11*b*e^(3*c))*e^(3*d*x)
 + b*e^(7*d*x + 7*c) - b*e^(d*x + c))/(a*b^2 - b^3 + (a*b^2*e^(8*c) - b^3*e
```

```
^(8*c))*e^(8*d*x) - 4*(a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*
e^(4*c) - 11*a*b^2*e^(4*c) + 3*b^3*e^(4*c))*e^(4*d*x) - 4*(a*b^2*e^(2*c) -
b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6250 vs. $2(169) = 338$.
time = 0.56, size = 6250, normalized size = 28.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cosh(d*x + c)^7 + 56*b*cosh(d*x + c)*sinh(d*x + c)^6 + 8*b*sinh(d
*x + c)^7 - 8*(4*a + b)*cosh(d*x + c)^5 + 8*(21*b*cosh(d*x + c)^2 - 4*a - b
)*sinh(d*x + c)^5 + 40*(7*b*cosh(d*x + c)^3 - (4*a + b)*cosh(d*x + c))*sinh
(d*x + c)^4 - 8*(4*a + b)*cosh(d*x + c)^3 + 8*(35*b*cosh(d*x + c)^4 - 10*(4
*a + b)*cosh(d*x + c)^2 - 4*a - b)*sinh(d*x + c)^3 + 8*(21*b*cosh(d*x + c)^
5 - 10*(4*a + b)*cosh(d*x + c)^3 - 3*(4*a + b)*cosh(d*x + c))*sinh(d*x + c)
^2 + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sin
h(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x
 + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x
+ c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b
^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 +
2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2
- (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(
d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh
(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^
4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sin
h(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*
b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)
^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4*b^2 - 3*a^3*b
^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 +
64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^
2*b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b
^4 - a*b^5)*d^2))*log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b +
 28*a*b^2 - 32*b^3)*cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)
*cosh(d*x + c)*sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*sinh(d*x
 + c)^2 + 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*cosh(d*x + c) +
(a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*sinh(d*x + c) - 2*((a^4*b^5 -
 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 +
3*a^2*b^7 - a*b^8)*d^3*sinh(d*x + c))*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 8
0*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*
b^9 - 6*a^2*b^10 + a*b^11)*d^4)))*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 -
```

```
a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5
 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*
d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))
) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh
(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x
+ c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x +
 c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^
3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 -
 (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d
*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(
d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4
 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh
(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b
^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^
3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4*b^2 - 3*a^3*b^
3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 6
4*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2
*b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^
4 - a*b^5)*d^2))*log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b +
28*a*b^2 - 32*b^3)*cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*
cosh(d*x + c)*sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*sinh(d*x
+ c)^2 - 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*cosh(d*x + c) + (
a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*sinh(d*x + c) - 2*((a^4*b^5 -
3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 + 3
*a^2*b^7 - a*b^8)*d^3*sinh(d*x + c))*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80
*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b
^9 - 6*a^2*b^10 + a*b^11)*d^4)))*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a
*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5
- 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d
^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2)))
 + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(
d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x +
 c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 984 vs. 2(169) = 338.
time = 0.69, size = 984, normalized size = 4.53

$$[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*(((4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 5*sqrt(a*b)*sqrt(-b^2 +
sqrt(a*b)*b)*a*b)*(a*b - b^2)^2*abs(b) + (4*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b
- 11*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^2 - 8*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^3
+ 15*sqrt(-b^2 + sqrt(a*b)*b)*a*b^4)*abs(-a*b + b^2)*abs(b) - (4*sqrt(a*b)*
sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^2 - 11*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^
3*b^3 + 17*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^5 - 10*sqrt(a*b)*sqrt(-b^
2 + sqrt(a*b)*b)*b^6)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(
-(a*b^2 - b^3 + sqrt((a^2*b - 2*a*b^2 + b^3)*(a*b^2 - b^3) + (a*b^2 - b^3)^
2))/(a*b^2 - b^3)))/((4*a^5*b^4 - 7*a^4*b^5 - 3*a^3*b^6 + 11*a^2*b^7 - 5*a*
b^8)*abs(-a*b + b^2)) - ((4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2 + 5*sqrt
(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b)*(a*b - b^2)^2*abs(b) - (4*sqrt(-b^2 - s
qrt(a*b)*b)*a^4*b - 11*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^2 - 8*sqrt(-b^2 - sqr
t(a*b)*b)*a^2*b^3 + 15*sqrt(-b^2 - sqrt(a*b)*b)*a*b^4)*abs(-a*b + b^2)*abs(
b) - (4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^2 - 11*sqrt(a*b)*sqrt(-b^2
 - sqrt(a*b)*b)*a^3*b^3 + 17*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^5 - 10*
sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^6)*abs(b))*arctan(1/2*(e^(d*x + c) + e
^(-d*x - c))/sqrt(-(a*b^2 - b^3 - sqrt((a^2*b - 2*a*b^2 + b^3)*(a*b^2 - b^3
) + (a*b^2 - b^3)^2))/(a*b^2 - b^3)))/((4*a^5*b^4 - 7*a^4*b^5 - 3*a^3*b^6 +
 11*a^2*b^7 - 5*a*b^8)*abs(-a*b + b^2)) - 4*(b*(e^(d*x + c) + e^(-d*x - c))
^3 - 4*a*(e^(d*x + c) + e^(-d*x - c)) - 4*b*(e^(d*x + c) + e^(-d*x - c)))/(
(b*(e^(d*x + c) + e^(-d*x - c))^4 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16
*a + 16*b)*(a*b - b^2)))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^5}{{\left(a-b\sinh\left(c+d\,x\right)^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4)^2, x)
```

3.244 $\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=186

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/4}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/4}d}-\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(\right.}$$

```
[Out] -1/4*cosh(d*x+c)*(2-cosh(d*x+c)^2)/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*
x+c)^4)-1/8*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(3/4)/d/a
^(1/2)/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(
1/2))^(1/2))/b^(3/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3294, 1192, 1180, 211, 214}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\,b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\,b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/8*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(Sqrt[a]*(Sqrt
[a] - Sqrt[b])^(3/2)*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt
[a] + Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/4)*d) - (Cosh[c +
 d*x]*(2 - Cosh[c + d*x]^2))/(4*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*
Cosh[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = -\frac{\mathrm{Subst}\left(\int \frac{1-x^2}{(a-b+2bx^2-bx^4)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + $$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + $$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/4}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/4}d} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.35, size = 422, normalized size = 2.27

$$-\frac{\frac{16(-5\cosh(c+dx)+\cosh(3(c+dx)))}{-8a+3b-4b\cosh(2(c+dx))+b\cosh(4(c+dx))}+\mathrm{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-c-dx-2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)+7c\#1^2+7dx\#1^2+14\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2-7c\#1^4-7dx\#1^4-14\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4+c\#1^6+dx\#1^6+2\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{32(a-b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/32*((16*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2
*(c + d*x)] + b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b
*#1^4 - 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(
c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 7*c*#1^2 + 7*d
*x*#1^2 + 14*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]
*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 7*c*#1^4 - 7*d*x*#1^4 - 14*Log[-Cosh[(c
+ d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1
]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
 Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3
*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/((a - b)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(144) = 288$.

time = 4.10, size = 322, normalized size = 1.73

method	result
derivativedivides	$\frac{\frac{-\frac{\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a-b)}-\frac{(3a-8b)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a-b)}+\frac{5\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)}-\frac{1}{2(a-b)}}{a\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{a\left(-\frac{\left(\sqrt{ab}-b\right)a}{}\right.}{}}{d}$
default	$\frac{\frac{-\frac{\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a-b)}-\frac{(3a-8b)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a-b)}+\frac{5\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)}-\frac{1}{2(a-b)}}{a\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{a\left(-\frac{\left(\sqrt{ab}-b\right)a}{}\right.}{}}{d}$
risch	$\frac{e^{dx+c}\left(e^{6dx+6c}-5\,e^{4dx+4c}-5\,e^{2dx+2c}+1\right)}{2(a-b)d\left(-b\,e^{8dx+8c}+4b\,e^{6dx+6c}+16a\,e^{4dx+4c}-6b\,e^{4dx+4c}+4b\,e^{2dx+2c}-b\right)}+\Bigg(\_R=\mathrm{RootOf}\big(-1+(65536a^5b^3d^4-1966$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(8*(-1/16/(a-b)*tanh(1/2*d*x+1/2*c)^6-1/16*(3*a-8*b)/a/(a-b)*tanh(1/2*d
*x+1/2*c)^4+5/16/(a-b)*tanh(1/2*d*x+1/2*c)^2-1/16/(a-b))/(a*tanh(1/2*d*x+1/
2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh(1/2*d*
```

```
x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)+1/2/(a-b)*a*(-1/4*((a*b)^(1/2)-b)/a
/b/(-(a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b
)^(1/2)+2*a)/(-(a*b)^(1/2)*a-a*b)^(1/2))+1/4*(-(a*b)^(1/2)-b)/a/b/((a*b)^(1
/2)*a-a*b)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)-2*a)/(
(a*b)^(1/2)*a-a*b)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e^(7*d*x + 7*c) - 5*e^(5*d*x + 5*c) - 5*e^(3*d*x + 3*c) + e^(d*x + c)
)/(a*b*d - b^2*d + (a*b*d*e^(8*c) - b^2*d*e^(8*c))*e^(8*d*x) - 4*(a*b*d*e^(
6*c) - b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*d*e^(4*c) - 11*a*b*d*e^(4*c) + 3
*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a*b*d*e^(2*c) - b^2*d*e^(2*c))*e^(2*d*x)) -
1/8*integrate(4*(e^(7*d*x + 7*c) - 7*e^(5*d*x + 5*c) + 7*e^(3*d*x + 3*c) -
e^(d*x + c))/(a*b - b^2 + (a*b*e^(8*c) - b^2*e^(8*c))*e^(8*d*x) - 4*(a*b*e^
(6*c) - b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^2*e^(4*c) - 11*a*b*e^(4*c) + 3*b^2*
e^(4*c))*e^(4*d*x) - 4*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5238 vs. $2(141) = 282$.
time = 0.49, size = 5238, normalized size = 28.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*cosh(d*x + c)^7 + 56*cosh(d*x + c)*sinh(d*x + c)^6 + 8*sinh(d*x +
c)^7 + 8*(21*cosh(d*x + c)^2 - 5)*sinh(d*x + c)^5 - 40*cosh(d*x + c)^5 + 40
*(7*cosh(d*x + c)^3 - 5*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(7*cosh(d*x + c
)^4 - 10*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^3 - 40*cosh(d*x + c)^3 + 8*(21*
cosh(d*x + c)^5 - 50*cosh(d*x + c)^3 - 15*cosh(d*x + c))*sinh(d*x + c)^2 -
((a*b - b^2)*d*cosh(d*x + c)^8 + 8*(a*b - b^2)*d*cosh(d*x + c)*sinh(d*x + c
)^7 + (a*b - b^2)*d*sinh(d*x + c)^8 - 4*(a*b - b^2)*d*cosh(d*x + c)^6 + 4*(
7*(a*b - b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^2
 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^3 -
 3*(a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b - b^2)*d*cosh(
d*x + c)^4 - 30*(a*b - b^2)*d*cosh(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)
*sinh(d*x + c)^4 - 4*(a*b - b^2)*d*cosh(d*x + c)^2 + 8*(7*(a*b - b^2)*d*cos
h(d*x + c)^5 - 10*(a*b - b^2)*d*cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^6 - 15*
```

```
(a*b - b^2)*d*cosh(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^
2 - (a*b - b^2)*d)*sinh(d*x + c)^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*cosh(
d*x + c)^7 - 3*(a*b - b^2)*d*cosh(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*c
osh(d*x + c)^3 - (a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b
- 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 -
 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)
) + 3*a + b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*c
osh(d*x + c)^2 + 2*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c) + (a + 3*b)*sinh(d
*x + c)^2 + 2*(2*(a^2*b + 3*a*b^2)*d*cosh(d*x + c) + 2*(a^2*b + 3*a*b^2)*d*
sinh(d*x + c) - ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*cosh(d*x + c
) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sinh(d*x + c))*sqrt((a^2
+ 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b
^7 - 6*a^2*b^8 + a*b^9)*d^4)))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^
4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a
^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b
^2 + 3*a^2*b^3 - a*b^4)*d^2)) + a + 3*b) + ((a*b - b^2)*d*cosh(d*x + c)^8 +
 8*(a*b - b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b - b^2)*d*sinh(d*x + c
)^8 - 4*(a*b - b^2)*d*cosh(d*x + c)^6 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^2
- (a*b - b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x +
c)^4 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^3 - 3*(a*b - b^2)*d*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(35*(a*b - b^2)*d*cosh(d*x + c)^4 - 30*(a*b - b^2)*d*co
sh(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)*sinh(d*x + c)^4 - 4*(a*b - b^2)
*d*cosh(d*x + c)^2 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^5 - 10*(a*b - b^2)*d*
cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^6 - 15*(a*b - b^2)*d*cosh(d*x + c)^4 -
3*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)
^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*cosh(d*x + c)^7 - 3*(a*b - b^2)*d*cos
h(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^3 - (a*b - b^2)*d*c
osh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)
*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4
*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2
 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cosh(d*x + c)^2 + 2*(a + 3*b)*cos
h(d*x + c)*sinh(d*x + c) + (a + 3*b)*sinh(d*x + c)^2 - 2*(2*(a^2*b + 3*a*b^
2)*d*cosh(d*x + c) + 2*(a^2*b + 3*a*b^2)*d*sinh(d*x + c) - ((a^5*b^2 - 2*a^
4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*cosh(d*x + c) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2
*b^5 - a*b^6)*d^3*sinh(d*x + c))*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a
^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)))*s
qrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2
)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8
+ a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) +
a + 3*b) - ((a*b - b^2)*d*cosh(d*x + c)^8 + 8*(a*b - b^2)*d*cosh(d*x + c)*s
inh(d*x + c)^7 + (a*b - b^2)*d*sinh(d*x + c)^8 - 4*(a*b - b^2)*d*cosh(d*x +
 c)^6 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^6
 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a*b - b^2)*d*cosh(d
*x + c)^3 - 3*(a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b - b
```

```
^2)*d*cosh(d*x + c)^4 - 30*(a*b - b^2)*d*cosh(d*x + c)^2 - (8*a^2 - 11*a*b
+ 3*b^2)*d)*sinh(d*x + c)^4 - 4*(a*b - b^2)*d*cosh(d*x + c)^2 + 8*(7*(a*b -
 b^2)*d*cosh(d*x + c)^5 - 10*(a*b - b^2)*d*cosh(d*x + c)^3 - (8*a^2 - 11*a*
b + 3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b - b^2)*d*cosh(d*x +
 c)^6 - 15*(a*b - b^2)*d*cosh(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*cos
h(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^2 + (a*b - b^2)*d + 8*((a*b - b
^2)*d*cosh(d*x + c)^7 - 3*(a*b - b^2)*d*cosh(d*x + c)^5 - (8*a^2 - 11*a*b +
 3*b^2)*d*cosh(d*x + c)^3 - (a*b - b^2)*d*cosh(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(141) = 282.
time = 0.63, size = 861, normalized size = 4.63

$$\frac{\frac{\left(\left(4\,\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,a^2+5\,\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,ab\right)(a-b)^2|b|-2\left(4\sqrt{-b^2+\sqrt{ab}\,b}\,a^3b+\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b^2-5\sqrt{-b^2+\sqrt{ab}\,b}\,ab^3\right)|-a+b||b|+\left(4\,\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,a^3b-3\,\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b^2-6\,\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,ab^3+5\,\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,b^4\right)|b|\right)\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{ab-b^2+\sqrt{(a^2-2\,ab+b^2)(ab-b^2)+(ab-b^2)^2}}{ab-b^2}}}\right)}{(4\,a^5b^3-7\,a^4b^4-3\,a^3b^5+11\,a^2b^6-5\,ab^7)|-a+b|}-\frac{\left(\left(4\,\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,a^2+5\,\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,ab\right)(a-b)^2|b|+2\left(4\sqrt{-b^2-\sqrt{ab}\,b}\,a^3b+\sqrt{-b^2-\sqrt{ab}\,b}\,a^2b^2-5\sqrt{-b^2-\sqrt{ab}\,b}\,ab^3\right)|-a+b||b|+\left(4\,\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,a^3b-3\,\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,a^2b^2-6\,\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,ab^3+5\,\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,b^4\right)|b|\right)\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{ab-b^2-\sqrt{(a^2-2\,ab+b^2)(ab-b^2)+(ab-b^2)^2}}{ab-b^2}}}\right)}{(4\,a^5b^3-7\,a^4b^4-3\,a^3b^5+11\,a^2b^6-5\,ab^7)|-a+b|}-\frac{4\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-8\,e^{(dx+c)}-8\,e^{(-dx-c)}\right)}{\left(b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4-8\,b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-16\,a+16\,b\right)(a-b)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/8*(((4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 5*sqrt(a*b)*sqrt(-b^2 + s
qrt(a*b)*b)*a*b)*(a - b)^2*abs(b) - 2*(4*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b + s
qrt(-b^2 + sqrt(a*b)*b)*a^2*b^2 - 5*sqrt(-b^2 + sqrt(a*b)*b)*a*b^3)*abs(-a
+ b)*abs(b) + (4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b - 3*sqrt(a*b)*sqr
t(-b^2 + sqrt(a*b)*b)*a^2*b^2 - 6*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^3
+ 5*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^4)*abs(b))*arctan(1/2*(e^(d*x + c)
 + e^(-d*x - c))/sqrt(-(a*b - b^2 + sqrt((a^2 - 2*a*b + b^2)*(a*b - b^2) +
(a*b - b^2)^2))/(a*b - b^2)))/((4*a^5*b^3 - 7*a^4*b^4 - 3*a^3*b^5 + 11*a^2*
b^6 - 5*a*b^7)*abs(-a + b)) - ((4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2 +
5*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b)*(a - b)^2*abs(b) + 2*(4*sqrt(-b^2
 - sqrt(a*b)*b)*a^3*b + sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^2 - 5*sqrt(-b^2 - sq
rt(a*b)*b)*a*b^3)*abs(-a + b)*abs(b) + (4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b
)*a^3*b - 3*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^2 - 6*sqrt(a*b)*sqrt(-
b^2 - sqrt(a*b)*b)*a*b^3 + 5*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^4)*abs(b)
)*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a*b - b^2 - sqrt((a^2 - 2*
a*b + b^2)*(a*b - b^2) + (a*b - b^2)^2))/(a*b - b^2)))/((4*a^5*b^3 - 7*a^4*
b^4 - 3*a^3*b^5 + 11*a^2*b^6 - 5*a*b^7)*abs(-a + b)) - 4*((e^(d*x + c) + e^
```

```
(-d*x - c))^3 - 8*e^(d*x + c) - 8*e^(-d*x - c))/((b*(e^(d*x + c) + e^(-d*x
- c))^4 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)*(a - b)))/d
```

Mupad **[F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)^3}{{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4)^2, x)
```

3.245 $\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=221

$$\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt[4]{b}\,d}+\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt[4]{b}\,d}+\frac{\cosh(c}{4a(a-b)d\,(a-}$$

```
[Out] 1/4*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/a/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*co
sh(d*x+c)^4)+1/8*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a^(
1/2)-2*b^(1/2))/a^(3/2)/b^(1/4)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctanh(b^(1/
4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(3*a^(1/2)+2*b^(1/2))/a^(3/2)/b^(1/
4)/d/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A]

time = 0.22, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3294, 1106, 1180, 211, 214}

$$\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4ad(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((3*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(8*a^(3/2)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(1/4)*d) + ((3*Sqrt[a] + 2*Sq
rt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(3/2)
*(Sqrt[a] + Sqrt[b])^(3/2)*b^(1/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c +
d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a-b+2bx^2-bx^4)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\Big(\int \frac{2(a-b)b+4}{}}{}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\Big(\Big(3\sqrt{a}-2\sqrt{b}\Big)}{}$$

$$= \frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt[4]{b}\,d} + \frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\Bigg(\frac{\sqrt[4]{b}}{\sqrt{\sqrt{}}}}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.27, size = 597, normalized size = 2.70

$$\frac{\frac{32\cosh(c+dx)(2a+b-b\cosh(2(c+dx)))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))}+\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-bc-bdx-2b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)+12ac\#1^2-5bc\#1^2+12adx\#1^2-5bdx\#1^2+24a\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2-10b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^2-12ac\#1^4+5bc\#1^4-12adx\#1^4+5bdx\#1^4-24a\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4+10b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^4+bc\#1^6+bdx\#1^6+2b\log\left(-\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\#1-\sinh\left(\frac{1}{2}(c+dx)\right)\#1\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{32a(a-b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((32*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2
*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b
*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(b*c) - b*d*x - 2*b*Log[-Cosh[(c + d*x)/2]
- Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 12*a*c
*#1^2 - 5*b*c*#1^2 + 12*a*d*x*#1^2 - 5*b*d*x*#1^2 + 24*a*Log[-Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^
2 - 10*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
- Sinh[(c + d*x)/2]*#1]*#1^2 - 12*a*c*#1^4 + 5*b*c*#1^4 - 12*a*d*x*#1^4 + 5
*b*d*x*#1^4 - 24*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 10*b*Log[-Cosh[(c + d*x)/2] - Sinh
[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + b*c*#1^
6 + b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
 d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 -
3*b*#1^5 + b*#1^7) & ])/(32*a*(a - b)*d)
```

Maple [A]
time = 5.01, size = 339, normalized size = 1.53

method	result
derivativedivides	$\frac{\frac{-\frac{(-2b+a)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a-b)}+\frac{(3a-8b)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a-b)}-\frac{(3a+2b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a-b)}+\frac{2}{4a-4b}}{a\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{-\frac{\left(-\sqrt{ab}+3a-\right.}{}}{}}{d}$
default	$\frac{\frac{-\frac{(-2b+a)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a-b)}+\frac{(3a-8b)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a-b)}-\frac{(3a+2b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a-b)}+\frac{2}{4a-4b}}{a\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{-\frac{\left(-\sqrt{ab}+3a-\right.}{}}{}}{d}$
risch	$\frac{e^{dx+c}\left(-b\,e^{6dx+6c}+4a\,e^{4dx+4c}+b\,e^{4dx+4c}+4a\,e^{2dx+2c}+b\,e^{2dx+2c}-b\right)}{2ad(a-b)\left(-b\,e^{8dx+8c}+4b\,e^{6dx+6c}+16a\,e^{4dx+4c}-6b\,e^{4dx+4c}+4b\,e^{2dx+2c}-b\right)}+\left(\_R=\text{RootOf}\left(\left(65536a^9b\,d^4-196608\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/4*(-2*b+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^6+1/4*(3*a-8*b)/a/(a-b)*t
anh(1/2*d*x+1/2*c)^4-1/4*(3*a+2*b)/a/(a-b)*tanh(1/2*d*x+1/2*c)^2+1/4/(a-b))
```

```
/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c)
^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)+1/2/(a-b)*(-1/4*
(-(a*b)^(1/2)+3*a-2*b)/a/(-(a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(-2*a*tanh(1
/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(a*b)^(1/2)*a-a*b)^(1/2))+1/4*((a*b)^(
1/2)+3*a-2*b)/a/((a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*
c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*e^(5*c) + b*e^(5*c))*e^(5*d*x) + (4*a*e^(3*c) + b*e^(3*c))*e^(3*
d*x) - b*e^(7*d*x + 7*c) - b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(
8*c) - a*b^2*d*e^(8*c))*e^(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c))*e
^(6*d*x) - 2*(8*a^3*d*e^(4*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))*e^(
4*d*x) - 4*(a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(-
((12*a*e^(5*c) - 5*b*e^(5*c))*e^(5*d*x) - (12*a*e^(3*c) - 5*b*e^(3*c))*e^(3
*d*x) - b*e^(7*d*x + 7*c) + b*e^(d*x + c))/(a^2*b - a*b^2 + (a^2*b*e^(8*c)
- a*b^2*e^(8*c))*e^(8*d*x) - 4*(a^2*b*e^(6*c) - a*b^2*e^(6*c))*e^(6*d*x) -
2*(8*a^3*e^(4*c) - 11*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c))*e^(4*d*x) - 4*(a^2*b
*e^(2*c) - a*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6018 vs. 2(173) = 346.
time = 0.54, size = 6018, normalized size = 27.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cosh(d*x + c)^7 + 56*b*cosh(d*x + c)*sinh(d*x + c)^6 + 8*b*sinh(d
*x + c)^7 - 8*(4*a + b)*cosh(d*x + c)^5 + 8*(21*b*cosh(d*x + c)^2 - 4*a - b
)*sinh(d*x + c)^5 + 40*(7*b*cosh(d*x + c)^3 - (4*a + b)*cosh(d*x + c))*sinh
(d*x + c)^4 - 8*(4*a + b)*cosh(d*x + c)^3 + 8*(35*b*cosh(d*x + c)^4 - 10*(4
*a + b)*cosh(d*x + c)^2 - 4*a - b)*sinh(d*x + c)^3 + 8*(21*b*cosh(d*x + c)^
5 - 10*(4*a + b)*cosh(d*x + c)^3 - 3*(4*a + b)*cosh(d*x + c))*sinh(d*x + c)
^2 + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)
*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*
cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*
d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(
7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*si
```

```
nh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2
)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(
a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 -
 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a
^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x
 + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*
b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3
 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a
^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^
5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5
*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c
)^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c)*sinh(d*x + c) + (81*a^2 -
81*a*b + 20*b^2)*sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 + 2*((27*a^4 -
24*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2*b^2)*d*s
inh(d*x + c) - 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d
^3*cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*
d^3*sinh(d*x + c))*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15
*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*sqrt(-((a^
6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^
9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^
7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*
d^2))) - ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x
+ c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2
)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b
^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 +
 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a
*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 -
 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)
^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 1
5*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh
(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((
a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8
*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x +
 c))*sinh(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((
81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 1
5*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3
*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cosh(d*x
 + c)^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c)*sinh(d*x + c) + (81*a^
2 - 81*a*b + 20*b^2)*sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 - 2*((27*a^
4 - 24*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2*b^2)
*d*sinh(d*x + c) - 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^
```

```
5)*d^3*cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b
^5)*d^3*sinh(d*x + c))*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2
+ 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*sqrt(-
((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/
((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^
3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b
^3)*d^2))) + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a
*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b -
 a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)
^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1054 vs. 2(173) = 346.
time = 0.60, size = 1054, normalized size = 4.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*(((4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b + 5*sqrt(a*b)*sqrt(-b^2 +
sqrt(a*b)*b)*b^2)*(a^2 - a*b)^2*abs(b) - (12*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b
 - sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^2 - 16*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^3 +
 5*sqrt(-b^2 + sqrt(a*b)*b)*a*b^4)*abs(-a^2 + a*b)*abs(b) + (12*sqrt(a*b)*s
qrt(-b^2 + sqrt(a*b)*b)*a^5*b - 17*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b
^2 - 12*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^3 + 27*sqrt(a*b)*sqrt(-b^2
 + sqrt(a*b)*b)*a^2*b^4 - 10*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^5)*abs(
b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^2*b - a*b^2 + sqrt((a^
3 - 2*a^2*b + a*b^2)*(a^2*b - a*b^2) + (a^2*b - a*b^2)^2))/(a^2*b - a*b^2))
)/((4*a^6*b^3 - 7*a^5*b^4 - 3*a^4*b^5 + 11*a^3*b^6 - 5*a^2*b^7)*abs(-a^2 +
a*b)) + ((4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b + 5*sqrt(a*b)*sqrt(-b^2
- sqrt(a*b)*b)*b^2)*(a^2 - a*b)^2*abs(b) - (12*sqrt(-b^2 - sqrt(a*b)*b)*a^4
*b - sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^2 - 16*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^3
 + 5*sqrt(-b^2 - sqrt(a*b)*b)*a*b^4)*abs(-a^2 + a*b)*abs(b) + (12*sqrt(a*b)
*sqrt(-b^2 - sqrt(a*b)*b)*a^5*b - 17*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^4
```

```
*b^2 - 12*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^3 + 27*sqrt(a*b)*sqrt(-b
^2 - sqrt(a*b)*b)*a^2*b^4 - 10*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^5)*ab
s(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^2*b - a*b^2 - sqrt((
a^3 - 2*a^2*b + a*b^2)*(a^2*b - a*b^2) + (a^2*b - a*b^2)^2))/(a^2*b - a*b^2
)))/((4*a^6*b^3 - 7*a^5*b^4 - 3*a^4*b^5 + 11*a^3*b^6 - 5*a^2*b^7)*abs(-a^2
+ a*b)) - 4*(b*(e^(d*x + c) + e^(-d*x - c))^3 - 4*a*(e^(d*x + c) + e^(-d*x
- c)) - 4*b*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^
4 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)*(a^2 - a*b)))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)}{{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4)^2, x)
```

3.246 $\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=325

$$-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{\sqrt[4]{b}\,\tanh^{-1}\left(\right.}{8a^{3/2}\left(\sqrt{a}\right.}$$

```
[Out] -arctanh(cosh(d*x+c))/a^2/d-1/4*b*cosh(d*x+c)*(2-cosh(d*x+c)^2)/a/(a-b)/d/(
a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)-1/8*b^(1/4)*arctan(b^(1/4)*cosh(d*x+
c)/(a^(1/2)-b^(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*b^(1/4)*a
rctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)+b^(1
/2))^(3/2)-1/2*b^(1/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/
a^2/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1
/2)+b^(1/2))^(1/2))/a^2/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3294, 1252, 213, 1192, 1180, 211, 214}

$$-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4ad(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/8*(b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(a^(
3/2)*(Sqrt[a] - Sqrt[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])
/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cosh
[c + d*x]]/(a^2*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a]
+ Sqrt[b]]])/(8*a^(3/2)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^
(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] + Sqrt[b
]]*d) - (b*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b
*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1252

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)^2}\,dx,x,\cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\Big(\int \Big(-\frac{1}{a^2(-1+x^2)}+\frac{b-bx^2}{a(a-b+2bx^2-bx^4)^2}+\frac{b-bx^2}{a^2(a-b+2bx^2-bx^4)}\Big)\,dx,x,\cosh(c+}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\cosh(c+dx)\right)}{a^2d} - \frac{\text{Subst}\Big(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx,x,\cosh(c+d}{a^2d}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+\right.}$$

$$= -\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2d} + \frac{\sqrt[4]{b}\,\tanh^{-1}}{2a^2\sqrt{\sqrt{a}}}$$

$$= -\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(c}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.63, size = 761, normalized size = 2.34

$$\frac{\frac{16ab(-5\cosh(c+dx)+\cosh(3(c+dx)))}{(a-b)(8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx)))}+32\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{b\,\text{RootSum}[\text{[illegible]}]}{a-b}}{32a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((16*a*b*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/((a - b)*(8*a - 3*b + 4*b*
Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + 32*Log[Tanh[(c + d*x)/2]] - (b*
RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-5*a*c
 + 4*b*c - 5*a*d*x + 4*b*d*x - 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 8*b*Log[-Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 1
9*a*c*#1^2 - 12*b*c*#1^2 + 19*a*d*x*#1^2 - 12*b*d*x*#1^2 + 38*a*Log[-Cosh[(
c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*
#1]*#1^2 - 24*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)
/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 19*a*c*#1^4 + 12*b*c*#1^4 - 19*a*d*x*
```

```
#1^4 + 12*b*d*x*#1^4 - 38*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Co
sh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 24*b*Log[-Cosh[(c + d*x)/
2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4
+ 5*a*c*#1^6 - 4*b*c*#1^6 + 5*a*d*x*#1^6 - 4*b*d*x*#1^6 + 10*a*Log[-Cosh[(c
 + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#
1]*#1^6 - 8*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2
]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^
5 + b*#1^7) & ])/(a - b))/(32*a^2*d)
```

Maple [A]

time = 2.80, size = 364, normalized size = 1.12

method	result
derivativedivides	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{16b\left(\frac{-\frac{a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32(a-b)}-\frac{(3a-8b)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32(a-b)}+\frac{5a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32(a-b)}-\frac{a}{32(a-b)}}{a\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{}$
default	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{16b\left(\frac{-\frac{a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32(a-b)}-\frac{(3a-8b)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32(a-b)}+\frac{5a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32(a-b)}-\frac{a}{32(a-b)}}{a\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{}$
risch	$\frac{b\,\mathrm{e}^{dx+c}\left(\mathrm{e}^{6dx+6c}-5\,\mathrm{e}^{4dx+4c}-5\,\mathrm{e}^{2dx+2c}+1\right)}{2a(a-b)d\left(-b\,\mathrm{e}^{8dx+8c}+4b\,\mathrm{e}^{6dx+6c}+16a\,\mathrm{e}^{4dx+4c}-6b\,\mathrm{e}^{4dx+4c}+4b\,\mathrm{e}^{2dx+2c}-b\right)}+\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)}{a^2d}-\frac{\ln\left(\mathrm{e}^{dx+c}+1\right)}{a^2d}+2\left(\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^2*ln(tanh(1/2*d*x+1/2*c))+16*b/a^2*((-1/32*a/(a-b)*tanh(1/2*d*x+1/
2*c)^6-1/32*(3*a-8*b)/(a-b)*tanh(1/2*d*x+1/2*c)^4+5/32*a/(a-b)*tanh(1/2*d*x
+1/2*c)^2-1/32*a/(a-b))/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+
6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c
)^2+a)+1/32/(a-b)*a*(-1/4*(5*(a*b)^(1/2)*a-4*(a*b)^(1/2)*b-a*b)/a/b/(-(a*b)
```

```
^(1/2)*a-a*b)^(1/2)*arctan(1/4*(-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*
a)/(-(a*b)^(1/2)*a-a*b)^(1/2))+1/4*(-5*(a*b)^(1/2)*a+4*(a*b)^(1/2)*b-a*b)/a
/b/((a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^
(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2)))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*e^(7*d*x + 7*c) - 5*b*e^(5*d*x + 5*c) - 5*b*e^(3*d*x + 3*c) + b*e^(
d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*e^(8*d*x
) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e^(4*c) -
11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^(2*c) - a*
b^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log((e^
(d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/4*((5*a*b*e^(7*c) - 4*b^2*e^
(7*c))*e^(7*d*x) - (19*a*b*e^(5*c) - 12*b^2*e^(5*c))*e^(5*d*x) + (19*a*b*e^
(3*c) - 12*b^2*e^(3*c))*e^(3*d*x) - (5*a*b*e^c - 4*b^2*e^c)*e^(d*x))/(a^3*b
 - a^2*b^2 + (a^3*b*e^(8*c) - a^2*b^2*e^(8*c))*e^(8*d*x) - 4*(a^3*b*e^(6*c)
 - a^2*b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^4*e^(4*c) - 11*a^3*b*e^(4*c) + 3*a^2
*b^2*e^(4*c))*e^(4*d*x) - 4*(a^3*b*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x
)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7793 vs. $2(244) = 488$.
time = 0.68, size = 7793, normalized size = 23.98

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a*b*cosh(d*x + c)^7 + 56*a*b*cosh(d*x + c)*sinh(d*x + c)^6 + 8*a*b
*sinh(d*x + c)^7 - 40*a*b*cosh(d*x + c)^5 + 8*(21*a*b*cosh(d*x + c)^2 - 5*a
*b)*sinh(d*x + c)^5 - 40*a*b*cosh(d*x + c)^3 + 40*(7*a*b*cosh(d*x + c)^3 -
5*a*b*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(7*a*b*cosh(d*x + c)^4 - 10*a*b*c
osh(d*x + c)^2 - a*b)*sinh(d*x + c)^3 + 8*a*b*cosh(d*x + c) + 8*(21*a*b*cos
h(d*x + c)^5 - 50*a*b*cosh(d*x + c)^3 - 15*a*b*cosh(d*x + c))*sinh(d*x + c)
^2 + ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2)*d*cosh(d*x
+ c)*sinh(d*x + c)^7 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^8 - 4*(a^3*b - a^2
*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b
 - a^2*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*
x + c)^4 + 8*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(a^3*b - a^2*b^2)*d
```

```
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4
 - 30*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*
d)*sinh(d*x + c)^4 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^3*b -
a^2*b^2)*d*cosh(d*x + c)^5 - 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - (8*a^
4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3*b -
a^2*b^2)*d*cosh(d*x + c)^6 - 15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 3*(8*
a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d
*x + c)^2 + (a^3*b - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 -
3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*co
sh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a
^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 12
41*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*
b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 + 16*b^
3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-625*a^3*b + 1125*a^2*b
^2 - 664*a*b^3 + 128*b^4 - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)
*cosh(d*x + c)^2 - 2*(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*cosh(
d*x + c)*sinh(d*x + c) - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*s
inh(d*x + c)^2 + 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d*
cosh(d*x + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d*sinh
(d*x + c) - ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*
cosh(d*x + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d
^3*sinh(d*x + c))*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4
 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a
^8*b^5 + a^7*b^6)*d^4)))*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*s
qrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 -
 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d
^4)) + 35*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)
*d^2))) - ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2)*d*cosh
(d*x + c)*sinh(d*x + c)^7 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^8 - 4*(a^3*b
 - a^2*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (
a^3*b - a^2*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*co
sh(d*x + c)^4 + 8*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(a^3*b - a^2*b
^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*cosh(d*x +
 c)^4 - 30*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b + 3*a^2*
b^2)*d)*sinh(d*x + c)^4 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^3
*b - a^2*b^2)*d*cosh(d*x + c)^5 - 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 -
(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3
*b - a^2*b^2)*d*cosh(d*x + c)^6 - 15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 -
3*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*s
inh(d*x + c)^2 + (a^3*b - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*cosh(d*x + c)
^7 - 3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)
*d*cosh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt
(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2
 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*
a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 +
```

```
16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-625*a^3*b + 1125*
a^2*b^2 - 664*a*b^3 + 128*b^4 - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128
*b^4)*cosh(d*x + c)^2 - 2*(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*
cosh(d*x + c)*sinh(d*x + c) - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b
^4)*sinh(d*x + c)^2 - 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^
4)*d*cosh(d*x + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d
*sinh(d*x + c) - ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)
*d^3*cosh(d*x + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b
^4)*d^3*sinh(d*x + c))*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*
a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4
- 6*a^8*b^5 + a^7*b^6)*d^4)))*sqrt(-((a^7 - 3*a...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4373 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1116 vs. 2(244) = 488.
time = 0.55, size = 1116, normalized size = 3.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/8*(((20*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 9*sqrt(a*b)*sqrt(-b^2 +
sqrt(a*b)*b)*a*b - 20*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^2)*(a^3 - a^2*b)
^2*abs(b) - 2*(12*sqrt(-b^2 + sqrt(a*b)*b)*a^5*b - 5*sqrt(-b^2 + sqrt(a*b)*
b)*a^4*b^2 - 17*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^3 + 10*sqrt(-b^2 + sqrt(a*b)
*b)*a^2*b^4)*abs(-a^3 + a^2*b)*abs(b) + (4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*
b)*a^7*b - 3*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^6*b^2 - 6*sqrt(a*b)*sqrt(
-b^2 + sqrt(a*b)*b)*a^5*b^3 + 5*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^4)
*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^3*b - a^2*b^2 + s
qrt((a^4 - 2*a^3*b + a^2*b^2)*(a^3*b - a^2*b^2) + (a^3*b - a^2*b^2)^2))/(a^
3*b - a^2*b^2)))/((4*a^8*b^2 - 7*a^7*b^3 - 3*a^6*b^4 + 11*a^5*b^5 - 5*a^4*b
^6)*abs(-a^3 + a^2*b)) - ((20*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2 + 9*sq
rt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b - 20*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b
)*b^2)*(a^3 - a^2*b)^2*abs(b) + 2*(12*sqrt(-b^2 - sqrt(a*b)*b)*a^5*b - 5*sq
rt(-b^2 - sqrt(a*b)*b)*a^4*b^2 - 17*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^3 + 10*s
qrt(-b^2 - sqrt(a*b)*b)*a^2*b^4)*abs(-a^3 + a^2*b)*abs(b) + (4*sqrt(a*b)*sq
```

```
rt(-b^2 - sqrt(a*b)*b)*a^7*b - 3*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^6*b^2
 - 6*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^5*b^3 + 5*sqrt(a*b)*sqrt(-b^2 - s
qrt(a*b)*b)*a^4*b^4)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-
(a^3*b - a^2*b^2 - sqrt((a^4 - 2*a^3*b + a^2*b^2)*(a^3*b - a^2*b^2) + (a^3*
b - a^2*b^2)^2))/(a^3*b - a^2*b^2)))/((4*a^8*b^2 - 7*a^7*b^3 - 3*a^6*b^4 +
11*a^5*b^5 - 5*a^4*b^6)*abs(-a^3 + a^2*b)) - 4*(b*(e^(d*x + c) + e^(-d*x -
c))^3 - 8*b*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^
4 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)*(a^2 - a*b)) - 4*log(
e^(d*x + c) + e^(-d*x - c) + 2)/a^2 + 4*log(e^(d*x + c) + e^(-d*x - c) - 2)
/a^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(c+d\,x\right)\,{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)^2), x)
```

3.247 $\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=320

$$\frac{x}{b^2}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d}+\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\sqrt{\sqrt{}}\right)}{2\sqrt{\sqrt{a}}}$$

```
[Out] x/b^2+1/8*a^(1/4)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/b^(3
/2)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*a^(1/4)*arctanh((a^(1/2)+b^(1/2))^(1/2)*t
anh(d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)+b^(1/2))^(3/2)-1/2*a^(1/4)*arctanh((
a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/b^2/d/(a^(1/2)-b^(1/2))^(1/2)-1
/2*a^(1/4)*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/b^2/d/(a^(1
/2)+b^(1/2))^(1/2)-1/4*tanh(d*x+c)/(a-b)/b/d+1/4*tanh(d*x+c)^5/b/d/(a-2*a*t
anh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A]
time = 0.34, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3296, 1327, 1289, 12, 1136, 1180, 214, 1301, 213}

$$\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\tanh^5(c+dx)}{4bd\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\tanh(c+dx)}{4bd(a-b)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] x/b^2 - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/
(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b
]]*Tanh[c + d*x])/a^(1/4)])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - (a^(1
/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[
a] + Sqrt[b]]*b^2*d) - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d
*x])/a^(1/4)])/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tanh[c + d*x]/(4*(
a - b)*b*d) + Tanh[c + d*x]^5/(4*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tan
h[c + d*x]^4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1327

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[-f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
```

```
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[d^2*(f
^4/(c*d^2 - b*d*e + a*e^2)), Int[(f*x)^(m - 4)*((a + b*x^2 + c*x^4)^(p + 1)
/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{(1-x^2)(a-2ax^2+(a-b)x^4)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4(a-ax^2)}{(a-2ax^2+(a-b)x^4)^2}\,dx, x, \tanh(c+dx)\right)}{bd} - \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a-2ax^2+(a-b)x}\right.}{bd}$$

$$= \frac{\tanh^5(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int -\frac{2abx^4}{a-2ax^2+(a-b}\right.}{8a}$$

$$= \frac{\tanh^5(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{t}\right.}{b^2d}$$

$$= \frac{x}{b^2} - \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{\tanh^5(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} +$$

$$= \frac{x}{b^2} - \frac{\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d} - \frac{\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}}{\sqrt[4]{a}}\right.}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^2}$$

$$= \frac{x}{b^2} - \frac{\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d} + \frac{\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}}{\sqrt[4]{a}}\right.}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b}$$

Mathematica [A]

time = 3.39, size = 262, normalized size = 0.82

$$\frac{8(c+dx)+\frac{\sqrt{a}\left(4\sqrt{a}-5\sqrt{b}\right)\mathrm{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\,\sqrt{b}}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{-a+\sqrt{a}\,\sqrt{b}}}-\frac{\sqrt{a}\left(4\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\,\sqrt{b}}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{a+\sqrt{a}\,\sqrt{b}}}+\frac{2ab(-6\sinh(2(c+dx))+\sinh(4(c+dx)))}{(a-b)(8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx)))}}{8b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (8*(c + d*x) + (Sqrt[a]*(4*Sqrt[a] - 5*Sqrt[b])*ArcTan[((Sqrt[a] - Sqrt[b])
*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a +
 Sqrt[a]*Sqrt[b]]) - (Sqrt[a]*(4*Sqrt[a] + 5*Sqrt[b])*ArcTanh[((Sqrt[a] + S
qrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqr
t[a + Sqrt[a]*Sqrt[b]]) + (2*a*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)])
)/((a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])))/(8*b
^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.12, size = 331, normalized size = 1.03 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/b^2*a*((-1/4*b/(a-b)*tanh(1/2*d*x+1
/2*c)^7+5/4*b/(a-b)*tanh(1/2*d*x+1/2*c)^5+5/4*b/(a-b)*tanh(1/2*d*x+1/2*c)^3
-1/4*b/(a-b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x
+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2
*d*x+1/2*c)^2+a)+1/32/(a-b)*sum(((4*a-5*b)*_R^6+(-12*a+19*b)*_R^4+(12*a-19*
b)*_R^2-4*a+5*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1
/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))+1/b^2*ln(
tanh(1/2*d*x+1/2*c)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a*b*d*e^(8*c) - b^2*d*e^(8*c))*x*e^(8*d*x) + a*b + 2*(a*b*d - b^2*d
)*x + (a*b*e^(6*c) - 8*(a*b*d*e^(6*c) - b^2*d*e^(6*c))*x)*e^(6*d*x) - (8*a^
2*e^(4*c) - 3*a*b*e^(4*c) + 4*(8*a^2*d*e^(4*c) - 11*a*b*d*e^(4*c) + 3*b^2*d
*e^(4*c))*x)*e^(4*d*x) - (5*a*b*e^(2*c) + 8*(a*b*d*e^(2*c) - b^2*d*e^(2*c))
*x)*e^(2*d*x))/(a*b^3*d - b^4*d + (a*b^3*d*e^(8*c) - b^4*d*e^(8*c))*e^(8*d*
```

```
x) - 4*(a*b^3*d*e^(6*c) - b^4*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b^2*d*e^(4*c)
 - 11*a*b^3*d*e^(4*c) + 3*b^4*d*e^(4*c))*e^(4*d*x) - 4*(a*b^3*d*e^(2*c) - b
^4*d*e^(2*c))*e^(2*d*x)) + 1/256*integrate(256*(a*b*e^(6*d*x + 6*c) + a*b*e
^(2*d*x + 2*c) + 2*(8*a^2*e^(4*c) - 11*a*b*e^(4*c))*e^(4*d*x))/(a*b^3 - b^4
 + (a*b^3*e^(8*c) - b^4*e^(8*c))*e^(8*d*x) - 4*(a*b^3*e^(6*c) - b^4*e^(6*c)
)*e^(6*d*x) - 2*(8*a^2*b^2*e^(4*c) - 11*a*b^3*e^(4*c) + 3*b^4*e^(4*c))*e^(4
*d*x) - 4*(a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6944 vs. $2(240) = 480$.
time = 0.67, size = 6944, normalized size = 21.70

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*(a*b - b^2)*d*x*cosh(d*x + c)^8 + 128*(a*b - b^2)*d*x*cosh(d*x + c
)*sinh(d*x + c)^7 + 16*(a*b - b^2)*d*x*sinh(d*x + c)^8 - 8*(8*(a*b - b^2)*d
*x - a*b)*cosh(d*x + c)^6 + 8*(56*(a*b - b^2)*d*x*cosh(d*x + c)^2 - 8*(a*b
- b^2)*d*x + a*b)*sinh(d*x + c)^6 + 16*(56*(a*b - b^2)*d*x*cosh(d*x + c)^3
- 3*(8*(a*b - b^2)*d*x - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(4*(8*a^2
- 11*a*b + 3*b^2)*d*x + 8*a^2 - 3*a*b)*cosh(d*x + c)^4 + 8*(140*(a*b - b^2)
*d*x*cosh(d*x + c)^4 - 4*(8*a^2 - 11*a*b + 3*b^2)*d*x - 15*(8*(a*b - b^2)*d
*x - a*b)*cosh(d*x + c)^2 - 8*a^2 + 3*a*b)*sinh(d*x + c)^4 + 32*(28*(a*b -
b^2)*d*x*cosh(d*x + c)^5 - 5*(8*(a*b - b^2)*d*x - a*b)*cosh(d*x + c)^3 - (4
*(8*a^2 - 11*a*b + 3*b^2)*d*x + 8*a^2 - 3*a*b)*cosh(d*x + c))*sinh(d*x + c)
^3 + 16*(a*b - b^2)*d*x - 8*(8*(a*b - b^2)*d*x + 5*a*b)*cosh(d*x + c)^2 + 8
*(56*(a*b - b^2)*d*x*cosh(d*x + c)^6 - 15*(8*(a*b - b^2)*d*x - a*b)*cosh(d*
x + c)^4 - 8*(a*b - b^2)*d*x - 6*(4*(8*a^2 - 11*a*b + 3*b^2)*d*x + 8*a^2 -
3*a*b)*cosh(d*x + c)^2 - 5*a*b)*sinh(d*x + c)^2 + ((a*b^3 - b^4)*d*cosh(d*x
 + c)^8 + 8*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^3 - b^4)*d
*sinh(d*x + c)^8 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^6 + 4*(7*(a*b^3 - b^4)*d
*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b^2 - 11*a*b
^3 + 3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 3*(a
*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^3 - b^4)*d*cosh(d
*x + c)^4 - 30*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*
b^4)*d)*sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + 8*(7*(a*b^3 -
 b^4)*d*cosh(d*x + c)^5 - 10*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (8*a^2*b^2 -
 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^3 - b^4)*d*
cosh(d*x + c)^6 - 15*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 3*(8*a^2*b^2 - 11*a*
b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a*b^3
- b^4)*d + 8*((a*b^3 - b^4)*d*cosh(d*x + c)^7 - 3*(a*b^3 - b^4)*d*cosh(d*x
+ c)^5 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d
*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)
```

```
*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((
a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b
^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6
- b^7)*d^2))*log(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b
^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)
/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12
+ b^13)*d^4)) + 128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3 - (128*a^3 - 664
*a^2*b + 1125*a*b^2 - 625*b^3)*cosh(d*x + c)^2 - 2*(128*a^3 - 664*a^2*b + 1
125*a*b^2 - 625*b^3)*cosh(d*x + c)*sinh(d*x + c) - (128*a^3 - 664*a^2*b + 1
125*a*b^2 - 625*b^3)*sinh(d*x + c)^2 + 2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2
*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450
*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15
*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4
- 125*b^5)*d)*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5
 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b
^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a
^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) - (
(a*b^3 - b^4)*d*cosh(d*x + c)^8 + 8*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x
+ c)^7 + (a*b^3 - b^4)*d*sinh(d*x + c)^8 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^
6 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^6
 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*
d*cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
5*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 30*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8
*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*cosh(d*
x + c)^2 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 10*(a*b^3 - b^4)*d*cosh(d
*x + c)^3 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 15*(a*b^3 - b^4)*d*cosh(d*x + c)^
4 - 3*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*s
inh(d*x + c)^2 + (a*b^3 - b^4)*d + 8*((a*b^3 - b^4)*d*cosh(d*x + c)^7 - 3*(
a*b^3 - b^4)*d*cosh(d*x + c)^5 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x
+ c)^3 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^3*b^4 - 3*
a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450
*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15
*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^
4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a
^2*b^5 - 91*a*b^6 + 25*b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1
450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 +
 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 128*a^3 - 664*a^2*b + 1125*a*b^2 -
625*b^3 - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 6...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**8/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.05, size = 149, normalized size = 0.47

$$\frac{\frac{abe^{(6\,dx+6\,c)}-8\,a^{2}e^{(4\,dx+4\,c)}+3\,abe^{(4\,dx+4\,c)}-5\,abe^{(2\,dx+2\,c)}+ab}{(ab^{2}-b^{3})\left(be^{(8\,dx+8\,c)}-4\,be^{(6\,dx+6\,c)}-16\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}-4\,be^{(2\,dx+2\,c)}+b\right)}+\frac{2\,(dx+c)}{b^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a*b*e^(6*d*x + 6*c) - 8*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) -
 5*a*b*e^(2*d*x + 2*c) + a*b)/((a*b^2 - b^3)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*
d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*
c) + b)) + 2*(d*x + c)/b^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sinh(c+d\,x)}^8}{{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^8/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^8/(a - b*sinh(c + d*x)^4)^2, x)
```

3.248 $\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=233

$$\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d}-\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/2}d}$$

```
[Out] 1/8*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)-3*b^(1/
2))/a^(1/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctanh((a^(1/2)+b^(1/2))
^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)+3*b^(1/2))/a^(1/4)/b^(3/2)/d/(a^(1/2
)+b^(1/2))^(3/2)+1/4*tanh(d*x+c)/(a-b)/b/d+1/4*sech(d*x+c)^2*tanh(d*x+c)^3/
b/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A]
time = 0.23, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3296, 1134, 1293, 1180, 214}

$$\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\,b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\,b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh(c+dx)}{4bd(a-b)}+\frac{\tanh^3(c+dx)\text{sech}^2(c+dx)}{4bd\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((2*Sqrt[a] - 3*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^
(1/4)])/(8*a^(1/4)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - ((2*Sqrt[a] + 3*S
qrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(1/4
)*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) + Tanh[c + d*x]/(4*(a - b)*b*d) + (S
ech[c + d*x]^2*Tanh[c + d*x]^3)/(4*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*T
anh[c + d*x]^4))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
```

```
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}^2(c+dx)\tanh^3(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{x^2(6a-2ax^2)}{a-2ax^2+(a-b)}\right.}{8}$$

$$= \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{\operatorname{sech}^2(c+dx)\tanh^3(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\text{Subs}}{}$$

$$= \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{\operatorname{sech}^2(c+dx)\tanh^3(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\left(a -\right.}{}$$

$$= \frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d} - \frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tan}{8\sqrt[4]{a}\left(\right.}$$

Mathematica [A]
time = 1.84, size = 238, normalized size = 1.02

$$\frac{\frac{\sqrt{b}\left(-2a+\sqrt{a}\sqrt{b}+3b\right)\text{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}} + \frac{\sqrt{b}\left(-2a-\sqrt{a}\sqrt{b}+3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}} - \frac{4b(-2a-b+b\cosh(2(c+dx)))\sinh(2(c+dx))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))}}{8(a-b)b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((Sqrt[b]*(-2*a + Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c
 + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]
*(-2*a - Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])
/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (4*b*(-2*a - b + b
*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] -
 b*Cosh[4*(c + d*x)]))/(8*(a - b)*b^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.95, size = 305, normalized size = 1.31

method	result

derivativedivides	$\frac{-\frac{128\left(-\frac{a\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256b(a-b)}+\frac{(a+4b)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256b(a-b)}+\frac{(a+4b)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256b(a-b)}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{256b(a-b)}\right)}{a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{\_\_R=\mathrm{RootOf}\left(a\_\_Z\right.}{}}{d}$
default	$\frac{-\frac{128\left(-\frac{a\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256b(a-b)}+\frac{(a+4b)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256b(a-b)}+\frac{(a+4b)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256b(a-b)}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{256b(a-b)}\right)}{a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{\_\_R=\mathrm{RootOf}\left(a\_\_Z\right.}{}}{d}$
risch	$\frac{2a\,\mathrm{e}^{6dx+6c}-b\,\mathrm{e}^{6dx+6c}-8a\,\mathrm{e}^{4dx+4c}+3b\,\mathrm{e}^{4dx+4c}-2a\,\mathrm{e}^{2dx+2c}-3b\,\mathrm{e}^{2dx+2c}+b}{2bd(a-b)\left(-b\,\mathrm{e}^{8dx+8c}+4b\,\mathrm{e}^{6dx+6c}+16a\,\mathrm{e}^{4dx+4c}-6b\,\mathrm{e}^{4dx+4c}+4b\,\mathrm{e}^{2dx+2c}-b\right)}+\left(\sum_{\_\_R=\mathrm{RootOf}\left((65536a^{4}b^{6}d^{4}-196608a\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-128*(-1/256*a/b/(a-b)*tanh(1/2*d*x+1/2*c)^7+1/256/b*(a+4*b)/(a-b)*tan
h(1/2*d*x+1/2*c)^5+1/256/b*(a+4*b)/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/256*a/b/(a
-b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6
+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*
c)^2+a)-1/16/b/(a-b)*sum((-a*_R^6+(-5*a+12*b)*_R^4+(5*a-12*b)*_R^2+a)/(_R^7
*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*
_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*e^(6*c) - b*e^(6*c))*e^(6*d*x) - (8*a*e^(4*c) - 3*b*e^(4*c))*e^(
4*d*x) - (2*a*e^(2*c) + 3*b*e^(2*c))*e^(2*d*x) + b)/(a*b^2*d - b^3*d + (a*b
^2*d*e^(8*c) - b^3*d*e^(8*c))*e^(8*d*x) - 4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c
))*e^(6*d*x) - 2*(8*a^2*b*d*e^(4*c) - 11*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))
*e^(4*d*x) - 4*(a*b^2*d*e^(2*c) - b^3*d*e^(2*c))*e^(2*d*x)) + 1/64*integrat
e(64*((2*a*e^(6*c) - 3*b*e^(6*c))*e^(6*d*x) + (2*a*e^(2*c) - 3*b*e^(2*c))*e
^(2*d*x) + 6*b*e^(4*d*x + 4*c))/(a*b^2 - b^3 + (a*b^2*e^(8*c) - b^3*e^(8*c)
)*e^(8*d*x) - 4*(a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*e^(4*c
) - 11*a*b^2*e^(4*c) + 3*b^3*e^(4*c))*e^(4*d*x) - 4*(a*b^2*e^(2*c) - b^3*e^
(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6045 vs. $2(181) = 362$.

time = 0.63, size = 6045, normalized size = 25.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*(2*a - b)*cosh(d*x + c)^6 + 48*(2*a - b)*cosh(d*x + c)*sinh(d*x +
c)^5 + 8*(2*a - b)*sinh(d*x + c)^6 - 8*(8*a - 3*b)*cosh(d*x + c)^4 + 8*(15*
(2*a - b)*cosh(d*x + c)^2 - 8*a + 3*b)*sinh(d*x + c)^4 + 32*(5*(2*a - b)*co
sh(d*x + c)^3 - (8*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(2*a + 3*b)*
cosh(d*x + c)^2 + 8*(15*(2*a - b)*cosh(d*x + c)^4 - 6*(8*a - 3*b)*cosh(d*x
+ c)^2 - 2*a - 3*b)*sinh(d*x + c)^2 - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*
(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x +
c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x +
c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*
cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30
*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x
 + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x
 + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 -
15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(
d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2
 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b -
11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d
*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90
*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7
 - 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^
4 + 3*a*b^5 - b^6)*d^2))*log(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*
b^4 + 9*a*b^5)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 +
15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + (20*a^2 -
 81*a*b + 81*b^2)*cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*cosh(d*x +
 c)*sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*sinh(d*x + c)^2 - 20*a^2 + 8
1*a*b - 81*b^2 + 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^
7)*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 -
 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 2*(5*a^3*b - 19*a^2*b
^2 + 18*a*b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25
*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 1
5*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 -
 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(
a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c
)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c
)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*c
osh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*c
```

```
osh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*
(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x
+ c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x
+ c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 1
5*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d
*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2
- b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 1
1*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*
x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*
a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7
- 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4
 + 3*a*b^5 - b^6)*d^2))*log(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b
^4 + 9*a*b^5)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 1
5*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + (20*a^2 -
81*a*b + 81*b^2)*cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*cosh(d*x +
c)*sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*sinh(d*x + c)^2 - 20*a^2 + 81
*a*b - 81*b^2 - 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7
)*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 -
20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 2*(5*a^3*b - 19*a^2*b^
2 + 18*a*b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*
a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15
*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 -
3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a
*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)
^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)
^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*co
sh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b...
```

Sympy **[F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac **[A]**
time = 1.10, size = 153, normalized size = 0.66

$$-\frac{2\,ae^{(6\,dx+6\,c)} - be^{(6\,dx+6\,c)} - 8\,ae^{(4\,dx+4\,c)} + 3\,be^{(4\,dx+4\,c)} - 2\,ae^{(2\,dx+2\,c)} - 3\,be^{(2\,dx+2\,c)} + b}{2\,(ab - b^2)(be^{(8\,dx+8\,c)} - 4\,be^{(6\,dx+6\,c)} - 16\,ae^{(4\,dx+4\,c)} + 6\,be^{(4\,dx+4\,c)} - 4\,be^{(2\,dx+2\,c)} + b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*e^(6*d*x + 6*c) - b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e
^(4*d*x + 4*c) - 2*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + b)/((a*b - b^2
)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(
4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^6}{{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4)^2, x)
```

3.249 $\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=195

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}\,d}-\frac{\tanh(c+dx)}{4a(a-b)d}+\frac{\quad}{4ad\left(a-2a\tanh^2\right.}$$

```
[Out] 1/8*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)
-b^(1/2))^(3/2)/b^(1/2)-1/8*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(
1/4))/a^(3/4)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(3/2)-1/4*tanh(d*x+c)/a/(a-b)/d+1
/4*tanh(d*x+c)^5/a/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A]
time = 0.18, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$,
Rules used = {3296, 1289, 12, 1136, 1180, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh^5(c+dx)}{4ad\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\tanh(c+dx)}{4ad(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a
] - Sqrt[b])^(3/2)*Sqrt[b]*d) - ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d
*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - Tanh[c + d*
x]/(4*a*(a - b)*d) + Tanh[c + d*x]^5/(4*a*d*(a - 2*a*Tanh[c + d*x]^2 + (a -
 b)*Tanh[c + d*x]^4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4(1-x^2)}{(a-2ax^2+(a-b)x^4)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh^5(c+dx)}{4ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int -\frac{2bx^4}{a-2ax^2+(a-b}\right.}{8}$$

$$= \frac{\tanh^5(c+dx)}{4ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{x^4}{a-2ax^2+(a-b)x}\right.}{4}$$

$$= -\frac{\tanh(c+dx)}{4a(a-b)d} + \frac{\tanh^5(c+dx)}{4ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\text{Sub}}{}$$

$$= -\frac{\tanh(c+dx)}{4a(a-b)d} + \frac{\tanh^5(c+dx)}{4ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{(\sqrt{}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}\,d} -$$

Mathematica [A]
time = 3.03, size = 225, normalized size = 1.15

$$-\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)\text{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{2(-6\sinh(2(c+dx))+\sinh(4(c+dx)))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))}}{8(a-b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/8*(((Sqrt[a] + Sqrt[b])*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[
-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((Sq
rt[a] - Sqrt[b])*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[
a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) - (2*(-6*Sinh[2*(
c + d*x)] + Sinh[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh
[4*(c + d*x)]))/((a - b)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.65, size = 263, normalized size = 1.35

method	result
derivativedivides	$\frac{-\frac{32\left(\frac{\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a-64b}-\frac{5\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64(a-b)}-\frac{5\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64(a-b)}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a-64b}\right)}{a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{\_\_R=\text{RootOf}\left(a\_\right.}{}}{d}$
default	$\frac{-\frac{32\left(\frac{\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a-64b}-\frac{5\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64(a-b)}-\frac{5\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64(a-b)}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a-64b}\right)}{a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{\_\_R=\text{RootOf}\left(a\_\right.}{}}{d}$
risch	$\frac{-b\,\mathrm{e}^{6dx+6c}+8a\,\mathrm{e}^{4dx+4c}-3b\,\mathrm{e}^{4dx+4c}+5b\,\mathrm{e}^{2dx+2c}-b}{2b(a-b)d\left(-b\,\mathrm{e}^{8dx+8c}+4b\,\mathrm{e}^{6dx+6c}+16a\,\mathrm{e}^{4dx+4c}-6b\,\mathrm{e}^{4dx+4c}+4b\,\mathrm{e}^{2dx+2c}-b\right)}+\left(\_\_R=\text{RootOf}\left(1+(65536a^{6}b^{2}d^{4}-196\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-32*(1/64/(a-b)*tanh(1/2*d*x+1/2*c)^7-5/64/(a-b)*tanh(1/2*d*x+1/2*c)^5
-5/64/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/64/(a-b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1
/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)-1/16/(a-b)*sum((_R^6-7*_R^
4+7*_R^2-1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)
-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((8*a*e^(4*c) - 3*b*e^(4*c))*e^(4*d*x) - b*e^(6*d*x + 6*c) + 5*b*e^(2*
d*x + 2*c) - b)/(a*b^2*d - b^3*d + (a*b^2*d*e^(8*c) - b^3*d*e^(8*c))*e^(8*d
*x) - 4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*d*e^(4*c)
- 11*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))*e^(4*d*x) - 4*(a*b^2*d*e^(2*c) - b^
3*d*e^(2*c))*e^(2*d*x)) + 1/16*integrate(16*(e^(6*d*x + 6*c) - 6*e^(4*d*x +
 4*c) + e^(2*d*x + 2*c))/(a*b - b^2 + (a*b*e^(8*c) - b^2*e^(8*c))*e^(8*d*x)
 - 4*(a*b*e^(6*c) - b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^2*e^(4*c) - 11*a*b*e^(4
*c) + 3*b^2*e^(4*c))*e^(4*d*x) - 4*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)),
x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5658 vs. $2(151) = 302$.
time = 0.53, size = 5658, normalized size = 29.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cosh(d*x + c)^6 + 48*b*cosh(d*x + c)*sinh(d*x + c)^5 + 8*b*sinh(d
*x + c)^6 - 8*(8*a - 3*b)*cosh(d*x + c)^4 + 8*(15*b*cosh(d*x + c)^2 - 8*a +
 3*b)*sinh(d*x + c)^4 + 32*(5*b*cosh(d*x + c)^3 - (8*a - 3*b)*cosh(d*x + c)
)*sinh(d*x + c)^3 - 40*b*cosh(d*x + c)^2 + 8*(15*b*cosh(d*x + c)^4 - 6*(8*a
 - 3*b)*cosh(d*x + c)^2 - 5*b)*sinh(d*x + c)^2 + ((a*b^2 - b^3)*d*cosh(d*x
+ c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*
sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*
cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2
+ 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^
2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x
+ c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*
d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)
*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b
^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*
x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b
^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d
+ 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 -
(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x +
 c))*sinh(d*x + c))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt(
(9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^
5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b
^3 - a*b^4)*d^2))*log(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*sqrt((9*a
^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^
5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a + b)*cosh(d*x + c)^2 + 2*(3*a + b)*co
sh(d*x + c)*sinh(d*x + c) + (3*a + b)*sinh(d*x + c)^2 + 2*(2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*
b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - (
3*a^3 + 4*a^2*b + a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d
^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4
 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 +
 3*a^2*b^3 - a*b^4)*d^2)) - 3*a - b) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8
*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x +
 c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x +
 c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d
*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3
0*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*
x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*
x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3
)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 -
 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh
```

```
(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^
2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b -
 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(
d*x + c))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6
*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*
a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4
)*d^2))*log(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*sqrt((9*a^2 + 6*a*b
 + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*
b^6 + a^3*b^7)*d^4)) + (3*a + b)*cosh(d*x + c)^2 + 2*(3*a + b)*cosh(d*x + c
)*sinh(d*x + c) + (3*a + b)*sinh(d*x + c)^2 - 2*(2*(a^6*b - 3*a^5*b^2 + 3*a
^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a
^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - (3*a^3 + 4*
a^2*b + a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9
*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*
b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d^2)) - 3*a - b) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 -
b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*
(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a
*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x
+ c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 -
 b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 -
 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 -
 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d
*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2
 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.83, size = 128, normalized size = 0.66

$$\frac{be^{(6\,dx+6\,c)} - 8\,ae^{(4\,dx+4\,c)} + 3\,be^{(4\,dx+4\,c)} - 5\,be^{(2\,dx+2\,c)} + b}{2\,(ab-b^2)(be^{(8\,dx+8\,c)} - 4\,be^{(6\,dx+6\,c)} - 16\,ae^{(4\,dx+4\,c)} + 6\,be^{(4\,dx+4\,c)} - 4\,be^{(2\,dx+2\,c)} + b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 5*b*e^
(2*d*x + 2*c) + b)/((a*b - b^2)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) -
16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)^4}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4)^2, x)
```

3.250 $\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^2}\,dx$

Optimal. Leaf size=220

$$-\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}\,d}+\frac{\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}\,d}$$

```
[Out] -1/8*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)-b^(1/2
))/a^(5/4)/d/(a^(1/2)-b^(1/2))^(3/2)/b^(1/2)+1/8*arctanh((a^(1/2)+b^(1/2))^
(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)+b^(1/2))/a^(5/4)/d/b^(1/2)/(a^(1/2)+b
^(1/2))^(3/2)+1/4*tanh(d*x+c)*(a-(a+b)*tanh(d*x+c)^2)/a/(a-b)/d/(a-2*a*tanh
(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A]
time = 0.22, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3296, 1347, 1180, 214}

$$-\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4ad(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/8*((2*Sqrt[a] - Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])
/a^(1/4)])/(a^(5/4)*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]*d) + ((2*Sqrt[a] + Sq
rt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(5/4)
*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) + (Tanh[c + d*x]*(a - (a + b)*Tanh[c
+ d*x]^2))/(4*a*(a - b)*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^
4))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(1-x^2\right)^2}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{\frac{2a^2b}{a-b}-}{a-2ax}\right.}{}$$

$$= \frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\left(2a-\sqrt{a}\sqrt{b}\right.}{}$$

$$= -\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}\,d} + \frac{\left(2\sqrt{a}+\sqrt{b}\right)\tanh}{8a^{5/4}\left(\sqrt{\right.}}$$

Mathematica [A]

time = 1.47, size = 253, normalized size = 1.15

$$\frac{\frac{\sqrt{a}\left(2a+\sqrt{a}\sqrt{b}-b\right)\text{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\sqrt{a}\left(2a-\sqrt{a}\sqrt{b}-b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{4\sqrt{a}\,(2a+b-b\cosh(2(c+dx)))\sinh(2(c+dx))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))}}{8a^{3/2}(a-b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((Sqrt[a]*(2*a + Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c +
d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (
Sqrt[a]*(2*a - Sqrt[a]*Sqrt[b] - b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d
*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (4*S
qrt[a]*(2*a + b - b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*
Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]))/(8*a^(3/2)*(a - b)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.21, size = 297, normalized size = 1.35

method	result
derivativedivides	$\dfrac{-\frac{8\left(-\frac{\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{16(a-b)}+\frac{(a+4b)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a(a-b)}+\frac{(a+4b)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a(a-b)}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{16(a-b)}\right)}{a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{\_R=\text{RootOf}\left(a\_\right.}{}}{d}$
default	$\dfrac{-\frac{8\left(-\frac{\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{16(a-b)}+\frac{(a+4b)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a(a-b)}+\frac{(a+4b)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a(a-b)}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{16(a-b)}\right)}{a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-\frac{\_R=\text{RootOf}\left(a\_\right.}{}}{d}$
risch	$\frac{2a\,\mathrm{e}^{6dx+6c}-b\,\mathrm{e}^{6dx+6c}-8a\,\mathrm{e}^{4dx+4c}+3b\,\mathrm{e}^{4dx+4c}-2a\,\mathrm{e}^{2dx+2c}-3b\,\mathrm{e}^{2dx+2c}+b}{2ad(a-b)\left(-b\,\mathrm{e}^{8dx+8c}+4b\,\mathrm{e}^{6dx+6c}+16a\,\mathrm{e}^{4dx+4c}-6b\,\mathrm{e}^{4dx+4c}+4b\,\mathrm{e}^{2dx+2c}-b\right)}+\left(\sum_{\_R=\text{RootOf}\left(\left(65536a^{8}b^{2}d^{4}-19660\right.\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-8*(-1/16/(a-b)*tanh(1/2*d*x+1/2*c)^7+1/16*(a+4*b)/a/(a-b)*tanh(1/2*d*
x+1/2*c)^5+1/16*(a+4*b)/a/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/16/(a-b)*tanh(1/2*d
*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*
d*x+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)-1/16/a
/(a-b)*sum((-a*_R^6+(11*a-4*b)*_R^4+(-11*a+4*b)*_R^2+a)/(_R^7*a-3*_R^5*a+3*
_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+
(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*e^(6*c) - b*e^(6*c))*e^(6*d*x) - (8*a*e^(4*c) - 3*b*e^(4*c))*e^(
4*d*x) - (2*a*e^(2*c) + 3*b*e^(2*c))*e^(2*d*x) + b)/(a^2*b*d - a*b^2*d + (a
^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*e^(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*
e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e^(4*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^
(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) - 1/4*i
ntegrate(4*((2*a*e^(6*c) - b*e^(6*c))*e^(6*d*x) - 2*(4*a*e^(4*c) - b*e^(4*c
))*e^(4*d*x) + (2*a*e^(2*c) - b*e^(2*c))*e^(2*d*x))/(a^2*b - a*b^2 + (a^2*b
*e^(8*c) - a*b^2*e^(8*c))*e^(8*d*x) - 4*(a^2*b*e^(6*c) - a*b^2*e^(6*c))*e^(
6*d*x) - 2*(8*a^3*e^(4*c) - 11*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c))*e^(4*d*x) -
 4*(a^2*b*e^(2*c) - a*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6525 vs. $2(171) = 342$.
time = 0.64, size = 6525, normalized size = 29.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*(2*a - b)*cosh(d*x + c)^6 + 48*(2*a - b)*cosh(d*x + c)*sinh(d*x +
c)^5 + 8*(2*a - b)*sinh(d*x + c)^6 - 8*(8*a - 3*b)*cosh(d*x + c)^4 + 8*(15*
(2*a - b)*cosh(d*x + c)^2 - 8*a + 3*b)*sinh(d*x + c)^4 + 32*(5*(2*a - b)*co
sh(d*x + c)^3 - (8*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(2*a + 3*b)*
cosh(d*x + c)^2 + 8*(15*(2*a - b)*cosh(d*x + c)^4 - 6*(8*a - 3*b)*cosh(d*x
+ c)^2 - 2*a - 3*b)*sinh(d*x + c)^2 + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 +
8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(
d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*c
osh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b +
 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a
^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*co
sh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b +
3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a
^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8
*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b -
 a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3
 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)
^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b -
a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 -
 (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^5*b - 3*a^4*b^2
+ 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3
+ b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6
```

```
*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3
- a^2*b^4)*d^2))*log(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4
)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a
^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))
 + 32*a^3 - 28*a^2*b + 9*a*b^2 - b^3 - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*
cosh(d*x + c)^2 - 2*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cosh(d*x + c)*sinh(
d*x + c) - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*sinh(d*x + c)^2 + 2*((3*a^8*
b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^
3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*
a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 2*(8*a^5 - 5*a^4*b + a^
3*b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4
 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b
^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b
^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))) - ((a^2*b - a*b^2)*d*
cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*
b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(
a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(
8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh
(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a
^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8
*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*
x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*co
sh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x
 + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2
)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x +
c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d
*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((
a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a
^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 +
15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^
4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*log(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7
*a^4*b^3 + a^3*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b
^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^
6 + a^5*b^7)*d^4)) + 32*a^3 - 28*a^2*b + 9*a*b^2 - b^3 - (32*a^3 - 28*a^2*b
 + 9*a*b^2 - b^3)*cosh(d*x + c)^2 - 2*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*c
osh(d*x + c)*sinh(d*x + c) - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*sinh(d*x +
 c)^2 - 2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sq
rt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2
+ 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 2*(8*
a^5 - 5*a^4*b + a^3*b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4
)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a
^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))
 - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))) - (
(a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.65, size = 152, normalized size = 0.69

$$-\frac{2\,ae^{(6\,dx+6\,c)}-be^{(6\,dx+6\,c)}-8\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-2\,ae^{(2\,dx+2\,c)}-3\,be^{(2\,dx+2\,c)}+b}{2\,(a^2-ab)\big(be^{(8\,dx+8\,c)}-4\,be^{(6\,dx+6\,c)}-16\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}-4\,be^{(2\,dx+2\,c)}+b\big)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*e^(6*d*x + 6*c) - b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e
^(4*d*x + 4*c) - 2*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + b)/((a^2 - a*b
)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(
4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sinh(c+d\,x)}^2}{{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4)^2, x)
```

3.251 $\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}$$

```
[Out] 1/8*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(4*a^(1/2)-3*b^(1/
2))/a^(7/4)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctanh((a^(1/2)+b^(1/2))^(1/2)*t
anh(d*x+c)/a^(1/4))*(4*a^(1/2)+3*b^(1/2))/a^(7/4)/d/(a^(1/2)+b^(1/2))^(3/2)
-1/4*b*tanh(d*x+c)*(1-2*tanh(d*x+c)^2)/a/(a-b)/d/(a-2*a*tanh(d*x+c)^2+(a-b)
*tanh(d*x+c)^4)
```

Rubi [A]

time = 0.18, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3288, 1219, 1180, 214}

$$\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tanh(c+dx)\left(1-2\tanh^2(c+dx)\right)}{4ad(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sinh[c + d*x]^4)^(-2),x]
```

```
[Out] ((4*Sqrt[a] - 3*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^
(1/4)])/(8*a^(7/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((4*Sqrt[a] + 3*Sqrt[b])*
ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(7/4)*(Sqrt[
a] + Sqrt[b])^(3/2)*d) - (b*Tanh[c + d*x]*(1 - 2*Tanh[c + d*x]^2))/(4*a*(a
- b)*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)\left(1-2\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \text{Subst}\Big(\int -\frac{2a}{}$$

$$= -\frac{b\tanh(c+dx)\left(1-2\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \Big(4a-\sqrt{a}\,\sqrt{}$$

$$= \frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} + \frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tanh}{8a^{7/4}\Big(}$$

Mathematica [A]

time = 2.05, size = 230, normalized size = 1.10

$$\frac{-\frac{\left(4a+\sqrt{a}\,\sqrt{b}-3b\right)\mathrm{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\,\sqrt{b}}}+\frac{\left(4a-\sqrt{a}\,\sqrt{b}-3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{a}\,b(-6\sinh(2(c+dx))+\sinh(4(c+dx)))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))}}{8a^{3/2}(a-b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sinh[c + d*x]^4)^(-2),x]
```

```
[Out] (-(((4*a + Sqrt[a]*Sqrt[b] - 3*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x]
)/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]]) + ((4*a - Sqrt[a
]*Sqrt[b] - 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[
a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (2*Sqrt[a]*b*(-6*Sinh[2*(c + d*x)
] + Sinh[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c +
d*x)]))/(8*a^(3/2)*(a - b)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.05, size = 307, normalized size = 1.46

method	result
derivativedivides	$-\frac{2\left(\frac{b\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a(a-b)}-\frac{5b\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a(a-b)}-\frac{5b\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a(a-b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a-b)}\right)}{a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-$ $\_\_R=\mathrm{RootOf}\big(a\_$
default	$-\frac{2\left(\frac{b\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a(a-b)}-\frac{5b\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a(a-b)}-\frac{5b\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a(a-b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a-b)}\right)}{a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16b\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}-$ $\_\_R=\mathrm{RootOf}\big(a\_$
risch	$\frac{-b\,\mathrm{e}^{6dx+6c}+8a\,\mathrm{e}^{4dx+4c}-3b\,\mathrm{e}^{4dx+4c}+5b\,\mathrm{e}^{2dx+2c}-b}{2ad(a-b)\left(-b\,\mathrm{e}^{8dx+8c}+4b\,\mathrm{e}^{6dx+6c}+16a\,\mathrm{e}^{4dx+4c}-6b\,\mathrm{e}^{4dx+4c}+4b\,\mathrm{e}^{2dx+2c}-b\right)}+\Bigg($ $\_\_R=\mathrm{RootOf}\big((65536a^{10}d^{4}-196608$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(1/4*b/a/(a-b)*tanh(1/2*d*x+1/2*c)^7-5/4*b/a/(a-b)*tanh(1/2*d*x+1/2
*c)^5-5/4*b/a/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/4*b/a/(a-b)*tanh(1/2*d*x+1/2*c)
)/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c
)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)-1/16/a/(a-b)*su
m(((4*a-3*b)*_R^6+(-12*a+5*b)*_R^4+(12*a-5*b)*_R^2-4*a+3*b)/(_R^7*a-3*_R^5*
a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_
Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((8*a*e^(4*c) - 3*b*e^(4*c))*e^(4*d*x) - b*e^(6*d*x + 6*c) + 5*b*e^(2*
d*x + 2*c) - b)/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*e^
(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e^(4
*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^(2*c
) - a*b^2*d*e^(2*c))*e^(2*d*x)) + integrate(-(2*(8*a*e^(4*c) - 5*b*e^(4*c))
*e^(4*d*x) - b*e^(6*d*x + 6*c) - b*e^(2*d*x + 2*c))/(a^2*b - a*b^2 + (a^2*b
*e^(8*c) - a*b^2*e^(8*c))*e^(8*d*x) - 4*(a^2*b*e^(6*c) - a*b^2*e^(6*c))*e^(
6*d*x) - 2*(8*a^3*e^(4*c) - 11*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c))*e^(4*d*x) -
 4*(a^2*b*e^(2*c) - a*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6522 vs. $2(164) = 328$.
time = 0.64, size = 6522, normalized size = 31.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cosh(d*x + c)^6 + 48*b*cosh(d*x + c)*sinh(d*x + c)^5 + 8*b*sinh(d
*x + c)^6 - 8*(8*a - 3*b)*cosh(d*x + c)^4 + 8*(15*b*cosh(d*x + c)^2 - 8*a +
 3*b)*sinh(d*x + c)^4 + 32*(5*b*cosh(d*x + c)^3 - (8*a - 3*b)*cosh(d*x + c)
)*sinh(d*x + c)^3 - 40*b*cosh(d*x + c)^2 + 8*(15*b*cosh(d*x + c)^4 - 6*(8*a
 - 3*b)*cosh(d*x + c)^2 - 5*b)*sinh(d*x + c)^2 - ((a^2*b - a*b^2)*d*cosh(d*
x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b
^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b -
 a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 -
 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x +
c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b -
a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 -
11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^
2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x
+ c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*
(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4
 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*si
nh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 -
3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d
```

*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^6 - 3
*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2
*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 +
15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 -
3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(384*a^3*b - 680*a^2*b^2 + 405*a*b^
3 - 81*b^4 + 2*(16*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^
2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^1
3 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6
)*d^4)) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)^2 -
2*(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)*sinh(d*x + c
) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*sinh(d*x + c)^2 + 2*(2*(
2*a^10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1
392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^1
1*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + (120*a^5*b
- 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*
b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^
4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*
a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4
*b^2 - a^3*b^3)*d^2))) + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*
b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 -
4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^
2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*c
osh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4
 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*s
inh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)
*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2
*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cos
h(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b +
 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b -
 a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh
(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b
^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3
*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^
5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 +
 a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^
3*b^3)*d^2))*log(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4 + 2*(16*a^8 -
 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*sqrt((576*a^4*b - 1392
*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b
^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - (384*a^3*b - 6
80*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)^2 - 2*(384*a^3*b - 680*a^2*b
^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)*sinh(d*x + c) - (384*a^3*b - 680*a^2
*b^2 + 405*a*b^3 - 81*b^4)*sinh(d*x + c)^2 - 2*(2*(2*a^10 - 7*a^9*b + 9*a^8
*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b
^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15

```
*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + (120*a^5*b - 217*a^4*b^2 + 132*a^3*
b^3 - 27*a^2*b^4)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt(
(576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a
^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 127, normalized size = 0.60

$$\frac{be^{(6\,dx+6\,c)}-8\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-5\,be^{(2\,dx+2\,c)}+b}{2\,(a^2-ab)(be^{(8\,dx+8\,c)}-4\,be^{(6\,dx+6\,c)}-16\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}-4\,be^{(2\,dx+2\,c)}+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 5*b*e^
(2*d*x + 2*c) + b)/((a^2 - a*b)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) -
16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(1/(a - b*sinh(c + d*x)^4)^2, x)
```

3.252 $\int \frac{\mathbf{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=237

$$-\frac{\left(6\sqrt{a}-5\sqrt{b}\right)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}+\frac{\left(6\sqrt{a}+5\sqrt{b}\right)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\ \mathrm{t}}{\sqrt[4]{a}}\right.}{8a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}$$

```
[Out] -coth(d*x+c)/a^2/d-1/8*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))
*(6*a^(1/2)-5*b^(1/2))*b^(1/2)/a^(9/4)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctan
h((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*b^(1/2)*(6*a^(1/2)+5*b^(1/2)
)/a^(9/4)/d/(a^(1/2)+b^(1/2))^(3/2)+1/4*b*tanh(d*x+c)*(a-(a+b)*tanh(d*x+c)^
2)/a^2/(a-b)/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A]
time = 0.38, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3296, 1348, 1678, 1180, 214}

$$-\frac{\sqrt{b}\left(6\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{b}\left(6\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2d(a-b)\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\coth(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/8*((6*Sqrt[a] - 5*Sqrt[b])*Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh
[c + d*x])/a^(1/4)])/(a^(9/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((6*Sqrt[a] +
5*Sqrt[b])*Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]
)/(8*a^(9/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) - Coth[c + d*x]/(a^2*d) + (b*Tanh
[c + d*x]*(a - (a + b)*Tanh[c + d*x]^2))/(4*a^2*(a - b)*d*(a - 2*a*Tanh[c +
 d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1348

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p +
1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5)
 - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] &
& ILtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{x^2(a-2ax^2+(a-b)x^4)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \text{Subst}\Bigg(\int \frac{-8}{}$$

$$= \frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \text{Subst}\Big(\int \Big(-$$

$$= -\frac{\coth(c+dx)}{a^2d} + \frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx\right.}$$

$$= -\frac{\coth(c+dx)}{a^2d} + \frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx\right.}$$

$$= -\frac{\left(6\sqrt{a}-5\sqrt{b}\right)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} + \Big(6\sqrt{a}+5\sqrt{}$$

Mathematica [A]
time = 1.40, size = 272, normalized size = 1.15

$$\frac{\frac{\left(6a\sqrt{b}-5\sqrt{a}\,b\right)\text{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\left(6a\sqrt{b}+5\sqrt{a}\,b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{a+\sqrt{a}\sqrt{b}}}-8\sqrt{a}\coth(c+dx)+\frac{4\sqrt{a}\,b(2a+b-b\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b)(8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx)))}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (((6*a*Sqrt[b] - 5*Sqrt[a]*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sq
rt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a + Sqrt[a]*Sqrt[b]])
 + ((6*a*Sqrt[b] + 5*Sqrt[a]*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])
/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqrt[a + Sqrt[a]*Sqrt[b]]
) - 8*Sqrt[a]*Coth[c + d*x] + (4*Sqrt[a]*b*(2*a + b - b*Cosh[2*(c + d*x)])*
Sinh[2*(c + d*x)])/((a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(
c + d*x)])))/(8*a^(5/2)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.41, size = 326, normalized size = 1.38 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^2*tanh(1/2*d*x+1/2*c)-16*b/a^2*((-1/32*a/(a-b)*tanh(1/2*d*x+1/2
*c)^7+1/32*(a+4*b)/(a-b)*tanh(1/2*d*x+1/2*c)^5+1/32*(a+4*b)/(a-b)*tanh(1/2*
d*x+1/2*c)^3-1/32*a/(a-b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^8-4*a
*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4
-4*a*tanh(1/2*d*x+1/2*c)^2+a)+1/256/(a-b)*sum((-a*_R^6+(27*a-20*b)*_R^4+(-2
7*a+20*b)*_R^2+a)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+
1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a)))-1/2/a^2/
tanh(1/2*d*x+1/2*c))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b - 5*b^2 + (6*a*b*e^(8*c) - 5*b^2*e^(8*c))*e^(8*d*x) - 2*(13*a*b*
e^(6*c) - 10*b^2*e^(6*c))*e^(6*d*x) - 2*(32*a^2*e^(4*c) - 47*a*b*e^(4*c) +
15*b^2*e^(4*c))*e^(4*d*x) - 2*(7*a*b*e^(2*c) - 10*b^2*e^(2*c))*e^(2*d*x))/(
a^3*b*d - a^2*b^2*d - (a^3*b*d*e^(10*c) - a^2*b^2*d*e^(10*c))*e^(10*d*x) +
5*(a^3*b*d*e^(8*c) - a^2*b^2*d*e^(8*c))*e^(8*d*x) + 2*(8*a^4*d*e^(6*c) - 13
*a^3*b*d*e^(6*c) + 5*a^2*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^4*d*e^(4*c) - 13
*a^3*b*d*e^(4*c) + 5*a^2*b^2*d*e^(4*c))*e^(4*d*x) - 5*(a^3*b*d*e^(2*c) - a^
2*b^2*d*e^(2*c))*e^(2*d*x)) - 4*integrate(1/4*((6*a*b*e^(6*c) - 5*b^2*e^(6*
c))*e^(6*d*x) - 2*(8*a*b*e^(4*c) - 5*b^2*e^(4*c))*e^(4*d*x) + (6*a*b*e^(2*c
) - 5*b^2*e^(2*c))*e^(2*d*x))/(a^3*b - a^2*b^2 + (a^3*b*e^(8*c) - a^2*b^2*e
^(8*c))*e^(8*d*x) - 4*(a^3*b*e^(6*c) - a^2*b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^
4*e^(4*c) - 11*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c))*e^(4*d*x) - 4*(a^3*b*e^(2
*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8824 vs. 2(188) = 376.
time = 0.74, size = 8824, normalized size = 37.23

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*(6*a*b - 5*b^2)*cosh(d*x + c)^8 + 64*(6*a*b - 5*b^2)*cosh(d*x + c)
*sinh(d*x + c)^7 + 8*(6*a*b - 5*b^2)*sinh(d*x + c)^8 - 16*(13*a*b - 10*b^2)
```

```
*cosh(d*x + c)^6 + 16*(14*(6*a*b - 5*b^2)*cosh(d*x + c)^2 - 13*a*b + 10*b^2
)*sinh(d*x + c)^6 + 32*(14*(6*a*b - 5*b^2)*cosh(d*x + c)^3 - 3*(13*a*b - 10
*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 16*(32*a^2 - 47*a*b + 15*b^2)*cosh(d
*x + c)^4 + 16*(35*(6*a*b - 5*b^2)*cosh(d*x + c)^4 - 15*(13*a*b - 10*b^2)*c
osh(d*x + c)^2 - 32*a^2 + 47*a*b - 15*b^2)*sinh(d*x + c)^4 + 64*(7*(6*a*b -
 5*b^2)*cosh(d*x + c)^5 - 5*(13*a*b - 10*b^2)*cosh(d*x + c)^3 - (32*a^2 - 4
7*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*(7*a*b - 10*b^2)*cosh(d
*x + c)^2 + 16*(14*(6*a*b - 5*b^2)*cosh(d*x + c)^6 - 15*(13*a*b - 10*b^2)*c
osh(d*x + c)^4 - 6*(32*a^2 - 47*a*b + 15*b^2)*cosh(d*x + c)^2 - 7*a*b + 10*
b^2)*sinh(d*x + c)^2 + ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^10 + 10*(a^3*b -
a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3*b - a^2*b^2)*d*sinh(d*x + c
)^10 - 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 5*(9*(a^3*b - a^2*b^2)*d*cos
h(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3*b +
 5*a^2*b^2)*d*cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 -
 (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^3*b - a^2*b
^2)*d*cosh(d*x + c)^4 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 1
3*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*
d*cosh(d*x + c)^4 + 4*(63*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 70*(a^3*b -
 a^2*b^2)*d*cosh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 175*(a^
3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh
(d*x + c)^2 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^4 + 5*(a^3*b
- a^2*b^2)*d*cosh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 -
35*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 5*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d
*cosh(d*x + c)^3 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + (45*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 - 140*(a^3*b - a^2*b^2)*d
*cosh(d*x + c)^6 - 30*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 12
*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + 5*(a^3*b - a^2*b^2)*d)*
sinh(d*x + c)^2 - (a^3*b - a^2*b^2)*d + 2*(5*(a^3*b - a^2*b^2)*d*cosh(d*x +
 c)^9 - 20*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 6*(8*a^4 - 13*a^3*b + 5*a^
2*b^2)*d*cosh(d*x + c)^5 + 4*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)
^3 + 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^7 - 3*a^
6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2
*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3
+ 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)
/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(1728*a^3*b^2 - 3684*a^2*b
^3 + 2625*a*b^4 - 625*b^5 + 2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b
^3 + 25*a^5*b^4)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 345
0*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*
b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b
^4 - 625*b^5)*cosh(d*x + c)^2 - 2*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4
 - 625*b^5)*cosh(d*x + c)*sinh(d*x + c) - (1728*a^3*b^2 - 3684*a^2*b^3 + 26
25*a*b^4 - 625*b^5)*sinh(d*x + c)^2 + 2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 -
 22*a^8*b^3 + 5*a^7*b^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b
^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 +
```

```
15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 2*(144*a^6*b - 303*a^5*b^2 + 21
3*a^4*b^3 - 50*a^3*b^4)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2
*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/(
(a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a
^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*d^2))) - ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^10 + 10*(a^3*b - a^2*
b^2)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^10
 - 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 5*(9*(a^3*b - a^2*b^2)*d*cosh(d*
x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3*b + 5*a
^2*b^2)*d*cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - (a^
3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^3*b - a^2*b^2)*
d*cosh(d*x + c)^4 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 13*a^
3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*co
sh(d*x + c)^4 + 4*(63*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 70*(a^3*b - a^2
*b^2)*d*cosh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))
*sinh(d*x + c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 175*(a^3*b
- a^2*b^2)*d*cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x
 + c)^2 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^4 + 5*(a^3*b - a^
2*b^2)*d*cosh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 35*(
a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 5*(8*a^4 -...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4373 deep
```

Giac [A]
time = 0.49, size = 238, normalized size = 1.00

$$-\frac{6\,abe^{(8\,dx+8\,c)}-5\,b^2e^{(8\,dx+8\,c)}-26\,abe^{(6\,dx+6\,c)}+20\,b^2e^{(6\,dx+6\,c)}-64\,a^2e^{(4\,dx+4\,c)}+94\,abe^{(4\,dx+4\,c)}-30\,b^2e^{(4\,dx+4\,c)}-14\,abe^{(2\,dx+2\,c)}+20\,b^2e^{(2\,dx+2\,c)}+4\,ab-5\,b^2}{2\,(a^3-a^2b)(be^{(10\,dx+10\,c)}-5\,be^{(8\,dx+8\,c)}-16\,ae^{(6\,dx+6\,c)}+10\,be^{(6\,dx+6\,c)}+16\,ae^{(4\,dx+4\,c)}-10\,be^{(4\,dx+4\,c)}+5\,be^{(2\,dx+2\,c)}-b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(6*a*b*e^(8*d*x + 8*c) - 5*b^2*e^(8*d*x + 8*c) - 26*a*b*e^(6*d*x + 6*c
) + 20*b^2*e^(6*d*x + 6*c) - 64*a^2*e^(4*d*x + 4*c) + 94*a*b*e^(4*d*x + 4*c
) - 30*b^2*e^(4*d*x + 4*c) - 14*a*b*e^(2*d*x + 2*c) + 20*b^2*e^(2*d*x + 2*c
) + 4*a*b - 5*b^2)/((a^3 - a^2*b)*(b*e^(10*d*x + 10*c) - 5*b*e^(8*d*x + 8*c
) - 16*a*e^(6*d*x + 6*c) + 10*b*e^(6*d*x + 6*c) + 16*a*e^(4*d*x + 4*c) - 10
*b*e^(4*d*x + 4*c) + 5*b*e^(2*d*x + 2*c) - b)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(c+d\,x\right)^2\,{\left(a-b\,\sinh\left(c+d\,x\right)^4\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)^2), x)
```

3.253 $\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=315

$$\frac{\left(5a-14\sqrt{a}\sqrt{b}+12b\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{9/4}d}+\frac{\left(5a+14\sqrt{a}\sqrt{b}+12b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{9/4}d}+$$

```
[Out] 1/8*a*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/(a-b)/b^2/d/(a-b+2*b*cosh(d*x+c)^2-
b*cosh(d*x+c)^4)^2-1/32*cosh(d*x+c)*(9*a^2-11*a*b-10*b^2-2*(2*a-5*b)*b*cosh
(d*x+c)^2)/(a-b)^2/b^2/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)+1/64*arcta
n(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(5*a+12*b-14*a^(1/2)*b^(1/2)
)/b^(9/4)/d/a^(1/2)/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctanh(b^(1/4)*cosh(d*x+c
)/(a^(1/2)+b^(1/2))^(1/2))*(5*a+12*b+14*a^(1/2)*b^(1/2))/b^(9/4)/d/a^(1/2)/
(a^(1/2)+b^(1/2))^(5/2)
```

Rubi [A]

time = 0.42, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3294, 1219, 1692, 1180, 211, 214}

$$-\frac{\cosh(c+dx)\left(9a^2-2b(2a-5b)\cosh^2(c+dx)-11ab-10b^2\right)}{32b^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}+\frac{\left(-14\sqrt{a}\sqrt{b}+5a+12b\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(14\sqrt{a}\sqrt{b}+5a+12b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}b^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b^2d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((5*a - 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt
[a] - Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(9/4)*d) + ((5*a +
 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(9/4)*d) + (a*Cosh[c + d
*x]*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b^2*d*(a - b + 2*b*Cosh[c + d*x
]^2 - b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(9*a^2 - 11*a*b - 10*b^2 - 2*(
2*a - 5*b)*b*Cosh[c + d*x]^2))/(32*(a - b)^2*b^2*d*(a - b + 2*b*Cosh[c + d*
x]^2 - b*Cosh[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{(a-b+2bx^2-bx^4)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{2a\left(a^2+a\right.}{b}\right.}{}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\left(9\right.}{32(a-b)^2b^2d}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\left(9\right.}{32(a-b)^2b^2d}$$

$$= \frac{\left(5a-14\sqrt{a}\sqrt{b}+12b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{9/4}d} + \frac{\left(5a+14\sqrt{a}\sqrt{b}+12\right.}{64\sqrt{a}\left(\sqrt{\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.22, size = 1021, normalized size = 3.24

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cosh[c + d*x]*(-9*a^2 + 13*a*b + 5*b^2 + (2*a - 5*b)*b*Cosh[2*(c + d*x
)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a
- b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[
2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 - RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*a*b*c + 5*b^2*c - 2*a*b*d*x + 5*b^2*d*
x - 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
 - Sinh[(c + d*x)/2]*#1] + 10*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2
] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 10*a^2*c*#1^2 + 28*a*b*c
*#1^2 - 39*b^2*c*#1^2 - 10*a^2*d*x*#1^2 + 28*a*b*d*x*#1^2 - 39*b^2*d*x*#1^2
 - 20*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
 - Sinh[(c + d*x)/2]*#1]*#1^2 + 56*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d
*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 78*b^2*Log[-Co
```

```
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^2 + 10*a^2*c*#1^4 - 28*a*b*c*#1^4 + 39*b^2*c*#1^4 + 10*a^2*d*x*#
1^4 - 28*a*b*d*x*#1^4 + 39*b^2*d*x*#1^4 + 20*a^2*Log[-Cosh[(c + d*x)/2] - S
inh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 56*a
*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh
[(c + d*x)/2]*#1]*#1^4 + 78*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
+ Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 2*a*b*c*#1^6 - 5*b^2*
c*#1^6 + 2*a*b*d*x*#1^6 - 5*b^2*d*x*#1^6 + 4*a*b*Log[-Cosh[(c + d*x)/2] - S
inh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 10*b
^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh
[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7)
 & ])/(128*(a - b)^2*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(263) = 526$.

time = 10.76, size = 652, normalized size = 2.07

method	result
derivativedivides	$\frac{\frac{a(5a^2-11ab+12b^2)(\tanh^{14}(\frac{dx}{2}+\frac{c}{2}))}{16b^2(a^2-2ab+b^2)}-\frac{a(35a^2-85ab+104b^2)(\tanh^{12}(\frac{dx}{2}+\frac{c}{2}))}{16b^2(a^2-2ab+b^2)}+\frac{(105a^3-407a^2b+652a\,b^2-320b^3)(\tanh^{10}(\frac{dx}{2}}{16b^2(a^2-2ab+b^2)}}{(a(\tanh^8(}$
default	$\frac{\frac{a(5a^2-11ab+12b^2)(\tanh^{14}(\frac{dx}{2}+\frac{c}{2}))}{16b^2(a^2-2ab+b^2)}-\frac{a(35a^2-85ab+104b^2)(\tanh^{12}(\frac{dx}{2}+\frac{c}{2}))}{16b^2(a^2-2ab+b^2)}+\frac{(105a^3-407a^2b+652a\,b^2-320b^3)(\tanh^{10}(\frac{dx}{2}}{16b^2(a^2-2ab+b^2)}}{(a(\tanh^8(}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(512*(1/8192*a*(5*a^2-11*a*b+12*b^2)/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^14-1/8192*a/b^2*(35*a^2-85*a*b+104*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^12+1/8192/b^2*(105*a^3-407*a^2*b+652*a*b^2-320*b^3)/(a^2-2*a*b+b^2)*
tanh(1/2*d*x+1/2*c)^10-1/8192*(175*a^3-865*a^2*b+1696*a*b^2-1408*b^3)/b^2/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+1/8192*(175*a^3-849*a^2*b+756*a*b^2+32
0*b^3)/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/8192*a*(105*a^2-383*a*b+
248*b^2)/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/8192*(35*a^2-77*a*b-12
*b^2)*a/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/8192*a^2*(5*a-11*b)/b^2
/(a^2-2*a*b+b^2))/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*ta
nh(1/2*d*x+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)
```

```
^2+1/16/b^2/(a^2-2*a*b+b^2)*a*(1/4*(-4*(a*b)^(1/2)*a+10*(a*b)^(1/2)*b+5*a^2
-11*a*b+12*b^2)/a/((a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/
2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2))-1/4*(4*(a*b)^(1/2)*a-1
0*(a*b)^(1/2)*b+5*a^2-11*a*b+12*b^2)/a/(-(a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/
4*(-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(a*b)^(1/2)*a-a*b)^(1/2)
)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((2*a*b^2*e^(15*c) - 5*b^3*e^(15*c))*e^(15*d*x) - (18*a^2*b*e^(13*c) -
 20*a*b^2*e^(13*c) - 25*b^3*e^(13*c))*e^(13*d*x) + 3*(18*a^2*b*e^(11*c) - 8
*a*b^2*e^(11*c) - 15*b^3*e^(11*c))*e^(11*d*x) + (160*a^3*e^(9*c) - 388*a^2*
b*e^(9*c) + 2*a*b^2*e^(9*c) + 25*b^3*e^(9*c))*e^(9*d*x) + (160*a^3*e^(7*c)
- 388*a^2*b*e^(7*c) + 2*a*b^2*e^(7*c) + 25*b^3*e^(7*c))*e^(7*d*x) + 3*(18*a
^2*b*e^(5*c) - 8*a*b^2*e^(5*c) - 15*b^3*e^(5*c))*e^(5*d*x) - (18*a^2*b*e^(3
*c) - 20*a*b^2*e^(3*c) - 25*b^3*e^(3*c))*e^(3*d*x) + (2*a*b^2*e^c - 5*b^3*e
^c)*e^(d*x))/(a^2*b^4*d - 2*a*b^5*d + b^6*d + (a^2*b^4*d*e^(16*c) - 2*a*b^5
*d*e^(16*c) + b^6*d*e^(16*c))*e^(16*d*x) - 8*(a^2*b^4*d*e^(14*c) - 2*a*b^5*
d*e^(14*c) + b^6*d*e^(14*c))*e^(14*d*x) - 4*(8*a^3*b^3*d*e^(12*c) - 23*a^2*
b^4*d*e^(12*c) + 22*a*b^5*d*e^(12*c) - 7*b^6*d*e^(12*c))*e^(12*d*x) + 8*(16
*a^3*b^3*d*e^(10*c) - 39*a^2*b^4*d*e^(10*c) + 30*a*b^5*d*e^(10*c) - 7*b^6*d
*e^(10*c))*e^(10*d*x) + 2*(128*a^4*b^2*d*e^(8*c) - 352*a^3*b^3*d*e^(8*c) +
355*a^2*b^4*d*e^(8*c) - 166*a*b^5*d*e^(8*c) + 35*b^6*d*e^(8*c))*e^(8*d*x) +
 8*(16*a^3*b^3*d*e^(6*c) - 39*a^2*b^4*d*e^(6*c) + 30*a*b^5*d*e^(6*c) - 7*b^
6*d*e^(6*c))*e^(6*d*x) - 4*(8*a^3*b^3*d*e^(4*c) - 23*a^2*b^4*d*e^(4*c) + 22
*a*b^5*d*e^(4*c) - 7*b^6*d*e^(4*c))*e^(4*d*x) - 8*(a^2*b^4*d*e^(2*c) - 2*a*
b^5*d*e^(2*c) + b^6*d*e^(2*c))*e^(2*d*x)) - 1/512*integrate(64*((2*a*b*e^(7
*c) - 5*b^2*e^(7*c))*e^(7*d*x) + (10*a^2*e^(5*c) - 28*a*b*e^(5*c) + 39*b^2*
e^(5*c))*e^(5*d*x) - (10*a^2*e^(3*c) - 28*a*b*e^(3*c) + 39*b^2*e^(3*c))*e^(
3*d*x) - (2*a*b*e^c - 5*b^2*e^c)*e^(d*x))/(a^2*b^3 - 2*a*b^4 + b^5 + (a^2*b
^3*e^(8*c) - 2*a*b^4*e^(8*c) + b^5*e^(8*c))*e^(8*d*x) - 4*(a^2*b^3*e^(6*c)
- 2*a*b^4*e^(6*c) + b^5*e^(6*c))*e^(6*d*x) - 2*(8*a^3*b^2*e^(4*c) - 19*a^2*
b^3*e^(4*c) + 14*a*b^4*e^(4*c) - 3*b^5*e^(4*c))*e^(4*d*x) - 4*(a^2*b^3*e^(2
*c) - 2*a*b^4*e^(2*c) + b^5*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 21541 vs. 2(264) = 528.
time = 0.85, size = 21541, normalized size = 68.38

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(16*(2*a*b^2 - 5*b^3)*cosh(d*x + c)^15 + 240*(2*a*b^2 - 5*b^3)*cosh(
d*x + c)*sinh(d*x + c)^14 + 16*(2*a*b^2 - 5*b^3)*sinh(d*x + c)^15 - 16*(18*
a^2*b - 20*a*b^2 - 25*b^3)*cosh(d*x + c)^13 - 16*(18*a^2*b - 20*a*b^2 - 25*
b^3 - 105*(2*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 208*(35*(2*
a*b^2 - 5*b^3)*cosh(d*x + c)^3 - (18*a^2*b - 20*a*b^2 - 25*b^3)*cosh(d*x +
c))*sinh(d*x + c)^12 + 48*(18*a^2*b - 8*a*b^2 - 15*b^3)*cosh(d*x + c)^11 +
48*(455*(2*a*b^2 - 5*b^3)*cosh(d*x + c)^4 + 18*a^2*b - 8*a*b^2 - 15*b^3 - 2
6*(18*a^2*b - 20*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 176*(2
73*(2*a*b^2 - 5*b^3)*cosh(d*x + c)^5 - 26*(18*a^2*b - 20*a*b^2 - 25*b^3)*co
sh(d*x + c)^3 + 3*(18*a^2*b - 8*a*b^2 - 15*b^3)*cosh(d*x + c))*sinh(d*x + c
)^10 + 16*(160*a^3 - 388*a^2*b + 2*a*b^2 + 25*b^3)*cosh(d*x + c)^9 + 16*(50
05*(2*a*b^2 - 5*b^3)*cosh(d*x + c)^6 - 715*(18*a^2*b - 20*a*b^2 - 25*b^3)*c
osh(d*x + c)^4 + 160*a^3 - 388*a^2*b + 2*a*b^2 + 25*b^3 + 165*(18*a^2*b - 8
*a*b^2 - 15*b^3)*cosh(d*x ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**9/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1091 vs. $2(264) = 528$.
time = 0.98, size = 1091, normalized size = 3.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*((20*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b - 35*sqrt(-b^2 - sqrt(a*b)*b)*a^3*
b^2 + 13*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^3 + 110*sqrt(-b^2 - sqrt(a*b)*b)*a*
b^4 - 4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b - sqrt(a*b)*sqrt(-b^2 - sq
rt(a*b)*b)*a^2*b^2 - 43*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^3 - 60*sqrt(
a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^4)*abs(b)*arctan(1/2*(e^(d*x + c) + e^(-d*x
 - c))/sqrt(-(a^2*b^3 - 2*a*b^4 + b^5 + sqrt((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4
 - b^5)*(a^2*b^3 - 2*a*b^4 + b^5) + (a^2*b^3 - 2*a*b^4 + b^5)^2))/(a^2*b^3
- 2*a*b^4 + b^5)))/(4*a^5*b^5 - 7*a^4*b^6 - 3*a^3*b^7 + 11*a^2*b^8 - 5*a*b^
```

```
9) + (20*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b - 35*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b
^2 + 13*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^3 + 110*sqrt(-b^2 + sqrt(a*b)*b)*a*b
^4 - 4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b - sqrt(a*b)*sqrt(-b^2 + sqr
t(a*b)*b)*a^2*b^2 - 43*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^3 - 60*sqrt(a
*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^4)*abs(b)*arctan(1/2*(e^(d*x + c) + e^(-d*x
- c))/sqrt(-(a^2*b^3 - 2*a*b^4 + b^5 - sqrt((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4
- b^5)*(a^2*b^3 - 2*a*b^4 + b^5) + (a^2*b^3 - 2*a*b^4 + b^5)^2))/(a^2*b^3 -
 2*a*b^4 + b^5)))/(4*a^5*b^5 - 7*a^4*b^6 - 3*a^3*b^7 + 11*a^2*b^8 - 5*a*b^9
) - 8*(2*a*b^2*(e^(d*x + c) + e^(-d*x - c))^7 - 5*b^3*(e^(d*x + c) + e^(-d*
x - c))^7 - 18*a^2*b*(e^(d*x + c) + e^(-d*x - c))^5 + 6*a*b^2*(e^(d*x + c)
+ e^(-d*x - c))^5 + 60*b^3*(e^(d*x + c) + e^(-d*x - c))^5 + 144*a^2*b*(e^(d
*x + c) + e^(-d*x - c))^3 - 96*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 240*b
^3*(e^(d*x + c) + e^(-d*x - c))^3 + 160*a^3*(e^(d*x + c) + e^(-d*x - c)) -
640*a^2*b*(e^(d*x + c) + e^(-d*x - c)) + 160*a*b^2*(e^(d*x + c) + e^(-d*x -
 c)) + 320*b^3*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c
))^4 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)^2*(a^2*b^2 - 2*a*b
^3 + b^4)))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^9}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^9/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^9/(a - b*sinh(c + d*x)^4)^3, x)
```

3.254 $\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=290

$$\frac{3\left(\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{7/4}d}-\frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{7/4}d}-\frac{a\cos}{8(a-b)bd\,(a-}$$

```
[Out] -1/8*a*cosh(d*x+c)*(2-cosh(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cosh(d*x+c)^2-b*cos
h(d*x+c)^4)^2+1/32*cosh(d*x+c)*(5*a-17*b-3*(a-3*b)*cosh(d*x+c)^2)/(a-b)^2/b
/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)+3/64*arctan(b^(1/4)*cosh(d*x+c)/
(a^(1/2)-b^(1/2))^(1/2))*(a^(1/2)-2*b^(1/2))/b^(7/4)/d/a^(1/2)/(a^(1/2)-b^(
1/2))^(5/2)-3/64*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(a^(1
/2)+2*b^(1/2))/b^(7/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(5/2)
```

Rubi [A]

time = 0.36, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3294, 1219, 1192, 1180, 211, 214}

$$\frac{3\left(\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\,b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\,b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cosh(c+dx)\left(-3(a-3b)\cosh^2(c+dx)+5a-17b\right)}{32bd(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8bd(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (3*(Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(7/4)*d) - (3*(Sqrt[a] + 2*S
qrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*Sqrt[
a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(7/4)*d) - (a*Cosh[c + d*x]*(2 - Cosh[c + d*
x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2)
+ (Cosh[c + d*x]*(5*a - 17*b - 3*(a - 3*b)*Cosh[c + d*x]^2))/(32*(a - b)^2*
b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\text{Subst}\Big(\int \frac{4a(a-}{(a-}}{}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\cosh(c+}{32(a-b)^2bd\,(}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\cosh(c+}{32(a-b)^2bd\,(}$$

$$= \frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{7/4}d} - \frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\Bigg(}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.96, size = 802, normalized size = 2.77

$$-\frac{32\cosh(c+dx)(-7a+25b+3(a-3b)\cosh(2(c+dx)))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))} + \frac{512a(a-b)(-5\cosh(c+dx)+\cosh(3(c+dx)))}{(-8a+3b-4b\cosh(2(c+dx))+b\cosh(4(c+dx)))^2} - 3\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \text{[illegible]}\right] \over 256(a-b)^2bd$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((-32*Cosh[c + d*x]*(-7*a + 25*b + 3*(a - 3*b)*Cosh[2*(c + d*x)]))/(8*a - 3
*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a - b)*(-5*Cosh
[c + d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cos
h[4*(c + d*x)])^2 - 3*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^
6 + b*#1^8 & , (a*c - 3*b*c + a*d*x - 3*b*d*x + 2*a*Log[-Cosh[(c + d*x)/2]
- Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 6*b*Lo
g[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
 d*x)/2]*#1] - 3*a*c*#1^2 + 17*b*c*#1^2 - 3*a*d*x*#1^2 + 17*b*d*x*#1^2 - 6*
a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[
(c + d*x)/2]*#1]*#1^2 + 34*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + C
osh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 3*a*c*#1^4 - 17*b*c*#1^4
 + 3*a*d*x*#1^4 - 17*b*d*x*#1^4 + 6*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*
x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 34*b*Log[-Cosh[
```

```
(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]
*#1]*#1^4 - a*c*#1^6 + 3*b*c*#1^6 - a*d*x*#1^6 + 3*b*d*x*#1^6 - 2*a*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^6 + 6*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*
b*#1^5 + b*#1^7) & ])/(256*(a - b)^2*b*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(238) = 476$.
time = 11.00, size = 585, normalized size = 2.02 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(128*(-3/1024*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14-3/1024/b*(a-10*b
)*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+1/1024/b*(16*a^2-111*a*b+80*b^2)
/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10-1/1024*(35*a^3-26*a^2*b-64*a*b^2+25
6*b^3)/a/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+1/1024*(40*a^2+95*a*b-336*
b^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/1024*(25*a^2+54*a*b-64*b^2)/
b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/1024*(8*a+19*b)*a/b/(a^2-2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^2-1/1024*(2*b+a)*a/b/(a^2-2*a*b+b^2))/(a*tanh(1/2*d*
x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh(1/
2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)^2+3/8/b/(a^2-2*a*b+b^2)*a*(1/8*
((a*b)^(1/2)*a-3*(a*b)^(1/2)*b-2*b^2)/a/b/((a*b)^(1/2)*a-a*b)^(1/2)*arctan(
1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2)
)-1/8*(-(a*b)^(1/2)*a+3*(a*b)^(1/2)*b-2*b^2)/a/b/(-(a*b)^(1/2)*a-a*b)^(1/2)
*arctan(1/4*(-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(a*b)^(1/2)*a-
a*b)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(a*b*e^(15*c) - 3*b^2*e^(15*c))*e^(15*d*x) - (23*a*b*e^(13*c) - 77*
b^2*e^(13*c))*e^(13*d*x) + (16*a^2*e^(11*c) + 131*a*b*e^(11*c) - 177*b^2*e^
(11*c))*e^(11*d*x) - (144*a^2*e^(9*c) + 367*a*b*e^(9*c) - 109*b^2*e^(9*c))*
e^(9*d*x) - (144*a^2*e^(7*c) + 367*a*b*e^(7*c) - 109*b^2*e^(7*c))*e^(7*d*x)
 + (16*a^2*e^(5*c) + 131*a*b*e^(5*c) - 177*b^2*e^(5*c))*e^(5*d*x) - (23*a*b
*e^(3*c) - 77*b^2*e^(3*c))*e^(3*d*x) + 3*(a*b*e^c - 3*b^2*e^c)*e^(d*x))/(a^
2*b^3*d - 2*a*b^4*d + b^5*d + (a^2*b^3*d*e^(16*c) - 2*a*b^4*d*e^(16*c) + b^
5*d*e^(16*c))*e^(16*d*x) - 8*(a^2*b^3*d*e^(14*c) - 2*a*b^4*d*e^(14*c) + b^5
*d*e^(14*c))*e^(14*d*x) - 4*(8*a^3*b^2*d*e^(12*c) - 23*a^2*b^3*d*e^(12*c) +
```

```
 22*a*b^4*d*e^(12*c) - 7*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16*a^3*b^2*d*e^(10
*c) - 39*a^2*b^3*d*e^(10*c) + 30*a*b^4*d*e^(10*c) - 7*b^5*d*e^(10*c))*e^(10
*d*x) + 2*(128*a^4*b*d*e^(8*c) - 352*a^3*b^2*d*e^(8*c) + 355*a^2*b^3*d*e^(8
*c) - 166*a*b^4*d*e^(8*c) + 35*b^5*d*e^(8*c))*e^(8*d*x) + 8*(16*a^3*b^2*d*e
^(6*c) - 39*a^2*b^3*d*e^(6*c) + 30*a*b^4*d*e^(6*c) - 7*b^5*d*e^(6*c))*e^(6*
d*x) - 4*(8*a^3*b^2*d*e^(4*c) - 23*a^2*b^3*d*e^(4*c) + 22*a*b^4*d*e^(4*c) -
 7*b^5*d*e^(4*c))*e^(4*d*x) - 8*(a^2*b^3*d*e^(2*c) - 2*a*b^4*d*e^(2*c) + b^
5*d*e^(2*c))*e^(2*d*x)) + 1/128*integrate(24*((a*e^(7*c) - 3*b*e^(7*c))*e^(
7*d*x) - (3*a*e^(5*c) - 17*b*e^(5*c))*e^(5*d*x) + (3*a*e^(3*c) - 17*b*e^(3*
c))*e^(3*d*x) - (a*e^c - 3*b*e^c)*e^(d*x))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*
b^2*e^(8*c) - 2*a*b^3*e^(8*c) + b^4*e^(8*c))*e^(8*d*x) - 4*(a^2*b^2*e^(6*c)
 - 2*a*b^3*e^(6*c) + b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^3*b*e^(4*c) - 19*a^2*b
^2*e^(4*c) + 14*a*b^3*e^(4*c) - 3*b^4*e^(4*c))*e^(4*d*x) - 4*(a^2*b^2*e^(2*
c) - 2*a*b^3*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 20362 vs. $2(234) = 468$.
time = 0.74, size = 20362, normalized size = 70.21

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(24*(a*b - 3*b^2)*cosh(d*x + c)^15 + 360*(a*b - 3*b^2)*cosh(d*x + c)*
sinh(d*x + c)^14 + 24*(a*b - 3*b^2)*sinh(d*x + c)^15 - 8*(23*a*b - 77*b^2)*
cosh(d*x + c)^13 + 8*(315*(a*b - 3*b^2)*cosh(d*x + c)^2 - 23*a*b + 77*b^2)*
sinh(d*x + c)^13 + 104*(105*(a*b - 3*b^2)*cosh(d*x + c)^3 - (23*a*b - 77*b^
2)*cosh(d*x + c))*sinh(d*x + c)^12 + 8*(16*a^2 + 131*a*b - 177*b^2)*cosh(d*
x + c)^11 + 8*(4095*(a*b - 3*b^2)*cosh(d*x + c)^4 - 78*(23*a*b - 77*b^2)*co
sh(d*x + c)^2 + 16*a^2 + 131*a*b - 177*b^2)*sinh(d*x + c)^11 + 88*(819*(a*b
 - 3*b^2)*cosh(d*x + c)^5 - 26*(23*a*b - 77*b^2)*cosh(d*x + c)^3 + (16*a^2
+ 131*a*b - 177*b^2)*cosh(d*x + c))*sinh(d*x + c)^10 - 8*(144*a^2 + 367*a*b
 - 109*b^2)*cosh(d*x + c)^9 + 8*(15015*(a*b - 3*b^2)*cosh(d*x + c)^6 - 715*
(23*a*b - 77*b^2)*cosh(d*x + c)^4 + 55*(16*a^2 + 131*a*b - 177*b^2)*cosh(d*
x + c)^2 - 144*a^2 - 367*a*b + 109*b^2)*sinh(d*x + c)^9 + 24*(6435*(a*b - 3
*b^2)*cosh(d*x + c)^7 - 429*(23*a*b - 77*b^2)*cosh(d*x + c)^5 + 55*(16*a^2
+ 131*a*b - 177*b^2)*cosh ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

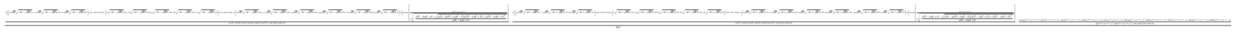
Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1515 vs. $2(234) = 468$.
time = 1.03, size = 1515, normalized size = 5.22



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/64*(3*((4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3 - 7*sqrt(a*b)*sqrt(-b^2
 - sqrt(a*b)*b)*a^2*b - 15*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^2)*(a^2*b
 - 2*a*b^2 + b^3)^2*abs(b) - (4*sqrt(-b^2 - sqrt(a*b)*b)*a^5*b^2 - 23*sqrt(
-b^2 - sqrt(a*b)*b)*a^4*b^3 + 9*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^4 + 35*sqrt(
-b^2 - sqrt(a*b)*b)*a^2*b^5 - 25*sqrt(-b^2 - sqrt(a*b)*b)*a*b^6)*abs(a^2*b
- 2*a*b^2 + b^3)*abs(b) - 2*(4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^5*b^4 -
 11*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^5 + 4*sqrt(a*b)*sqrt(-b^2 - sq
rt(a*b)*b)*a^3*b^6 + 14*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^7 - 16*sqr
t(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^8 + 5*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b
)*b^9)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^2*b^2 - 2*a
*b^3 + b^4 + sqrt((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*(a^2*b^2 - 2*a*b^3 +
b^4) + (a^2*b^2 - 2*a*b^3 + b^4)^2))/(a^2*b^2 - 2*a*b^3 + b^4)))/((4*a^7*b^
5 - 15*a^6*b^6 + 15*a^5*b^7 + 10*a^4*b^8 - 30*a^3*b^9 + 21*a^2*b^10 - 5*a*b
^11)*abs(a^2*b - 2*a*b^2 + b^3)) - 3*((4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)
*a^3 - 7*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b - 15*sqrt(a*b)*sqrt(-b^2
+ sqrt(a*b)*b)*a*b^2)*(a^2*b - 2*a*b^2 + b^3)^2*abs(b) + (4*sqrt(-b^2 + sqr
t(a*b)*b)*a^5*b^2 - 23*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^3 + 9*sqrt(-b^2 + sqr
t(a*b)*b)*a^3*b^4 + 35*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^5 - 25*sqrt(-b^2 + sq
rt(a*b)*b)*a*b^6)*abs(a^2*b - 2*a*b^2 + b^3)*abs(b) - 2*(4*sqrt(a*b)*sqrt(-
b^2 + sqrt(a*b)*b)*a^5*b^4 - 11*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^5
+ 4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^6 + 14*sqrt(a*b)*sqrt(-b^2 + s
qrt(a*b)*b)*a^2*b^7 - 16*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^8 + 5*sqrt(
a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^9)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*
x - c))/sqrt(-(a^2*b^2 - 2*a*b^3 + b^4 - sqrt((a^3*b - 3*a^2*b^2 + 3*a*b^3
- b^4)*(a^2*b^2 - 2*a*b^3 + b^4) + (a^2*b^2 - 2*a*b^3 + b^4)^2))/(a^2*b^2 -
 2*a*b^3 + b^4)))/((4*a^7*b^5 - 15*a^6*b^6 + 15*a^5*b^7 + 10*a^4*b^8 - 30*a
^3*b^9 + 21*a^2*b^10 - 5*a*b^11)*abs(a^2*b - 2*a*b^2 + b^3)) - 4*(3*a*b*(e^
(d*x + c) + e^(-d*x - c))^7 - 9*b^2*(e^(d*x + c) + e^(-d*x - c))^7 - 44*a*b
*(e^(d*x + c) + e^(-d*x - c))^5 + 140*b^2*(e^(d*x + c) + e^(-d*x - c))^5 +
16*a^2*(e^(d*x + c) + e^(-d*x - c))^3 + 288*a*b*(e^(d*x + c) + e^(-d*x - c)
)^3 - 688*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 192*a^2*(e^(d*x + c) + e^(-d
*x - c)) - 896*a*b*(e^(d*x + c) + e^(-d*x - c)) + 1088*b^2*(e^(d*x + c) + e
^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^4 - 8*b*(e^(d*x + c) + e^(-d
*x - c))^2 - 16*a + 16*b)^2*(a^2*b - 2*a*b^2 + b^3)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^7}{{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4)^3, x)
```

3.255 $\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=313

$$-\frac{\left(3a-10\sqrt{a}\sqrt{b}+4b\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{5/4}d}-\frac{\left(3a+10\sqrt{a}\sqrt{b}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{5/4}d}+$$

```
[Out] 1/8*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cosh(d*x+c)^2-b*co
sh(d*x+c)^4)^2-1/32*cosh(d*x+c)*(a^2-11*a*b-2*b^2+2*b*(2*a+b)*cosh(d*x+c)^2
)/a/(a-b)^2/b/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)-1/64*arctan(b^(1/4)
*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a+4*b-10*a^(1/2)*b^(1/2))/a^(3/2)/
b^(5/4)/d/(a^(1/2)-b^(1/2))^(5/2)-1/64*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)
+b^(1/2))^(1/2))*(3*a+4*b+10*a^(1/2)*b^(1/2))/a^(3/2)/b^(5/4)/d/(a^(1/2)+b^
(1/2))^(5/2)
```

Rubi [A]

time = 0.39, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3294, 1219, 1192, 1180, 211, 214}

$$-\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(10\sqrt{a}\sqrt{b}+3a+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cosh(c+dx)\left(a^2+2b(2a+b)\cosh^2(c+dx)-11ab-2b^2\right)}{32abd(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8bd(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/64*((3*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt
[Sqrt[a] - Sqrt[b]]])/(a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(5/4)*d) - ((3*a
 + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]])/(64*a^(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(5/4)*d) + (Cosh[c + d*
x]*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2
 - b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(a^2 - 11*a*b - 2*b^2 + 2*b*(2*a
+ b)*Cosh[c + d*x]^2))/(32*a*(a - b)^2*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b
*Cosh[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a-b+2bx^2-bx^4)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

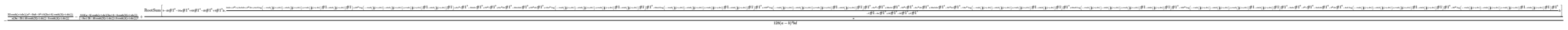
$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\text{Subst}\Big(\int \frac{2a(a-7b)}{(a-b+2bx}}{1}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\,(a^2}{32a(a-b)^2bd\,(a}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\,(a^2}{32a(a-b)^2bd\,(a}$$

$$= -\frac{\left(3a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{5/4}d} - \frac{\left(3a+10\sqrt{a}\sqrt{b}+4b\right.}{64a^{3/2}\left(\sqrt{a}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.36, size = 1019, normalized size = 3.26



Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/128*((32*Cosh[c + d*x]*(a^2 - 9*a*b - b^2 + b*(2*a + b)*Cosh[2*(c + d*x)
]))/(a*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) - (512*(a
 - b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh
[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 + RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (2*a*b*c + b^2*c + 2*a*b*d*x + b^2*d*x +
4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - S
inh[(c + d*x)/2]*#1] + 2*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + C
osh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*a^2*c*#1^2 - 32*a*b*c*#1^2
+ 5*b^2*c*#1^2 + 6*a^2*d*x*#1^2 - 32*a*b*d*x*#1^2 + 5*b^2*d*x*#1^2 + 12*a^2
*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(
c + d*x)/2]*#1]*#1^2 - 64*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 10*b^2*Log[-Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
```

```
1^2 - 6*a^2*c*#1^4 + 32*a*b*c*#1^4 - 5*b^2*c*#1^4 - 6*a^2*d*x*#1^4 + 32*a*b
*d*x*#1^4 - 5*b^2*d*x*#1^4 - 12*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 64*a*b*Log[-Cosh[
(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]
*#1]*#1^4 - 10*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 2*a*b*c*#1^6 - b^2*c*#1^6 - 2*a*b*
d*x*#1^6 - b^2*d*x*#1^6 - 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
+ Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 2*b^2*Log[-Cosh[(c +
d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*
#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ]/a)/((a - b)^2
*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(261) = 522$.
time = 10.68, size = 650, normalized size = 2.08

method	result
derivativedivides	$\frac{-\frac{\left(3a^{2}-13ab+4b^{2}\right)\left(\tanh^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16b\left(a^{2}-2ab+b^{2}\right)}+\frac{3\left(7a^{2}-33ab+8b^{2}\right)\left(\tanh^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16b\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(63a^{3}-225a^{2}b+68a\,b^{2}+64b^{3}\right)\left(\tanh^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16b\left(a^{2}-2ab+b^{2}\right)a}+}{\left(a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}$
default	$\frac{-\frac{\left(3a^{2}-13ab+4b^{2}\right)\left(\tanh^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16b\left(a^{2}-2ab+b^{2}\right)}+\frac{3\left(7a^{2}-33ab+8b^{2}\right)\left(\tanh^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16b\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(63a^{3}-225a^{2}b+68a\,b^{2}+64b^{3}\right)\left(\tanh^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16b\left(a^{2}-2ab+b^{2}\right)a}+}{\left(a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(32*(-1/512*(3*a^2-13*a*b+4*b^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
14+3/512/b*(7*a^2-33*a*b+8*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-1/51
2/b*(63*a^3-225*a^2*b+68*a*b^2+64*b^3)/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c
)^10+3/512*(35*a^3-61*a^2*b+32*a*b^2+128*b^3)/a/b/(a^2-2*a*b+b^2)*tanh(1/2*
d*x+1/2*c)^8-1/512/a*(105*a^3+9*a^2*b-452*a*b^2-64*b^3)/b/(a^2-2*a*b+b^2)*t
anh(1/2*d*x+1/2*c)^6+3/512*(21*a^2+29*a*b-40*b^2)/b/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^4-1/512*(21*a^2+37*a*b-4*b^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^2+3/512*a*(a+b)/b/(a^2-2*a*b+b^2))/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(
1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*t
anh(1/2*d*x+1/2*c)^2+a)^2+1/16/b/(a^2-2*a*b+b^2)*(1/4*(4*(a*b)^(1/2)*a+2*(a
*b)^(1/2)*b-3*a^2+13*a*b-4*b^2)/a/((a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(2*a
*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2))-1/4*(-
4*(a*b)^(1/2)*a-2*(a*b)^(1/2)*b-3*a^2+13*a*b-4*b^2)/a/(-(a*b)^(1/2)*a-a*b)^
```

```
(1/2)*arctan(1/4*(-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(a*b)^(1/
2)*a-a*b)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((2*a*b^2*e^(15*c) + b^3*e^(15*c))*e^(15*d*x) + (2*a^2*b*e^(13*c) - 24*
a*b^2*e^(13*c) - 5*b^3*e^(13*c))*e^(13*d*x) - (70*a^2*b*e^(11*c) - 76*a*b^2
*e^(11*c) - 9*b^3*e^(11*c))*e^(11*d*x) + (96*a^3*e^(9*c) + 164*a^2*b*e^(9*c
) - 54*a*b^2*e^(9*c) - 5*b^3*e^(9*c))*e^(9*d*x) + (96*a^3*e^(7*c) + 164*a^2
*b*e^(7*c) - 54*a*b^2*e^(7*c) - 5*b^3*e^(7*c))*e^(7*d*x) - (70*a^2*b*e^(5*c
) - 76*a*b^2*e^(5*c) - 9*b^3*e^(5*c))*e^(5*d*x) + (2*a^2*b*e^(3*c) - 24*a*b
^2*e^(3*c) - 5*b^3*e^(3*c))*e^(3*d*x) + (2*a*b^2*e^c + b^3*e^c)*e^(d*x))/(a
^3*b^3*d - 2*a^2*b^4*d + a*b^5*d + (a^3*b^3*d*e^(16*c) - 2*a^2*b^4*d*e^(16*
c) + a*b^5*d*e^(16*c))*e^(16*d*x) - 8*(a^3*b^3*d*e^(14*c) - 2*a^2*b^4*d*e^(
14*c) + a*b^5*d*e^(14*c))*e^(14*d*x) - 4*(8*a^4*b^2*d*e^(12*c) - 23*a^3*b^3
*d*e^(12*c) + 22*a^2*b^4*d*e^(12*c) - 7*a*b^5*d*e^(12*c))*e^(12*d*x) + 8*(1
6*a^4*b^2*d*e^(10*c) - 39*a^3*b^3*d*e^(10*c) + 30*a^2*b^4*d*e^(10*c) - 7*a*
b^5*d*e^(10*c))*e^(10*d*x) + 2*(128*a^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c)
 + 355*a^3*b^3*d*e^(8*c) - 166*a^2*b^4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))*e^(8
*d*x) + 8*(16*a^4*b^2*d*e^(6*c) - 39*a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^(6*
c) - 7*a*b^5*d*e^(6*c))*e^(6*d*x) - 4*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3*d*e
^(4*c) + 22*a^2*b^4*d*e^(4*c) - 7*a*b^5*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^3*d
*e^(2*c) - 2*a^2*b^4*d*e^(2*c) + a*b^5*d*e^(2*c))*e^(2*d*x)) + 1/32*integra
te(4*((2*a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (6*a^2*e^(5*c) - 32*a*b*e^(
5*c) + 5*b^2*e^(5*c))*e^(5*d*x) - (6*a^2*e^(3*c) - 32*a*b*e^(3*c) + 5*b^2*e
^(3*c))*e^(3*d*x) - (2*a*b*e^c + b^2*e^c)*e^(d*x))/(a^3*b^2 - 2*a^2*b^3 + a
*b^4 + (a^3*b^2*e^(8*c) - 2*a^2*b^3*e^(8*c) + a*b^4*e^(8*c))*e^(8*d*x) - 4*
(a^3*b^2*e^(6*c) - 2*a^2*b^3*e^(6*c) + a*b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^4*
b*e^(4*c) - 19*a^3*b^2*e^(4*c) + 14*a^2*b^3*e^(4*c) - 3*a*b^4*e^(4*c))*e^(4
*d*x) - 4*(a^3*b^2*e^(2*c) - 2*a^2*b^3*e^(2*c) + a*b^4*e^(2*c))*e^(2*d*x)),
 x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 22506 vs. $2(262) = 524$.
time = 0.83, size = 22506, normalized size = 71.90

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(16*(2*a*b^2 + b^3)*cosh(d*x + c)^15 + 240*(2*a*b^2 + b^3)*cosh(d*x +
 c)*sinh(d*x + c)^14 + 16*(2*a*b^2 + b^3)*sinh(d*x + c)^15 + 16*(2*a^2*b -
24*a*b^2 - 5*b^3)*cosh(d*x + c)^13 + 16*(2*a^2*b - 24*a*b^2 - 5*b^3 + 105*(
2*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 208*(35*(2*a*b^2 + b^3)*
cosh(d*x + c)^3 + (2*a^2*b - 24*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)
^12 - 16*(70*a^2*b - 76*a*b^2 - 9*b^3)*cosh(d*x + c)^11 + 16*(1365*(2*a*b^2
 + b^3)*cosh(d*x + c)^4 - 70*a^2*b + 76*a*b^2 + 9*b^3 + 78*(2*a^2*b - 24*a*
b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 176*(273*(2*a*b^2 + b^3)*c
osh(d*x + c)^5 + 26*(2*a^2*b - 24*a*b^2 - 5*b^3)*cosh(d*x + c)^3 - (70*a^2*
b - 76*a*b^2 - 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + 16*(96*a^3 + 164*a^
2*b - 54*a*b^2 - 5*b^3)*cosh(d*x + c)^9 + 16*(5005*(2*a*b^2 + b^3)*cosh(d*x
 + c)^6 + 715*(2*a^2*b - 24*a*b^2 - 5*b^3)*cosh(d*x + c)^4 + 96*a^3 + 164*a
^2*b - 54*a*b^2 - 5*b^3 - 55*(70*a^2*b - 76*a*b^2 - 9*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^9 + 48*(21 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1797 vs. $2(262) = 524$.
time = 0.93, size = 1797, normalized size = 5.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*((2*(a^3*b - 2*a^2*b^2 + a*b^3)^2*(8*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b
)*a^2 + 14*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b + 5*sqrt(a*b)*sqrt(-b^2 -
 sqrt(a*b)*b)*b^2)*abs(b) - (12*sqrt(-b^2 - sqrt(a*b)*b)*a^6*b - 77*sqrt(-b
^2 - sqrt(a*b)*b)*a^5*b^2 + 41*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^3 + 111*sqrt(
-b^2 - sqrt(a*b)*b)*a^3*b^4 - 97*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^5 + 10*sqrt
(-b^2 - sqrt(a*b)*b)*a*b^6)*abs(a^3*b - 2*a^2*b^2 + a*b^3)*abs(b) - (12*sqr
t(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^8*b^2 - 85*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b
)*b)*a^7*b^3 + 171*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^6*b^4 - 54*sqrt(a*b
)*sqrt(-b^2 - sqrt(a*b)*b)*a^5*b^5 - 214*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)
*a^4*b^6 + 279*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^7 - 129*sqrt(a*b)*s
qrt(-b^2 - sqrt(a*b)*b)*a^2*b^8 + 20*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b
^9)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^3*b^2 - 2*a^2*
```

```
b^3 + a*b^4 + sqrt((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*(a^3*b^2 - 2*a^2
*b^3 + a*b^4) + (a^3*b^2 - 2*a^2*b^3 + a*b^4)^2))/(a^3*b^2 - 2*a^2*b^3 + a*
b^4)))/((4*a^8*b^4 - 15*a^7*b^5 + 15*a^6*b^6 + 10*a^5*b^7 - 30*a^4*b^8 + 21
*a^3*b^9 - 5*a^2*b^10)*abs(a^3*b - 2*a^2*b^2 + a*b^3)) + (2*(a^3*b - 2*a^2*
b^2 + a*b^3)^2*(8*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 14*sqrt(a*b)*sqr
t(-b^2 + sqrt(a*b)*b)*a*b + 5*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^2)*abs(b
) - (12*sqrt(-b^2 + sqrt(a*b)*b)*a^6*b - 77*sqrt(-b^2 + sqrt(a*b)*b)*a^5*b^
2 + 41*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^3 + 111*sqrt(-b^2 + sqrt(a*b)*b)*a^3*
b^4 - 97*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^5 + 10*sqrt(-b^2 + sqrt(a*b)*b)*a*b
^6)*abs(a^3*b - 2*a^2*b^2 + a*b^3)*abs(b) - (12*sqrt(a*b)*sqrt(-b^2 + sqrt(
a*b)*b)*a^8*b^2 - 85*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^7*b^3 + 171*sqrt(
a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^6*b^4 - 54*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*
b)*a^5*b^5 - 214*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^6 + 279*sqrt(a*b)
*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^7 - 129*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*
a^2*b^8 + 20*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^9)*abs(b))*arctan(1/2*(
e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^3*b^2 - 2*a^2*b^3 + a*b^4 - sqrt((a^4*
b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*(a^3*b^2 - 2*a^2*b^3 + a*b^4) + (a^3*b^2
 - 2*a^2*b^3 + a*b^4)^2))/(a^3*b^2 - 2*a^2*b^3 + a*b^4)))/((4*a^8*b^4 - 15*
a^7*b^5 + 15*a^6*b^6 + 10*a^5*b^7 - 30*a^4*b^8 + 21*a^3*b^9 - 5*a^2*b^10)*a
bs(a^3*b - 2*a^2*b^2 + a*b^3)) + 8*(2*a*b^2*(e^(d*x + c) + e^(-d*x - c))^7
+ b^3*(e^(d*x + c) + e^(-d*x - c))^7 + 2*a^2*b*(e^(d*x + c) + e^(-d*x - c))
^5 - 38*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 - 12*b^3*(e^(d*x + c) + e^(-d*
x - c))^5 - 80*a^2*b*(e^(d*x + c) + e^(-d*x - c))^3 + 224*a*b^2*(e^(d*x + c
) + e^(-d*x - c))^3 + 48*b^3*(e^(d*x + c) + e^(-d*x - c))^3 + 96*a^3*(e^(d*
x + c) + e^(-d*x - c)) + 384*a^2*b*(e^(d*x + c) + e^(-d*x - c)) - 416*a*b^2
*(e^(d*x + c) + e^(-d*x - c)) - 64*b^3*(e^(d*x + c) + e^(-d*x - c)))/((b*(e
^(d*x + c) + e^(-d*x - c))^4 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a +
16*b)^2*(a^3*b - 2*a^2*b^2 + a*b^3)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^5}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4)^3, x)
```

3.256 $\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=288

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/4}d}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/4}d}-\frac{\mathrm{co}}{8(a-b)d\left(a-\right.}$$

```
[Out] -1/8*cosh(d*x+c)*(2-cosh(d*x+c)^2)/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*
x+c)^4)^2-1/32*cosh(d*x+c)*(11*a+b-(5*a+b)*cosh(d*x+c)^2)/a/(a-b)^2/d/(a-b+
2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)-1/64*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)
-b^(1/2))^(1/2))*(5*a^(1/2)-2*b^(1/2))/a^(3/2)/b^(3/4)/d/(a^(1/2)-b^(1/2))^
(5/2)+1/64*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(5*a^(1/2)+
2*b^(1/2))/a^(3/2)/b^(3/4)/d/(a^(1/2)+b^(1/2))^(5/2)
```

Rubi [A]
time = 0.38, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3294, 1192, 1180, 211, 214}

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cosh(c+dx)\left(-\left((5a+b)\cosh^2(c+dx)\right)+11a+b\right)}{32ad(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/64*((5*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a]
- Sqrt[b]]])/(a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/4)*d) + ((5*Sqrt[a] +
2*Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*a^
(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/4)*d) - (Cosh[c + d*x]*(2 - Cosh[c + d
*x]^2))/(8*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) -
 (Cosh[c + d*x]*(11*a + b - (5*a + b)*Cosh[c + d*x]^2))/(32*a*(a - b)^2*d*(
a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{(a-b+2bx^2-bx^4)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\text{Subst}\Big(\int \frac{-12a}{(a-b+}}{}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+}{32a(a-b)^2d\,(}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+}{32a(a-b)^2d\,(}$$

$$= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/4}d} + \frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\Big(}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.89, size = 802, normalized size = 2.78

$$\frac{32\cosh(c+dx)(-17a-b+(5a+b)\cosh(2(c+dx)))}{a(8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx)))} + \frac{512(a-b)(-5\cosh(c+dx)+\cosh(3(c+dx)))}{(-8a+3b-4b\cosh(2(c+dx))+b\cosh(4(c+dx)))^2} + \frac{\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \text{[illegible]}\right]}{256(a-b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cosh[c + d*x]*(-17*a - b + (5*a + b)*Cosh[2*(c + d*x)]))/(a*(8*a - 3*b
 + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (512*(a - b)*(-5*Cosh[c
+ d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4
*(c + d*x)])^2 + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b
*#1^8 & , (5*a*c + b*c + 5*a*d*x + b*d*x + 10*a*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 2*b*Log[-C
osh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x
)/2]*#1] - 47*a*c*#1^2 + 5*b*c*#1^2 - 47*a*d*x*#1^2 + 5*b*d*x*#1^2 - 94*a*L
og[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
+ d*x)/2]*#1]*#1^2 + 10*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh
[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 47*a*c*#1^4 - 5*b*c*#1^4 +
47*a*d*x*#1^4 - 5*b*d*x*#1^4 + 94*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 10*b*Log[-Cosh[(c
```

```
 + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#
1]*#1^4 - 5*a*c*#1^6 - b*c*#1^6 - 5*a*d*x*#1^6 - b*d*x*#1^6 - 10*a*Log[-Cos
h[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/
2]*#1]*#1^6 - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*
x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b
*#1^5 + b*#1^7) & ]/a)/(256*(a - b)^2*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(236) = 472$.
time = 10.73, size = 593, normalized size = 2.06 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(8*(-1/64*(4*a-b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14+1/64*(a^2+58*a
*b-32*b^2)/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^12+3/64/a*(20*a^2-73*a*b+4
8*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10-1/64/a^2*(175*a^3-550*a^2*b+8
32*a*b^2-256*b^3)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+1/64/a*(220*a^2-533
*a*b+112*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/64*(141*a^2-158*a*b+3
2*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/64*(44*a-17*b)/(a^2-2*a*b+
b^2)*tanh(1/2*d*x+1/2*c)^2-1/64*(5*a-2*b)/(a^2-2*a*b+b^2))/(a*tanh(1/2*d*x+
1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh(1/2*
d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)^2+1/8/(a^2-2*a*b+b^2)*(-1/8*(5*(a
*b)^(1/2)*a+(a*b)^(1/2)*b-8*a*b+2*b^2)/a/b/(-(a*b)^(1/2)*a-a*b)^(1/2)*arcta
n(1/4*(-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(a*b)^(1/2)*a-a*b)^(
1/2))+1/8*(-5*(a*b)^(1/2)*a-(a*b)^(1/2)*b-8*a*b+2*b^2)/a/b/((a*b)^(1/2)*a-a
*b)^(1/2)*arctan(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(
1/2)*a-a*b)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((5*a*b*e^(15*c) + b^2*e^(15*c))*e^(15*d*x) - (49*a*b*e^(13*c) + 5*b^
2*e^(13*c))*e^(13*d*x) - 3*(48*a^2*e^(11*c) - 55*a*b*e^(11*c) - 3*b^2*e^(11
*c))*e^(11*d*x) + (784*a^2*e^(9*c) - 377*a*b*e^(9*c) - 5*b^2*e^(9*c))*e^(9*
d*x) + (784*a^2*e^(7*c) - 377*a*b*e^(7*c) - 5*b^2*e^(7*c))*e^(7*d*x) - 3*(4
8*a^2*e^(5*c) - 55*a*b*e^(5*c) - 3*b^2*e^(5*c))*e^(5*d*x) - (49*a*b*e^(3*c)
 + 5*b^2*e^(3*c))*e^(3*d*x) + (5*a*b*e^c + b^2*e^c)*e^(d*x))/(a^3*b^2*d - 2
*a^2*b^3*d + a*b^4*d + (a^3*b^2*d*e^(16*c) - 2*a^2*b^3*d*e^(16*c) + a*b^4*d
*e^(16*c))*e^(16*d*x) - 8*(a^3*b^2*d*e^(14*c) - 2*a^2*b^3*d*e^(14*c) + a*b^
4*d*e^(14*c))*e^(14*d*x) - 4*(8*a^4*b*d*e^(12*c) - 23*a^3*b^2*d*e^(12*c) +
```

```
22*a^2*b^3*d*e^(12*c) - 7*a*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^4*b*d*e^(1
0*c) - 39*a^3*b^2*d*e^(10*c) + 30*a^2*b^3*d*e^(10*c) - 7*a*b^4*d*e^(10*c))*
e^(10*d*x) + 2*(128*a^5*d*e^(8*c) - 352*a^4*b*d*e^(8*c) + 355*a^3*b^2*d*e^(
8*c) - 166*a^2*b^3*d*e^(8*c) + 35*a*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^4*b*
d*e^(6*c) - 39*a^3*b^2*d*e^(6*c) + 30*a^2*b^3*d*e^(6*c) - 7*a*b^4*d*e^(6*c)
)*e^(6*d*x) - 4*(8*a^4*b*d*e^(4*c) - 23*a^3*b^2*d*e^(4*c) + 22*a^2*b^3*d*e^
(4*c) - 7*a*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^2*d*e^(2*c) - 2*a^2*b^3*d*e
^(2*c) + a*b^4*d*e^(2*c))*e^(2*d*x)) - 1/8*integrate(1/2*((5*a*e^(7*c) + b*
e^(7*c))*e^(7*d*x) - (47*a*e^(5*c) - 5*b*e^(5*c))*e^(5*d*x) + (47*a*e^(3*c)
 - 5*b*e^(3*c))*e^(3*d*x) - (5*a*e^c + b*e^c)*e^(d*x))/(a^3*b - 2*a^2*b^2 +
 a*b^3 + (a^3*b*e^(8*c) - 2*a^2*b^2*e^(8*c) + a*b^3*e^(8*c))*e^(8*d*x) - 4*
(a^3*b*e^(6*c) - 2*a^2*b^2*e^(6*c) + a*b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^4*e^
(4*c) - 19*a^3*b*e^(4*c) + 14*a^2*b^2*e^(4*c) - 3*a*b^3*e^(4*c))*e^(4*d*x)
- 4*(a^3*b*e^(2*c) - 2*a^2*b^2*e^(2*c) + a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 20961 vs. $2(233) = 466$.
time = 0.74, size = 20961, normalized size = 72.78

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(8*(5*a*b + b^2)*cosh(d*x + c)^15 + 120*(5*a*b + b^2)*cosh(d*x + c)*
sinh(d*x + c)^14 + 8*(5*a*b + b^2)*sinh(d*x + c)^15 - 8*(49*a*b + 5*b^2)*co
sh(d*x + c)^13 + 8*(105*(5*a*b + b^2)*cosh(d*x + c)^2 - 49*a*b - 5*b^2)*sin
h(d*x + c)^13 + 104*(35*(5*a*b + b^2)*cosh(d*x + c)^3 - (49*a*b + 5*b^2)*co
sh(d*x + c))*sinh(d*x + c)^12 - 24*(48*a^2 - 55*a*b - 3*b^2)*cosh(d*x + c)^
11 + 24*(455*(5*a*b + b^2)*cosh(d*x + c)^4 - 26*(49*a*b + 5*b^2)*cosh(d*x +
 c)^2 - 48*a^2 + 55*a*b + 3*b^2)*sinh(d*x + c)^11 + 88*(273*(5*a*b + b^2)*c
osh(d*x + c)^5 - 26*(49*a*b + 5*b^2)*cosh(d*x + c)^3 - 3*(48*a^2 - 55*a*b -
 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^10 + 8*(784*a^2 - 377*a*b - 5*b^2)*cos
h(d*x + c)^9 + 8*(5005*(5*a*b + b^2)*cosh(d*x + c)^6 - 715*(49*a*b + 5*b^2)
*cosh(d*x + c)^4 - 165*(48*a^2 - 55*a*b - 3*b^2)*cosh(d*x + c)^2 + 784*a^2
- 377*a*b - 5*b^2)*sinh(d*x + c)^9 + 72*(715*(5*a*b + b^2)*cosh(d*x + c)^7
- 143*(49*a*b + 5*b^2)*cosh(d*x + c)^5 - 55*(48*a^2 - 55*a*b - 3*b^2)*cosh(
d*x + c)^3 + (784*a^2 - 3 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1580 vs. 2(233) = 466.
time = 0.75, size = 1580, normalized size = 5.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*(((20*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2 + 29*sqrt(a*b)*sqrt(-b^2
- sqrt(a*b)*b)*a*b + 5*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^2)*(a^3 - 2*a^2
*b + a*b^2)^2*abs(b) - (52*sqrt(-b^2 - sqrt(a*b)*b)*a^5*b - 43*sqrt(-b^2 -
sqrt(a*b)*b)*a^4*b^2 - 75*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^3 + 71*sqrt(-b^2 -
 sqrt(a*b)*b)*a^2*b^4 - 5*sqrt(-b^2 - sqrt(a*b)*b)*a*b^5)*abs(a^3 - 2*a^2*b
 + a*b^2)*abs(b) + 2*(16*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^7*b - 48*sqrt
(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^6*b^2 + 27*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)
*b)*a^5*b^3 + 52*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^4 - 78*sqrt(a*b)*
sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^5 + 36*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^
2*b^6 - 5*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^7)*abs(b))*arctan(1/2*(e^(
d*x + c) + e^(-d*x - c))/sqrt(-(a^3*b - 2*a^2*b^2 + a*b^3 + sqrt((a^4 - 3*a
^3*b + 3*a^2*b^2 - a*b^3)*(a^3*b - 2*a^2*b^2 + a*b^3) + (a^3*b - 2*a^2*b^2
+ a*b^3)^2))/(a^3*b - 2*a^2*b^2 + a*b^3)))/((4*a^8*b^3 - 15*a^7*b^4 + 15*a^
6*b^5 + 10*a^5*b^6 - 30*a^4*b^7 + 21*a^3*b^8 - 5*a^2*b^9)*abs(a^3 - 2*a^2*b
 + a*b^2)) - ((20*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 29*sqrt(a*b)*sqr
t(-b^2 + sqrt(a*b)*b)*a*b + 5*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^2)*(a^3
- 2*a^2*b + a*b^2)^2*abs(b) + (52*sqrt(-b^2 + sqrt(a*b)*b)*a^5*b - 43*sqrt(
-b^2 + sqrt(a*b)*b)*a^4*b^2 - 75*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^3 + 71*sqrt
(-b^2 + sqrt(a*b)*b)*a^2*b^4 - 5*sqrt(-b^2 + sqrt(a*b)*b)*a*b^5)*abs(a^3 -
2*a^2*b + a*b^2)*abs(b) + 2*(16*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^7*b -
48*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^6*b^2 + 27*sqrt(a*b)*sqrt(-b^2 + sq
rt(a*b)*b)*a^5*b^3 + 52*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^4 - 78*sqr
t(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^5 + 36*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b
)*b)*a^2*b^6 - 5*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^7)*abs(b))*arctan(1
/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^3*b - 2*a^2*b^2 + a*b^3 - sqrt((a^
4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(a^3*b - 2*a^2*b^2 + a*b^3) + (a^3*b - 2*a
^2*b^2 + a*b^3)^2))/(a^3*b - 2*a^2*b^2 + a*b^3)))/((4*a^8*b^3 - 15*a^7*b^4
+ 15*a^6*b^5 + 10*a^5*b^6 - 30*a^4*b^7 + 21*a^3*b^8 - 5*a^2*b^9)*abs(a^3 -
2*a^2*b + a*b^2)) - 4*(5*a*b*(e^(d*x + c) + e^(-d*x - c))^7 + b^2*(e^(d*x +
 c) + e^(-d*x - c))^7 - 84*a*b*(e^(d*x + c) + e^(-d*x - c))^5 - 12*b^2*(e^(
d*x + c) + e^(-d*x - c))^5 - 144*a^2*(e^(d*x + c) + e^(-d*x - c))^3 + 480*a
*b*(e^(d*x + c) + e^(-d*x - c))^3 + 48*b^2*(e^(d*x + c) + e^(-d*x - c))^3 +
 1216*a^2*(e^(d*x + c) + e^(-d*x - c)) - 1152*a*b*(e^(d*x + c) + e^(-d*x -
c)) - 64*b^2*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))
```

```
^4 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)^2*(a^3 - 2*a^2*b + a
*b^2)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^3}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4)^3, x)
```

3.257 $\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=313

$$\frac{3\left(7a-10\sqrt{a}\sqrt{b}+4b\right)\mathrm{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt[4]{b}\,d}+\frac{3\left(7a+10\sqrt{a}\sqrt{b}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt[4]{b}\,d}+$$

```
[Out] 1/8*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/a/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*co
sh(d*x+c)^4)^2+1/32*cosh(d*x+c)*((7*a-3*b)*(a+2*b)-6*(2*a-b)*b*cosh(d*x+c)^
2)/a^2/(a-b)^2/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)+3/64*arctan(b^(1/4
)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(7*a+4*b-10*a^(1/2)*b^(1/2))/a^(5/2)
/b^(1/4)/d/(a^(1/2)-b^(1/2))^(5/2)+3/64*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2
)+b^(1/2))^(1/2))*(7*a+4*b+10*a^(1/2)*b^(1/2))/a^(5/2)/b^(1/4)/d/(a^(1/2)+b
^(1/2))^(5/2)
```

Rubi [A]

time = 0.38, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3294, 1106, 1192, 1180, 211, 214}

$$\frac{3\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\mathrm{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{3\left(10\sqrt{a}\sqrt{b}+7a+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cosh(c+dx)\left((7a-3b)(a+2b)-6b(2a-b)\cosh^2(c+dx)\right)}{32a^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8ad(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (3*(7*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqr
t[a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(1/4)*d) + (3*(7*
a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a]
+ Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(1/4)*d) + (Cosh[c + d
*x]*(a + b - b*Cosh[c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^
2 - b*Cosh[c + d*x]^4)^2) + (Cosh[c + d*x]*((7*a - 3*b)*(a + 2*b) - 6*(2*a
- b)*b*Cosh[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cosh[c + d*x]^2 -
 b*Cosh[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a-b+2bx^2-bx^4)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{2(a-b)b+}{}\right.}{}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\cosh(c+dx)\left((7\right.}{32a^2(a-b)^2d\,(}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\cosh(c+dx)\left((7\right.}{32a^2(a-b)^2d\,(}$$

$$= \frac{3\left(7a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt[4]{b}\,d} + \frac{3\left(7a+10\sqrt{a}\sqrt{b}+4\right.}{64a^{5/2}\left(\sqrt{a}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.98, size = 1018, normalized size = 3.25

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cosh[c + d*x]*(7*a^2 + 5*a*b - 3*b^2 + 3*b*(-2*a + b)*Cosh[2*(c + d*x)
]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a -
 b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2
*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 + 3*RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*a*b*c + b^2*c - 2*a*b*d*x + b^2*d*x -
 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
Sinh[(c + d*x)/2]*#1] + 2*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 14*a^2*c*#1^2 - 12*a*b*c*#1^
2 + 5*b^2*c*#1^2 + 14*a^2*d*x*#1^2 - 12*a*b*d*x*#1^2 + 5*b^2*d*x*#1^2 + 28*
a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sin
h[(c + d*x)/2]*#1]*#1^2 - 24*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
 + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 10*b^2*Log[-Cosh[(c
+ d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1
```

```
]*#1^2 - 14*a^2*c*#1^4 + 12*a*b*c*#1^4 - 5*b^2*c*#1^4 - 14*a^2*d*x*#1^4 + 1
2*a*b*d*x*#1^4 - 5*b^2*d*x*#1^4 - 28*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 24*a*b*Log[-
Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*
x)/2]*#1]*#1^4 - 10*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 2*a*b*c*#1^6 - b^2*c*#1^6 + 2
*a*b*d*x*#1^6 - b^2*d*x*#1^6 + 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 2*b^2*Log[-Cosh[
(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]
*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(128*a^
2*(a - b)^2*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(261) = 522$.

time = 10.00, size = 641, normalized size = 2.05 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/32*(11*a^2-37*a*b+20*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
14+1/32/a*(77*a^2-283*a*b+152*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-1
/32/a^2*(231*a^3-857*a^2*b+788*a*b^2-192*b^3)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^10+1/32/a^2*(385*a^3-1231*a^2*b+1888*a*b^2-640*b^3)/(a^2-2*a*b+b^2)*
tanh(1/2*d*x+1/2*c)^8-1/32/a^2*(385*a^3-831*a^2*b-148*a*b^2+192*b^3)/(a^2-2
*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+1/32/a*(231*a^2-209*a*b+8*b^2)/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^4-1/32*(77*a^2-3*a*b-20*b^2)/a/(a^2-2*a*b+b^2)*tanh
(1/2*d*x+1/2*c)^2+1/32*(11*a-5*b)/(a^2-2*a*b+b^2))/(a*tanh(1/2*d*x+1/2*c)^8
-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*
c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)^2+3/16/a/(a^2-2*a*b+b^2)*(1/4*(4*(a*b)^(1
/2)*a-2*(a*b)^(1/2)*b+7*a^2-9*a*b+4*b^2)/a/((a*b)^(1/2)*a-a*b)^(1/2)*arctan
(1/4*(2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2
))-1/4*(-4*(a*b)^(1/2)*a+2*(a*b)^(1/2)*b+7*a^2-9*a*b+4*b^2)/a/(-(a*b)^(1/2)
*a-a*b)^(1/2)*arctan(1/4*(-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)+2*a)/(-(
a*b)^(1/2)*a-a*b)^(1/2))))
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(2*a*b^2*e^(15*c) - b^3*e^(15*c))*e^(15*d*x) - (14*a^2*b*e^(13*c) +
28*a*b^2*e^(13*c) - 15*b^3*e^(13*c))*e^(13*d*x) - (86*a^2*b*e^(11*c) - 128*
a*b^2*e^(11*c) + 27*b^3*e^(11*c))*e^(11*d*x) + (352*a^3*e^(9*c) - 60*a^2*b*
```

```
e^(9*c) - 106*a*b^2*e^(9*c) + 15*b^3*e^(9*c))*e^(9*d*x) + (352*a^3*e^(7*c)
- 60*a^2*b*e^(7*c) - 106*a*b^2*e^(7*c) + 15*b^3*e^(7*c))*e^(7*d*x) - (86*a^
2*b*e^(5*c) - 128*a*b^2*e^(5*c) + 27*b^3*e^(5*c))*e^(5*d*x) - (14*a^2*b*e^(
3*c) + 28*a*b^2*e^(3*c) - 15*b^3*e^(3*c))*e^(3*d*x) + 3*(2*a*b^2*e^c - b^3*
e^c)*e^(d*x))/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(16*c) -
2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*c))*e^(16*d*x) - 8*(a^4*b^2*d*e^(14*
c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e^(14*c))*e^(14*d*x) - 4*(8*a^5*b*d*e
^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^3*b^3*d*e^(12*c) - 7*a^2*b^4*d*e^(12
*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c) - 39*a^4*b^2*d*e^(10*c) + 30*a^3*b
^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e^(10*d*x) + 2*(128*a^6*d*e^(8*c) - 3
52*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8*c) - 166*a^3*b^3*d*e^(8*c) + 35*a^2
*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b*d*e^(6*c) - 39*a^4*b^2*d*e^(6*c) +
30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6*c))*e^(6*d*x) - 4*(8*a^5*b*d*e^(4*c
) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d*e^(4*c) - 7*a^2*b^4*d*e^(4*c))*e^(4
*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^3*d*e^(2*c) + a^2*b^4*d*e^(2*c))*e^(
2*d*x)) + 1/2*integrate(3/4*((2*a*b*e^(7*c) - b^2*e^(7*c))*e^(7*d*x) - (14*
a^2*e^(5*c) - 12*a*b*e^(5*c) + 5*b^2*e^(5*c))*e^(5*d*x) + (14*a^2*e^(3*c) -
 12*a*b*e^(3*c) + 5*b^2*e^(3*c))*e^(3*d*x) - (2*a*b*e^c - b^2*e^c)*e^(d*x))
/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^4*b*e^(8*c) - 2*a^3*b^2*e^(8*c) + a^2*b^
3*e^(8*c))*e^(8*d*x) - 4*(a^4*b*e^(6*c) - 2*a^3*b^2*e^(6*c) + a^2*b^3*e^(6*
c))*e^(6*d*x) - 2*(8*a^5*e^(4*c) - 19*a^4*b*e^(4*c) + 14*a^3*b^2*e^(4*c) -
3*a^2*b^3*e^(4*c))*e^(4*d*x) - 4*(a^4*b*e^(2*c) - 2*a^3*b^2*e^(2*c) + a^2*b
^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 22332 vs. $2(263) = 526$.
time = 0.83, size = 22332, normalized size = 71.35

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(48*(2*a*b^2 - b^3)*cosh(d*x + c)^15 + 720*(2*a*b^2 - b^3)*cosh(d*x +
 c)*sinh(d*x + c)^14 + 48*(2*a*b^2 - b^3)*sinh(d*x + c)^15 - 16*(14*a^2*b +
 28*a*b^2 - 15*b^3)*cosh(d*x + c)^13 - 16*(14*a^2*b + 28*a*b^2 - 15*b^3 - 3
15*(2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 208*(105*(2*a*b^2 -
b^3)*cosh(d*x + c)^3 - (14*a^2*b + 28*a*b^2 - 15*b^3)*cosh(d*x + c))*sinh(d
*x + c)^12 - 16*(86*a^2*b - 128*a*b^2 + 27*b^3)*cosh(d*x + c)^11 + 16*(4095
*(2*a*b^2 - b^3)*cosh(d*x + c)^4 - 86*a^2*b + 128*a*b^2 - 27*b^3 - 78*(14*a
^2*b + 28*a*b^2 - 15*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 176*(819*(2*a
*b^2 - b^3)*cosh(d*x + c)^5 - 26*(14*a^2*b + 28*a*b^2 - 15*b^3)*cosh(d*x +
c)^3 - (86*a^2*b - 128*a*b^2 + 27*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + 16
*(352*a^3 - 60*a^2*b - 106*a*b^2 + 15*b^3)*cosh(d*x + c)^9 + 16*(15015*(2*a
*b^2 - b^3)*cosh(d*x + c)^6 - 715*(14*a^2*b + 28*a*b^2 - 15*b^3)*cosh(d*x +
```

```
 c)^4 + 352*a^3 - 60*a^2*b - 106*a*b^2 + 15*b^3 - 55*(86*a^2*b - 128*a*b^2
+ 27*b^3)*cosh(d*x + c)^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. 2(263) = 526.
time = 0.70, size = 1127, normalized size = 3.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*(3*(28*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b + 15*sqrt(-b^2 - sqrt(a*b)*b)*a^
3*b^2 - 17*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^3 + 10*sqrt(-b^2 - sqrt(a*b)*b)*a
*b^4 - 44*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b - 11*sqrt(a*b)*sqrt(-b^2
 - sqrt(a*b)*b)*a^2*b^2 + 39*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^3 - 20*
sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^4)*abs(b)*arctan(1/2*(e^(d*x + c) + e^
(-d*x - c))/sqrt(-(a^4*b - 2*a^3*b^2 + a^2*b^3 + sqrt((a^5 - 3*a^4*b + 3*a^
3*b^2 - a^2*b^3)*(a^4*b - 2*a^3*b^2 + a^2*b^3) + (a^4*b - 2*a^3*b^2 + a^2*b
^3)^2))/(a^4*b - 2*a^3*b^2 + a^2*b^3)))/(4*a^7*b^3 - 7*a^6*b^4 - 3*a^5*b^5
+ 11*a^4*b^6 - 5*a^3*b^7) + 3*(28*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b + 15*sqrt(
-b^2 + sqrt(a*b)*b)*a^3*b^2 - 17*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^3 + 10*sqrt
(-b^2 + sqrt(a*b)*b)*a*b^4 - 44*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b -
11*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^2 + 39*sqrt(a*b)*sqrt(-b^2 + sq
rt(a*b)*b)*a*b^3 - 20*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^4)*abs(b)*arctan
(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^4*b - 2*a^3*b^2 + a^2*b^3 - sqrt
((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*(a^4*b - 2*a^3*b^2 + a^2*b^3) + (a^4
*b - 2*a^3*b^2 + a^2*b^3)^2))/(a^4*b - 2*a^3*b^2 + a^2*b^3)))/(4*a^7*b^3 -
7*a^6*b^4 - 3*a^5*b^5 + 11*a^4*b^6 - 5*a^3*b^7) + 8*(6*a*b^2*(e^(d*x + c) +
 e^(-d*x - c))^7 - 3*b^3*(e^(d*x + c) + e^(-d*x - c))^7 - 14*a^2*b*(e^(d*x
+ c) + e^(-d*x - c))^5 - 70*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 + 36*b^3*(
e^(d*x + c) + e^(-d*x - c))^5 - 16*a^2*b*(e^(d*x + c) + e^(-d*x - c))^3 + 3
52*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 144*b^3*(e^(d*x + c) + e^(-d*x -
c))^3 + 352*a^3*(e^(d*x + c) + e^(-d*x - c)) + 128*a^2*b*(e^(d*x + c) + e^(
-d*x - c)) - 672*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 192*b^3*(e^(d*x + c)
```

```
+ e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^4 - 8*b*(e^(d*x + c) + e^
(-d*x - c))^2 - 16*a + 16*b)^2*(a^4 - 2*a^3*b + a^2*b^2)))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)}{{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4)^3,x)
[Out] int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4)^3, x)
```

3.258 $\int \frac{\mathbf{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=617

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\ \text{cosh}}{\sqrt{\sqrt{a}\ -}}\right.}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}}$$

```
[Out] -arctanh(cosh(d*x+c))/a^3/d-1/8*b*cosh(d*x+c)*(2-cosh(d*x+c)^2)/a/(a-b)/d/(
a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)^2-1/4*b*cosh(d*x+c)*(2-cosh(d*x+c)^2
)/a^2/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)-1/32*b*cosh(d*x+c)*(1
1*a+b-(5*a+b)*cosh(d*x+c)^2)/a^2/(a-b)^2/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*
x+c)^4)-1/64*b^(1/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(5
*a^(1/2)-2*b^(1/2))/a^(5/2)/d/(a^(1/2)-b^(1/2))^(5/2)-1/8*b^(1/4)*arctan(b^
(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^(5/2)/d/(a^(1/2)-b^(1/2))^(3/2
)+1/8*b^(1/4)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(5/2)/
d/(a^(1/2)+b^(1/2))^(3/2)+1/64*b^(1/4)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)
+b^(1/2))^(1/2))*(5*a^(1/2)+2*b^(1/2))/a^(5/2)/d/(a^(1/2)+b^(1/2))^(5/2)-1/
2*b^(1/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^3/d/(a^(1/2
)-b^(1/2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^
(1/2))/a^3/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]

time = 0.66, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3294, 1252, 213, 1192, 1180, 211, 214}

$$-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{b}\left(5\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\sqrt[4]{b}\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{5/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^3d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d}-\frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4a^2d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{b\cosh(c+dx)\left(-\left((5a+b)\cosh^2(c+dx)\right)+11a+b\right)}{32a^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8ad(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/64*((5*Sqrt[a] - 2*Sqrt[b])*b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[
Sqrt[a] - Sqrt[b]]])/(a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*d) - (b^(1/4)*ArcTa
n[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(5/2)*(Sqrt[a] - S
qrt[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - S
qrt[b]]])/(2*a^3*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cosh[c + d*x]]/(a^3*d
) + (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a
^(5/2)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*
x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^3*Sqrt[Sqrt[a] + Sqrt[b]]*d) + ((5*Sqrt[
a] + 2*Sqrt[b])*b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt
[b]]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b*Cosh[c + d*x]*(2 - Cos
```

```
h[c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x
]^4)^2) - (b*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*a^2*(a - b)*d*(a - b +
 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)) - (b*Cosh[c + d*x]*(11*a + b - (
5*a + b)*Cosh[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cosh[c + d*x]^2
 - b*Cosh[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1252

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\Big(\int \Big(-\frac{1}{a^3(-1+x^2)} + \frac{b-bx^2}{a(a-b+2bx^2-bx^4)^3} + \frac{b-bx^2}{a^2(a-b+2bx^2-bx^4)^2} + \frac{b-bx^2}{a^3(a-b+2bx^2-b}}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(c+dx)\right)}{a^3d} - \frac{\operatorname{Subst}\Big(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+}{a^3d}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d} - \frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8a(a-b)d\,\big(a-b+2b\cosh^2(c+dx)-b\cosh^4(c}$$

$$= -\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{\sqrt[4]{b}\,\tanh^{-1}\Bigg(}{2a^3\sqrt{\sqrt{a}}}$$

$$= -\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(}{}$$

$$= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} - \frac{\sqrt[4]{b}\,\tan^{-1}\Bigg(\frac{\sqrt[4]{b}\cosh(c+}{\sqrt{\sqrt{a}-}}}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.85, size = 1189, normalized size = 1.93

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((32*a*b*Cosh[c + d*x]*(-41*a + 23*b + (13*a - 7*b)*Cosh[2*(c + d*x)]))/((a
 - b)^2*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (512*a
^2*b*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/((a - b)*(-8*a + 3*b - 4*b*Cos
h[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2) + 256*Log[Tanh[(c + d*x)/2]] - (b*
RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-45*a^
2*c + 71*a*b*c - 32*b^2*c - 45*a^2*d*x + 71*a*b*d*x - 32*b^2*d*x - 90*a^2*L
og[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
+ d*x)/2]*#1] + 142*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 64*b^2*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 199*a^2*c*
#1^2 - 253*a*b*c*#1^2 + 96*b^2*c*#1^2 + 199*a^2*d*x*#1^2 - 253*a*b*d*x*#1^2
 + 96*b^2*d*x*#1^2 + 398*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + C
osh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 506*a*b*Log[-Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
1^2 + 192*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2
]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 199*a^2*c*#1^4 + 253*a*b*c*#1^4 - 96*b^
2*c*#1^4 - 199*a^2*d*x*#1^4 + 253*a*b*d*x*#1^4 - 96*b^2*d*x*#1^4 - 398*a^2*
Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
 + d*x)/2]*#1]*#1^4 + 506*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 192*b^2*Log[-Cosh[(c +
d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*
#1^4 + 45*a^2*c*#1^6 - 71*a*b*c*#1^6 + 32*b^2*c*#1^6 + 45*a^2*d*x*#1^6 - 71
*a*b*d*x*#1^6 + 32*b^2*d*x*#1^6 + 90*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 142*a*b*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d
*x)/2]*#1]*#1^6 + 64*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[
(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^
3 - 3*b*#1^5 + b*#1^7) & ])/(a - b)^2)/(256*a^3*d)
```

Maple [**A**]
time = 4.05, size = 647, normalized size = 1.05

method	result

derivativedivides	$8b\left(\frac{-\frac{a^{2}(8a-5b)\left(\tanh^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64\left(a^{2}-2ab+b^{2}\right)}+\frac{a\left(5a^{2}+86ab-64b^{2}\right)\left(\tanh^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a^{2}-128ab+64b^{2}}+\frac{a\left(104a^{2}-327ab+208b^{2}\right)\left(\tanh^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a^{2}-128ab+64b^{2}}-\frac{3(105}{}}{\left(a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a(\right.}\right.$
default	$8b\left(\frac{-\frac{a^{2}(8a-5b)\left(\tanh^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64\left(a^{2}-2ab+b^{2}\right)}+\frac{a\left(5a^{2}+86ab-64b^{2}\right)\left(\tanh^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a^{2}-128ab+64b^{2}}+\frac{a\left(104a^{2}-327ab+208b^{2}\right)\left(\tanh^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a^{2}-128ab+64b^{2}}-\frac{3(105}{}}{\left(a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a(\right.}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(8/a^3*b*((-1/64*a^2*(8*a-5*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14+1
/64*a*(5*a^2+86*a*b-64*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+1/64*a*(
104*a^2-327*a*b+208*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10-3/64*(105*a
^3-358*a^2*b+576*a*b^2-256*b^3)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+1/64*
a*(400*a^2-1161*a*b+560*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/64*a*(
257*a^2-370*a*b+128*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/64*(80*a-5
3*b)*a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-3/64*a^2*(3*a-2*b)/(a^2-2*a*
b+b^2))/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x
+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)^2+1/64/(a
^2-2*a*b+b^2)*a*(1/8*(-45*(a*b)^(1/2)*a^2+71*a*b*(a*b)^(1/2)-32*(a*b)^(1/2)
*b^2-16*a^2*b+10*a*b^2)/a/b/((a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(2*a*tanh(
1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)-2*a)/((a*b)^(1/2)*a-a*b)^(1/2))-1/8*(45*(a*b
)^(1/2)*a^2-71*a*b*(a*b)^(1/2)+32*(a*b)^(1/2)*b^2-16*a^2*b+10*a*b^2)/a/b/(-
(a*b)^(1/2)*a-a*b)^(1/2)*arctan(1/4*(-2*a*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/
2)+2*a)/(-(a*b)^(1/2)*a-a*b)^(1/2))))+1/a^3*ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((13*a*b^2*e^(15*c) - 7*b^3*e^(15*c))*e^(15*d*x) - (121*a*b^2*e^(13*c
) - 67*b^3*e^(13*c))*e^(13*d*x) - (272*a^2*b*e^(11*c) - 461*a*b^2*e^(11*c)
+ 159*b^3*e^(11*c))*e^(11*d*x) + (1424*a^2*b*e^(9*c) - 1121*a*b^2*e^(9*c) +
 99*b^3*e^(9*c))*e^(9*d*x) + (1424*a^2*b*e^(7*c) - 1121*a*b^2*e^(7*c) + 99*
b^3*e^(7*c))*e^(7*d*x) - (272*a^2*b*e^(5*c) - 461*a*b^2*e^(5*c) + 159*b^3*e
^(5*c))*e^(5*d*x) - (121*a*b^2*e^(3*c) - 67*b^3*e^(3*c))*e^(3*d*x) + (13*a*
b^2*e^c - 7*b^3*e^c)*e^(d*x))/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b
^2*d*e^(16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*c))*e^(16*d*x) - 8*(
a^4*b^2*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e^(14*c))*e^(14*d*x)
- 4*(8*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^3*b^3*d*e^(12*c) - 7
*a^2*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c) - 39*a^4*b^2*d*e^(
10*c) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e^(10*d*x) + 2*(128*a
^6*d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8*c) - 166*a^3*b^3*d*
e^(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b*d*e^(6*c) - 39*a^4*
b^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6*c))*e^(6*d*x) - 4*(
8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d*e^(4*c) - 7*a^2*b^4
*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^3*d*e^(2*c) + a^2*b^
4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^3*d) + log((e^(d
*x + c) - 1)*e^(-c))/(a^3*d) - 2*integrate(1/32*((45*a^2*b*e^(7*c) - 71*a*b
^2*e^(7*c) + 32*b^3*e^(7*c))*e^(7*d*x) - (199*a^2*b*e^(5*c) - 253*a*b^2*e^(
5*c) + 96*b^3*e^(5*c))*e^(5*d*x) + (199*a^2*b*e^(3*c) - 253*a*b^2*e^(3*c) +
 96*b^3*e^(3*c))*e^(3*d*x) - (45*a^2*b*e^c - 71*a*b^2*e^c + 32*b^3*e^c)*e^(
d*x))/(a^5*b - 2*a^4*b^2 + a^3*b^3 + (a^5*b*e^(8*c) - 2*a^4*b^2*e^(8*c) + a
^3*b^3*e^(8*c))*e^(8*d*x) - 4*(a^5*b*e^(6*c) - 2*a^4*b^2*e^(6*c) + a^3*b^3*
e^(6*c))*e^(6*d*x) - 2*(8*a^6*e^(4*c) - 19*a^5*b*e^(4*c) + 14*a^4*b^2*e^(4*
c) - 3*a^3*b^3*e^(4*c))*e^(4*d*x) - 4*(a^5*b*e^(2*c) - 2*a^4*b^2*e^(2*c) +
a^3*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 28586 vs. $2(481) = 962$.
time = 1.43, size = 28586, normalized size = 46.33

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(8*(13*a^2*b^2 - 7*a*b^3)*cosh(d*x + c)^15 + 120*(13*a^2*b^2 - 7*a*b
^3)*cosh(d*x + c)*sinh(d*x + c)^14 + 8*(13*a^2*b^2 - 7*a*b^3)*sinh(d*x + c)
^15 - 8*(121*a^2*b^2 - 67*a*b^3)*cosh(d*x + c)^13 - 8*(121*a^2*b^2 - 67*a*b
^3 - 105*(13*a^2*b^2 - 7*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 104*(35
*(13*a^2*b^2 - 7*a*b^3)*cosh(d*x + c)^3 - (121*a^2*b^2 - 67*a*b^3)*cosh(d*x
```

```
 + c))*sinh(d*x + c)^12 - 8*(272*a^3*b - 461*a^2*b^2 + 159*a*b^3)*cosh(d*x
+ c)^11 + 8*(1365*(13*a^2*b^2 - 7*a*b^3)*cosh(d*x + c)^4 - 272*a^3*b + 461*
a^2*b^2 - 159*a*b^3 - 78*(121*a^2*b^2 - 67*a*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^11 + 88*(273*(13*a^2*b^2 - 7*a*b^3)*cosh(d*x + c)^5 - 26*(121*a^2*b^2
 - 67*a*b^3)*cosh(d*x + c)^3 - (272*a^3*b - 461*a^2*b^2 + 159*a*b^3)*cosh(d
*x + c))*sinh(d*x + c)^10 + 8*(1424*a^3*b - 1121*a^2*b^2 + 99*a*b^3)*cosh(d
*x + c)^9 + 8*(5005*(13*a^2*b^2 - 7*a*b^3)*cosh(d*x + c)^6 - 715*(121*a^2*b
^2 - 67*a*b^3)*cosh(d*x + c)^4 + 1424*a^3*b - 1121*a^2*b^2 + 99*a*b^3 - 55*
(272*a^3*b - 461*a^2*b^2  ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

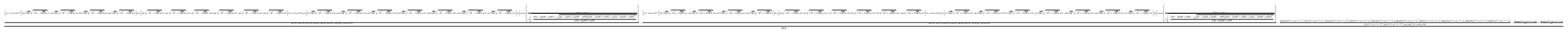
Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1783 vs. 2(481) = 962.
time = 0.64, size = 1783, normalized size = 2.89



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*(((a^5 - 2*a^4*b + a^3*b^2)^2*(180*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*
a^3 - 59*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b - 227*sqrt(a*b)*sqrt(-b^2
 - sqrt(a*b)*b)*a*b^2 + 160*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^3)*abs(b)
- (244*sqrt(-b^2 - sqrt(a*b)*b)*a^8*b - 507*sqrt(-b^2 - sqrt(a*b)*b)*a^7*b^
2 + 5*sqrt(-b^2 - sqrt(a*b)*b)*a^6*b^3 + 695*sqrt(-b^2 - sqrt(a*b)*b)*a^5*b
^4 - 597*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^5 + 160*sqrt(-b^2 - sqrt(a*b)*b)*a^
3*b^6)*abs(a^5 - 2*a^4*b + a^3*b^2)*abs(b) + 2*(32*sqrt(a*b)*sqrt(-b^2 - sq
rt(a*b)*b)*a^12*b - 108*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^11*b^2 + 87*sq
rt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^10*b^3 + 92*sqrt(a*b)*sqrt(-b^2 - sqrt(a
*b)*b)*a^9*b^4 - 198*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^8*b^5 + 120*sqrt(
a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^7*b^6 - 25*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*
b)*a^6*b^7)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^5*b -
2*a^4*b^2 + a^3*b^3 + sqrt((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*(a^5*b - 2
*a^4*b^2 + a^3*b^3) + (a^5*b - 2*a^4*b^2 + a^3*b^3)^2))/(a^5*b - 2*a^4*b^2
+ a^3*b^3)))/((4*a^12*b^2 - 15*a^11*b^3 + 15*a^10*b^4 + 10*a^9*b^5 - 30*a^8
*b^6 + 21*a^7*b^7 - 5*a^6*b^8)*abs(a^5 - 2*a^4*b + a^3*b^2)) - ((a^5 - 2*a^
4*b + a^3*b^2)^2*(180*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3 - 59*sqrt(a*b)
```

```
*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b - 227*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*
b^2 + 160*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^3)*abs(b) + (244*sqrt(-b^2 +
 sqrt(a*b)*b)*a^8*b - 507*sqrt(-b^2 + sqrt(a*b)*b)*a^7*b^2 + 5*sqrt(-b^2 +
sqrt(a*b)*b)*a^6*b^3 + 695*sqrt(-b^2 + sqrt(a*b)*b)*a^5*b^4 - 597*sqrt(-b^2
 + sqrt(a*b)*b)*a^4*b^5 + 160*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^6)*abs(a^5 - 2
*a^4*b + a^3*b^2)*abs(b) + 2*(32*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^12*b
- 108*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^11*b^2 + 87*sqrt(a*b)*sqrt(-b^2
+ sqrt(a*b)*b)*a^10*b^3 + 92*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^9*b^4 - 1
98*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^8*b^5 + 120*sqrt(a*b)*sqrt(-b^2 + s
qrt(a*b)*b)*a^7*b^6 - 25*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^6*b^7)*abs(b)
)*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^5*b - 2*a^4*b^2 + a^3*b^
3 - sqrt((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*(a^5*b - 2*a^4*b^2 + a^3*b^3
) + (a^5*b - 2*a^4*b^2 + a^3*b^3)^2))/(a^5*b - 2*a^4*b^2 + a^3*b^3)))/((4*a
^12*b^2 - 15*a^11*b^3 + 15*a^10*b^4 + 10*a^9*b^5 - 30*a^8*b^6 + 21*a^7*b^7
- 5*a^6*b^8)*abs(a^5 - 2*a^4*b + a^3*b^2)) - 4*(13*a*b^2*(e^(d*x + c) + e^(
-d*x - c))^7 - 7*b^3*(e^(d*x + c) + e^(-d*x - c))^7 - 212*a*b^2*(e^(d*x + c
) + e^(-d*x - c))^5 + 116*b^3*(e^(d*x + c) + e^(-d*x - c))^5 - 272*a^2*b*(e
^(d*x + c) + e^(-d*x - c))^3 + 1248*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 -
592*b^3*(e^(d*x + c) + e^(-d*x - c))^3 + 2240*a^2*b*(e^(d*x + c) + e^(-d*x
- c)) - 3200*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 960*b^3*(e^(d*x + c) + e^
(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^4 - 8*b*(e^(d*x + c) + e^(-d*
x - c))^2 - 16*a + 16*b)^2*(a^4 - 2*a^3*b + a^2*b^2)) - 32*log(e^(d*x + c)
+ e^(-d*x - c) + 2)/a^3 + 32*log(e^(d*x + c) + e^(-d*x - c) - 2)/a^3)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh (c+d x) \left(a-b \sinh (c+d x)^4\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)^3), x)
```

3.259 $\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=319

$$-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right.}{64a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}d}$$

```
[Out] -1/64*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)-5*b^(
1/2))/a^(3/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctanh((a^(1/2)+b^(1/
2))^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)+5*b^(1/2))/a^(3/4)/b^(3/2)/d/(a^(
1/2)+b^(1/2))^(5/2)-1/32*(a+5*b)*tanh(d*x+c)/a/(a-b)^2/b/d-1/32*tanh(d*x+c)
^3/a/(a-b)/b/d+1/8*tanh(d*x+c)^9/a/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)
^4)^2-1/32*sech(d*x+c)^2*tanh(d*x+c)^5/a/b/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tan
h(d*x+c)^4)
```

Rubi [A]
time = 0.38, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = $\{3296, 1289, 12, 1134, 1293, 1180, 214\}$

$$-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tanh^3(c+dx)}{32abd(a-b)}+\frac{\tanh^9(c+dx)}{8ad\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}-\frac{(a+5b)\tanh(c+dx)}{32abd(a-b)^2}-\frac{\tanh^5(c+dx)\text{sech}^2(c+dx)}{32abd\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/64*((2*Sqrt[a] - 5*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*
x])/a^(1/4)])/(a^(3/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((2*Sqrt[a] +
 5*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a
^(3/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - ((a + 5*b)*Tanh[c + d*x])/(32
*a*(a - b)^2*b*d) - Tanh[c + d*x]^3/(32*a*(a - b)*b*d) + Tanh[c + d*x]^9/(8
*a*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - (Sech[c + d*x
]^2*Tanh[c + d*x]^5)/(32*a*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c +
d*x]^4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
```

```
IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\frac{\text{Subst}\left(\int \frac{x^8(1-x^2)}{(a-2ax^2+(a-b)x^4)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}+\frac{\text{Subst}\Big(\int -\frac{2bx}{(a-2ax^2+(}}{}$$

$$=\frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}-\frac{\text{Subst}\Big(\int \frac{x^8}{(a-2ax^2+(a-}}{}$$

$$=\frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}-\frac{\text{sech}^2(}{32abd\left(a-2a\tanh^2\right.}$$

$$=-\frac{\tanh^3(c+dx)}{32a(a-b)bd}+\frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}-$$

$$=-\frac{(a+5b)\tanh(c+dx)}{32a(a-b)^2bd}-\frac{\tanh^3(c+dx)}{32a(a-b)bd}+\frac{\tanh^9(c+d}{8ad\left(a-2a\tanh^2(c+dx)+(a\right.}$$

$$=-\frac{(a+5b)\tanh(c+dx)}{32a(a-b)^2bd}-\frac{\tanh^3(c+dx)}{32a(a-b)bd}+\frac{\tanh^9(c+d}{8ad\left(a-2a\tanh^2(c+dx)+(a\right.}$$

$$=-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\text{t}}{64a^{3/4}}$$

Mathematica [A]
time = 2.89, size = 331, normalized size = 1.04

$$\frac{\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{b}\,\text{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\left(2a^{3/2}\sqrt{b}+ab-8\sqrt{a}\,b^{3/2}+5b^2\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{8b(5a-14b+(-2a+5b)\cosh(2(c+dx)))\sinh(2(c+dx))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))}+\frac{64a(a-b)b(-6\sinh(2(c+dx))+\sinh(4(c+dx)))}{(-8a+3b-4b\cosh(2(c+dx))+b\cosh(4(c+dx)))^2}}{64(a-b)^2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (((2*Sqrt[a] - 5*Sqrt[b])*(Sqrt[a] + Sqrt[b])^2*Sqrt[b]*ArcTan[((Sqrt[a] -
Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqr
t[a]*Sqrt[b]]) + ((2*a^(3/2)*Sqrt[b] + a*b - 8*Sqrt[a]*b^(3/2) + 5*b^2)*Arc
Tanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[
a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) + (8*b*(5*a - 14*b + (-2*a + 5*b)*Cosh[2*(c +
 d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c
 + d*x)]) + (64*a*(a - b)*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))/(-8
*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2)/(64*(a - b)^2*b^
2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.50, size = 535, normalized size = 1.68

method	result
derivativedivides	$-\frac{512\left(\frac{a(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192b\left(a^{2}-2ab+b^{2}\right)}-\frac{(5a+49b)a\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8192b\left(a^{2}-2ab+b^{2}\right)}+\frac{3\left(3a^{2}+55ab-48b^{2}\right)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8192b\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(5a^{2}+377ab-784b^{2}\right)\left(\tanh^{7}(\right.}{8192b\left(a^{2}-2ab+b^{2}\right)}\right.}{\left(a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\right.}$
default	$-\frac{512\left(\frac{a(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192b\left(a^{2}-2ab+b^{2}\right)}-\frac{(5a+49b)a\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8192b\left(a^{2}-2ab+b^{2}\right)}+\frac{3\left(3a^{2}+55ab-48b^{2}\right)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8192b\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(5a^{2}+377ab-784b^{2}\right)\left(\tanh^{7}(\right.}{8192b\left(a^{2}-2ab+b^{2}\right)}\right.}{\left(a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-512*(1/8192*a*(a+5*b)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/8192*(5
*a+49*b)*a/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+3/8192/b*(3*a^2+55*a*b-4
8*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-1/8192*(5*a^2+377*a*b-784*b^2)
/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/8192*(5*a^2+377*a*b-784*b^2)/b/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+3/8192/b*(3*a^2+55*a*b-48*b^2)/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^11-1/8192*(5*a+49*b)*a/b/(a^2-2*a*b+b^2)*tanh(
1/2*d*x+1/2*c)^13+1/8192*a*(a+5*b)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15
)/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c
)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)^2-1/128/b/(a^2-
2*a*b+b^2)*sum(((a+5*b)*_R^6+(5*a-47*b)*_R^4+(-5*a+47*b)*_R^2-a-5*b)/(_R^7*
a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_
Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(2*a*b^2 - 5*b^3 + (a*b^2*e^(14*c) - 4*b^3*e^(14*c))*e^(14*d*x) - (32*
a^2*b*e^(12*c) - 58*a*b^2*e^(12*c) - b^3*e^(12*c))*e^(12*d*x) + 3*(48*a^2*b
*e^(10*c) - 73*a*b^2*e^(10*c) + 20*b^3*e^(10*c))*e^(10*d*x) + (256*a^3*e^(8
*c) - 832*a^2*b*e^(8*c) + 550*a*b^2*e^(8*c) - 175*b^3*e^(8*c))*e^(8*d*x) +
(112*a^2*b*e^(6*c) - 533*a*b^2*e^(6*c) + 220*b^3*e^(6*c))*e^(6*d*x) - (32*a
^2*b*e^(4*c) - 158*a*b^2*e^(4*c) + 141*b^3*e^(4*c))*e^(4*d*x) - (17*a*b^2*e
^(2*c) - 44*b^3*e^(2*c))*e^(2*d*x))/(a^2*b^4*d - 2*a*b^5*d + b^6*d + (a^2*b
^4*d*e^(16*c) - 2*a*b^5*d*e^(16*c) + b^6*d*e^(16*c))*e^(16*d*x) - 8*(a^2*b^
4*d*e^(14*c) - 2*a*b^5*d*e^(14*c) + b^6*d*e^(14*c))*e^(14*d*x) - 4*(8*a^3*b
^3*d*e^(12*c) - 23*a^2*b^4*d*e^(12*c) + 22*a*b^5*d*e^(12*c) - 7*b^6*d*e^(12
*c))*e^(12*d*x) + 8*(16*a^3*b^3*d*e^(10*c) - 39*a^2*b^4*d*e^(10*c) + 30*a*b
^5*d*e^(10*c) - 7*b^6*d*e^(10*c))*e^(10*d*x) + 2*(128*a^4*b^2*d*e^(8*c) - 3
52*a^3*b^3*d*e^(8*c) + 355*a^2*b^4*d*e^(8*c) - 166*a*b^5*d*e^(8*c) + 35*b^6
*d*e^(8*c))*e^(8*d*x) + 8*(16*a^3*b^3*d*e^(6*c) - 39*a^2*b^4*d*e^(6*c) + 30
*a*b^5*d*e^(6*c) - 7*b^6*d*e^(6*c))*e^(6*d*x) - 4*(8*a^3*b^3*d*e^(4*c) - 23
*a^2*b^4*d*e^(4*c) + 22*a*b^5*d*e^(4*c) - 7*b^6*d*e^(4*c))*e^(4*d*x) - 8*(a
^2*b^4*d*e^(2*c) - 2*a*b^5*d*e^(2*c) + b^6*d*e^(2*c))*e^(2*d*x)) - 1/256*in
tegrate(64*((a*e^(6*c) - 4*b*e^(6*c))*e^(6*d*x) + (a*e^(2*c) - 4*b*e^(2*c))
*e^(2*d*x) + 18*b*e^(4*d*x + 4*c))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*b^2*e^(8
*c) - 2*a*b^3*e^(8*c) + b^4*e^(8*c))*e^(8*d*x) - 4*(a^2*b^2*e^(6*c) - 2*a*b
^3*e^(6*c) + b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^3*b*e^(4*c) - 19*a^2*b^2*e^(4*
c) + 14*a*b^3*e^(4*c) - 3*b^4*e^(4*c))*e^(4*d*x) - 4*(a^2*b^2*e^(2*c) - 2*a
*b^3*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 20486 vs. $2(263) = 526$.
time = 0.91, size = 20486, normalized size = 64.22

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(16*(a*b^2 - 4*b^3)*cosh(d*x + c)^14 + 224*(a*b^2 - 4*b^3)*cosh(d*x
+ c)*sinh(d*x + c)^13 + 16*(a*b^2 - 4*b^3)*sinh(d*x + c)^14 - 16*(32*a^2*b
- 58*a*b^2 - b^3)*cosh(d*x + c)^12 - 16*(32*a^2*b - 58*a*b^2 - b^3 - 91*(a*
b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 64*(91*(a*b^2 - 4*b^3)*cos
h(d*x + c)^3 - 3*(32*a^2*b - 58*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^1
1 + 48*(48*a^2*b - 73*a*b^2 + 20*b^3)*cosh(d*x + c)^10 + 16*(1001*(a*b^2 -
4*b^3)*cosh(d*x + c)^4 + 144*a^2*b - 219*a*b^2 + 60*b^3 - 66*(32*a^2*b - 58
*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 32*(1001*(a*b^2 - 4*b^3)*
cosh(d*x + c)^5 - 110*(32*a^2*b - 58*a*b^2 - b^3)*cosh(d*x + c)^3 + 15*(48*
a^2*b - 73*a*b^2 + 20*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 16*(256*a^3 - 8
32*a^2*b + 550*a*b^2 - 175*b^3)*cosh(d*x + c)^8 + 16*(3003*(a*b^2 - 4*b^3)*
```

```
cosh(d*x + c)^6 - 495*(32*a^2*b - 58*a*b^2 - b^3)*cosh(d*x + c)^4 + 256*a^3
 - 832*a^2*b + 550*a*b^2 - 175*b^3 + 135*(48*a^2*b - 73*a*b^2 + 20*b^3)*cos
h(d*x + c)^2)*sinh(d*x +  ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**8/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.49, size = 389, normalized size = 1.22

$$-\frac{ab^{2}e^{(14\,dx+14\,c)} - 4\,b^{3}e^{(14\,dx+14\,c)} - 32\,a^{2}be^{(12\,dx+12\,c)} + 58\,ab^{2}e^{(12\,dx+12\,c)} + b^{3}e^{(12\,dx+12\,c)} + 144\,a^{2}be^{(10\,dx+10\,c)} - 219\,ab^{2}e^{(10\,dx+10\,c)} + 60\,b^{3}e^{(10\,dx+10\,c)} + 256\,a^{3}e^{(8\,dx+8\,c)} - 832\,a^{2}be^{(8\,dx+8\,c)} + 550\,ab^{2}e^{(8\,dx+8\,c)} - 175\,b^{3}e^{(8\,dx+8\,c)} + 112\,a^{2}be^{(6\,dx+6\,c)} - 533\,ab^{2}e^{(6\,dx+6\,c)} + 220\,b^{3}e^{(6\,dx+6\,c)} - 32\,a^{2}be^{(4\,dx+4\,c)} + 158\,ab^{2}e^{(4\,dx+4\,c)} - 141\,b^{3}e^{(4\,dx+4\,c)} - 17\,ab^{2}e^{(2\,dx+2\,c)} + 44\,b^{3}e^{(2\,dx+2\,c)} + 2\,ab^{2} - 5\,b^{3}}{8\,(a^{2}b^{2} - 2\,ab^{3} + b^{4})(be^{(8\,dx+8\,c)} - 4\,be^{(6\,dx+6\,c)} - 16\,ae^{(4\,dx+4\,c)} + 6\,be^{(4\,dx+4\,c)} - 4\,be^{(2\,dx+2\,c)} + b)^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/8*(a*b^2*e^(14*d*x + 14*c) - 4*b^3*e^(14*d*x + 14*c) - 32*a^2*b*e^(12*d*
x + 12*c) + 58*a*b^2*e^(12*d*x + 12*c) + b^3*e^(12*d*x + 12*c) + 144*a^2*b*
e^(10*d*x + 10*c) - 219*a*b^2*e^(10*d*x + 10*c) + 60*b^3*e^(10*d*x + 10*c)
+ 256*a^3*e^(8*d*x + 8*c) - 832*a^2*b*e^(8*d*x + 8*c) + 550*a*b^2*e^(8*d*x
+ 8*c) - 175*b^3*e^(8*d*x + 8*c) + 112*a^2*b*e^(6*d*x + 6*c) - 533*a*b^2*e^
(6*d*x + 6*c) + 220*b^3*e^(6*d*x + 6*c) - 32*a^2*b*e^(4*d*x + 4*c) + 158*a*
b^2*e^(4*d*x + 4*c) - 141*b^3*e^(4*d*x + 4*c) - 17*a*b^2*e^(2*d*x + 2*c) +
44*b^3*e^(2*d*x + 2*c) + 2*a*b^2 - 5*b^3)/((a^2*b^2 - 2*a*b^3 + b^4)*(b*e^(
8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x +
4*c) - 4*b*e^(2*d*x + 2*c) + b)^2*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^8}{{\left(a-b\,\sinh\left(c+d\,x\right)^4\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^8/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^8/(a - b*sinh(c + d*x)^4)^3, x)
```

3.260 $\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=345

$$\frac{\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d}-\frac{\left(4a+10\sqrt{a}\sqrt{b}+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{}}}{\sqrt[4]{}}\right.}{64a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}d}$$

```
[Out] 1/64*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(4*a+3*b-10*a^(1/
2)*b^(1/2))/a^(5/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(5/2)-1/64*arctanh((a^(1/2)
+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(4*a+3*b+10*a^(1/2)*b^(1/2))/a^(5/4)/b
^(3/2)/d/(a^(1/2)+b^(1/2))^(5/2)+1/8*tanh(d*x+c)*(a*(a+3*b)-(a^2+6*a*b+b^2)
*tanh(d*x+c)^2)/(a-b)^3/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^2+1/32*
tanh(d*x+c)*(2*a*(a^2-a*b-8*b^2)/(a-b)^3-(2*a^2+15*a*b+3*b^2)*tanh(d*x+c)^2
/(a-b)^2)/a/b/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A]
time = 0.52, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3296, 1347, 1692, 1180, 214}

$$\frac{\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tanh(c+dx)\left(\frac{2a\left(a^2-ab-8b^2\right)}{(a-b)^3}-\frac{\left(2a^2+15ab+3b^2\right)\tanh^2(c+dx)}{(a-b)^2}\right)}{32abd\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+\frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8d(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c +
 d*x])/a^(1/4)])/(64*a^(5/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) - ((4*a +
 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/
a^(1/4)])/(64*a^(5/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) + (Tanh[c + d*x]
*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*(a - b)^3*d*(a - 2
*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (Tanh[c + d*x]*((2*a*(a^
2 - a*b - 8*b^2))/(a - b)^3 - ((2*a^2 + 15*a*b + 3*b^2)*Tanh[c + d*x]^2)/(a
 - b)^2))/(32*a*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^6\left(1-x^2\right)^2}{\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\mathrm{Subst}\Bigg(\int \frac{2a^3}{(a}}{}$$

$$= \frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{\tanh(c+dx)}{32abd\,(a-2}$$

$$= \frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{\tanh(c+dx)}{32abd\,(a-2}$$

$$= \frac{\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} - \frac{\left(4a+10\sqrt{a}}{}$$

Mathematica [A]

time = 2.33, size = 351, normalized size = 1.02

$$-\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\mathrm{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{a\sqrt{-a+\sqrt{a}\sqrt{b}}\,b^{3/2}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(4a+10\sqrt{a}\sqrt{b}+3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{a\sqrt{a+\sqrt{a}\sqrt{b}}\,b^{3/2}}+\frac{4\left(4a^2-19ab-3b^2+3b(a+b)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{ab(8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx)))}-\frac{128(a-b)(2a+b-b\cosh(2(c+dx)))\sinh(2(c+dx))}{b(-8a+3b-4b\cosh(2(c+dx))+b\cosh(4(c+dx)))^2}}{64(a-b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/64*(((Sqrt[a] + Sqrt[b])^2*(4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((Sqr
t[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(a*Sqrt[-a + Sq
rt[a]*Sqrt[b]]*b^(3/2)) + ((Sqrt[a] - Sqrt[b])^2*(4*a + 10*Sqrt[a]*Sqrt[b]
+ 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]
]])/(a*Sqrt[a + Sqrt[a]*Sqrt[b]]*b^(3/2)) + (4*(4*a^2 - 19*a*b - 3*b^2 + 3*
b*(a + b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a*b*(8*a - 3*b + 4*b*Cosh[
2*(c + d*x)] - b*Cosh[4*(c + d*x)])) - (128*(a - b)*(2*a + b - b*Cosh[2*(c
+ d*x)])*Sinh[2*(c + d*x)])/(b*(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh
[4*(c + d*x)])^2))/((a - b)^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.80, size = 594, normalized size = 1.72

method	result
derivativedivides	$-\frac{128\left(-\frac{(2b+a)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{1024b\left(a^2-2ab+b^2\right)}+\frac{\left(5a^2+24ab-2b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024b\left(a^2-2ab+b^2\right)}-\frac{\left(9a^2+76ab-70b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024b\left(a^2-2ab+b^2\right)}+\frac{\left(5a^3+54a^2b-164a\,b^2-\right.}{1024ab\left(a^2\right.}\right.}{\left(a\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^6\right.\right.}$
default	$-\frac{128\left(-\frac{(2b+a)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{1024b\left(a^2-2ab+b^2\right)}+\frac{\left(5a^2+24ab-2b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024b\left(a^2-2ab+b^2\right)}-\frac{\left(9a^2+76ab-70b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024b\left(a^2-2ab+b^2\right)}+\frac{\left(5a^3+54a^2b-164a\,b^2-\right.}{1024ab\left(a^2\right.}\right.}{\left(a\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^6\right.\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-128*(-1/1024*(2*b+a)*a/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+1/1024*(
5*a^2+24*a*b-2*b^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-1/1024/b*(9*a^2
+76*a*b-70*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/1024*(5*a^3+54*a^2*
b-164*a*b^2-96*b^3)/a/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+1/1024*(5*a^3
+54*a^2*b-164*a*b^2-96*b^3)/a/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-1/102
4/b*(9*a^2+76*a*b-70*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11+1/1024*(5*
a^2+24*a*b-2*b^2)/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13-1/1024*(2*b+a)*a
/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15)/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tan
h(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a
*tanh(1/2*d*x+1/2*c)^2+a)^2-1/64/a/b/(a^2-2*a*b+b^2)*sum((a*(-2*b-a)*_R^6+(
-5*a^2+32*a*b-6*b^2)*_R^4+(5*a^2-32*a*b+6*b^2)*_R^2+a^2+2*a*b)/(_R^7*a-3*_R
^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*
a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*a*b^2 + 3*b^3 - (4*a^2*b*e^(14*c) - 13*a*b^2*e^(14*c) + 3*b^3*e^(1
4*c))*e^(14*d*x) + 3*(8*a^2*b*e^(12*c) - 33*a*b^2*e^(12*c) + 7*b^3*e^(12*c)
)*e^(12*d*x) - (64*a^3*e^(10*c) + 68*a^2*b*e^(10*c) - 225*a*b^2*e^(10*c) +
63*b^3*e^(10*c))*e^(10*d*x) + 3*(128*a^3*e^(8*c) + 32*a^2*b*e^(8*c) - 61*a*
b^2*e^(8*c) + 35*b^3*e^(8*c))*e^(8*d*x) + (64*a^3*e^(6*c) + 452*a^2*b*e^(6*
```

```
c) - 9*a*b^2*e^(6*c) - 105*b^3*e^(6*c))*e^(6*d*x) - 3*(40*a^2*b*e^(4*c) - 2
9*a*b^2*e^(4*c) - 21*b^3*e^(4*c))*e^(4*d*x) + (4*a^2*b*e^(2*c) - 37*a*b^2*e
^(2*c) - 21*b^3*e^(2*c))*e^(2*d*x))/(a^3*b^3*d - 2*a^2*b^4*d + a*b^5*d + (a
^3*b^3*d*e^(16*c) - 2*a^2*b^4*d*e^(16*c) + a*b^5*d*e^(16*c))*e^(16*d*x) - 8
*(a^3*b^3*d*e^(14*c) - 2*a^2*b^4*d*e^(14*c) + a*b^5*d*e^(14*c))*e^(14*d*x)
- 4*(8*a^4*b^2*d*e^(12*c) - 23*a^3*b^3*d*e^(12*c) + 22*a^2*b^4*d*e^(12*c) -
 7*a*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16*a^4*b^2*d*e^(10*c) - 39*a^3*b^3*d*e
^(10*c) + 30*a^2*b^4*d*e^(10*c) - 7*a*b^5*d*e^(10*c))*e^(10*d*x) + 2*(128*a
^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c) + 355*a^3*b^3*d*e^(8*c) - 166*a^2*b^
4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))*e^(8*d*x) + 8*(16*a^4*b^2*d*e^(6*c) - 39*
a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^(6*c) - 7*a*b^5*d*e^(6*c))*e^(6*d*x) - 4
*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3*d*e^(4*c) + 22*a^2*b^4*d*e^(4*c) - 7*a*b
^5*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^3*d*e^(2*c) - 2*a^2*b^4*d*e^(2*c) + a*b^
5*d*e^(2*c))*e^(2*d*x)) + 1/64*integrate(8*((4*a^2*e^(6*c) - 13*a*b*e^(6*c)
 + 3*b^2*e^(6*c))*e^(6*d*x) + 6*(7*a*b*e^(4*c) - b^2*e^(4*c))*e^(4*d*x) + (
4*a^2*e^(2*c) - 13*a*b*e^(2*c) + 3*b^2*e^(2*c))*e^(2*d*x))/(a^3*b^2 - 2*a^2
*b^3 + a*b^4 + (a^3*b^2*e^(8*c) - 2*a^2*b^3*e^(8*c) + a*b^4*e^(8*c))*e^(8*d
*x) - 4*(a^3*b^2*e^(6*c) - 2*a^2*b^3*e^(6*c) + a*b^4*e^(6*c))*e^(6*d*x) - 2
*(8*a^4*b*e^(4*c) - 19*a^3*b^2*e^(4*c) + 14*a^2*b^3*e^(4*c) - 3*a*b^4*e^(4*
c))*e^(4*d*x) - 4*(a^3*b^2*e^(2*c) - 2*a^2*b^3*e^(2*c) + a*b^4*e^(2*c))*e^(
2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 22729 vs. $2(292) = 584$.
time = 1.20, size = 22729, normalized size = 65.88

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(8*(4*a^2*b - 13*a*b^2 + 3*b^3)*cosh(d*x + c)^14 + 112*(4*a^2*b - 13*
a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^13 + 8*(4*a^2*b - 13*a*b^2 + 3*b
^3)*sinh(d*x + c)^14 - 24*(8*a^2*b - 33*a*b^2 + 7*b^3)*cosh(d*x + c)^12 - 8
*(24*a^2*b - 99*a*b^2 + 21*b^3 - 91*(4*a^2*b - 13*a*b^2 + 3*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^12 + 32*(91*(4*a^2*b - 13*a*b^2 + 3*b^3)*cosh(d*x + c)
^3 - 9*(8*a^2*b - 33*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + 8*(64
*a^3 + 68*a^2*b - 225*a*b^2 + 63*b^3)*cosh(d*x + c)^10 + 8*(1001*(4*a^2*b -
 13*a*b^2 + 3*b^3)*cosh(d*x + c)^4 + 64*a^3 + 68*a^2*b - 225*a*b^2 + 63*b^3
 - 198*(8*a^2*b - 33*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 16*
(1001*(4*a^2*b - 13*a*b^2 + 3*b^3)*cosh(d*x + c)^5 - 330*(8*a^2*b - 33*a*b^
2 + 7*b^3)*cosh(d*x + c)^3 + 5*(64*a^3 + 68*a^2*b - 225*a*b^2 + 63*b^3)*cos
h(d*x + c))*sinh(d*x + c)^9 - 24*(128*a^3 + 32*a^2*b - 61*a*b^2 + 35*b^3)*c
osh(d*x + c)^8 + 24*(1001*(4*a^2*b - 13*a*b^2 + 3*b^3)*cosh(d*x + c)^6 - 49
5*(8*a^2*b - 33*a*b^2 + 7 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.27, size = 451, normalized size = 1.31

$$\frac{4\,a^{2}be^{\left(14\,dx+14\,c\right)} - 13\,ab^{2}e^{\left(14\,dx+14\,c\right)} + 3\,b^{3}e^{\left(14\,dx+14\,c\right)} - 24\,a^{2}be^{\left(12\,dx+12\,c\right)} + 99\,ab^{2}e^{\left(12\,dx+12\,c\right)} - 21\,b^{3}e^{\left(12\,dx+12\,c\right)} + 64\,a^{3}e^{\left(10\,dx+10\,c\right)} + 68\,a^{2}be^{\left(10\,dx+10\,c\right)} - 225\,ab^{2}e^{\left(10\,dx+10\,c\right)} + 63\,b^{3}e^{\left(10\,dx+10\,c\right)} - 384\,a^{3}e^{\left(8\,dx+8\,c\right)} - 96\,a^{2}be^{\left(8\,dx+8\,c\right)} + 183\,ab^{2}e^{\left(8\,dx+8\,c\right)} - 105\,b^{3}e^{\left(8\,dx+8\,c\right)} - 64\,a^{3}e^{\left(6\,dx+6\,c\right)} - 452\,a^{2}be^{\left(6\,dx+6\,c\right)} + 9\,ab^{2}e^{\left(6\,dx+6\,c\right)} + 105\,b^{3}e^{\left(6\,dx+6\,c\right)} + 120\,a^{2}be^{\left(4\,dx+4\,c\right)} - 87\,ab^{2}e^{\left(4\,dx+4\,c\right)} - 63\,b^{3}e^{\left(4\,dx+4\,c\right)} - 4\,a^{2}be^{\left(2\,dx+2\,c\right)} + 37\,ab^{2}e^{\left(2\,dx+2\,c\right)} + 21\,b^{3}e^{\left(2\,dx+2\,c\right)} - 3\,ab^{2} - 3\,b^{3}}{16\,{\left(a^{3}b - 2\,a^{2}b^{2} + ab^{3}\right)}{\left(be^{\left(8\,dx+8\,c\right)} - 4\,be^{\left(6\,dx+6\,c\right)} - 16\,ae^{\left(4\,dx+4\,c\right)} + 6\,be^{\left(4\,dx+4\,c\right)} - 4\,be^{\left(2\,dx+2\,c\right)} + b\right)}^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/16*(4*a^2*b*e^(14*d*x + 14*c) - 13*a*b^2*e^(14*d*x + 14*c) + 3*b^3*e^(14*
d*x + 14*c) - 24*a^2*b*e^(12*d*x + 12*c) + 99*a*b^2*e^(12*d*x + 12*c) - 21*
b^3*e^(12*d*x + 12*c) + 64*a^3*e^(10*d*x + 10*c) + 68*a^2*b*e^(10*d*x + 10*
c) - 225*a*b^2*e^(10*d*x + 10*c) + 63*b^3*e^(10*d*x + 10*c) - 384*a^3*e^(8*
d*x + 8*c) - 96*a^2*b*e^(8*d*x + 8*c) + 183*a*b^2*e^(8*d*x + 8*c) - 105*b^3
*e^(8*d*x + 8*c) - 64*a^3*e^(6*d*x + 6*c) - 452*a^2*b*e^(6*d*x + 6*c) + 9*a
*b^2*e^(6*d*x + 6*c) + 105*b^3*e^(6*d*x + 6*c) + 120*a^2*b*e^(4*d*x + 4*c)
- 87*a*b^2*e^(4*d*x + 4*c) - 63*b^3*e^(4*d*x + 4*c) - 4*a^2*b*e^(2*d*x + 2*
c) + 37*a*b^2*e^(2*d*x + 2*c) + 21*b^3*e^(2*d*x + 2*c) - 3*a*b^2 - 3*b^3)/(
(a^3*b - 2*a^2*b^2 + a*b^3)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a
*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)^2*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{sinh}\left(c+d\,x\right)}^6}{{\left(a-b\,{\mathrm{sinh}\left(c+d\,x\right)}^4\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4)^3, x)
```

3.261 $\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=314

$$\frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}\,d}-\frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b}\,d}$$

```
[Out] 3/64*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)-b^(1/2
))/a^(7/4)/d/(a^(1/2)-b^(1/2))^(5/2)/b^(1/2)-3/64*arctanh((a^(1/2)+b^(1/2))
^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)+b^(1/2))/a^(7/4)/d/b^(1/2)/(a^(1/2)+
b^(1/2))^(5/2)-1/8*b*tanh(d*x+c)*(3*a+b-4*(a+b)*tanh(d*x+c)^2)/(a-b)^3/d/(a
-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^2-1/32*tanh(d*x+c)*((9*a^2-24*a*b-b
^2)/(a-b)^3-(17*a+3*b)*tanh(d*x+c)^2/(a-b)^2)/a/d/(a-2*a*tanh(d*x+c)^2+(a-b
)*tanh(d*x+c)^4)
```

Rubi [A]

time = 0.48, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3296, 1347, 1692, 1180, 214}

$$\frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tanh(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}-\frac{(17a+3b)\tanh^2(c+dx)}{(a-b)^2}\right)}{32ad\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{b\tanh(c+dx)\left(-4(a+b)\tanh^2(c+dx)+3a+b\right)}{8d(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (3*(2*Sqrt[a] - Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^
(1/4)])/(64*a^(7/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - (3*(2*Sqrt[a] +
Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(7
/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tanh[c + d*x]*(3*a + b - 4*(a
 + b)*Tanh[c + d*x]^2))/(8*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*T
anh[c + d*x]^4)^2) - (Tanh[c + d*x]*((9*a^2 - 24*a*b - b^2)/(a - b)^3 - ((1
7*a + 3*b)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a*d*(a - 2*a*Tanh[c + d*x]^2 +
(a - b)*Tanh[c + d*x]^4))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^4(1-x^2)^3}{(a-2ax^2+(a-b)x^4)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\mathrm{Subst}\Bigg(\int -}{}$$

$$= -\frac{b\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\tanh(c-}{32ad\,(a-}$$

$$= -\frac{b\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\tanh(c-}{32ad\,(a-}$$

$$= \frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}\,d} - \frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tan}{64a^{7/4}\Big(}$$

Mathematica [A]
time = 3.41, size = 316, normalized size = 1.01

$$\frac{-\frac{3\left(2a^{3/2}+3a\sqrt{b}-b^{3/2}\right)\mathrm{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{a^{3/2}\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{3\left(2a^{3/2}-3a\sqrt{b}+b^{3/2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{a^{3/2}\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{8(-7a-2b+(2a+b)\cosh(2(c+dx)))\sinh(2(c+dx))}{a(8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx)))} + \frac{64(a-b)(-6\sinh(2(c+dx))+\sinh(4(c+dx)))}{(-8a+3b-4b\cosh(2(c+dx))+b\cosh(4(c+dx)))^2}}{64(a-b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((-3*(2*a^(3/2) + 3*a*Sqrt[b] - b^(3/2))*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c
 + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[-a + Sqrt[a]*Sqrt[b]]*S
qrt[b]) - (3*(2*a^(3/2) - 3*a*Sqrt[b] + b^(3/2))*ArcTanh[((Sqrt[a] + Sqrt[b
])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[a + Sqrt[a]*Sqr
t[b]]*Sqrt[b]) + (8*(-7*a - 2*b + (2*a + b)*Cosh[2*(c + d*x)])*Sinh[2*(c +
d*x)])/(a*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (64*
(a - b)*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[
2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2)/(64*(a - b)^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.32, size = 523, normalized size = 1.67

method	result
derivativedivides	$-\frac{32\left(\frac{3(3a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{512\left(a^2-2ab+b^2\right)}-\frac{(77a-23b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512\left(a^2-2ab+b^2\right)}+\frac{\left(177a^2-131ab-16b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512\left(a^2-2ab+b^2\right)a}-\frac{\left(109a^2-367ab-144b^2\right)\left(\tanh^7\right.}{512a\left(a^2-2ab+b^2\right)}\right.}{\left(a\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}$
default	$-\frac{32\left(\frac{3(3a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{512\left(a^2-2ab+b^2\right)}-\frac{(77a-23b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512\left(a^2-2ab+b^2\right)}+\frac{\left(177a^2-131ab-16b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512\left(a^2-2ab+b^2\right)a}-\frac{\left(109a^2-367ab-144b^2\right)\left(\tanh^7\right.}{512a\left(a^2-2ab+b^2\right)}\right.}{\left(a\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-32*(3/512*(3*a-b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/512*(77*a-23*
b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/512*(177*a^2-131*a*b-16*b^2)/(a^
2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^5-1/512*(109*a^2-367*a*b-144*b^2)/a/(a^2
-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/512*(109*a^2-367*a*b-144*b^2)/a/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+1/512*(177*a^2-131*a*b-16*b^2)/(a^2-2*a*b+b^
2)/a*tanh(1/2*d*x+1/2*c)^11-1/512*(77*a-23*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^13+3/512*(3*a-b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15)/(a*tanh(1/2
*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh
(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)^2-3/128/(a^2-2*a*b+b^2)/a*su
m(((3*a-b)*_R^6+(-17*a+3*b)*_R^4+(17*a-3*b)*_R^2-3*a+b)/(_R^7*a-3*_R^5*a+3*
_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+
(6*a-16*b)*_Z^4-4*a*_Z^2+a)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*a*b^2*e^(14*d*x + 14*c) + 2*a*b^2 + b^3 - 3*(10*a*b^2*e^(12*c) - b^3
*e^(12*c))*e^(12*d*x) - (80*a^2*b*e^(10*c) - 111*a*b^2*e^(10*c) + 16*b^3*e^
(10*c))*e^(10*d*x) + (256*a^3*e^(8*c) - 64*a^2*b*e^(8*c) - 26*a*b^2*e^(8*c)
 + 35*b^3*e^(8*c))*e^(8*d*x) + (336*a^2*b*e^(6*c) - 95*a*b^2*e^(6*c) - 40*b
^3*e^(6*c))*e^(6*d*x) - (64*a^2*b*e^(4*c) - 54*a*b^2*e^(4*c) - 25*b^3*e^(4*
c))*e^(4*d*x) - (19*a*b^2*e^(2*c) + 8*b^3*e^(2*c))*e^(2*d*x))/(a^3*b^3*d -
2*a^2*b^4*d + a*b^5*d + (a^3*b^3*d*e^(16*c) - 2*a^2*b^4*d*e^(16*c) + a*b^5*
d*e^(16*c))*e^(16*d*x) - 8*(a^3*b^3*d*e^(14*c) - 2*a^2*b^4*d*e^(14*c) + a*b
```

```
^5*d*e^(14*c))*e^(14*d*x) - 4*(8*a^4*b^2*d*e^(12*c) - 23*a^3*b^3*d*e^(12*c)
 + 22*a^2*b^4*d*e^(12*c) - 7*a*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16*a^4*b^2*d
*e^(10*c) - 39*a^3*b^3*d*e^(10*c) + 30*a^2*b^4*d*e^(10*c) - 7*a*b^5*d*e^(10
*c))*e^(10*d*x) + 2*(128*a^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c) + 355*a^3*
b^3*d*e^(8*c) - 166*a^2*b^4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))*e^(8*d*x) + 8*(
16*a^4*b^2*d*e^(6*c) - 39*a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^(6*c) - 7*a*b^
5*d*e^(6*c))*e^(6*d*x) - 4*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3*d*e^(4*c) + 22
*a^2*b^4*d*e^(4*c) - 7*a*b^5*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^3*d*e^(2*c) -
2*a^2*b^4*d*e^(2*c) + a*b^5*d*e^(2*c))*e^(2*d*x)) + 1/16*integrate(-12*(2*(
4*a*e^(4*c) - b*e^(4*c))*e^(4*d*x) - a*e^(6*d*x + 6*c) - a*e^(2*d*x + 2*c))
/(a^3*b - 2*a^2*b^2 + a*b^3 + (a^3*b*e^(8*c) - 2*a^2*b^2*e^(8*c) + a*b^3*e^
(8*c))*e^(8*d*x) - 4*(a^3*b*e^(6*c) - 2*a^2*b^2*e^(6*c) + a*b^3*e^(6*c))*e^
(6*d*x) - 2*(8*a^4*e^(4*c) - 19*a^3*b*e^(4*c) + 14*a^2*b^2*e^(4*c) - 3*a*b^
3*e^(4*c))*e^(4*d*x) - 4*(a^3*b*e^(2*c) - 2*a^2*b^2*e^(2*c) + a*b^3*e^(2*c)
)*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 21932 vs. $2(264) = 528$.
time = 0.91, size = 21932, normalized size = 69.85

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(48*a*b^2*cosh(d*x + c)^14 + 672*a*b^2*cosh(d*x + c)*sinh(d*x + c)^13
 + 48*a*b^2*sinh(d*x + c)^14 - 48*(10*a*b^2 - b^3)*cosh(d*x + c)^12 + 48*(9
1*a*b^2*cosh(d*x + c)^2 - 10*a*b^2 + b^3)*sinh(d*x + c)^12 + 192*(91*a*b^2*
cosh(d*x + c)^3 - 3*(10*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 16*(
80*a^2*b - 111*a*b^2 + 16*b^3)*cosh(d*x + c)^10 + 16*(3003*a*b^2*cosh(d*x +
 c)^4 - 80*a^2*b + 111*a*b^2 - 16*b^3 - 198*(10*a*b^2 - b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^10 + 32*(3003*a*b^2*cosh(d*x + c)^5 - 330*(10*a*b^2 - b^3)
*cosh(d*x + c)^3 - 5*(80*a^2*b - 111*a*b^2 + 16*b^3)*cosh(d*x + c))*sinh(d*
x + c)^9 + 16*(256*a^3 - 64*a^2*b - 26*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 16
*(9009*a*b^2*cosh(d*x + c)^6 - 1485*(10*a*b^2 - b^3)*cosh(d*x + c)^4 + 256*
a^3 - 64*a^2*b - 26*a*b^2 + 35*b^3 - 45*(80*a^2*b - 111*a*b^2 + 16*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^8 + 128*(1287*a*b^2*cosh(d*x + c)^7 - 297*(10*a
*b^2 - b^3)*cosh(d*x + c)^5 - 15*(80*a^2*b - 111*a*b^2 + 16*b^3)*cosh(d*x +
 c)^3 + (256*a^3 - 64*a^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.98, size = 362, normalized size = 1.15

$$\frac{3\,ab^2e^{(14\,dx+14\,c)} - 30\,ab^2e^{(12\,dx+12\,c)} + 3\,b^3e^{(12\,dx+12\,c)} - 80\,a^2be^{(10\,dx+10\,c)} + 111\,ab^2e^{(10\,dx+10\,c)} - 16\,b^3e^{(10\,dx+10\,c)} + 256\,a^3e^{(8\,dx+8\,c)} - 64\,a^2be^{(8\,dx+8\,c)} - 26\,ab^2e^{(8\,dx+8\,c)} + 35\,b^3e^{(8\,dx+8\,c)} + 336\,a^2be^{(6\,dx+6\,c)} - 95\,ab^2e^{(6\,dx+6\,c)} - 40\,b^3e^{(6\,dx+6\,c)} - 64\,a^2be^{(4\,dx+4\,c)} + 54\,ab^2e^{(4\,dx+4\,c)} + 25\,b^3e^{(4\,dx+4\,c)} - 19\,ab^2e^{(2\,dx+2\,c)} - 8\,b^3e^{(2\,dx+2\,c)} + 2\,ab^2 + b^3}{8\,(a^3b - 2\,a^2b^2 + ab^3)\big(be^{(8\,dx+8\,c)} - 4\,be^{(6\,dx+6\,c)} - 16\,ae^{(4\,dx+4\,c)} + 6\,be^{(4\,dx+4\,c)} - 4\,be^{(2\,dx+2\,c)} + b\big)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*a*b^2*e^(14*d*x + 14*c) - 30*a*b^2*e^(12*d*x + 12*c) + 3*b^3*e^(12*d
*x + 12*c) - 80*a^2*b*e^(10*d*x + 10*c) + 111*a*b^2*e^(10*d*x + 10*c) - 16*
b^3*e^(10*d*x + 10*c) + 256*a^3*e^(8*d*x + 8*c) - 64*a^2*b*e^(8*d*x + 8*c)
- 26*a*b^2*e^(8*d*x + 8*c) + 35*b^3*e^(8*d*x + 8*c) + 336*a^2*b*e^(6*d*x +
6*c) - 95*a*b^2*e^(6*d*x + 6*c) - 40*b^3*e^(6*d*x + 6*c) - 64*a^2*b*e^(4*d*
x + 4*c) + 54*a*b^2*e^(4*d*x + 4*c) + 25*b^3*e^(4*d*x + 4*c) - 19*a*b^2*e^(
2*d*x + 2*c) - 8*b^3*e^(2*d*x + 2*c) + 2*a*b^2 + b^3)/((a^3*b - 2*a^2*b^2 +
 a*b^3)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6
*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)^2*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sinh(c+d\,x)}^4}{{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4)^3, x)
```

3.262 $\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=348

$$-\frac{\left(12a-14\sqrt{a}\sqrt{b}+5b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}\,d}+\frac{\left(12a+14\sqrt{a}\sqrt{b}+5b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}}}{}\right.}{64a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{}}$$

```
[Out] -1/64*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(12*a+5*b-14*a^(
1/2)*b^(1/2))/a^(9/4)/d/(a^(1/2)-b^(1/2))^(5/2)/b^(1/2)+1/64*arctanh((a^(1/
2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(12*a+5*b+14*a^(1/2)*b^(1/2))/a^(9/4
)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(5/2)+1/8*b*tanh(d*x+c)*(a*(a+3*b)-(a^2+6*a*b
+b^2)*tanh(d*x+c)^2)/a/(a-b)^3/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^
2+1/32*tanh(d*x+c)*(2*a*(5*a^2-9*a*b-4*b^2)/(a-b)^3-5*(2*a^2+3*a*b-b^2)*tan
h(d*x+c)^2/(a-b)^2)/a^2/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A]
time = 0.49, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3296, 1347, 1692, 1180, 214}

$$-\frac{\left(-14\sqrt{a}\sqrt{b}+12a+5b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(14\sqrt{a}\sqrt{b}+12a+5b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tanh(c+dx)\left(\frac{2a\left(5a^2-9ab-4b^2\right)}{(a-b)^3}-\frac{5\left(2a^2+3ab-b^2\right)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+\frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8ad(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/64*((12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*T
anh[c + d*x])/a^(1/4)])/(a^(9/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) + ((1
2*a + 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d
*x])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) + (b*Tanh[c
 + d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*a*(a - b)^3
*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (Tanh[c + d*x]*
((2*a*(5*a^2 - 9*a*b - 4*b^2))/(a - b)^3 - (5*(2*a^2 + 3*a*b - b^2)*Tanh[c
+ d*x]^2)/(a - b)^2))/(32*a^2*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c +
 d*x]^4))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(1-x^2\right)^4}{\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} -$$

$$= \frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} +$$

$$= \frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} +$$

$$= -\frac{\left(12a-14\sqrt{a}\sqrt{b}+5b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}\,d} +$$

Mathematica [A]
time = 3.40, size = 343, normalized size = 0.99

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(12a-14\sqrt{a}\sqrt{b}+5b\right)\text{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(12a+14\sqrt{a}\sqrt{b}+5b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{4\left(12a^2+11ab-5b^2+b(-11a+5b)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))} + \frac{128a(a-b)(2a+b-b\cosh(2(c+dx)))\sinh(2(c+dx))}{(-8a+3b-4b\cosh(2(c+dx))+b\cosh(4(c+dx)))^2}}{64a^2(a-b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (((Sqrt[a] + Sqrt[b])^2*(12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[((Sqrt[a]
- Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[-a + Sqrt[a]*S
qrt[b]]*Sqrt[b]) + ((Sqrt[a] - Sqrt[b])^2*(12*a + 14*Sqrt[a]*Sqrt[b] + 5*b)
*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(S
qrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (4*(12*a^2 + 11*a*b - 5*b^2 + b*(-11*a
+ 5*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d
*x)] - b*Cosh[4*(c + d*x)]) + (128*a*(a - b)*(2*a + b - b*Cosh[2*(c + d*x)]
)*Sinh[2*(c + d*x)])/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*
x)])^2)/(64*a^2*(a - b)^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.98, size = 583, normalized size = 1.68

method	result
derivativedivides	$-\frac{\frac{8\left(-\frac{(5a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64\left(a^2-2ab+b^2\right)}+\frac{\left(25a^2+20ab-18b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a\left(a^2-2ab+b^2\right)}-\frac{3\left(15a^2+8ab-18b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a\left(a^2-2ab+b^2\right)}+\frac{\left(25a^3+2a^2b-388a\,b^2\right.}{64a^2\left(a\right.}\right.}{\left(a\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\mathrm{ta}\right.\right.}}{}$
default	$-\frac{\frac{8\left(-\frac{(5a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64\left(a^2-2ab+b^2\right)}+\frac{\left(25a^2+20ab-18b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a\left(a^2-2ab+b^2\right)}-\frac{3\left(15a^2+8ab-18b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a\left(a^2-2ab+b^2\right)}+\frac{\left(25a^3+2a^2b-388a\,b^2\right.}{64a^2\left(a\right.}\right.}{\left(a\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\left(\mathrm{ta}\right.\right.}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-8*(-1/64*(5*a-2*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+1/64*(25*a^2+2
0*a*b-18*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-3/64/a*(15*a^2+8*a*b-
18*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/64*(25*a^3+2*a^2*b-388*a*b^
2+160*b^3)/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+1/64*(25*a^3+2*a^2*b-3
88*a*b^2+160*b^3)/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-3/64/a*(15*a^2+
8*a*b-18*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11+1/64*(25*a^2+20*a*b-18
*b^2)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13-1/64*(5*a-2*b)/(a^2-2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^15)/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)
^6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/
2*c)^2+a)^2-1/64/a^2/(a^2-2*a*b+b^2)*sum((a*(-5*a+2*b)*_R^6+(39*a^2-28*a*b+
10*b^2)*_R^4+(-39*a^2+28*a*b-10*b^2)*_R^2+5*a^2-2*a*b)/(_R^7*a-3*_R^5*a+3*_
R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(
6*a-16*b)*_Z^4-4*a*_Z^2+a)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(11*a*b^2 - 5*b^3 + (12*a^2*b*e^(14*c) - 11*a*b^2*e^(14*c) + 5*b^3*e^
(14*c))*e^(14*d*x) - (104*a^2*b*e^(12*c) - 85*a*b^2*e^(12*c) + 35*b^3*e^(12
*c))*e^(12*d*x) - (320*a^3*e^(10*c) - 652*a^2*b*e^(10*c) + 407*a*b^2*e^(10*
c) - 105*b^3*e^(10*c))*e^(10*d*x) + (1408*a^3*e^(8*c) - 1696*a^2*b*e^(8*c)
+ 865*a*b^2*e^(8*c) - 175*b^3*e^(8*c))*e^(8*d*x) + (320*a^3*e^(6*c) + 756*a
^2*b*e^(6*c) - 849*a*b^2*e^(6*c) + 175*b^3*e^(6*c))*e^(6*d*x) - (248*a^2*b*
e^(4*c) - 383*a*b^2*e^(4*c) + 105*b^3*e^(4*c))*e^(4*d*x) - (12*a^2*b*e^(2*c
```

```
) + 77*a*b^2*e^(2*c) - 35*b^3*e^(2*c))*e^(2*d*x))/(a^4*b^2*d - 2*a^3*b^3*d
+ a^2*b^4*d + (a^4*b^2*d*e^(16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*
c))*e^(16*d*x) - 8*(a^4*b^2*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e
^(14*c))*e^(14*d*x) - 4*(8*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^
3*b^3*d*e^(12*c) - 7*a^2*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c
) - 39*a^4*b^2*d*e^(10*c) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e
^(10*d*x) + 2*(128*a^6*d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8
*c) - 166*a^3*b^3*d*e^(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b
*d*e^(6*c) - 39*a^4*b^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6
*c))*e^(6*d*x) - 4*(8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d
*e^(4*c) - 7*a^2*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^
3*d*e^(2*c) + a^2*b^4*d*e^(2*c))*e^(2*d*x)) - 1/4*integrate(1/2*((12*a^2*e^
(6*c) - 11*a*b*e^(6*c) + 5*b^2*e^(6*c))*e^(6*d*x) - 2*(32*a^2*e^(4*c) - 19*
a*b*e^(4*c) + 5*b^2*e^(4*c))*e^(4*d*x) + (12*a^2*e^(2*c) - 11*a*b*e^(2*c) +
 5*b^2*e^(2*c))*e^(2*d*x))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^4*b*e^(8*c) -
2*a^3*b^2*e^(8*c) + a^2*b^3*e^(8*c))*e^(8*d*x) - 4*(a^4*b*e^(6*c) - 2*a^3*b
^2*e^(6*c) + a^2*b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^5*e^(4*c) - 19*a^4*b*e^(4*
c) + 14*a^3*b^2*e^(4*c) - 3*a^2*b^3*e^(4*c))*e^(4*d*x) - 4*(a^4*b*e^(2*c) -
 2*a^3*b^2*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 23355 vs. $2(296) = 592$.
time = 1.33, size = 23355, normalized size = 67.11

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(8*(12*a^2*b - 11*a*b^2 + 5*b^3)*cosh(d*x + c)^14 + 112*(12*a^2*b -
11*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^13 + 8*(12*a^2*b - 11*a*b^2 +
 5*b^3)*sinh(d*x + c)^14 - 8*(104*a^2*b - 85*a*b^2 + 35*b^3)*cosh(d*x + c)^
12 - 8*(104*a^2*b - 85*a*b^2 + 35*b^3 - 91*(12*a^2*b - 11*a*b^2 + 5*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^12 + 32*(91*(12*a^2*b - 11*a*b^2 + 5*b^3)*cosh
(d*x + c)^3 - 3*(104*a^2*b - 85*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c
)^11 - 8*(320*a^3 - 652*a^2*b + 407*a*b^2 - 105*b^3)*cosh(d*x + c)^10 + 8*(
1001*(12*a^2*b - 11*a*b^2 + 5*b^3)*cosh(d*x + c)^4 - 320*a^3 + 652*a^2*b -
407*a*b^2 + 105*b^3 - 66*(104*a^2*b - 85*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^10 + 16*(1001*(12*a^2*b - 11*a*b^2 + 5*b^3)*cosh(d*x + c)^5 -
110*(104*a^2*b - 85*a*b^2 + 35*b^3)*cosh(d*x + c)^3 - 5*(320*a^3 - 652*a^2*
b + 407*a*b^2 - 105*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 8*(1408*a^3 - 169
6*a^2*b + 865*a*b^2 - 175*b^3)*cosh(d*x + c)^8 + 8*(3003*(12*a^2*b - 11*a*b
^2 + 5*b^3)*cosh(d*x + c) ...
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.71, size = 449, normalized size = 1.29

$$-\frac{12\,a^{2}be^{(14\,dx+14\,c)}-11\,ab^{2}e^{(14\,dx+14\,c)}+5\,b^{3}e^{(14\,dx+14\,c)}-104\,a^{2}be^{(12\,dx+12\,c)}+85\,ab^{2}e^{(12\,dx+12\,c)}-35\,b^{3}e^{(12\,dx+12\,c)}-320\,a^{3}e^{(10\,dx+10\,c)}+652\,a^{2}be^{(10\,dx+10\,c)}-407\,ab^{2}e^{(10\,dx+10\,c)}+105\,b^{3}e^{(10\,dx+10\,c)}+1408\,a^{3}e^{(8\,dx+8\,c)}-1696\,a^{2}be^{(8\,dx+8\,c)}+865\,ab^{2}e^{(8\,dx+8\,c)}-175\,b^{3}e^{(8\,dx+8\,c)}+320\,a^{3}e^{(6\,dx+6\,c)}+756\,a^{2}be^{(6\,dx+6\,c)}-849\,ab^{2}e^{(6\,dx+6\,c)}+175\,b^{3}e^{(6\,dx+6\,c)}-248\,a^{2}be^{(4\,dx+4\,c)}+383\,ab^{2}e^{(4\,dx+4\,c)}-105\,b^{3}e^{(4\,dx+4\,c)}-12\,a^{2}be^{(2\,dx+2\,c)}-77\,ab^{2}e^{(2\,dx+2\,c)}+35\,b^{3}e^{(2\,dx+2\,c)}+11\,ab^{2}-5\,b^{3}}{16\,(a^{4}-2\,a^{3}b+a^{2}b^{2})(be^{(8\,dx+8\,c)}-4\,be^{(6\,dx+6\,c)}-16\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}-4\,be^{(2\,dx+2\,c)}+b)^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(12*a^2*b*e^(14*d*x + 14*c) - 11*a*b^2*e^(14*d*x + 14*c) + 5*b^3*e^(1
4*d*x + 14*c) - 104*a^2*b*e^(12*d*x + 12*c) + 85*a*b^2*e^(12*d*x + 12*c) -
35*b^3*e^(12*d*x + 12*c) - 320*a^3*e^(10*d*x + 10*c) + 652*a^2*b*e^(10*d*x
+ 10*c) - 407*a*b^2*e^(10*d*x + 10*c) + 105*b^3*e^(10*d*x + 10*c) + 1408*a^
3*e^(8*d*x + 8*c) - 1696*a^2*b*e^(8*d*x + 8*c) + 865*a*b^2*e^(8*d*x + 8*c)
- 175*b^3*e^(8*d*x + 8*c) + 320*a^3*e^(6*d*x + 6*c) + 756*a^2*b*e^(6*d*x +
6*c) - 849*a*b^2*e^(6*d*x + 6*c) + 175*b^3*e^(6*d*x + 6*c) - 248*a^2*b*e^(4
*d*x + 4*c) + 383*a*b^2*e^(4*d*x + 4*c) - 105*b^3*e^(4*d*x + 4*c) - 12*a^2*
b*e^(2*d*x + 2*c) - 77*a*b^2*e^(2*d*x + 2*c) + 35*b^3*e^(2*d*x + 2*c) + 11*
a*b^2 - 5*b^3)/((a^4 - 2*a^3*b + a^2*b^2)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x
 + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c)
+ b)^2*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{sinh}\left(c+d\,x\right)}^2}{{\left(a-b\,{\mathrm{sinh}\left(c+d\,x\right)}^4\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4)^3, x)
```

3.263 $\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=320

$$\frac{\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}+\frac{\left(32a+50\sqrt{a}\sqrt{b}+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}}}{}\right.}{64a^{11/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}$$

```
[Out] 1/64*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(32*a+21*b-50*a^(
1/2)*b^(1/2))/a^(11/4)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctanh((a^(1/2)+b^(1
/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(32*a+21*b+50*a^(1/2)*b^(1/2))/a^(11/4)/d/(
a^(1/2)+b^(1/2))^(5/2)-1/8*b^2*tanh(d*x+c)*(3*a+b-4*(a+b)*tanh(d*x+c)^2)/a/
(a-b)^3/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^2-1/32*b*tanh(d*x+c)*((
17*a^2-40*a*b+7*b^2)/(a-b)^3-(33*a-13*b)*tanh(d*x+c)^2/(a-b)^2)/a^2/d/(a-2*
a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A]
time = 0.47, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3288, 1219, 1692, 1180, 214}

$$\frac{\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(50\sqrt{a}\sqrt{b}+32a+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{b\tanh(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}-\frac{(33a-13b)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{b^2\tanh(c+dx)\left(-4(a+b)\tanh^2(c+dx)+3a+b\right)}{8ad(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sinh[c + d*x]^4)^(-3),x]
```

```
[Out] ((32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c
 + d*x])/a^(1/4)])/(64*a^(11/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + ((32*a + 50*
Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(
1/4)])/(64*a^(11/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b^2*Tanh[c + d*x]*(3*a
+ b - 4*(a + b)*Tanh[c + d*x]^2))/(8*a*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2
 + (a - b)*Tanh[c + d*x]^4)^2) - (b*Tanh[c + d*x]*((17*a^2 - 40*a*b + 7*b^2
)/(a - b)^3 - ((33*a - 13*b)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^2*d*(a - 2*
a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\Bigl(\int \frac{\left(1-x^2\right)^5}{\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\Bigr)}{d}$$

$$= -\frac{b^2\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8a(a-b)^3 d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{Subst}\Bigl(\int}{}$$

$$= -\frac{b^2\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8a(a-b)^3 d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{b\tanh(c}{32a^2d\,(a\,-}$$

$$= -\frac{b^2\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8a(a-b)^3 d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{b\tanh(c}{32a^2d\,(a\,-}$$

$$= \frac{\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} + \frac{\left(32a+50\right.}{}$$

Mathematica [A]
time = 2.18, size = 333, normalized size = 1.04

$$\frac{-\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\text{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(32a+50\sqrt{a}\sqrt{b}+21b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{8\sqrt{a}\,b(-19a+10b+(6a-3b)\cosh(2(c+dx)))\sinh(2(c+dx))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))}+\frac{64a^{3/2}(a-b)b(-6\sinh(2(c+dx))+\sinh(4(c+dx)))}{(-8a+3b-4b\cosh(2(c+dx))+b\cosh(4(c+dx)))^2}}{64a^{5/2}(a-b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sinh[c + d*x]^4)^(-3),x]
```

```
[Out] (-(((Sqrt[a] + Sqrt[b])^2*(32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[((Sqrt[
a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]
*Sqrt[b]]) + ((Sqrt[a] - Sqrt[b])^2*(32*a + 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcT
anh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a
+ Sqrt[a]*Sqrt[b]] + (8*Sqrt[a]*b*(-19*a + 10*b + (6*a - 3*b)*Cosh[2*(c + d
*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c +
 d*x)]) + (64*a^(3/2)*(a - b)*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))
/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2)/(64*a^(5/2)*
(a - b)^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.78, size = 577, normalized size = 1.80

method	result
derivativedivides	$-\frac{2\left(\frac{b(17a-11b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32a\left(a^{2}-2ab+b^{2}\right)}-\frac{(149a-95b)b\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a\left(a^{2}-2ab+b^{2}\right)}+\frac{b\left(345a^{2}-427ab+112b^{2}\right)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a^{2}\left(a^{2}-2ab+b^{2}\right)}-\frac{(213a^{2}-1111ab+496b^{2}}{32a^{2}(a^{2}-2}\right.}{(a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a(\tanh^{6}(}$
default	$-\frac{2\left(\frac{b(17a-11b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32a\left(a^{2}-2ab+b^{2}\right)}-\frac{(149a-95b)b\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a\left(a^{2}-2ab+b^{2}\right)}+\frac{b\left(345a^{2}-427ab+112b^{2}\right)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a^{2}\left(a^{2}-2ab+b^{2}\right)}-\frac{(213a^{2}-1111ab+496b^{2}}{32a^{2}(a^{2}-2}\right.}{(a\left(\tanh^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a(\tanh^{6}(}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(1/32*b*(17*a-11*b)/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/32*(149
*a-95*b)/a*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/32/a^2*b*(345*a^2-427*
a*b+112*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-1/32*(213*a^2-1111*a*b+4
96*b^2)/a^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/32*(213*a^2-1111*a*b+
496*b^2)/a^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+1/32/a^2*b*(345*a^2-42
7*a*b+112*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11-1/32*(149*a-95*b)/a*b
/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13+1/32*b*(17*a-11*b)/a/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^15)/(a*tanh(1/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^
6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*tanh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2
*c)^2+a)^2-1/128/(a^2-2*a*b+b^2)/a^2*sum(((32*a^2-47*a*b+21*b^2)*_R^6+(-96*
a^2+85*a*b-31*b^2)*_R^4+(96*a^2-85*a*b+31*b^2)*_R^2-32*a^2+47*a*b-21*b^2)/(
_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootO
f(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(6*a*b^2 - 3*b^3 + (7*a*b^2*e^(14*c) - 4*b^3*e^(14*c))*e^(14*d*x) - (32
*a^2*b*e^(12*c) + 2*a*b^2*e^(12*c) - 7*b^3*e^(12*c))*e^(12*d*x) - (16*a^2*b
*e^(10*c) - 3*a*b^2*e^(10*c) - 28*b^3*e^(10*c))*e^(10*d*x) + 3*(256*a^3*e^(
8*c) - 320*a^2*b*e^(8*c) + 166*a*b^2*e^(8*c) - 35*b^3*e^(8*c))*e^(8*d*x) +
(784*a^2*b*e^(6*c) - 723*a*b^2*e^(6*c) + 140*b^3*e^(6*c))*e^(6*d*x) - (160*
a^2*b*e^(4*c) - 266*a*b^2*e^(4*c) + 91*b^3*e^(4*c))*e^(4*d*x) - (55*a*b^2*e
^(2*c) - 28*b^3*e^(2*c))*e^(2*d*x))/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d +
```

```
(a^4*b^2*d*e^(16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*c))*e^(16*d*x)
 - 8*(a^4*b^2*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e^(14*c))*e^(14
*d*x) - 4*(8*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^3*b^3*d*e^(12*
c) - 7*a^2*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c) - 39*a^4*b^2
*d*e^(10*c) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e^(10*d*x) + 2*
(128*a^6*d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8*c) - 166*a^3*
b^3*d*e^(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b*d*e^(6*c) - 3
9*a^4*b^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6*c))*e^(6*d*x)
 - 4*(8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d*e^(4*c) - 7*a
^2*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^3*d*e^(2*c) +
a^2*b^4*d*e^(2*c))*e^(2*d*x)) + integrate(1/4*((7*a*b*e^(6*c) - 4*b^2*e^(6*
c))*e^(6*d*x) - 2*(32*a^2*e^(4*c) - 40*a*b*e^(4*c) + 17*b^2*e^(4*c))*e^(4*d
*x) + (7*a*b*e^(2*c) - 4*b^2*e^(2*c))*e^(2*d*x))/(a^4*b - 2*a^3*b^2 + a^2*b
^3 + (a^4*b*e^(8*c) - 2*a^3*b^2*e^(8*c) + a^2*b^3*e^(8*c))*e^(8*d*x) - 4*(a
^4*b*e^(6*c) - 2*a^3*b^2*e^(6*c) + a^2*b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^5*e^
(4*c) - 19*a^4*b*e^(4*c) + 14*a^3*b^2*e^(4*c) - 3*a^2*b^3*e^(4*c))*e^(4*d*x
) - 4*(a^4*b*e^(2*c) - 2*a^3*b^2*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 23125 vs. $2(274) = 548$.
time = 1.38, size = 23125, normalized size = 72.27

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(16*(7*a*b^2 - 4*b^3)*cosh(d*x + c)^14 + 224*(7*a*b^2 - 4*b^3)*cosh(d
*x + c)*sinh(d*x + c)^13 + 16*(7*a*b^2 - 4*b^3)*sinh(d*x + c)^14 - 16*(32*a
^2*b + 2*a*b^2 - 7*b^3)*cosh(d*x + c)^12 - 16*(32*a^2*b + 2*a*b^2 - 7*b^3 -
 91*(7*a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 64*(91*(7*a*b^2 -
 4*b^3)*cosh(d*x + c)^3 - 3*(32*a^2*b + 2*a*b^2 - 7*b^3)*cosh(d*x + c))*sin
h(d*x + c)^11 - 16*(16*a^2*b - 3*a*b^2 - 28*b^3)*cosh(d*x + c)^10 + 16*(100
1*(7*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 16*a^2*b + 3*a*b^2 + 28*b^3 - 66*(32*
a^2*b + 2*a*b^2 - 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 32*(1001*(7*a*
b^2 - 4*b^3)*cosh(d*x + c)^5 - 110*(32*a^2*b + 2*a*b^2 - 7*b^3)*cosh(d*x +
c)^3 - 5*(16*a^2*b - 3*a*b^2 - 28*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 48*
(256*a^3 - 320*a^2*b + 166*a*b^2 - 35*b^3)*cosh(d*x + c)^8 + 48*(1001*(7*a*
b^2 - 4*b^3)*cosh(d*x + c)^6 - 165*(32*a^2*b + 2*a*b^2 - 7*b^3)*cosh(d*x +
c)^4 + 256*a^3 - 320*a^2*b + 166*a*b^2 - 35*b^3 - 15*(16*a^2*b - 3*a*b^2 -
28*b^3)*cosh(d*x + c)^2)* ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 391, normalized size = 1.22

$$\frac{7\,ab^2e^{(14\,dx+14\,c)} - 4\,b^3e^{(14\,dx+14\,c)} - 32\,a^2be^{(12\,dx+12\,c)} - 2\,ab^2e^{(12\,dx+12\,c)} + 7\,b^3e^{(12\,dx+12\,c)} - 16\,a^2be^{(10\,dx+10\,c)} + 3\,ab^2e^{(10\,dx+10\,c)} + 28\,b^3e^{(10\,dx+10\,c)} + 768\,a^3e^{(8\,dx+8\,c)} - 960\,a^2be^{(8\,dx+8\,c)} + 498\,ab^2e^{(8\,dx+8\,c)} - 105\,b^3e^{(8\,dx+8\,c)} + 784\,a^2be^{(6\,dx+6\,c)} - 723\,ab^2e^{(6\,dx+6\,c)} + 140\,b^3e^{(6\,dx+6\,c)} - 160\,a^2be^{(4\,dx+4\,c)} + 266\,ab^2e^{(4\,dx+4\,c)} - 91\,b^3e^{(4\,dx+4\,c)} - 55\,ab^2e^{(2\,dx+2\,c)} + 28\,b^3e^{(2\,dx+2\,c)} + 6\,ab^2 - 3\,b^3}{8\,(a^4 - 2\,a^3b + a^2b^2)(be^{(8\,dx+8\,c)} - 4\,be^{(6\,dx+6\,c)} - 16\,ae^{(4\,dx+4\,c)} + 6\,be^{(4\,dx+4\,c)} - 4\,be^{(2\,dx+2\,c)} + b)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/8*(7*a*b^2*e^(14*d*x + 14*c) - 4*b^3*e^(14*d*x + 14*c) - 32*a^2*b*e^(12*d
*x + 12*c) - 2*a*b^2*e^(12*d*x + 12*c) + 7*b^3*e^(12*d*x + 12*c) - 16*a^2*b
*e^(10*d*x + 10*c) + 3*a*b^2*e^(10*d*x + 10*c) + 28*b^3*e^(10*d*x + 10*c) +
 768*a^3*e^(8*d*x + 8*c) - 960*a^2*b*e^(8*d*x + 8*c) + 498*a*b^2*e^(8*d*x +
 8*c) - 105*b^3*e^(8*d*x + 8*c) + 784*a^2*b*e^(6*d*x + 6*c) - 723*a*b^2*e^(
6*d*x + 6*c) + 140*b^3*e^(6*d*x + 6*c) - 160*a^2*b*e^(4*d*x + 4*c) + 266*a*
b^2*e^(4*d*x + 4*c) - 91*b^3*e^(4*d*x + 4*c) - 55*a*b^2*e^(2*d*x + 2*c) + 2
8*b^3*e^(2*d*x + 2*c) + 6*a*b^2 - 3*b^3)/((a^4 - 2*a^3*b + a^2*b^2)*(b*e^(8
*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4
*c) - 4*b*e^(2*d*x + 2*c) + b)^2*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a-b\,{\mathrm{sinh}\left(c+d\,x\right)}^4\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(1/(a - b*sinh(c + d*x)^4)^3, x)
```

3.264 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=359

$$-\frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}+\frac{3\sqrt{b}\left(20a+34\sqrt{a}\sqrt{b}+15b\right)\tanh}{64a^{13/4}\left(\sqrt{a}+\right.}$$

```
[Out] -coth(d*x+c)/a^3/d-3/64*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4)
)*b^(1/2)*(20*a+15*b-34*a^(1/2)*b^(1/2))/a^(13/4)/d/(a^(1/2)-b^(1/2))^(5/2)
+3/64*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*b^(1/2)*(20*a+15
*b+34*a^(1/2)*b^(1/2))/a^(13/4)/d/(a^(1/2)+b^(1/2))^(5/2)+1/8*b^2*tanh(d*x+
c)*(a*(a+3*b)-(a^2+6*a*b+b^2)*tanh(d*x+c)^2)/a^2/(a-b)^3/d/(a-2*a*tanh(d*x+
c)^2+(a-b)*tanh(d*x+c)^4)^2+1/32*b*tanh(d*x+c)*(2*a^2*(9*a-17*b)/(a-b)^3-(1
8*a^2+15*a*b-13*b^2)*tanh(d*x+c)^2/(a-b)^2)/a^3/d/(a-2*a*tanh(d*x+c)^2+(a-b
)*tanh(d*x+c)^4)
```

Rubi [A]
time = 0.84, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3296, 1348, 1683, 1678, 1180, 214}

$$-\frac{3\sqrt{b}\left(-34\sqrt{a}\sqrt{b}+20a+15b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{3\sqrt{b}\left(34\sqrt{a}\sqrt{b}+20a+15b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\coth(c+dx)}{a^3d}+\frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2d(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}+\frac{b\tanh(c+dx)\left(\frac{2a^2(9a-17b)}{(a-b)^3}-\frac{\left(18a^2+15ab-13b^2\right)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^3d\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt
[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + (
3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b
]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - Cot
h[c + d*x]/(a^3*d) + (b^2*Tanh[c + d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*
Tanh[c + d*x]^2))/(8*a^2*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tan
h[c + d*x]^4)^2) + (b*Tanh[c + d*x]*((2*a^2*(9*a - 17*b))/(a - b)^3 - ((18*
a^2 + 15*a*b - 13*b^2)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^3*d*(a - 2*a*Tanh
[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1348

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p +
1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5)
 - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] &
& ILtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
```

```
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^6}{x^2\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{b\tanh(c+}{32a^3d\,(a-}$$

$$= \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{b\tanh(c+}{32a^3d\,(a-}$$

$$= -\frac{\coth(c+dx)}{a^3d} + \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+d\right.}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+d\right.}$$

$$= -\frac{\coth(c+dx)}{a^3d} + \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+d\right.}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+d\right.}$$

$$= -\frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} + \frac{3\sqrt{}}{}$$

Mathematica [A]
time = 2.41, size = 357, normalized size = 0.99

$$\frac{\frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\text{ArcTan}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)^2\sqrt{-a+\sqrt{a}\sqrt{b}}} + \frac{3\sqrt{b}\left(20a+34\sqrt{a}\sqrt{b}+15b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{a+\sqrt{a}\sqrt{b}}} - 64\coth(c+dx) + \frac{4b\left(28a^2+3ab-13b^2+b(-19a+13b)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{(a-b)^2(8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx)))} + \frac{128ab(2a+b-b\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b)(-8a+3b-4b\cosh(2(c+dx))+b\cosh(4(c+dx)))^2}}{64a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[((Sqrt[a] - Sqrt[b])*
Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])^2*Sqrt[-a
+ Sqrt[a]*Sqrt[b]]) + (3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh
[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a]
+ Sqrt[b])^2*Sqrt[a + Sqrt[a]*Sqrt[b]]) - 64*Coth[c + d*x] + (4*b*(28*a^2 +
 3*a*b - 13*b^2 + b*(-19*a + 13*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((
a - b)^2*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (128*
a*b*(2*a + b - b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a - b)*(-8*a + 3*b
 - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2))/(64*a^3*d)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.18, size = 608, normalized size = 1.69

method	result
derivativedivides	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{8b\left(\frac{-\frac{3a^2(3a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64\left(a^2-2ab+b^2\right)}+\frac{\left(45a^2+16ab-34b^2\right)a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a^2-128ab+64b^2}-\frac{a\left(81a^2-28ab-38b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64\left(a^2-2ab+b^2\right)}}{}\right.}{}$
default	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{8b\left(\frac{-\frac{3a^2(3a-2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64\left(a^2-2ab+b^2\right)}+\frac{\left(45a^2+16ab-34b^2\right)a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a^2-128ab+64b^2}-\frac{a\left(81a^2-28ab-38b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64\left(a^2-2ab+b^2\right)}}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^3*tanh(1/2*d*x+1/2*c)-8/a^3*b*((-3/64*a^2*(3*a-2*b)/(a^2-2*a*b+
b^2)*tanh(1/2*d*x+1/2*c)+1/64*(45*a^2+16*a*b-34*b^2)*a/(a^2-2*a*b+b^2)*tanh
(1/2*d*x+1/2*c)^3-1/64*a*(81*a^2-28*a*b-38*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d*
x+1/2*c)^5+1/64*(45*a^3-50*a^2*b-612*a*b^2+416*b^3)/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^7+1/64*(45*a^3-50*a^2*b-612*a*b^2+416*b^3)/(a^2-2*a*b+b^2)*tan
h(1/2*d*x+1/2*c)^9-1/64*a*(81*a^2-28*a*b-38*b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^11+1/64*(45*a^2+16*a*b-34*b^2)*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2
*c)^13-3/64*a^2*(3*a-2*b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15)/(a*tanh(1
/2*d*x+1/2*c)^8-4*a*tanh(1/2*d*x+1/2*c)^6+6*a*tanh(1/2*d*x+1/2*c)^4-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*a*tanh(1/2*d*x+1/2*c)^2+a)^2+3/512/(a^2-2*a*b+b^2)*su
m((a*(-3*a+2*b)*_R^6+(49*a^2-72*a*b+30*b^2)*_R^4+(-49*a^2+72*a*b-30*b^2)*_R
^2+3*a^2-2*a*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/
```

```
2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a)))-1/2/a^3/ta
nh(1/2*d*x+1/2*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(32*a^2*b^2 - 83*a*b^3 + 45*b^4 + 3*(20*a^2*b^2*e^(16*c) - 33*a*b^3*e^
(16*c) + 15*b^4*e^(16*c))*e^(16*d*x) - 12*(43*a^2*b^2*e^(14*c) - 68*a*b^3*e
^(14*c) + 30*b^4*e^(14*c))*e^(14*d*x) - 4*(400*a^3*b*e^(12*c) - 1137*a^2*b^
2*e^(12*c) + 1031*a*b^3*e^(12*c) - 315*b^4*e^(12*c))*e^(12*d*x) + 12*(592*a
^3*b*e^(10*c) - 1237*a^2*b^2*e^(10*c) + 886*a*b^3*e^(10*c) - 210*b^4*e^(10*
c))*e^(10*d*x) + 2*(4096*a^4*e^(8*c) - 12192*a^3*b*e^(8*c) + 13634*a^2*b^2*
e^(8*c) - 7113*a*b^3*e^(8*c) + 1575*b^4*e^(8*c))*e^(8*d*x) + 4*(880*a^3*b*e
^(6*c) - 2855*a^2*b^2*e^(6*c) + 2512*a*b^3*e^(6*c) - 630*b^4*e^(6*c))*e^(6*
d*x) - 4*(256*a^3*b*e^(4*c) - 823*a^2*b^2*e^(4*c) + 903*a*b^3*e^(4*c) - 315
*b^4*e^(4*c))*e^(4*d*x) - 12*(19*a^2*b^2*e^(2*c) - 54*a*b^3*e^(2*c) + 30*b^
4*e^(2*c))*e^(2*d*x))/(a^5*b^2*d - 2*a^4*b^3*d + a^3*b^4*d - (a^5*b^2*d*e^(
18*c) - 2*a^4*b^3*d*e^(18*c) + a^3*b^4*d*e^(18*c))*e^(18*d*x) + 9*(a^5*b^2*
d*e^(16*c) - 2*a^4*b^3*d*e^(16*c) + a^3*b^4*d*e^(16*c))*e^(16*d*x) + 4*(8*a
^6*b*d*e^(14*c) - 25*a^5*b^2*d*e^(14*c) + 26*a^4*b^3*d*e^(14*c) - 9*a^3*b^4
*d*e^(14*c))*e^(14*d*x) - 4*(40*a^6*b*d*e^(12*c) - 101*a^5*b^2*d*e^(12*c) +
 82*a^4*b^3*d*e^(12*c) - 21*a^3*b^4*d*e^(12*c))*e^(12*d*x) - 2*(128*a^7*d*e
^(10*c) - 416*a^6*b*d*e^(10*c) + 511*a^5*b^2*d*e^(10*c) - 286*a^4*b^3*d*e^(
10*c) + 63*a^3*b^4*d*e^(10*c))*e^(10*d*x) + 2*(128*a^7*d*e^(8*c) - 416*a^6*
b*d*e^(8*c) + 511*a^5*b^2*d*e^(8*c) - 286*a^4*b^3*d*e^(8*c) + 63*a^3*b^4*d*
e^(8*c))*e^(8*d*x) + 4*(40*a^6*b*d*e^(6*c) - 101*a^5*b^2*d*e^(6*c) + 82*a^4
*b^3*d*e^(6*c) - 21*a^3*b^4*d*e^(6*c))*e^(6*d*x) - 4*(8*a^6*b*d*e^(4*c) - 2
5*a^5*b^2*d*e^(4*c) + 26*a^4*b^3*d*e^(4*c) - 9*a^3*b^4*d*e^(4*c))*e^(4*d*x)
 - 9*(a^5*b^2*d*e^(2*c) - 2*a^4*b^3*d*e^(2*c) + a^3*b^4*d*e^(2*c))*e^(2*d*x
)) - 4*integrate(3/32*((20*a^2*b*e^(6*c) - 33*a*b^2*e^(6*c) + 15*b^3*e^(6*c
))*e^(6*d*x) - 2*(32*a^2*b*e^(4*c) - 41*a*b^2*e^(4*c) + 15*b^3*e^(4*c))*e^(
4*d*x) + (20*a^2*b*e^(2*c) - 33*a*b^2*e^(2*c) + 15*b^3*e^(2*c))*e^(2*d*x))/
(a^5*b - 2*a^4*b^2 + a^3*b^3 + (a^5*b*e^(8*c) - 2*a^4*b^2*e^(8*c) + a^3*b^3
*e^(8*c))*e^(8*d*x) - 4*(a^5*b*e^(6*c) - 2*a^4*b^2*e^(6*c) + a^3*b^3*e^(6*c
))*e^(6*d*x) - 2*(8*a^6*e^(4*c) - 19*a^5*b*e^(4*c) + 14*a^4*b^2*e^(4*c) - 3
*a^3*b^3*e^(4*c))*e^(4*d*x) - 4*(a^5*b*e^(2*c) - 2*a^4*b^2*e^(2*c) + a^3*b^
3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28429 vs. $2(306) = 612$.

time = 1.48, size = 28429, normalized size = 79.19

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(24*(20*a^2*b^2 - 33*a*b^3 + 15*b^4)*cosh(d*x + c)^16 + 384*(20*a^2*
b^2 - 33*a*b^3 + 15*b^4)*cosh(d*x + c)*sinh(d*x + c)^15 + 24*(20*a^2*b^2 -
33*a*b^3 + 15*b^4)*sinh(d*x + c)^16 - 96*(43*a^2*b^2 - 68*a*b^3 + 30*b^4)*c
osh(d*x + c)^14 - 96*(43*a^2*b^2 - 68*a*b^3 + 30*b^4 - 30*(20*a^2*b^2 - 33*
a*b^3 + 15*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^14 + 1344*(10*(20*a^2*b^2 -
33*a*b^3 + 15*b^4)*cosh(d*x + c)^3 - (43*a^2*b^2 - 68*a*b^3 + 30*b^4)*cosh(
d*x + c))*sinh(d*x + c)^13 - 32*(400*a^3*b - 1137*a^2*b^2 + 1031*a*b^3 - 31
5*b^4)*cosh(d*x + c)^12 + 32*(1365*(20*a^2*b^2 - 33*a*b^3 + 15*b^4)*cosh(d*
x + c)^4 - 400*a^3*b + 1137*a^2*b^2 - 1031*a*b^3 + 315*b^4 - 273*(43*a^2*b^
2 - 68*a*b^3 + 30*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 384*(273*(20*a^2
*b^2 - 33*a*b^3 + 15*b^4)*cosh(d*x + c)^5 - 91*(43*a^2*b^2 - 68*a*b^3 + 30*
b^4)*cosh(d*x + c)^3 - (400*a^3*b - 1137*a^2*b^2 + 1031*a*b^3 - 315*b^4)*co
sh(d*x + c))*sinh(d*x + c)^11 + 96*(592*a^3*b - 1237*a^2*b^2 + 886*a*b^3 -
210*b^4)*cosh(d*x + c)^10 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.59, size = 486, normalized size = 1.35

$$-\frac{\frac{28\,a^{2}b^{2}e^{(14\,dx+14\,c)}-35\,ab^{3}e^{(14\,dx+14\,c)}+13\,b^{4}e^{(14\,dx+14\,c)}-232\,a^{2}b^{2}e^{(12\,dx+12\,c)}+269\,ab^{3}e^{(12\,dx+12\,c)}-91\,b^{4}e^{(12\,dx+12\,c)}-576\,a^{3}be^{(10\,dx+10\,c)}+1372\,a^{2}b^{2}e^{(10\,dx+10\,c)}-1039\,ab^{3}e^{(10\,dx+10\,c)}+273\,b^{4}e^{(10\,dx+10\,c)}+2432\,a^{3}be^{(8\,dx+8\,c)}-3488\,a^{2}b^{2}e^{(8\,dx+8\,c)}+1913\,ab^{3}e^{(8\,dx+8\,c)}-455\,b^{4}e^{(8\,dx+8\,c)}+576\,a^{3}be^{(6\,dx+6\,c)}+1060\,a^{2}b^{2}e^{(6\,dx+6\,c)}-1689\,ab^{3}e^{(6\,dx+6\,c)}+455\,b^{4}e^{(6\,dx+6\,c)}-376\,a^{2}b^{2}e^{(4\,dx+4\,c)}+679\,ab^{3}e^{(4\,dx+4\,c)}-273\,b^{4}e^{(4\,dx+4\,c)}-28\,a^{2}b^{2}e^{(2\,dx+2\,c)}-117\,ab^{3}e^{(2\,dx+2\,c)}+91\,b^{4}e^{(2\,dx+2\,c)}+19\,ab^{3}-13\,b^{4}}{(a^{5}-2\,a^{4}b+a^{3}b^{2})\left(be^{(8\,dx+8\,c)}-4\,be^{(6\,dx+6\,c)}-16\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}-4\,be^{(2\,dx+2\,c)}+b\right)^{2}}+\frac{32}{a^{3}\left(e^{(2\,dx+2\,c)}-1\right)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*((28*a^2*b^2*e^(14*d*x + 14*c) - 35*a*b^3*e^(14*d*x + 14*c) + 13*b^4*
e^(14*d*x + 14*c) - 232*a^2*b^2*e^(12*d*x + 12*c) + 269*a*b^3*e^(12*d*x + 1
2*c) - 91*b^4*e^(12*d*x + 12*c) - 576*a^3*b*e^(10*d*x + 10*c) + 1372*a^2*b^
2*e^(10*d*x + 10*c) - 1039*a*b^3*e^(10*d*x + 10*c) + 273*b^4*e^(10*d*x + 10
*c) + 2432*a^3*b*e^(8*d*x + 8*c) - 3488*a^2*b^2*e^(8*d*x + 8*c) + 1913*a*b^
```

```
3*e^(8*d*x + 8*c) - 455*b^4*e^(8*d*x + 8*c) + 576*a^3*b*e^(6*d*x + 6*c) + 1
060*a^2*b^2*e^(6*d*x + 6*c) - 1689*a*b^3*e^(6*d*x + 6*c) + 455*b^4*e^(6*d*x
 + 6*c) - 376*a^2*b^2*e^(4*d*x + 4*c) + 679*a*b^3*e^(4*d*x + 4*c) - 273*b^4
*e^(4*d*x + 4*c) - 28*a^2*b^2*e^(2*d*x + 2*c) - 117*a*b^3*e^(2*d*x + 2*c) +
 91*b^4*e^(2*d*x + 2*c) + 19*a*b^3 - 13*b^4)/((a^5 - 2*a^4*b + a^3*b^2)*(b*
e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x
 + 4*c) - 4*b*e^(2*d*x + 2*c) + b)^2) + 32/(a^3*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(c+d\,x\right)^2\,{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)^3), x)
```

3.265 $\int \frac{1}{1-\sinh^4(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

```
[Out] 1/4*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/2*tanh(x)
```

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3288, 396, 212}

$$\frac{\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1-\sinh^4(x)}\,dx &= \text{Subst}\left(\int \frac{1-x^2}{1-2x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 24, normalized size = 0.96

$$\frac{1}{4}\left(\sqrt{2}\,\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)+2\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.
time = 0.43, size = 55, normalized size = 2.20

method	result	size
risch	$-\frac{1}{1+e^{2x}}+\frac{\sqrt{2}\,\ln\left(e^{2x}-3+2\sqrt{2}\right)}{8}-\frac{\sqrt{2}\,\ln\left(e^{2x}-3-2\sqrt{2}\right)}{8}$	46
default	$\frac{\tanh\left(\frac{x}{2}\right)}{\tanh^{2}\left(\frac{x}{2}\right)+1}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{4}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{4}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^4),x,method=_RETURNVERBOSE)
```

```
[Out] tanh(1/2*x)/(tanh(1/2*x)^2+1)+1/4*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(
1/2))+1/4*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(17) = 34$.
time = 0.47, size = 69, normalized size = 2.76

$$\frac{1}{8}\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{8}\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)+\frac{1}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/8*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + 1/(e^(-2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(17) = 34$.
time = 0.37, size = 113, normalized size = 4.52

$$\frac{\left(\sqrt{2}\cosh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^{2}+\sqrt{2}\right)\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^{2}-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^{2}-2\sqrt{2}+3}{\cosh(x)^{2}+\sinh(x)^{2}-3}\right)-8}{8\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="fricas")
```

```
[Out] 1/8*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + s
qrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(
x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 -
3)) - 8)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 908 vs. $2(20) = 40$.
time = 2.85, size = 908, normalized size = 36.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**4),x)
```

```
[Out] 3064704*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/
2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt
(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**
2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 3064704*log(tanh
(x/2) - 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2
 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt(2)*log(tanh(x/2) - 1 + sqrt(
2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(
2) + 17336584) + 3064704*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1225881
6*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 1733658
4) + 2167073*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(12258816*sq
rt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) +
 3064704*log(tanh(x/2) + 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 1733
6584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt(2)*log(tanh
(x/2) + 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2
 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2) -
```

```
 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 1
2258816*sqrt(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2
)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt
(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(12258816*sq
rt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) -
 3064704*log(tanh(x/2) - sqrt(2) - 1)/(12258816*sqrt(2)*tanh(x/2)**2 + 1733
6584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh
(x/2) - sqrt(2) + 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584
*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt
(2) + 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**
2 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2)
+ 1)/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt
(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt(2) + 1)/(12258816*sqrt(2)*ta
nh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 1225881
6*sqrt(2)*tanh(x/2)/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2
+ 12258816*sqrt(2) + 17336584) + 17336584*tanh(x/2)/(12258816*sqrt(2)*tanh(
x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.
time = 0.41, size = 48, normalized size = 1.92

$$-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-\frac{1}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - 1/(e^(2*x) + 1)
```

Mupad [**B**]
time = 0.16, size = 63, normalized size = 2.52

$$\frac{\sqrt{2}\,\ln\left(2\,e^{2x}+\frac{\sqrt{2}\,(12\,e^{2x}-4)}{8}\right)}{8}-\frac{\sqrt{2}\,\ln\left(2\,e^{2x}-\frac{\sqrt{2}\,(12\,e^{2x}-4)}{8}\right)}{8}-\frac{1}{e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^4 - 1),x)
```

```
[Out] (2^(1/2)*log(2*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - (2^(1/2)*log(
2*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - 1/(exp(2*x) + 1)
```

3.266 $\int \frac{1}{1+\sinh^4(x)}\,dx$

Optimal. Leaf size=176

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{1+\sqrt{2}}-2\tanh(x)}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{1+\sqrt{2}}+2\tanh(x)}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}-\frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(\sqrt{2}-2\sqrt{1+\sqrt{2}}\right.$$

```
[Out] -1/4*arctan(((1+2^(1/2))^(1/2)-2*tanh(x))/(2^(1/2)-1)^(1/2))/(1+2^(1/2))^(1
/2)+1/4*arctan(((1+2^(1/2))^(1/2)+2*tanh(x))/(2^(1/2)-1)^(1/2))/(1+2^(1/2))
^(1/2)-1/8*ln(2^(1/2)-2*(1+2^(1/2))^(1/2)*tanh(x)+2*tanh(x)^2)*(1+2^(1/2))^
(1/2)+1/8*ln(1+(2+2*2^(1/2))^(1/2)*tanh(x)+2^(1/2)*tanh(x)^2)*(1+2^(1/2))^(
1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3288, 1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{1+\sqrt{2}}-2\tanh(x)}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\text{ArcTan}\left(\frac{2\tanh(x)+\sqrt{1+\sqrt{2}}}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{1+\sqrt{2}}}-\frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(2\tanh^2(x)-2\sqrt{1+\sqrt{2}}\tanh(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(\sqrt{2}\tanh^2(x)+\sqrt{2\left(1+\sqrt{2}\right)}\tanh(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^4)^(-1),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[1 + Sqrt[2]] - 2*Tanh[x])/Sqrt[-1 + Sqrt[2]]]/Sqrt[1 + Sq
rt[2]] + ArcTan[(Sqrt[1 + Sqrt[2]] + 2*Tanh[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt
[1 + Sqrt[2]]) - (Sqrt[1 + Sqrt[2]]*Log[Sqrt[2] - 2*Sqrt[1 + Sqrt[2]]*Tanh[
x] + 2*Tanh[x]^2])/8 + (Sqrt[1 + Sqrt[2]]*Log[1 + Sqrt[2*(1 + Sqrt[2])]*Tan
h[x] + Sqrt[2]*Tanh[x]^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{1+\sinh^4(x)}\,dx = \text{Subst}\left(\int \frac{1-x^2}{1-2x^2+2x^4}\,dx, x, \tanh(x)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{1+\sqrt{2}}-\left(1+\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}-\sqrt{1+\sqrt{2}}\,x+x^2}\,dx, x, \tanh(x)\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{1+\sqrt{2}}+\left(1+\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}+\sqrt{1+\sqrt{2}}\,x+x^2}\right.}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

$$= \frac{1}{8}\sqrt{3-2\sqrt{2}}\,\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{2}}-\sqrt{1+\sqrt{2}}\,x+x^2}\,dx, x, \tanh(x)\right) + \frac{1}{8}\sqrt{3-2\sqrt{2}}\,\text{S}$$

$$= -\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(\sqrt{2}-2\sqrt{1+\sqrt{2}}\,\tanh(x)+2\tanh^2(x)\right) + \frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(1\right.$$

$$= -\frac{1}{4}\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{1+\sqrt{2}}-2\tanh(x)}{\sqrt{-1+\sqrt{2}}}\right) + \frac{1}{4}\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{1+}}{\sqrt{}}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 45, normalized size = 0.26

$$\frac{\tanh^{-1}\left(\sqrt{1-i}\,\tanh(x)\right)}{2\sqrt{1-i}} + \frac{\tanh^{-1}\left(\sqrt{1+i}\,\tanh(x)\right)}{2\sqrt{1+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^4)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[1 - I]*Tanh[x]]/(2*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + I]*Tanh[x]]
/(2*Sqrt[1 + I])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.54, size = 44, normalized size = 0.25

method	result	size
risch	$\sum_{\_R=\text{RootOf}(512\_Z^4-32\_Z^2+1)} \_R\ln\left(256\_R^3-64\_R^2+e^{2x}+1\right)$	36

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^{4}-2\_Z^{2}+1\right)}\_R\ln\left(\tanh^{2}\left(\frac{x}{2}\right)+\left(-4\_R^{3}+4\_R\right)\tanh\left(\frac{x}{2}\right)+1\right)\right)}{4}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R*ln(tanh(1/2*x)^2+(-4*_R^3+4*_R)*tanh(1/2*x)+1),_R=RootOf(2*_Z^4-
2*_Z^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sinh(x)^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(124) = 248$.
time = 0.38, size = 596, normalized size = 3.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^4),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log((2^(3/4)*e^(2*x) - 2^(1
/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x)
 + 5) - 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log(-(2^(3/4)*e^(2*
x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) -
2*e^(2*x) + 5) - 1/4*2^(1/4)*sqrt(-2*sqrt(2) + 4)*arctan(1/14*(sqrt(2)*(5*s
qrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) - 1/28*(2*sqrt(2)*(5*sqrt(2) + 6) - (2
^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sqrt(2) + 4) +
 16*sqrt(2) + 8)*sqrt(-(2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*
sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) - 1/14*sqrt(2)*(3*sqrt(
2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*e^(2
*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(-2*sqrt(2)
+ 4) - 1/7*sqrt(2) + 3/7) - 1/4*2^(1/4)*sqrt(-2*sqrt(2) + 4)*arctan(-1/14*(
sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) + 1/28*(2*sqrt(2)*(5*sqrt(
2) + 6) + (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sq
rt(2) + 4) + 16*sqrt(2) + 8)*sqrt((2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4
))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) + 1/14*sqrt(
```

```
2)*(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2)
 + 6))*e^(2*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(
-2*sqrt(2) + 4) + 1/7*sqrt(2) - 3/7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**4),x)
```

```
[Out] Timed out
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.45, size = 281, normalized size = 1.60

$$\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2\sqrt{2}-2}\left(-\frac{i}{\sqrt{2}-1}+1\right)\log\left((20i+10)\sqrt{2}e^{(2x)}+10\sqrt{2}\sqrt{10\sqrt{2}+14}-50\sqrt{2}-(2i-14)\sqrt{10\sqrt{2}+14}+(28i+14)e^{(2x)}-70\right)-\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2\sqrt{2}-2}\left(-\frac{i}{\sqrt{2}-1}+1\right)\log\left((20i+10)\sqrt{2}e^{(2x)}-10\sqrt{2}\sqrt{10\sqrt{2}+14}-50\sqrt{2}+(2i-14)\sqrt{10\sqrt{2}+14}+(28i+14)e^{(2x)}-70\right)+\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2\sqrt{2}+2}\left(-\frac{i}{\sqrt{2}+1}+1\right)\log\left(2\sqrt{2}e^{(2x)}+2\sqrt{2}\sqrt{2\sqrt{2}-2}-(4i+2)\sqrt{2}+(2i-2)\sqrt{2\sqrt{2}-2}-2e^{(2x)}+4i+2\right)-\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2\sqrt{2}+2}\left(-\frac{i}{\sqrt{2}+1}+1\right)\log\left(2\sqrt{2}e^{(2x)}-2\sqrt{2}\sqrt{2\sqrt{2}-2}-(4i+2)\sqrt{2}-(2i-2)\sqrt{2\sqrt{2}-2}-2e^{(2x)}+4i+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^4),x, algorithm="giac")
```

```
[Out] (1/16*I + 1/16)*sqrt(2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1)*log((20*I + 10)*
sqrt(2)*e^(2*x) + 10*sqrt(2)*sqrt(10*sqrt(2) + 14) - 50*sqrt(2) - (2*I - 14
)*sqrt(10*sqrt(2) + 14) + (28*I + 14)*e^(2*x) - 70) - (1/16*I + 1/16)*sqrt(
2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1)*log((20*I + 10)*sqrt(2)*e^(2*x) - 10*
sqrt(2)*sqrt(10*sqrt(2) + 14) - 50*sqrt(2) + (2*I - 14)*sqrt(10*sqrt(2) + 1
4) + (28*I + 14)*e^(2*x) - 70) + (1/16*I + 1/16)*sqrt(2*sqrt(2) + 2)*(-I/(s
qrt(2) + 1) + 1)*log(2*sqrt(2)*e^(2*x) + 2*sqrt(2)*sqrt(2*sqrt(2) - 2) - (4
*I + 2)*sqrt(2) + (2*I - 2)*sqrt(2*sqrt(2) - 2) - 2*e^(2*x) + 4*I + 2) - (1
/16*I + 1/16)*sqrt(2*sqrt(2) + 2)*(-I/(sqrt(2) + 1) + 1)*log(2*sqrt(2)*e^(2
*x) - 2*sqrt(2)*sqrt(2*sqrt(2) - 2) - (4*I + 2)*sqrt(2) - (2*I - 2)*sqrt(2*
sqrt(2) - 2) - 2*e^(2*x) + 4*I + 2)
```

Mupad [**B**]
time = 1.15, size = 205, normalized size = 1.16

$$-\frac{\sqrt{2}\,\sqrt{1-\mathrm{i}}\,\ln\left(\mathrm{e}^{2x}\,(436273152+91291648\mathrm{i})+\sqrt{2}\,\sqrt{1-\mathrm{i}}\,(-9830400+56623104\mathrm{i})+\sqrt{2}\,\sqrt{1-\mathrm{i}}\,\mathrm{e}^{2x}\,(-218890240-149422080\mathrm{i})-21168128-94306304\mathrm{i}\right)}{8}+\frac{\sqrt{2}\,\sqrt{1-\mathrm{i}}\,\ln\left(\mathrm{e}^{2x}\,(436273152+91291648\mathrm{i})+\sqrt{2}\,\sqrt{1-\mathrm{i}}\,(9830400-56623104\mathrm{i})+\sqrt{2}\,\sqrt{1-\mathrm{i}}\,\mathrm{e}^{2x}\,(218890240+149422080\mathrm{i})-21168128-94306304\mathrm{i}\right)}{8}-\frac{\sqrt{2}\,\sqrt{1+1\mathrm{i}}\,\ln\left(\mathrm{e}^{2x}\,(436273152-91291648\mathrm{i})+\sqrt{2}\,\sqrt{1+1\mathrm{i}}\,(-9830400-56623104\mathrm{i})+\sqrt{2}\,\sqrt{1+1\mathrm{i}}\,\mathrm{e}^{2x}\,(-218890240+149422080\mathrm{i})-21168128+94306304\mathrm{i}\right)}{8}+\frac{\sqrt{2}\,\sqrt{1+1\mathrm{i}}\,\ln\left(\mathrm{e}^{2x}\,(436273152-91291648\mathrm{i})+\sqrt{2}\,\sqrt{1+1\mathrm{i}}\,(9830400+56623104\mathrm{i})+\sqrt{2}\,\sqrt{1+1\mathrm{i}}\,\mathrm{e}^{2x}\,(218890240-149422080\mathrm{i})-21168128+94306304\mathrm{i}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4 + 1),x)
```

```
[Out] (2^(1/2)*(1 - 1i)^(1/2)*log(exp(2*x)*(436273152 + 91291648i) + 2^(1/2)*(1 -
 1i)^(1/2)*(9830400 - 56623104i) + 2^(1/2)*(1 - 1i)^(1/2)*exp(2*x)*(2188902
40 + 149422080i) - (21168128 + 94306304i)))/8 - (2^(1/2)*(1 - 1i)^(1/2)*log
(exp(2*x)*(436273152 + 91291648i) - 2^(1/2)*(1 - 1i)^(1/2)*(9830400 - 56623
```

```
104i) - 2^(1/2)*(1 - 1i)^(1/2)*exp(2*x)*(218890240 + 149422080i) - (2116812
8 + 94306304i)))/8 - (2^(1/2)*(1 + 1i)^(1/2)*log(exp(2*x)*(436273152 - 9129
1648i) - 2^(1/2)*(1 + 1i)^(1/2)*(9830400 + 56623104i) - 2^(1/2)*(1 + 1i)^(1
/2)*exp(2*x)*(218890240 - 149422080i) - (21168128 - 94306304i)))/8 + (2^(1/
2)*(1 + 1i)^(1/2)*log(exp(2*x)*(436273152 - 91291648i) + 2^(1/2)*(1 + 1i)^(
1/2)*(9830400 + 56623104i) + 2^(1/2)*(1 + 1i)^(1/2)*exp(2*x)*(218890240 - 1
49422080i) - (21168128 - 94306304i)))/8
```

3.267 $\int \frac{1}{a+b\sinh^5(x)}\,dx$

Optimal. Leaf size=435

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+b^{2/5}}}+\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{(-1)^{9/10}\left(\sqrt[5]{-1}\,\sqrt[5]{b}+\sqrt[5]{a}\,\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{-(-1)^{4/5}a^{2/5}+\sqrt[5]{-1}\,b^{2/5}}}\right)}{5a^{4/5}\sqrt{-(-1)^{4/5}a^{2/5}+\sqrt[5]{-1}\,b^{2/5}}}+\frac{2\sqrt[5]{-1}\,\tanh^{-1}\left(\frac{\sqrt[5]{b}}{\sqrt{\ }}\right.}{5a^{4/5}\sqrt{(-1)^2}}$$

```
[Out] -2/5*(-1)^(9/10)*arctanh((I*b^(1/5)-(-1)^(9/10)*a^(1/5)*tanh(1/2*x))/(-(-1)
^(4/5)*a^(2/5)-b^(2/5))^(1/2))/a^(4/5)/(-(-1)^(4/5)*a^(2/5)-b^(2/5))^(1/2)-
2/5*arctanh((b^(1/5)-a^(1/5)*tanh(1/2*x))/(a^(2/5)+b^(2/5))^(1/2))/a^(4/5)/
(a^(2/5)+b^(2/5))^(1/2)+2/5*(-1)^(1/5)*arctanh((b^(1/5)+(-1)^(1/5)*a^(1/5)*
tanh(1/2*x))/((-1)^(2/5)*a^(2/5)+b^(2/5))^(1/2))/a^(4/5)/((-1)^(2/5)*a^(2/5
)+b^(2/5))^(1/2)+2/5*(-1)^(9/10)*arctanh((-1)^(9/10)*((-1)^(1/5)*b^(1/5)+a^
(1/5)*tanh(1/2*x))/(-(-1)^(4/5)*a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/2))/a^(4/5)/
(-(-1)^(4/5)*a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/2)+2/5*(-1)^(9/10)*arctanh((-1)
^(3/10)*(b^(1/5)+(-1)^(3/5)*a^(1/5)*tanh(1/2*x))/(-(-1)^(4/5)*a^(2/5)+(-1)^
(3/5)*b^(2/5))^(1/2))/a^(4/5)/(-(-1)^(4/5)*a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2
)
```

Rubi [A]

time = 0.68, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3292, 2739, 632, 212, 210}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+b^{2/5}}}+\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{(-1)^{9/10}\left(\sqrt[5]{a}\,\tanh\left(\frac{x}{2}\right)+\sqrt[5]{-1}\,\sqrt[5]{b}\right)}{\sqrt{\sqrt[5]{-1}\,b^{2/5}-(-1)^{4/5}a^{2/5}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{-1}\,b^{2/5}-(-1)^{4/5}a^{2/5}}}+\frac{2\sqrt[5]{-1}\,\tanh^{-1}\left(\frac{\sqrt[5]{-1}\,\sqrt[5]{a}\,\tanh\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}+\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{(-1)^{3/10}\left((-1)^{3/5}\sqrt[5]{a}\,\tanh\left(\frac{x}{2}\right)+\sqrt[5]{b}\right)}{\sqrt{(-1)^{3/5}b^{2/5}-(-1)^{4/5}a^{2/5}}}\right)}{5a^{4/5}\sqrt{(-1)^{3/5}b^{2/5}-(-1)^{4/5}a^{2/5}}}-\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{-(-1)^{9/10}\sqrt[5]{a}\,\tanh\left(\frac{x}{2}\right)+i\sqrt[5]{b}}{\sqrt{-(-1)^{4/5}a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{-(-1)^{4/5}a^{2/5}-b^{2/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x]^5)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b^(1/5) - a^(1/5)*Tanh[x/2])/Sqrt[a^(2/5) + b^(2/5)]])/(5*a^(4
/5)*Sqrt[a^(2/5) + b^(2/5)]) + (2*(-1)^(9/10)*ArcTanh[((-1)^(9/10)*((-1)^(1
/5)*b^(1/5) + a^(1/5)*Tanh[x/2]))/Sqrt[-((-1)^(4/5)*a^(2/5)) + (-1)^(1/5)*b
^(2/5)]])/(5*a^(4/5)*Sqrt[-((-1)^(4/5)*a^(2/5)) + (-1)^(1/5)*b^(2/5)]) + (2
*(-1)^(1/5)*ArcTanh[(b^(1/5) + (-1)^(1/5)*a^(1/5)*Tanh[x/2])/Sqrt[(-1)^(2/5
)*a^(2/5) + b^(2/5)]])/(5*a^(4/5)*Sqrt[(-1)^(2/5)*a^(2/5) + b^(2/5)]) + (2*
(-1)^(9/10)*ArcTanh[((-1)^(3/10)*(b^(1/5) + (-1)^(3/5)*a^(1/5)*Tanh[x/2]))/
Sqrt[-((-1)^(4/5)*a^(2/5)) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[-((-1)^(
4/5)*a^(2/5)) + (-1)^(3/5)*b^(2/5)]) - (2*(-1)^(9/10)*ArcTanh[(I*b^(1/5) -
(-1)^(9/10)*a^(1/5)*Tanh[x/2])/Sqrt[-((-1)^(4/5)*a^(2/5)) - b^(2/5)]])/(5*a
^(4/5)*Sqrt[-((-1)^(4/5)*a^(2/5)) - b^(2/5)])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\sinh^5(x)}\,dx = \int \left(-\frac{(-1)^{9/10}}{5a^{4/5}\left(-(-1)^{9/10}\sqrt[5]{a}-i\sqrt[5]{b}\sinh(x)\right)}-\frac{(-1)^{9/10}}{5a^{4/5}\left(-(-1)^{9/10}\sqrt[5]{a}-\sqrt[10]{-1}\sqrt[5]{b}\,\mathrm{si}\right.}\right.$$

$$=-\frac{(-1)^{9/10}\int \frac{1}{-(-1)^{9/10}\sqrt[5]{a}-i\sqrt[5]{b}\sinh(x)}\,dx}{5a^{4/5}}-\frac{(-1)^{9/10}\int \frac{1}{-(-1)^{9/10}\sqrt[5]{a}-\sqrt[10]{-1}\sqrt[5]{b}\sinh(x)}\,d}{5a^{4/5}}$$

$$=-\frac{\left(2(-1)^{9/10}\right)\mathrm{Subst}\left(\int \frac{1}{-(-1)^{9/10}\sqrt[5]{a}-2i\sqrt[5]{b}\,x+(-1)^{9/10}\sqrt[5]{a}\,x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{5a^{4/5}}-\frac{\left(2(-1)\right.}{}$$

$$=\frac{\left(4(-1)^{9/10}\right)\mathrm{Subst}\left(\int \frac{1}{-4(-1)^{4/5}\left(a^{2/5}+b^{2/5}\right)-x^2}\,dx,x,-2(-1)^{9/10}\sqrt[5]{b}+2(-1)^{9/10}\sqrt[5]{a}\,\mathrm{tanh}\right.}{5a^{4/5}}$$

$$=-\frac{2(-1)^{7/10}\tan^{-1}\left(\frac{i\sqrt[5]{b}+(-1)^{7/10}\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)}{\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+b^{2/5}}}-\frac{2(-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.22, size = 141, normalized size = 0.32

$$\frac{8}{5}\mathrm{RootSum}\left[-b+5b\#1^2-10b\#1^4+32a\#1^5+10b\#1^6-5b\#1^8+b\#1^{10}\&,\frac{x\#1^3+2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^3}{b-4b\#1^2+16a\#1^3+6b\#1^4-4b\#1^6+b\#1^8}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^5)^(-1),x]
```

```
[Out] (8*RootSum[-b + 5*b*#1^2 - 10*b*#1^4 + 32*a*#1^5 + 10*b*#1^6 - 5*b*#1^8 + b
*#1^10 & , (x*#1^3 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2
]*#1]*#1^3)/(b - 4*b*#1^2 + 16*a*#1^3 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8) & ])/
5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.35, size = 113, normalized size = 0.26

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{10}-5a\_Z^{8}+10a\_Z^{6}-32b\_Z^{5}-10a\_Z^{4}+5a\_Z^{2}-a\right)}\frac{\left(-\_R^{8}+4\_R^{6}-6\_R^{4}+4\_R^{2}-1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^{9}a-4\_R^{7}a+6\_R^{5}a-16\_R^{4}b-4\_R^{3}a+\_Ra}\right)}{5}$

risch	$\sum_{\_R=\mathrm{RootOf}\left(-1+(9765625a^{10}+9765625a^{8}b^{2})\_Z^{10}-1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}-6250a^{4}\_Z^{4}+125\_Z^{2}a^{2}\right)} \_R\ln\left(\mathrm{e}^{x}+\left(\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*sum((-_R^8+4*_R^6-6*_R^4+4*_R^2-1)/(_R^9*a-4*_R^7*a+6*_R^5*a-16*_R^4*b-
4*_R^3*a+_R*a)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^10*a-5*_Z^8*a+10*_Z^6*a-32*_
Z^5*b-10*_Z^4*a+5*_Z^2*a-a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^5),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sinh(x)^5 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sinh^{5}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**5),x)
```

```
[Out] Integral(1/(a + b*sinh(x)**5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^5),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sinh(x)^5 + a), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x)^5),x)
```

```
[Out] \text{Hanged}
```

3.268 $\int \frac{1}{a+b\sinh^6(x)}\,dx$

Optimal. Leaf size=175

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}{\sqrt[6]{a}}\right.}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{}}}$$

```
[Out] 1/3*arctanh((a^(1/3)-b^(1/3))^(1/2)*tanh(x)/a^(1/6))/a^(5/6)/(a^(1/3)-b^(1/
3))^(1/2)+1/3*arctanh((a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)*tanh(x)/a^(1/6))/a
^(5/6)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctanh((a^(1/3)-(-1)^(2/3)*b
^(1/3))^(1/2)*tanh(x)/a^(1/6))/a^(5/6)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3290, 3260, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x]^6)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[a^(1/3) - b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3)
- b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tanh[x])/a^(1/6)]
/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) - (
-1)^(2/3)*b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b
^(1/3)])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\sinh^6(x)}\,dx = \frac{\int \frac{1}{1+\frac{\sqrt[3]{b}\sinh^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sinh^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{(-1)^{2/3}\sqrt[3]{b}\sinh^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{1}{1-\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tanh(x)\right)}{3a} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tanh(\right.}{3a}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\tanh^{-1}}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 134, normalized size = 0.77

$$\frac{16}{3}\mathrm{RootSum}\left[b-6b\#1+15b\#1^2+64a\#1^3-20b\#1^3+15b\#1^4-6b\#1^5+b\#1^6\&, \frac{x\#1^2+\log(-\cosh(x)-\sinh(x)+\cosh(x)\#1-\sinh(x)\#1)\#1^2}{-b+5b\#1+32a\#1^2-10b\#1^2+10b\#1^3-5b\#1^4+b\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^6)^(-1),x]
```

```
[Out] (16*RootSum[b - 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6*
b*#1^5 + b*#1^6 & , (x*#1^2 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x]
*#1]*#1^2)/(-b + 5*b*#1 + 32*a*#1^2 - 10*b*#1^2 + 10*b*#1^3 - 5*b*#1^4 + b*
#1^5) & ])/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.86, size = 128, normalized size = 0.73

method	result

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{12}-6a\_Z^{10}+15a\_Z^{8}+(-20a+64b)\_Z^{6}+15a\_Z^{4}-6a\_Z^{2}+a\right)} \frac{\left(-\_R^{10}+5\_R^{8}-10\_R^{6}+10\_R^{4}-5\_R^{2}+1\right)\ln}{\_R^{11}a-5\_R^{9}a+10\_R^{7}a-10\_R^{5}a+32\_R^{5}b+}\right.}{6}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(-1+(46656a^{6}-46656a^{5}b)\_Z^{6}-3888a^{4}\_Z^{4}+108\_Z^{2}a^{2}\right)} \_R\ln\left(\mathrm{e}^{2x}+\left(-\frac{15552a^{6}}{b}+15552a^{5}\right)\_R^{5}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum((-_R^10+5*_R^8-10*_R^6+10*_R^4-5*_R^2+1)/(_R^11*a-5*_R^9*a+10*_R^7*
a-10*_R^5*a+32*_R^5*b+5*_R^3*a-_R*a)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^12-6
*a*_Z^10+15*a*_Z^8+(-20*a+64*b)*_Z^6+15*a*_Z^4-6*a*_Z^2+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^6),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sinh(x)^6 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.42, size = 16401, normalized size = 93.72

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^6),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(1/2)*sqrt(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a^4 - a^3*b) - 1/(a^
2 - a*b)^2)/(1/(a^6 - a^5*b) - 3/((a^4 - a^3*b)*(a^2 - a*b)) + 2/(a^2 - a*b
)^3 + b/((a - b)^2*a^5))^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(a^6 - a^5*
b) - 3/((a^4 - a^3*b)*(a^2 - a*b)) + 2/(a^2 - a*b)^3 + b/((a - b)^2*a^5))^(
1/3) + 2/(a^2 - a*b))*log(1/2*(a^5 - a^4*b)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*(1/(a^4 - a^3*b) - 1/(a^2 - a*b)^2)/(1/(a^6 - a^5*b) - 3/((a^4 - a^3*b)*(a
^2 - a*b)) + 2/(a^2 - a*b)^3 + b/((a - b)^2*a^5))^(1/3) - (1/2)^(1/3)*(I*sq
rt(3) + 1)*(1/(a^6 - a^5*b) - 3/((a^4 - a^3*b)*(a^2 - a*b)) + 2/(a^2 - a*b)
^3 + b/((a - b)^2*a^5))^(1/3) + 2/(a^2 - a*b))^2 + b*cosh(x)^2 + 2*b*cosh(x
)*sinh(x) + b*sinh(x)^2 + 1/2*sqrt(1/2)*((a^6 - a^5*b)*(2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(1/(a^4 - a^3*b) - 1/(a^2 - a*b)^2)/(1/(a^6 - a^5*b) - 3/((a^4
- a^3*b)*(a^2 - a*b)) + 2/(a^2 - a*b)^3 + b/((a - b)^2*a^5))^(1/3) - (1/2)^
```

```
(1/3)*(I*sqrt(3) + 1)*(1/(a^6 - a^5*b) - 3/((a^4 - a^3*b)*(a^2 - a*b)) + 2/
(a^2 - a*b)^3 + b/((a - b ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \sinh^{6}{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**6),x)
```

```
[Out] Integral(1/(a + b*sinh(x)**6), x)
```

Giac [A]
time = 0.46, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^6),x, algorithm="giac")
```

```
[Out] 0
```

Mupad [B]
time = 58.56, size = 857, normalized size = 4.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x)^6),x)
```

```
[Out] symsum(log(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 108*a^2*d^
2 + 1, d, k)*(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 108*a^2
*d^2 + 1, d, k)*(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 108*
a^2*d^2 + 1, d, k)*(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 1
08*a^2*d^2 + 1, d, k)*((14591662792680407O4*(327680*a^7*exp(2*x) + 298496*a
^6*b - 65536*a^7 + 158*a^2*b^5 - 91315*a^3*b^4 + 348176*a^4*b^3 - 489952*a^
5*b^2 - 196*a^2*b^5*exp(2*x) + 274019*a^3*b^4*exp(2*x) - 1132876*a^4*b^3*ex
p(2*x) + 1770440*a^5*b^2*exp(2*x) - 1239040*a^6*b*exp(2*x)))/(b^10*(a - b)^
3) + (17509995351216488448*root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*
d^4 - 108*a^2*d^2 + 1, d, k)*(262144*a^7*exp(2*x) + 203520*a^6*b - 65536*a^
7 - 453*a^3*b^4 + 72022*a^4*b^3 - 209472*a^5*b^2 + 630*a^3*b^4*exp(2*x) - 2
54512*a^4*b^3*exp(2*x) + 767508*a^5*b^2*exp(2*x) - 775680*a^6*b*exp(2*x)))/
(b^10*(a - b)^2)) - (4863887597560135568*(655360*a^5*exp(2*x) + 9*a*b^4 + 37
0176*a^4*b - 196608*a^5 - 24408*a^2*b^3 - 149088*a^3*b^2 + 63676*a^2*b^3*ex
```

```
p(2*x) + 526248*a^3*b^2*exp(2*x) - 10*a*b^4*exp(2*x) - 1245184*a^4*b*exp(2*
x)))/(b^10*(a - b)^2)) - (40532396646334464*(655360*a^5*exp(2*x) + b^5*exp(
2*x) + 24677*a*b^4 + 773120*a^4*b - 262144*a^5 - b^5 + 198071*a^2*b^3 - 733
696*a^3*b^2 - 477713*a^2*b^3*exp(2*x) + 1770640*a^3*b^2*exp(2*x) - 53861*a*
b^4*exp(2*x) - 1894400*a^4*b*exp(2*x)))/(b^10*(a - b)^3)) + (13510798882111
488*(655360*a^3*exp(2*x) - 11382*b^3*exp(2*x) - 144416*a*b^2 + 269056*a^2*b
 - 131072*a^3 + 6459*b^3 + 677524*a*b^2*exp(2*x) - 1321472*a^2*b*exp(2*x)))
/(b^10*(a - b)^2)) - (1125899906842624*(851968*a^4*exp(2*x) + 6006*b^4*exp(
2*x) + 211497*a*b^3 + 597504*a^3*b - 196608*a^4 - 3840*b^4 - 608544*a^2*b^2
 + 2562504*a^2*b^2*exp(2*x) - 864565*a*b^3*exp(2*x) - 2555904*a^3*b*exp(2*x
)))/(b^10*(a - b)^2*(a*b - a^2)))*root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 38
88*a^4*d^4 - 108*a^2*d^2 + 1, d, k), k, 1, 6)
```

3.269 $\int \frac{1}{a+b\sinh^8(x)}\,dx$

Optimal. Leaf size=245

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tanh(x}{\sqrt[8]{-a}}\right.}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}$$

```
[Out] -1/4*arctanh(((-a)^(1/4)-b^(1/4))^(1/2)*tanh(x)/(-a)^(1/8))/(-a)^(7/8)/((-a
)^(1/4)-b^(1/4))^(1/2)-1/4*arctanh(((-a)^(1/4)-I*b^(1/4))^(1/2)*tanh(x)/(-a
)^(1/8))/(-a)^(7/8)/((-a)^(1/4)-I*b^(1/4))^(1/2)-1/4*arctanh(((-a)^(1/4)+I*
b^(1/4))^(1/2)*tanh(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+I*b^(1/4))^(1/2)-
1/4*arctanh(((-a)^(1/4)+b^(1/4))^(1/2)*tanh(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)
^(1/4)+b^(1/4))^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3290, 3260, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\tanh(x)}{(-a)^{5/8}}\right)}{4(-a)^{3/8}\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x]^8)^(-1),x]
```

```
[Out] -1/4*ArcTanh[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Tanh[x])/(-a)^(1/8)]/((-a)^(7/8)
*Sqrt[(-a)^(1/4) - I*b^(1/4)]) - ArcTanh[(Sqrt[(-a)^(1/4) + I*b^(1/4)]*Tanh
[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + I*b^(1/4)]) - ArcTanh[(Sqr
t[(-a)^(1/4) + b^(1/4)]*Tanh[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4)
+ b^(1/4)]) - ArcTanh[(Sqrt[(-a)^(5/4) + a*b^(1/4)]*Tanh[x])/(-a)^(5/8)]/(4
*(-a)^(3/8)*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\sinh^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sinh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sinh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\sinh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\sinh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \tanh(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \tanh(x\right.}{4a}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}} - \frac{\tanh}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.19, size = 160, normalized size = 0.65

$$16\text{RootSum}\left[b-8b\#1+28b\#1^2-56b\#1^3+256a\#1^4+70b\#1^4-56b\#1^5+28b\#1^6-8b\#1^7+b\#1^8\&, \frac{x\#1^3+\log(-\cosh(x)-\sinh(x)+\cosh(x)\#1-\sinh(x)\#1)\#1^3}{-b+7b\#1-21b\#1^2+128a\#1^3+35b\#1^3-35b\#1^4+21b\#1^5-7b\#1^6+b\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^8)^(-1),x]
```

```
[Out] 16*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 - 56
*b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (x*#1^3 + Log[-Cosh[x] - Sinh[x
] + Cosh[x]*#1 - Sinh[x]*#1]*#1^3)/(-b + 7*b*#1 - 21*b*#1^2 + 128*a*#1^3 +
35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.09, size = 162, normalized size = 0.66

---

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{16}-8a\_Z^{14}+28a\_Z^{12}-56a\_Z^{10}+(70a+256b)\_Z^{8}-56a\_Z^{6}+28a\_Z^{4}-8a\_Z^{2}+a\right)} \frac{\left(-\_R^{14}+7\_R^{12}-21\_R^{10}+35\right.}{\_R^{15}a-7\_R^{13}a+21\_R^{11}a-35}\right.}{8}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(16777216a^{8}+16777216a^{7}b)\_Z^{8}-1048576a^{6}\_Z^{6}+24576a^{4}\_Z^{4}-256\_Z^{2}a^{2}\right)} \_R\ln\left(\mathrm{e}^{2x}+\left(\frac{4194304a^{8}}{b}+41\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)^8),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum((-_R^14+7*_R^12-21*_R^10+35*_R^8-35*_R^6+21*_R^4-7*_R^2+1)/(_R^15*a
-7*_R^13*a+21*_R^11*a-35*_R^9*a+35*_R^7*a+128*_R^7*b-21*_R^5*a+7*_R^3*a-_R*
a)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^16-8*a*_Z^14+28*a*_Z^12-56*a*_Z^10+(70
*a+256*b)*_Z^8-56*a*_Z^6+28*a*_Z^4-8*a*_Z^2+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sinh(x)^8 + a), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 661332 vs. $2(165) = 330$.
time = 3.54, size = 661332, normalized size = 2699.31

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^8),x, algorithm="fricas")
```

```
[Out] 1/192*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(((a^3*sqrt(-(2*a*b*sqrt(-b/a) - a*b
+ b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) + a^2*b*sqrt(-(2*a*b*sqrt(-b
/a) - a*b + b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) + 3*a)*sqrt(-b/a)
- b)^2*a/((a^3 + a^2*b)^2*b) - 3*(2*a^2*b*sqrt(-(2*a*b*sqrt(-b/a) - a*b + b
^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) - (2*a^3*sqrt(-(2*a*b*sqrt(-b/a
) - a*b + b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) + 3*a)*sqrt(-b/a) +
b)/((a^5 + a^4*b)*sqrt(-b/a)))/(-1/1572864*(2*a^2*b*sqrt(-(2*a*b*sqrt(-b/a)
 - a*b + b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) - (2*a^3*sqrt(-(2*a*b
*sqrt(-b/a) - a*b + b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) + 3*a)*sqr
```

```
t(-b/a) + b)*((a^3*sqrt(-(2*a*b*sqrt(-b/a) - a*b + b^2)/((a^6 + 2*a^5*b + a
^4*b^2)*sqrt(-b/a))) + a^2*b*sqrt(-(2*a*b*sqrt(-b/a) - a*b + b^2)/((a^6 + 2
*a^5*b + a^4*b^2)*sqrt(-b/a))) + 3*a)*sqrt(-b/a) - b)*a/((a^5 + a^4*b)*(a^3
 + a^2*b)*b) - 1/524288*(2*a^2*b*sqrt(-(2*a*b*sqrt(-b/a) - a*b + b^2)/((a^6
 + 2*a^5*b + a^4*b^2)*sqr ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \sinh^{8}{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**8),x)
```

```
[Out] Integral(1/(a + b*sinh(x)**8), x)
```

Giac [**A**]
time = 0.53, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^8),x, algorithm="giac")
```

```
[Out] 0
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x)^8),x)
```

```
[Out] \text{Hanged}
```

3.270 $\int \frac{1}{1+\sinh^5(x)}\,dx$

Optimal. Leaf size=242

$$-\frac{2(-1)^{3/5}\text{ArcTan}\left(\frac{1+(-1)^{3/5}\tanh\left(\frac{x}{2}\right)}{\sqrt{-1+\sqrt[5]{-1}}}\right)}{5\sqrt{-1+\sqrt[5]{-1}}}+\frac{2(-1)^{9/10}\text{ArcTan}\left(\frac{i-(-1)^{9/10}\tanh\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{4/5}}}\right)}{5\sqrt{1+(-1)^{4/5}}}-\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right.$$

```
[Out] -1/5*arctanh(1/2*(1-tanh(1/2*x))*2^(1/2))*2^(1/2)-2/5*(-1)^(3/5)*arctan((1+
(-1)^(3/5)*tanh(1/2*x))/(-1+(-1)^(1/5))^(1/2))/(-1+(-1)^(1/5))^(1/2)+2/5*(-
1)^(9/10)*arctanh((-1)^(7/10)*(1+(-1)^(1/5)*tanh(1/2*x))/(-(-1)^(2/5)*(1+(-
1)^(2/5)))^(1/2))/(-(-1)^(2/5)*(1+(-1)^(2/5)))^(1/2)-2/5*(-1)^(4/5)*arctanh
((1-(-1)^(4/5)*tanh(1/2*x))/(1-(-1)^(3/5))^(1/2))/(1-(-1)^(3/5))^(1/2)+2/5*
(-1)^(9/10)*arctan((I-(-1)^(9/10)*tanh(1/2*x))/(1+(-1)^(4/5))^(1/2))/(1+(-1
)^(4/5))^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3292, 2739, 632, 210, 212, 631}

$$-\frac{2(-1)^{3/5}\text{ArcTan}\left(\frac{(-1)^{3/5}\tanh\left(\frac{x}{2}\right)+1}{\sqrt{\sqrt[5]{-1}-1}}\right)}{5\sqrt{\sqrt[5]{-1}-1}}+\frac{2(-1)^{9/10}\text{ArcTan}\left(\frac{-(-1)^{9/10}\tanh\left(\frac{x}{2}\right)+i}{\sqrt{1+(-1)^{4/5}}}\right)}{5\sqrt{1+(-1)^{4/5}}}-\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{(-1)^{7/10}\left(\sqrt[5]{-1}\tanh\left(\frac{x}{2}\right)+1\right)}{\sqrt{-(-1)^{2/5}\left(1+(-1)^{2/5}\right)}}\right)}{5\sqrt{-(-1)^{2/5}\left(1+(-1)^{2/5}\right)}}-\frac{2(-1)^{4/5}\tanh^{-1}\left(\frac{1-(-1)^{4/5}\tanh\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{3/5}}}\right)}{5\sqrt{1-(-1)^{3/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^5)^(-1),x]
```

```
[Out] (-2*(-1)^(3/5)*ArcTan[(1 + (-1)^(3/5)*Tanh[x/2])/Sqrt[-1 + (-1)^(1/5)]])/(5
*Sqrt[-1 + (-1)^(1/5)]) + (2*(-1)^(9/10)*ArcTan[(I - (-1)^(9/10)*Tanh[x/2])
/Sqrt[1 + (-1)^(4/5)]])/(5*Sqrt[1 + (-1)^(4/5)]) - (Sqrt[2]*ArcTanh[(1 - Ta
nh[x/2])/Sqrt[2]])/5 + (2*(-1)^(9/10)*ArcTanh[((-1)^(7/10)*(1 + (-1)^(1/5)*
Tanh[x/2]))/Sqrt[-((-1)^(2/5)*(1 + (-1)^(2/5)))]])/(5*Sqrt[-((-1)^(2/5)*(1
+ (-1)^(2/5)))]) - (2*(-1)^(4/5)*ArcTanh[(1 - (-1)^(4/5)*Tanh[x/2])/Sqrt[1
- (-1)^(3/5)]])/(5*Sqrt[1 - (-1)^(3/5)])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\sinh^5(x)}\,dx &= \int \left(-\frac{(-1)^{9/10}}{5\left(-(-1)^{9/10}-i\sinh(x)\right)} - \frac{(-1)^{9/10}}{5\left(-(-1)^{9/10}-\sqrt[10]{-1}\,\sinh(x)\right)} - \frac{(-1}{5\left(-(-1)^{9/10}+(\right.}\right. \\
&= -\left(\frac{1}{5}(-1)^{9/10}\int \frac{1}{-(-1)^{9/10}-i\sinh(x)}\,dx\right) - \frac{1}{5}(-1)^{9/10}\int \frac{1}{-(-1)^{9/10}-\sqrt[10]{-1}\,\sinh(x} \\
&= -\left(\frac{1}{5}\left(2(-1)^{9/10}\right)\text{Subst}\left(\int \frac{1}{-(-1)^{9/10}-2ix+(-1)^{9/10}x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\right) - \frac{1}{5}(2(- \\
&= -\left(\frac{2}{5}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, 1-\tanh\left(\frac{x}{2}\right)\right)\right) + \frac{1}{5}\left(4(-1)^{9/10}\right)\text{Subst}\left(\int \frac{}{4(-1)^{3/5}\left(1-\right.}\right. \\
&= \frac{2(-1)^{9/10}\tan^{-1}\left(\frac{i-(-1)^{9/10}\tanh\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{4/5}}}\right)}{5\sqrt{1+(-1)^{4/5}}} - \frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - \frac{2\sqrt[10]{-1}\,\tanh^{}}{5\sqrt{}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.69, size = 439, normalized size = 1.81

$$\frac{1}{10}\left(2\sqrt{2}\tanh^{-1}\left(\frac{-1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\text{RootSum}\left[1+2\#1+2\#1^3+14\#1^4-2\#1^5-2\#1^7+\#1^8\&, \frac{-x-2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)-4x\#1-8\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1-9x\#1^2-18\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^2-24x\#1^3-48\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^3+9x\#1^4+18\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^4-4x\#1^5-8\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^5+x\#1^6+2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^6}{1+3\#1^2+28\#1^3-5\#1^4-7\#1^6+4\#1^7}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^5)^(-1),x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[(-1 + Tanh[x/2])/Sqrt[2]] - RootSum[1 + 2*#1 + 2*#1^3 +
14*#1^4 - 2*#1^5 - 2*#1^7 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + C
osh[x/2]*#1 - Sinh[x/2]*#1] - 4*x*#1 - 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[
x/2]*#1 - Sinh[x/2]*#1]*#1 - 9*x*#1^2 - 18*Log[-Cosh[x/2] - Sinh[x/2] + Cos
h[x/2]*#1 - Sinh[x/2]*#1]*#1^2 - 24*x*#1^3 - 48*Log[-Cosh[x/2] - Sinh[x/2]
+ Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^3 + 9*x*#1^4 + 18*Log[-Cosh[x/2] - Sinh[x
/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 - 4*x*#1^5 - 8*Log[-Cosh[x/2] - Sin
h[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^5 + x*#1^6 + 2*Log[-Cosh[x/2] - Si
nh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(1 + 3*#1^2 + 28*#1^3 - 5*#1^4
 - 7*#1^6 + 4*#1^7) & ])/10
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.67, size = 124, normalized size = 0.51

method	result

risch	$\Big(\sum_{\_R=\text{RootOf}(390625\_Z^8-31250\_Z^6+2500\_Z^4-75\_Z^2+1)} \_R\ln\big(-15625\_R^6+3125\_R^5+625\_R^4-125$
default	$\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{5}+\frac{2\Big(\sum_{\_R=\text{RootOf}(\_Z^8+2\_Z^7+2\_Z^5+14\_Z^4-2\_Z^3-2\_Z+1)}\frac{(-2\_R^6-3\_R^5+2\_R^4+}{4\_R^7+7\_}}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+2/5*sum((-2*_R^6-3*_R^5+
2*_R^4+2*_R^3-2*_R^2-3*_R+2)/(4*_R^7+7*_R^6+5*_R^4+28*_R^3-3*_R^2-1)*ln(tan
h(1/2*x)-_R),_R=RootOf(_Z^8+2*_Z^7+2*_Z^5+14*_Z^4-2*_Z^3-2*_Z+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^5),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(2)*log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) - integrate(2/5*
(e^(7*x) - 4*e^(6*x) + 9*e^(5*x) - 24*e^(4*x) - 9*e^(3*x) - 4*e^(2*x) - e^x
)/(e^(8*x) - 2*e^(7*x) - 2*e^(5*x) + 14*e^(4*x) + 2*e^(3*x) + 2*e^x + 1), x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3507 vs. $2(161) = 322$.
time = 0.53, size = 3507, normalized size = 14.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^5),x, algorithm="fricas")
```

```
[Out] -1/200*sqrt(2)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5)
 + 20)*(8*sqrt(5) + 24)^(1/4)*(3*sqrt(5) - 5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt
(5) + 3)*arctan(1/40*sqrt(2)*((11*sqrt(5) - 25)*e^x - 7*sqrt(5) + 15)*sqrt(
2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 1/80*sqrt(2)*(sqrt(2)*((11*sqrt(5) - 25)
*e^x + 4*sqrt(5) - 10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*((3*sqrt(5
) - 5)*e^x + 7*sqrt(5) - 15)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt(5) + 3) + 1/128
00*(80*sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 40*
sqrt(2)*(sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2
```

```
*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 3))*sqrt(sqrt(5) + 3) + sqrt(2*sqrt(2)*(2*s
qrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*(7*sqrt(5) - 15)*
sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5)
+ 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sqrt(2)*(17*sqrt(5) - 35)*sqrt(2*sqrt(5)
+ 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5
) + 24)^(1/4)) + 320*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 4))*sqrt(-20*sqrt(2)*sq
rt(sqrt(5) + 3)*(sqrt(5) - 3) + 40*(sqrt(5) - 1)*e^x - 2*(sqrt(2)*((2*sqrt(
5) - 5)*e^x - 3*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5) - 5)*e^x - 3*sq
rt(5) + 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 2
0)*(8*sqrt(5) + 24)^(1/4) + 80*e^(2*x) + 80) + 1/640*sqrt(2*sqrt(2)*(2*sqrt
(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*((3*sqrt(5) - 7)*e^x
 + 8*sqrt(5) - 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*((5*sqrt(5) -
11)*e^x + 2*sqrt(5) - 4)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 24)^(3/4) + 4*(s
qrt(2)*((7*sqrt(5) - 17)*e^x + 8*sqrt(5) - 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqr
t(5) + 3) + 2*((5*sqrt(5) - 11)*e^x + 5*sqrt(5) - 9)*sqrt(2*sqrt(5) + 5))*(
8*sqrt(5) + 24)^(1/4)) + 1/20*(2*(4*sqrt(5) - 5)*e^x - sqrt(5) + 5)*sqrt(2*
sqrt(5) + 5)) - 1/200*sqrt(2)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) +
 3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*(3*sqrt(5) - 5)*sqrt(2*sqrt(5)
 + 5)*sqrt(sqrt(5) + 3)*arctan(-1/40*sqrt(2)*((11*sqrt(5) - 25)*e^x - 7*sqr
t(5) + 15)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) - 1/80*sqrt(2)*(sqrt(2)*((
11*sqrt(5) - 25)*e^x + 4*sqrt(5) - 10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3
) + 2*((3*sqrt(5) - 5)*e^x + 7*sqrt(5) - 15)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt
(5) + 3) - 1/12800*(80*sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sq
rt(5) + 3) + 40*sqrt(2)*(sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(
sqrt(5) + 3) + 2*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 3))*sqrt(sqrt(5) + 3) - sqr
t(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*(
7*sqrt(5) - 15)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)
*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sqrt(2)*(17*sqrt(5) - 35)
*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5)
 + 5))*(8*sqrt(5) + 24)^(1/4)) + 320*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 4))*sqr
t(-20*sqrt(2)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) + 40*(sqrt(5) - 1)*e^x + 2*(s
qrt(2)*((2*sqrt(5) - 5)*e^x - 3*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5)
 - 5)*e^x - 3*sqrt(5) + 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3)
 - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 80*e^(2*x) + 80) + 1/640*sqrt(2
*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*((3*
sqrt(5) - 7)*e^x + 8*sqrt(5) - 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) +
2*((5*sqrt(5) - 11)*e^x + 2*sqrt(5) - 4)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) +
24)^(3/4) + 4*(sqrt(2)*((7*sqrt(5) - 17)*e^x + 8*sqrt(5) - 18)*sqrt(2*sqrt(
5) + 5)*sqrt(sqrt(5) + 3) + 2*((5*sqrt(5) - 11)*e^x + 5*sqrt(5) - 9)*sqrt(2
*sqrt(5) + 5))*(8*sqrt(5) + 24)^(1/4)) - 1/20*(2*(4*sqrt(5) - 5)*e^x - sqrt
(5) + 5)*sqrt(2*sqrt(5) + 5)) + 1/400*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5)
 + 24) + 4*sqrt(5) + 20)*(3*sqrt(5) + 5)*sqrt(-2*sqrt(5) + 5)*(-8*sqrt(5) +
 24)^(3/4)*arctan(-1/1280*((4*(5*sqrt(5) + 11)*e^x + ((3*sqrt(5) + 7)*e^x +
 8*sqrt(5) + 18)*sqrt(-8*sqrt(5) + 24) + 8*sqrt(5) + 16)*(-8*sqrt(5) + 24)^
(3/4) + 4*(4*(5*sqrt(5) + 11)*e^x + ((7*sqrt(5) + 17)*e^x + 8*sqrt(5) + 18)
```

```
*sqrt(-8*sqrt(5) + 24) + 20*sqrt(5) + 36)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2
*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5)
+ 1/25600*((((7*sqrt(5) + 15)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8
*sqrt(5) + 24)^(3/4) + 4*((17*sqrt(5) + 35)*sqrt(-8*sqrt(5) + 24) + 44*sqrt
(5) + 100)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) +
 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) + 20*(((5*sqrt(5) + 11)*sqrt(-8
*sqrt(5) + 24) + 4*sqrt(5) + 12)*sqrt(-8*sqrt(5) + 24) + 4*(5*sqrt(5) + 11)
*sqrt(-8*sqrt(5) + 24) + 32*sqrt(5) + 128)*sqrt(-2*sqrt(5) + 5))*sqrt(-40*(
sqrt(5) + 1)*e^x + (4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x - 3*sqrt(5)
- 5)*sqrt(-8*sqrt(5) + 24) - 6*sqrt(5) - 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*
sqrt(5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 10*(sqrt(5) + 3)*
sqrt(-8*sqrt(5) + 24) + 80*e^(2*x) + 80) - 1/320*(32*(4*sqrt(5) + 5)*e^x +
4*((11*sqrt(5) + 25)*e^x - 7*sqrt(5) - 15)*sqrt(-8*sqrt(5) + 24) + (4*(3*sq
rt(5) + 5)*e^x + ((11*sqrt(5) + 25)*e^x + 4*sqr...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sinh(x)+1)\left(\sinh^4(x)-\sinh^3(x)+\sinh^2(x)-\sinh(x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**5),x)
```

```
[Out] Integral(1/((sinh(x) + 1)*(sinh(x)**4 - sinh(x)**3 + sinh(x)**2 - sinh(x) +
 1)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4946 vs. 2(161) = 322.
time = 2.25, size = 4946, normalized size = 20.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^5),x, algorithm="giac")
```

```
[Out] 8/25*5^(3/4)*sqrt(-1/32*sqrt(5) + 5/64)*arctan(-5*(5^(3/4) + sqrt(5) + 5^(1
/4) - 4*e^x + 1)/(5^(3/4)*sqrt(-2*sqrt(5) + 5) + 5*sqrt(5)*sqrt(-2*sqrt(5)
+ 5) + 5*5^(1/4)*sqrt(-2*sqrt(5) + 5) + 5*sqrt(-2*sqrt(5) + 5))) - 8/25*5^(
3/4)*sqrt(-1/32*sqrt(5) + 5/64)*arctan(-5*(5^(3/4) - sqrt(5) + 5^(1/4) + 4*
e^x - 1)/(5^(3/4)*sqrt(-2*sqrt(5) + 5) - 5*sqrt(5)*sqrt(-2*sqrt(5) + 5) + 5
*5^(1/4)*sqrt(-2*sqrt(5) + 5) - 5*sqrt(-2*sqrt(5) + 5))) - 1/10*sqrt(sqrt(5
) + 2)*log((302427386195713850867712*sqrt(5)*(2*sqrt(5) + 5)^3 + 1728156492
54693629067264*(2*sqrt(5) + 5)^(7/2) + 226820539646785388150784*sqrt(5)*(2*
sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) + 151213693097856925433856*(2*sqrt(5)
+ 5)^3*sqrt(sqrt(5) + 2) + 708814186396204337971 20*sqrt(5)*(2*sqrt(5) + 5)^
```

```
2*(sqrt(5) + 2) + 567051349116963470376 96*(2*sqrt(5) + 5)^(5/2)*(sqrt(5) +
2) + 118135697732700722995 20*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3
/2) + 118135697732700722995 20*(2*sqrt(5) + 5)^2*(sqrt(5) + 2)^(3/2) + 11075
22166244069278080*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 + 147669622165875
9037440*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^2 + 553761083122034639 04*sqrt(5
)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 1107522166244069278 08*(2*sqrt(5
) + 5)*(sqrt(5) + 2)^(5/2) + 115366892317090549 8*sqrt(5)*(sqrt(5) + 2)^3 +
461467569268362199 2*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^3 + 82404923083636107
*(sqrt(5) + 2)^(7/2) - 622619531678741564620800*sqrt(5)*(2*sqrt(5) + 5)^(5/
2) - 415079687785827709747200*(2*sqrt(5) + 5)^3 - 389137207299213477888000*
sqrt(5)*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 311309765839370782310400*(2*s
qrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) - 97284301824803369472000*sqrt(5)*(2*sq
rt(5) + 5)^(3/2)*(sqrt(5) + 2) - 97284301824803369472000*(2*sqrt(5) + 5)^2*
(sqrt(5) + 2) - 12160537728100421184000*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) +
2)^(3/2) - 16214050304133894912000*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/2
) - 760033608006276324000*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 152
0067216012552648000*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 19000840200156908100*
sqrt(5)*(sqrt(5) + 2)^(5/2) - 76003360800627632400*sqrt(2*sqrt(5) + 5)*(sqr
t(5) + 2)^(5/2) - 1583403350013075675*(sqrt(5) + 2)^3 - 346430300390652274 6
101760*sqrt(5)*(2*sqrt(5) + 5)^2 - 2015373937635933569712128*(2*sqrt(5) + 5
)^(5/2) - 1732151501953261373050880*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt
(5) + 2) - 1259608711022458481070080*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) -
324778406616236507447040*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2) - 3149021777
55614620267520*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 270648672180197089539 2
0*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 39362772219451827533440
*(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 845777100563115904810*sqrt(5)*(sqrt(
5) + 2)^2 - 2460173263715739220840*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 61
504331592893480521*(sqrt(5) + 2)^(5/2) + 3959703717250098693214208*sqrt(5)*
(2*sqrt(5) + 5)^(3/2) + 2662579692919387100254208*(2*sqrt(5) + 5)^2 + 14848
88893968787009955328*sqrt(5)*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 1331289846
459693550127104*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5) + 2) + 18561111174609837
6244416*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2) + 249616846211192540648 83
2*(2*sqrt(5) + 5)*(sqrt(5) + 2) + 7733796322754099010184*sqrt(5)*(sqrt(5) +
 2)^(3/2) + 20801403850932711720736*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2)
 + 650043870341647241273*(sqrt(5) + 2)^2 + 1099194045690938228328 2816*sqrt(
5)*(2*sqrt(5) + 5) + 8567053742081103206220288*(2*sqrt(5) + 5)^(3/2) + 2747
985114227345570820704*sqrt(5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 32126
45153280413702332608*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 171749069639209098
176294*sqrt(5)*(sqrt(5) + 2) + 401580644160051712791576*sqrt(2*sqrt(5) + 5)
*(sqrt(5) + 2) + 16732526840002154699649*(sqrt(5) + 2)^(3/2) - 255726977589
9525489493536*sqrt(5)*sqrt(2*sqrt(5) + 5) - 3196587219874406861866 92*sqrt(5
)*sqrt(sqrt(5) + 2) + 3984277521156257144 2672*sqrt(2*sqrt(5) + 5)*sqrt(sqrt
(5) + 2) - 3308326863346966249269767*sqrt(5) - 4580301686563984868886360*sq
rt(2*sqrt(5) + 5) - 5725377108204981086 10795*sqrt(sqrt(5) + 2) + 2850824269
841065226382633)^2 + 64*(2432282250124078193049 6*sqrt(5)*(2*sqrt(5) + 5)^3
```

```
+ 13898755714994732531712*(2*sqrt(5) + 5)^(7/2) + 18242116875930586447872*s
qrt(5)*(2*sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) + 12161411250620390965248*(2
*sqrt(5) + 5)^3*sqrt(sqrt(5) + 2) + 5700661523728308264960*sqrt(5)*(2*sqrt(
5) + 5)^2*(sqrt(5) + 2) + 4560529218982646611968*(2*sqrt(5) + 5)^(5/2)*(sqr
t(5) + 2) + 950110253954718044160*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) +
2)^(3/2) + 950110253954718044160*(2*sqrt(5) + 5)^2*(sqrt(5) + 2)^(3/2) + 89
07283630825481664 0*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 + 11876378174433
9755520*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^2 + 4453641815412740832*sqrt(5)
*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 8907283630825481664*(2*sqrt(5) +
 5)*(sqrt(5) + 2)^(5/2) + 92784204487765434*sqrt(5)*(sqrt(5) + 2)^3 + 37113
6817951061736*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^3 + 6627443177697531*(sqrt(
5) + 2)^(7/2) - 13726081827177108602880*sqrt(5)...
```

Mupad [**F(-1)**]

time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^5 + 1),x)
```

```
[Out] \text{Hanged}
```

3.271 $\int \frac{1}{1+\sinh^6(x)}\,dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tanh(x)}{3}$$

```
[Out] 1/3*arctanh((1+(-1)^(1/3))^(1/2)*tanh(x))/(1+(-1)^(1/3))^(1/2)+1/3*arctanh(
(1-(-1)^(2/3))^(1/2)*tanh(x))/(1-(-1)^(2/3))^(1/2)+1/3*tanh(x)
```

Rubi [A]
time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3290, 3260, 212, 3254, 3852, 8}

$$\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tanh(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^6)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[1 + (-1)^(1/3)]*Tanh[x]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTanh[Sq
rt[1 - (-1)^(2/3)]*Tanh[x]]/(3*Sqrt[1 - (-1)^(2/3)]) + Tanh[x]/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sinh^6(x)}\,dx &= \frac{1}{3}\int \frac{1}{1+\sinh^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\,\sinh^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1+(-1)^{2/3}\sinh^2(x)}\,dx\\ &= \frac{1}{3}\int \operatorname{sech}^2(x)\,dx+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\left(1+\sqrt[3]{-1}\right)x^2}\,dx,x,\tanh(x)\right)+\frac{1}{3}\text{Subst}\Bigg(\int \frac{}{1-}\\ &= \frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{1}{3}i\text{Subst}(\int\\ &= \frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tanh(x)}{3}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 87, normalized size = 1.23

$$\frac{1}{18}\left(\sqrt[4]{-3}\left(\left(-3-i\sqrt{3}\right)\text{ArcTan}\left(\frac{1}{2}\sqrt[4]{-3}\left(1+i\sqrt{3}\right)\tanh(x)\right)-\left(3i+\sqrt{3}\right)\text{ArcTan}\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}\left(3+i\sqrt{3}\right)\tanh(x)\right)\right)+6\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^6)^(-1),x]
```

```
[Out] ((-3)^(1/4)*((-3 - I*Sqrt[3])*ArcTan[((-3)^(1/4)*(1 + I*Sqrt[3])*Tanh[x])/2
] - (3*I + Sqrt[3])*ArcTan[((-1/3)^(1/4)*(3 + I*Sqrt[3])*Tanh[x])/2]) + 6*T
anh[x])/18
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.59, size = 61, normalized size = 0.86

method	result	size
risch	$-\frac{2}{3(1+e^{2x})}+\left(\sum_{\_R=\operatorname{RootOf}(3888\_Z^4-108\_Z^2+1)}\_R\ln\left(1296\_R^3-216\_R^2+e^{2x}+1\right)\right)$	47
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4-3\_Z^2+1\right)}\_R\ln\left(\tanh^2\left(\frac{x}{2}\right)+\left(-6\_R^3+6\_R\right)\tanh\left(\frac{x}{2}\right)+1\right)\right)}{6}+\frac{2\tanh\left(\frac{x}{2}\right)}{3\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum(_R*ln(tanh(1/2*x)^2+(-6*_R^3+6*_R)*tanh(1/2*x)+1),_R=RootOf(3*_Z^4-
3*_Z^2+1))+2/3*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^6),x, algorithm="maxima")
```

```
[Out] -2/3/(e^(2*x) + 1) - integrate(4/3*(e^(6*x) - 10*e^(4*x) + e^(2*x))/(e^(8*x
) - 8*e^(6*x) + 30*e^(4*x) - 8*e^(2*x) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(49) = 98$.
time = 0.41, size = 692, normalized size = 9.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^6),x, algorithm="fricas")
```

```
[Out] -1/144*(4*(12^(1/4)*sqrt(6)*e^(2*x) + 12^(1/4)*sqrt(6))*sqrt(-4*sqrt(3) + 8
)*arctan((sqrt(3) + 2)*e^(2*x) + 1/216*sqrt(-6*(12^(1/4)*sqrt(6)*(sqrt(3) +
 3)*e^(2*x) - 12^(1/4)*sqrt(6)*(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*
sqrt(3) + 36*e^(4*x) - 144*e^(2*x) + 252)*((12^(3/4)*sqrt(6)*(sqrt(3) + 3)
+ 3*12^(1/4)*sqrt(6)*(sqrt(3) + 3))*sqrt(-4*sqrt(3) + 8) - 36*sqrt(3) - 72)
 - 2/3*sqrt(3)*(2*sqrt(3) - 3) - 1/36*(12^(3/4)*sqrt(6)*(sqrt(3) - 3) + (12
^(3/4)*sqrt(6)*(sqrt(3) + 3) + 3*12^(1/4)*sqrt(6)*(sqrt(3) + 3))*e^(2*x) +
3*12^(1/4)*sqrt(6)*(sqrt(3) - 3))*sqrt(-4*sqrt(3) + 8) - 2*sqrt(3) + 4) + 4
```

```
*(12^(1/4)*sqrt(6)*e^(2*x) + 12^(1/4)*sqrt(6))*sqrt(-4*sqrt(3) + 8)*arctan(
-(sqrt(3) + 2)*e^(2*x) + 1/216*sqrt(6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3)*e^(2*
x) - 12^(1/4)*sqrt(6)*(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sqrt(3) +
 36*e^(4*x) - 144*e^(2*x) + 252)*((12^(3/4)*sqrt(6)*(sqrt(3) + 3) + 3*12^(1
/4)*sqrt(6)*(sqrt(3) + 3))*sqrt(-4*sqrt(3) + 8) + 36*sqrt(3) + 72) + 2/3*sq
rt(3)*(2*sqrt(3) - 3) - 1/36*(12^(3/4)*sqrt(6)*(sqrt(3) - 3) + (12^(3/4)*sq
rt(6)*(sqrt(3) + 3) + 3*12^(1/4)*sqrt(6)*(sqrt(3) + 3))*e^(2*x) + 3*12^(1/4
)*sqrt(6)*(sqrt(3) - 3))*sqrt(-4*sqrt(3) + 8) + 2*sqrt(3) - 4) - (12^(1/4)*
sqrt(6)*(sqrt(3) + 2)*e^(2*x) + 12^(1/4)*sqrt(6)*(sqrt(3) + 2))*sqrt(-4*sqr
t(3) + 8)*log(6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3)*e^(2*x) - 12^(1/4)*sqrt(6)*
(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sqrt(3) + 36*e^(4*x) - 144*e^(2
*x) + 252) + (12^(1/4)*sqrt(6)*(sqrt(3) + 2)*e^(2*x) + 12^(1/4)*sqrt(6)*(sq
rt(3) + 2))*sqrt(-4*sqrt(3) + 8)*log(-6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3)*e^(
2*x) - 12^(1/4)*sqrt(6)*(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sqrt(3)
 + 36*e^(4*x) - 144*e^(2*x) + 252) + 96)/(e^(2*x) + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**6),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 10, normalized size = 0.14

$$-\frac{2}{3\,(e^{(2\,x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^6),x, algorithm="giac")
```

```
[Out] -2/3/(e^(2*x) + 1)
```

Mupad [**B**]
time = 4.21, size = 325, normalized size = 4.58

$$-\ln\left(\frac{1061158912\,e^{2x}}{27}+\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\,\left(\frac{548405248}{27}+\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\,\left(\frac{3870294016}{9}+\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\,\left(19788726272\,e^{2x}-2864709632\right)-\frac{21515730944\,e^{2x}}{9}\right)-\frac{2539651072\,e^{2x}}{9}\right)-\frac{351797248}{81}\right)\,\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}-\ln\left(\frac{1061158912\,e^{2x}}{27}+\sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}\,\left(\frac{548405248}{27}+\sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}\,\left(\frac{3870294016}{9}+\sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}\,\left(19788726272\,e^{2x}-2864709632\right)-\frac{21515730944\,e^{2x}}{9}\right)-\frac{2539651072\,e^{2x}}{9}\right)-\frac{351797248}{81}\right)\,\sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}+\ln\left(\frac{1061158912\,e^{2x}}{27}-\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\,\left(\frac{548405248}{27}+\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\,\left(\frac{21515730944\,e^{2x}}{9}+\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\,\left(19788726272\,e^{2x}-2864709632\right)-\frac{3870294016}{9}\right)-\frac{2539651072\,e^{2x}}{9}\right)-\frac{351797248}{81}\right)\,\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}+\ln\left(\frac{1061158912\,e^{2x}}{27}-\sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}\,\left(\frac{548405248}{27}+\sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}\,\left(\frac{21515730944\,e^{2x}}{9}+\sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}\,\left(19788726272\,e^{2x}-2864709632\right)-\frac{3870294016}{9}\right)-\frac{2539651072\,e^{2x}}{9}\right)-\frac{351797248}{81}\right)\,\sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}-\frac{2}{3\,(e^{2x}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^6 + 1),x)
```

```
[Out] log((1061158912*exp(2*x))/27 - (1/72 - (3^(1/2)*1i)/216)^(1/2)*((1/72 - (3^
(1/2)*1i)/216)^(1/2)*((21515730944*exp(2*x))/9 + (1/72 - (3^(1/2)*1i)/216)^
```

```
(1/2)*(19788726272*exp(2*x) - 2864709632) - 3870294016/9) - (2539651072*exp
(2*x))/9 + 548405248/27) - 351797248/81)*(1/72 - (3^(1/2)*1i)/216)^(1/2) -
log((1061158912*exp(2*x))/27 + ((3^(1/2)*1i)/216 + 1/72)^(1/2)*(((3^(1/2)*1
i)/216 + 1/72)^(1/2)*(((3^(1/2)*1i)/216 + 1/72)^(1/2)*(19788726272*exp(2*x)
 - 2864709632) - (21515730944*exp(2*x))/9 + 3870294016/9) - (2539651072*exp
(2*x))/9 + 548405248/27) - 351797248/81)*((3^(1/2)*1i)/216 + 1/72)^(1/2) -
log((1061158912*exp(2*x))/27 + (1/72 - (3^(1/2)*1i)/216)^(1/2)*((1/72 - (3^
(1/2)*1i)/216)^(1/2)*((1/72 - (3^(1/2)*1i)/216)^(1/2)*(19788726272*exp(2*x)
 - 2864709632) - (21515730944*exp(2*x))/9 + 3870294016/9) - (2539651072*exp
(2*x))/9 + 548405248/27) - 351797248/81)*(1/72 - (3^(1/2)*1i)/216)^(1/2) +
log((1061158912*exp(2*x))/27 - ((3^(1/2)*1i)/216 + 1/72)^(1/2)*(((3^(1/2)*1
i)/216 + 1/72)^(1/2)*((21515730944*exp(2*x))/9 + ((3^(1/2)*1i)/216 + 1/72)^
(1/2)*(19788726272*exp(2*x) - 2864709632) - 3870294016/9) - (2539651072*exp
(2*x))/9 + 548405248/27) - 351797248/81)*((3^(1/2)*1i)/216 + 1/72)^(1/2) -
2/(3*(exp(2*x) + 1))
```

3.272 $\int \frac{1}{1+\sinh^8(x)}\,dx$

Optimal. Leaf size=129

$$\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\,\tanh(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\,\tanh(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{3/4}}\,\tanh(x)\right)}{4\sqrt{1-(-1)^{3/4}}}+$$

```
[Out] 1/4*arctanh((1-(-1)^(1/4))^(1/2)*tanh(x))/(1-(-1)^(1/4))^(1/2)+1/4*arctanh(
(1+(-1)^(1/4))^(1/2)*tanh(x))/(1+(-1)^(1/4))^(1/2)+1/4*arctanh((1-(-1)^(3/4
))^(1/2)*tanh(x))/(1-(-1)^(3/4))^(1/2)+1/4*arctanh((1+(-1)^(3/4))^(1/2)*tan
h(x))/(1+(-1)^(3/4))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3290, 3260, 212}

$$\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\,\tanh(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\,\tanh(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{3/4}}\,\tanh(x)\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\tanh^{-1}\left(\sqrt{1+(-1)^{3/4}}\,\tanh(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^8)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[1 - (-1)^(1/4)]*Tanh[x]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTanh[Sq
rt[1 + (-1)^(1/4)]*Tanh[x]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTanh[Sqrt[1 - (-1
)^(3/4)]*Tanh[x]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTanh[Sqrt[1 + (-1)^(3/4)]*T
anh[x]]/(4*Sqrt[1 + (-1)^(3/4)])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
```

```
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{1+\sinh^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\,\sinh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt[4]{-1}\,\sinh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\sinh^2(x)}$$

$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\left(1-\sqrt[4]{-1}\right)x^2}\,dx, x, \tanh(x)\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\left(1+\sqrt[4]{-1}\right)x^2}\,dx,\right.$$

$$= \frac{\tanh^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\,\tanh(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\,\tanh(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.10, size = 127, normalized size = 0.98

$$16\text{RootSum}\left[1-8\#1+28\#1^2-56\#1^3+326\#1^4-56\#1^5+28\#1^6-8\#1^7+\#1^8\&, \frac{x\#1^3+\log(-\cosh(x)-\sinh(x)+\cosh(x)\#1-\sinh(x)\#1)\#1^3}{-1+7\#1-21\#1^2+163\#1^3-35\#1^4+21\#1^5-7\#1^6+\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^8)^(-1),x]
```

```
[Out] 16*RootSum[1 - 8*#1 + 28*#1^2 - 56*#1^3 + 326*#1^4 - 56*#1^5 + 28*#1^6 - 8*
#1^7 + #1^8 & , (x*#1^3 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x]*#1]
*#1^3)/(-1 + 7*#1 - 21*#1^2 + 163*#1^3 - 35*#1^4 + 21*#1^5 - 7*#1^6 + #1^7)
 & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.61, size = 64, normalized size = 0.50

method	result
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(2\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+1\right)}\_R\ln\left(\tanh^2\left(\frac{x}{2}\right)+\left(-4\_R^7+8\_R^5-12\_R^3+8\_R\right)\tanh\left(\frac{x}{2}\right)+1\right)\right)}{8}$
risch	$\sum_{\_R=\text{RootOf}\left(33554432\_Z^8-1048576\_Z^6+24576\_Z^4-256\_Z^2+1\right)}\_R\ln\left(8388608\_R^7-1048576\_R^6-131072\_\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^8),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum(_R*ln(tanh(1/2*x)^2+(-4*_R^7+8*_R^5-12*_R^3+8*_R)*tanh(1/2*x)+1),_R
=RootOf(2*_Z^8-4*_Z^6+6*_Z^4-4*_Z^2+1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(sinh(x)^8 + 1), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3773 vs. $2(89) = 178$.
time = 0.49, size = 3773, normalized size = 29.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^8),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2)
 + 4)^(3/4)*sqrt(2*sqrt(2) + 3)*(sqrt(2) - 1)*arctan(1/31*(2*(13*sqrt(2) -
20)*e^(2*x) + 23*sqrt(2) - 33)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 1/
496*(32*(10*sqrt(2) - 13)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + (((355*
sqrt(2) - 508)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 6*(59*sqrt(2) - 86
)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(3/4) + 4*((82*sqrt(2) - 119)*sqrt(2
*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + (85*sqrt(2) - 126)*sqrt(2*sqrt(2) + 3))
*(2*sqrt(2) + 4)^(1/4))*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt
(2) + 8) + 4*((76*sqrt(2) - 105)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) +
2*(53*sqrt(2) - 72)*sqrt(2*sqrt(2) + 3))*sqrt(2*sqrt(2) + 4) + 16*(23*sqrt(
2) - 33)*sqrt(2*sqrt(2) + 3))*sqrt(4*(sqrt(2) - 1)*e^(2*x) - (2*(sqrt(2) -
1)*e^(2*x) + ((sqrt(2) - 2)*e^(2*x) - 5*sqrt(2) + 6)*sqrt(2*sqrt(2) + 4) -
6*sqrt(2) + 6)*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*
(2*sqrt(2) + 4)^(1/4) - 4*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2) - 4*sqrt(2) + 2
*e^(4*x) + 10) + 1/248*((((254*sqrt(2) - 355)*e^(2*x) - 102*sqrt(2) + 145)*
sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 2*(3*(43*sqrt(2) - 59)*e^(2*x) -
23*sqrt(2) + 33)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt
(2) - 164)*e^(2*x) - 39*sqrt(2) + 60)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) +
3) + 2*((63*sqrt(2) - 85)*e^(2*x) - 17*sqrt(2) + 19)*sqrt(2*sqrt(2) + 3))*(
2*sqrt(2) + 4)^(1/4))*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2
) + 8) + 1/124*(((105*sqrt(2) - 152)*e^(2*x) + 13*sqrt(2) - 20)*sqrt(2*sqrt
(2) + 4)*sqrt(2*sqrt(2) + 3) + 4*((36*sqrt(2) - 53)*e^(2*x) - 23*sqrt(2) +
33)*sqrt(2*sqrt(2) + 3))*sqrt(2*sqrt(2) + 4) + 1/31*((33*sqrt(2) - 46)*e^(2
*x) - 3*sqrt(2) + 7)*sqrt(2*sqrt(2) + 3)) + 1/16*sqrt(2*sqrt(2*sqrt(2) + 4)
```

```
*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2) + 4)^(3/4)*sqrt(2*sqrt(2) + 3)
*(sqrt(2) - 1)*arctan(-1/31*(2*(13*sqrt(2) - 20)*e^(2*x) + 23*sqrt(2) - 33)
*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) - 1/496*(32*(10*sqrt(2) - 13)*sqrt
(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) - (((355*sqrt(2) - 508)*sqrt(2*sqrt(2)
 + 4)*sqrt(2*sqrt(2) + 3) + 6*(59*sqrt(2) - 86)*sqrt(2*sqrt(2) + 3))*(2*sqrt
(2) + 4)^(3/4) + 4*((82*sqrt(2) - 119)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) +
 3) + (85*sqrt(2) - 126)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(1/4))*sqrt(2
*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8) + 4*((76*sqrt(2) - 10
5)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 2*(53*sqrt(2) - 72)*sqrt(2*sqr
t(2) + 3))*sqrt(2*sqrt(2) + 4) + 16*(23*sqrt(2) - 33)*sqrt(2*sqrt(2) + 3))*
sqrt(4*(sqrt(2) - 1)*e^(2*x) + (2*(sqrt(2) - 1)*e^(2*x) + ((sqrt(2) - 2)*e^
(2*x) - 5*sqrt(2) + 6)*sqrt(2*sqrt(2) + 4) - 6*sqrt(2) + 6)*sqrt(2*sqrt(2*s
qrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2) + 4)^(1/4) - 4*sqrt
(2*sqrt(2) + 4)*(sqrt(2) - 2) - 4*sqrt(2) + 2*e^(4*x) + 10) + 1/248*((((254
*sqrt(2) - 355)*e^(2*x) - 102*sqrt(2) + 145)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqr
t(2) + 3) + 2*(3*(43*sqrt(2) - 59)*e^(2*x) - 23*sqrt(2) + 33)*sqrt(2*sqrt(2
) + 3))*(2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt(2) - 164)*e^(2*x) - 39*sqrt(2
) + 60)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 2*((63*sqrt(2) - 85)*e^(2
*x) - 17*sqrt(2) + 19)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(1/4))*sqrt(2*s
qrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8) - 1/124*(((105*sqrt(2)
 - 152)*e^(2*x) + 13*sqrt(2) - 20)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) +
 4*((36*sqrt(2) - 53)*e^(2*x) - 23*sqrt(2) + 33)*sqrt(2*sqrt(2) + 3))*sqrt(
2*sqrt(2) + 4) - 1/31*((33*sqrt(2) - 46)*e^(2*x) - 3*sqrt(2) + 7)*sqrt(2*sq
rt(2) + 3)) - 1/16*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2)
 + 8)*(sqrt(2) + 1)*(-2*sqrt(2) + 4)^(3/4)*sqrt(-2*sqrt(2) + 3)*arctan(1/31
*(2*(13*sqrt(2) + 20)*e^(2*x) + 23*sqrt(2) + 33)*sqrt(-2*sqrt(2) + 4)*sqrt(
-2*sqrt(2) + 3) - 1/496*(32*(10*sqrt(2) + 13)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*
sqrt(2) + 3) - (((355*sqrt(2) + 508)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) +
 3) + 6*(59*sqrt(2) + 86)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(3/4) + 4*
((82*sqrt(2) + 119)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + (85*sqrt(2)
 + 126)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(1/4))*sqrt(-2*(2*sqrt(2) +
3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8) + 4*((76*sqrt(2) + 105)*sqrt(-2*sq
rt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*(53*sqrt(2) + 72)*sqrt(-2*sqrt(2) + 3))
*sqrt(-2*sqrt(2) + 4) + 16*(23*sqrt(2) + 33)*sqrt(-2*sqrt(2) + 3))*sqrt(-4*
(sqrt(2) + 1)*e^(2*x) - (2*(sqrt(2) + 1)*e^(2*x) + ((sqrt(2) + 2)*e^(2*x) -
 5*sqrt(2) - 6)*sqrt(-2*sqrt(2) + 4) - 6*sqrt(2) - 6)*sqrt(-2*(2*sqrt(2) +
3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*(-2*sqrt(2) + 4)^(1/4) + 4*(sqrt(2
) + 2)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 2*e^(4*x) + 10) - 1/248*((((254*s
qrt(2) + 355)*e^(2*x) - 102*sqrt(2) - 145)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqr
t(2) + 3) + 2*(3*(43*sqrt(2) + 59)*e^(2*x) - 23*sqrt(2) - 33)*sqrt(-2*sqrt(
2) + 3))*(-2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt(2) + 164)*e^(2*x) - 39*sqrt
(2) - 60)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*((63*sqrt(2) + 85)*
e^(2*x) - 17*sqrt(2) - 19)*sqrt(-2*sqrt(2) + 3)...
```

Sympy **[F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh^8(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**8),x)
```

```
[Out] Integral(1/(sinh(x)**8 + 1), x)
```

Giac [A]
time = 0.42, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^8),x, algorithm="giac")
```

```
[Out] 0
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

$$\text{Hanged}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^8 + 1),x)
```

```
[Out] \text{Hanged}
```

3.273 $\int \frac{1}{1-\sinh^5(x)}\,dx$

Optimal. Leaf size=228

$$-\frac{2\sqrt[10]{-1}\,\text{ArcTan}\left(\frac{i+\sqrt[10]{-1}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}-\frac{2\tanh^{-1}\left(\frac{(-1)^{3/5}-\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}+\frac{1}{5}\sqrt{2}\,\tanh^{-1}\left(\frac{1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+$$

```
[Out] 1/5*arctanh(1/2*(1+tanh(1/2*x))*2^(1/2))*2^(1/2)-2/5*(-1)^(1/10)*arctan((I+
(-1)^(1/10)*tanh(1/2*x))/(1-(-1)^(1/5))^(1/2))/(1-(-1)^(1/5))^(1/2)-2/5*arc
tanh(((-1)^(3/5)-tanh(1/2*x))/(1-(-1)^(1/5))^(1/2))/(1-(-1)^(1/5))^(1/2)+2/
5*arctanh(((-1)^(4/5)+tanh(1/2*x))/(1-(-1)^(3/5))^(1/2))/(1-(-1)^(3/5))^(1/
2)-2/5*(-1)^(1/10)*arctanh((-1)^(3/10)*(1+(-1)^(4/5)*tanh(1/2*x))/((-1)^(1/
5)+(-1)^(3/5))^(1/2))/((-1)^(1/5)+(-1)^(3/5))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3292, 2739, 632, 210, 631, 212}

$$-\frac{2\sqrt[10]{-1}\,\text{ArcTan}\left(\frac{\sqrt[10]{-1}\,\tanh\left(\frac{x}{2}\right)+i}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}-\frac{2\tanh^{-1}\left(\frac{(-1)^{3/5}-\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}+\frac{1}{5}\sqrt{2}\,\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)+\frac{2\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+(-1)^{4/5}}{\sqrt{1-(-1)^{3/5}}}\right)}{5\sqrt{1-(-1)^{3/5}}}-\frac{2\sqrt[10]{-1}\,\tanh^{-1}\left(\frac{(-1)^{3/10}\left((-1)^{4/5}\tanh\left(\frac{x}{2}\right)+1\right)}{\sqrt{\sqrt[5]{-1}+(-1)^{3/5}}}\right)}{5\sqrt{\sqrt[5]{-1}+(-1)^{3/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^5)^(-1),x]
```

```
[Out] (-2*(-1)^(1/10)*ArcTan[(I + (-1)^(1/10)*Tanh[x/2])/Sqrt[1 - (-1)^(1/5)]])/(
5*Sqrt[1 - (-1)^(1/5)]) - (2*ArcTanh[((-1)^(3/5) - Tanh[x/2])/Sqrt[1 - (-1)
^(1/5)]])/(5*Sqrt[1 - (-1)^(1/5)]) + (Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[
2]])/5 + (2*ArcTanh[((-1)^(4/5) + Tanh[x/2])/Sqrt[1 - (-1)^(3/5)]])/(5*Sqrt
[1 - (-1)^(3/5)]) - (2*(-1)^(1/10)*ArcTanh[((-1)^(3/10)*(1 + (-1)^(4/5)*Tan
h[x/2]))/Sqrt[(-1)^(1/5) + (-1)^(3/5)]])/(5*Sqrt[(-1)^(1/5) + (-1)^(3/5)])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sinh^5(x)}\,dx &= \int \left(\frac{\sqrt[10]{-1}}{5\left(\sqrt[10]{-1}-i\sinh(x)\right)}+\frac{\sqrt[10]{-1}}{5\left(\sqrt[10]{-1}-\sqrt[10]{-1}\sinh(x)\right)}+\frac{\sqrt[10]{-1}}{5\left(\sqrt[10]{-1}+(-1)^{3/10}\sinh\right.}\right.\\
&=\frac{1}{5}\sqrt[10]{-1}\int \frac{1}{\sqrt[10]{-1}-i\sinh(x)}\,dx+\frac{1}{5}\sqrt[10]{-1}\int \frac{1}{\sqrt[10]{-1}-\sqrt[10]{-1}\sinh(x)}\,dx+\frac{1}{5}\sqrt[10]{-1}\\
&=\frac{1}{5}\left(2\sqrt[10]{-1}\right)\text{Subst}\left(\int \frac{1}{\sqrt[10]{-1}-2ix-\sqrt[10]{-1}x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{5}\left(2\sqrt[10]{-1}\right)\text{Subst}\\
&=\frac{2}{5}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,1+\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{5}\left(4\sqrt[10]{-1}\right)\text{Subst}\left(\int \frac{1}{-4\left(1-\sqrt[5]{-1}\right)-x}\right.\\
&=-\frac{2\sqrt[10]{-1}\tan^{-1}\left(\frac{i+\sqrt[10]{-1}\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}-\frac{2\tanh^{-1}\left(\frac{(-1)^{3/5}-\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}+\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{1}{}\right.\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.69, size = 437, normalized size = 1.92

$$\frac{1}{10}\left(2\sqrt{2}\tanh^{-1}\left(\frac{1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\text{RootSum}\left[1-2\#1-2\#1^3+14\#1^4+2\#1^5+2\#1^7+\#1^8\&,\frac{-x-2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)+4x\#1+8\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1-9x\#1^2-18\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^2+24x\#1^3+48\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^3+9x\#1^4+18\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^4+4x\#1^5+8\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^5+x\#1^6+2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^6}{-1-3\#1^2+28\#1^3+5\#1^4+7\#1^6+4\#1^7}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^5)^(-1),x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[2]] + RootSum[1 - 2*#1 - 2*#1^3 + 1
4*#1^4 + 2*#1^5 + 2*#1^7 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Co
sh[x/2]*#1 - Sinh[x/2]*#1] + 4*x*#1 + 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x
/2]*#1 - Sinh[x/2]*#1]*#1 - 9*x*#1^2 - 18*Log[-Cosh[x/2] - Sinh[x/2] + Cosh
[x/2]*#1 - Sinh[x/2]*#1]*#1^2 + 24*x*#1^3 + 48*Log[-Cosh[x/2] - Sinh[x/2] +
 Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^3 + 9*x*#1^4 + 18*Log[-Cosh[x/2] - Sinh[x/
2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 + 4*x*#1^5 + 8*Log[-Cosh[x/2] - Sinh
[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^5 + x*#1^6 + 2*Log[-Cosh[x/2] - Sin
h[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-1 - 3*#1^2 + 28*#1^3 + 5*#1^4
 + 7*#1^6 + 4*#1^7) & ])/10
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.70, size = 124, normalized size = 0.54

method	result

risch	$\Bigg(\sum_{\_R=\operatorname{RootOf}(390625\_Z^8-31250\_Z^6+2500\_Z^4-75\_Z^2+1)} \_R\ln\left(15625\_R^6-3125\_R^5-625\_R^4+125\_\right.$
default	$\frac{2\left(\sum_{\_R=\operatorname{RootOf}(\_Z^8-2\_Z^7-2\_Z^5+14\_Z^4+2\_Z^3+2\_Z+1)} \frac{(-2\_R^6+3\_R^5+2\_R^4-2\_R^3-2\_R^2+3\_R+2)\ln\left(\tanh\left(\frac{x}{2}\right)-\right.}{4\_R^7-7\_R^6-5\_R^4+28\_R^3+3\_R^2+1}\right.}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*sum((-2*_R^6+3*_R^5+2*_R^4-2*_R^3-2*_R^2+3*_R+2)/(4*_R^7-7*_R^6-5*_R^4+
28*_R^3+3*_R^2+1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^8-2*_Z^7-2*_Z^5+14*_Z^4+2
*_Z^3+2*_Z+1))+1/5*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^5),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(2)*log(-(sqrt(2) - e^x + 1)/(sqrt(2) + e^x - 1)) + integrate(2/5
*(e^(7*x) + 4*e^(6*x) + 9*e^(5*x) + 24*e^(4*x) - 9*e^(3*x) + 4*e^(2*x) - e^
x)/(e^(8*x) + 2*e^(7*x) + 2*e^(5*x) + 14*e^(4*x) - 2*e^(3*x) - 2*e^x + 1),
x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3500 vs. $2(153) = 306$.
time = 0.54, size = 3500, normalized size = 15.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^5),x, algorithm="fricas")
```

```
[Out] 1/200*sqrt(2)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5)
+ 20)*(8*sqrt(5) + 24)^(1/4)*(3*sqrt(5) - 5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(
5) + 3)*arctan(1/40*sqrt(2)*((11*sqrt(5) - 25)*e^x + 7*sqrt(5) - 15)*sqrt(2
*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 1/80*sqrt(2)*(sqrt(2)*((11*sqrt(5) - 25)*
e^x - 4*sqrt(5) + 10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*((3*sqrt(5)
 - 5)*e^x - 7*sqrt(5) + 15)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt(5) + 3) + 1/1280
0*(80*sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 40*s
qrt(2)*(sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*
```

```
sqrt(2*sqrt(5) + 5)*(sqrt(5) - 3))*sqrt(sqrt(5) + 3) + sqrt(2*sqrt(2)*(2*sq
rt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*(7*sqrt(5) - 15)*s
qrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5) +
 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sqrt(2)*(17*sqrt(5) - 35)*sqrt(2*sqrt(5) +
 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5)
 + 24)^(1/4)) + 320*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 4))*sqrt(-20*sqrt(2)*sqr
t(sqrt(5) + 3)*(sqrt(5) - 3) - 40*(sqrt(5) - 1)*e^x + 2*(sqrt(2)*((2*sqrt(5
) - 5)*e^x + 3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5) - 5)*e^x + 3*sqr
t(5) - 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20
)*(8*sqrt(5) + 24)^(1/4) + 80*e^(2*x) + 80) + 1/640*sqrt(2*sqrt(2)*(2*sqrt(
5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*((3*sqrt(5) - 7)*e^x
- 8*sqrt(5) + 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*((5*sqrt(5) - 1
1)*e^x - 2*sqrt(5) + 4)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sq
rt(2)*((7*sqrt(5) - 17)*e^x - 8*sqrt(5) + 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt
(5) + 3) + 2*((5*sqrt(5) - 11)*e^x - 5*sqrt(5) + 9)*sqrt(2*sqrt(5) + 5))*(8
*sqrt(5) + 24)^(1/4)) + 1/20*(2*(4*sqrt(5) - 5)*e^x + sqrt(5) - 5)*sqrt(2*s
qrt(5) + 5)) + 1/200*sqrt(2)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) +
3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*(3*sqrt(5) - 5)*sqrt(2*sqrt(5)
+ 5)*sqrt(sqrt(5) + 3)*arctan(-1/40*sqrt(2)*((11*sqrt(5) - 25)*e^x + 7*sqrt
(5) - 15)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) - 1/80*sqrt(2)*(sqrt(2)*((1
1*sqrt(5) - 25)*e^x - 4*sqrt(5) + 10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3)
 + 2*((3*sqrt(5) - 5)*e^x - 7*sqrt(5) + 15)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt(
5) + 3) - 1/12800*(80*sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqr
t(5) + 3) + 40*sqrt(2)*(sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(s
qrt(5) + 3) + 2*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 3))*sqrt(sqrt(5) + 3) - sqrt
(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*(7
*sqrt(5) - 15)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*
sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sqrt(2)*(17*sqrt(5) - 35)*
sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5)
+ 5))*(8*sqrt(5) + 24)^(1/4)) + 320*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 4))*sqrt
(-20*sqrt(2)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) - 40*(sqrt(5) - 1)*e^x - 2*(sq
rt(2)*((2*sqrt(5) - 5)*e^x + 3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5)
- 5)*e^x + 3*sqrt(5) - 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3)
- 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 80*e^(2*x) + 80) + 1/640*sqrt(2*
sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*((3*s
qrt(5) - 7)*e^x - 8*sqrt(5) + 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2
*((5*sqrt(5) - 11)*e^x - 2*sqrt(5) + 4)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 2
4)^(3/4) + 4*(sqrt(2)*((7*sqrt(5) - 17)*e^x - 8*sqrt(5) + 18)*sqrt(2*sqrt(5
) + 5)*sqrt(sqrt(5) + 3) + 2*((5*sqrt(5) - 11)*e^x - 5*sqrt(5) + 9)*sqrt(2*
sqrt(5) + 5))*(8*sqrt(5) + 24)^(1/4)) - 1/20*(2*(4*sqrt(5) - 5)*e^x + sqrt(
5) - 5)*sqrt(2*sqrt(5) + 5)) - 1/400*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5)
+ 24) + 4*sqrt(5) + 20)*(3*sqrt(5) + 5)*sqrt(-2*sqrt(5) + 5)*(-8*sqrt(5) +
24)^(3/4)*arctan(-1/1280*((4*(5*sqrt(5) + 11)*e^x + ((3*sqrt(5) + 7)*e^x -
8*sqrt(5) - 18)*sqrt(-8*sqrt(5) + 24) - 8*sqrt(5) - 16)*(-8*sqrt(5) + 24)^(
3/4) + 4*(4*(5*sqrt(5) + 11)*e^x + ((7*sqrt(5) + 17)*e^x - 8*sqrt(5) - 18)*
```

```
sqrt(-8*sqrt(5) + 24) - 20*sqrt(5) - 36)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*
sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) +
 1/25600*((((7*sqrt(5) + 15)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8*
sqrt(5) + 24)^(3/4) + 4*((17*sqrt(5) + 35)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(
5) + 100)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) +
24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) - 20*(((5*sqrt(5) + 11)*sqrt(-8*
sqrt(5) + 24) + 4*sqrt(5) + 12)*sqrt(-8*sqrt(5) + 24) + 4*(5*sqrt(5) + 11)*
sqrt(-8*sqrt(5) + 24) + 32*sqrt(5) + 128)*sqrt(-2*sqrt(5) + 5))*sqrt(40*(sq
rt(5) + 1)*e^x + (4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x + 3*sqrt(5) +
5)*sqrt(-8*sqrt(5) + 24) + 6*sqrt(5) + 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sq
rt(5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 10*(sqrt(5) + 3)*sq
rt(-8*sqrt(5) + 24) + 80*e^(2*x) + 80) + 1/320*(32*(4*sqrt(5) + 5)*e^x + 4*
((11*sqrt(5) + 25)*e^x + 7*sqrt(5) + 15)*sqrt(-8*sqrt(5) + 24) + (4*(3*sqrt
(5) + 5)*e^x + ((11*sqrt(5) + 25)*e^x - 4*sqrt(...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sinh^5(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**5),x)
```

```
[Out] -Integral(1/(sinh(x)**5 - 1), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4948 vs. 2(153) = 306.
time = 2.25, size = 4948, normalized size = 21.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^5),x, algorithm="giac")
```

```
[Out] -8/25*5^(3/4)*sqrt(-1/32*sqrt(5) + 5/64)*arctan(5*(5^(3/4) + sqrt(5) + 5^(1
/4) + 4*e^x + 1)/(5^(3/4)*sqrt(-2*sqrt(5) + 5) + 5*sqrt(5)*sqrt(-2*sqrt(5)
+ 5) + 5*5^(1/4)*sqrt(-2*sqrt(5) + 5) + 5*sqrt(-2*sqrt(5) + 5))) + 8/25*5^(
3/4)*sqrt(-1/32*sqrt(5) + 5/64)*arctan(5*(5^(3/4) - sqrt(5) + 5^(1/4) - 4*e
^x - 1)/(5^(3/4)*sqrt(-2*sqrt(5) + 5) - 5*sqrt(5)*sqrt(-2*sqrt(5) + 5) + 5*
5^(1/4)*sqrt(-2*sqrt(5) + 5) - 5*sqrt(-2*sqrt(5) + 5))) - 1/10*sqrt(sqrt(5)
 + 2)*log((302427386195713850867712*sqrt(5)*(2*sqrt(5) + 5)^3 + 17281564925
4693629067264*(2*sqrt(5) + 5)^(7/2) + 226820539646785388150784*sqrt(5)*(2*s
qrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) + 151213693097856925433856*(2*sqrt(5) +
 5)^3*sqrt(sqrt(5) + 2) + 70881418639620433797120*sqrt(5)*(2*sqrt(5) + 5)^2
*(sqrt(5) + 2) + 56705134911696347037696*(2*sqrt(5) + 5)^(5/2)*(sqrt(5) + 2
```

```
) + 11813569773270072299520*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/
2) + 11813569773270072299520*(2*sqrt(5) + 5)^2*(sqrt(5) + 2)^(3/2) + 110752
2166244069278080*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 + 1476696221658759
037440*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^2 + 55376108312203463904*sqrt(5)
*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 110752216624406927808*(2*sqrt(5)
 + 5)*(sqrt(5) + 2)^(5/2) + 1153668923170905498*sqrt(5)*(sqrt(5) + 2)^3 + 4
614675692683621992*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^3 + 82404923083636107*
(sqrt(5) + 2)^(7/2) - 622619531678741564620800*sqrt(5)*(2*sqrt(5) + 5)^(5/2
) - 415079687785827709747200*(2*sqrt(5) + 5)^3 - 389137207299213477888000*s
qrt(5)*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 311309765839370782310400*(2*sq
rt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) - 97284301824803369472000*sqrt(5)*(2*sqr
t(5) + 5)^(3/2)*(sqrt(5) + 2) - 97284301824803369472000*(2*sqrt(5) + 5)^2*(
sqrt(5) + 2) - 12160537728100421184000*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2
)^(3/2) - 16214050304133894912000*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/2)
 - 760033608006276324000*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 1520
067216012552648000*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 19000840200156908100*s
qrt(5)*(sqrt(5) + 2)^(5/2) - 76003360800627632400*sqrt(2*sqrt(5) + 5)*(sqrt
(5) + 2)^(5/2) - 1583403350013075675*(sqrt(5) + 2)^3 - 34643030039065227461
01760*sqrt(5)*(2*sqrt(5) + 5)^2 - 2015373937635933569712128*(2*sqrt(5) + 5)
^(5/2) - 1732151501953261373050880*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(
5) + 2) - 1259608711022458481070080*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 3
24778406616236507447040*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2) - 31490217775
5614620267520*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 27064867218019708953920
*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 39362772219451827533440*
(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 845777100563115904810*sqrt(5)*(sqrt(5
) + 2)^2 - 2460173263715739220840*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 615
04331592893480521*(sqrt(5) + 2)^(5/2) + 3959703717250098693214208*sqrt(5)*(
2*sqrt(5) + 5)^(3/2) + 2662579692919387100254208*(2*sqrt(5) + 5)^2 + 148488
8893968787009955328*sqrt(5)*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 13312898464
59693550127104*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5) + 2) + 18561111746098376
244416*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2) + 249616846211192540648832
*(2*sqrt(5) + 5)*(sqrt(5) + 2) + 7733796322754099010184*sqrt(5)*(sqrt(5) +
2)^(3/2) + 20801403850932711720736*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2)
+ 650043870341647241273*(sqrt(5) + 2)^2 + 10991940456909382283282816*sqrt(5
)*(2*sqrt(5) + 5) + 8567053742081103206220288*(2*sqrt(5) + 5)^(3/2) + 27479
85114227345570820704*sqrt(5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 321264
5153280413702332608*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 1717490696392090981
76294*sqrt(5)*(sqrt(5) + 2) + 401580644160051712791576*sqrt(2*sqrt(5) + 5)*
(sqrt(5) + 2) + 16732526840002154699649*(sqrt(5) + 2)^(3/2) - 2557269775899
525489493536*sqrt(5)*sqrt(2*sqrt(5) + 5) - 319658721987440686186692*sqrt(5)
*sqrt(sqrt(5) + 2) + 39842775211562571442672*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(
5) + 2) - 3308326863346966249269767*sqrt(5) - 4580301686563984868886360*sqr
t(2*sqrt(5) + 5) - 572537710820498108610795*sqrt(sqrt(5) + 2) + 28508242698
41065226382633)^2 + 64*(24322822501240781930496*sqrt(5)*(2*sqrt(5) + 5)^3 +
 13898755714994732531712*(2*sqrt(5) + 5)^(7/2) + 18242116875930586447872*sq
```

```
rt(5)*(2*sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) + 1216141125062039096 5248*(2*
sqrt(5) + 5)^3*sqrt(sqrt(5) + 2) + 5700661523728308264960*sqrt(5)*(2*sqrt(5
) + 5)^2*(sqrt(5) + 2) + 4560529218982646611968*(2*sqrt(5) + 5)^(5/2)*(sqrt
(5) + 2) + 950110253954718044160*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2
)^(3/2) + 950110253954718044160*(2*sqrt(5) + 5)^2*(sqrt(5) + 2)^(3/2) + 890
72836308254816640*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 + 118763781744339
755520*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^2 + 4453641815412740832*sqrt(5)*
sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 8907283630825481664*(2*sqrt(5) +
5)*(sqrt(5) + 2)^(5/2) + 927842044877654 34*sqrt(5)*(sqrt(5) + 2)^3 + 371136
817951061736*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^3 + 6627443177697531*(sqrt(5
) + 2)^(7/2) - 137260818271771086 02880*sqrt(5)*...
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^5 - 1),x)
```

```
[Out] \text{Hanged}
```

3.274 $\int \frac{1}{1-\sinh^6(x)}\,dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

```
[Out] 1/6*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/3*arctanh((1-(-1)^(1/3))^(1/2)*tanh(
x))/(1-(-1)^(1/3))^(1/2)+1/3*arctanh((1+(-1)^(2/3))^(1/2)*tanh(x))/(1+(-1)^
(2/3))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3290, 3260, 212}

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^6)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(3*Sqrt[2]) + ArcTanh[Sqrt[1 - (-1)^(1/3)]*Tanh[x]
]/(3*Sqrt[1 - (-1)^(1/3)]) + ArcTanh[Sqrt[1 + (-1)^(2/3)]*Tanh[x]]/(3*Sqrt[
1 + (-1)^(2/3)])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1-\sinh^6(x)}\,dx &= \frac{1}{3}\int \frac{1}{1-\sinh^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1+\sqrt[3]{-1}\,\sinh^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sinh^2(x)}\,dx \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\tanh(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\left(1-\sqrt[3]{-1}\,\right)x^2}\,dx,x,\tanh(x)\right) \\
&= \frac{\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\,\tanh(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+(-1)^{2/3}}\right.}{3\sqrt{1+(-1)^{2/3}}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.34, size = 70, normalized size = 0.84

$$\frac{1}{6}\left(-\text{ArcTan}(\text{csch}(x)\text{sech}(x))+i\sqrt{3}\left(\text{ArcTan}\left(\frac{1-2i\tanh(x)}{\sqrt{3}}\right)-\text{ArcTan}\left(\frac{1+2i\tanh(x)}{\sqrt{3}}\right)\right)+\sqrt{2}\,\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^6)^(-1),x]
```

```
[Out] (-ArcTan[Csch[x]*Sech[x]] + I*Sqrt[3]*(ArcTan[(1 - (2*I)*Tanh[x])/Sqrt[3]]
- ArcTan[(1 + (2*I)*Tanh[x])/Sqrt[3]]) + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]])/
6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.62, size = 160, normalized size = 1.93

method	result
risch	$\left(\sum_{\_R=\text{RootOf}\left(1296\_Z^4-36\_Z^2+1\right)}\_R\ln\left(432\_R^3-72\_R^2+e^{2x}+1\right)\right)+\frac{\sqrt{2}\,\ln\left(e^{2x}-3+2\sqrt{2}\right)}{12}-\frac{\sqrt{2}}{}$
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4-2\_Z^3+2\_Z^2+2\_Z+1\right)}\frac{\left(-\_R^2+\_R+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3-3\_R^2+2\_R+1}\right)}{3}+\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{6}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sum((-_R^2+_R+1)/(2*_R^3-3*_R^2+2*_R+1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z
^4-2*_Z^3+2*_Z^2+2*_Z+1))+1/6*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2)
)+1/3*sum((-_R^2-_R+1)/(2*_R^3+3*_R^2+2*_R-1)*ln(tanh(1/2*x)-_R),_R=RootOf(
```

```
_Z^4+2*_Z^3+2*_Z^2-2*_Z+1))+1/6*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/
2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^6),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(2)*log(-(sqrt(2) - e^x + 1)/(sqrt(2) + e^x - 1)) + 1/12*sqrt(2)*
log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) + integrate(1/3*(e^(3*x) + 4*
e^(2*x) - e^x)/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x) - integrat
e(1/3*(e^(3*x) - 4*e^(2*x) - e^x)/(e^(4*x) - 2*e^(3*x) + 2*e^(2*x) + 2*e^x
+ 1), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(57) = 114.
time = 0.41, size = 155, normalized size = 1.87

$$-\frac{1}{12}\sqrt{3}\log\left(16\sqrt{3}+4e^{(4x)}+28\right)+\frac{1}{12}\sqrt{3}\log\left(-16\sqrt{3}+4e^{(4x)}+28\right)+\frac{1}{12}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2}-3\right)e^{(2x)}-12\sqrt{2}+e^{(4x)}+17}{e^{(4x)}-6e^{(2x)}+1}\right)-\frac{1}{3}\arctan\left(-\left(\sqrt{3}+2\right)e^{(2x)}+\frac{1}{2}\left(\sqrt{3}+2\right)\sqrt{-16\sqrt{3}+4e^{(4x)}+28}\right)+\frac{1}{3}\arctan\left(-\left(\sqrt{3}-2\right)e^{(2x)}+\sqrt{4\sqrt{3}+e^{(4x)}+7}\left(\sqrt{3}-2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^6),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*log(16*sqrt(3) + 4*e^(4*x) + 28) + 1/12*sqrt(3)*log(-16*sqrt(
3) + 4*e^(4*x) + 28) + 1/12*sqrt(2)*log((2*(2*sqrt(2) - 3)*e^(2*x) - 12*sqr
t(2) + e^(4*x) + 17)/(e^(4*x) - 6*e^(2*x) + 1)) - 1/3*arctan(-(sqrt(3) + 2)
*e^(2*x) + 1/2*(sqrt(3) + 2)*sqrt(-16*sqrt(3) + 4*e^(4*x) + 28)) + 1/3*arct
an(-(sqrt(3) - 2)*e^(2*x) + sqrt(4*sqrt(3) + e^(4*x) + 7)*(sqrt(3) - 2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**6),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(57) = 114.
time = 0.43, size = 143, normalized size = 1.72

$$-\frac{1}{36}\left(\left(2\sqrt{3}-3\right)e^{(4x)}+2\sqrt{3}-3\right)\arctan\left(\frac{e^{(2x)}}{\sqrt{3}+2}\right)+\frac{1}{36}\left(\left(2\sqrt{3}+3\right)e^{(4x)}+2\sqrt{3}+3\right)\arctan\left(-\frac{e^{(2x)}}{\sqrt{3}-2}\right)-\frac{1}{12}\sqrt{3}\log\left(\left(\sqrt{3}+2\right)^{2}+e^{(4x)}\right)+\frac{1}{12}\sqrt{3}\log\left(\left(\sqrt{3}-2\right)^{2}+e^{(4x)}\right)-\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^6),x, algorithm="giac")
```

```
[Out] -1/36*((2*sqrt(3) - 3)*e^(4*x) + 2*sqrt(3) - 3)*arctan(e^(2*x)/(sqrt(3) + 2
)) + 1/36*((2*sqrt(3) + 3)*e^(4*x) + 2*sqrt(3) + 3)*arctan(-e^(2*x)/(sqrt(3
) - 2)) - 1/12*sqrt(3)*log((sqrt(3) + 2)^2 + e^(4*x)) + 1/12*sqrt(3)*log((s
qrt(3) - 2)^2 + e^(4*x)) - 1/12*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)
/abs(4*sqrt(2) + 2*e^(2*x) - 6))
```

Mupad [B]
time = 2.71, size = 285, normalized size = 3.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^6 - 1),x)
```

```
[Out] (log(exp(2*x)*(14009449395540459520 + 6177144285775790080i) + 3^(1/2)*(9556
07545932677120 - 2167269359741829120i) - 3^(1/2)*exp(2*x)*(8088359377641144
320 + 3566375915854233600i) - (1655160823988879360 - 3753820658157486080i))
*1i)/12 - (log(exp(2*x)*(14009449395540459520 - 6177144285775790080i) + 3^(
1/2)*(955607545932677120 + 2167269359741829120i) - 3^(1/2)*exp(2*x)*(808835
9377641144320 - 3566375915854233600i) - (1655160823988879360 + 375382065815
7486080i))*1i)/12 + atan((14009449395540459520*exp(2*x) - 95560754593267712
0*3^(1/2) + 8088359377641144320*3^(1/2)*exp(2*x) - 1655160823988879360)/(61
77144285775790080*exp(2*x) + 2167269359741829120*3^(1/2) + 3566375915854233
600*3^(1/2)*exp(2*x) + 3753820658157486080))/6 - (3^(1/2)*log((617714428577
5790080*exp(2*x) - 2167269359741829120*3^(1/2) - 3566375915854233600*3^(1/2
)*exp(2*x) + 3753820658157486080)^2 + (14009449395540459520*exp(2*x) + 9556
07545932677120*3^(1/2) - 8088359377641144320*3^(1/2)*exp(2*x) - 16551608239
88879360)^2))/12 + (3^(1/2)*log((6177144285775790080*exp(2*x) + 21672693597
41829120*3^(1/2) + 3566375915854233600*3^(1/2)*exp(2*x) + 37538206581574860
80)^2 + (14009449395540459520*exp(2*x) - 955607545932677120*3^(1/2) + 80883
59377641144320*3^(1/2)*exp(2*x) - 1655160823988879360)^2))/12 + (2^(1/2)*lo
g(17674880313941032960*exp(2*x) - 2144322552070144000*2^(1/2) + 12498027726
650736640*2^(1/2)*exp(2*x) - 3032530035220152320))/12 - (2^(1/2)*log(176748
80313941032960*exp(2*x) + 2144322552070144000*2^(1/2) - 1249802772665073664
0*2^(1/2)*exp(2*x) - 3032530035220152320))/12
```

3.275 $\int \frac{1}{1-\sinh^8(x)}\,dx$

Optimal. Leaf size=69

$$\frac{\tanh^{-1}\left(\sqrt{1-i}\,\tanh(x)\right)}{4\sqrt{1-i}}+\frac{\tanh^{-1}\left(\sqrt{1+i}\,\tanh(x)\right)}{4\sqrt{1+i}}+\frac{\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{4\sqrt{2}}+\frac{\tanh(x)}{4}$$

```
[Out] 1/4*arctanh((1-I)^(1/2)*tanh(x))/(1-I)^(1/2)+1/4*arctanh((1+I)^(1/2)*tanh(x
))/(1+I)^(1/2)+1/8*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/4*tanh(x)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3290, 3260, 212, 3254, 3852, 8}

$$\frac{\tanh^{-1}\left(\sqrt{1-i}\,\tanh(x)\right)}{4\sqrt{1-i}}+\frac{\tanh^{-1}\left(\sqrt{1+i}\,\tanh(x)\right)}{4\sqrt{1+i}}+\frac{\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{4\sqrt{2}}+\frac{\tanh(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^8)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[1 - I]*Tanh[x]]/(4*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + I]*Tanh[x]]
/(4*Sqrt[1 + I]) + ArcTanh[Sqrt[2]*Tanh[x]]/(4*Sqrt[2]) + Tanh[x]/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1-\sinh^8(x)}\,dx &= \frac{1}{4}\int \frac{1}{1-\sinh^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1-i\sinh^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1+i\sinh^2(x)}\,dx+\frac{1}{4}\int \\
&= \frac{1}{4}\int \operatorname{sech}^2(x)\,dx+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\tanh(x)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-(1+i)x^2}\,d\right. \\
&= \frac{\tanh^{-1}\left(\sqrt{1-i}\,\tanh(x)\right)}{4\sqrt{1-i}}+\frac{\tanh^{-1}\left(\sqrt{1+i}\,\tanh(x)\right)}{4\sqrt{1+i}}+\frac{\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{4\sqrt{2}}+ \\
&= \frac{\tanh^{-1}\left(\sqrt{1-i}\,\tanh(x)\right)}{4\sqrt{1-i}}+\frac{\tanh^{-1}\left(\sqrt{1+i}\,\tanh(x)\right)}{4\sqrt{1+i}}+\frac{\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{4\sqrt{2}}+
\end{aligned}
$$

Mathematica [**A**]
time = 0.37, size = 64, normalized size = 0.93

$$
\frac{1}{8}\left(\frac{2\tanh^{-1}\left(\sqrt{1-i}\,\tanh(x)\right)}{\sqrt{1-i}}+\frac{2\tanh^{-1}\left(\sqrt{1+i}\,\tanh(x)\right)}{\sqrt{1+i}}+\sqrt{2}\,\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)+2\tanh(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^8)^(-1),x]
```

```
[Out] ((2*ArcTanh[Sqrt[1 - I]*Tanh[x]])/Sqrt[1 - I] + (2*ArcTanh[Sqrt[1 + I]*Tanh
[x]])/Sqrt[1 + I] + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/8
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.64, size = 99, normalized size = 1.43

method	result

risch	$-\frac{1}{2(1+e^{2x})}+\frac{\sqrt{2}\,\ln\left(e^{2x}-3+2\sqrt{2}\right)}{16}-\frac{\sqrt{2}\,\ln\left(e^{2x}-3-2\sqrt{2}\right)}{16}+\left(\sum_{\_R=\mathrm{RootOf}(8192\_Z^4-128\_Z^2+1)}\_R\ln\left(204\right.\right.$
default	$\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{8}+\frac{\tanh\left(\frac{x}{2}\right)}{2\left(\tanh^2\left(\frac{x}{2}\right)\right)+2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^4-2\_Z^2+1)}\_R\ln\left(\tanh^2\left(\frac{x}{2}\right)+\left(-4\_R^3+4\_R\right.\right.\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^8),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+1/2*tanh(1/2*x)/(tanh(1/
2*x)^2+1)+1/8*sum(_R*ln(tanh(1/2*x)^2+(-4*_R^3+4*_R)*tanh(1/2*x)+1),_R=Root
Of(2*_Z^4-2*_Z^2+1))+1/8*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^8),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*log(-(sqrt(2) - e^x + 1)/(sqrt(2) + e^x - 1)) + 1/16*sqrt(2)*
log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) - 1/2/(e^(2*x) + 1) + 8*integ
rate(e^(4*x)/(e^(8*x) - 4*e^(6*x) + 22*e^(4*x) - 4*e^(2*x) + 1), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(41) = 82$.
time = 0.41, size = 708, normalized size = 10.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^8),x, algorithm="fricas")
```

```
[Out] -1/32*(4*(2^(1/4)*e^(2*x) + 2^(1/4))*sqrt(-2*sqrt(2) + 4)*arctan(1/14*(sqrt
(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) - 1/28*(2*sqrt(2)*(5*sqrt(2) +
 6) - (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sqrt(2
) + 4) + 16*sqrt(2) + 8)*sqrt(-(2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*
sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) - 1/14*sqrt(2)*
(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) +
6))*e^(2*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(-2*
sqrt(2) + 4) - 1/7*sqrt(2) + 3/7) + 4*(2^(1/4)*e^(2*x) + 2^(1/4))*sqrt(-2*s
qrt(2) + 4)*arctan(-1/14*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x)
```

```
+ 1/28*(2*sqrt(2)*(5*sqrt(2) + 6) + (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(
5*sqrt(2) + 6))*sqrt(-2*sqrt(2) + 4) + 16*sqrt(2) + 8)*sqrt((2^(3/4)*e^(2*x
) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2
*e^(2*x) + 5) + 1/14*sqrt(2)*(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) +
11) + 2*2^(1/4)*(5*sqrt(2) + 6))*e^(2*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1
/4)*(3*sqrt(2) - 2))*sqrt(-2*sqrt(2) + 4) + 1/7*sqrt(2) - 3/7) - (2^(1/4)*(
sqrt(2) + 1)*e^(2*x) + 2^(1/4)*(sqrt(2) + 1))*sqrt(-2*sqrt(2) + 4)*log((2^(
3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) +
e^(4*x) - 2*e^(2*x) + 5) + (2^(1/4)*(sqrt(2) + 1)*e^(2*x) + 2^(1/4)*(sqrt(2
) + 1))*sqrt(-2*sqrt(2) + 4)*log(-(2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4
))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) - 2*(sqrt(2)
*e^(2*x) + sqrt(2))*log((2*(2*sqrt(2) - 3)*e^(2*x) - 12*sqrt(2) + e^(4*x) +
 17)/(e^(4*x) - 6*e^(2*x) + 1)) + 16)/(e^(2*x) + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**8),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.50, size = 48, normalized size = 0.70

$$-\frac{1}{16}\,\sqrt{2}\,\log\left(\frac{\left|-4\,\sqrt{2}+2\,e^{(2\,x)}-6\right|}{\left|4\,\sqrt{2}+2\,e^{(2\,x)}-6\right|}\right)-\frac{1}{2\,\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^8),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - 1/2/(e^(2*x) + 1)
```

Mupad [**B**]
time = 4.81, size = 273, normalized size = 3.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^8 - 1),x)
```

```
[Out] (2^(1/2)*log(582732658686033920*exp(2*x) - 706973263556771184*2^(1/2) + 4120
54214575915008*2^(1/2)*exp(2*x) - 999811177544441728))/16 - (2^(1/2)*log(582
```

```
732658686033920*exp(2*x) + 70697326355677184*2^(1/2) - 412054214575915008*2
^(1/2)*exp(2*x) - 99981117754441728))/16 - 1/(2*(exp(2*x) + 1)) - (2^(1/2)*
(1 - 1i)^(1/2)*log(exp(2*x)*(155613434002538496 + 429723297714798592i) - 2^
(1/2)*(1 - 1i)^(1/2)*(54684829282729984 - 21956972328779776i) + 2^(1/2)*(1
- 1i)^(1/2)*exp(2*x)*(12296353929494528 - 271474128182050816i) + (708364832
96067584 - 69311013991743488i)))/16 + (2^(1/2)*(1 - 1i)^(1/2)*log(exp(2*x)*
(155613434002538496 + 429723297714798592i) + 2^(1/2)*(1 - 1i)^(1/2)*(546848
29282729984 - 21956972328779776i) - 2^(1/2)*(1 - 1i)^(1/2)*exp(2*x)*(122963
53929494528 - 271474128182050816i) + (70836483296067584 - 69311013991743488
i)))/16 - (2^(1/2)*(1 + 1i)^(1/2)*log(exp(2*x)*(155613434002538496 - 429723
297714798592i) - 2^(1/2)*(1 + 1i)^(1/2)*(54684829282729984 + 21956972328779
776i) + 2^(1/2)*(1 + 1i)^(1/2)*exp(2*x)*(12296353929494528 + 27147412818205
0816i) + (70836483296067584 + 69311013991743488i)))/16 + (2^(1/2)*(1 + 1i)^
(1/2)*log(exp(2*x)*(155613434002538496 - 429723297714798592i) + 2^(1/2)*(1
+ 1i)^(1/2)*(54684829282729984 + 21956972328779776i) - 2^(1/2)*(1 + 1i)^(1/
2)*exp(2*x)*(12296353929494528 + 271474128182050816i) + (70836483296067584
+ 69311013991743488i)))/16
```

3.276 $\int \frac{\cosh^5(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\sinh(x)}{a}+\frac{\sinh^3(x)}{3a}$$

```
[Out] sinh(x)/a+1/3*sinh(x)^3/a
```

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {3254, 2713}

$$\frac{\sinh^3(x)}{3a}+\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(a + a*Sinh[x]^2),x]
```

```
[Out] Sinh[x]/a + Sinh[x]^3/(3*a)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^5(x)}{a+a\sinh^2(x)}\,dx&=\frac{\int \cosh^3(x)\,dx}{a}\\&=\frac{i\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(x)\right)}{a}\\&=\frac{\sinh(x)}{a}+\frac{\sinh^3(x)}{3a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.06

$$\frac{\frac{3\sinh(x)}{4}+\frac{1}{12}\sinh(3x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(a + a*Sinh[x]^2),x]
```

```
[Out] ((3*Sinh[x])/4 + Sinh[3*x]/12)/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(16) = 32$.
time = 0.42, size = 67, normalized size = 3.72

method	result	size
risch	$\frac{e^{3x}}{24a}+\frac{3\,e^{x}}{8a}-\frac{3\,e^{-x}}{8a}-\frac{e^{-3x}}{24a}$	36
default	$\frac{-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}}{a}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a*(-1/6/(tanh(1/2*x)+1)^3+1/4/(tanh(1/2*x)+1)^2-1/2/(tanh(1/2*x)+1)-1/6/(
tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2-1/2/(tanh(1/2*x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.
time = 0.26, size = 34, normalized size = 1.89

$$\frac{\left(9\,e^{(-2\,x)}+1\right)e^{(3\,x)}}{24\,a}-\frac{9\,e^{(-x)}+e^{(-3\,x)}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/24*(9*e^(-2*x) + 1)*e^(3*x)/a - 1/24*(9*e^(-x) + e^(-3*x))/a
```

Fricas [A]
time = 0.37, size = 20, normalized size = 1.11

$$\frac{\sinh\left(x\right)^{3}+3\left(\cosh\left(x\right)^{2}+3\right)\sinh\left(x\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(x)^3 + 3*(cosh(x)^2 + 3)*sinh(x))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(12) = 24$.
time = 2.76, size = 124, normalized size = 6.89

$$-\frac{6\tanh^{5}\left(\frac{x}{2}\right)}{3a\tanh^{6}\left(\frac{x}{2}\right)-9a\tanh^{4}\left(\frac{x}{2}\right)+9a\tanh^{2}\left(\frac{x}{2}\right)-3a}+\frac{4\tanh^{3}\left(\frac{x}{2}\right)}{3a\tanh^{6}\left(\frac{x}{2}\right)-9a\tanh^{4}\left(\frac{x}{2}\right)+9a\tanh^{2}\left(\frac{x}{2}\right)-3a}-\frac{6\tanh\left(\frac{x}{2}\right)}{3a\tanh^{6}\left(\frac{x}{2}\right)-9a\tanh^{4}\left(\frac{x}{2}\right)+9a\tanh^{2}\left(\frac{x}{2}\right)-3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(a+a*sinh(x)**2),x)
```

```
[Out] -6*tanh(x/2)**5/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)**2 - 3
*a) + 4*tanh(x/2)**3/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)**
2 - 3*a) - 6*tanh(x/2)/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)
**2 - 3*a)
```

Giac [A]
time = 0.41, size = 29, normalized size = 1.61

$$-\frac{\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)}-e^{(3\,x)}-9\,e^{x}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)/a
```

Mupad [B]
time = 1.36, size = 35, normalized size = 1.94

$$\frac{\mathrm{e}^{3\,x}}{24\,a}-\frac{\mathrm{e}^{-3\,x}}{24\,a}-\frac{3\,\mathrm{e}^{-x}}{8\,a}+\frac{3\,\mathrm{e}^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a + a*sinh(x)^2),x)
```

```
[Out] exp(3*x)/(24*a) - exp(-3*x)/(24*a) - (3*exp(-x))/(8*a) + (3*exp(x))/(8*a)
```

3.277 $\int \frac{\cosh^4(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=20

$$\frac{x}{2a}+\frac{\cosh(x)\sinh(x)}{2a}$$

```
[Out] 1/2*x/a+1/2*cosh(x)*sinh(x)/a
```

Rubi [A]

time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {3254, 2715, 8}

$$\frac{x}{2a}+\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + a*Sinh[x]^2),x]
```

```
[Out] x/(2*a) + (Cosh[x]*Sinh[x])/(2*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^4(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \cosh^2(x)\,dx}{a}\\ &= \frac{\cosh(x)\sinh(x)}{2a}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{\cosh(x)\sinh(x)}{2a}\end{aligned}$$

Mathematica [**A**]
time = 0.00, size = 18, normalized size = 0.90

$$\frac{\frac{x}{2}+\frac{1}{4}\sinh(2x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + a*Sinh[x]^2),x]
```

```
[Out] (x/2 + Sinh[2*x]/4)/a
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(16) = 32.
time = 0.43, size = 65, normalized size = 3.25

method	result	size
risch	$\frac{x}{2a}+\frac{e^{2x}}{8a}-\frac{e^{-2x}}{8a}$	26
default	$\frac{-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{2}{4\tanh\left(\frac{x}{2}\right)+4}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{2}{4\tanh\left(\frac{x}{2}\right)-4}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}}{a}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a*(-1/4/(tanh(1/2*x)+1)^2+1/4/(tanh(1/2*x)+1)+1/4*ln(tanh(1/2*x)+1)+1/4/(
tanh(1/2*x)-1)^2+1/4/(tanh(1/2*x)-1)-1/4*ln(tanh(1/2*x)-1))
```

Maxima [**A**]
time = 0.26, size = 25, normalized size = 1.25

$$\frac{x}{2\,a}+\frac{e^{(2\,x)}}{8\,a}-\frac{e^{(-2\,x)}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*x/a + 1/8*e^(2*x)/a - 1/8*e^(-2*x)/a
```

Fricas [**A**]
time = 0.37, size = 12, normalized size = 0.60

$$\frac{\cosh\left(x\right)\sinh\left(x\right)+x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(x)*sinh(x) + x)/a
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(14) = 28$.
time = 1.56, size = 153, normalized size = 7.65

$$\frac{x\tanh^4\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}-\frac{2x\tanh^2\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}+\frac{x}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}+\frac{2\tanh^3\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}+\frac{2\tanh\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+a*sinh(x)**2),x)
```

```
[Out] x*tanh(x/2)**4/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 2*x*tanh(x/2)*
*2/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + x/(2*a*tanh(x/2)**4 - 4*a*
tanh(x/2)**2 + 2*a) + 2*tanh(x/2)**3/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 +
 2*a) + 2*tanh(x/2)/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
```

Giac [**A**]
time = 0.42, size = 28, normalized size = 1.40

$$-\frac{\left(2\,e^{(2\,x)}+1\right)e^{(-2\,x)}-4\,x-e^{(2\,x)}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))/a
```

Mupad [**B**]
time = 1.33, size = 25, normalized size = 1.25

$$\frac{\mathrm{e}^{2\,x}}{8\,a}-\frac{\mathrm{e}^{-2\,x}}{8\,a}+\frac{x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + a*sinh(x)^2),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) + x/(2*a)
```

3.278 $\int \frac{\cosh^3(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=6

$$\frac{\sinh(x)}{a}$$

`[Out] sinh(x)/a`

Rubi [A]
time = 0.03, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3254, 2717}

$$\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + a*Sinh[x]^2),x]
```

```
[Out] Sinh[x]/a
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \cosh(x)\,dx}{a}\\ &= \frac{\sinh(x)}{a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + a*Sinh[x]^2),x]
```

```
[Out] Sinh[x]/a
```

Maple [A]
time = 0.37, size = 7, normalized size = 1.17

method	result	size
derivativedivides	$\frac{\sinh(x)}{a}$	7
default	$\frac{\sinh(x)}{a}$	7
risch	$\frac{e^{x}}{2a}-\frac{e^{-x}}{2a}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] sinh(x)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.27, size = 17, normalized size = 2.83

$$-\frac{e^{(-x)}}{2\,a}+\frac{e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x)/a + 1/2*e^x/a
```

Fricas [A]
time = 0.38, size = 6, normalized size = 1.00

$$\frac{\sinh\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] sinh(x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(3) = 6$.
time = 0.90, size = 17, normalized size = 2.83

$$-\frac{2\tanh\left(\frac{x}{2}\right)}{a\tanh^{2}\left(\frac{x}{2}\right)-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+a*sinh(x)**2),x)
```

```
[Out] -2*tanh(x/2)/(a*tanh(x/2)**2 - a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.
time = 0.41, size = 14, normalized size = 2.33

$$-\frac{e^{(-x)} - e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*(e^(-x) - e^x)/a
```

Mupad [B]
time = 1.29, size = 6, normalized size = 1.00

$$\frac{\sinh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + a*sinh(x)^2),x)
```

```
[Out] sinh(x)/a
```

3.279 $\int \frac{\cosh^2(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=5

$$\frac{x}{a}$$

```
[Out] x/a
```

Rubi [A]
time = 0.03, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3254, 8}

$$\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + a*Sinh[x]^2),x]
```

```
[Out] x/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int 1\,dx}{a}\\ &= \frac{x}{a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + a*Sinh[x]^2),x]
```

```
[Out] x/a
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.51, size = 11, normalized size = 2.20

method	result	size
risch	$\frac{x}{a}$	6
default	$\frac{2\,\mathrm{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a*arctanh(tanh(1/2*x))
```

Maxima [A]
time = 0.29, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] x/a
```

Fricas [A]
time = 0.48, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] x/a
```

Sympy [A]
time = 0.47, size = 2, normalized size = 0.40

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+a*sinh(x)**2),x)
```

```
[Out] x/a
```

Giac [**A**]
time = 0.41, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] x/a
```

Mupad [**B**]
time = 1.26, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + a*sinh(x)^2),x)
```

```
[Out] x/a
```

3.280 $\int \frac{\cosh(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=7

$$\frac{\text{ArcTan}(\sinh(x))}{a}$$

```
[Out] arctan(sinh(x))/a
```

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3254, 3855}

$$\frac{\text{ArcTan}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + a*Sinh[x]^2),x]
```

```
[Out] ArcTan[Sinh[x]]/a
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \operatorname{sech}(x)\,dx}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.71

$$\frac{2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + a*Sinh[x]^2),x]
```

```
[Out] (2*ArcTan[Tanh[x/2]])/a
```

Maple [A]
time = 0.32, size = 8, normalized size = 1.14

method	result	size
derivativedivides	$\frac{\arctan(\sinh(x))}{a}$	8
default	$\frac{\arctan(\sinh(x))}{a}$	8
risch	$\frac{i\ln(\mathrm{e}^{x}+i)}{a}-\frac{i\ln(\mathrm{e}^{x}-i)}{a}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(sinh(x))/a
```

Maxima [A]
time = 0.47, size = 10, normalized size = 1.43

$$-\frac{2\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(-x))/a
```

Fricas [A]
time = 0.39, size = 11, normalized size = 1.57

$$\frac{2\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(x) + sinh(x))/a
```

Sympy [A]
time = 0.08, size = 5, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\sinh\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sinh(x)**2),x)
```

```
[Out] atan(sinh(x))/a
```

Giac [A]
time = 0.40, size = 8, normalized size = 1.14

$$\frac{2\arctan\left(e^{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)/a
```

Mupad [B]
time = 0.07, size = 7, normalized size = 1.00

$$\frac{\operatorname{atan}(\sinh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + a*sinh(x)^2),x)
```

```
[Out] atan(sinh(x))/a
```

3.281 $\int \frac{\operatorname{sech}(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=22

$$\frac{\operatorname{ArcTan}(\sinh(x))}{2a}+\frac{\operatorname{sech}(x)\tanh(x)}{2a}$$

```
[Out] 1/2*arctan(sinh(x))/a+1/2*sech(x)*tanh(x)/a
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {3254, 3853, 3855}

$$\frac{\operatorname{ArcTan}(\sinh(x))}{2a}+\frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + a*Sinh[x]^2),x]
```

```
[Out] ArcTan[Sinh[x]]/(2*a) + (Sech[x]*Tanh[x])/(2*a)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \operatorname{sech}^3(x)\,dx}{a}\\ &= \frac{\operatorname{sech}(x)\tanh(x)}{2a}+\frac{\int \operatorname{sech}(x)\,dx}{2a}\\ &= \frac{\tan^{-1}(\sinh(x))}{2a}+\frac{\operatorname{sech}(x)\tanh(x)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.91

$$\frac{\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2}\operatorname{sech}(x)\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + a*Sinh[x]^2),x]
```

```
[Out] (ArcTan[Tanh[x/2]] + (Sech[x]*Tanh[x])/2)/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(18) = 36.
time = 0.61, size = 40, normalized size = 1.82

method	result	size
default	$\frac{\frac{2\left(-\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{2}+\frac{\tanh\left(\frac{x}{2}\right)}{2}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	40
risch	$\frac{e^{x}\left(e^{2x}-1\right)}{\left(1+e^{2x}\right)^2 a}+\frac{i\ln\left(e^{x}+i\right)}{2a}-\frac{i\ln\left(e^{x}-i\right)}{2a}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a*((-1/2*tanh(1/2*x)^3+1/2*tanh(1/2*x))/(tanh(1/2*x)^2+1)^2+1/2*arctan(ta
nh(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(18) = 36.
time = 0.50, size = 40, normalized size = 1.82

$$\frac{e^{(-x)}-e^{(-3\,x)}}{2\,ae^{(-2\,x)}+ae^{(-4\,x)}+a}-\frac{\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] (e^(-x) - e^(-3*x))/(2*a*e^(-2*x) + a*e^(-4*x) + a) - arctan(e^(-x))/a
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(18) = 36$.
time = 0.40, size = 151, normalized size = 6.86

$$\frac{\cosh\left(x\right)^{3} + 3\,\cosh\left(x\right)\sinh\left(x\right)^{2} + \sinh\left(x\right)^{3} + \left(\cosh\left(x\right)^{4} + 4\,\cosh\left(x\right)\sinh\left(x\right)^{3} + \sinh\left(x\right)^{4} + 2\,\left(3\,\cosh\left(x\right)^{2} + 1\right)\sinh\left(x\right)^{2} + 2\,\cosh\left(x\right)^{2} + 4\,\left(\cosh\left(x\right)^{3} + \cosh\left(x\right)\right)\sinh\left(x\right) + 1\right)\arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right) + \left(3\,\cosh\left(x\right)^{2} - 1\right)\sinh\left(x\right) - \cosh\left(x\right)}{a\cosh\left(x\right)^{4} + 4\,a\cosh\left(x\right)\sinh\left(x\right)^{3} + a\sinh\left(x\right)^{4} + 2\,a\cosh\left(x\right)^{2} + 2\,\left(3\,a\cosh\left(x\right)^{2} + a\right)\sinh\left(x\right)^{2} + 4\,\left(a\cosh\left(x\right)^{3} + a\cosh\left(x\right)\right)\sinh\left(x\right) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] (cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh(
x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)
^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (3*cosh(x)^2 - 1)*si
nh(x) - cosh(x))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*c
osh(x)^2 + 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*si
nh(x) + a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}\left(x\right)}{\sinh^{2}\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)**2),x)
```

```
[Out] Integral(sech(x)/(sinh(x)**2 + 1), x)/a
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(18) = 36$.
time = 0.41, size = 52, normalized size = 2.36

$$\frac{\pi + 2\,\arctan\left(\frac{1}{2}\,\left(e^{\left(2\,x\right)} - 1\right)e^{\left(-x\right)}\right)}{4\,a} - \frac{e^{\left(-x\right)} - e^{x}}{\left(\left(e^{\left(-x\right)} - e^{x}\right)^{2} + 4\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a - (e^(-x) - e^x)/(((e^(-x)
- e^x)^2 + 4)*a)
```

Mupad [**B**]
time = 1.30, size = 54, normalized size = 2.45

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}} - \frac{2\,\mathrm{e}^{x}}{a\,\left(2\,\mathrm{e}^{2\,x} + \mathrm{e}^{4\,x} + 1\right)} + \frac{\mathrm{e}^{x}}{a\,\left(\mathrm{e}^{2\,x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + a*sinh(x)^2)),x)
```

```
[Out] atan((exp(x)*(a^2)^(1/2))/a)/(a^2)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(
4*x) + 1)) + exp(x)/(a*(exp(2*x) + 1))
```

3.282 $\int \frac{\operatorname{sech}^3(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=35

$$\frac{3\text{ArcTan}(\sinh(x))}{8a}+\frac{3\operatorname{sech}(x)\tanh(x)}{8a}+\frac{\operatorname{sech}^3(x)\tanh(x)}{4a}$$

```
[Out] 3/8*arctan(sinh(x))/a+3/8*sech(x)*tanh(x)/a+1/4*sech(x)^3*tanh(x)/a
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3254, 3853, 3855}

$$\frac{3\text{ArcTan}(\sinh(x))}{8a}+\frac{\tanh(x)\operatorname{sech}^3(x)}{4a}+\frac{3\tanh(x)\operatorname{sech}(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + a*Sinh[x]^2),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(8*a) + (3*Sech[x]*Tanh[x])/(8*a) + (Sech[x]^3*Tanh[x])
/(4*a)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \operatorname{sech}^5(x)\,dx}{a} \\
&= \frac{\operatorname{sech}^3(x)\tanh(x)}{4a} + \frac{3\int \operatorname{sech}^3(x)\,dx}{4a} \\
&= \frac{3\operatorname{sech}(x)\tanh(x)}{8a} + \frac{\operatorname{sech}^3(x)\tanh(x)}{4a} + \frac{3\int \operatorname{sech}(x)\,dx}{8a} \\
&= \frac{3\tan^{-1}(\sinh(x))}{8a} + \frac{3\operatorname{sech}(x)\tanh(x)}{8a} + \frac{\operatorname{sech}^3(x)\tanh(x)}{4a}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 0.97

$$
\frac{\frac{3}{4}\mathrm{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{3}{8}\operatorname{sech}(x)\tanh(x) + \frac{1}{4}\operatorname{sech}^3(x)\tanh(x)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + a*Sinh[x]^2),x]
```

```
[Out] ((3*ArcTan[Tanh[x/2]])/4 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4)/a
```

Maple [A]
time = 0.59, size = 56, normalized size = 1.60

method	result	size
default	$\frac{\frac{2\left(-\frac{5\left(\tanh^{7}\left(\frac{x}{2}\right)\right)}{8}+\frac{3\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{8}-\frac{3\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{8}+\frac{5\tanh\left(\frac{x}{2}\right)}{8}\right)}{\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4}}{a}$	56
risch	$\frac{\mathrm{e}^{x}\left(3\,\mathrm{e}^{6x}+11\,\mathrm{e}^{4x}-11\,\mathrm{e}^{2x}-3\right)}{4\left(1+\mathrm{e}^{2x}\right)^{4}a}+\frac{3i\ln\left(\mathrm{e}^{x}+i\right)}{8a}-\frac{3i\ln\left(\mathrm{e}^{x}-i\right)}{8a}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a*((-5/8*tanh(1/2*x)^7+3/8*tanh(1/2*x)^5-3/8*tanh(1/2*x)^3+5/8*tanh(1/2*x
))/(tanh(1/2*x)^2+1)^4+3/8*arctan(tanh(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(29) = 58$.
time = 0.49, size = 69, normalized size = 1.97

$$
\frac{3\,e^{(-x)} + 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} - 3\,e^{(-7\,x)}}{4\,\left(4\,ae^{(-2\,x)} + 6\,ae^{(-4\,x)} + 4\,ae^{(-6\,x)} + ae^{(-8\,x)} + a\right)} - \frac{3\,\arctan\left(e^{(-x)}\right)}{4\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*e^(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*a*e^(-2*x) + 6*a
*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) - 3/4*arctan(e^(-x))/a
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(29) = 58$.
time = 0.37, size = 488, normalized size = 13.94

$$\frac{3 \cosh\left(x\right)^{7} + 21 \cosh\left(x\right) \sinh\left(x\right)^{6} + 3 \sinh\left(x\right)^{7} + \left(63 \cosh\left(x\right)^{2} + 11\right) \sinh\left(x\right)^{5} + 11 \cosh\left(x\right)^{5} + 5 \left(21 \cosh\left(x\right)^{3} + 11 \cosh\left(x\right)\right) \sinh\left(x\right)^{4} + \left(105 \cosh\left(x\right)^{4} + 110 \cosh\left(x\right)^{2} - 11\right) \sinh\left(x\right)^{3} - 11 \cosh\left(x\right)^{3} + \left(63 \cosh\left(x\right)^{5} + 110 \cosh\left(x\right)^{3} - 33 \cosh\left(x\right)\right) \sinh\left(x\right)^{2} + 3 \left(\cosh\left(x\right)^{8} + 8 \cosh\left(x\right) \sinh\left(x\right)^{7} + \sinh\left(x\right)^{8} + 4 \left(7 \cosh\left(x\right)^{2} + 1\right) \sinh\left(x\right)^{6} + 4 \cosh\left(x\right)^{6} + 8 \left(7 \cosh\left(x\right)^{3} + 3 \cosh\left(x\right)\right) \sinh\left(x\right)^{5} + 2 \left(35 \cosh\left(x\right)^{4} + 30 \cosh\left(x\right)^{2} + 3\right) \sinh\left(x\right)^{4} + 6 \cosh\left(x\right)^{4} + 8 \left(7 \cosh\left(x\right)^{5} + 10 \cosh\left(x\right)^{3} + 3 \cosh\left(x\right)\right) \sinh\left(x\right)^{3} + 4 \left(7 \cosh\left(x\right)^{6} + 15 \cosh\left(x\right)^{4} + 9 \cosh\left(x\right)^{2} + 1\right) \sinh\left(x\right)^{2} + 4 \cosh\left(x\right)^{2} + 8 \left(\cosh\left(x\right)^{7} + 3 \cosh\left(x\right)^{5} + 3 \cosh\left(x\right)^{3} + \cosh\left(x\right)\right) \sinh\left(x\right) + 1\right) \arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right) + \left(21 \cosh\left(x\right)^{6} + 55 \cosh\left(x\right)^{4} - 33 \cosh\left(x\right)^{2} - 3\right) \sinh\left(x\right) - 3 \cosh\left(x\right)}{4 \left(a \cosh\left(x\right)^{8} + 8 a \cosh\left(x\right) \sinh\left(x\right)^{7} + a \sinh\left(x\right)^{8} + 4 a \cosh\left(x\right)^{6} + 4 \left(7 a \cosh\left(x\right)^{2} + a\right) \sinh\left(x\right)^{6} + 8 \left(7 a \cosh\left(x\right)^{3} + 3 a \cosh\left(x\right)\right) \sinh\left(x\right)^{5} + 6 a \cosh\left(x\right)^{4} + 2 \left(35 a \cosh\left(x\right)^{4} + 30 a \cosh\left(x\right)^{2} + 3 a\right) \sinh\left(x\right)^{4} + 8 \left(7 a \cosh\left(x\right)^{5} + 10 a \cosh\left(x\right)^{3} + 3 a \cosh\left(x\right)\right) \sinh\left(x\right)^{3} + 4 a \cosh\left(x\right)^{2} + 4 \left(7 a \cosh\left(x\right)^{6} + 15 a \cosh\left(x\right)^{4} + 9 a \cosh\left(x\right)^{2} + a\right) \sinh\left(x\right)^{2} + 8 \left(a \cosh\left(x\right)^{7} + 3 a \cosh\left(x\right)^{5} + 3 a \cosh\left(x\right)^{3} + a \cosh\left(x\right)\right) \sinh\left(x\right) + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(x)^7 + 21*cosh(x)*sinh(x)^6 + 3*sinh(x)^7 + (63*cosh(x)^2 + 11)
*sinh(x)^5 + 11*cosh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*sinh(x)^4 + (105*
cosh(x)^4 + 110*cosh(x)^2 - 11)*sinh(x)^3 - 11*cosh(x)^3 + (63*cosh(x)^5 +
110*cosh(x)^3 - 33*cosh(x))*sinh(x)^2 + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7
+ sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3
+ 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*
cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cos
h(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(
x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + s
inh(x)) + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*sinh(x) - 3*cosh
(x))/(a*cosh(x)^8 + 8*a*cosh(x)*sinh(x)^7 + a*sinh(x)^8 + 4*a*cosh(x)^6 + 4
*(7*a*cosh(x)^2 + a)*sinh(x)^6 + 8*(7*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^5
+ 6*a*cosh(x)^4 + 2*(35*a*cosh(x)^4 + 30*a*cosh(x)^2 + 3*a)*sinh(x)^4 + 8*(
7*a*cosh(x)^5 + 10*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 4*a*cosh(x)^2 + 4
*(7*a*cosh(x)^6 + 15*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*sinh(x)^2 + 8*(a*cosh
(x)^7 + 3*a*cosh(x)^5 + 3*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}^{3}\left(x\right)}{\sinh^{2}\left(x\right)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+a*sinh(x)**2),x)
```

```
[Out] Integral(sech(x)**3/(sinh(x)**2 + 1), x)/a
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(29) = 58$.
time = 0.42, size = 67, normalized size = 1.91

$$\frac{3 \left(\pi + 2 \arctan\left(\frac{1}{2} \left(e^{(2 x)} - 1\right) e^{(-x)}\right)\right)}{16 a} - \frac{3 \left(e^{(-x)} - e^{x}\right)^{3} + 20 e^{(-x)} - 20 e^{x}}{4 \left(\left(e^{(-x)} - e^{x}\right)^{2} + 4\right)^{2} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] 3/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a - 1/4*(3*(e^(-x) - e^x)^3
+ 20*e^(-x) - 20*e^x)/(((e^(-x) - e^x)^2 + 4)^2*a)
```

Mupad [B]
time = 1.29, size = 118, normalized size = 3.37

$$\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{a^2}}{a}\right)}{4\,\sqrt{a^2}}-\frac{4\,\mathrm{e}^{3\,x}}{a\,\left(4\,\mathrm{e}^{2\,x}+6\,\mathrm{e}^{4\,x}+4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}-\frac{2\,\mathrm{e}^x}{a\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)}+\frac{\mathrm{e}^x}{2\,a\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)}+\frac{3\,\mathrm{e}^x}{4\,a\,\left(\mathrm{e}^{2\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + a*sinh(x)^2)),x)
```

```
[Out] (3*atan((exp(x)*(a^2)^(1/2))/a))/(4*(a^2)^(1/2)) - (4*exp(3*x))/(a*(4*exp(2
*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) - (2*exp(x))/(a*(3*exp(2*x)
+ 3*exp(4*x) + exp(6*x) + 1)) + exp(x)/(2*a*(2*exp(2*x) + exp(4*x) + 1)) +
(3*exp(x))/(4*a*(exp(2*x) + 1))
```

3.283 $\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=89

$$\frac{1}{16}(6a-b)x+\frac{(6a-b)\cosh(c+dx)\sinh(c+dx)}{16d}+\frac{(6a-b)\cosh^3(c+dx)\sinh(c+dx)}{24d}+\frac{b\cosh^5(c+dx)\sinh(}{6d}$$

```
[Out] 1/16*(6*a-b)*x+1/16*(6*a-b)*cosh(d*x+c)*sinh(d*x+c)/d+1/24*(6*a-b)*cosh(d*x
+c)^3*sinh(d*x+c)/d+1/6*b*cosh(d*x+c)^5*sinh(d*x+c)/d
```

Rubi [A]

time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3270, 393, 205, 212}

$$\frac{(6a-b)\sinh(c+dx)\cosh^3(c+dx)}{24d}+\frac{(6a-b)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x(6a-b)+\frac{b\sinh(c+dx)\cosh^5(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((6*a - b)*x)/16 + ((6*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) + ((6*a -
 b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d) + (b*Cosh[c + d*x]^5*Sinh[c + d*x
])/(6*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{(1-x^2)^4}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{b\cosh^5(c+dx)\sinh(c+dx)}{6d}+\frac{(6a-b)\text{Subst}\left(\int \frac{1}{(1-x^2)^3}\,dx,x,\right.}{6d}\\ &= \frac{(6a-b)\cosh^3(c+dx)\sinh(c+dx)}{24d}+\frac{b\cosh^5(c+dx)\sinh(c+}{6d}\\ &= \frac{(6a-b)\cosh(c+dx)\sinh(c+dx)}{16d}+\frac{(6a-b)\cosh^3(c+dx)\sin}{24d}\\ &= \frac{1}{16}(6a-b)x+\frac{(6a-b)\cosh(c+dx)\sinh(c+dx)}{16d}+\frac{(6a-b)\,c}{}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 63, normalized size = 0.71

$$\frac{72ac+72adx-12bdx+(48a-3b)\sinh(2(c+dx))+3(2a+b)\sinh(4(c+dx))+b\sinh(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (72*a*c + 72*a*d*x - 12*b*d*x + (48*a - 3*b)*Sinh[2*(c + d*x)] + 3*(2*a + b
)*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 1.70, size = 67, normalized size = 0.75

method	result
default	$\frac{\left(-\frac{b}{32}+\frac{a}{2}\right)\sinh(2dx+2c)}{2d}+\frac{\left(\frac{b}{16}+\frac{a}{8}\right)\sinh(4dx+4c)}{4d}+\frac{3ax}{8}-\frac{bx}{16}+\frac{b\sinh(6dx+6c)}{192d}$
risch	$-\frac{bx}{16}+\frac{3ax}{8}+\frac{b\,\mathrm{e}^{6dx+6c}}{384d}+\frac{\mathrm{e}^{4dx+4c}a}{64d}+\frac{\mathrm{e}^{4dx+4c}b}{128d}-\frac{\mathrm{e}^{2dx+2c}b}{128d}+\frac{\mathrm{e}^{2dx+2c}a}{8d}+\frac{\mathrm{e}^{-2dx-2c}b}{128d}-\frac{\mathrm{e}^{-2dx-2c}a}{8d}-\frac{\mathrm{e}^{-4dx-4c}a}{64d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1/32*b+1/2*a)*sinh(2*d*x+2*c)/d+1/4*(1/16*b+1/8*a)*sinh(4*d*x+4*c)/d+
3/8*a*x-1/16*b*x+1/192*b*sinh(6*d*x+6*c)/d
```

Maxima [A]
time = 0.27, size = 152, normalized size = 1.71

$$\frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{1}{384}\,b\left(\frac{\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+1\right)e^{(6\,dx+6\,c)}}{d}-\frac{24\,(dx+c)}{d}+\frac{3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-e^{(-6\,dx-6\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + 1/384*b*((3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c
) + 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d + (3*e^(-2*d*x - 2*c) - 3*e^(-4*d
*x - 4*c) - e^(-6*d*x - 6*c))/d)
```

Fricas [A]
time = 0.37, size = 117, normalized size = 1.31

$$\frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+2\,\left(5\,b\cosh\left(dx+c\right)^{3}+3\,\left(2\,a+b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+6\,\left(6\,a-b\right)dx+3\,\left(b\cosh\left(dx+c\right)^{5}+2\,\left(2\,a+b\right)\cosh\left(dx+c\right)^{3}+\left(16\,a-b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 + 3*(2*a +
 b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(6*a - b)*d*x + 3*(b*cosh(d*x + c)^5
 + 2*(2*a + b)*cosh(d*x + c)^3 + (16*a - b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(76) = 152$.
time = 0.46, size = 250, normalized size = 2.81

$$\begin{cases} \frac{3ax\sinh^{4}{\left(c+dx\right)}}{8}-\frac{3ax\sinh^{2}{\left(c+dx\right)}\cosh^{2}{\left(c+dx\right)}}{4}+\frac{3ax\cosh^{4}{\left(c+dx\right)}}{8}-\frac{3a\sinh^{3}{\left(c+dx\right)}\cosh{\left(c+dx\right)}}{8d}+\frac{5a\sinh{\left(c+dx\right)}\cosh^{3}{\left(c+dx\right)}}{8d}+\frac{bx\sinh^{6}{\left(c+dx\right)}}{16}-\frac{3bx\sinh^{4}{\left(c+dx\right)}\cosh^{2}{\left(c+dx\right)}}{16}+\frac{3bx\sinh^{2}{\left(c+dx\right)}\cosh^{4}{\left(c+dx\right)}}{16}-\frac{bx\cosh^{6}{\left(c+dx\right)}}{16}-\frac{b\sinh^{5}{\left(c+dx\right)}\cosh{\left(c+dx\right)}}{16d}+\frac{b\sinh^{3}{\left(c+dx\right)}\cosh^{3}{\left(c+dx\right)}}{6d}+\frac{b\sinh{\left(c+dx\right)}\cosh^{5}{\left(c+dx\right)}}{16d} & \text{for}\: d \neq 0 \\x\left(a+b\sinh^{2}{\left(c\right)}\right)\cosh^{4}{\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 - 3*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
+ 5*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + b*x*sinh(c + d*x)**6/16 - 3*b*
```

```
x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 3*b*x*sinh(c + d*x)**2*cosh(c + d*
x)**4/16 - b*x*cosh(c + d*x)**6/16 - b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d
) + b*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + b*sinh(c + d*x)*cosh(c + d*
x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c)**4, True))
```

Giac [A]

time = 0.42, size = 121, normalized size = 1.36

$$\frac{1}{16}\left(6\,a-b\right)x+\frac{be^{\left(6\,dx+6\,c\right)}}{384\,d}+\frac{\left(2\,a+b\right)e^{\left(4\,dx+4\,c\right)}}{128\,d}+\frac{\left(16\,a-b\right)e^{\left(2\,dx+2\,c\right)}}{128\,d}-\frac{\left(16\,a-b\right)e^{\left(-2\,dx-2\,c\right)}}{128\,d}-\frac{\left(2\,a+b\right)e^{\left(-4\,dx-4\,c\right)}}{128\,d}-\frac{be^{\left(-6\,dx-6\,c\right)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*(6*a - b)*x + 1/384*b*e^(6*d*x + 6*c)/d + 1/128*(2*a + b)*e^(4*d*x + 4
*c)/d + 1/128*(16*a - b)*e^(2*d*x + 2*c)/d - 1/128*(16*a - b)*e^(-2*d*x - 2
*c)/d - 1/128*(2*a + b)*e^(-4*d*x - 4*c)/d - 1/384*b*e^(-6*d*x - 6*c)/d
```

Mupad [B]

time = 1.43, size = 76, normalized size = 0.85

$$\frac{12\,a\sinh\left(2\,c+2\,d\,x\right)+\frac{3\,a\sinh\left(4\,c+4\,d\,x\right)}{2}-\frac{3\,b\sinh\left(2\,c+2\,d\,x\right)}{4}+\frac{3\,b\sinh\left(4\,c+4\,d\,x\right)}{4}+\frac{b\sinh\left(6\,c+6\,d\,x\right)}{4}+18\,a\,d\,x-3\,b\,d\,x}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (12*a*sinh(2*c + 2*d*x) + (3*a*sinh(4*c + 4*d*x))/2 - (3*b*sinh(2*c + 2*d*x
))/4 + (3*b*sinh(4*c + 4*d*x))/4 + (b*sinh(6*c + 6*d*x))/4 + 18*a*d*x - 3*b
*d*x)/(48*d)
```

3.284 $\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=46

$$\frac{a\sinh(c+dx)}{d}+\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{b\sinh^5(c+dx)}{5d}$$

```
[Out] a*sinh(d*x+c)/d+1/3*(a+b)*sinh(d*x+c)^3/d+1/5*b*sinh(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3269, 380}

$$\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Sinh[c + d*x])/d + ((a + b)*Sinh[c + d*x]^3)/(3*d) + (b*Sinh[c + d*x]^5)
/(5*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+bx^2\right)\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a+(a+b)x^2+bx^4\right)\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{a\sinh(c+dx)}{d}+\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{b\sinh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [**A**]
time = 0.11, size = 48, normalized size = 1.04

$$\frac{(100a - 11b + 4(5a + 2b)\cosh(2(c + dx)) + 3b\cosh(4(c + dx)))\sinh(c + dx)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((100*a - 11*b + 4*(5*a + 2*b)*Cosh[2*(c + d*x)] + 3*b*Cosh[4*(c + d*x)])*S
inh[c + d*x])/(120*d)
```

Maple [**A**]
time = 1.64, size = 55, normalized size = 1.20

method	result
default	$\frac{\left(-\frac{b}{8}+\frac{3a}{4}\right)\sinh(dx+c)}{d} + \frac{\left(\frac{b}{16}+\frac{a}{4}\right)\sinh(3dx+3c)}{3d} + \frac{b\sinh(5dx+5c)}{80d}$
risch	$\frac{b\,\mathrm{e}^{5dx+5c}}{160d} + \frac{\mathrm{e}^{3dx+3c}a}{24d} + \frac{\mathrm{e}^{3dx+3c}b}{96d} + \frac{3a\,\mathrm{e}^{dx+c}}{8d} - \frac{b\,\mathrm{e}^{dx+c}}{16d} - \frac{3\,\mathrm{e}^{-dx-c}a}{8d} + \frac{\mathrm{e}^{-dx-c}b}{16d} - \frac{\mathrm{e}^{-3dx-3c}a}{24d} - \frac{\mathrm{e}^{-3dx-3c}b}{96d} - \frac{b\,\mathrm{e}^{-5}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8*b+3/4*a)*sinh(d*x+c)/d+1/3*(1/16*b+1/4*a)*sinh(3*d*x+3*c)/d+1/80*b*si
nh(5*d*x+5*c)/d
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(42) = 84$.
time = 0.26, size = 136, normalized size = 2.96

$$\frac{1}{480}\,b\left(\frac{\left(5\,e^{(-2\,dx-2\,c)} - 30\,e^{(-4\,dx-4\,c)} + 3\right)e^{(5\,dx+5\,c)}}{d} + \frac{30\,e^{(-dx-c)} - 5\,e^{(-3\,dx-3\,c)} - 3\,e^{(-5\,dx-5\,c)}}{d}\right) + \frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*b*((5*e^(-2*d*x - 2*c) - 30*e^(-4*d*x - 4*c) + 3)*e^(5*d*x + 5*c)/d +
 (30*e^(-d*x - c) - 5*e^(-3*d*x - 3*c) - 3*e^(-5*d*x - 5*c))/d) + 1/24*a*(e
^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Fricas [**A**]
time = 0.38, size = 82, normalized size = 1.78

$$\frac{3\,b\sinh\left(dx+c\right)^{5} + 5\left(6\,b\cosh\left(dx+c\right)^{2} + 4\,a + b\right)\sinh\left(dx+c\right)^{3} + 15\left(b\cosh\left(dx+c\right)^{4} + (4\,a+b)\cosh\left(dx+c\right)^{2} + 12\,a - 2\,b\right)\sinh\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/240*(3*b*sinh(d*x + c)^5 + 5*(6*b*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c
)^3 + 15*(b*cosh(d*x + c)^4 + (4*a + b)*cosh(d*x + c)^2 + 12*a - 2*b)*sinh(
d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(39) = 78$.
time = 0.28, size = 85, normalized size = 1.85

$$\begin{cases} -\frac{2a\sinh^3(c+dx)}{3d} + \frac{a\sinh(c+dx)\cosh^2(c+dx)}{d} - \frac{2b\sinh^5(c+dx)}{15d} + \frac{b\sinh^3(c+dx)\cosh^2(c+dx)}{3d} & \text{for } d \neq 0 \\ x\left(a + b\sinh^2(c)\right)\cosh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((-2*a*sinh(c + d*x)**3/(3*d) + a*sinh(c + d*x)*cosh(c + d*x)**2/d
 - 2*b*sinh(c + d*x)**5/(15*d) + b*sinh(c + d*x)**3*cosh(c + d*x)**2/(3*d),
 Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c)**3, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(42) = 84$.
time = 0.42, size = 108, normalized size = 2.35

$$\frac{be^{(5\,dx+5\,c)}}{160\,d} + \frac{(4\,a+b)e^{(3\,dx+3\,c)}}{96\,d} + \frac{(6\,a-b)e^{(dx+c)}}{16\,d} - \frac{(6\,a-b)e^{(-dx-c)}}{16\,d} - \frac{(4\,a+b)e^{(-3\,dx-3\,c)}}{96\,d} - \frac{be^{(-5\,dx-5\,c)}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/160*b*e^(5*d*x + 5*c)/d + 1/96*(4*a + b)*e^(3*d*x + 3*c)/d + 1/16*(6*a -
b)*e^(d*x + c)/d - 1/16*(6*a - b)*e^(-d*x - c)/d - 1/96*(4*a + b)*e^(-3*d*x
 - 3*c)/d - 1/160*b*e^(-5*d*x - 5*c)/d
```

Mupad [**B**]
time = 1.34, size = 48, normalized size = 1.04

$$\frac{15\,a\sinh(c+d\,x) + 5\,a\sinh(c+d\,x)^3 + 5\,b\sinh(c+d\,x)^3 + 3\,b\sinh(c+d\,x)^5}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (15*a*sinh(c + d*x) + 5*a*sinh(c + d*x)^3 + 5*b*sinh(c + d*x)^3 + 3*b*sinh(
c + d*x)^5)/(15*d)
```

3.285 $\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{1}{8}(4a-b)x+\frac{(4a-b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

```
[Out] 1/8*(4*a-b)*x+1/8*(4*a-b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b*cosh(d*x+c)^3*sin
h(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3270, 393, 205, 212}

$$\frac{(4a-b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(4a-b)+\frac{b\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((4*a - b)*x)/8 + ((4*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c
 + d*x]^3*Sinh[c + d*x])/(4*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx=\frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{(1-x^2)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{b\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{(4a-b)\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx,x,\text{ta}\right.}{4d}$$

$$=\frac{(4a-b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh^3(c+dx)\sinh(c+d}{4d}$$

$$=\frac{1}{8}(4a-b)x+\frac{(4a-b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh^3(c+}{}$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 0.70

$$\frac{16ac+16adx-4bdx+8a\sinh(2(c+dx))+b\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (16*a*c + 16*a*d*x - 4*b*d*x + 8*a*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)])
/(32*d)
```

Maple [A]
time = 1.04, size = 70, normalized size = 1.15

method	result	size
derivativedivides	$\frac{b\left(\frac{\sinh(dx+c)\left(\cosh^3(dx+c)\right)}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	70
default	$\frac{b\left(\frac{\sinh(dx+c)\left(\cosh^3(dx+c)\right)}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	70
risch	$\frac{ax}{2}-\frac{bx}{8}+\frac{e^{4dx+4c}b}{64d}+\frac{e^{2dx+2c}a}{8d}-\frac{e^{-2dx-2c}a}{8d}-\frac{e^{-4dx-4c}b}{64d}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1
/8*c)+a*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.30, size = 76, normalized size = 1.25

$$\frac{1}{8}\,a\left(4\,x+\frac{e^{\left(2\,dx+2\,c\right)}}{d}-\frac{e^{\left(-2\,dx-2\,c\right)}}{d}\right)-\frac{1}{64}\,b\left(\frac{8\,\left(dx+c\right)}{d}-\frac{e^{\left(4\,dx+4\,c\right)}}{d}+\frac{e^{\left(-4\,dx-4\,c\right)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/64*b*(8*(d*x + c)/
d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Fricas [A]
time = 0.36, size = 59, normalized size = 0.97

$$\frac{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\left(4\,a-b\right)dx+\left(b\cosh\left(dx+c\right)^{3}+4\,a\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a - b)*d*x + (b*cosh(d*x + c)^3 +
 4*a*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(49) = 98$.
time = 0.19, size = 150, normalized size = 2.46

$$\begin{cases} -\frac{ax\sinh^{2}{\left(c+dx\right)}}{2}+\frac{ax\cosh^{2}{\left(c+dx\right)}}{2}+\frac{a\sinh{\left(c+dx\right)}\cosh{\left(c+dx\right)}}{2d}-\frac{bx\sinh^{4}{\left(c+dx\right)}}{8}+\frac{bx\sinh^{2}{\left(c+dx\right)}\cosh^{2}{\left(c+dx\right)}}{4}-\frac{bx\cosh^{4}{\left(c+dx\right)}}{8}+\frac{b\sinh^{3}{\left(c+dx\right)}\cosh{\left(c+dx\right)}}{8d}+\frac{b\sinh{\left(c+dx\right)}\cosh^{3}{\left(c+dx\right)}}{8d} & \text{for}\: d\neq 0 \\ x\left(a+b\sinh^{2}{\left(c\right)}\right)\cosh^{2}{\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((-a*x*sinh(c + d*x)**2/2 + a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*
x)*cosh(c + d*x)/(2*d) - b*x*sinh(c + d*x)**4/8 + b*x*sinh(c + d*x)**2*cosh
(c + d*x)**2/4 - b*x*cosh(c + d*x)**4/8 + b*sinh(c + d*x)**3*cosh(c + d*x)/
(8*d) + b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sinh(c
)**2)*cosh(c)**2, True))
```

Giac [A]
time = 0.42, size = 71, normalized size = 1.16

$$\frac{1}{8}\left(4\,a-b\right)x+\frac{be^{\left(4\,dx+4\,c\right)}}{64\,d}+\frac{ae^{\left(2\,dx+2\,c\right)}}{8\,d}-\frac{ae^{\left(-2\,dx-2\,c\right)}}{8\,d}-\frac{be^{\left(-4\,dx-4\,c\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*a - b)*x + 1/64*b*e^(4*d*x + 4*c)/d + 1/8*a*e^(2*d*x + 2*c)/d - 1/8*
a*e^(-2*d*x - 2*c)/d - 1/64*b*e^(-4*d*x - 4*c)/d
```

Mupad [B]
time = 0.10, size = 38, normalized size = 0.62

$$\frac{a\,x}{2}-\frac{b\,x}{8}+\frac{\frac{a\,\mathrm{sinh}\left(2\,c+2\,d\,x\right)}{4}+\frac{b\,\mathrm{sinh}\left(4\,c+4\,d\,x\right)}{32}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (a*x)/2 - (b*x)/8 + ((a*sinh(2*c + 2*d*x))/4 + (b*sinh(4*c + 4*d*x))/32)/d
```

3.286 $\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}$$

```
[Out] a*sinh(d*x+c)/d+1/3*b*sinh(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.053$, Rules used = {3269}

$$\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Sinh[c + d*x])/d + (b*Sinh[c + d*x]^3)/(3*d)
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx&=\frac{\operatorname{Subst}\left(\int \left(a+bx^2\right)\,dx,x,\sinh(c+dx)\right)}{d}\\&=\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.39

$$\frac{a\cosh(dx)\sinh(c)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x])/d + (b*Sinh[c + d*x]^3)/(3*
d)
```

Maple [A]
time = 0.57, size = 25, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\frac{b\left(\sinh^3(dx+c)\right)}{3}+a\sinh(dx+c)}{d}$	25
default	$\frac{\frac{b\left(\sinh^3(dx+c)\right)}{3}+a\sinh(dx+c)}{d}$	25
risch	$\frac{e^{3dx+3c}b}{24d}+\frac{a\,e^{dx+c}}{2d}-\frac{b\,e^{dx+c}}{8d}-\frac{e^{-dx-c}a}{2d}+\frac{e^{-dx-c}b}{8d}-\frac{e^{-3dx-3c}b}{24d}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b*sinh(d*x+c)^3+a*sinh(d*x+c))
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.93

$$\frac{b\sinh\left(dx+c\right)^3}{3\,d}+\frac{a\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*b*sinh(d*x + c)^3/d + a*sinh(d*x + c)/d
```

Fricas [A]
time = 0.58, size = 41, normalized size = 1.46

$$\frac{b\sinh\left(dx+c\right)^3+3\left(b\cosh\left(dx+c\right)^2+4\,a-b\right)\sinh\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(b*sinh(d*x + c)^3 + 3*(b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))/d
```

Sympy [A]
time = 0.11, size = 36, normalized size = 1.29

$$\begin{cases}\frac{a\sinh\left(c+dx\right)}{d}+\frac{b\sinh^3\left(c+dx\right)}{3d} & \text{for } d\neq 0\\ x\left(a+b\sinh^2\left(c\right)\right)\cosh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)/d + b*sinh(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a +
 b*sinh(c)**2)*cosh(c), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(26) = 52$.
time = 0.41, size = 70, normalized size = 2.50

$$\frac{be^{(3\,dx+3\,c)}}{24\,d} + \frac{(4\,a-b)e^{(dx+c)}}{8\,d} - \frac{(4\,a-b)e^{(-dx-c)}}{8\,d} - \frac{be^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*b*e^(3*d*x + 3*c)/d + 1/8*(4*a - b)*e^(d*x + c)/d - 1/8*(4*a - b)*e^(-
d*x - c)/d - 1/24*b*e^(-3*d*x - 3*c)/d
```

Mupad [**B**]
time = 0.09, size = 25, normalized size = 0.89

$$\frac{\sinh(c+d\,x)\,\left(b\sinh(c+d\,x)^2+3\,a\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (sinh(c + d*x)*(3*a + b*sinh(c + d*x)^2))/(3*d)
```

3.287 $\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{(a-b)\text{ArcTan}(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}$$

```
[Out] (a-b)*arctan(sinh(d*x+c))/d+b*sinh(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3269, 396, 209}

$$\frac{(a-b)\text{ArcTan}(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - b)*ArcTan[Sinh[c + d*x]])/d + (b*Sinh[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a+bx^2}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{b\sinh(c+dx)}{d}+\frac{(a-b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{(a-b)\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 1.32

$$\frac{a\text{ArcTan}(\sinh(c+dx))}{d}-\frac{b\text{ArcTan}(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/d - (b*ArcTan[Sinh[c + d*x]])/d + (b*Sinh[c + d*x
])/d
```

Maple [A]
time = 0.83, size = 34, normalized size = 1.21

method	result	size
derivativedivides	$\frac{2a\arctan\left(e^{dx+c}\right)+b\left(\sinh(dx+c)-2\arctan\left(e^{dx+c}\right)\right)}{d}$	34
default	$\frac{2a\arctan\left(e^{dx+c}\right)+b\left(\sinh(dx+c)-2\arctan\left(e^{dx+c}\right)\right)}{d}$	34
risch	$\frac{b\,e^{dx+c}}{2d}-\frac{e^{-dx-c}b}{2d}+\frac{i\ln\left(e^{dx+c}+i\right)a}{d}-\frac{i\ln\left(e^{dx+c}+i\right)b}{d}-\frac{i\ln\left(e^{dx+c}-i\right)a}{d}+\frac{i\ln\left(e^{dx+c}-i\right)b}{d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a*arctan(exp(d*x+c))+b*(sinh(d*x+c)-2*arctan(exp(d*x+c))))
```

Maxima [A]
time = 0.49, size = 56, normalized size = 2.00

$$\frac{1}{2}b\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)+\frac{a\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a*arcta
n(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(28) = 56$.
time = 0.39, size = 101, normalized size = 3.61

$$\frac{b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}+4\left(\left(a-b\right)\cosh\left(dx+c\right)+\left(a-b\right)\sinh\left(dx+c\right)\right)\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)-b}{2\left(d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^
2 + 4*((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*arctan(cosh(d*x + c)
+ sinh(d*x + c)) - b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^{2}\left(c+dx\right)\right)\operatorname{sech}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x), x)
```

Giac [A]
time = 0.43, size = 40, normalized size = 1.43

$$\frac{4\left(a-b\right)\arctan\left(e^{\left(dx+c\right)}\right)+be^{\left(dx+c\right)}-be^{\left(-dx-c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(4*(a - b)*arctan(e^(d*x + c)) + b*e^(d*x + c) - b*e^(-d*x - c))/d
```

Mupad [B]
time = 1.78, size = 88, normalized size = 3.14

$$\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a\,\sqrt{d^{2}}-b\,\sqrt{d^{2}}\right)}{d\,\sqrt{a^{2}-2\,a\,b+b^{2}}}\right)\,\sqrt{a^{2}-2\,a\,b+b^{2}}}{\sqrt{d^{2}}}-\frac{b\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{b\,\mathrm{e}^{c+d\,x}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/cosh(c + d*x),x)
```

```
[Out] (2*atan((exp(d*x)*exp(c)*(a*(d^2)^(1/2) - b*(d^2)^(1/2)))/(d*(a^2 - 2*a*b +
 b^2)^(1/2)))*(a^2 - 2*a*b + b^2)^(1/2))/(d^2)^(1/2) - (b*exp(- c - d*x))/(
2*d) + (b*exp(c + d*x))/(2*d)
```

3.288 $\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=19

$$bx+\frac{(a-b)\tanh(c+dx)}{d}$$

```
[Out] b*x+(a-b)*tanh(d*x+c)/d
```

Rubi [A]

time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3270, 396, 212}

$$\frac{(a-b)\tanh(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] b*x + ((a - b)*Tanh[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{(a-b)\tanh(c+dx)}{d} + \frac{b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= bx + \frac{(a-b)\tanh(c+dx)}{d}$$

Mathematica [**A**]
time = 0.02, size = 36, normalized size = 1.89

$$\frac{b\tanh^{-1}(\tanh(c+dx))}{d} + \frac{a\tanh(c+dx)}{d} - \frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
[Out] (b*ArcTanh[Tanh[c + d*x]])/d + (a*Tanh[c + d*x])/d - (b*Tanh[c + d*x])/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.
time = 1.81, size = 43, normalized size = 2.26

method	result	size
risch	$bx - \frac{2a}{d\left(1+e^{2dx+2c}\right)} + \frac{2b}{d\left(1+e^{2dx+2c}\right)}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] b*x-2/d/(1+exp(2*d*x+2*c))*a+2/d/(1+exp(2*d*x+2*c))*b
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.
time = 0.30, size = 47, normalized size = 2.47

$$b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right) + \frac{2\,a}{d(e^{(-2\,dx-2\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
[Out] b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 2*a/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.
time = 0.38, size = 41, normalized size = 2.16

$$\frac{(bdx - a + b)\cosh(dx + c) + (a - b)\sinh(dx + c)}{d\cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] ((b*d*x - a + b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^2(c + dx)\right)\operatorname{sech}^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**2, x)
```

Giac [**A**]
time = 0.41, size = 32, normalized size = 1.68

$$\frac{(dx + c)b - \frac{2(a-b)}{e^{(2dx+2c)}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] ((d*x + c)*b - 2*(a - b)/(e^(2*d*x + 2*c) + 1))/d
```

Mupad [**B**]
time = 0.80, size = 27, normalized size = 1.42

$$b\,x - \frac{2\,(a - b)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^2,x)
```

```
[Out] b*x - (2*(a - b))/(d*(exp(2*c + 2*d*x) + 1))
```

3.289 $\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=42

$$\frac{(a+b)\mathrm{ArcTan}(\sinh(c+dx))}{2d}+\frac{(a-b)\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] 1/2*(a+b)*arctan(sinh(d*x+c))/d+1/2*(a-b)*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3269, 393, 209}

$$\frac{(a+b)\mathrm{ArcTan}(\sinh(c+dx))}{2d}+\frac{(a-b)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((a - b)*Sech[c + d*x]*Tanh[c + d*x
])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-b)\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \mathrm{s}\right.}{2d}$$

$$= \frac{(a+b)\tan^{-1}(\sinh(c+dx))}{2d} + \frac{(a-b)\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 1.69

$$\frac{a\text{ArcTan}(\sinh(c+dx))}{2d} + \frac{b\text{ArcTan}(\sinh(c+dx))}{2d} + \frac{a\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/(2*d) + (b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*Sech
[c + d*x]*Tanh[c + d*x])/(2*d) - (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Maple [C] Result contains complex when optimal does not.
time = 2.00, size = 109, normalized size = 2.60

method	result	size
risch	$\frac{e^{dx+c}(a-b)\left(e^{2dx+2c}-1\right)}{d\left(1+e^{2dx+2c}\right)^2}+\frac{i\ln\left(e^{dx+c}+i\right)a}{2d}+\frac{i\ln\left(e^{dx+c}+i\right)b}{2d}-\frac{i\ln\left(e^{dx+c}-i\right)a}{2d}-\frac{i\ln\left(e^{dx+c}-i\right)b}{2d}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] exp(d*x+c)*(a-b)*(exp(2*d*x+2*c)-1)/d/(1+exp(2*d*x+2*c))^2+1/2*I/d*ln(exp(d
*x+c)+I)*a+1/2*I/d*ln(exp(d*x+c)+I)*b-1/2*I/d*ln(exp(d*x+c)-I)*a-1/2*I/d*ln
(exp(d*x+c)-I)*b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(38) = 76.
time = 0.47, size = 136, normalized size = 3.24

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) - a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a*(arctan(e^(-d*x - c))/d - (e^(-d*x
 - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(38) = 76$.
time = 0.38, size = 324, normalized size = 7.71

$$\frac{(a-b)\cosh(dx+c)^3+3(a-b)\cosh(dx+c)\sinh(dx+c)^2+(a-b)\sinh(dx+c)^3+((a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2(a+b)\cosh(dx+c)^2+2(3(a+b)\cosh(dx+c)^2+a+b)\sinh(dx+c)^2+4((a+b)\cosh(dx+c)^3+(a+b)\cosh(dx+c))\sinh(dx+c)+a+b)\arctan(\cosh(dx+c)+\sinh(dx+c))-(a-b)\cosh(dx+c)+(3(a-b)\cosh(dx+c)^2-a+b)\sinh(dx+c)}{d\cosh(dx+c)^4+4d\cosh(dx+c)\sinh(dx+c)^3+d\sinh(dx+c)^4+2d\cosh(dx+c)^2+2(3d\cosh(dx+c)^2+d)\sinh(dx+c)^2+4(d\cosh(dx+c)^3+d\cosh(dx+c))\sinh(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] ((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b
)*sinh(d*x + c)^3 + ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a
 + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3
 + (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*arctan(cosh(d*x + c) + sin
h(d*x + c)) - (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 - a + b)*s
inh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*si
nh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^2(c+dx)\right)\operatorname{sech}^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(38) = 76$.
time = 0.42, size = 105, normalized size = 2.50

$$\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)(a+b)+\frac{4\left(a\left(e^{(dx+c)}-e^{(-dx-c)}\right)-b\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a + b) + 4*(a
*(e^(d*x + c) - e^(-d*x - c)) - b*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x +
c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [B]
time = 0.12, size = 127, normalized size = 3.02

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a\,\sqrt{d^2}+b\,\sqrt{d^2}\right)}{d\,\sqrt{a^2+2\,a\,b+b^2}}\right)\,\sqrt{a^2+2\,a\,b+b^2}}{\sqrt{d^2}}+\frac{\mathrm{e}^{c+d\,x}\,\left(a-b\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\mathrm{e}^{c+d\,x}\,\left(a-b\right)}{d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(2*a*b + a^2 + b
^2)^(1/2)))*(2*a*b + a^2 + b^2)^(1/2))/(d^2)^(1/2) + (exp(c + d*x)*(a - b))
/(d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(a - b))/(d*(2*exp(2*c + 2*d*
x) + exp(4*c + 4*d*x) + 1))
```

3.290 $\int \mathbf{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=32

$$\frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}$$

```
[Out] a*tanh(d*x+c)/d-1/3*(a-b)*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = $\{3270\}$

$$\frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d - ((a - b)*Tanh[c + d*x]^3)/(3*d)
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \left(a-(a-b)x^2\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.38

$$\frac{a\tanh(c+dx)}{d}-\frac{a\tanh^3(c+dx)}{3d}+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^3)/(3*d)
```

Maple [A]
time = 1.54, size = 48, normalized size = 1.50

method	result	size
risch	$-\frac{2\left(3b\,\mathrm{e}^{4dx+4c}+6a\,\mathrm{e}^{2dx+2c}+2a+b\right)}{3d\left(1+\mathrm{e}^{2dx+2c}\right)^{3}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(3*b*exp(4*d*x+4*c)+6*a*exp(2*d*x+2*c)+2*a+b)/d/(1+exp(2*d*x+2*c))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(30) = 60$.
time = 0.28, size = 185, normalized size = 5.78

$$\frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)+\frac{2}{3}\,b\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) + 1))) + 2/3*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e
^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(
-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(30) = 60$.
time = 0.36, size = 159, normalized size = 4.97

$$-\frac{4\,\left((a+2\,b)\cosh\left(dx+c\right)^{2}-2\,(a-b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a+2\,b)\sinh\left(dx+c\right)^{2}+3\,a\right)}{3\,\left(d\cosh\left(dx+c\right)^{4}+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{4}+4\,d\cosh\left(dx+c\right)^{2}+2\,\left(3\,d\cosh\left(dx+c\right)^{2}+2\,d\right)\sinh\left(dx+c\right)^{2}+4\,\left(d\cosh\left(dx+c\right)^{3}+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4/3*((a + 2*b)*cosh(d*x + c)^2 - 2*(a - b)*cosh(d*x + c)*sinh(d*x + c) + (
a + 2*b)*sinh(d*x + c)^2 + 3*a)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh
(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c
)^2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d
*x + c) + 3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^{2}\left(c+dx\right)\right)\operatorname{sech}^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**4, x)
```

Giac [A]
time = 0.40, size = 47, normalized size = 1.47

$$-\frac{2\left(3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+2\,a+b\right)}{3\,d(e^{(2\,dx+2\,c)}+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 2*a + b)/(d*(e^(2*d*x + 2
*c) + 1)^3)
```

Mupad [B]
time = 0.82, size = 47, normalized size = 1.47

$$-\frac{2\left(2\,a+b+6\,a\,\mathrm{e}^{2\,c+2\,d\,x}+3\,b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^4,x)
```

```
[Out] -(2*(2*a + b + 6*a*exp(2*c + 2*d*x) + 3*b*exp(4*c + 4*d*x)))/(3*d*(exp(2*c
+ 2*d*x) + 1)^3)
```

3.291 $\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=70

$$\frac{(3a+b)\text{ArcTan}(\sinh(c+dx))}{8d}+\frac{(3a+b)\text{sech}(c+dx)\tanh(c+dx)}{8d}+\frac{(a-b)\text{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

```
[Out] 1/8*(3*a+b)*arctan(sinh(d*x+c))/d+1/8*(3*a+b)*sech(d*x+c)*tanh(d*x+c)/d+1/4
*(a-b)*sech(d*x+c)^3*tanh(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3269, 393, 205, 209}

$$\frac{(3a+b)\text{ArcTan}(\sinh(c+dx))}{8d}+\frac{(a-b)\tanh(c+dx)\text{sech}^3(c+dx)}{4d}+\frac{(3a+b)\tanh(c+dx)\text{sech}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((3*a + b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((3*a + b)*Sech[c + d*x]*Tanh[c +
 d*x])/(8*d) + ((a - b)*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\Big(\int \frac{a+bx^2}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\Big)}{d}\\
&= \frac{(a-b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d} + \frac{(3a+b)\operatorname{Subst}\Big(\int \frac{1}{(1+x^2)^2}}{4d}\\
&= \frac{(3a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{(a-b)\operatorname{sech}^3(c+dx)\tan}{4d}\\
&= \frac{(3a+b)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{(3a+b)\operatorname{sech}(c+dx)\tanh(c+}{8d}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 60, normalized size = 0.86

$$\frac{(3a+b)\operatorname{ArcTan}(\sinh(c+dx)) + (3a+b)\operatorname{sech}(c+dx)\tanh(c+dx) + 2(a-b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((3*a + b)*ArcTan[Sinh[c + d*x]] + (3*a + b)*Sech[c + d*x]*Tanh[c + d*x] +
2*(a - b)*Sech[c + d*x]^3*Tanh[c + d*x])/(8*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.29, size = 172, normalized size = 2.46

method	result
risch	$\frac{e^{dx+c}\left(3a\,e^{6dx+6c}+b\,e^{6dx+6c}+11a\,e^{4dx+4c}-7b\,e^{4dx+4c}-11a\,e^{2dx+2c}+7b\,e^{2dx+2c}-3a-b\right)}{4d\left(1+e^{2dx+2c}\right)^4} + \frac{3i\ln\left(e^{dx+c}+i\right)a}{8d} + \frac{i\ln\left(e^{dx+c}+i\right)b}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp(d*x+c)*(3*a*exp(6*d*x+6*c)+b*exp(6*d*x+6*c)+11*a*exp(4*d*x+4*c)-7*b
*exp(4*d*x+4*c)-11*a*exp(2*d*x+2*c)+7*b*exp(2*d*x+2*c)-3*a-b)/d/(1+exp(2*d*
x+2*c))^4+3/8*I/d*ln(exp(d*x+c)+I)*a+1/8*I/d*ln(exp(d*x+c)+I)*b-3/8*I/d*ln(
exp(d*x+c)-I)*a-1/8*I/d*ln(exp(d*x+c)-I)*b
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(64) = 128$.
time $= 0.47$, size $= 228$, normalized size $= 3.26$

$$-\frac{1}{4}\,a\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)-\frac{1}{4}\,b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7\,e^{(-3\,dx-3\,c)}+7\,e^{(-5\,dx-5\,c)}-e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) -
11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/4*b*(arctan(e
^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c) -
e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
 - 6*c) + e^(-8*d*x - 8*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(64) = 128$.
time $= 0.38$, size $= 1046$, normalized size $= 14.94$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((3*a + b)*cosh(d*x + c)^7 + 7*(3*a + b)*cosh(d*x + c)*sinh(d*x + c)^6
+ (3*a + b)*sinh(d*x + c)^7 + (11*a - 7*b)*cosh(d*x + c)^5 + (21*(3*a + b)*
cosh(d*x + c)^2 + 11*a - 7*b)*sinh(d*x + c)^5 + 5*(7*(3*a + b)*cosh(d*x + c
)^3 + (11*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (11*a - 7*b)*cosh(d*x +
 c)^3 + (35*(3*a + b)*cosh(d*x + c)^4 + 10*(11*a - 7*b)*cosh(d*x + c)^2 - 1
1*a + 7*b)*sinh(d*x + c)^3 + (21*(3*a + b)*cosh(d*x + c)^5 + 10*(11*a - 7*b
)*cosh(d*x + c)^3 - 3*(11*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a +
 b)*cosh(d*x + c)^8 + 8*(3*a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + b)
*sinh(d*x + c)^8 + 4*(3*a + b)*cosh(d*x + c)^6 + 4*(7*(3*a + b)*cosh(d*x +
c)^2 + 3*a + b)*sinh(d*x + c)^6 + 8*(7*(3*a + b)*cosh(d*x + c)^3 + 3*(3*a +
 b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a + b)*cosh(d*x + c)^4 + 2*(35*(3
*a + b)*cosh(d*x + c)^4 + 30*(3*a + b)*cosh(d*x + c)^2 + 9*a + 3*b)*sinh(d*
x + c)^4 + 8*(7*(3*a + b)*cosh(d*x + c)^5 + 10*(3*a + b)*cosh(d*x + c)^3 +
3*(3*a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a + b)*cosh(d*x + c)^2 +
4*(7*(3*a + b)*cosh(d*x + c)^6 + 15*(3*a + b)*cosh(d*x + c)^4 + 9*(3*a + b)
*cosh(d*x + c)^2 + 3*a + b)*sinh(d*x + c)^2 + 8*((3*a + b)*cosh(d*x + c)^7
+ 3*(3*a + b)*cosh(d*x + c)^5 + 3*(3*a + b)*cosh(d*x + c)^3 + (3*a + b)*cos
h(d*x + c))*sinh(d*x + c) + 3*a + b)*arctan(cosh(d*x + c) + sinh(d*x + c))
- (3*a + b)*cosh(d*x + c) + (7*(3*a + b)*cosh(d*x + c)^6 + 5*(11*a - 7*b)*c
osh(d*x + c)^4 - 3*(11*a - 7*b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c))/(
```

```
d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 +
 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d
*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4
 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 +
8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)
^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*
cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^2\left(c + dx\right)\right)\operatorname{sech}^5\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(64) = 128.
time = 0.44, size = 153, normalized size = 2.19

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)(3a+b) + \frac{4\left(3a\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+20a\left(e^{(dx+c)}-e^{(-dx-c)}\right)-4b\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a + b) + 4
*(3*a*(e^(d*x + c) - e^(-d*x - c))^3 + b*(e^(d*x + c) - e^(-d*x - c))^3 + 2
0*a*(e^(d*x + c) - e^(-d*x - c)) - 4*b*(e^(d*x + c) - e^(-d*x - c)))/((e^(d
*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B]
time = 0.83, size = 280, normalized size = 4.00

$$\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(3a\sqrt{d^2}+b\sqrt{d^2}\right)}{d\sqrt{9a^2+6ab+b^2}}\right)\sqrt{9a^2+6ab+b^2}}{4\sqrt{d^2}} - \frac{\frac{b\,e^{5c+5dx}}{d}+\frac{2\,e^{3c+3dx}\,(2a-b)}{d}+\frac{b\,e^{c+dx}}{d}}{4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1} + \frac{e^{c+dx}\,(3a+b)}{4d\,\left(e^{2c+2dx}+1\right)} + \frac{e^{c+dx}\,(a-3b)}{2d\,\left(2\,e^{2c+2dx}+e^{4c+4dx}+1\right)} - \frac{2\,e^{c+dx}\,(a-b)}{d\,\left(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^5,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(3*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(6*a*b + 9*a^2
 + b^2)^(1/2)))*(6*a*b + 9*a^2 + b^2)^(1/2))/(4*(d^2)^(1/2)) - ((b*exp(5*c
```

```
+ 5*d*x))/d + (2*exp(3*c + 3*d*x)*(2*a - b))/d + (b*exp(c + d*x))/d)/(4*exp
(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x)
+ 1) + (exp(c + d*x)*(3*a + b))/(4*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x
)*(a - 3*b))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*exp(c +
 d*x)*(a - b))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*
x) + 1))
```

3.292 $\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=54

$$\frac{a\tanh(c+dx)}{d}-\frac{(2a-b)\tanh^3(c+dx)}{3d}+\frac{(a-b)\tanh^5(c+dx)}{5d}$$

```
[Out] a*tanh(d*x+c)/d-1/3*(2*a-b)*tanh(d*x+c)^3/d+1/5*(a-b)*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3270, 380}

$$\frac{(a-b)\tanh^5(c+dx)}{5d}-\frac{(2a-b)\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d - ((2*a - b)*Tanh[c + d*x]^3)/(3*d) + ((a - b)*Tanh[c +
 d*x]^5)/(5*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx&=\frac{\operatorname{Subst}\left(\int \left(1-x^2\right)\left(a-(a-b)x^2\right)\,dx,x,\tanh(c+dx)\right)}{d}\\&=\frac{\operatorname{Subst}\left(\int \left(a-(2a-b)x^2+(a-b)x^4\right)\,dx,x,\tanh(c+dx)\right)}{d}\\&=\frac{a\tanh(c+dx)}{d}-\frac{(2a-b)\tanh^3(c+dx)}{3d}+\frac{(a-b)\tanh^5(c+}{5d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 102, normalized size = 1.89

$$\frac{a\tanh(c+dx)}{d}+\frac{2b\tanh(c+dx)}{15d}+\frac{b\operatorname{sech}^2(c+dx)\tanh(c+dx)}{15d}-\frac{b\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}-\frac{2a\tanh^3(c+dx)}{3d}+\frac{a\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (2*b*Tanh[c + d*x])/(15*d) + (b*Sech[c + d*x]^2*Tanh[
c + d*x])/(15*d) - (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d) - (2*a*Tanh[c +
d*x]^3)/(3*d) + (a*Tanh[c + d*x]^5)/(5*d)
```

Maple [A]
time = 1.44, size = 84, normalized size = 1.56

method	result	size
risch	$-\frac{4\left(15b\,\mathrm{e}^{6dx+6c}+40a\,\mathrm{e}^{4dx+4c}-5b\,\mathrm{e}^{4dx+4c}+20a\,\mathrm{e}^{2dx+2c}+5b\,\mathrm{e}^{2dx+2c}+4a+b\right)}{15d\left(1+\mathrm{e}^{2dx+2c}\right)^5}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/15*(15*b*exp(6*d*x+6*c)+40*a*exp(4*d*x+4*c)-5*b*exp(4*d*x+4*c)+20*a*exp(
2*d*x+2*c)+5*b*exp(2*d*x+2*c)+4*a+b)/d/(1+exp(2*d*x+2*c))^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(50) = 100$.
time = 0.27, size = 486, normalized size = 9.00

$$\frac{16}{15}a\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{10e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{1}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)+\frac{4}{15}b\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}-\frac{5e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{15e^{(-6dx-6c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{1}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 16/15*a*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) + 4/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c
) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10
*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d
*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(
```

```
5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*
x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(50) = 100$.
time = 0.48, size = 343, normalized size = 6.35

$$-\frac{8\left(2\left(a+4b\right)\cosh\left(dx+c\right)^{3}+6\left(a+4b\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}-\left(2a-7b\right)\sinh\left(dx+c\right)^{3}+30a\cosh\left(dx+c\right)-\left(3\left(2a-7b\right)\cosh\left(dx+c\right)^{2}-10a+5b\right)\sinh\left(dx+c\right)\right)}{15\left(d\cosh\left(dx+c\right)^{7}+7d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{7}+5d\cosh\left(dx+c\right)^{5}+\left(21d\cosh\left(dx+c\right)^{2}+5d\right)\sinh\left(dx+c\right)^{5}+5\left(7d\cosh\left(dx+c\right)^{3}+5d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}+11d\cosh\left(dx+c\right)^{3}+\left(35d\cosh\left(dx+c\right)^{4}+50d\cosh\left(dx+c\right)^{2}+9d\right)\sinh\left(dx+c\right)^{3}+\left(21d\cosh\left(dx+c\right)^{5}+50d\cosh\left(dx+c\right)^{3}+33d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+15d\cosh\left(dx+c\right)+\left(7d\cosh\left(dx+c\right)^{6}+25d\cosh\left(dx+c\right)^{4}+27d\cosh\left(dx+c\right)^{2}+5d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/15*(2*(a + 4*b)*cosh(d*x + c)^3 + 6*(a + 4*b)*cosh(d*x + c)*sinh(d*x + c
)^2 - (2*a - 7*b)*sinh(d*x + c)^3 + 30*a*cosh(d*x + c) - (3*(2*a - 7*b)*cos
h(d*x + c)^2 - 10*a + 5*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x
 + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 + (21*d*cos
h(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5*d*cosh(d*x
 + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 50*
d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 50*d*cos
h(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cosh(d*x + c) + (
7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5*d)*si
nh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 83, normalized size = 1.54

$$-\frac{4\left(15\,be^{\left(6\,dx+6\,c\right)}+40\,ae^{\left(4\,dx+4\,c\right)}-5\,be^{\left(4\,dx+4\,c\right)}+20\,ae^{\left(2\,dx+2\,c\right)}+5\,be^{\left(2\,dx+2\,c\right)}+4\,a+b\right)}{15\,d{\left(e^{\left(2\,dx+2\,c\right)}+1\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -4/15*(15*b*e^(6*d*x + 6*c) + 40*a*e^(4*d*x + 4*c) - 5*b*e^(4*d*x + 4*c) +
20*a*e^(2*d*x + 2*c) + 5*b*e^(2*d*x + 2*c) + 4*a + b)/(d*(e^(2*d*x + 2*c) +
 1)^5)
```

Mupad [B]
time = 0.82, size = 298, normalized size = 5.52

$$-\frac{\frac{8\,b\,e^{2\,c+2\,d\,x}}{5\,d}+\frac{8\,b\,e^{6\,c+6\,d\,x}}{5\,d}+\frac{16\,e^{4\,c+4\,d\,x}\,(2\,a-b)}{5\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1}-\frac{\frac{2\,b}{5\,d}+\frac{6\,b\,e^{4\,c+4\,d\,x}}{5\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(2\,a-b)}{5\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}-\frac{\frac{8\,(2\,a-b)}{15\,d}+\frac{4\,b\,e^{2\,c+2\,d\,x}}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}-\frac{2\,b}{5\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^6,x)
```

```
[Out] - ((8*b*exp(2*c + 2*d*x))/(5*d) + (8*b*exp(6*c + 6*d*x))/(5*d) + (16*exp(4*
c + 4*d*x)*(2*a - b))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10
*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*b)/(
5*d) + (6*b*exp(4*c + 4*d*x))/(5*d) + (8*exp(2*c + 2*d*x)*(2*a - b))/(5*d))
/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c +
8*d*x) + 1) - ((8*(2*a - b))/(15*d) + (4*b*exp(2*c + 2*d*x))/(5*d))/(3*exp(
2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*b)/(5*d*(2*e
xp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.293 $\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=159

$$\frac{1}{128}\left(48a^2-16ab+3b^2\right)x+\frac{(48a^2-16ab+3b^2)\cosh(c+dx)\sinh(c+dx)}{128d}+\frac{(48a^2-16ab+3b^2)\cosh^3(c+}{192d}$$

```
[Out] 1/128*(48*a^2-16*a*b+3*b^2)*x+1/128*(48*a^2-16*a*b+3*b^2)*cosh(d*x+c)*sinh(
d*x+c)/d+1/192*(48*a^2-16*a*b+3*b^2)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/48*(10*a
-3*b)*b*cosh(d*x+c)^5*sinh(d*x+c)/d+1/8*b*cosh(d*x+c)^7*sinh(d*x+c)*(a-(a-b
)*tanh(d*x+c)^2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3270, 424, 393, 205, 212}

$$\frac{(48a^2-16ab+3b^2)\sinh(c+dx)\cosh^3(c+dx)}{192d}+\frac{(48a^2-16ab+3b^2)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x\left(48a^2-16ab+3b^2\right)+\frac{b(10a-3b)\sinh(c+dx)\cosh^5(c+dx)}{48d}+\frac{b\sinh(c+dx)\cosh^7(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((48*a^2 - 16*a*b + 3*b^2)*x)/128 + ((48*a^2 - 16*a*b + 3*b^2)*Cosh[c + d*x
]*Sinh[c + d*x])/(128*d) + ((48*a^2 - 16*a*b + 3*b^2)*Cosh[c + d*x]^3*Sinh[
c + d*x])/(192*d) + ((10*a - 3*b)*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) +
 (b*Cosh[c + d*x]^7*Sinh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(8*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^5}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^7(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{8d} - \text{Sul}$$

$$= \frac{(10a-3b)b\cosh^5(c+dx)\sinh(c+dx)}{48d} + b\cosh^7(c+dx)\sinh($$

$$= \frac{\left(48a^2-16ab+3b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{192d} + (10a-3b)b$$

$$= \frac{\left(48a^2-16ab+3b^2\right)\cosh(c+dx)\sinh(c+dx)}{128d} + (48a^2-16ab$$

$$= \frac{1}{128}\left(48a^2-16ab+3b^2\right)x + \frac{\left(48a^2-16ab+3b^2\right)\cosh(c+dx)s}{128d}$$

Mathematica [A]
time = 0.23, size = 98, normalized size = 0.62

$$\frac{24\left(48a^2-16ab+3b^2\right)(c+dx)+96a(8a-b)\sinh(2(c+dx))+24\left(4a^2+4ab-b^2\right)\sinh(4(c+dx))+32ab\sinh(6(c+dx))+3b^2\sinh(8(c+dx))}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (24*(48*a^2 - 16*a*b + 3*b^2)*(c + d*x) + 96*a*(8*a - b)*Sinh[2*(c + d*x)]
+ 24*(4*a^2 + 4*a*b - b^2)*Sinh[4*(c + d*x)] + 32*a*b*Sinh[6*(c + d*x)] + 3
*b^2*Sinh[8*(c + d*x)])/(3072*d)
```

Maple [A]
time = 1.92, size = 105, normalized size = 0.66

method	result
default	$\frac{\left(-\frac{1}{16}ab+\frac{1}{2}a^2\right)\sinh(2dx+2c)}{2d}+\frac{\left(-\frac{1}{32}b^2+\frac{1}{8}ab+\frac{1}{8}a^2\right)\sinh(4dx+4c)}{4d}+\frac{3a^2x}{8}+\frac{3b^2x}{128}-\frac{abx}{8}+\frac{b^2\sinh(8dx+8c)}{1024d}+\frac{ab\sinh(6d}{96d}$
risch	$\frac{3b^2x}{128}-\frac{abx}{8}+\frac{3a^2x}{8}+\frac{b^2\mathrm{e}^{8dx+8c}}{2048d}+\frac{ab\,\mathrm{e}^{6dx+6c}}{192d}+\frac{\mathrm{e}^{4dx+4c}a^2}{64d}+\frac{\mathrm{e}^{4dx+4c}ab}{64d}-\frac{\mathrm{e}^{4dx+4c}b^2}{256d}+\frac{\mathrm{e}^{2dx+2c}a^2}{8d}-\frac{\mathrm{e}^{2dx+2c}ab}{64d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1/16*a*b+1/2*a^2)*sinh(2*d*x+2*c)/d+1/4*(-1/32*b^2+1/8*a*b+1/8*a^2)*s
inh(4*d*x+4*c)/d+3/8*a^2*x+3/128*b^2*x-1/8*a*b*x+1/1024*b^2*sinh(8*d*x+8*c)
/d+1/96*a*b*sinh(6*d*x+6*c)/d
```

Maxima [A]
time = 0.27, size = 225, normalized size = 1.42

$$\frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{2048}b^2\left(\frac{\left(8e^{(-4dx-4c)}-1\right)e^{(8dx+8c)}}{d}-\frac{48(dx+c)}{d}-\frac{8e^{(-4dx-4c)}-e^{(-8dx-8c)}}{d}\right)+\frac{1}{192}ab\left(\frac{\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+1\right)e^{(6dx+6c)}}{d}-\frac{24(dx+c)}{d}+\frac{3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-e^{(-6dx-6c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/2048*b^2*((8*e^(-4*d*x - 4*c) - 1)*e^(8*d*x
+ 8*c)/d - 48*(d*x + c)/d - (8*e^(-4*d*x - 4*c) - e^(-8*d*x - 8*c))/d) + 1/
192*a*b*((3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + 1)*e^(6*d*x + 6*c)/d -
24*(d*x + c)/d + (3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c
))/d)
```

Fricas [A]
time = 0.38, size = 212, normalized size = 1.33

$$\frac{3b^2\cosh(dx+c)\sinh(dx+c)^7+3\left(7b^2\cosh(dx+c)^3+8ab\cosh(dx+c)\right)\sinh(dx+c)^5+\left(21b^2\cosh(dx+c)^5+80ab\cosh(dx+c)^3+12\left(4a^2+4ab-b^2\right)\cosh(dx+c)\right)\sinh(dx+c)^3+3\left(48a^2-16ab+3b^2\right)dx+3\left(b^2\cosh(dx+c)^7+8ab\cosh(dx+c)^5+4\left(4a^2+4ab-b^2\right)\cosh(dx+c)^3+8\left(8a^2-ab\right)\cosh(dx+c)\right)\sinh(dx+c)}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^2*cosh(d*x + c)^3 + 8*a
*b*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^2*cosh(d*x + c)^5 + 80*a*b*cosh(d
```

```
*x + c)^3 + 12*(4*a^2 + 4*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(48
*a^2 - 16*a*b + 3*b^2)*d*x + 3*(b^2*cosh(d*x + c)^7 + 8*a*b*cosh(d*x + c)^5
 + 4*(4*a^2 + 4*a*b - b^2)*cosh(d*x + c)^3 + 8*(8*a^2 - a*b)*cosh(d*x + c))
*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(146) = 292$.
time = 1.11, size = 481, normalized size = 3.03

$$\begin{cases} \frac{3a^2x\sinh^4(c+dx)}{8} - \frac{3a^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3a^2x\cosh^4(c+dx)}{8} - \frac{3a^2\sinh^3(c+dx)\cosh(c+dx)}{8d} + \frac{5a^2\sinh(c+dx)\cosh^3(c+dx)}{8d} + \frac{abx\sinh^6(c+dx)}{8} - \frac{3abx\sinh^4(c+dx)\cosh^2(c+dx)}{8} + \frac{3abx\sinh^2(c+dx)\cosh^4(c+dx)}{8} - \frac{abx\cosh^6(c+dx)}{8} - \frac{ab\sinh^5(c+dx)\cosh(c+dx)}{8d} + \frac{ab\sinh^3(c+dx)\cosh^3(c+dx)}{3d} + \frac{ab\sinh(c+dx)\cosh^5(c+dx)}{8d} + \frac{3b^2x\sinh^8(c+dx)}{128} - \frac{3b^2x\sinh^6(c+dx)\cosh^2(c+dx)}{32} + \frac{9b^2x\sinh^4(c+dx)\cosh^4(c+dx)}{64} - \frac{3b^2x\sinh^2(c+dx)\cosh^6(c+dx)}{32} + \frac{3b^2x\cosh^8(c+dx)}{128} - \frac{3b^2\sinh^7(c+dx)\cosh(c+dx)}{128d} + \frac{11b^2\sinh^5(c+dx)\cosh^3(c+dx)}{128d} + \frac{11b^2\sinh^3(c+dx)\cosh^5(c+dx)}{128d} - \frac{3b^2\sinh(c+dx)\cosh^7(c+dx)}{128d} & \text{for}\ d \neq 0 \\ x\left(a+b\sinh^2(c)\right)^2\cosh^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((3*a**2*x*sinh(c + d*x)**4/8 - 3*a**2*x*sinh(c + d*x)**2*cosh(c +
 d*x)**2/4 + 3*a**2*x*cosh(c + d*x)**4/8 - 3*a**2*sinh(c + d*x)**3*cosh(c +
 d*x)/(8*d) + 5*a**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + a*b*x*sinh(c +
d*x)**6/8 - 3*a*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/8 + 3*a*b*x*sinh(c +
d*x)**2*cosh(c + d*x)**4/8 - a*b*x*cosh(c + d*x)**6/8 - a*b*sinh(c + d*x)**
5*cosh(c + d*x)/(8*d) + a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3*d) + a*b*s
inh(c + d*x)*cosh(c + d*x)**5/(8*d) + 3*b**2*x*sinh(c + d*x)**8/128 - 3*b**
2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 9*b**2*x*sinh(c + d*x)**4*cosh(c
 + d*x)**4/64 - 3*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 3*b**2*x*co
sh(c + d*x)**8/128 - 3*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) + 11*b**
2*sinh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 11*b**2*sinh(c + d*x)**3*cosh
(c + d*x)**5/(128*d) - 3*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d,
 0)), (x*(a + b*sinh(c)**2)**2*cosh(c)**4, True))
```

Giac [A]
time = 0.41, size = 191, normalized size = 1.20

$$\frac{1}{128}\left(48a^2 - 16ab + 3b^2\right)x + \frac{b^2e^{(8dx+8c)}}{2048d} + \frac{abe^{(6dx+6c)}}{192d} - \frac{abe^{(-6dx-6c)}}{192d} - \frac{b^2e^{(-8dx-8c)}}{2048d} + \frac{(4a^2+4ab-b^2)e^{(4dx+4c)}}{256d} + \frac{(8a^2-ab)e^{(2dx+2c)}}{64d} - \frac{(8a^2-ab)e^{(-2dx-2c)}}{64d} - \frac{(4a^2+4ab-b^2)e^{(-4dx-4c)}}{256d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/128*(48*a^2 - 16*a*b + 3*b^2)*x + 1/2048*b^2*e^(8*d*x + 8*c)/d + 1/192*a*
b*e^(6*d*x + 6*c)/d - 1/192*a*b*e^(-6*d*x - 6*c)/d - 1/2048*b^2*e^(-8*d*x -
 8*c)/d + 1/256*(4*a^2 + 4*a*b - b^2)*e^(4*d*x + 4*c)/d + 1/64*(8*a^2 - a*b
)*e^(2*d*x + 2*c)/d - 1/64*(8*a^2 - a*b)*e^(-2*d*x - 2*c)/d - 1/256*(4*a^2
+ 4*a*b - b^2)*e^(-4*d*x - 4*c)/d
```

Mupad [B]
time = 0.36, size = 121, normalized size = 0.76

$$\frac{96a^2\sinh(2c+2dx) + 12a^2\sinh(4c+4dx) - 3b^2\sinh(4c+4dx) + \frac{3b^2\sinh(8c+8dx)}{8} - 12ab\sinh(2c+2dx) + 12ab\sinh(4c+4dx) + 4ab\sinh(6c+6dx) + 144a^2dx + 9b^2dx - 48abdx}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (96*a^2*sinh(2*c + 2*d*x) + 12*a^2*sinh(4*c + 4*d*x) - 3*b^2*sinh(4*c + 4*d
*x) + (3*b^2*sinh(8*c + 8*d*x))/8 - 12*a*b*sinh(2*c + 2*d*x) + 12*a*b*sinh(
4*c + 4*d*x) + 4*a*b*sinh(6*c + 6*d*x) + 144*a^2*d*x + 9*b^2*d*x - 48*a*b*d
*x)/(384*d)
```

3.294 $\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=74

$$\frac{a^2\sinh(c+dx)}{d}+\frac{a(a+2b)\sinh^3(c+dx)}{3d}+\frac{b(2a+b)\sinh^5(c+dx)}{5d}+\frac{b^2\sinh^7(c+dx)}{7d}$$

```
[Out] a^2*sinh(d*x+c)/d+1/3*a*(a+2*b)*sinh(d*x+c)^3/d+1/5*b*(2*a+b)*sinh(d*x+c)^5
/d+1/7*b^2*sinh(d*x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3269, 380}

$$\frac{a^2\sinh(c+dx)}{d}+\frac{b(2a+b)\sinh^5(c+dx)}{5d}+\frac{a(a+2b)\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sinh[c + d*x])/d + (a*(a + 2*b)*Sinh[c + d*x]^3)/(3*d) + (b*(2*a + b)*
Sinh[c + d*x]^5)/(5*d) + (b^2*Sinh[c + d*x]^7)/(7*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx&=\frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+bx^2\right)^2\,dx,x,\sinh(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(a^2+a(a+2b)x^2+b(2a+b)x^4+b^2x^6\right)\,dx,x,\sinh(c\right.}{d}\\&=\frac{a^2\sinh(c+dx)}{d}+\frac{a(a+2b)\sinh^3(c+dx)}{3d}+\frac{b(2a+b)\sinh^5(c}{5d}\end{aligned}$$

Mathematica [**A**]
time = 0.16, size = 83, normalized size = 1.12

$$\frac{(2800a^2-616ab+102b^2+(560a^2+448ab-111b^2)\cosh(2(c+dx))+6(28a-b)b\cosh(4(c+dx))+15b^2\cosh(6(c+dx)))\sinh(c+dx)}{3360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((2800*a^2 - 616*a*b + 102*b^2 + (560*a^2 + 448*a*b - 111*b^2)*Cosh[2*(c +
d*x)] + 6*(28*a - b)*b*Cosh[4*(c + d*x)] + 15*b^2*Cosh[6*(c + d*x)])*Sinh[c
 + d*x])/(3360*d)
```

Maple [**A**]
time = 1.84, size = 97, normalized size = 1.31

method	result
default	$\frac{\left(-\frac{1}{64}b^2+\frac{1}{8}ab\right)\sinh(5dx+5c)}{5d}+\frac{\left(-\frac{3}{64}b^2+\frac{1}{8}ab+\frac{1}{4}a^2\right)\sinh(3dx+3c)}{3d}+\frac{\left(\frac{3}{64}b^2-\frac{1}{4}ab+\frac{3}{4}a^2\right)\sinh(dx+c)}{d}+\frac{b^2\sinh(7dx+7c)}{448d}$
risch	$\frac{b^2 e^{7dx+7c}}{896d}+\frac{b\,e^{5dx+5c}a}{80d}-\frac{b^2 e^{5dx+5c}}{640d}+\frac{e^{3dx+3c}a^2}{24d}+\frac{e^{3dx+3c}ab}{48d}-\frac{e^{3dx+3c}b^2}{128d}+\frac{3\,e^{dx+c}a^2}{8d}-\frac{ab\,e^{dx+c}}{8d}+\frac{3\,e^{dx+c}b^2}{128d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(-1/64*b^2+1/8*a*b)*sinh(5*d*x+5*c)/d+1/3*(-3/64*b^2+1/8*a*b+1/4*a^2)*s
inh(3*d*x+3*c)/d+(3/64*b^2-1/4*a*b+3/4*a^2)*sinh(d*x+c)/d+1/448*b^2*sinh(7*
d*x+7*c)/d
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(68) = 136$.
time = 0.27, size = 242, normalized size = 3.27

$$-\frac{1}{4480}\,b^2\left(\frac{\left(7\,e^{(-2\,dx-2\,c)}+35\,e^{(-4\,dx-4\,c)}-105\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{105\,e^{(-dx-c)}-35\,e^{(-3\,dx-3\,c)}-7\,e^{(-5\,dx-5\,c)}+5\,e^{(-7\,dx-7\,c)}}{d}\right)+\frac{1}{240}\,ab\left(\frac{\left(5\,e^{(-2\,dx-2\,c)}-30\,e^{(-4\,dx-4\,c)}+3\right)e^{(5\,dx+5\,c)}}{d}+\frac{30\,e^{(-dx-c)}-5\,e^{(-3\,dx-3\,c)}-3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{1}{24}\,a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4480*b^2*((7*e^(-2*d*x - 2*c) + 35*e^(-4*d*x - 4*c) - 105*e^(-6*d*x - 6*
c) - 5)*e^(7*d*x + 7*c)/d + (105*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) - 7*e^(
-5*d*x - 5*c) + 5*e^(-7*d*x - 7*c))/d) + 1/240*a*b*((5*e^(-2*d*x - 2*c) - 3
0*e^(-4*d*x - 4*c) + 3)*e^(5*d*x + 5*c)/d + (30*e^(-d*x - c) - 5*e^(-3*d*x
- 3*c) - 3*e^(-5*d*x - 5*c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x +
c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(68) = 136$.
time = 0.37, size = 188, normalized size = 2.54

$$\frac{15\,b^2\sinh(dx+c)^7+21\left(15\,b^2\cosh(dx+c)^2+8\,ab-b^2\right)\sinh(dx+c)^5+35\left(15\,b^2\cosh(dx+c)^4+6\,(8\,ab-b^2)\cosh(dx+c)^2+16\,a^2+8\,ab-3\,b^2\right)\sinh(dx+c)^3+105\left(b^2\cosh(dx+c)^6+(8\,ab-b^2)\cosh(dx+c)^4+(16\,a^2+8\,ab-3\,b^2)\cosh(dx+c)^2+48\,a^2-16\,ab+3\,b^2\right)\sinh(dx+c)}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(15*b^2*sinh(d*x + c)^7 + 21*(15*b^2*cosh(d*x + c)^2 + 8*a*b - b^2)*
sinh(d*x + c)^5 + 35*(15*b^2*cosh(d*x + c)^4 + 6*(8*a*b - b^2)*cosh(d*x + c
)^2 + 16*a^2 + 8*a*b - 3*b^2)*sinh(d*x + c)^3 + 105*(b^2*cosh(d*x + c)^6 +
(8*a*b - b^2)*cosh(d*x + c)^4 + (16*a^2 + 8*a*b - 3*b^2)*cosh(d*x + c)^2 +
48*a^2 - 16*a*b + 3*b^2)*sinh(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(63) = 126$.
time = 0.67, size = 136, normalized size = 1.84

$$\begin{cases} -\frac{2a^2\sinh^3(c+dx)}{3d} + \frac{a^2\sinh(c+dx)\cosh^2(c+dx)}{d} - \frac{4ab\sinh^5(c+dx)}{15d} + \frac{2ab\sinh^3(c+dx)\cosh^2(c+dx)}{3d} - \frac{2b^2\sinh^7(c+dx)}{35d} + \frac{b^2\sinh^5(c+dx)\cosh^2(c+dx)}{5d} & \text{for } d \neq 0 \\ x\left(a+b\sinh^2(c)\right)^2\cosh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-2*a**2*sinh(c + d*x)**3/(3*d) + a**2*sinh(c + d*x)*cosh(c + d*x
)**2/d - 4*a*b*sinh(c + d*x)**5/(15*d) + 2*a*b*sinh(c + d*x)**3*cosh(c + d*
x)**2/(3*d) - 2*b**2*sinh(c + d*x)**7/(35*d) + b**2*sinh(c + d*x)**5*cosh(c
 + d*x)**2/(5*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c)**3, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(68) = 136$.
time = 0.42, size = 196, normalized size = 2.65

$$\frac{b^2e^{(7\,dx+7\,c)}}{896\,d} - \frac{b^2e^{(-7\,dx-7\,c)}}{896\,d} + \frac{(8\,ab-b^2)e^{(5\,dx+5\,c)}}{640\,d} + \frac{(16\,a^2+8\,ab-3\,b^2)e^{(3\,dx+3\,c)}}{384\,d} + \frac{(48\,a^2-16\,ab+3\,b^2)e^{(dx+c)}}{128\,d} - \frac{(48\,a^2-16\,ab+3\,b^2)e^{(-dx-c)}}{128\,d} - \frac{(16\,a^2+8\,ab-3\,b^2)e^{(-3\,dx-3\,c)}}{384\,d} - \frac{(8\,ab-b^2)e^{(-5\,dx-5\,c)}}{640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/896*b^2*e^(7*d*x + 7*c)/d - 1/896*b^2*e^(-7*d*x - 7*c)/d + 1/640*(8*a*b -
 b^2)*e^(5*d*x + 5*c)/d + 1/384*(16*a^2 + 8*a*b - 3*b^2)*e^(3*d*x + 3*c)/d
+ 1/128*(48*a^2 - 16*a*b + 3*b^2)*e^(d*x + c)/d - 1/128*(48*a^2 - 16*a*b +
3*b^2)*e^(-d*x - c)/d - 1/384*(16*a^2 + 8*a*b - 3*b^2)*e^(-3*d*x - 3*c)/d -
 1/640*(8*a*b - b^2)*e^(-5*d*x - 5*c)/d
```

Mupad [**B**]
time = 0.96, size = 80, normalized size = 1.08

$$\frac{35\,a^2\,\mathrm{sinh}(c+d\,x)^3+105\,a^2\,\mathrm{sinh}(c+d\,x)+42\,a\,b\,\mathrm{sinh}(c+d\,x)^5+70\,a\,b\,\mathrm{sinh}(c+d\,x)^3+15\,b^2\,\mathrm{sinh}(c+d\,x)^7+21\,b^2\,\mathrm{sinh}(c+d\,x)^5}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (105*a^2*sinh(c + d*x) + 35*a^2*sinh(c + d*x)^3 + 21*b^2*sinh(c + d*x)^5 +
15*b^2*sinh(c + d*x)^7 + 70*a*b*sinh(c + d*x)^3 + 42*a*b*sinh(c + d*x)^5)/(
105*d)
```

3.295 $\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=119

$$\frac{1}{16}\left(8a^2-4ab+b^2\right)x+\frac{\left(8a^2-4ab+b^2\right)\cosh(c+dx)\sinh(c+dx)}{16d}+\frac{(8a-3b)b\cosh^3(c+dx)\sinh(c+dx)}{24d}+$$

```
[Out] 1/16*(8*a^2-4*a*b+b^2)*x+1/16*(8*a^2-4*a*b+b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1
/24*(8*a-3*b)*b*cosh(d*x+c)^3*sinh(d*x+c)/d+1/6*b*cosh(d*x+c)^5*sinh(d*x+c)
*(a-(a-b)*tanh(d*x+c)^2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3270, 424, 393, 205, 212}

$$\frac{\left(8a^2-4ab+b^2\right)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x\left(8a^2-4ab+b^2\right)+\frac{b(8a-3b)\sinh(c+dx)\cosh^3(c+dx)}{24d}+\frac{b\sinh(c+dx)\cosh^5(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 4*a*b + b^2)*x)/16 + ((8*a^2 - 4*a*b + b^2)*Cosh[c + d*x]*Sinh[c
 + d*x])/(16*d) + ((8*a - 3*b)*b*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d) + (b*
Cosh[c + d*x]^5*Sinh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(6*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^5(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{6d} - \frac{\text{Su}}{}$$

$$= \frac{(8a-3b)b\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b\cosh^5(c+dx)\sinh(}{}$$

$$= \frac{\left(8a^2-4ab+b^2\right)\cosh(c+dx)\sinh(c+dx)}{16d} + \frac{(8a-3b)b\cosh}{}$$

$$= \frac{1}{16}\left(8a^2-4ab+b^2\right)x + \frac{\left(8a^2-4ab+b^2\right)\cosh(c+dx)\sinh(c}{16d}$$

Mathematica [A]
time = 0.21, size = 79, normalized size = 0.66

$$\frac{12\left(8a^2-4ab+b^2\right)(c+dx)+3\left(16a^2-b^2\right)\sinh(2(c+dx))+3(4a-b)b\sinh(4(c+dx))+b^2\sinh(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (12*(8*a^2 - 4*a*b + b^2)*(c + d*x) + 3*(16*a^2 - b^2)*Sinh[2*(c + d*x)] +
3*(4*a - b)*b*Sinh[4*(c + d*x)] + b^2*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A]

time = 1.25, size = 134, normalized size = 1.13

method	result
derivativedivides	$\frac{b^2\left(\frac{\left(\sinh^3(dx+c)\right)\left(\cosh^3(dx+c)\right)}{6}-\frac{\sinh(dx+c)\left(\cosh^3(dx+c)\right)}{8}+\frac{\cosh(dx+c)\sinh(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+2ab\left(\frac{\sinh(dx+c)\left(\cosh^3(dx+c}{4}}{d}$
default	$\frac{b^2\left(\frac{\left(\sinh^3(dx+c)\right)\left(\cosh^3(dx+c)\right)}{6}-\frac{\sinh(dx+c)\left(\cosh^3(dx+c)\right)}{8}+\frac{\cosh(dx+c)\sinh(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+2ab\left(\frac{\sinh(dx+c)\left(\cosh^3(dx+c}{4}}{d}$
risch	$\frac{a^2x}{2}-\frac{abx}{4}+\frac{b^2x}{16}+\frac{b^2\mathrm{e}^{6dx+6c}}{384d}+\frac{\mathrm{e}^{4dx+4c}ab}{32d}-\frac{\mathrm{e}^{4dx+4c}b^2}{128d}+\frac{\mathrm{e}^{2dx+2c}a^2}{8d}-\frac{\mathrm{e}^{2dx+2c}b^2}{128d}-\frac{\mathrm{e}^{-2dx-2c}a^2}{8d}+\frac{\mathrm{e}^{-2d}}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/6*sinh(d*x+c)^3*cosh(d*x+c)^3-1/8*sinh(d*x+c)*cosh(d*x+c)^3+1/1
6*cosh(d*x+c)*sinh(d*x+c)+1/16*d*x+1/16*c)+2*a*b*(1/4*sinh(d*x+c)*cosh(d*x+
c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c)+a^2*(1/2*cosh(d*x+c)*sinh(d
*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]

time = 0.27, size = 171, normalized size = 1.44

$$\frac{1}{8}a^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{384}b^2\left(\frac{\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}-\frac{24(dx+c)}{d}-\frac{3e^{(-2dx-2c)}+3e^{(-4dx-4c)}-e^{(-6dx-6c)}}{d}\right)-\frac{1}{32}ab\left(\frac{8(dx+c)}{d}-\frac{e^{(4dx+4c)}}{d}+\frac{e^{(-4dx-4c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/384*b^2*((3*e^(-
2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d -
 (3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d) - 1/32*a*b
*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Fricas [A]

time = 0.38, size = 143, normalized size = 1.20

$$\frac{3b^2\cosh(dx+c)\sinh(dx+c)^5+2\left(5b^2\cosh(dx+c)^3+3\left(4ab-b^2\right)\cosh(dx+c)\right)\sinh(dx+c)^3+6\left(8a^2-4ab+b^2\right)dx+3\left(b^2\cosh(dx+c)^5+2\left(4ab-b^2\right)\cosh(dx+c)^3+\left(16a^2-b^2\right)\cosh(dx+c)\right)\sinh(dx+c)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^2*cosh(d*x + c)^3 + 3*(4
*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(8*a^2 - 4*a*b + b^2)*d*x +
3*(b^2*cosh(d*x + c)^5 + 2*(4*a*b - b^2)*cosh(d*x + c)^3 + (16*a^2 - b^2)*c
osh(d*x + c))*sinh(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(105) = 210$.

time = 0.49, size = 314, normalized size = 2.64

$$\begin{cases} -\frac{a^2x\sinh^2(c+dx)}{2} + \frac{a^2x\cosh^2(c+dx)}{2} + \frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d} - \frac{abx\sinh^4(c+dx)}{4} + \frac{abx\sinh^2(c+dx)\cosh^2(c+dx)}{2} - \frac{abx\cosh^4(c+dx)}{4} + \frac{ab\sinh^3(c+dx)\cosh(c+dx)}{4d} + \frac{ab\sinh(c+dx)\cosh^3(c+dx)}{4d} - \frac{b^2x\sinh^6(c+dx)}{16} + \frac{3b^2x\sinh^4(c+dx)\cosh^2(c+dx)}{16} - \frac{3b^2x\sinh^2(c+dx)\cosh^4(c+dx)}{16} + \frac{b^2x\cosh^6(c+dx)}{16} + \frac{b^2\sinh^5(c+dx)\cosh(c+dx)}{16d} + \frac{b^2\sinh^3(c+dx)\cosh^3(c+dx)}{6d} - \frac{b^2\sinh(c+dx)\cosh^5(c+dx)}{16d} & \text{for } d \neq 0 \\ x\left(a + b\sinh^2(c)\right)^2\cosh^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-a**2*x*sinh(c + d*x)**2/2 + a**2*x*cosh(c + d*x)**2/2 + a**2*si
nh(c + d*x)*cosh(c + d*x)/(2*d) - a*b*x*sinh(c + d*x)**4/4 + a*b*x*sinh(c +
 d*x)**2*cosh(c + d*x)**2/2 - a*b*x*cosh(c + d*x)**4/4 + a*b*sinh(c + d*x)*
*3*cosh(c + d*x)/(4*d) + a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d) - b**2*x*
sinh(c + d*x)**6/16 + 3*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 - 3*b**
2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 + b**2*x*cosh(c + d*x)**6/16 + b**
2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) + b**2*sinh(c + d*x)**3*cosh(c + d*
x)**3/(6*d) - b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a
+ b*sinh(c)**2)**2*cosh(c)**2, True))
```

Giac [A]

time = 0.42, size = 149, normalized size = 1.25

$$\frac{1}{16}\left(8a^2 - 4ab + b^2\right)x + \frac{b^2e^{(6dx+6c)}}{384d} - \frac{b^2e^{(-6dx-6c)}}{384d} + \frac{(4ab - b^2)e^{(4dx+4c)}}{128d} + \frac{(16a^2 - b^2)e^{(2dx+2c)}}{128d} - \frac{(16a^2 - b^2)e^{(-2dx-2c)}}{128d} - \frac{(4ab - b^2)e^{(-4dx-4c)}}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^2 - 4*a*b + b^2)*x + 1/384*b^2*e^(6*d*x + 6*c)/d - 1/384*b^2*e^(-
6*d*x - 6*c)/d + 1/128*(4*a*b - b^2)*e^(4*d*x + 4*c)/d + 1/128*(16*a^2 - b^
2)*e^(2*d*x + 2*c)/d - 1/128*(16*a^2 - b^2)*e^(-2*d*x - 2*c)/d - 1/128*(4*a
*b - b^2)*e^(-4*d*x - 4*c)/d
```

Mupad [B]

time = 0.22, size = 95, normalized size = 0.80

$$\frac{12\,a^2\sinh(2\,c + 2\,d\,x) - \frac{3\,b^2\sinh(2\,c+2\,d\,x)}{4} - \frac{3\,b^2\sinh(4\,c+4\,d\,x)}{4} + \frac{b^2\sinh(6\,c+6\,d\,x)}{4} + 3\,a\,b\sinh(4\,c + 4\,d\,x) + 24\,a^2\,d\,x + 3\,b^2\,d\,x - 12\,a\,b\,d\,x}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (12*a^2*sinh(2*c + 2*d*x) - (3*b^2*sinh(2*c + 2*d*x))/4 - (3*b^2*sinh(4*c +
 4*d*x))/4 + (b^2*sinh(6*c + 6*d*x))/4 + 3*a*b*sinh(4*c + 4*d*x) + 24*a^2*d
*x + 3*b^2*d*x - 12*a*b*d*x)/(48*d)
```

3.296 $\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^5(c+dx)}{5d}$$

```
[Out] a^2*sinh(d*x+c)/d+2/3*a*b*sinh(d*x+c)^3/d+1/5*b^2*sinh(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3269, 200}

$$\frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sinh[c + d*x])/d + (2*a*b*Sinh[c + d*x]^3)/(3*d) + (b^2*Sinh[c + d*x]^
5)/(5*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \left(a+bx^2\right)^2\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^2+2abx^2+b^2x^4\right)\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]

time = 0.02, size = 49, normalized size = 1.00

$$\frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sinh[c + d*x])/d + (2*a*b*Sinh[c + d*x]^3)/(3*d) + (b^2*Sinh[c + d*x]^
5)/(5*d)
```

Maple [A]

time = 0.72, size = 41, normalized size = 0.84

method	result
derivativedivides	$\frac{\frac{b^2\left(\sinh^5(dx+c)\right)}{5}+\frac{2ab\left(\sinh^3(dx+c)\right)}{3}+a^2\sinh(dx+c)}{d}$
default	$\frac{\frac{b^2\left(\sinh^5(dx+c)\right)}{5}+\frac{2ab\left(\sinh^3(dx+c)\right)}{3}+a^2\sinh(dx+c)}{d}$
risch	$\frac{b^2\mathrm{e}^{5dx+5c}}{160d}+\frac{\mathrm{e}^{3dx+3c}ab}{12d}-\frac{\mathrm{e}^{3dx+3c}b^2}{32d}+\frac{\mathrm{e}^{dx+c}a^2}{2d}-\frac{ab\,\mathrm{e}^{dx+c}}{4d}+\frac{\mathrm{e}^{dx+c}b^2}{16d}-\frac{\mathrm{e}^{-dx-c}a^2}{2d}+\frac{\mathrm{e}^{-dx-c}ab}{4d}-\frac{\mathrm{e}^{-dx-}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*b^2*sinh(d*x+c)^5+2/3*a*b*sinh(d*x+c)^3+a^2*sinh(d*x+c))
```

Maxima [A]

time = 0.26, size = 45, normalized size = 0.92

$$\frac{b^2\sinh\left(dx+c\right)^5}{5\,d}+\frac{2\,ab\sinh\left(dx+c\right)^3}{3\,d}+\frac{a^2\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*sinh(d*x + c)^5/d + 2/3*a*b*sinh(d*x + c)^3/d + a^2*sinh(d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(45) = 90$.

time = 0.38, size = 106, normalized size = 2.16

$$\frac{3\,b^2\sinh\left(dx+c\right)^5+5\,\left(6\,b^2\cosh\left(dx+c\right)^2+8\,ab-3\,b^2\right)\sinh\left(dx+c\right)^3+15\,\left(b^2\cosh\left(dx+c\right)^4+\left(8\,ab-3\,b^2\right)\cosh\left(dx+c\right)^2+16\,a^2-8\,ab+2\,b^2\right)\sinh\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^2*sinh(d*x + c)^5 + 5*(6*b^2*cosh(d*x + c)^2 + 8*a*b - 3*b^2)*si
nh(d*x + c)^3 + 15*(b^2*cosh(d*x + c)^4 + (8*a*b - 3*b^2)*cosh(d*x + c)^2 +
 16*a^2 - 8*a*b + 2*b^2)*sinh(d*x + c))/d
```

Sympy [A]
time = 0.28, size = 58, normalized size = 1.18

$$\begin{cases} \frac{a^2 \sinh(c+dx)}{d} + \frac{2ab\sinh^3(c+dx)}{3d} + \frac{b^2\sinh^5(c+dx)}{5d} & \text{for } d \neq 0 \\ x\left(a + b\sinh^2(c)\right)^2 \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*sinh(c + d*x)**3/(3*d) + b**2*sinh(
c + d*x)**5/(5*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(45) = 90.
time = 0.41, size = 134, normalized size = 2.73

$$\frac{b^2 e^{(5\,dx+5\,c)}}{160\,d} - \frac{b^2 e^{(-5\,dx-5\,c)}}{160\,d} + \frac{(8\,ab-3\,b^2)e^{(3\,dx+3\,c)}}{96\,d} + \frac{(8\,a^2-4\,ab+b^2)e^{(dx+c)}}{16\,d} - \frac{(8\,a^2-4\,ab+b^2)e^{(-dx-c)}}{16\,d} - \frac{(8\,ab-3\,b^2)e^{(-3\,dx-3\,c)}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/160*b^2*e^(5*d*x + 5*c)/d - 1/160*b^2*e^(-5*d*x - 5*c)/d + 1/96*(8*a*b -
3*b^2)*e^(3*d*x + 3*c)/d + 1/16*(8*a^2 - 4*a*b + b^2)*e^(d*x + c)/d - 1/16*
(8*a^2 - 4*a*b + b^2)*e^(-d*x - c)/d - 1/96*(8*a*b - 3*b^2)*e^(-3*d*x - 3*c
)/d
```

Mupad [B]
time = 0.83, size = 42, normalized size = 0.86

$$\frac{\sinh(c+d\,x)\,\left(15\,a^2+10\,a\,b\,\sinh(c+d\,x)^2+3\,b^2\,\sinh(c+d\,x)^4\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (sinh(c + d*x)*(15*a^2 + 3*b^2*sinh(c + d*x)^4 + 10*a*b*sinh(c + d*x)^2))/(
15*d)
```

3.297 $\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=55

$$\frac{(a-b)^2\text{ArcTan}(\sinh(c+dx))}{d}+\frac{(2a-b)b\sinh(c+dx)}{d}+\frac{b^2\sinh^3(c+dx)}{3d}$$

```
[Out] (a-b)^2*arctan(sinh(d*x+c))/d+(2*a-b)*b*sinh(d*x+c)/d+1/3*b^2*sinh(d*x+c)^3
/d
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3269, 398, 209}

$$\frac{(a-b)^2\text{ArcTan}(\sinh(c+dx))}{d}+\frac{b(2a-b)\sinh(c+dx)}{d}+\frac{b^2\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)^2*ArcTan[Sinh[c + d*x]])/d + ((2*a - b)*b*Sinh[c + d*x])/d + (b^2*
Sinh[c + d*x]^3)/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{d}$$

$$=\frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right)\,dx,x,\sinh(c+dx)\right)}{d}$$

$$=\frac{(2a-b)b\sinh(c+dx)}{d}+\frac{b^2\sinh^3(c+dx)}{3d}+ (a-b)^2\text{Subst}\left(\int \frac{1}{1+}\right.$$

$$=\frac{(a-b)^2\tan^{-1}(\sinh(c+dx))}{d}+\frac{(2a-b)b\sinh(c+dx)}{d}+ b^2\sinh$$

Mathematica [A]
time = 0.19, size = 70, normalized size = 1.27

$$\frac{\sinh(c+dx)\left(\frac{3(a-b)^2\tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}}+b\left(6a+b\left(-3+\sinh^2(c+dx)\right)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (Sinh[c + d*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c
+ d*x]^2] + b*(6*a + b*(-3 + Sinh[c + d*x]^2))))/(3*d)
```

Maple [A]
time = 0.98, size = 70, normalized size = 1.27

method	result
derivativedivides	$\frac{2a^2\arctan\left(e^{dx+c}\right)+2ab\left(\sinh(dx+c)-2\arctan\left(e^{dx+c}\right)\right)+b^2\left(\frac{\left(\sinh^3(dx+c)\right)}{3}-\sinh(dx+c)+2\arctan\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{2a^2\arctan\left(e^{dx+c}\right)+2ab\left(\sinh(dx+c)-2\arctan\left(e^{dx+c}\right)\right)+b^2\left(\frac{\left(\sinh^3(dx+c)\right)}{3}-\sinh(dx+c)+2\arctan\left(e^{dx+c}\right)\right)}{d}$
risch	$\frac{e^{3dx+3c}b^2}{24d}+\frac{ab\,e^{dx+c}}{d}-\frac{5\,e^{dx+c}b^2}{8d}-\frac{e^{-dx-c}ab}{d}+\frac{5\,e^{-dx-c}b^2}{8d}-\frac{e^{-3dx-3c}b^2}{24d}+\frac{i\ln\left(e^{dx+c}+i\right)a^2}{d}-\frac{2i\ln\left(e^{dx+c}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2*arctan(exp(d*x+c))+2*a*b*(sinh(d*x+c)-2*arctan(exp(d*x+c)))+b^2*
(1/3*sinh(d*x+c)^3-sinh(d*x+c)+2*arctan(exp(d*x+c))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(53) = 106$.
time = 0.49, size = 133, normalized size = 2.42

$$-\frac{1}{24}b^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)}-1\right)e^{(3\,dx+3\,c)}}{d}-\frac{15\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{48\arctan\left(e^{(-dx-c)}\right)}{d}\right)+ab\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)+\frac{a^2\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*b^2*((15*e^(-2*d*x - 2*c) - 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) -
 e^(-3*d*x - 3*c))/d + 48*arctan(e^(-d*x - c))/d) + a*b*(4*arctan(e^(-d*x -
 c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a^2*arctan(sinh(d*x + c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(53) = 106$.
time = 0.39, size = 446, normalized size = 8.11

$$\frac{b^2\cosh\left(dx+c\right)^6+6\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+b^2\sinh\left(dx+c\right)^6+3\left(8\,ab-5\,b^2\right)\cosh\left(dx+c\right)^4+3\left(5\,b^2\cosh\left(dx+c\right)^2+8\,ab-5\,b^2\right)\sinh\left(dx+c\right)^4+4\left(5\,b^2\cosh\left(dx+c\right)^3+3\left(8\,ab-5\,b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3-3\left(8\,ab-5\,b^2\right)\cosh\left(dx+c\right)^2+3\left(5\,b^2\cosh\left(dx+c\right)^4+6\left(8\,ab-5\,b^2\right)\cosh\left(dx+c\right)^2-8\,ab+5\,b^2\right)\sinh\left(dx+c\right)^2-b^2+48\left(\left(a^2-2\,ab+b^2\right)\cosh\left(dx+c\right)^3+3\left(a^2-2\,ab+b^2\right)\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)+3\left(a^2-2\,ab+b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+\left(a^2-2\,ab+b^2\right)\sinh\left(dx+c\right)^3\right)\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)+6\left(b^2\cosh\left(dx+c\right)^5+2\left(8\,ab-5\,b^2\right)\cosh\left(dx+c\right)^3-\left(8\,ab-5\,b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{24\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+d\sinh\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(
d*x + c)^6 + 3*(8*a*b - 5*b^2)*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 +
 8*a*b - 5*b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 + 3*(8*a*b - 5*b
^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(8*a*b - 5*b^2)*cosh(d*x + c)^2 + 3*
(5*b^2*cosh(d*x + c)^4 + 6*(8*a*b - 5*b^2)*cosh(d*x + c)^2 - 8*a*b + 5*b^2)
*sinh(d*x + c)^2 - b^2 + 48*((a^2 - 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 -
 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2 - 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^3)*arctan(cosh(d
*x + c) + sinh(d*x + c)) + 6*(b^2*cosh(d*x + c)^5 + 2*(8*a*b - 5*b^2)*cosh(
d*x + c)^3 - (8*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)
^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2
+ d*sinh(d*x + c)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^2\left(c+dx\right)\right)^2\operatorname{sech}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)**2*sech(c + d*x), x)
```

Giac [A]
time = 0.42, size = 102, normalized size = 1.85

$$\frac{b^{2}e^{(3\,dx+3\,c)}+24\,abe^{(dx+c)}-15\,b^{2}e^{(dx+c)}+48\,(a^{2}-2\,ab+b^{2})\arctan\left(e^{(dx+c)}\right)-\left(24\,abe^{(2\,dx+2\,c)}-15\,b^{2}e^{(2\,dx+2\,c)}+b^{2}\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(b^2*e^(3*d*x + 3*c) + 24*a*b*e^(d*x + c) - 15*b^2*e^(d*x + c) + 48*(a
^2 - 2*a*b + b^2)*arctan(e^(d*x + c)) - (24*a*b*e^(2*d*x + 2*c) - 15*b^2*e^
(2*d*x + 2*c) + b^2)*e^(-3*d*x - 3*c))/d
```

Mupad [B]
time = 0.17, size = 182, normalized size = 3.31

$$\frac{b^{2}\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{b^{2}\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{\mathrm{e}^{-c-d\,x}\,(8\,a\,b-5\,b^{2})}{8\,d}+\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a^{2}\,\sqrt{d^{2}}+b^{2}\,\sqrt{d^{2}}-2\,a\,b\,\sqrt{d^{2}}\right)}{d\,\sqrt{a^{4}-4\,a^{3}\,b+6\,a^{2}\,b^{2}-4\,a\,b^{3}+b^{4}}}\right)\,\sqrt{a^{4}-4\,a^{3}\,b+6\,a^{2}\,b^{2}-4\,a\,b^{3}+b^{4}}}{\sqrt{d^{2}}}+\frac{b\,\mathrm{e}^{c+d\,x}\,(8\,a-5\,b)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x),x)
```

```
[Out] (b^2*exp(3*c + 3*d*x))/(24*d) - (b^2*exp(- 3*c - 3*d*x))/(24*d) - (exp(- c
- d*x)*(8*a*b - 5*b^2))/(8*d) + (2*atan((exp(d*x)*exp(c)*(a^2*(d^2)^(1/2) +
 b^2*(d^2)^(1/2) - 2*a*b*(d^2)^(1/2)))/(d*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
6*a^2*b^2)^(1/2)))*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)^(1/2))/(d^2)
^(1/2) + (b*exp(c + d*x)*(8*a - 5*b))/(8*d)
```

3.298 $\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{1}{2}(4a-3b)bx+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{(a-b)^2\tanh(c+dx)}{d}$$

```
[Out] 1/2*(4*a-3*b)*b*x+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d+(a-b)^2*tanh(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3270, 398, 393, 212}

$$\frac{(a-b)^2\tanh(c+dx)}{d}+\frac{1}{2}bx(4a-3b)+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((4*a - 3*b)*b*x)/2 + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + ((a - b)^2*
Tanh[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
```

```
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a-(a-b)x^2)^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left((a-b)^2+\frac{(2a-b)b-2(a-b)bx^2}{(1-x^2)^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a-b)^2\tanh(c+dx)}{d} + \frac{\operatorname{Subst}\Big(\int \frac{(2a-b)b-2(a-b)bx^2}{(1-x^2)^2}\,dx, x, \tanh(c}{d}$$

$$= \frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{(a-b)^2\tanh(c+dx)}{d} + \frac{((4a-}{}$$

$$= \frac{1}{2}(4a-3b)bx + \frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{(a-b)^2\tanh(c}{d}$$

Mathematica [**A**]
time = 0.23, size = 50, normalized size = 0.94

$$\frac{2(4a-3b)b(c+dx)+b^2\sinh(2(c+dx))+4(a-b)^2\tanh(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*(4*a - 3*b)*b*(c + d*x) + b^2*Sinh[2*(c + d*x)] + 4*(a - b)^2*Tanh[c + d
*x])/(4*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(49) = 98$.
time = 1.59, size = 109, normalized size = 2.06

method	result	size
risch	$2abx-\frac{3b^2x}{2}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}b^2}{8d}-\frac{2a^2}{d(1+e^{2dx+2c})}+\frac{4ab}{d(1+e^{2dx+2c})}-\frac{2b^2}{d(1+e^{2dx+2c})}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b*x-3/2*b^2*x+1/8/d*exp(2*d*x+2*c)*b^2-1/8/d*exp(-2*d*x-2*c)*b^2-2/d/(1
+exp(2*d*x+2*c))*a^2+4/d/(1+exp(2*d*x+2*c))*a*b-2/d/(1+exp(2*d*x+2*c))*b^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(49) = 98$.
time = 0.27, size = 119, normalized size = 2.25

$$2\,ab\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)-\frac{1}{8}\,b^2\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}+1}{d\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}\right)}\right)+\frac{2\,a^2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) - 1/8*b^2*(12*(d*x + c)/d +
e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4
*d*x - 4*c)))) + 2*a^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [A]
time = 0.39, size = 97, normalized size = 1.83

$$\frac{b^2\sinh\left(dx+c\right)^3+4\left(\left(4\,ab-3\,b^2\right)dx-2\,a^2+4\,ab-2\,b^2\right)\cosh\left(dx+c\right)+\left(3\,b^2\cosh\left(dx+c\right)^2+8\,a^2-16\,ab+9\,b^2\right)\sinh\left(dx+c\right)}{8\,d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*sinh(d*x + c)^3 + 4*((4*a*b - 3*b^2)*d*x - 2*a^2 + 4*a*b - 2*b^2)*
cosh(d*x + c) + (3*b^2*cosh(d*x + c)^2 + 8*a^2 - 16*a*b + 9*b^2)*sinh(d*x +
 c))/(d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^2\left(c+dx\right)\right)^2\operatorname{sech}^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)**2*sech(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(49) = 98$.
time = 0.43, size = 131, normalized size = 2.47

$$\frac{b^2e^{(2\,dx+2\,c)}+4\left(4\,ab-3\,b^2\right)(dx+c)-\frac{4\,abe^{(4\,dx+4\,c)}-3\,b^2e^{(4\,dx+4\,c)}+16\,a^2e^{(2\,dx+2\,c)}-28\,abe^{(2\,dx+2\,c)}+14\,b^2e^{(2\,dx+2\,c)}+b^2}{e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(b^2*e^(2*d*x + 2*c) + 4*(4*a*b - 3*b^2)*(d*x + c) - (4*a*b*e^(4*d*x +
4*c) - 3*b^2*e^(4*d*x + 4*c) + 16*a^2*e^(2*d*x + 2*c) - 28*a*b*e^(2*d*x + 2
*c) + 14*b^2*e^(2*d*x + 2*c) + b^2)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c)))/d
```

Mupad [B]
time = 0.88, size = 75, normalized size = 1.42

$$\frac{b\,x\,\left(4\,a-3\,b\right)}{2}-\frac{b^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}+\frac{b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}-\frac{2\,\left(a^2-2\,a\,b+b^2\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^2,x)
```

```
[Out] (b*x*(4*a - 3*b))/2 - (b^2*exp(- 2*c - 2*d*x))/(8*d) + (b^2*exp(2*c + 2*d*x
))/(8*d) - (2*(a^2 - 2*a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1))
```

3.299 $\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=64

$$\frac{(a-b)(a+3b)\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{b^2\sinh(c+dx)}{d}+\frac{(a-b)^2\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] 1/2*(a-b)*(a+3*b)*arctan(sinh(d*x+c))/d+b^2*sinh(d*x+c)/d+1/2*(a-b)^2*sech(
d*x+c)*tanh(d*x+c)/d
```

Rubi [A]

time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 398, 393, 209}

$$\frac{(a+3b)(a-b)\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{(a-b)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}+\frac{b^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)*(a + 3*b)*ArcTan[Sinh[c + d*x]])/(2*d) + (b^2*Sinh[c + d*x])/d + (
(a - b)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^2}{(1+x^2)^2}\,dx,x,\sinh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(b^2+\frac{a^2-b^2+2(a-b)bx^2}{(1+x^2)^2}\right)dx,x,\sinh(c+dx)\right)}{d}\\
&= \frac{b^2\sinh(c+dx)}{d}+\frac{\operatorname{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{(1+x^2)^2}\,dx,x,\sinh(c+dx)\right)}{d}\\
&= \frac{b^2\sinh(c+dx)}{d}+\frac{(a-b)^2\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}+((a-b\\
&= \frac{(a-b)(a+3b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^2\sinh(c+dx)}{d}+(a-b
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 11.26, size = 253, normalized size = 3.95

$$\frac{\operatorname{csch}^3(c+dx)\left(-64\,{}_4F_3\left(\tfrac{3}{2},2,2,2;1,1,\tfrac{9}{2};-\sinh^2(c+dx)\right)\sinh^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2-35\left(a^2\left(375+37\sinh^2(c+dx)\right)+b^2\sinh^4(c+dx)\left(303+61\sinh^2(c+dx)\right)+2ab\sinh^2(c+dx)\left(375+61\sinh^2(c+dx)\right)\right)+\frac{105\tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)\left(b^2\sinh^4(c+dx)(101+54\sinh^2(c+dx)+\sinh^4(c+dx))+2ab\sinh^2(c+dx)(125+62\sinh^2(c+dx)+\sinh^4(c+dx))+a^2(125+54\sinh^2(c+dx)+9\sinh^4(c+dx))\right)}{\sqrt{-\sinh^2(c+dx)}}\right)}{1680d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (Csch[c + d*x]^3*(-64*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[
c + d*x]^2]*Sinh[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2 - 35*(a^2*(375 + 37*S
inh[c + d*x]^2) + b^2*Sinh[c + d*x]^4*(303 + 61*Sinh[c + d*x]^2) + 2*a*b*Si
nh[c + d*x]^2*(375 + 61*Sinh[c + d*x]^2)) + (105*ArcTanh[Sqrt[-Sinh[c + d*x
]^2]]*(b^2*Sinh[c + d*x]^4*(101 + 54*Sinh[c + d*x]^2 + Sinh[c + d*x]^4) + 2
*a*b*Sinh[c + d*x]^2*(125 + 62*Sinh[c + d*x]^2 + Sinh[c + d*x]^4) + a^2*(12
5 + 54*Sinh[c + d*x]^2 + 9*Sinh[c + d*x]^4)))/Sqrt[-Sinh[c + d*x]^2]))/(168
0*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.47, size = 190, normalized size = 2.97

method	result
risch	$\frac{e^{dx+c}b^2}{2d}-\frac{e^{-dx-c}b^2}{2d}+\frac{e^{dx+c}(a^2-2ab+b^2)(e^{2dx+2c}-1)}{d(1+e^{2dx+2c})^2}+\frac{i\ln(e^{dx+c}+i)a^2}{2d}+\frac{i\ln(e^{dx+c}+i)ab}{d}-\frac{3i\ln(e^{dx+c}+i)b^2}{2d}-\frac{i\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*exp(d*x+c)*b^2-1/2/d*exp(-d*x-c)*b^2+exp(d*x+c)*(a^2-2*a*b+b^2)*(exp(
2*d*x+2*c)-1)/d/(1+exp(2*d*x+2*c))^2+1/2*I/d*ln(exp(d*x+c)+I)*a^2+I/d*ln(ex
p(d*x+c)+I)*a*b-3/2*I/d*ln(exp(d*x+c)+I)*b^2-1/2*I/d*ln(exp(d*x+c)-I)*a^2-I
/d*ln(exp(d*x+c)-I)*a*b+3/2*I/d*ln(exp(d*x+c)-I)*b^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(60) = 120$.
time = 0.51, size = 234, normalized size = 3.66

$$\frac{1}{2}b^2\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4e^{(-2dx-2c)}-e^{(-4dx-4c)}+1}{d(e^{(-dx-c)}+2e^{(-3dx-3c)}+e^{(-5dx-5c)})}\right)-2ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)-a^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) -
e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5
*c)))) - 2*a*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/
(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a^2*(arctan(e^(-d*x - c)
)/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x
- 4*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(60) = 120$.
time = 0.38, size = 759, normalized size = 11.86

$$\frac{b^2\cosh(dx+c)^6+6b^2\cosh(dx+c)\sinh(dx+c)^5+b^2\sinh(dx+c)^6+(2a^2-4ab+3b^2)\cosh(dx+c)^4+(15b^2\cosh(dx+c)^2+2a^2-4ab+3b^2)\sinh(dx+c)^4+4(5b^2\cosh(dx+c)^3+(2a^2-4ab+3b^2)\cosh(dx+c))\sinh(dx+c)^3-(2a^2-4ab+3b^2)\cosh(dx+c)^2+(15b^2\cosh(dx+c)^4+6(2a^2-4ab+3b^2)\cosh(dx+c)^2-2a^2+4ab-3b^2)\sinh(dx+c)^2-b^2+2((a^2+2ab-3b^2)\cosh(dx+c)^5+5(a^2+2ab-3b^2)\cosh(dx+c)\sinh(dx+c)^4+(a^2+2ab-3b^2)\sinh(dx+c)^5+2(a^2+2ab-3b^2)\cosh(dx+c)^3+2(5(a^2+2ab-3b^2)\cosh(dx+c)^2+a^2+2ab-3b^2)\sinh(dx+c)^3+2(5(a^2+2ab-3b^2)\cosh(dx+c)^3+3(a^2+2ab-3b^2)\cosh(dx+c))\sinh(dx+c)^2+(a^2+2ab-3b^2)\cosh(dx+c)+(5(a^2+2ab-3b^2)\cosh(dx+c)^4+6(a^2+2ab-3b^2)\cosh(dx+c)^2+a^2+2ab-3b^2)\sinh(dx+c))\arctan(\cosh(dx+c)+\sinh(dx+c))+2(3b^2\cosh(dx+c)^5+2(2a^2-4ab+3b^2)\cosh(dx+c)^3-(2a^2-4ab+3b^2)\cosh(dx+c))\sinh(dx+c)}{2(d\cosh(dx+c)^5+5d\cosh(dx+c)\sinh(dx+c)^4+d\sinh(dx+c)^5+2d\cosh(dx+c)^3+2(5d\cosh(dx+c)^2+d)\sinh(dx+c)^3+2(5d\cosh(dx+c)^3+3d\cosh(dx+c))\sinh(dx+c)^2+d\cosh(dx+c)+(5d\cosh(dx+c)^4+6d\cosh(dx+c)^2+d)\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d
*x + c)^6 + (2*a^2 - 4*a*b + 3*b^2)*cosh(d*x + c)^4 + (15*b^2*cosh(d*x + c)
^2 + 2*a^2 - 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 + (2
*a^2 - 4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (2*a^2 - 4*a*b + 3*b
^2)*cosh(d*x + c)^2 + (15*b^2*cosh(d*x + c)^4 + 6*(2*a^2 - 4*a*b + 3*b^2)*c
osh(d*x + c)^2 - 2*a^2 + 4*a*b - 3*b^2)*sinh(d*x + c)^2 - b^2 + 2*((a^2 + 2
*a*b - 3*b^2)*cosh(d*x + c)^5 + 5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)*sinh(
```

```
d*x + c)^4 + (a^2 + 2*a*b - 3*b^2)*sinh(d*x + c)^5 + 2*(a^2 + 2*a*b - 3*b^2
)*cosh(d*x + c)^3 + 2*(5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*
b - 3*b^2)*sinh(d*x + c)^3 + 2*(5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^3 + 3
*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (a^2 + 2*a*b - 3*b^
2)*cosh(d*x + c) + (5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*
b - 3*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b - 3*b^2)*sinh(d*x + c))*arctan(cos
h(d*x + c) + sinh(d*x + c)) + 2*(3*b^2*cosh(d*x + c)^5 + 2*(2*a^2 - 4*a*b +
 3*b^2)*cosh(d*x + c)^3 - (2*a^2 - 4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x +
 c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x +
c)^5 + 2*d*cosh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 +
2*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x +
c) + (5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^{2}\left(c+dx\right)\right)^{2}\operatorname{sech}^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
[Out] Integral((a + b*sinh(c + d*x)**2)**2*sech(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(60) = 120.
time = 0.43, size = 163, normalized size = 2.55

$$\frac{2\,b^{2}\left(e^{(dx+c)}-e^{(-dx-c)}\right)+\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^{2}+2\,ab-3\,b^{2}\right)+\frac{4\left(a^{2}\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)+b^{2}\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^{2}+4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
[Out] 1/4*(2*b^2*(e^(d*x + c) - e^(-d*x - c)) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*
c) - 1)*e^(-d*x - c)))*(a^2 + 2*a*b - 3*b^2) + 4*(a^2*(e^(d*x + c) - e^(-d*
x - c)) - 2*a*b*(e^(d*x + c) - e^(-d*x - c)) + b^2*(e^(d*x + c) - e^(-d*x -
 c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [**B**]
time = 0.88, size = 220, normalized size = 3.44

$$\frac{b^{2}\,\mathrm{e}^{c+d\,x}}{2\,d}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a^{2}\,\sqrt{d^{2}}-3\,b^{2}\,\sqrt{d^{2}}+2\,a\,b\,\sqrt{d^{2}}\right)}{d\,\sqrt{a^{4}+4\,a^{3}\,b-2\,a^{2}\,b^{2}-12\,a\,b^{3}+9\,b^{4}}}\right)\,\sqrt{a^{4}+4\,a^{3}\,b-2\,a^{2}\,b^{2}-12\,a\,b^{3}+9\,b^{4}}}{\sqrt{d^{2}}}-\frac{b^{2}\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{\mathrm{e}^{c+d\,x}\,\left(a^{2}-2\,a\,b+b^{2}\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\mathrm{e}^{c+d\,x}\,\left(a^{2}-2\,a\,b+b^{2}\right)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^3,x)
```

```
[Out] (b^2*exp(c + d*x))/(2*d) + (atan((exp(d*x)*exp(c)*(a^2*(d^2)^(1/2) - 3*b^2*
(d^2)^(1/2) + 2*a*b*(d^2)^(1/2)))/(d*(4*a^3*b - 12*a*b^3 + a^4 + 9*b^4 - 2*
a^2*b^2)^(1/2)))*(4*a^3*b - 12*a*b^3 + a^4 + 9*b^4 - 2*a^2*b^2)^(1/2))/(d^2
)^(1/2) - (b^2*exp(- c - d*x))/(2*d) + (exp(c + d*x)*(a^2 - 2*a*b + b^2))/(
d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(a^2 - 2*a*b + b^2))/(d*(2*exp(
2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.300 $\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=47

$$b^2x+\frac{(a^2-b^2)\tanh(c+dx)}{d}-\frac{(a-b)^2\tanh^3(c+dx)}{3d}$$

```
[Out] b^2*x+(a^2-b^2)*tanh(d*x+c)/d-1/3*(a-b)^2*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3270, 398, 212}

$$\frac{(a^2-b^2)\tanh(c+dx)}{d}-\frac{(a-b)^2\tanh^3(c+dx)}{3d}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] b^2*x + ((a^2 - b^2)*Tanh[c + d*x])/d - ((a - b)^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^2-b^2-(a-b)^2x^2+\frac{b^2}{1-x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{(a^2-b^2)\tanh(c+dx)}{d}-\frac{(a-b)^2\tanh^3(c+dx)}{3d}+\frac{b^2\text{Subst}(\int}{}\\ &= b^2x+\frac{(a^2-b^2)\tanh(c+dx)}{d}-\frac{(a-b)^2\tanh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 57, normalized size = 1.21

$$\frac{3b^2(c+dx)+(a-b)(2a+b+(a+2b)\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\tanh(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (3*b^2*(c + d*x) + (a - b)*(2*a + b + (a + 2*b)*Cosh[2*(c + d*x)])*Sech[c +
 d*x]^2*Tanh[c + d*x])/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(45) = 90$.
time = 1.67, size = 92, normalized size = 1.96

method	result	size
risch	$b^2x-\frac{4\left(3ab\,e^{4dx+4c}-3b^2e^{4dx+4c}+3a^2e^{2dx+2c}-3b^2e^{2dx+2c}+a^2+ab-2b^2\right)}{3d\left(1+e^{2dx+2c}\right)^3}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x-4/3*(3*a*b*exp(4*d*x+4*c)-3*b^2*exp(4*d*x+4*c)+3*a^2*exp(2*d*x+2*c)-3
*b^2*exp(2*d*x+2*c)+a^2+a*b-2*b^2)/d/(1+exp(2*d*x+2*c))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(45) = 90$.
time = 0.32, size = 267, normalized size = 5.68

$$\frac{1}{3}b^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)+\frac{4}{3}a^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)+\frac{4}{3}ab\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4/3*a^2
*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x
- 6*c) + 1))) + 4/3*a*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-
4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(45) = 90$.
time = 0.37, size = 200, normalized size = 4.26

$$\frac{\left(3\,b^2dx-2\,a^2-2\,ab+4\,b^2\right)\cosh\left(dx+c\right)^3+3\left(3\,b^2dx-2\,a^2-2\,ab+4\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+2\left(a^2+ab-2\,b^2\right)\sinh\left(dx+c\right)^3+3\left(3\,b^2dx-2\,a^2-2\,ab+4\,b^2\right)\cosh\left(dx+c\right)+6\left(\left(a^2+ab-2\,b^2\right)\cosh\left(dx+c\right)^2+a^2-ab\right)\sinh\left(dx+c\right)}{3\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+3\,d\cosh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b^2*d*x - 2*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*b^2*d*x - 2
*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*b - 2*b^2)
*sinh(d*x + c)^3 + 3*(3*b^2*d*x - 2*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c) + 6*
((a^2 + a*b - 2*b^2)*cosh(d*x + c)^2 + a^2 - a*b)*sinh(d*x + c))/(d*cosh(d*
x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(45) = 90$.
time = 0.43, size = 98, normalized size = 2.09

$$\frac{3\,(dx+c)b^2-\frac{4\left(3\,abe^{(4\,dx+4\,c)}-3\,b^2e^{(4\,dx+4\,c)}+3\,a^2e^{(2\,dx+2\,c)}-3\,b^2e^{(2\,dx+2\,c)}+a^2+ab-2\,b^2\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*b^2 - 4*(3*a*b*e^(4*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 3
*a^2*e^(2*d*x + 2*c) - 3*b^2*e^(2*d*x + 2*c) + a^2 + a*b - 2*b^2)/(e^(2*d*x
 + 2*c) + 1)^3)/d
```

Mupad [B]
time = 0.84, size = 194, normalized size = 4.13

$$b^2\,x-\frac{\frac{4\,\left(a\,b-b^2\right)}{3\,d}-\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b-a^2\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a\,b-b^2\right)}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}+\frac{\frac{4\,\left(a\,b-a^2\right)}{3\,d}-\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b-b^2\right)}{3\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{4\,\left(a\,b-b^2\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^4,x)
```

```
[Out] b^2*x - ((4*(a*b - b^2))/(3*d) - (8*exp(2*c + 2*d*x)*(a*b - a^2))/(3*d) + (
4*exp(4*c + 4*d*x)*(a*b - b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*
d*x) + exp(6*c + 6*d*x) + 1) + ((4*(a*b - a^2))/(3*d) - (4*exp(2*c + 2*d*x)
*(a*b - b^2))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (4*(a*b
- b^2))/(3*d*(exp(2*c + 2*d*x) + 1))
```

3.301 $\int \mathbf{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=96

$$\frac{(3a^2+2ab+3b^2)\,\text{ArcTan}(\sinh(c+dx))}{8d}+\frac{3(a^2-b^2)\,\text{sech}(c+dx)\tanh(c+dx)}{8d}+\frac{(a-b)\text{sech}^3(c+dx)\,\big(a+b}{}$$

```
[Out] 1/8*(3*a^2+2*a*b+3*b^2)*arctan(sinh(d*x+c))/d+3/8*(a^2-b^2)*sech(d*x+c)*tan
h(d*x+c)/d+1/4*(a-b)*sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 424, 393, 209}

$$\frac{(3a^2+2ab+3b^2)\,\text{ArcTan}(\sinh(c+dx))}{8d}+\frac{3(a^2-b^2)\tanh(c+dx)\text{sech}(c+dx)}{8d}+\frac{(a-b)\tanh(c+dx)\text{sech}^3(c+dx)\,\left(a+b\sinh^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 2*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (3*(a^2 - b^2)*Sech
[c + d*x]*Tanh[c + d*x])/(8*d) + ((a - b)*Sech[c + d*x]^3*(a + b*Sinh[c + d
*x]^2)*Tanh[c + d*x])/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^2}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{(a-b)\operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\tanh(c+dx)}{4d} + \text{Su}\\
&= \frac{3(a^2-b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{(a-b)\operatorname{sech}^3(c+dx)\,(}{}\\
&= \frac{(3a^2+2ab+3b^2)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{3(a^2-b^2)\operatorname{sech}(c+}{8d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.52, size = 303, normalized size = 3.16

$$
-\frac{\operatorname{csch}^3(c+dx)\left(128\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{9}{2};-\sinh^2(c+dx)\right)\sinh^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2+128\,{}_4F_3\left(\frac{3}{2},2,2,2;1,1,\frac{9}{2};-\sinh^2(c+dx)\right)\sinh^6(c+dx)\left(7a^2+12ab\sinh^2(c+dx)+5b^2\sinh^4(c+dx)\right)+35\left(3375a^2+a(657a+4643b+607b\cosh(2(c+dx)))\sinh^2(c+dx)+1947b^2\sinh^4(c+dx)+485b^2\sinh^6(c+dx)\right)-\frac{105\tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)(1125a^2+2a(297a+875b)\sinh^2(c+dx)+(37a^2+988ab+649b^2)\sinh^4(c+dx)+2b(11a+189b)\sinh^6(c+dx)+9b^2\sinh^8(c+dx))}{\sqrt{-\sinh^2(c+dx)}}\right)}{6720d}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/6720*(Csch[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1
, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2 + 128*H
ypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*
x]^6*(7*a^2 + 12*a*b*Sinh[c + d*x]^2 + 5*b^2*Sinh[c + d*x]^4) + 35*(3375*a^
2 + a*(657*a + 4643*b + 607*b*Cosh[2*(c + d*x)])*Sinh[c + d*x]^2 + 1947*b^2
*Sinh[c + d*x]^4 + 485*b^2*Sinh[c + d*x]^6) - (105*ArcTanh[Sqrt[-Sinh[c + d
*x]^2]]*(1125*a^2 + 2*a*(297*a + 875*b)*Sinh[c + d*x]^2 + (37*a^2 + 988*a*b
 + 649*b^2)*Sinh[c + d*x]^4 + 2*b*(11*a + 189*b)*Sinh[c + d*x]^6 + 9*b^2*Si
nh[c + d*x]^8))/Sqrt[-Sinh[c + d*x]^2]))/d
```

Maple [C] Result contains complex when optimal does not.
time = 1.67, size = 276, normalized size = 2.88

method	result

risch	$\frac{e^{dx+c}\left(3a^2e^{6dx+6c}+2ab\,e^{6dx+6c}-5b^2e^{6dx+6c}+11a^2e^{4dx+4c}-14ab\,e^{4dx+4c}+3b^2e^{4dx+4c}-11a^2e^{2dx+2c}+14ab\,e^{2dx+2c}-3b^2e^{2dx+2c}-3a\right.}{4d\left(1+e^{2dx+2c}\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp(d*x+c)*(3*a^2*exp(6*d*x+6*c)+2*a*b*exp(6*d*x+6*c)-5*b^2*exp(6*d*x+6
*c)+11*a^2*exp(4*d*x+4*c)-14*a*b*exp(4*d*x+4*c)+3*b^2*exp(4*d*x+4*c)-11*a^2
*exp(2*d*x+2*c)+14*a*b*exp(2*d*x+2*c)-3*b^2*exp(2*d*x+2*c)-3*a^2-2*a*b+5*b^
2)/d/(1+exp(2*d*x+2*c))^4+3/8*I/d*ln(exp(d*x+c)+I)*a^2+1/4*I/d*ln(exp(d*x+c
)+I)*a*b+3/8*I/d*ln(exp(d*x+c)+I)*b^2-3/8*I/d*ln(exp(d*x+c)-I)*a^2-1/4*I/d*
ln(exp(d*x+c)-I)*a*b-3/8*I/d*ln(exp(d*x+c)-I)*b^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(90) = 180.
time = 0.50, size = 347, normalized size = 3.61

$$-\frac{1}{4}\,b^2\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5\,e^{(-dx-c)}-3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)-\frac{1}{4}\,a^2\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)-\frac{1}{2}\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7\,e^{(-3\,dx-3\,c)}+7\,e^{(-5\,dx-5\,c)}-e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*x - 3*c) +
 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/4*a^2*(3*arct
an(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x -
 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4
*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/2*a*b*(arctan(e^(-d*x - c))
/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c) - e^(-7*d*x -
7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^
(-8*d*x - 8*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. 2(90) = 180.
time = 0.39, size = 1472, normalized size = 15.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^2 + 2*a*b - 5*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 + 2*a*b - 5*b^2)*co
sh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 2*a*b - 5*b^2)*sinh(d*x + c)^7 + (11
*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 2*a*b - 5*b^2)*cosh(d
```

```
*x + c)^2 + 11*a^2 - 14*a*b + 3*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 2*a*b
- 5*b^2)*cosh(d*x + c)^3 + (11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*
x + c)^4 - (11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^3 + (35*(3*a^2 + 2*a*b -
 5*b^2)*cosh(d*x + c)^4 + 10*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^2 - 11
*a^2 + 14*a*b - 3*b^2)*sinh(d*x + c)^3 + (21*(3*a^2 + 2*a*b - 5*b^2)*cosh(d
*x + c)^5 + 10*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^3 - 3*(11*a^2 - 14*a
*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^2 + 2*a*b + 3*b^2)*cosh(
d*x + c)^8 + 8*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a
^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x +
c)^6 + 4*(7*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b + 3*b^2
)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(3*a^2
 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2 + 2*a*b + 3*b^2
)*cosh(d*x + c)^4 + 2*(35*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 30*(3*a
^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 9*a^2 + 6*a*b + 9*b^2)*sinh(d*x + c)^
4 + 8*(7*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(3*a^2 + 2*a*b + 3*b^
2)*cosh(d*x + c)^3 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 + 2*a*b + 3*b^
2)*cosh(d*x + c)^6 + 15*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 9*(3*a^2
+ 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^2 +
 3*a^2 + 2*a*b + 3*b^2 + 8*((3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^7 + 3*(3*
a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^5 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x +
 c)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d
*x + c) + sinh(d*x + c)) - (3*a^2 + 2*a*b - 5*b^2)*cosh(d*x + c) + (7*(3*a^
2 + 2*a*b - 5*b^2)*cosh(d*x + c)^6 + 5*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x +
 c)^4 - 3*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^2 - 3*a^2 - 2*a*b + 5*b^2
)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d
*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d
*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6
*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*
sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(
d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 +
15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh
(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*
sinh(d*x + c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(90) = 180.

time = 0.44, size = 218, normalized size = 2.27

$$\frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\,\left(e^{\left(2\,dx+2\,c\right)} - 1\right)e^{\left(-dx-c\right)}\right)\right)\left(3\,a^{2} + 2\,ab + 3\,b^{2}\right) + \frac{4\,\left(3\,a^{2}\left(e^{\left(dx+c\right)}-e^{\left(-dx-c\right)}\right)^{3}+2\,ab\left(e^{\left(dx+c\right)}-e^{\left(-dx-c\right)}\right)^{3}-5\,b^{2}\left(e^{\left(dx+c\right)}-e^{\left(-dx-c\right)}\right)^{3}+20\,a^{2}\left(e^{\left(dx+c\right)}-e^{\left(-dx-c\right)}\right)-8\,ab\left(e^{\left(dx+c\right)}-e^{\left(-dx-c\right)}\right)-12\,b^{2}\left(e^{\left(dx+c\right)}-e^{\left(-dx-c\right)}\right)\right)}{\left(\left(e^{\left(dx+c\right)}-e^{\left(-dx-c\right)}\right)^{2}+4\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a^2 + 2*a*
b + 3*b^2) + 4*(3*a^2*(e^(d*x + c) - e^(-d*x - c))^3 + 2*a*b*(e^(d*x + c) -
 e^(-d*x - c))^3 - 5*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 20*a^2*(e^(d*x +
c) - e^(-d*x - c)) - 8*a*b*(e^(d*x + c) - e^(-d*x - c)) - 12*b^2*(e^(d*x +
c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B]
time = 0.89, size = 327, normalized size = 3.41

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(3\,a^2\,\sqrt{d^2}+3\,b^2\,\sqrt{d^2}+2\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{9\,a^4+12\,a^3\,b+22\,a^2\,b^2+12\,a\,b^3+9\,b^4}}\right)\,\sqrt{9\,a^4+12\,a^3\,b+22\,a^2\,b^2+12\,a\,b^3+9\,b^4}}{4\,\sqrt{d^2}}-\frac{6\,{\mathrm{e}}^{c+d\,x}\,\left(a^2-2\,a\,b+b^2\right)}{d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)}+\frac{4\,{\mathrm{e}}^{c+d\,x}\,\left(a^2-2\,a\,b+b^2\right)}{d\,\left(4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1\right)}+\frac{{\mathrm{e}}^{c+d\,x}\,\left(a^2-10\,a\,b+9\,b^2\right)}{2\,d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}+\frac{{\mathrm{e}}^{c+d\,x}\,\left(3\,a^2+2\,a\,b-5\,b^2\right)}{4\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^5,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(3*a^2*(d^2)^(1/2) + 3*b^2*(d^2)^(1/2) + 2*a*b*(d^2)
^(1/2)))/(d*(12*a*b^3 + 12*a^3*b + 9*a^4 + 9*b^4 + 22*a^2*b^2)^(1/2)))*(12*
a*b^3 + 12*a^3*b + 9*a^4 + 9*b^4 + 22*a^2*b^2)^(1/2))/(4*(d^2)^(1/2)) - (6*
exp(c + d*x)*(a^2 - 2*a*b + b^2))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) + 1)) + (4*exp(c + d*x)*(a^2 - 2*a*b + b^2))/(d*(4*ex
p(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x)
 + 1)) + (exp(c + d*x)*(a^2 - 10*a*b + 9*b^2))/(2*d*(2*exp(2*c + 2*d*x) + e
xp(4*c + 4*d*x) + 1)) + (exp(c + d*x)*(2*a*b + 3*a^2 - 5*b^2))/(4*d*(exp(2*
c + 2*d*x) + 1))
```

3.302 $\int \mathbf{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=57

$$\frac{a^2\tanh(c+dx)}{d}-\frac{2a(a-b)\tanh^3(c+dx)}{3d}+\frac{(a-b)^2\tanh^5(c+dx)}{5d}$$

```
[Out] a^2*tanh(d*x+c)/d-2/3*a*(a-b)*tanh(d*x+c)^3/d+1/5*(a-b)^2*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3270, 200}

$$\frac{a^2\tanh(c+dx)}{d}+\frac{(a-b)^2\tanh^5(c+dx)}{5d}-\frac{2a(a-b)\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d - (2*a*(a - b)*Tanh[c + d*x]^3)/(3*d) + ((a - b)^2*Ta
nh[c + d*x]^5)/(5*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int\left(a-(a-b)x^2\right)^2\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int\left(a^2-2a(a-b)x^2+(a-b)^2x^4\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a^2\tanh(c+dx)}{d}-\frac{2a(a-b)\tanh^3(c+dx)}{3d}+\frac{(a-b)^2\tanh^5(}{5d}\end{aligned}$$

Mathematica [**A**]
time = 0.32, size = 69, normalized size = 1.21

$$\frac{\left(8a^2+4ab+3b^2+2\left(2a^2+ab-3b^2\right)\operatorname{sech}^2(c+dx)+3(a-b)^2\operatorname{sech}^4(c+dx)\right)\tanh(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 4*a*b + 3*b^2 + 2*(2*a^2 + a*b - 3*b^2)*Sech[c + d*x]^2 + 3*(a -
b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(15*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(53) = 106$.
time = 1.70, size = 129, normalized size = 2.26

method	result	size
risch	$-\frac{2\left(15b^2 e^{8dx+8c}+60ab\,e^{6dx+6c}+80a^2 e^{4dx+4c}-20ab\,e^{4dx+4c}+30b^2 e^{4dx+4c}+40a^2 e^{2dx+2c}+20ab\,e^{2dx+2c}+8a^2+4ab+3b^2\right)}{15d\left(1+e^{2dx+2c}\right)^5}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(15*b^2*exp(8*d*x+8*c)+60*a*b*exp(6*d*x+6*c)+80*a^2*exp(4*d*x+4*c)-20
*a*b*exp(4*d*x+4*c)+30*b^2*exp(4*d*x+4*c)+40*a^2*exp(2*d*x+2*c)+20*a*b*exp(
2*d*x+2*c)+8*a^2+4*a*b+3*b^2)/d/(1+exp(2*d*x+2*c))^5
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(53) = 106$.
time = 0.27, size = 698, normalized size = 12.25

$$\frac{16}{15}a^2\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{10e^{(-4dx-4c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{1}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)+\frac{8}{15}ab\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}-\frac{5e^{(-4dx-4c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{15e^{(-6dx-6c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{1}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)+\frac{2}{5}b^2\left(\frac{10e^{(-4dx-4c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{5e^{(-8dx-8c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{1}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 16/15*a^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*
e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*
x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) + 1))) + 8/15*a*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
```

```
c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
 + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/
(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2/5*b^2*(10*e^(-4*d*x - 4*c)/(d*
(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d
*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(53) = 106$.
time $= 0.39$, size $= 403$, normalized size $= 7.07$

$$-\frac{4\left(\left(4\,a^{2}+2\,ab+9\,b^{2}\right)\cosh\left(dx+c\right)^{4}-8\left(2\,a^{2}+ab-3\,b^{2}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\left(4\,a^{2}+2\,ab+9\,b^{2}\right)\sinh\left(dx+c\right)^{4}+20\left(a^{2}+2\,ab\right)\cosh\left(dx+c\right)^{2}+2\left(3\left(4\,a^{2}+2\,ab+9\,b^{2}\right)\cosh\left(dx+c\right)^{2}+10\,a^{2}+20\,ab\right)\sinh\left(dx+c\right)^{2}+40\,a^{2}-10\,ab+15\,b^{2}-8\left(\left(2\,a^{2}+ab-3\,b^{2}\right)\cosh\left(dx+c\right)^{3}+5\left(a^{2}-ab\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)}{15\left(d\cosh\left(dx+c\right)^{6}+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+d\sinh\left(dx+c\right)^{6}+6\,d\cosh\left(dx+c\right)^{4}+3\left(5\,d\cosh\left(dx+c\right)^{2}+2\,d\right)\sinh\left(dx+c\right)^{4}+4\left(5\,d\cosh\left(dx+c\right)^{3}+4\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+15\,d\cosh\left(dx+c\right)^{2}+3\left(5\,d\cosh\left(dx+c\right)^{4}+12\,d\cosh\left(dx+c\right)^{2}+5\,d\right)\sinh\left(dx+c\right)^{2}+2\left(3\,d\cosh\left(dx+c\right)^{5}+8\,d\cosh\left(dx+c\right)^{3}+5\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+10\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/15*((4*a^2 + 2*a*b + 9*b^2)*cosh(d*x + c)^4 - 8*(2*a^2 + a*b - 3*b^2)*co
sh(d*x + c)*sinh(d*x + c)^3 + (4*a^2 + 2*a*b + 9*b^2)*sinh(d*x + c)^4 + 20*
(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(4*a^2 + 2*a*b + 9*b^2)*cosh(d*x + c)^
2 + 10*a^2 + 20*a*b)*sinh(d*x + c)^2 + 40*a^2 - 10*a*b + 15*b^2 - 8*((2*a^2
 + a*b - 3*b^2)*cosh(d*x + c)^3 + 5*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c
))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)
^6 + 6*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^4 +
4*(5*d*cosh(d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x
 + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c
)^2 + 2*(3*d*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sin
h(d*x + c) + 10*d)
```

Sympy [**F(-2)**]
time $= 0.00$, size $= 0$, normalized size $= 0.00$

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(53) = 106$.
time $= 0.44$, size $= 128$, normalized size $= 2.25$

$$-\frac{2\left(15\,b^{2}e^{\left(8\,dx+8\,c\right)}+60\,abe^{\left(6\,dx+6\,c\right)}+80\,a^{2}e^{\left(4\,dx+4\,c\right)}-20\,abe^{\left(4\,dx+4\,c\right)}+30\,b^{2}e^{\left(4\,dx+4\,c\right)}+40\,a^{2}e^{\left(2\,dx+2\,c\right)}+20\,abe^{\left(2\,dx+2\,c\right)}+8\,a^{2}+4\,ab+3\,b^{2}\right)}{15\,d\left(e^{\left(2\,dx+2\,c\right)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*b^2*e^(8*d*x + 8*c) + 60*a*b*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x +
4*c) - 20*a*b*e^(4*d*x + 4*c) + 30*b^2*e^(4*d*x + 4*c) + 40*a^2*e^(2*d*x +
2*c) + 20*a*b*e^(2*d*x + 2*c) + 8*a^2 + 4*a*b + 3*b^2)/(d*(e^(2*d*x + 2*c)
+ 1)^5)
```

Mupad [B]
time = 0.86, size = 464, normalized size = 8.14

$$-\frac{\frac{2\,\left(8\,a^2-8\,a\,b+3\,b^2\right)}{15\,d}+\frac{2\,b^2\,{\mathrm{e}}^{4\,c+4\,d\,x}}{5\,d}+\frac{4\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a-b\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{\frac{2\,b^2}{5\,d}+\frac{2\,b^2\,{\mathrm{e}}^{8\,c+8\,d\,x}}{5\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(8\,a^2-8\,a\,b+3\,b^2\right)}{5\,d}+\frac{8\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a-b\right)}{5\,d}+\frac{8\,b\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(2\,a-b\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{\frac{2\,b\,\left(2\,a-b\right)}{5\,d}+\frac{2\,b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,b\,\left(2\,a-b\right)}{5\,d}+\frac{2\,b^2\,{\mathrm{e}}^{6\,c+6\,d\,x}}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(8\,a^2-8\,a\,b+3\,b^2\right)}{5\,d}+\frac{6\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,a-b\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{2\,b^2}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^6,x)
```

```
[Out] - ((2*(8*a^2 - 8*a*b + 3*b^2))/(15*d) + (2*b^2*exp(4*c + 4*d*x))/(5*d) + (4
*b*exp(2*c + 2*d*x)*(2*a - b))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d
*x) + exp(6*c + 6*d*x) + 1) - ((2*b^2)/(5*d) + (2*b^2*exp(8*c + 8*d*x))/(5*
d) + (4*exp(4*c + 4*d*x)*(8*a^2 - 8*a*b + 3*b^2))/(5*d) + (8*b*exp(2*c + 2*
d*x)*(2*a - b))/(5*d) + (8*b*exp(6*c + 6*d*x)*(2*a - b))/(5*d))/(5*exp(2*c
+ 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) +
 exp(10*c + 10*d*x) + 1) - ((2*b*(2*a - b))/(5*d) + (2*b^2*exp(2*c + 2*d*x)
)/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*b*(2*a - b))/(5*
d) + (2*b^2*exp(6*c + 6*d*x))/(5*d) + (2*exp(2*c + 2*d*x)*(8*a^2 - 8*a*b +
3*b^2))/(5*d) + (6*b*exp(4*c + 4*d*x)*(2*a - b))/(5*d))/(4*exp(2*c + 2*d*x)
 + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (2*b^2
)/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.303 $\int \text{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=131

$$\frac{(5a^2+2ab+b^2)\,\text{ArcTan}(\sinh(c+dx))}{16d}+\frac{(5a^2+2ab+b^2)\,\text{sech}(c+dx)\tanh(c+dx)}{16d}+\frac{(a-b)(5a+3b)\text{sech}}{2}$$

```
[Out] 1/16*(5*a^2+2*a*b+b^2)*arctan(sinh(d*x+c))/d+1/16*(5*a^2+2*a*b+b^2)*sech(d*
x+c)*tanh(d*x+c)/d+1/24*(a-b)*(5*a+3*b)*sech(d*x+c)^3*tanh(d*x+c)/d+1/6*(a-
b)*sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3269, 424, 393, 205, 209}

$$\frac{(5a^2+2ab+b^2)\,\text{ArcTan}(\sinh(c+dx))}{16d}+\frac{(5a^2+2ab+b^2)\tanh(c+dx)\text{sech}(c+dx)}{16d}+\frac{(a-b)(5a+3b)\tanh(c+dx)\text{sech}^3(c+dx)}{24d}+\frac{(a-b)\tanh(c+dx)\text{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((5*a^2 + 2*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((5*a^2 + 2*a*b + b^
2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) + ((a - b)*(5*a + 3*b)*Sech[c + d*x]
^3*Tanh[c + d*x])/(24*d) + ((a - b)*Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)
*Tanh[c + d*x])/(6*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{(1+x^2)^4}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{(a-b)\operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)\tanh(c+dx)}{6d} + \text{Sub}\\
&= \frac{(a-b)(5a+3b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} + \frac{(a-b)\operatorname{sech}^5(c+}{}\\
&= \frac{(5a^2+2ab+b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} + \frac{(a-b)(5a+3b)\text{s}}{}\\
&= \frac{(5a^2+2ab+b^2)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{(5a^2+2ab+b^2)\operatorname{sech}(c}{16}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 9.20, size = 715, normalized size = 5.46

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (Csch[c + d*x]^3*(65625*a^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]] + 36855*a^2*Arc
Tanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 + 91875*a*b*ArcTanh[Sqrt[-Sinh
[c + d*x]^2]]*Sinh[c + d*x]^2 + 1680*a^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Si
nh[c + d*x]^4 + 54180*a*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 +
 32970*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 + 1365*a*b*ArcTa
nh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 + 19845*b^2*ArcTanh[Sqrt[-Sinh[c
 + d*x]^2]]*Sinh[c + d*x]^6 + 525*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[
c + d*x]^8 - 65625*a^2*Sqrt[-Sinh[c + d*x]^2] + 14980*a^2*(-Sinh[c + d*x]^2
)^(3/2) + 91875*a*b*(-Sinh[c + d*x]^2)^(3/2) + 8855*b^2*Sinh[c + d*x]^4*(-S
inh[c + d*x]^2)^(3/2) + 16*a^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1,
1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2) +
 32*a*b*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c
+ d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2) + 16*b^2*HypergeometricP
FQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]
^8*(-Sinh[c + d*x]^2)^(3/2) - 23555*a*b*(-Sinh[c + d*x]^2)^(5/2) - 32970*b^
2*(-Sinh[c + d*x]^2)^(5/2) + 32*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1,
 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2)*(5*a^2
 + 9*a*b*Sinh[c + d*x]^2 + 4*b^2*Sinh[c + d*x]^4) + 4*HypergeometricPFQ[{3/
2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]
^2)^(3/2)*(155*a^2 + 242*a*b*Sinh[c + d*x]^2 + 95*b^2*Sinh[c + d*x]^4)))/(2
520*d*Sqrt[-Sinh[c + d*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 1.77, size = 358, normalized size = 2.73

method	result
risch	$\frac{e^{dx+c}(15a^2 e^{10dx+10c}+6ab\, e^{10dx+10c}+3b^2 e^{10dx+10c}+85a^2 e^{8dx+8c}+34ab\, e^{8dx+8c}-47b^2 e^{8dx+8c}+198a^2 e^{6dx+6c}-228ab\, e^{6dx+6c}+78}{24d(1+e^{2dx+2c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*exp(d*x+c)*(15*a^2*exp(10*d*x+10*c)+6*a*b*exp(10*d*x+10*c)+3*b^2*exp(1
0*d*x+10*c)+85*a^2*exp(8*d*x+8*c)+34*a*b*exp(8*d*x+8*c)-47*b^2*exp(8*d*x+8*
c)+198*a^2*exp(6*d*x+6*c)-228*a*b*exp(6*d*x+6*c)+78*b^2*exp(6*d*x+6*c)-198*
a^2*exp(4*d*x+4*c)+228*a*b*exp(4*d*x+4*c)-78*b^2*exp(4*d*x+4*c)-85*a^2*exp(
2*d*x+2*c)-34*a*b*exp(2*d*x+2*c)+47*b^2*exp(2*d*x+2*c)-15*a^2-6*a*b-3*b^2)/
d/(1+exp(2*d*x+2*c))^6+5/16*I/d*ln(exp(d*x+c)+I)*a^2+1/8*I/d*ln(exp(d*x+c)+
I)*a*b+1/16*I/d*ln(exp(d*x+c)+I)*b^2-5/16*I/d*ln(exp(d*x+c)-I)*a^2-1/8*I/d*
ln(exp(d*x+c)-I)*a*b-1/16*I/d*ln(exp(d*x+c)-I)*b^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(123) = 246$.
time = 0.48, size = 483, normalized size = 3.69

$$-\frac{1}{24}\,a^2\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15\,e^{(-dx-c)}+85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}-198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9\,dx-9\,c)}-15\,e^{(-11\,dx-11\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)-\frac{1}{12}\,ab\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+17\,e^{(-3\,dx-3\,c)}-114\,e^{(-5\,dx-5\,c)}+114\,e^{(-7\,dx-7\,c)}-17\,e^{(-9\,dx-9\,c)}-3\,e^{(-11\,dx-11\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)-\frac{1}{24}\,b^2\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}-47\,e^{(-3\,dx-3\,c)}+78\,e^{(-5\,dx-5\,c)}-78\,e^{(-7\,dx-7\,c)}+47\,e^{(-9\,dx-9\,c)}-3\,e^{(-11\,dx-11\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*a^2*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*
c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15
*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-
6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) - 1/12*a*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 17*e^(-
3*d*x - 3*c) - 114*e^(-5*d*x - 5*c) + 114*e^(-7*d*x - 7*c) - 17*e^(-9*d*x -
 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c)
+ 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12
*d*x - 12*c) + 1))) - 1/24*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c)
- 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78*e^(-7*d*x - 7*c) + 47*e^(-
9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x
- 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) +
 e^(-12*d*x - 12*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2824 vs. $2(123) = 246$.
time = 0.40, size = 2824, normalized size = 21.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 + 33*(5*a^2 + 2*a*b + b^2)*c
osh(d*x + c)*sinh(d*x + c)^10 + 3*(5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^11 +
(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^9 + (165*(5*a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + 85*a^2 + 34*a*b - 47*b^2)*sinh(d*x + c)^9 + 9*(55*(5*a^2 + 2
*a*b + b^2)*cosh(d*x + c)^3 + (85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c))*sin
h(d*x + c)^8 + 6*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^7 + 6*(165*(5*a^2
 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c
)^2 + 33*a^2 - 38*a*b + 13*b^2)*sinh(d*x + c)^7 + 42*(33*(5*a^2 + 2*a*b + b
^2)*cosh(d*x + c)^5 + 2*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^3 + (33*a^
2 - 38*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 6*(33*a^2 - 38*a*b +
13*b^2)*cosh(d*x + c)^5 + 6*(231*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 21
*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^4 + 21*(33*a^2 - 38*a*b + 13*b^2)
*cosh(d*x + c)^2 - 33*a^2 + 38*a*b - 13*b^2)*sinh(d*x + c)^5 + 6*(165*(5*a^
2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 21*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x +
 c)^5 + 35*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^3 - 5*(33*a^2 - 38*a*b
+ 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (85*a^2 + 34*a*b - 47*b^2)*cosh(
d*x + c)^3 + (495*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 84*(85*a^2 + 34*a
*b - 47*b^2)*cosh(d*x + c)^6 + 210*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)
^4 - 60*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^2 - 85*a^2 - 34*a*b + 47*b
```

```
^2)*sinh(d*x + c)^3 + 3*(55*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 12*(85*
a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^7 + 42*(33*a^2 - 38*a*b + 13*b^2)*cosh
(d*x + c)^5 - 20*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^3 - (85*a^2 + 34*
a*b - 47*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((5*a^2 + 2*a*b + b^2)*cos
h(d*x + c)^12 + 12*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (
5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^12 + 6*(5*a^2 + 2*a*b + b^2)*cosh(d*x +
c)^10 + 6*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*
sinh(d*x + c)^10 + 20*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(5*a^2
+ 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(5*a^2 + 2*a*b + b^2)*co
sh(d*x + c)^8 + 15*(33*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 18*(5*a^2 +
2*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 24*(3
3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 30*(5*a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^3 + 5*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(5*a^
2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(231*(5*a^2 + 2*a*b + b^2)*cosh(d*x +
c)^6 + 315*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 105*(5*a^2 + 2*a*b + b^2
)*cosh(d*x + c)^2 + 25*a^2 + 10*a*b + 5*b^2)*sinh(d*x + c)^6 + 24*(33*(5*a^
2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 63*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5
 + 35*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(5*a^2 + 2*a*b + b^2)*cosh(
d*x + c))*sinh(d*x + c)^5 + 15*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(
33*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 84*(5*a^2 + 2*a*b + b^2)*cosh(d*
x + c)^6 + 70*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 20*(5*a^2 + 2*a*b + b
^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(5*a^2
+ 2*a*b + b^2)*cosh(d*x + c)^9 + 36*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 +
 42*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 20*(5*a^2 + 2*a*b + b^2)*cosh(d
*x + c)^3 + 3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(5*a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 6*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x +
c)^10 + 45*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 70*(5*a^2 + 2*a*b + b^2)
*cosh(d*x + c)^6 + 50*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(5*a^2 + 2
*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + 5*a^2
+ 2*a*b + b^2 + 12*((5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 + 5*(5*a^2 + 2*a
*b + b^2)*cosh(d*x + c)^9 + 10*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 10*(
5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 5*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c
)^3 + (5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x +
 c) + sinh(d*x + c)) - 3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c) + 3*(11*(5*a^2
 + 2*a*b + b^2)*cosh(d*x + c)^10 + 3*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x +
c)^8 + 14*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^6 - 10*(33*a^2 - 38*a*b
+ 13*b^2)*cosh(d*x + c)^4 - (85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^2 - 5*
a^2 - 2*a*b - b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*
sinh(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh
(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x
 + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 +
18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d
*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^
6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^
2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5
```

```
+ 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x
 + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x +
 c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(123) = 246.
time = 0.45, size = 291, normalized size = 2.22

$$\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(5a^2+2ab+b^2\right)+\frac{4\left(15a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+6ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+3b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+160a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+64ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-32b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+528a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)-96ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)-48b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(5*a^2 + 2*
a*b + b^2) + 4*(15*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 6*a*b*(e^(d*x + c)
- e^(-d*x - c))^5 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 160*a^2*(e^(d*x
+ c) - e^(-d*x - c))^3 + 64*a*b*(e^(d*x + c) - e^(-d*x - c))^3 - 32*b^2*(e^
(d*x + c) - e^(-d*x - c))^3 + 528*a^2*(e^(d*x + c) - e^(-d*x - c)) - 96*a*b
*(e^(d*x + c) - e^(-d*x - c)) - 48*b^2*(e^(d*x + c) - e^(-d*x - c)))/((e^(d
*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

Mupad [**B**]
time = 0.92, size = 582, normalized size = 4.44

$$\frac{\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\left(5a^2\sqrt{d^2}+b^2\sqrt{d^2}+2ab\sqrt{d^2}\right)}{d\sqrt{25a^4+20a^3b+14a^2b^2+4ab^3+b^4}}\right)\sqrt{25a^4+20a^3b+14a^2b^2+4ab^3+b^4}}{8\sqrt{d^2}}-\frac{\frac{2b^2e^{c+dx}}{3d}+\frac{2b^2e^{9c+9dx}}{3d}+\frac{4e^{5c+5dx}\left(8a^2-8ab+3b^2\right)}{3d}+\frac{8be^{3c+3dx}(2a-b)}{3d}+\frac{8be^{7c+7dx}(2a-b)}{3d}}{6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1}-\frac{2e^{c+dx}\left(11a^2-26ab+15b^2\right)}{3d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}+\frac{e^{c+dx}\left(5a^2+2ab+b^2\right)}{8d\left(e^{2c+2dx}+1\right)}+\frac{16e^{c+dx}\left(a^2-2ab+b^2\right)}{3d\left(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1\right)}+\frac{e^{c+dx}\left(a^2-22ab+21b^2\right)}{3d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}+\frac{e^{c+dx}\left(5a^2+2ab-23b^2\right)}{12d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^7,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(5*a^2*(d^2)^(1/2) + b^2*(d^2)^(1/2) + 2*a*b*(d^2)^(
1/2)))/(d*(4*a*b^3 + 20*a^3*b + 25*a^4 + b^4 + 14*a^2*b^2)^(1/2)))*(4*a*b^3
 + 20*a^3*b + 25*a^4 + b^4 + 14*a^2*b^2)^(1/2))/(8*(d^2)^(1/2)) - ((2*b^2*e
xp(c + d*x))/(3*d) + (2*b^2*exp(9*c + 9*d*x))/(3*d) + (4*exp(5*c + 5*d*x)*(
8*a^2 - 8*a*b + 3*b^2))/(3*d) + (8*b*exp(3*c + 3*d*x)*(2*a - b))/(3*d) + (8
*b*exp(7*c + 7*d*x)*(2*a - b))/(3*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*
d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + e
```

```
xp(12*c + 12*d*x) + 1) - (2*exp(c + d*x)*(11*a^2 - 26*a*b + 15*b^2))/(3*d*(
4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*
d*x) + 1)) + (exp(c + d*x)*(2*a*b + 5*a^2 + b^2))/(8*d*(exp(2*c + 2*d*x) +
1)) + (16*exp(c + d*x)*(a^2 - 2*a*b + b^2))/(3*d*(5*exp(2*c + 2*d*x) + 10*e
xp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*
d*x) + 1)) + (exp(c + d*x)*(a^2 - 22*a*b + 21*b^2))/(3*d*(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*(2*a*b + 5*
a^2 - 23*b^2))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.304 $\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=238

$$\frac{3}{256}(4a-b)\left(8a^2-2ab+b^2\right)x+\frac{3(4a-b)\left(8a^2-2ab+b^2\right)\cosh(c+dx)\sinh(c+dx)}{256d}+(4a-b)\left(8a^2-2ab+\right.$$

```
[Out] 3/256*(4*a-b)*(8*a^2-2*a*b+b^2)*x+3/256*(4*a-b)*(8*a^2-2*a*b+b^2)*cosh(d*x+
c)*sinh(d*x+c)/d+1/128*(4*a-b)*(8*a^2-2*a*b+b^2)*cosh(d*x+c)^3*sinh(d*x+c)/
d+1/160*b*(44*a^2-28*a*b+5*b^2)*cosh(d*x+c)^5*sinh(d*x+c)/d+1/10*b*cosh(d*x
+c)^9*sinh(d*x+c)*(a-(a-b)*tanh(d*x+c)^2)^2/d+1/80*b*cosh(d*x+c)^7*sinh(d*x
+c)*(a*(10*a-b)-5*(a-b)*(2*a-b)*tanh(d*x+c)^2)/d
```

Rubi [A]

time = 0.22, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3270, 424, 540, 393, 205, 212}

$$\frac{b\left(44a^2-28ab+5b^2\right)\sinh(c+dx)\cosh^5(c+dx)}{160d}+\frac{(4a-b)\left(8a^2-2ab+b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{128d}+\frac{3(4a-b)\left(8a^2-2ab+b^2\right)\sinh(c+dx)\cosh(c+dx)}{256d}+\frac{3}{256}x(4a-b)\left(8a^2-2ab+b^2\right)+\frac{b\sinh(c+dx)\cosh^9(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{10d}+\frac{b\sinh(c+dx)\cosh^7(c+dx)\left(a(10a-b)-5(a-b)(2a-b)\tanh^2(c+dx)\right)}{80d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*(4*a - b)*(8*a^2 - 2*a*b + b^2)*x)/256 + (3*(4*a - b)*(8*a^2 - 2*a*b + b
^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) + ((4*a - b)*(8*a^2 - 2*a*b + b^2)
*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*d) + (b*(44*a^2 - 28*a*b + 5*b^2)*Cosh
[c + d*x]^5*Sinh[c + d*x])/(160*d) + (b*Cosh[c + d*x]^9*Sinh[c + d*x]*(a -
(a - b)*Tanh[c + d*x]^2)^2)/(10*d) + (b*Cosh[c + d*x]^7*Sinh[c + d*x]*(a*(1
0*a - b) - 5*(a - b)*(2*a - b)*Tanh[c + d*x]^2))/(80*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{(1-x^2)^6}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^9(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{10d} - \text{Su}$$

$$= \frac{b\cosh^9(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{10d} + b\,\text{c}$$

$$= \frac{b(44a^2-28ab+5b^2)\cosh^5(c+dx)\sinh(c+dx)}{160d} + b\cosh^9(c+$$

$$= \frac{(4a-b)\left(8a^2-2ab+b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{128d} + b(44a^2$$

$$= \frac{3(4a-b)\left(8a^2-2ab+b^2\right)\cosh(c+dx)\sinh(c+dx)}{256d} + (4a-b$$

$$= \frac{3}{256}(4a-b)\left(8a^2-2ab+b^2\right)x + \frac{3(4a-b)\left(8a^2-2ab+b^2\right)\text{co}}{256d}$$

Mathematica [A]

time = 0.39, size = 144, normalized size = 0.61

$$\frac{120(4a-b)\left(8a^2-2ab+b^2\right)(c+dx)+20\left(128a^3-24a^2b+b^3\right)\sinh(2(c+dx))+40\left(8a^3+12a^2b-6ab^2+b^3\right)\sinh(4(c+dx))-10b\left(-16a^2+b^2\right)\sinh(6(c+dx))+5(6a-b)b^2\sinh(8(c+dx))+2b^3\sinh(10(c+dx))}{10240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (120*(4*a - b)*(8*a^2 - 2*a*b + b^2)*(c + d*x) + 20*(128*a^3 - 24*a^2*b + b
^3)*Sinh[2*(c + d*x)] + 40*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*Sinh[4*(c + d
*x)] - 10*b*(-16*a^2 + b^2)*Sinh[6*(c + d*x)] + 5*(6*a - b)*b^2*Sinh[8*(c +
 d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(10240*d)
```

Maple [A]

time = 2.36, size = 165, normalized size = 0.69

method	result
default	$\frac{\left(-\frac{3}{512}b^3+\frac{3}{32}a^2b\right)\sinh(6dx+6c)}{6d}+\frac{\left(-\frac{1}{256}b^3+\frac{3}{128}a\,b^2\right)\sinh(8dx+8c)}{8d}+\frac{\left(\frac{1}{256}b^3-\frac{3}{32}a^2b+\frac{1}{2}a^3\right)\sinh(2dx+2c)}{2d}+\left(\frac{1}{64}b^3-\frac{3}{32}a\,b^2+\frac{3}{16}\right.$
risch	$-\frac{3a^2bx}{16}-\frac{3b\,\mathrm{e}^{2dx+2c}a^2}{128d}+\frac{\mathrm{e}^{4dx+4c}a^3}{64d}+\frac{\mathrm{e}^{2dx+2c}a^3}{8d}-\frac{\mathrm{e}^{-2dx-2c}a^3}{8d}-\frac{b^3\mathrm{e}^{-10dx-10c}}{10240d}-\frac{b^3\mathrm{e}^{-2dx-2c}}{1024d}-\frac{b^3\mathrm{e}^{-4dx-4c}}{512d}-\frac{3b^3x}{256}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-3/512*b^3+3/32*a^2*b)*sinh(6*d*x+6*c)/d+1/8*(-1/256*b^3+3/128*a*b^2)*
sinh(8*d*x+8*c)/d+1/2*(1/256*b^3-3/32*a^2*b+1/2*a^3)*sinh(2*d*x+2*c)/d+1/4*
(1/64*b^3-3/32*a*b^2+3/16*a^2*b+1/8*a^3)*sinh(4*d*x+4*c)/d+3/8*a^3*x-3/256*
b^3*x+9/128*a*b^2*x-3/16*a^2*b*x+1/5120*b^3*sinh(10*d*x+10*c)/d
```

Maxima [**A**]
time = 0.27, size = 363, normalized size = 1.53

$$\frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{20480}b^3\left(\frac{\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}-40e^{(-6dx-6c)}-20e^{(-8dx-8c)}-2\right)e^{(10dx+10c)}}{d}+\frac{240(dx+c)}{d}+\frac{20e^{(-2dx-2c)}+40e^{(-4dx-4c)}-10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+2e^{(-10dx-10c)}}{d}\right)-\frac{3}{2048}ab^2\left(\frac{\left(8e^{(-4dx-4c)}-1\right)e^{(8dx+8c)}}{d}-\frac{48(dx+c)}{d}-\frac{8e^{(-4dx-4c)}-e^{(-8dx-8c)}}{d}\right)+\frac{1}{128}a^2b\left(\frac{\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+1\right)e^{(6dx+6c)}}{d}-\frac{24(dx+c)}{d}+\frac{3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-e^{(-6dx-6c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((5*e^(-2*d*x - 2*c) + 10*e^(-4*d*
x - 4*c) - 40*e^(-6*d*x - 6*c) - 20*e^(-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)
/d + 240*(d*x + c)/d + (20*e^(-2*d*x - 2*c) + 40*e^(-4*d*x - 4*c) - 10*e^(-
6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 3/2048*a*b^2
*((8*e^(-4*d*x - 4*c) - 1)*e^(8*d*x + 8*c)/d - 48*(d*x + c)/d - (8*e^(-4*d*
x - 4*c) - e^(-8*d*x - 8*c))/d) + 1/128*a^2*b*((3*e^(-2*d*x - 2*c) - 3*e^(-
4*d*x - 4*c) + 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d + (3*e^(-2*d*x - 2*c)
- 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d)
```

Fricas [**A**]
time = 0.41, size = 376, normalized size = 1.58

$$\frac{5b^3\cosh(dx+c)\sinh(dx+c)^9+10\left(6b^3\cosh(dx+c)^3+(6ab^2-b^3)\cosh(dx+c)\right)\sinh(dx+c)^7+\left(126b^3\cosh(dx+c)^5+70(6ab^2-b^3)\cosh(dx+c)^3+15(16a^2b-b^3)\cosh(dx+c)\right)\sinh(dx+c)^5+10\left(6b^3\cosh(dx+c)^7+7(6ab^2-b^3)\cosh(dx+c)^5+5(16a^2b-b^3)\cosh(dx+c)^3+4(8a^3+12a^2b-6ab^2+b^3)\cosh(dx+c)\right)\sinh(dx+c)^3+30(32a^3-16a^2b+6ab^2-b^3)dx+5\left(b^3\cosh(dx+c)^9+2(6ab^2-b^3)\cosh(dx+c)^7+3(16a^2b-b^3)\cosh(dx+c)^5+8(8a^3+12a^2b-6ab^2+b^3)\cosh(dx+c)^3+2(128a^3-24a^2b+b^3)\cosh(dx+c)\right)\sinh(dx+c)}{2560d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2560*(5*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 10*(6*b^3*cosh(d*x + c)^3 + (
6*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (126*b^3*cosh(d*x + c)^5 +
70*(6*a*b^2 - b^3)*cosh(d*x + c)^3 + 15*(16*a^2*b - b^3)*cosh(d*x + c))*sin
h(d*x + c)^5 + 10*(6*b^3*cosh(d*x + c)^7 + 7*(6*a*b^2 - b^3)*cosh(d*x + c)^
5 + 5*(16*a^2*b - b^3)*cosh(d*x + c)^3 + 4*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^
3)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(32*a^3 - 16*a^2*b + 6*a*b^2 - b^3)*
d*x + 5*(b^3*cosh(d*x + c)^9 + 2*(6*a*b^2 - b^3)*cosh(d*x + c)^7 + 3*(16*a^
2*b - b^3)*cosh(d*x + c)^5 + 8*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*cosh(d*x
+ c)^3 + 2*(128*a^3 - 24*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 774 vs. 2(219) = 438.
time = 1.99, size = 774, normalized size = 3.25

$$\begin{cases} \text{[illegible]} & \text{for } d \neq 0 \\ x\left(a+b\sinh^2(c)\right)^3\cosh^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((3*a**3*x*sinh(c + d*x)**4/8 - 3*a**3*x*sinh(c + d*x)**2*cosh(c +
 d*x)**2/4 + 3*a**3*x*cosh(c + d*x)**4/8 - 3*a**3*sinh(c + d*x)**3*cosh(c +
 d*x)/(8*d) + 5*a**3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 3*a**2*b*x*sinh
(c + d*x)**6/16 - 9*a**2*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 9*a**2*
b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 3*a**2*b*x*cosh(c + d*x)**6/16 -
 3*a**2*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) + a**2*b*sinh(c + d*x)**3*c
osh(c + d*x)**3/(2*d) + 3*a**2*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + 9*
a*b**2*x*sinh(c + d*x)**8/128 - 9*a*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**
2/32 + 27*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 9*a*b**2*x*sinh(c
 + d*x)**2*cosh(c + d*x)**6/32 + 9*a*b**2*x*cosh(c + d*x)**8/128 - 9*a*b**2
*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) + 33*a*b**2*sinh(c + d*x)**5*cosh(c
 + d*x)**3/(128*d) + 33*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(128*d) -
9*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + 3*b**3*x*sinh(c + d*x)**1
0/256 - 15*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 15*b**3*x*sinh(c
+ d*x)**6*cosh(c + d*x)**4/128 - 15*b**3*x*sinh(c + d*x)**4*cosh(c + d*x)**
6/128 + 15*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256 - 3*b**3*x*cosh(c +
 d*x)**10/256 - 3*b**3*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) + 7*b**3*sinh
(c + d*x)**7*cosh(c + d*x)**3/(128*d) + b**3*sinh(c + d*x)**5*cosh(c + d*x)
**5/(10*d) - 7*b**3*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d) + 3*b**3*sinh
(c + d*x)*cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*cos
h(c)**4, True))
```

Giac [A]
time = 0.44, size = 293, normalized size = 1.23

$$\frac{b^{3}e^{(10\,dx+10\,c)}}{10240\,d}-\frac{b^{3}e^{(-10\,dx-10\,c)}}{10240\,d}+\frac{3}{256}\,(32\,a^{3}-16\,a^{2}b+6\,ab^{2}-b^{3})x+\frac{(6\,ab^{2}-b^{3})e^{(8\,dx+8\,c)}}{4096\,d}+\frac{(16\,a^{2}b-b^{3})e^{(6\,dx+6\,c)}}{2048\,d}+\frac{(8\,a^{3}+12\,a^{2}b-6\,ab^{2}+b^{3})e^{(4\,dx+4\,c)}}{512\,d}+\frac{(128\,a^{3}-24\,a^{2}b+b^{3})e^{(2\,dx+2\,c)}}{1024\,d}-\frac{(128\,a^{3}-24\,a^{2}b+b^{3})e^{(-2\,dx-2\,c)}}{1024\,d}-\frac{(8\,a^{3}+12\,a^{2}b-6\,ab^{2}+b^{3})e^{(-4\,dx-4\,c)}}{512\,d}-\frac{(16\,a^{2}b-b^{3})e^{(-6\,dx-6\,c)}}{2048\,d}-\frac{(6\,ab^{2}-b^{3})e^{(-8\,dx-8\,c)}}{4096\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10240*b^3*e^(10*d*x + 10*c)/d - 1/10240*b^3*e^(-10*d*x - 10*c)/d + 3/256*
(32*a^3 - 16*a^2*b + 6*a*b^2 - b^3)*x + 1/4096*(6*a*b^2 - b^3)*e^(8*d*x + 8
*c)/d + 1/2048*(16*a^2*b - b^3)*e^(6*d*x + 6*c)/d + 1/512*(8*a^3 + 12*a^2*b
 - 6*a*b^2 + b^3)*e^(4*d*x + 4*c)/d + 1/1024*(128*a^3 - 24*a^2*b + b^3)*e^(
2*d*x + 2*c)/d - 1/1024*(128*a^3 - 24*a^2*b + b^3)*e^(-2*d*x - 2*c)/d - 1/5
12*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*e^(-4*d*x - 4*c)/d - 1/2048*(16*a^2*b
 - b^3)*e^(-6*d*x - 6*c)/d - 1/4096*(6*a*b^2 - b^3)*e^(-8*d*x - 8*c)/d
```

Mupad [B]
time = 0.58, size = 209, normalized size = 0.88

$$\frac{320\,a^{3}\sinh(2\,c+2\,d\,x)+40\,a^{3}\sinh(4\,c+4\,d\,x)+\frac{5\,b^{3}\sinh(2\,c+2\,d\,x)}{2}+5\,b^{3}\sinh(4\,c+4\,d\,x)-\frac{5\,b^{3}\sinh(6\,c+6\,d\,x)}{4}-\frac{5\,b^{3}\sinh(8\,c+8\,d\,x)}{8}+\frac{b^{3}\sinh(10\,c+10\,d\,x)}{4}-60\,a^{2}\,b\sinh(2\,c+2\,d\,x)-30\,a\,b^{2}\sinh(4\,c+4\,d\,x)+60\,a^{2}\,b\sinh(4\,c+4\,d\,x)+20\,a^{2}\,b\sinh(6\,c+6\,d\,x)+\frac{15\,a\,b^{2}\sinh(8\,c+8\,d\,x)}{4}+480\,a^{3}\,d\,x-15\,b^{3}\,d\,x+90\,a\,b^{2}\,d\,x-240\,a^{2}\,b\,d\,x}{1280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (320*a^3*sinh(2*c + 2*d*x) + 40*a^3*sinh(4*c + 4*d*x) + (5*b^3*sinh(2*c + 2
*d*x))/2 + 5*b^3*sinh(4*c + 4*d*x) - (5*b^3*sinh(6*c + 6*d*x))/4 - (5*b^3*s
inh(8*c + 8*d*x))/8 + (b^3*sinh(10*c + 10*d*x))/4 - 60*a^2*b*sinh(2*c + 2*d
*x) - 30*a*b^2*sinh(4*c + 4*d*x) + 60*a^2*b*sinh(4*c + 4*d*x) + 20*a^2*b*si
nh(6*c + 6*d*x) + (15*a*b^2*sinh(8*c + 8*d*x))/4 + 480*a^3*d*x - 15*b^3*d*x
 + 90*a*b^2*d*x - 240*a^2*b*d*x)/(1280*d)
```

3.305 $\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=98

$$\frac{a^3\sinh(c+dx)}{d}+\frac{a^2(a+3b)\sinh^3(c+dx)}{3d}+\frac{3ab(a+b)\sinh^5(c+dx)}{5d}+\frac{b^2(3a+b)\sinh^7(c+dx)}{7d}+\frac{b^3\sinh^9(c+}{9d}$$

```
[Out] a^3*sinh(d*x+c)/d+1/3*a^2*(a+3*b)*sinh(d*x+c)^3/d+3/5*a*b*(a+b)*sinh(d*x+c)
^5/d+1/7*b^2*(3*a+b)*sinh(d*x+c)^7/d+1/9*b^3*sinh(d*x+c)^9/d
```

Rubi [A]

time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3269, 380}

$$\frac{a^3\sinh(c+dx)}{d}+\frac{a^2(a+3b)\sinh^3(c+dx)}{3d}+\frac{b^2(3a+b)\sinh^7(c+dx)}{7d}+\frac{3ab(a+b)\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Sinh[c + d*x])/d + (a^2*(a + 3*b)*Sinh[c + d*x]^3)/(3*d) + (3*a*b*(a +
 b)*Sinh[c + d*x]^5)/(5*d) + (b^2*(3*a + b)*Sinh[c + d*x]^7)/(7*d) + (b^3*S
inh[c + d*x]^9)/(9*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx&=\frac{\operatorname{Subst}\left(\int\left(1+x^2\right)\left(a+bx^2\right)^3\,dx,x,\sinh(c+dx)\right)}{d}\\&=\frac{\operatorname{Subst}\left(\int\left(a^3+a^2(a+3b)x^2+3ab(a+b)x^4+b^2(3a+b)x^6+b^{\right.}\right.}{d}\\&=\frac{a^3\sinh(c+dx)}{d}+\frac{a^2(a+3b)\sinh^3(c+dx)}{3d}+\frac{3ab(a+b)\sinh^5(}{5d}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 125, normalized size = 1.28

$$\frac{1890(32a^3-16a^2b+6ab^2-b^3)\sinh(c+dx)+420(16a^3+12a^2b-9ab^2+2b^3)\sinh(3(c+dx))+b(756a(4a-b)\sinh(5(c+dx))+5b(27(4a-b)\sinh(7(c+dx))+7b\sinh(9(c+dx))))}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (1890*(32*a^3 - 16*a^2*b + 6*a*b^2 - b^3)*Sinh[c + d*x] + 420*(16*a^3 + 12*
a^2*b - 9*a*b^2 + 2*b^3)*Sinh[3*(c + d*x)] + b*(756*a*(4*a - b)*Sinh[5*(c +
 d*x)] + 5*b*(27*(4*a - b)*Sinh[7*(c + d*x)] + 7*b*Sinh[9*(c + d*x)])))/(80
640*d)
```

Maple [A]
time = 2.20, size = 142, normalized size = 1.45

method	result
default	$\frac{\left(-\frac{3}{256}b^3+\frac{3}{64}a\,b^2\right)\sinh(7dx+7c)}{7d}+\frac{\left(-\frac{3}{64}a\,b^2+\frac{3}{16}a^2b\right)\sinh(5dx+5c)}{5d}+\frac{\left(-\frac{3}{128}b^3+\frac{9}{64}a\,b^2-\frac{3}{8}a^2b+\frac{3}{4}a^3\right)\sinh(dx+c)}{d}+$ $\left(\frac{1}{32}b^3-\frac{9}{64}\right.$
risch	$-\frac{3a\,b^2\mathrm{e}^{5dx+5c}}{640d}-\frac{3a\,b^2\mathrm{e}^{3dx+3c}}{128d}-\frac{3b\,\mathrm{e}^{dx+c}a^2}{16d}+\frac{b^3\mathrm{e}^{9dx+9c}}{4608d}+\frac{\mathrm{e}^{3dx+3c}a^3}{24d}+\frac{\mathrm{e}^{3dx+3c}b^3}{192d}+\frac{3a^3\mathrm{e}^{dx+c}}{8d}-\frac{b^3\mathrm{e}^{-9dx-9c}}{4608d}-$ $9a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(-3/256*b^3+3/64*a*b^2)*sinh(7*d*x+7*c)/d+1/5*(-3/64*a*b^2+3/16*a^2*b)*
sinh(5*d*x+5*c)/d+(-3/128*b^3+9/64*a*b^2-3/8*a^2*b+3/4*a^3)*sinh(d*x+c)/d+1
/3*(1/32*b^3-9/64*a*b^2+3/16*a^2*b+1/4*a^3)*sinh(3*d*x+3*c)/d+1/2304*b^3*si
nh(9*d*x+9*c)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(90) = 180.
time = 0.27, size = 349, normalized size = 3.56

$$-\frac{1}{32256}b^3\left(\frac{\left(27e^{(-2dx-2c)}-168e^{(-6dx-6c)}+378e^{(-8dx-8c)}-7\right)e^{(9dx+9c)}}{d}-\frac{378e^{(-dx-c)}-168e^{(-3dx-3c)}+27e^{(-7dx-7c)}-7e^{(-9dx-9c)}}{d}\right)-\frac{3}{4480}ab^2\left(\frac{\left(7e^{(-2dx-2c)}+35e^{(-4dx-4c)}-105e^{(-6dx-6c)}-5\right)e^{(7dx+7c)}}{d}+\frac{105e^{(-dx-c)}-35e^{(-3dx-3c)}-7e^{(-5dx-5c)}+5e^{(-7dx-7c)}}{d}\right)+\frac{1}{160}a^2b\left(\frac{\left(5e^{(-2dx-2c)}-30e^{(-4dx-4c)}+3\right)e^{(5dx+5c)}}{d}+\frac{30e^{(-dx-c)}-5e^{(-3dx-3c)}-3e^{(-5dx-5c)}}{d}\right)+\frac{1}{24}a^3\left(\frac{e^{(3dx+3c)}}{d}+\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}-\frac{e^{(-3dx-3c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/32256*b^3*((27*e^(-2*d*x - 2*c) - 168*e^(-6*d*x - 6*c) + 378*e^(-8*d*x -
 8*c) - 7)*e^(9*d*x + 9*c)/d - (378*e^(-d*x - c) - 168*e^(-3*d*x - 3*c) + 2
7*e^(-7*d*x - 7*c) - 7*e^(-9*d*x - 9*c))/d) - 3/4480*a*b^2*((7*e^(-2*d*x -
2*c) + 35*e^(-4*d*x - 4*c) - 105*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d +
(105*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) - 7*e^(-5*d*x - 5*c) + 5*e^(-7*d*x
 - 7*c))/d) + 1/160*a^2*b*((5*e^(-2*d*x - 2*c) - 30*e^(-4*d*x - 4*c) + 3)*e^
(5*d*x + 5*c)/d + (30*e^(-d*x - c) - 5*e^(-3*d*x - 3*c) - 3*e^(-5*d*x - 5*c
```

```
))/d) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d -
e^(-3*d*x - 3*c)/d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(90) = 180$.
time = 0.38, size = 324, normalized size = 3.31

$$\frac{35\,b^{3}\sinh\left(dx+c\right)^{9}+45\left(28\,b^{3}\cosh\left(dx+c\right)^{2}+12\,ab^{2}-3\,b^{3}\right)\sinh\left(dx+c\right)^{7}+63\left(70\,b^{3}\cosh\left(dx+c\right)^{4}+48\,a^{2}b-12\,ab^{2}+45\left(4\,ab^{2}-b^{3}\right)\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)^{5}+105\left(28\,b^{3}\cosh\left(dx+c\right)^{6}+45\left(4\,ab^{2}-b^{3}\right)\cosh\left(dx+c\right)^{4}+64\,a^{3}+48\,a^{2}b-36\,ab^{2}+8\,b^{3}+72\left(4\,a^{2}b-ab^{2}\right)\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)^{3}+315\left(b^{3}\cosh\left(dx+c\right)^{8}+3\left(4\,ab^{2}-b^{3}\right)\cosh\left(dx+c\right)^{6}+12\left(4\,a^{2}b-ab^{2}\right)\cosh\left(dx+c\right)^{4}+192\,a^{3}-96\,a^{2}b+36\,ab^{2}-6\,b^{3}+4\left(16\,a^{3}+12\,a^{2}b-9\,ab^{2}+2\,b^{3}\right)\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^3*sinh(d*x + c)^9 + 45*(28*b^3*cosh(d*x + c)^2 + 12*a*b^2 - 3
*b^3)*sinh(d*x + c)^7 + 63*(70*b^3*cosh(d*x + c)^4 + 48*a^2*b - 12*a*b^2 +
45*(4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 105*(28*b^3*cosh(d*x
+ c)^6 + 45*(4*a*b^2 - b^3)*cosh(d*x + c)^4 + 64*a^3 + 48*a^2*b - 36*a*b^2
+ 8*b^3 + 72*(4*a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 315*(b^3*
cosh(d*x + c)^8 + 3*(4*a*b^2 - b^3)*cosh(d*x + c)^6 + 12*(4*a^2*b - a*b^2)*
cosh(d*x + c)^4 + 192*a^3 - 96*a^2*b + 36*a*b^2 - 6*b^3 + 4*(16*a^3 + 12*a^
2*b - 9*a*b^2 + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(87) = 174$.
time = 1.38, size = 182, normalized size = 1.86

$$\begin{cases} -\frac{2a^{3}\sinh^{3}\left(c+dx\right)}{3d}+\frac{a^{3}\sinh\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{d}-\frac{2a^{2}b\sinh^{5}\left(c+dx\right)}{5d}+\frac{a^{2}b\sinh^{3}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{d}-\frac{6ab^{2}\sinh^{7}\left(c+dx\right)}{35d}+\frac{3ab^{2}\sinh^{5}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{5d}-\frac{2b^{3}\sinh^{9}\left(c+dx\right)}{63d}+\frac{b^{3}\sinh^{7}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{7d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^{2}\left(c\right)\right)^{3}\cosh^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((-2*a**3*sinh(c + d*x)**3/(3*d) + a**3*sinh(c + d*x)*cosh(c + d*x
)**2/d - 2*a**2*b*sinh(c + d*x)**5/(5*d) + a**2*b*sinh(c + d*x)**3*cosh(c +
 d*x)**2/d - 6*a*b**2*sinh(c + d*x)**7/(35*d) + 3*a*b**2*sinh(c + d*x)**5*c
osh(c + d*x)**2/(5*d) - 2*b**3*sinh(c + d*x)**9/(63*d) + b**3*sinh(c + d*x)
**7*cosh(c + d*x)**2/(7*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*cosh(c)**3,
 True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(90) = 180$.
time = 0.46, size = 286, normalized size = 2.92

$$\frac{b^{3}e^{\left(9\,dx+9\,c\right)}}{4608\,d}-\frac{b^{3}e^{\left(-9\,dx-9\,c\right)}}{4608\,d}+\frac{3\left(4\,ab^{2}-b^{3}\right)e^{\left(7\,dx+7\,c\right)}}{3584\,d}+\frac{3\left(4\,a^{2}b-ab^{2}\right)e^{\left(5\,dx+5\,c\right)}}{640\,d}+\frac{\left(16\,a^{3}+12\,a^{2}b-9\,ab^{2}+2\,b^{3}\right)e^{\left(3\,dx+3\,c\right)}}{384\,d}+\frac{3\left(32\,a^{3}-16\,a^{2}b+6\,ab^{2}-b^{3}\right)e^{\left(dx+c\right)}}{256\,d}-\frac{3\left(32\,a^{3}-16\,a^{2}b+6\,ab^{2}-b^{3}\right)e^{\left(-dx-c\right)}}{256\,d}-\frac{\left(16\,a^{3}+12\,a^{2}b-9\,ab^{2}+2\,b^{3}\right)e^{\left(-3\,dx-3\,c\right)}}{384\,d}-\frac{3\left(4\,a^{2}b-ab^{2}\right)e^{\left(-5\,dx-5\,c\right)}}{640\,d}-\frac{3\left(4\,ab^{2}-b^{3}\right)e^{\left(-7\,dx-7\,c\right)}}{3584\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4608*b^3*e^(9*d*x + 9*c)/d - 1/4608*b^3*e^(-9*d*x - 9*c)/d + 3/3584*(4*a*
b^2 - b^3)*e^(7*d*x + 7*c)/d + 3/640*(4*a^2*b - a*b^2)*e^(5*d*x + 5*c)/d +
1/384*(16*a^3 + 12*a^2*b - 9*a*b^2 + 2*b^3)*e^(3*d*x + 3*c)/d + 3/256*(32*a
^3 - 16*a^2*b + 6*a*b^2 - b^3)*e^(d*x + c)/d - 3/256*(32*a^3 - 16*a^2*b + 6
*a*b^2 - b^3)*e^(-d*x - c)/d - 1/384*(16*a^3 + 12*a^2*b - 9*a*b^2 + 2*b^3)*
e^(-3*d*x - 3*c)/d - 3/640*(4*a^2*b - a*b^2)*e^(-5*d*x - 5*c)/d - 3/3584*(4
*a*b^2 - b^3)*e^(-7*d*x - 7*c)/d
```

Mupad [B]

time = 0.29, size = 112, normalized size = 1.14

$$\frac{105\,a^3\sinh(c+d\,x)^3+315\,a^3\sinh(c+d\,x)+189\,a^2\,b\sinh(c+d\,x)^5+315\,a^2\,b\sinh(c+d\,x)^3+135\,a\,b^2\sinh(c+d\,x)^7+189\,a\,b^2\sinh(c+d\,x)^5+35\,b^3\sinh(c+d\,x)^9+45\,b^3\sinh(c+d\,x)^7}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (315*a^3*sinh(c + d*x) + 105*a^3*sinh(c + d*x)^3 + 45*b^3*sinh(c + d*x)^7 +
 35*b^3*sinh(c + d*x)^9 + 315*a^2*b*sinh(c + d*x)^3 + 189*a*b^2*sinh(c + d*
x)^5 + 189*a^2*b*sinh(c + d*x)^5 + 135*a*b^2*sinh(c + d*x)^7)/(315*d)
```

3.306 $\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=203

$$\frac{1}{128}\left(64a^3-48a^2b+24ab^2-5b^3\right)x+\frac{\left(64a^3-48a^2b+24ab^2-5b^3\right)\cosh(c+dx)\sinh(c+dx)}{128d}+\frac{b(88a^2-68a}{}$$

```
[Out] 1/128*(64*a^3-48*a^2*b+24*a*b^2-5*b^3)*x+1/128*(64*a^3-48*a^2*b+24*a*b^2-5*
b^3)*cosh(d*x+c)*sinh(d*x+c)/d+1/192*b*(88*a^2-68*a*b+15*b^2)*cosh(d*x+c)^3
*sinh(d*x+c)/d+1/8*b*cosh(d*x+c)^7*sinh(d*x+c)*(a-(a-b)*tanh(d*x+c)^2)^2/d+
1/48*b*cosh(d*x+c)^5*sinh(d*x+c)*(a*(8*a-b)-(8*a-5*b)*(a-b)*tanh(d*x+c)^2)/
d
```

Rubi [A]
time = 0.19, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3270, 424, 540, 393, 205, 212}

$$\frac{b\left(88a^2-68ab+15b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}+\frac{\left(64a^3-48a^2b+24ab^2-5b^3\right)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x\left(64a^3-48a^2b+24ab^2-5b^3\right)+\frac{b\sinh(c+dx)\cosh^7(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{8d}+\frac{b\sinh(c+dx)\cosh^5(c+dx)\left(a(8a-b)-(8a-5b)(a-b)\tanh^2(c+dx)\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*x)/128 + ((64*a^3 - 48*a^2*b + 24*a
*b^2 - 5*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (b*(88*a^2 - 68*a*b +
15*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) + (b*Cosh[c + d*x]^7*Sinh[c
+ d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(8*d) + (b*Cosh[c + d*x]^5*Sinh[c +
 d*x]*(a*(8*a - b) - (8*a - 5*b)*(a - b)*Tanh[c + d*x]^2))/(48*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^5}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^7(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{8d} - \frac{\text{Su}}{}$$

$$= \frac{b\cosh^7(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{8d} + \frac{b\,c}{}$$

$$= \frac{b(88a^2-68ab+15b^2)\cosh^3(c+dx)\sinh(c+dx)}{192d} + \frac{b\cosh^7(c+}{}$$

$$= \frac{(64a^3-48a^2b+24ab^2-5b^3)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b(88}{}$$

$$= \frac{1}{128}\left(64a^3-48a^2b+24ab^2-5b^3\right)x + \frac{(64a^3-48a^2b+24ab^2-}{}$$

Mathematica [A]
time = 0.24, size = 120, normalized size = 0.59

$$\frac{24(64a^3-48a^2b+24ab^2-5b^3)(c+dx)+48(16a^3-3ab^2+b^3)\sinh(2(c+dx))+24b(12a^2-6ab+b^2)\sinh(4(c+dx))+16(3a-b)b^2\sinh(6(c+dx))+3b^3\sinh(8(c+dx))}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (24*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*(c + d*x) + 48*(16*a^3 - 3*a*b^2
 + b^3)*Sinh[2*(c + d*x)] + 24*b*(12*a^2 - 6*a*b + b^2)*Sinh[4*(c + d*x)] +
 16*(3*a - b)*b^2*Sinh[6*(c + d*x)] + 3*b^3*Sinh[8*(c + d*x)])/(3072*d)
```

Maple [A]
time = 1.46, size = 216, normalized size = 1.06

method	result
derivativedivides	$\frac{b^3\left(\frac{\left(\sinh^5(dx+c)\right)\left(\cosh^3(dx+c)\right)}{8}-\frac{5\left(\sinh^3(dx+c)\right)\left(\cosh^3(dx+c)\right)}{48}+\frac{5\sinh(dx+c)\left(\cosh^3(dx+c)\right)}{64}-\frac{5\cosh(dx+c)\sinh(dx+c)}{128}-\frac{5dx}{128}-\right.}{}$
default	$\frac{b^3\left(\frac{\left(\sinh^5(dx+c)\right)\left(\cosh^3(dx+c)\right)}{8}-\frac{5\left(\sinh^3(dx+c)\right)\left(\cosh^3(dx+c)\right)}{48}+\frac{5\sinh(dx+c)\left(\cosh^3(dx+c)\right)}{64}-\frac{5\cosh(dx+c)\sinh(dx+c)}{128}-\frac{5dx}{128}-\right.}{}$
risch	$\frac{a^3x}{2}-\frac{3a^2bx}{8}+\frac{3a\,b^2x}{16}-\frac{5b^3x}{128}+\frac{b^3\mathrm{e}^{8dx+8c}}{2048d}+\frac{b^2\mathrm{e}^{6dx+6c}a}{128d}-\frac{b^3\mathrm{e}^{6dx+6c}}{384d}+\frac{3\,\mathrm{e}^{4dx+4c}a^2b}{64d}-\frac{3b^2\mathrm{e}^{4dx+4c}a}{128d}+b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/8*sinh(d*x+c)^5*cosh(d*x+c)^3-5/48*sinh(d*x+c)^3*cosh(d*x+c)^3+
5/64*sinh(d*x+c)*cosh(d*x+c)^3-5/128*cosh(d*x+c)*sinh(d*x+c)-5/128*d*x-5/12
8*c)+3*a*b^2*(1/6*sinh(d*x+c)^3*cosh(d*x+c)^3-1/8*sinh(d*x+c)*cosh(d*x+c)^3
+1/16*cosh(d*x+c)*sinh(d*x+c)+1/16*d*x+1/16*c)+3*a^2*b*(1/4*sinh(d*x+c)*cos
h(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c)+a^3*(1/2*cosh(d*x+c)*
sinh(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [**A**]
time = 0.27, size = 287, normalized size = 1.41

$$\frac{1}{8}\,a^{3}\left(4\,x+\frac{e^{\left(2\,dx+2\,c\right)}}{d}-\frac{e^{\left(-2\,dx-2\,c\right)}}{d}\right)-\frac{1}{6144}\,b^{3}\left(\frac{\left(16\,e^{\left(-2\,dx-2\,c\right)}-24\,e^{\left(-4\,dx-4\,c\right)}-48\,e^{\left(-6\,dx-6\,c\right)}-3\right)e^{\left(8\,dx+8\,c\right)}}{d}+\frac{240\,\left(dx+c\right)}{d}+\frac{48\,e^{\left(-2\,dx-2\,c\right)}+24\,e^{\left(-4\,dx-4\,c\right)}-16\,e^{\left(-6\,dx-6\,c\right)}+3\,e^{\left(-8\,dx-8\,c\right)}}{d}\right)-\frac{1}{128}\,ab^{2}\left(\frac{\left(3\,e^{\left(-2\,dx-2\,c\right)}+3\,e^{\left(-4\,dx-4\,c\right)}-1\right)e^{\left(6\,dx+6\,c\right)}}{d}-\frac{24\,\left(dx+c\right)}{d}-\frac{3\,e^{\left(-2\,dx-2\,c\right)}+3\,e^{\left(-4\,dx-4\,c\right)}-e^{\left(-6\,dx-6\,c\right)}}{d}\right)-\frac{3}{64}\,a^{2}b\left(\frac{8\,\left(dx+c\right)}{d}-\frac{e^{\left(4\,dx+4\,c\right)}}{d}+\frac{e^{\left(-4\,dx-4\,c\right)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/6144*b^3*((16*e^
(-2*d*x - 2*c) - 24*e^(-4*d*x - 4*c) - 48*e^(-6*d*x - 6*c) - 3)*e^(8*d*x +
8*c)/d + 240*(d*x + c)/d + (48*e^(-2*d*x - 2*c) + 24*e^(-4*d*x - 4*c) - 16*
e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c))/d) - 1/128*a*b^2*((3*e^(-2*d*x - 2*c
) + 3*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d - (3*e^(-2*d
*x - 2*c) + 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d) - 3/64*a^2*b*(8*(d*x
+ c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Fricas [**A**]
time = 0.39, size = 257, normalized size = 1.27

$$\frac{3\,b^{3}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{7}+3\,\left(7\,b^{3}\cosh\left(dx+c\right)^{3}+4\,\left(3\,ab^{2}-b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{5}+\left(21\,b^{3}\cosh\left(dx+c\right)^{5}+40\,\left(3\,ab^{2}-b^{3}\right)\cosh\left(dx+c\right)^{3}+12\,\left(12\,a^{2}b-6\,ab^{2}+b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+3\,\left(64\,a^{3}-48\,a^{2}b+24\,ab^{2}-5\,b^{3}\right)dx+3\,\left(b^{3}\cosh\left(dx+c\right)^{7}+4\,\left(3\,ab^{2}-b^{3}\right)\cosh\left(dx+c\right)^{5}+4\,\left(12\,a^{2}b-6\,ab^{2}+b^{3}\right)\cosh\left(dx+c\right)^{3}+4\,\left(16\,a^{3}-3\,ab^{2}+b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^3*cosh(d*x + c)^3 + 4*(
3*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^3*cosh(d*x + c)^5 + 4
0*(3*a*b^2 - b^3)*cosh(d*x + c)^3 + 12*(12*a^2*b - 6*a*b^2 + b^3)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 3*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*d*x + 3*(b
^3*cosh(d*x + c)^7 + 4*(3*a*b^2 - b^3)*cosh(d*x + c)^5 + 4*(12*a^2*b - 6*a*
b^2 + b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 3*a*b^2 + b^3)*cosh(d*x + c))*sinh
(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(190) = 380$.
time = 1.04, size = 559, normalized size = 2.75

$$\begin{cases} -\frac{a^{3}x\sinh^{2}\left(c+dx\right)}{2}+\frac{a^{3}x\cosh^{2}\left(c+dx\right)}{2}+\frac{a^{3}\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d}-\frac{3a^{2}bx\sinh^{4}\left(c+dx\right)}{8}+\frac{3a^{2}bx\sinh^{2}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{4}-\frac{3a^{2}bx\cosh^{4}\left(c+dx\right)}{8}+\frac{3a^{2}b\sinh^{3}\left(c+dx\right)\cosh\left(c+dx\right)}{8d}+\frac{3a^{2}b\sinh\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{8d}-\frac{3ab^{2}x\sinh^{6}\left(c+dx\right)}{16}+\frac{9ab^{2}x\sinh^{4}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{16}-\frac{9ab^{2}x\sinh^{2}\left(c+dx\right)\cosh^{4}\left(c+dx\right)}{16}+\frac{3ab^{2}x\cosh^{6}\left(c+dx\right)}{16}+\frac{3ab^{2}\sinh^{5}\left(c+dx\right)\cosh\left(c+dx\right)}{16d}+\frac{ab^{2}\sinh^{3}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{2d}-\frac{3ab^{2}\sinh\left(c+dx\right)\cosh^{5}\left(c+dx\right)}{16d}-\frac{5b^{3}x\sinh^{8}\left(c+dx\right)}{128}+\frac{5b^{3}x\sinh^{6}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{32}-\frac{15b^{3}x\sinh^{4}\left(c+dx\right)\cosh^{4}\left(c+dx\right)}{64}+\frac{5b^{3}x\sinh^{2}\left(c+dx\right)\cosh^{6}\left(c+dx\right)}{32}-\frac{5b^{3}x\cosh^{8}\left(c+dx\right)}{128}+\frac{5b^{3}\sinh^{7}\left(c+dx\right)\cosh\left(c+dx\right)}{128d}+\frac{73b^{3}\sinh^{5}\left(c+dx\right)\cosh^{3}\left(c+dx\right)}{384d}-\frac{55b^{3}\sinh^{3}\left(c+dx\right)\cosh^{5}\left(c+dx\right)}{384d}+\frac{5b^{3}\sinh\left(c+dx\right)\cosh^{7}\left(c+dx\right)}{128d} & \text{for}\: d \neq 0 \\ x\left(a+b\sinh^{2}\left(c\right)\right)^{3}\cosh^{2}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((-a**3*x*sinh(c + d*x)**2/2 + a**3*x*cosh(c + d*x)**2/2 + a**3*si
nh(c + d*x)*cosh(c + d*x)/(2*d) - 3*a**2*b*x*sinh(c + d*x)**4/8 + 3*a**2*b*
x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - 3*a**2*b*x*cosh(c + d*x)**4/8 + 3*a
**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + 3*a**2*b*sinh(c + d*x)*cosh(c
+ d*x)**3/(8*d) - 3*a*b**2*x*sinh(c + d*x)**6/16 + 9*a*b**2*x*sinh(c + d*x)
**4*cosh(c + d*x)**2/16 - 9*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 +
 3*a*b**2*x*cosh(c + d*x)**6/16 + 3*a*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(
16*d) + a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) - 3*a*b**2*sinh(c +
d*x)*cosh(c + d*x)**5/(16*d) - 5*b**3*x*sinh(c + d*x)**8/128 + 5*b**3*x*sin
h(c + d*x)**6*cosh(c + d*x)**2/32 - 15*b**3*x*sinh(c + d*x)**4*cosh(c + d*x
)**4/64 + 5*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 - 5*b**3*x*cosh(c +
 d*x)**8/128 + 5*b**3*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) + 73*b**3*sinh
(c + d*x)**5*cosh(c + d*x)**3/(384*d) - 55*b**3*sinh(c + d*x)**3*cosh(c + d
*x)**5/(384*d) + 5*b**3*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)),
(x*(a + b*sinh(c)**2)**3*cosh(c)**2, True))
```

Giac [**A**]
time = 0.43, size = 231, normalized size = 1.14

$$\frac{b^3e^{(8\,dx+8\,c)}}{2048\,d}-\frac{b^3e^{(-8\,dx-8\,c)}}{2048\,d}+\frac{1}{128}\,(64\,a^3-48\,a^2b+24\,ab^2-5\,b^3)x+\frac{(3\,ab^2-b^3)e^{(6\,dx+6\,c)}}{384\,d}+\frac{(12\,a^2b-6\,ab^2+b^3)e^{(4\,dx+4\,c)}}{256\,d}+\frac{(16\,a^3-3\,ab^2+b^3)e^{(2\,dx+2\,c)}}{128\,d}-\frac{(16\,a^3-3\,ab^2+b^3)e^{(-2\,dx-2\,c)}}{128\,d}-\frac{(12\,a^2b-6\,ab^2+b^3)e^{(-4\,dx-4\,c)}}{256\,d}-\frac{(3\,ab^2-b^3)e^{(-6\,dx-6\,c)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2048*b^3*e^(8*d*x + 8*c)/d - 1/2048*b^3*e^(-8*d*x - 8*c)/d + 1/128*(64*a^
3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*x + 1/384*(3*a*b^2 - b^3)*e^(6*d*x + 6*c)/
d + 1/256*(12*a^2*b - 6*a*b^2 + b^3)*e^(4*d*x + 4*c)/d + 1/128*(16*a^3 - 3*
a*b^2 + b^3)*e^(2*d*x + 2*c)/d - 1/128*(16*a^3 - 3*a*b^2 + b^3)*e^(-2*d*x -
 2*c)/d - 1/256*(12*a^2*b - 6*a*b^2 + b^3)*e^(-4*d*x - 4*c)/d - 1/384*(3*a*
b^2 - b^3)*e^(-6*d*x - 6*c)/d
```

Mupad [**B**]
time = 0.42, size = 166, normalized size = 0.82

$$\frac{96\,a^3\,\sinh(2\,c+2\,d\,x)+6\,b^3\,\sinh(2\,c+2\,d\,x)+3\,b^3\,\sinh(4\,c+4\,d\,x)-2\,b^3\,\sinh(6\,c+6\,d\,x)+\frac{3\,b^3\,\sinh(8\,c+8\,d\,x)}{8}-18\,a\,b^2\,\sinh(2\,c+2\,d\,x)-18\,a\,b^2\,\sinh(4\,c+4\,d\,x)+36\,a^2\,b\,\sinh(4\,c+4\,d\,x)+6\,a\,b^2\,\sinh(6\,c+6\,d\,x)+192\,a^3\,d\,x-15\,b^3\,d\,x+72\,a\,b^2\,d\,x-144\,a^2\,b\,d\,x}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (96*a^3*sinh(2*c + 2*d*x) + 6*b^3*sinh(2*c + 2*d*x) + 3*b^3*sinh(4*c + 4*d*
x) - 2*b^3*sinh(6*c + 6*d*x) + (3*b^3*sinh(8*c + 8*d*x))/8 - 18*a*b^2*sinh(
2*c + 2*d*x) - 18*a*b^2*sinh(4*c + 4*d*x) + 36*a^2*b*sinh(4*c + 4*d*x) + 6*
a*b^2*sinh(6*c + 6*d*x) + 192*a^3*d*x - 15*b^3*d*x + 72*a*b^2*d*x - 144*a^2
*b*d*x)/(384*d)
```

3.307 $\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=67

$$\frac{a^3\sinh(c+dx)}{d}+\frac{a^2b\sinh^3(c+dx)}{d}+\frac{3ab^2\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^7(c+dx)}{7d}$$

```
[Out] a^3*sinh(d*x+c)/d+a^2*b*sinh(d*x+c)^3/d+3/5*a*b^2*sinh(d*x+c)^5/d+1/7*b^3*s
inh(d*x+c)^7/d
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3269, 200}

$$\frac{a^3\sinh(c+dx)}{d}+\frac{a^2b\sinh^3(c+dx)}{d}+\frac{3ab^2\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Sinh[c + d*x])/d + (a^2*b*Sinh[c + d*x]^3)/d + (3*a*b^2*Sinh[c + d*x]^
5)/(5*d) + (b^3*Sinh[c + d*x]^7)/(7*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \left(a+bx^2\right)^3\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^3+3a^2bx^2+3ab^2x^4+b^3x^6\right)\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{a^3\sinh(c+dx)}{d}+\frac{a^2b\sinh^3(c+dx)}{d}+\frac{3ab^2\sinh^5(c+dx)}{5d}+\end{aligned}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 1.00

$$\frac{a^3\sinh(c+dx)}{d}+\frac{a^2b\sinh^3(c+dx)}{d}+\frac{3ab^2\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Sinh[c + d*x])/d + (a^2*b*Sinh[c + d*x]^3)/d + (3*a*b^2*Sinh[c + d*x]^
5)/(5*d) + (b^3*Sinh[c + d*x]^7)/(7*d)
```

Maple [A]
time = 0.82, size = 56, normalized size = 0.84

method	result
derivativedivides	$\frac{\frac{b^{3} \left(\sinh^{7}\left(d x +c \right)\right)}{7}+\frac{3 a \,b^{2} \left(\sinh^{5}\left(d x +c \right)\right)}{5}+a^{2} b \left(\sinh^{3}\left(d x +c \right)\right)+a^{3} \sinh \left(d x +c \right)}{d}$
default	$\frac{\frac{b^{3} \left(\sinh^{7}\left(d x +c \right)\right)}{7}+\frac{3 a \,b^{2} \left(\sinh^{5}\left(d x +c \right)\right)}{5}+a^{2} b \left(\sinh^{3}\left(d x +c \right)\right)+a^{3} \sinh \left(d x +c \right)}{d}$
risch	$\frac{b^{3} {\mathrm e}^{7 d x +7 c}}{896 d}+\frac{3 a \,b^{2} {\mathrm e}^{5 d x +5 c}}{160 d}-\frac{{\mathrm e}^{5 d x +5 c} b^{3}}{128 d}+\frac{{\mathrm e}^{3 d x +3 c} a^{2} b}{8 d}-\frac{3 a \,b^{2} {\mathrm e}^{3 d x +3 c}}{32 d}+\frac{3 \,{\mathrm e}^{3 d x +3 c} b^{3}}{128 d}+\frac{a^{3} {\mathrm e}^{d x +c}}{2 d}-\frac{3 b \,{\mathrm e}^{d x +c} a^{2}}{8 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/7*b^3*sinh(d*x+c)^7+3/5*a*b^2*sinh(d*x+c)^5+a^2*b*sinh(d*x+c)^3+a^3*
sinh(d*x+c))
```

Maxima [A]
time = 0.27, size = 63, normalized size = 0.94

$$\frac{b^3\sinh\left(dx+c\right)^7}{7\,d}+\frac{3\,ab^2\sinh\left(dx+c\right)^5}{5\,d}+\frac{a^2b\sinh\left(dx+c\right)^3}{d}+\frac{a^3\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*sinh(d*x + c)^7/d + 3/5*a*b^2*sinh(d*x + c)^5/d + a^2*b*sinh(d*x +
c)^3/d + a^3*sinh(d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(63) = 126.
time = 0.51, size = 209, normalized size = 3.12

$$\frac{5\,b^3\sinh\left(dx+c\right)^7+7\,\left(15\,b^3\cosh\left(dx+c\right)^2+12\,ab^2-5\,b^3\right)\sinh\left(dx+c\right)^5+35\,\left(5\,b^3\cosh\left(dx+c\right)^4+16\,a^2b-12\,ab^2+3\,b^3+2\,\left(12\,ab^2-5\,b^3\right)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)^3+35\,\left(b^3\cosh\left(dx+c\right)^6+\left(12\,ab^2-5\,b^3\right)\cosh\left(dx+c\right)^4+64\,a^3-48\,a^2b+24\,ab^2-5\,b^3+3\,\left(16\,a^2b-12\,ab^2+3\,b^3\right)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b^3*sinh(d*x + c)^7 + 7*(15*b^3*cosh(d*x + c)^2 + 12*a*b^2 - 5*b^
3)*sinh(d*x + c)^5 + 35*(5*b^3*cosh(d*x + c)^4 + 16*a^2*b - 12*a*b^2 + 3*b^
3 + 2*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 35*(b^3*cosh(d*
x + c)^6 + (12*a*b^2 - 5*b^3)*cosh(d*x + c)^4 + 64*a^3 - 48*a^2*b + 24*a*b^
2 - 5*b^3 + 3*(16*a^2*b - 12*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))
/d
```

Sympy [A]
time = 0.63, size = 75, normalized size = 1.12

$$\begin{cases} \frac{a^{3}\sinh{\left(c+dx \right)}}{d} + \frac{a^{2}b\sinh^{3}{\left(c+dx \right)}}{d} + \frac{3ab^{2}\sinh^{5}{\left(c+dx \right)}}{5d} + \frac{b^{3}\sinh^{7}{\left(c+dx \right)}}{7d} & \text{for}\: d \neq 0 \\ x\left(a + b\sinh^{2}{\left(c \right)}\right)^{3}\cosh{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)/d + a**2*b*sinh(c + d*x)**3/d + 3*a*b**2*sinh
(c + d*x)**5/(5*d) + b**3*sinh(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a + b*sinh
(c)**2)**3*cosh(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(63) = 126.
time = 0.43, size = 222, normalized size = 3.31

$$\frac{b^{3}e^{(7\,dx+7\,c)}}{896\,d} - \frac{b^{3}e^{(-7\,dx-7\,c)}}{896\,d} + \frac{(12\,ab^{2} - 5\,b^{3})e^{(5\,dx+5\,c)}}{640\,d} + \frac{(16\,a^{2}b - 12\,ab^{2} + 3\,b^{3})e^{(3\,dx+3\,c)}}{128\,d} + \frac{(64\,a^{3} - 48\,a^{2}b + 24\,ab^{2} - 5\,b^{3})e^{(dx+c)}}{128\,d} - \frac{(64\,a^{3} - 48\,a^{2}b + 24\,ab^{2} - 5\,b^{3})e^{(-dx-c)}}{128\,d} - \frac{(16\,a^{2}b - 12\,ab^{2} + 3\,b^{3})e^{(-3\,dx-3\,c)}}{128\,d} - \frac{(12\,ab^{2} - 5\,b^{3})e^{(-5\,dx-5\,c)}}{640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/896*b^3*e^(7*d*x + 7*c)/d - 1/896*b^3*e^(-7*d*x - 7*c)/d + 1/640*(12*a*b^
2 - 5*b^3)*e^(5*d*x + 5*c)/d + 1/128*(16*a^2*b - 12*a*b^2 + 3*b^3)*e^(3*d*x
 + 3*c)/d + 1/128*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*e^(d*x + c)/d - 1/
128*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*e^(-d*x - c)/d - 1/128*(16*a^2*b
 - 12*a*b^2 + 3*b^3)*e^(-3*d*x - 3*c)/d - 1/640*(12*a*b^2 - 5*b^3)*e^(-5*d*
x - 5*c)/d
```

Mupad [B]
time = 0.15, size = 58, normalized size = 0.87

$$\frac{\mathrm{sinh}\left(c+d\,x\right)\,\left(35\,a^3+35\,a^2\,b\,{\mathrm{sinh}\left(c+d\,x\right)}^2+21\,a\,b^2\,{\mathrm{sinh}\left(c+d\,x\right)}^4+5\,b^3\,{\mathrm{sinh}\left(c+d\,x\right)}^6\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (sinh(c + d*x)*(35*a^3 + 5*b^3*sinh(c + d*x)^6 + 35*a^2*b*sinh(c + d*x)^2 +
 21*a*b^2*sinh(c + d*x)^4))/(35*d)
```

3.308 $\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=86

$$\frac{(a-b)^3\operatorname{ArcTan}(\sinh(c+dx))}{d}+\frac{b(3a^2-3ab+b^2)\sinh(c+dx)}{d}+\frac{(3a-b)b^2\sinh^3(c+dx)}{3d}+\frac{b^3\sinh^5(c+dx)}{5d}$$

```
[Out] (a-b)^3*arctan(sinh(d*x+c))/d+b*(3*a^2-3*a*b+b^2)*sinh(d*x+c)/d+1/3*(3*a-b)
*b^2*sinh(d*x+c)^3/d+1/5*b^3*sinh(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3269, 398, 209}

$$\frac{b(3a^2-3ab+b^2)\sinh(c+dx)}{d}+\frac{(a-b)^3\operatorname{ArcTan}(\sinh(c+dx))}{d}+\frac{b^2(3a-b)\sinh^3(c+dx)}{3d}+\frac{b^3\sinh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - b)^3*ArcTan[Sinh[c + d*x]])/d + (b*(3*a^2 - 3*a*b + b^2)*Sinh[c + d*x
])/d + ((3*a - b)*b^2*Sinh[c + d*x]^3)/(3*d) + (b^3*Sinh[c + d*x]^5)/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(b(3a^2-3ab+b^2)+(3a-b)b^2x^2+b^3x^4+\frac{(a-b)^3}{1+x^2}\right)\,dx}{d}\\ &= \frac{b(3a^2-3ab+b^2)\sinh(c+dx)}{d}+\frac{(3a-b)b^2\sinh^3(c+dx)}{3d}+\\ &= \frac{(a-b)^3\tan^{-1}(\sinh(c+dx))}{d}+\frac{b(3a^2-3ab+b^2)\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 100, normalized size = 1.16

$$\frac{\sinh(c+dx)\left(\frac{15(a-b)^3\tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}}+b\left(45a^2+15ab\left(-3+\sinh^2(c+dx)\right)+b^2\left(15-5\sinh^2(c+dx)+3\sinh^4(c+dx)\right)\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Sinh[c + d*x]*((15*(a - b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c
 + d*x]^2] + b*(45*a^2 + 15*a*b*(-3 + Sinh[c + d*x]^2) + b^2*(15 - 5*Sinh[c
 + d*x]^2 + 3*Sinh[c + d*x]^4))))/(15*d)
```

Maple [A]
time = 1.00, size = 114, normalized size = 1.33

method	result
derivativedivides	$\frac{2a^3\arctan\left(e^{dx+c}\right)+3a^2b\left(\sinh(dx+c)-2\arctan\left(e^{dx+c}\right)\right)+3ab^2\left(\frac{\left(\sinh^3(dx+c)\right)}{3}-\sinh(dx+c)+2\arctan\left(e^{dx+c}\right)\right)+b^3}{d}$
default	$\frac{2a^3\arctan\left(e^{dx+c}\right)+3a^2b\left(\sinh(dx+c)-2\arctan\left(e^{dx+c}\right)\right)+3ab^2\left(\frac{\left(\sinh^3(dx+c)\right)}{3}-\sinh(dx+c)+2\arctan\left(e^{dx+c}\right)\right)+b^3}{d}$
risch	$\frac{e^{5dx+5c}b^3}{160d}-\frac{7\,e^{3dx+3c}b^3}{96d}+\frac{a\,b^2e^{3dx+3c}}{8d}+\frac{3b\,e^{dx+c}a^2}{2d}-\frac{15a\,e^{dx+c}b^2}{8d}+\frac{11b^3e^{dx+c}}{16d}-\frac{3\,e^{-dx-c}a^2b}{2d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^3*arctan(exp(d*x+c))+3*a^2*b*(sinh(d*x+c)-2*arctan(exp(d*x+c)))+3*
a*b^2*(1/3*sinh(d*x+c)^3-sinh(d*x+c)+2*arctan(exp(d*x+c)))+b^3*(1/5*sinh(d*
x+c)^5-1/3*sinh(d*x+c)^3+sinh(d*x+c)-2*arctan(exp(d*x+c))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(82) = 164$.
time = 0.47, size = 233, normalized size = 2.71

$$-\frac{1}{480}\,b^3\left(\frac{\left(35\,e^{(-2\,dx-2\,c)}-330\,e^{(-4\,dx-4\,c)}-3\right)e^{(5\,dx+5\,c)}}{d}+\frac{330\,e^{(-dx-c)}-35\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}}{d}-\frac{960\,\arctan\left(e^{(-dx-c)}\right)}{d}\right)-\frac{1}{8}\,ab^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)}-1\right)e^{(3\,dx+3\,c)}}{d}-\frac{15\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{48\,\arctan\left(e^{(-dx-c)}\right)}{d}\right)+\frac{3}{2}\,a^2b\left(\frac{4\,\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)+\frac{a^3\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/480*b^3*((35*e^(-2*d*x - 2*c) - 330*e^(-4*d*x - 4*c) - 3)*e^(5*d*x + 5*c
)/d + (330*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/d - 960
*arctan(e^(-d*x - c))/d) - 1/8*a*b^2*((15*e^(-2*d*x - 2*c) - 1)*e^(3*d*x +
3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arctan(e^(-d*x - c))/d
) + 3/2*a^2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) +
 a^3*arctan(sinh(d*x + c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(82) = 164$.
time = 0.39, size = 1114, normalized size = 12.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^3*cosh(d*x + c)^10 + 30*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^
3*sinh(d*x + c)^10 + 5*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^8 + 5*(27*b^3*cosh(
d*x + c)^2 + 12*a*b^2 - 7*b^3)*sinh(d*x + c)^8 + 40*(9*b^3*cosh(d*x + c)^3
+ (12*a*b^2 - 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(24*a^2*b - 30*a*b
^2 + 11*b^3)*cosh(d*x + c)^6 + 10*(63*b^3*cosh(d*x + c)^4 + 72*a^2*b - 90*a
*b^2 + 33*b^3 + 14*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*
(189*b^3*cosh(d*x + c)^5 + 70*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^3 + 45*(24*a
^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 30*(24*a^2*b - 3
0*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 10*(63*b^3*cosh(d*x + c)^6 + 35*(12*a*b
^2 - 7*b^3)*cosh(d*x + c)^4 - 72*a^2*b + 90*a*b^2 - 33*b^3 + 45*(24*a^2*b -
 30*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(9*b^3*cosh(d*x +
 c)^7 + 7*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^5 + 15*(24*a^2*b - 30*a*b^2 + 11
*b^3)*cosh(d*x + c)^3 - 3*(24*a^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c))*sin
h(d*x + c)^3 - 3*b^3 - 5*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^2 + 5*(27*b^3*cos
h(d*x + c)^8 + 28*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^6 + 90*(24*a^2*b - 30*a*
b^2 + 11*b^3)*cosh(d*x + c)^4 - 12*a*b^2 + 7*b^3 - 36*(24*a^2*b - 30*a*b^2
+ 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 960*((a^3 - 3*a^2*b + 3*a*b^2
- b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^4*
sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^3*sinh(d*x
 + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^2*sinh(d*x + c)^
```

```
3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (a^3
- 3*a^2*b + 3*a*b^2 - b^3)*sinh(d*x + c)^5)*arctan(cosh(d*x + c) + sinh(d*x
 + c)) + 10*(3*b^3*cosh(d*x + c)^9 + 4*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^7 +
 18*(24*a^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c)^5 - 12*(24*a^2*b - 30*a*b^
2 + 11*b^3)*cosh(d*x + c)^3 - (12*a*b^2 - 7*b^3)*cosh(d*x + c))*sinh(d*x +
c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*d*cosh(d*x
+ c)^3*sinh(d*x + c)^2 + 10*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*d*cosh(d*
x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^2\left(c + dx\right)\right)^3 \operatorname{sech}\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)**3*sech(c + d*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(82) = 164.
time = 0.44, size = 204, normalized size = 2.37

$$\frac{3\,b^3e^{(5\,dx+5\,c)} + 60\,ab^2e^{(3\,dx+3\,c)} - 35\,b^3e^{(3\,dx+3\,c)} + 720\,a^2be^{(dx+c)} - 900\,ab^2e^{(dx+c)} + 330\,b^3e^{(dx+c)} + 960\,(a^3 - 3\,a^2b + 3\,ab^2 - b^3)\arctan\left(e^{(dx+c)}\right) - \left(720\,a^2be^{(4\,dx+4\,c)} - 900\,ab^2e^{(4\,dx+4\,c)} + 330\,b^3e^{(4\,dx+4\,c)} + 60\,ab^2e^{(2\,dx+2\,c)} - 35\,b^3e^{(2\,dx+2\,c)} + 3\,b^3\right)e^{(-5\,dx-5\,c)}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/480*(3*b^3*e^(5*d*x + 5*c) + 60*a*b^2*e^(3*d*x + 3*c) - 35*b^3*e^(3*d*x +
 3*c) + 720*a^2*b*e^(d*x + c) - 900*a*b^2*e^(d*x + c) + 330*b^3*e^(d*x + c)
 + 960*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*arctan(e^(d*x + c)) - (720*a^2*b*e^(
4*d*x + 4*c) - 900*a*b^2*e^(4*d*x + 4*c) + 330*b^3*e^(4*d*x + 4*c) + 60*a*b
^2*e^(2*d*x + 2*c) - 35*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-5*d*x - 5*c))/d
```

Mupad [**B**]
time = 1.04, size = 294, normalized size = 3.42

$$\frac{\mathrm{e}^{c+d\,x}\,\left(24\,a^2\,b-30\,a\,b^2+11\,b^3\right)}{16\,d}-\frac{\mathrm{e}^{-c-d\,x}\,\left(24\,a^2\,b-30\,a\,b^2+11\,b^3\right)}{16\,d}+\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\left(a^3\,\sqrt{d^2}-b^3\,\sqrt{d^2}+3\,a\,b^2\,\sqrt{d^2}-3\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{a^6-6\,a^5\,b+15\,a^4\,b^2-20\,a^3\,b^3+15\,a^2\,b^4-6\,a\,b^5+b^6}}\right)\,\sqrt{a^6-6\,a^5\,b+15\,a^4\,b^2-20\,a^3\,b^3+15\,a^2\,b^4-6\,a\,b^5+b^6}}{\sqrt{d^2}}-\frac{b^3\,\mathrm{e}^{-5\,c-5\,d\,x}}{160\,d}+\frac{b^3\,\mathrm{e}^{5\,c+5\,d\,x}}{160\,d}-\frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}\,\left(12\,a-7\,b\right)}{96\,d}+\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}\,\left(12\,a-7\,b\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x),x)
```

```
[Out] (exp(c + d*x)*(24*a^2*b - 30*a*b^2 + 11*b^3))/(16*d) - (exp(- c - d*x)*(24*
a^2*b - 30*a*b^2 + 11*b^3))/(16*d) + (2*atan((exp(d*x)*exp(c)*(a^3*(d^2)^(1
/2) - b^3*(d^2)^(1/2) + 3*a*b^2*(d^2)^(1/2) - 3*a^2*b*(d^2)^(1/2)))/(d*(a^6
```

```
 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2)))*
(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2
))/(d^2)^(1/2) - (b^3*exp(- 5*c - 5*d*x))/(160*d) + (b^3*exp(5*c + 5*d*x))/
(160*d) - (b^2*exp(- 3*c - 3*d*x)*(12*a - 7*b))/(96*d) + (b^2*exp(3*c + 3*d
*x)*(12*a - 7*b))/(96*d)
```

3.309 $\int \mathbf{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=92

$$\frac{3}{8}b\left(8a^2-12ab+5b^2\right)x+\frac{3(4a-3b)b^2\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{(a-b)^3\,\mathrm{ta}}{}$$

```
[Out] 3/8*b*(8*a^2-12*a*b+5*b^2)*x+3/8*(4*a-3*b)*b^2*cosh(d*x+c)*sinh(d*x+c)/d+1/
4*b^3*cosh(d*x+c)^3*sinh(d*x+c)/d+(a-b)^3*tanh(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = $\{3270, 398, 1171, 393, 212\}$

$$\frac{3}{8}bx\left(8a^2-12ab+5b^2\right)+\frac{3b^2(4a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{(a-b)^3\tanh(c+dx)}{d}+\frac{b^3\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*b*(8*a^2 - 12*a*b + 5*b^2)*x)/8 + (3*(4*a - 3*b)*b^2*Cosh[c + d*x]*Sinh[
c + d*x])/(8*d) + (b^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) + ((a - b)^3*Ta
nh[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left((a-b)^3+\frac{b\left(3a^2-3ab+b^2\right)-3(a-b)(2a-b)bx^2+3(a-b)^2bx^4}{\left(1-x^2\right)^3}\right)\,d}{d}$$

$$= \frac{(a-b)^3\tanh(c+dx)}{d}+\frac{\text{Subst}\left(\int \frac{b\left(3a^2-3ab+b^2\right)-3(a-b)(2a-b)bx^2+}{\left(1-x^2\right)^3}\right.}{d}$$

$$= \frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{(a-b)^3\tanh(c+dx)}{d}-\text{Subs}$$

$$= \frac{3(4a-3b)b^2\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^3\cosh^3(c+dx)\sin}{4d}$$

$$= \frac{3}{8}b\left(8a^2-12ab+5b^2\right)x+\frac{3(4a-3b)b^2\cosh(c+dx)\sinh(c+}{8d}$$

Mathematica [A]
time = 0.40, size = 78, normalized size = 0.85

$$\frac{12b\left(8a^2-12ab+5b^2\right)(c+dx)+8(3a-2b)b^2\sinh(2(c+dx))+b^3\sinh(4(c+dx))+32(a-b)^3\tanh(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (12*b*(8*a^2 - 12*a*b + 5*b^2)*(c + d*x) + 8*(3*a - 2*b)*b^2*Sinh[2*(c + d*
x)] + b^3*Sinh[4*(c + d*x)] + 32*(a - b)^3*Tanh[c + d*x])/(32*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(86) = 172$.
time = 1.63, size = 212, normalized size = 2.30

method	result
risch	$3a^2bx-\frac{9a\,b^2x}{2}+\frac{15b^3x}{8}+\frac{b^3\mathrm{e}^{4dx+4c}}{64d}+\frac{3\,\mathrm{e}^{2dx+2c}a\,b^2}{8d}-\frac{b^3\mathrm{e}^{2dx+2c}}{4d}-\frac{3\,\mathrm{e}^{-2dx-2c}a\,b^2}{8d}+\frac{b^3\mathrm{e}^{-2dx-2c}}{4d}-\frac{b^3\mathrm{e}^{-4dx-4c}}{64d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a^2*b*x-9/2*a*b^2*x+15/8*b^3*x+1/64*b^3/d*exp(4*d*x+4*c)+3/8/d*exp(2*d*x+
2*c)*a*b^2-1/4*b^3/d*exp(2*d*x+2*c)-3/8/d*exp(-2*d*x-2*c)*a*b^2+1/4*b^3/d*e
xp(-2*d*x-2*c)-1/64*b^3/d*exp(-4*d*x-4*c)-2/d/(1+exp(2*d*x+2*c))*a^3+6/d/(1
+exp(2*d*x+2*c))*a^2*b-6/d/(1+exp(2*d*x+2*c))*a*b^2+2/d/(1+exp(2*d*x+2*c))*
b^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(86) = 172$.
time = 0.27, size = 215, normalized size = 2.34

$$3\,a^2b\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)+\frac{1}{64}\,b^3\left(\frac{120\,(dx+c)}{d}+\frac{16\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}}{d}-\frac{15\,e^{(-2\,dx-2\,c)}+144\,e^{(-4\,dx-4\,c)}-1}{d(e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)})}\right)-\frac{3}{8}\,ab^2\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)})}\right)+\frac{2\,a^3}{d(e^{(-2\,dx-2\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3*a^2*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 1/64*b^3*(120*(d*x + c)/
d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2*c) + 144
*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)))) - 3/8*a*b
^2*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(
-2*d*x - 2*c) + e^(-4*d*x - 4*c)))) + 2*a^3/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(86) = 172$.
time = 0.38, size = 178, normalized size = 1.93

$$\frac{b^3\sinh\left(dx+c\right)^5+\left(10\,b^3\cosh\left(dx+c\right)^2+24\,ab^2-15\,b^3\right)\sinh\left(dx+c\right)^3-8\left(8\,a^3-24\,a^2b+24\,ab^2-8\,b^3-3\left(8\,a^2b-12\,ab^2+5\,b^3\right)dx\right)\cosh\left(dx+c\right)+\left(5\,b^3\cosh\left(dx+c\right)^4+64\,a^3-192\,a^2b+216\,ab^2-80\,b^3+9\left(8\,ab^2-5\,b^3\right)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)}{64\,d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(b^3*sinh(d*x + c)^5 + (10*b^3*cosh(d*x + c)^2 + 24*a*b^2 - 15*b^3)*si
nh(d*x + c)^3 - 8*(8*a^3 - 24*a^2*b + 24*a*b^2 - 8*b^3 - 3*(8*a^2*b - 12*a*
b^2 + 5*b^3)*d*x)*cosh(d*x + c) + (5*b^3*cosh(d*x + c)^4 + 64*a^3 - 192*a^2
*b + 216*a*b^2 - 80*b^3 + 9*(8*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
))/(d*cosh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(86) = 172.
time = 0.46, size = 197, normalized size = 2.14

$$\frac{b^3e^{(4\,dx+4\,c)}+24\,ab^2e^{(2\,dx+2\,c)}-16\,b^3e^{(2\,dx+2\,c)}+24\,(8\,a^2b-12\,ab^2+5\,b^3)(dx+c)-\left(144\,a^2be^{(4\,dx+4\,c)}-216\,ab^2e^{(4\,dx+4\,c)}+90\,b^3e^{(4\,dx+4\,c)}+24\,ab^2e^{(2\,dx+2\,c)}-16\,b^3e^{(2\,dx+2\,c)}+b^3\right)e^{(-4\,dx-4\,c)}-\frac{128\,(a^3-3\,a^2b+3\,ab^2-b^3)}{e^{(2\,dx+2\,c)}+1}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(b^3*e^(4*d*x + 4*c) + 24*a*b^2*e^(2*d*x + 2*c) - 16*b^3*e^(2*d*x + 2*
c) + 24*(8*a^2*b - 12*a*b^2 + 5*b^3)*(d*x + c) - (144*a^2*b*e^(4*d*x + 4*c)
 - 216*a*b^2*e^(4*d*x + 4*c) + 90*b^3*e^(4*d*x + 4*c) + 24*a*b^2*e^(2*d*x +
 2*c) - 16*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x - 4*c) - 128*(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)/(e^(2*d*x + 2*c) + 1))/d
```

Mupad [**B**]
time = 0.98, size = 141, normalized size = 1.53

$$\frac{b^3\,\mathrm{e}^{4\,c+4\,d\,x}}{64\,d}-\frac{b^3\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,d}-\frac{2\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{3\,b\,x\,\left(8\,a^2-12\,a\,b+5\,b^2\right)}{8}-\frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}\,\left(3\,a-2\,b\right)}{8\,d}+\frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(3\,a-2\,b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^2,x)
```

```
[Out] (b^3*exp(4*c + 4*d*x))/(64*d) - (b^3*exp(- 4*c - 4*d*x))/(64*d) - (2*(3*a*b
^2 - 3*a^2*b + a^3 - b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (3*b*x*(8*a^2 - 12*
a*b + 5*b^2))/8 - (b^2*exp(- 2*c - 2*d*x)*(3*a - 2*b))/(8*d) + (b^2*exp(2*c
 + 2*d*x)*(3*a - 2*b))/(8*d)
```

3.310 $\int \text{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=91

$$\frac{(a-b)^2(a+5b)\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{(3a-2b)b^2\sinh(c+dx)}{d}+\frac{b^3\sinh^3(c+dx)}{3d}+\frac{(a-b)^3\text{sech}(c+dx)\,\text{ta}}{2d}$$

```
[Out] 1/2*(a-b)^2*(a+5*b)*arctan(sinh(d*x+c))/d+(3*a-2*b)*b^2*sinh(d*x+c)/d+1/3*b
^3*sinh(d*x+c)^3/d+1/2*(a-b)^3*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 398, 393, 209}

$$\frac{(a+5b)(a-b)^2\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{b^2(3a-2b)\sinh(c+dx)}{d}+\frac{(a-b)^3\tanh(c+dx)\text{sech}(c+dx)}{2d}+\frac{b^3\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - b)^2*(a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((3*a - 2*b)*b^2*Sinh[c
 + d*x])/d + (b^3*Sinh[c + d*x]^3)/(3*d) + ((a - b)^3*Sech[c + d*x]*Tanh[c
+ d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{(1+x^2)^2}\,dx,x,\sinh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left((3a-2b)b^2+b^3x^2+\frac{(a-b)^2(a+2b)+3(a-b)^2bx^2}{(1+x^2)^2}\right)\,dx,x,\text{s}\right.}{d}\\
&= \frac{(3a-2b)b^2\sinh(c+dx)}{d}+\frac{b^3\sinh^3(c+dx)}{3d}+\frac{\text{Subst}\left(\int \frac{(a-b)^2}{}\right.}{}\\
&= \frac{(3a-2b)b^2\sinh(c+dx)}{d}+\frac{b^3\sinh^3(c+dx)}{3d}+\frac{(a-b)^3\operatorname{sech}(c}{}\\
&= \frac{(a-b)^2(a+5b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(3a-2b)b^2\sinh(c+}{d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 5.73, size = 347, normalized size = 3.81

$$
\frac{\operatorname{csch}^5(c+dx)\left(-256\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{11}{2};-\sinh^2(c+dx)\right)\sinh^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3+21\left(36015a^3+5a^2(3224a+21609b)\sinh^2(c+dx)+3a(491a^2+16120ab+36015b^2)\sinh^4(c+dx)+3b(753a^2+18280ab+10805b^2)\sinh^6(c+dx)+b^2(2259a+17320b)\sinh^8(c+dx)+753b^3\sinh^{10}(c+dx)\right)-\frac{315\tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)\left(2401a^3+3a^2(625a+2401b)\sinh^2(c+dx)+3a(81a^2+1875ab+2401b^2)\sinh^4(c+dx)+(-47a^3+585a^2b+6057ab^2+2161b^3)\sinh^6(c+dx)+3b(a^2+243ab+625b^2)\sinh^8(c+dx)+3b^2(a+81b)\sinh^{10}(c+dx)+b^3\sinh^{12}(c+dx)\right)}{\sqrt{-\sinh^2(c+dx)}}\right)}{30240d}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c + d*x]^5*(-256*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}
, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3 + 21*(36015*a
^3 + 5*a^2*(3224*a + 21609*b)*Sinh[c + d*x]^2 + 3*a*(491*a^2 + 16120*a*b +
36015*b^2)*Sinh[c + d*x]^4 + 3*b*(753*a^2 + 18280*a*b + 10805*b^2)*Sinh[c +
 d*x]^6 + b^2*(2259*a + 17320*b)*Sinh[c + d*x]^8 + 753*b^3*Sinh[c + d*x]^10
) - (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(2401*a^3 + 3*a^2*(625*a + 2401*b)
*Sinh[c + d*x]^2 + 3*a*(81*a^2 + 1875*a*b + 2401*b^2)*Sinh[c + d*x]^4 + (-4
7*a^3 + 585*a^2*b + 6057*a*b^2 + 2161*b^3)*Sinh[c + d*x]^6 + 3*b*(a^2 + 243
*a*b + 625*b^2)*Sinh[c + d*x]^8 + 3*b^2*(a + 81*b)*Sinh[c + d*x]^10 + b^3*S
inh[c + d*x]^12))/Sqrt[-Sinh[c + d*x]^2]))/(30240*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.50, size = 311, normalized size = 3.42

method	result
risch	$\frac{e^{3dx+3c}b^3}{24d}+\frac{3a\,e^{dx+c}b^2}{2d}-\frac{9b^3e^{dx+c}}{8d}-\frac{3a\,e^{-dx-c}b^2}{2d}+\frac{9b^3e^{-dx-c}}{8d}-\frac{b^3e^{-3dx-3c}}{24d}+\frac{e^{dx+c}\left(a^3-3a^2b+3a\,b^2-b^3\right)\left(e^{2dx+2c}-1\right)}{d\left(1+e^{2dx+2c}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*b^3+3/2*a/d*exp(d*x+c)*b^2-9/8*b^3/d*exp(d*x+c)-3/2*a
/d*exp(-d*x-c)*b^2+9/8*b^3/d*exp(-d*x-c)-1/24*b^3/d*exp(-3*d*x-3*c)+exp(d*x
+c)*(a^3-3*a^2*b+3*a*b^2-b^3)*(exp(2*d*x+2*c)-1)/d/(1+exp(2*d*x+2*c))^2+1/2
*I/d*ln(exp(d*x+c)+I)*a^3+3/2*I/d*ln(exp(d*x+c)+I)*a^2*b-9/2*I/d*ln(exp(d*x
+c)+I)*a*b^2+5/2*I/d*ln(exp(d*x+c)+I)*b^3-1/2*I/d*ln(exp(d*x+c)-I)*a^3-3/2*
I/d*ln(exp(d*x+c)-I)*a^2*b+9/2*I/d*ln(exp(d*x+c)-I)*a*b^2-5/2*I/d*ln(exp(d*
x+c)-I)*b^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(85) = 170$.
time = 0.51, size = 357, normalized size = 3.92

$$\frac{1}{24}\,b^3\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}-1}{d(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\right)+\frac{3}{2}\,ab^2\left(\frac{6\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+1}{d(e^{(-dx-c)}+2\,e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)})}\right)-3\,a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)-a^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))
/d - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(
d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 3/2*a*b^2*
(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^(-4*d*
x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) -
 3*a^2*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*
e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a^3*(arctan(e^(-d*x - c))/d -
(e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)
 + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. $2(85) = 170$.
time = 0.42, size = 1679, normalized size = 18.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*sin
h(d*x + c)^10 + (36*a*b^2 - 25*b^3)*cosh(d*x + c)^8 + (45*b^3*cosh(d*x + c)
```

```
^2 + 36*a*b^2 - 25*b^3)*sinh(d*x + c)^8 + 8*(15*b^3*cosh(d*x + c)^3 + (36*a
*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(12*a^3 - 36*a^2*b + 54*a
*b^2 - 25*b^3)*cosh(d*x + c)^6 + 2*(105*b^3*cosh(d*x + c)^4 + 12*a^3 - 36*a
^2*b + 54*a*b^2 - 25*b^3 + 14*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^6 + 4*(63*b^3*cosh(d*x + c)^5 + 14*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^
3 + 3*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^
5 - 2*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)^4 + 2*(105*b^3*
cosh(d*x + c)^6 + 35*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^4 - 12*a^3 + 36*a^2*
b - 54*a*b^2 + 25*b^3 + 15*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 8*(15*b^3*cosh(d*x + c)^7 + 7*(36*a*b^2 - 25*b^3
)*cosh(d*x + c)^5 + 5*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)
^3 - (12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
 - b^3 - (36*a*b^2 - 25*b^3)*cosh(d*x + c)^2 + (45*b^3*cosh(d*x + c)^8 + 28
*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^6 + 30*(12*a^3 - 36*a^2*b + 54*a*b^2 - 2
5*b^3)*cosh(d*x + c)^4 - 36*a*b^2 + 25*b^3 - 12*(12*a^3 - 36*a^2*b + 54*a*b
^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*((a^3 + 3*a^2*b - 9*a*b^
2 + 5*b^3)*cosh(d*x + c)^7 + 7*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x +
 c)*sinh(d*x + c)^6 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*sinh(d*x + c)^7 + 2
*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + (2*a^3 + 6*a^2*b - 18*
a*b^2 + 10*b^3 + 21*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^5 + 5*(7*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 2*(a
^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + (a^3 + 3*a
^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (35*(a^3 + 3*a^2*b - 9*a*b^2 + 5*
b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3 + 20*(a^3 + 3*a^2*b
 - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (21*(a^3 + 3*a^2*b -
9*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 20*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cos
h(d*x + c)^3 + 3*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^2 + (7*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 10*(a^3 + 3
*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 3*(a^3 + 3*a^2*b - 9*a*b^2 + 5*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c))
+ 2*(5*b^3*cosh(d*x + c)^9 + 4*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^7 + 6*(12*
a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)^5 - 4*(12*a^3 - 36*a^2*b
+ 54*a*b^2 - 25*b^3)*cosh(d*x + c)^3 - (36*a*b^2 - 25*b^3)*cosh(d*x + c))*s
inh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*si
nh(d*x + c)^7 + 2*d*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x
 + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*c
osh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x
 + c)^3 + (21*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))
*sinh(d*x + c)^2 + (7*d*cosh(d*x + c)^6 + 10*d*cosh(d*x + c)^4 + 3*d*cosh(d
*x + c)^2)*sinh(d*x + c))
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(85) = 170.
time = 0.44, size = 247, normalized size = 2.71

$$\frac{b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 36\,ab^2\left(e^{(dx+c)} - e^{(-dx-c)}\right) - 24\,b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 6\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(a^3 + 3\,a^2b - 9\,ab^2 + 5\,b^3\right) + \frac{24\left(a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)-3\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+3\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)-b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 36*a*b^2*(e^(d*x + c) - e^(-d*x
- c)) - 24*b^3*(e^(d*x + c) - e^(-d*x - c)) + 6*(pi + 2*arctan(1/2*(e^(2*d*
x + 2*c) - 1)*e^(-d*x - c)))*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3) + 24*(a^3*(e
^(d*x + c) - e^(-d*x - c)) - 3*a^2*b*(e^(d*x + c) - e^(-d*x - c)) + 3*a*b^2
*(e^(d*x + c) - e^(-d*x - c)) - b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x
+ c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [**B**]
time = 2.28, size = 308, normalized size = 3.38

$$\frac{b^3\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{b^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}+\frac{3\,b^2\,\mathrm{e}^{c+d\,x}\,\left(4\,a-3\,b\right)}{8\,d}+\frac{\mathrm{e}^{c+d\,x}\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\mathrm{e}^{c+d\,x}\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{3\,b^2\,\mathrm{e}^{-c-d\,x}\,\left(4\,a-3\,b\right)}{8\,d}+\frac{\ln\left(-\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\left(a^3+3\,a^2\,b-9\,a\,b^2+5\,b^3\right)-{\left(a-b\right)}^2\,\left(a+5\,b\right)\,1{}\mathrm{i}\right)\,{\left(a-b\right)}^2\,\left(a+5\,b\right)\,1{}\mathrm{i}}{2\,d}-\frac{\ln\left(-\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\left(a^3+3\,a^2\,b-9\,a\,b^2+5\,b^3\right)+{\left(a-b\right)}^2\,\left(a+5\,b\right)\,1{}\mathrm{i}\right)\,{\left(a-b\right)}^2\,\left(a+5\,b\right)\,1{}\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^3,x)
```

```
[Out] (b^3*exp(3*c + 3*d*x))/(24*d) - (b^3*exp(- 3*c - 3*d*x))/(24*d) + (log(- (a
 - b)^2*(a + 5*b)*1i - exp(d*x)*exp(c)*(3*a^2*b - 9*a*b^2 + a^3 + 5*b^3))*(
a - b)^2*(a + 5*b)*1i)/(2*d) - (log((a - b)^2*(a + 5*b)*1i - exp(d*x)*exp(c
)*(3*a^2*b - 9*a*b^2 + a^3 + 5*b^3))*(a - b)^2*(a + 5*b)*1i)/(2*d) + (3*b^2
*exp(c + d*x)*(4*a - 3*b))/(8*d) + (exp(c + d*x)*(3*a*b^2 - 3*a^2*b + a^3 -
 b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(3*a*b^2 - 3*a^2*b + a^
3 - b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (3*b^2*exp(- c
- d*x)*(4*a - 3*b))/(8*d)
```

3.311 $\int \mathbf{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}(6a-5b)b^2x+\frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{(a-b)^2(a+2b)\tanh(c+dx)}{d}-\frac{(a-b)^3\tanh^3(c+dx)}{3d}$$

```
[Out] 1/2*(6*a-5*b)*b^2*x+1/2*b^3*cosh(d*x+c)*sinh(d*x+c)/d+(a-b)^2*(a+2*b)*tanh(
d*x+c)/d-1/3*(a-b)^3*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3270, 398, 393, 212}

$$\frac{1}{2}b^2x(6a-5b)-\frac{(a-b)^3\tanh^3(c+dx)}{3d}+\frac{(a-b)^2(a+2b)\tanh(c+dx)}{d}+\frac{b^3\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((6*a - 5*b)*b^2*x)/2 + (b^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + ((a - b)^
2*(a + 2*b)*Tanh[c + d*x])/d - ((a - b)^3*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left((a-b)^2(a+2b)-(a-b)^3x^2+\frac{(3a-2b)b^2-3(a-b)b^2x^2}{\left(1-x^2\right)^2}\right)}{d}\\ &= \frac{(a-b)^2(a+2b)\tanh(c+dx)}{d}-\frac{(a-b)^3\tanh^3(c+dx)}{3d}+\\ &= \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{(a-b)^2(a+2b)\tanh(c+dx)}{d}-\\ &= \frac{1}{2}(6a-5b)b^2x+\frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d}+\end{aligned}$$

Mathematica [**A**]
time = 0.75, size = 84, normalized size = 1.02

$$\frac{6(6a-5b)b^2(c+dx)+3b^3\sinh(2(c+dx))+2(a-b)^2(4a+5b+(2a+7b)\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\tanh(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (6*(6*a - 5*b)*b^2*(c + d*x) + 3*b^3*Sinh[2*(c + d*x)] + 2*(a - b)^2*(4*a +
 5*b + (2*a + 7*b)*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*Tanh[c + d*x])/(12*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(76) = 152.
time = 1.81, size = 177, normalized size = 2.16

method	result
risch	$3a\,b^2x-\frac{5b^3x}{2}+\frac{b^3\mathrm{e}^{2dx+2c}}{8d}-\frac{b^3\mathrm{e}^{-2dx-2c}}{8d}-\frac{2\left(9a^2b\,\mathrm{e}^{4dx+4c}-18a\,b^2\mathrm{e}^{4dx+4c}+9b^3\mathrm{e}^{4dx+4c}+6a^3\mathrm{e}^{2dx+2c}-18a\,b^2\mathrm{e}^{2dx+2c}+12b^3\mathrm{e}\right.}{3d\left(1+\mathrm{e}^{2dx+2c}\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a*b^2*x-5/2*b^3*x+1/8*b^3/d*exp(2*d*x+2*c)-1/8*b^3/d*exp(-2*d*x-2*c)-2/3*
(9*a^2*b*exp(4*d*x+4*c)-18*a*b^2*exp(4*d*x+4*c)+9*b^3*exp(4*d*x+4*c)+6*a^3*
exp(2*d*x+2*c)-18*a*b^2*exp(2*d*x+2*c)+12*b^3*exp(2*d*x+2*c)+2*a^3+3*a^2*b-
12*a*b^2+7*b^3)/d/(1+exp(2*d*x+2*c))^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(76) = 152$.
time = 0.29, size = 382, normalized size = 4.66

$$ab^2\left(3\,x + \frac{3\,c}{d} - \frac{4\,\left(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + 2\right)}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right) - \frac{1}{24}\,b^3\left(\frac{60\,(dx+c)}{d} + \frac{3\,e^{(-2\,dx-2\,c)}}{d} - \frac{121\,e^{(-2\,dx-2\,c)} + 201\,e^{(-4\,dx-4\,c)} + 147\,e^{(-6\,dx-6\,c)} + 3}{d(e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + 3\,e^{(-6\,dx-6\,c)} + e^{(-8\,dx-8\,c)})}\right) + \frac{4}{3}\,a^3\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)} + \frac{1}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right) + 2\,a^2b\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)} + \frac{1}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*
e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 1/24*b^3*
(60*(d*x + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*
d*x - 4*c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
 4*c) + 3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) + 4/3*a^3*(3*e^(-2*d*x - 2
*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) +
1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2
*a^2*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^
(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-
6*d*x - 6*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(76) = 152$.
time = 0.41, size = 321, normalized size = 3.91

$$\frac{3\,b^3\sinh\left(dx+c\right)^5 - 4\,(4\,a^3 + 6\,a^2b - 24\,ab^2 + 14\,b^3 - 3\,(6\,ab^2 - 5\,b^3)dx)\cosh\left(dx+c\right)^3 - 12\,(4\,a^3 + 6\,a^2b - 24\,ab^2 + 14\,b^3 - 3\,(6\,ab^2 - 5\,b^3)dx)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2 + \left(30\,b^3\cosh\left(dx+c\right)^2 + 16\,a^3 + 24\,a^2b - 96\,ab^2 + 65\,b^3\right)\sinh\left(dx+c\right)^3 - 12\,(4\,a^3 + 6\,a^2b - 24\,ab^2 + 14\,b^3 - 3\,(6\,ab^2 - 5\,b^3)dx)\cosh\left(dx+c\right) + 3\,\left(5\,b^3\cosh\left(dx+c\right)^4 + 16\,a^3 - 24\,a^2b + 10\,b^3 + (16\,a^3 + 24\,a^2b - 96\,ab^2 + 65\,b^3)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)}{24\,\left(d\cosh\left(dx+c\right)^3 + 3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2 + 3\,d\cosh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^3*sinh(d*x + c)^5 - 4*(4*a^3 + 6*a^2*b - 24*a*b^2 + 14*b^3 - 3*(6
*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)^3 - 12*(4*a^3 + 6*a^2*b - 24*a*b^2 + 14*
b^3 - 3*(6*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^2 + (30*b^3*cosh
(d*x + c)^2 + 16*a^3 + 24*a^2*b - 96*a*b^2 + 65*b^3)*sinh(d*x + c)^3 - 12*(
4*a^3 + 6*a^2*b - 24*a*b^2 + 14*b^3 - 3*(6*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c
) + 3*(5*b^3*cosh(d*x + c)^4 + 16*a^3 - 24*a^2*b + 10*b^3 + (16*a^3 + 24*a^
2*b - 96*a*b^2 + 65*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3
 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(76) = 152$.
time = 0.46, size = 208, normalized size = 2.54

$$\frac{3\,b^3e^{(2\,dx+2\,c)} + 12\,(6\,ab^2 - 5\,b^3)(dx + c) - 3\,\left(12\,ab^2e^{(2\,dx+2\,c)} - 10\,b^3e^{(2\,dx+2\,c)} + b^3\right)e^{(-2\,dx-2\,c)} - \frac{16\,\left(9\,a^2be^{(4\,dx+4\,c)} - 18\,ab^2e^{(4\,dx+4\,c)} + 9\,b^3e^{(4\,dx+4\,c)} + 6\,a^3e^{(2\,dx+2\,c)} - 18\,ab^2e^{(2\,dx+2\,c)} + 12\,b^3e^{(2\,dx+2\,c)} + 2\,a^3 + 3\,a^2b - 12\,ab^2 + 7\,b^3\right)}{\left(e^{(2\,dx+2\,c)} + 1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^3*e^(2*d*x + 2*c) + 12*(6*a*b^2 - 5*b^3)*(d*x + c) - 3*(12*a*b^2*
e^(2*d*x + 2*c) - 10*b^3*e^(2*d*x + 2*c) + b^3)*e^(-2*d*x - 2*c) - 16*(9*a^
2*b*e^(4*d*x + 4*c) - 18*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) + 6*
a^3*e^(2*d*x + 2*c) - 18*a*b^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 2
*a^3 + 3*a^2*b - 12*a*b^2 + 7*b^3)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [**B**]
time = 0.16, size = 273, normalized size = 3.33

$$\frac{b^2\,x\,(6\,a-5\,b)}{2} - \frac{\frac{2\,(a^2\,b-2\,a\,b^2+b^3)}{d} + \frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,(a^2\,b-2\,a\,b^2+b^3)}{d} + \frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(2\,a^3-3\,a^2\,b+b^3)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x} + 3\,\mathrm{e}^{4\,c+4\,d\,x} + \mathrm{e}^{6\,c+6\,d\,x} + 1} - \frac{\frac{2\,(2\,a^3-3\,a^2\,b+b^3)}{3\,d} + \frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(a^2\,b-2\,a\,b^2+b^3)}{d}}{2\,\mathrm{e}^{2\,c+2\,d\,x} + \mathrm{e}^{4\,c+4\,d\,x} + 1} - \frac{b^3\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d} + \frac{b^3\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d} - \frac{2\,(a^2\,b - 2\,a\,b^2 + b^3)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^4,x)
```

```
[Out] (b^2*x*(6*a - 5*b))/2 - ((2*(a^2*b - 2*a*b^2 + b^3))/d + (2*exp(4*c + 4*d*x
)*(a^2*b - 2*a*b^2 + b^3))/d + (4*exp(2*c + 2*d*x)*(2*a^3 - 3*a^2*b + b^3))
/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) -
((2*(2*a^3 - 3*a^2*b + b^3))/(3*d) + (2*exp(2*c + 2*d*x)*(a^2*b - 2*a*b^2 +
 b^3))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (b^3*exp(- 2*c - 2*
d*x))/(8*d) + (b^3*exp(2*c + 2*d*x))/(8*d) - (2*(a^2*b - 2*a*b^2 + b^3))/(d
*(exp(2*c + 2*d*x) + 1))
```

3.312 $\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{3(a-b)\left(4b^2+(a+b)^2\right)\text{ArcTan}(\sinh(c+dx))}{8d}+\frac{b^3\sinh(c+dx)}{d}+\frac{3(a-b)^2(a+3b)\text{sech}(c+dx)\tanh(c+}{8d}$$

```
[Out] 3/8*(a-b)*(4*b^2+(a+b)^2)*arctan(sinh(d*x+c))/d+b^3*sinh(d*x+c)/d+3/8*(a-b)
^2*(a+3*b)*sech(d*x+c)*tanh(d*x+c)/d+1/4*(a-b)^3*sech(d*x+c)^3*tanh(d*x+c)/
d
```

Rubi [A]

time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3269, 398, 1171, 393, 209}

$$\frac{3(a-b)\left((a+b)^2+4b^2\right)\text{ArcTan}(\sinh(c+dx))}{8d}+\frac{(a-b)^3\tanh(c+dx)\text{sech}^3(c+dx)}{4d}+\frac{3(a-b)^2(a+3b)\tanh(c+dx)\text{sech}(c+dx)}{8d}+\frac{b^3\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*(a - b)*(4*b^2 + (a + b)^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (b^3*Sinh[c +
 d*x])/d + (3*(a - b)^2*(a + 3*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) + ((a
- b)^3*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{\left(1+x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\Big(\int \Big(b^3+\frac{a^3-b^3+3b\left(a^2-b^2\right)x^2+3(a-b)b^2x^4}{\left(1+x^2\right)^3}\Big)\,dx, x, \sinh(c+dx)\Big)}{d}\\ &= \frac{b^3\sinh(c+dx)}{d}+\frac{\text{Subst}\Big(\int \frac{a^3-b^3+3b\left(a^2-b^2\right)x^2+3(a-b)b^2x^4}{\left(1+x^2\right)^3}\,dx, x, \text{sinl}}{d}\\ &= \frac{b^3\sinh(c+dx)}{d}+\frac{(a-b)^3\text{sech}^3(c+dx)\tanh(c+dx)}{4d}-\frac{\text{Subst}\Big(}{}\\ &= \frac{b^3\sinh(c+dx)}{d}+\frac{3(a-b)^2(a+3b)\text{sech}(c+dx)\tanh(c+dx)}{8d}-\\ &= \frac{3(a-b)\left(4b^2+(a+b)^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{b^3\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 9.39, size = 472, normalized size = 4.58

$$-\frac{\text{csch}^5(c+dx)\left(256\,{}_6F_5\left(\frac{3}{2},2,2,2,2,2;1,1,1,1,\frac{11}{2};-\sinh^2(c+dx)\right)\sinh^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3+384\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{11}{2};-\sinh^2(c+dx)\right)\sinh^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\left(7a+5b\sinh^2(c+dx)\right)-21\left(15ab^2\sinh^4(c+dx)\left(36015+21529\sinh^2(c+dx)+1128\sinh^4(c+dx)\right)+9a^2b\sinh^2(c+dx)\left(72030+41615\sinh^2(c+dx)+2131\sinh^4(c+dx)\right)+b^3\sinh^6(c+dx)\left(149460+90805\sinh^2(c+dx)+4887\sinh^4(c+dx)\right)+a^3\left(252105+140965\sinh^2(c+dx)+8226\sinh^4(c+dx)\right)\right)+\frac{315\tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)\left(a^3\left(16807+15000\sinh^2(c+dx)+2187\sinh^4(c+dx)-62\sinh^6(c+dx)\right)+9a^2b\sinh^2(c+dx)\left(4802+4375\sinh^2(c+dx)+640\sinh^4(c+dx)+3\sinh^6(c+dx)\right)+b^3\sinh^6(c+dx)\left(9964+9375\sinh^2(c+dx)+1458\sinh^4(c+dx)+7\sinh^6(c+dx)\right)+3ab^2\sinh^4(c+dx)\left(12005+11178\sinh^2(c+dx)+1701\sinh^4(c+dx)+8\sinh^6(c+dx)\right)\right)}{\sqrt{-\sinh^2(c+dx)}}\right)}{60480d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -1/60480*(Csch[c + d*x]^5*(256*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1,
1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3
 + 384*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]
^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^2*(7*a + 5*b*Sinh[c + d*x]^2) -
 21*(15*a*b^2*Sinh[c + d*x]^4*(36015 + 21529*Sinh[c + d*x]^2 + 1128*Sinh[c
+ d*x]^4) + 9*a^2*b*Sinh[c + d*x]^2*(72030 + 41615*Sinh[c + d*x]^2 + 2131*S
inh[c + d*x]^4) + b^3*Sinh[c + d*x]^6*(149460 + 90805*Sinh[c + d*x]^2 + 488
7*Sinh[c + d*x]^4) + a^3*(252105 + 140965*Sinh[c + d*x]^2 + 8226*Sinh[c + d
*x]^4)) + (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(a^3*(16807 + 15000*Sinh[c +
 d*x]^2 + 2187*Sinh[c + d*x]^4 - 62*Sinh[c + d*x]^6) + 9*a^2*b*Sinh[c + d*x
]^2*(4802 + 4375*Sinh[c + d*x]^2 + 640*Sinh[c + d*x]^4 + 3*Sinh[c + d*x]^6)
 + b^3*Sinh[c + d*x]^6*(9964 + 9375*Sinh[c + d*x]^2 + 1458*Sinh[c + d*x]^4
+ 7*Sinh[c + d*x]^6) + 3*a*b^2*Sinh[c + d*x]^4*(12005 + 11178*Sinh[c + d*x]
^2 + 1701*Sinh[c + d*x]^4 + 8*Sinh[c + d*x]^6)))/Sqrt[-Sinh[c + d*x]^2]))/d
```

Maple [C] Result contains complex when optimal does not.
time = 1.72, size = 409, normalized size = 3.97

method	result
risch	$\frac{b^{3} \mathrm{e}^{dx+c}}{2d}-\frac{b^{3} \mathrm{e}^{-dx-c}}{2d}+\frac{\mathrm{e}^{dx+c}\left(3a^{3}\mathrm{e}^{6dx+6c}+3a^{2}b\,\mathrm{e}^{6dx+6c}-15a\,b^{2}\mathrm{e}^{6dx+6c}+9b^{3}\mathrm{e}^{6dx+6c}+11a^{3}\mathrm{e}^{4dx+4c}-21a^{2}b\,\mathrm{e}^{4dx+4c}+9a\,b^{2}\mathrm{e}^{4dx+}\right.}{4d(1+\mathrm{e}^{2dx+})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^3/d*exp(d*x+c)-1/2*b^3/d*exp(-d*x-c)+1/4*exp(d*x+c)*(3*a^3*exp(6*d*x+
6*c)+3*a^2*b*exp(6*d*x+6*c)-15*a*b^2*exp(6*d*x+6*c)+9*b^3*exp(6*d*x+6*c)+11
*a^3*exp(4*d*x+4*c)-21*a^2*b*exp(4*d*x+4*c)+9*a*b^2*exp(4*d*x+4*c)+b^3*exp(
4*d*x+4*c)-11*a^3*exp(2*d*x+2*c)+21*a^2*b*exp(2*d*x+2*c)-9*a*b^2*exp(2*d*x+
2*c)-b^3*exp(2*d*x+2*c)-3*a^3-3*a^2*b+15*a*b^2-9*b^3)/d/(1+exp(2*d*x+2*c))^
4+3/8*I/d*ln(exp(d*x+c)+I)*a^3+3/8*I/d*ln(exp(d*x+c)+I)*a^2*b+9/8*I/d*ln(ex
p(d*x+c)+I)*a*b^2-15/8*I/d*ln(exp(d*x+c)+I)*b^3-3/8*I/d*ln(exp(d*x+c)-I)*a^
3-3/8*I/d*ln(exp(d*x+c)-I)*a^2*b-9/8*I/d*ln(exp(d*x+c)-I)*a*b^2+15/8*I/d*ln
(exp(d*x+c)-I)*b^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(97) = 194$.
time = 0.48, size = 489, normalized size = 4.75

$$\frac{1}{4}\,b^{3}\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{2\,e^{(-dx-c)}}{d}+\frac{17\,e^{(-2\,dx-2\,c)}+13\,e^{(-4\,dx-4\,c)}+7\,e^{(-6\,dx-6\,c)}-7\,e^{(-8\,dx-8\,c)}+2}{d(e^{(-dx-c)}+4\,e^{(-3\,dx-3\,c)}+6\,e^{(-5\,dx-5\,c)}+4\,e^{(-7\,dx-7\,c)}+e^{(-9\,dx-9\,c)})}\right)-\frac{3}{4}\,ab^{2}\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5\,e^{(-dx-c)}-3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)-\frac{1}{4}\,a^{3}\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)-\frac{3}{4}\,a^{2}b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7\,e^{(-3\,dx-3\,c)}+7\,e^{(-5\,dx-5\,c)}-e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*(15*arctan(e^(-d*x - c))/d - 2*e^(-d*x - c)/d + (17*e^(-2*d*x - 2*c
) + 13*e^(-4*d*x - 4*c) + 7*e^(-6*d*x - 6*c) - 7*e^(-8*d*x - 8*c) + 2)/(d*(
```

```
e^(-d*x - c) + 4*e^(-3*d*x - 3*c) + 6*e^(-5*d*x - 5*c) + 4*e^(-7*d*x - 7*c)
 + e^(-9*d*x - 9*c)))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x -
 c) - 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e
^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c
) + 1))) - 1/4*a^3*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d
*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c
) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 3/4
*a^2*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(
-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*
c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2245 vs. $2(97) = 194$.
time = 0.43, size = 2245, normalized size = 21.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*cosh(d*x + c)^10 + 20*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 2*b^3*
sinh(d*x + c)^10 + 3*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 3*(3
0*b^3*cosh(d*x + c)^2 + a^3 + a^2*b - 5*a*b^2 + 5*b^3)*sinh(d*x + c)^8 + 24
*(10*b^3*cosh(d*x + c)^3 + (a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c))*s
inh(d*x + c)^7 + (11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + (4
20*b^3*cosh(d*x + c)^4 + 11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3 + 84*(a^3 + a^
2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 6*(84*b^3*cosh(d*
x + c)^5 + 28*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (11*a^3 - 2
1*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (11*a^3 - 21*a^
2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + (420*b^3*cosh(d*x + c)^6 + 210*(a^
3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^4 - 11*a^3 + 21*a^2*b - 9*a*b^2
- 5*b^3 + 15*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 4*(60*b^3*cosh(d*x + c)^7 + 42*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*c
osh(d*x + c)^5 + 5*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^3 -
(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*b^
3 - 3*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 3*(30*b^3*cosh(d*x
+ c)^8 + 28*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 5*(11*a^3 - 2
1*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^4 - a^3 - a^2*b + 5*a*b^2 - 5*b^3
- 2*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 3*((a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^9 + 9*(a^3 + a^2*b + 3*a
*b^2 - 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + (a^3 + a^2*b + 3*a*b^2 - 5*b^
3)*sinh(d*x + c)^9 + 4*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 4*
(a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 9*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^7 + 28*(3*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x +
 c)^3 + (a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*
```

```
(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 6*(21*(a^3 + a^2*b + 3*a*
b^2 - 5*b^3)*cosh(d*x + c)^4 + a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 14*(a^3 + a^
2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(a^3 + a^2*
b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 70*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*c
osh(d*x + c)^3 + 15*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^4 + 4*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + 4*(21*(a^3 +
a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^6 + 35*(a^3 + a^2*b + 3*a*b^2 - 5*b^
3)*cosh(d*x + c)^4 + a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 15*(a^3 + a^2*b + 3*a*
b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(3*(a^3 + a^2*b + 3*a*b^
2 - 5*b^3)*cosh(d*x + c)^7 + 7*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c
)^5 + 5*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b + 3*
a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (a^3 + a^2*b + 3*a*b^2 - 5*
b^3)*cosh(d*x + c) + (9*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^8 + 2
8*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^6 + 30*(a^3 + a^2*b + 3*a*b
^2 - 5*b^3)*cosh(d*x + c)^4 + a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 12*(a^3 + a^2
*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c)
+ sinh(d*x + c)) + 2*(10*b^3*cosh(d*x + c)^9 + 12*(a^3 + a^2*b - 5*a*b^2 +
5*b^3)*cosh(d*x + c)^7 + 3*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x +
 c)^5 - 2*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^3 - 3*(a^3 +
a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^9 +
 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 4*d*cosh(d*x + c)^
7 + 4*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x + c)^3 +
 d*cosh(d*x + c))*sinh(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cosh(d*x
+ c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x + c)^
5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*d*cosh(d
*x + c)^3 + 4*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x
+ c)^2 + d)*sinh(d*x + c)^3 + 12*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5
 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c)
 + (9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 12*
d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(97) = 194$.
time = 0.46, size = 301, normalized size = 2.92

$$\frac{8\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+3\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^3+a^2b+3\,ab^2-5\,b^3\right)+\frac{4\left(3\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+3\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-15\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+9\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+20\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)-12\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-36\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+28\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(8*b^3*(e^(d*x + c) - e^(-d*x - c)) + 3*(pi + 2*arctan(1/2*(e^(2*d*x +
 2*c) - 1)*e^(-d*x - c)))*(a^3 + a^2*b + 3*a*b^2 - 5*b^3) + 4*(3*a^3*(e^(d*
x + c) - e^(-d*x - c))^3 + 3*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 - 15*a*b^
2*(e^(d*x + c) - e^(-d*x - c))^3 + 9*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 2
0*a^3*(e^(d*x + c) - e^(-d*x - c)) - 12*a^2*b*(e^(d*x + c) - e^(-d*x - c))
- 36*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 28*b^3*(e^(d*x + c) - e^(-d*x - c
)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B]

time = 0.19, size = 430, normalized size = 4.17

$$\frac{3\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\left(a^3\,\sqrt{d^2}-5\,b^3\,\sqrt{d^2}+3\,a\,b^2\,\sqrt{d^2}+a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{a^6+2\,a^5\,b+7\,a^4\,b^2-4\,a^3\,b^3-a^2\,b^4-30\,a\,b^5+25\,b^6}}\right)\,\sqrt{a^6+2\,a^5\,b+7\,a^4\,b^2-4\,a^3\,b^3-a^2\,b^4-30\,a\,b^5+25\,b^6}}{4\,\sqrt{d^2}}+\frac{b^3\,{\mathrm{e}}^{c+d\,x}}{2\,d}-\frac{b^3\,{\mathrm{e}}^{-c-d\,x}}{2\,d}+\frac{3\,{\mathrm{e}}^{c+d\,x}\,\left(a^3+a^2\,b-5\,a\,b^2+3\,b^3\right)}{4\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{{\mathrm{e}}^{c+d\,x}\,\left(a^3-15\,a^2\,b+27\,a\,b^2-13\,b^3\right)}{2\,d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}-\frac{6\,{\mathrm{e}}^{c+d\,x}\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)}+\frac{4\,{\mathrm{e}}^{c+d\,x}\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{d\,\left(4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^5,x)
```

```
[Out] (3*atan((exp(d*x)*exp(c)*(a^3*(d^2)^(1/2) - 5*b^3*(d^2)^(1/2) + 3*a*b^2*(d^
2)^(1/2) + a^2*b*(d^2)^(1/2)))/(d*(2*a^5*b - 30*a*b^5 + a^6 + 25*b^6 - a^2*
b^4 - 4*a^3*b^3 + 7*a^4*b^2)^(1/2)))*(2*a^5*b - 30*a*b^5 + a^6 + 25*b^6 - a
^2*b^4 - 4*a^3*b^3 + 7*a^4*b^2)^(1/2))/(4*(d^2)^(1/2)) + (b^3*exp(c + d*x))
/(2*d) - (b^3*exp(- c - d*x))/(2*d) + (3*exp(c + d*x)*(a^2*b - 5*a*b^2 + a^
3 + 3*b^3))/(4*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(27*a*b^2 - 15*a^2
*b + a^3 - 13*b^3))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (6*
exp(c + d*x)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(d*(3*exp(2*c + 2*d*x) + 3*ex
p(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*exp(c + d*x)*(3*a*b^2 - 3*a^2*
b + a^3 - b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6
*d*x) + exp(8*c + 8*d*x) + 1))
```

3.313 $\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=74

$$b^3x+\frac{(a^3-b^3)\tanh(c+dx)}{d}-\frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d}+\frac{(a-b)^3\tanh^5(c+dx)}{5d}$$

```
[Out] b^3*x+(a^3-b^3)*tanh(d*x+c)/d-1/3*(a-b)^2*(2*a+b)*tanh(d*x+c)^3/d+1/5*(a-b)
^3*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {3270, 398, 212}

$$\frac{(a^3-b^3)\tanh(c+dx)}{d}+\frac{(a-b)^3\tanh^5(c+dx)}{5d}-\frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] b^3*x + ((a^3 - b^3)*Tanh[c + d*x])/d - ((a - b)^2*(2*a + b)*Tanh[c + d*x]^
3)/(3*d) + ((a - b)^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(a^3-b^3-(a-b)^2(2a+b)x^2+(a-b)^3x^4+\frac{b^3}{1-x^2}\right)\,d}{d}$$

$$= \frac{(a^3-b^3)\tanh(c+dx)}{d} - \frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d} + \frac{(a}{}$$

$$= b^3x + \frac{(a^3-b^3)\tanh(c+dx)}{d} - \frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d}$$

Mathematica [**A**]
time = 0.55, size = 86, normalized size = 1.16

$$\frac{15b^3(c+dx)+(a-b)\left(8a^2+14ab+23b^2+\left(4a^2+7ab-11b^2\right)\operatorname{sech}^2(c+dx)+3(a-b)^2\operatorname{sech}^4(c+dx)\right)\tanh(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (15*b^3*(c + d*x) + (a - b)*(8*a^2 + 14*a*b + 23*b^2 + (4*a^2 + 7*a*b - 11*
b^2)*Sech[c + d*x]^2 + 3*(a - b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(15*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(70) = 140.
time = 1.75, size = 207, normalized size = 2.80

method	result
risch	$b^3x-\frac{2\left(45a\,b^2\mathrm{e}^{8dx+8c}-45b^3\mathrm{e}^{8dx+8c}+90a^2b\,\mathrm{e}^{6dx+6c}-90b^3\mathrm{e}^{6dx+6c}+80a^3\mathrm{e}^{4dx+4c}-30a^2b\,\mathrm{e}^{4dx+4c}+90a\,b^2\mathrm{e}^{4dx+4c}-140b^3\mathrm{e}^{4dx+4c}+4\right.}{15d\left(1+\mathrm{e}^{2dx+2c}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*x-2/15*(45*a*b^2*exp(8*d*x+8*c)-45*b^3*exp(8*d*x+8*c)+90*a^2*b*exp(6*d*
x+6*c)-90*b^3*exp(6*d*x+6*c)+80*a^3*exp(4*d*x+4*c)-30*a^2*b*exp(4*d*x+4*c)+
90*a*b^2*exp(4*d*x+4*c)-140*b^3*exp(4*d*x+4*c)+40*a^3*exp(2*d*x+2*c)+30*a^2
*b*exp(2*d*x+2*c)-70*b^3*exp(2*d*x+2*c)+8*a^3+6*a^2*b+9*a*b^2-23*b^3)/d/(1+
exp(2*d*x+2*c))^5
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 824 vs. 2(70) = 140.
time = 0.29, size = 824, normalized size = 11.14

$$\frac{1}{15}b^3\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)+\frac{16}{15}a^3\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{10e^{(-4dx-4c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{1}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)+\frac{4}{5}a^2b\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}-\frac{5e^{(-4dx-4c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{15e^{(-6dx-6c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{1}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)+\frac{6}{5}ab^2\left(\frac{10e^{(-4dx-4c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{5e^{(-8dx-8c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{1}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1))) + 16/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c
) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/5*a^2*b*(5*e^(-2*d*x - 2*c)/(
d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8
*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x
 - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 6/5*a*b^2*(10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x - 8*c)
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(70) = 140$.
time = 0.38, size = 530, normalized size = 7.16

$$\frac{(15\,b^3dx - 8\,a^3 - 6\,a^2b - 9\,ab^2 + 23\,b^3)\cosh\left(dx+c\right)^5 + 5\,(15\,b^3dx - 8\,a^3 - 6\,a^2b - 9\,ab^2 + 23\,b^3)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4 + (8\,a^3 + 6\,a^2b + 9\,ab^2 - 23\,b^3)\sinh\left(dx+c\right)^5 + 5\,(15\,b^3dx - 8\,a^3 - 6\,a^2b - 9\,ab^2 + 23\,b^3)\cosh\left(dx+c\right)^3 + 5\,\left(8\,a^3 + 6\,a^2b - 9\,ab^2 - 5\,b^3 + 2\,(8\,a^3 + 6\,a^2b + 9\,ab^2 - 23\,b^3)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)^3 + 5\,\left(2\,(15\,b^3dx - 8\,a^3 - 6\,a^2b - 9\,ab^2 + 23\,b^3)\cosh\left(dx+c\right)^3 + 3\,(15\,b^3dx - 8\,a^3 - 6\,a^2b - 9\,ab^2 + 23\,b^3)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2 + 10\,(15\,b^3dx - 8\,a^3 - 6\,a^2b - 9\,ab^2 + 23\,b^3)\cosh\left(dx+c\right) + 5\,\left((8\,a^3 + 6\,a^2b + 9\,ab^2 - 23\,b^3)\cosh\left(dx+c\right)^4 + 16\,a^3 - 24\,a^2b + 18\,ab^2 - 10\,b^3 + 3\,(8\,a^3 + 6\,a^2b - 9\,ab^2 - 5\,b^3)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)}{15\,\left(d\cosh\left(dx+c\right)^5 + 5\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4 + 5\,d\cosh\left(dx+c\right)^3 + 5\,\left(2\,d\cosh\left(dx+c\right)^3 + 3\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2 + 10\,d\cosh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*((15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh(d*x + c)^5 + 5
*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh(d*x + c)*sinh(d*x +
 c)^4 + (8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)*sinh(d*x + c)^5 + 5*(15*b^3*d*
x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh(d*x + c)^3 + 5*(8*a^3 + 6*a^2*
b - 9*a*b^2 - 5*b^3 + 2*(8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 5*(2*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)
*cosh(d*x + c)^3 + 3*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh
(d*x + c))*sinh(d*x + c)^2 + 10*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 2
3*b^3)*cosh(d*x + c) + 5*((8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)*cosh(d*x + c
)^4 + 16*a^3 - 24*a^2*b + 18*a*b^2 - 10*b^3 + 3*(8*a^3 + 6*a^2*b - 9*a*b^2
```

```
- 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x
+ c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*c
osh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(70) = 140.
time = 0.48, size = 213, normalized size = 2.88

$$\frac{15\,(dx+c)b^3-\frac{2\,\left(45\,ab^2e^{(8\,dx+8\,c)}-45\,b^3e^{(8\,dx+8\,c)}+90\,a^2be^{(6\,dx+6\,c)}-90\,b^3e^{(6\,dx+6\,c)}+80\,a^3e^{(4\,dx+4\,c)}-30\,a^2be^{(4\,dx+4\,c)}+90\,ab^2e^{(4\,dx+4\,c)}-140\,b^3e^{(4\,dx+4\,c)}+40\,a^3e^{(2\,dx+2\,c)}+30\,a^2be^{(2\,dx+2\,c)}-70\,b^3e^{(2\,dx+2\,c)}+8\,a^3+6\,a^2b+9\,ab^2-23\,b^3\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)*b^3 - 2*(45*a*b^2*e^(8*d*x + 8*c) - 45*b^3*e^(8*d*x + 8*
c) + 90*a^2*b*e^(6*d*x + 6*c) - 90*b^3*e^(6*d*x + 6*c) + 80*a^3*e^(4*d*x +
4*c) - 30*a^2*b*e^(4*d*x + 4*c) + 90*a*b^2*e^(4*d*x + 4*c) - 140*b^3*e^(4*d
*x + 4*c) + 40*a^3*e^(2*d*x + 2*c) + 30*a^2*b*e^(2*d*x + 2*c) - 70*b^3*e^(2
*d*x + 2*c) + 8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)/(e^(2*d*x + 2*c) + 1)^5)/
d
```

Mupad [**B**]
time = 0.84, size = 563, normalized size = 7.61

$$b^3\,x-\frac{\frac{2\,\left(8\,a^3-12\,a^2\,b+9\,a\,b^2-5\,b^3\right)}{15\,d}-\frac{12\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b^2-a^2\,b\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a\,b^2-b^3\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}+\frac{\frac{6\,\left(a\,b^2-a^2\,b\right)}{5\,d}-\frac{6\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b^2-b^3\right)}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}+\frac{\frac{6\,\left(a\,b^2-a^2\,b\right)}{5\,d}+\frac{18\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a\,b^2-a^2\,b\right)}{5\,d}-\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(8\,a^3-12\,a^2\,b+9\,a\,b^2-5\,b^3\right)}{5\,d}-\frac{6\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a\,b^2-b^3\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{6\,\left(a\,b^2-b^3\right)}{5\,d}-\frac{24\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b^2-a^2\,b\right)}{5\,d}-\frac{24\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a\,b^2-a^2\,b\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(8\,a^3-12\,a^2\,b+9\,a\,b^2-5\,b^3\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(a\,b^2-b^3\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{6\,\left(a\,b^2-b^3\right)}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^6,x)
```

```
[Out] b^3*x - ((2*(9*a*b^2 - 12*a^2*b + 8*a^3 - 5*b^3))/(15*d) - (12*exp(2*c + 2*
d*x)*(a*b^2 - a^2*b))/(5*d) + (6*exp(4*c + 4*d*x)*(a*b^2 - b^3))/(5*d))/(3*
exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((6*(a*b^2
- a^2*b))/(5*d) - (6*exp(2*c + 2*d*x)*(a*b^2 - b^3))/(5*d))/(2*exp(2*c + 2*
d*x) + exp(4*c + 4*d*x) + 1) + ((6*(a*b^2 - a^2*b))/(5*d) + (18*exp(4*c + 4
*d*x)*(a*b^2 - a^2*b))/(5*d) - (2*exp(2*c + 2*d*x)*(9*a*b^2 - 12*a^2*b + 8*
a^3 - 5*b^3))/(5*d) - (6*exp(6*c + 6*d*x)*(a*b^2 - b^3))/(5*d))/(4*exp(2*c
+ 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)
```

```
- ((6*(a*b^2 - b^3))/(5*d) - (24*exp(2*c + 2*d*x)*(a*b^2 - a^2*b))/(5*d) -
(24*exp(6*c + 6*d*x)*(a*b^2 - a^2*b))/(5*d) + (4*exp(4*c + 4*d*x)*(9*a*b^2
- 12*a^2*b + 8*a^3 - 5*b^3))/(5*d) + (6*exp(8*c + 8*d*x)*(a*b^2 - b^3))/(5*
d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp
(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (6*(a*b^2 - b^3))/(5*d*(exp(2*c +
 2*d*x) + 1))
```

3.314 $\int \mathbf{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=154

$$\frac{(a+b)\left(5a^2-2ab+5b^2\right)\text{ArcTan}(\sinh(c+dx))}{16d}+\frac{(a-b)\left(15a^2+14ab+15b^2\right)\text{sech}(c+dx)\tanh(c+dx)}{48d}+$$

```
[Out] 1/16*(a+b)*(5*a^2-2*a*b+5*b^2)*arctan(sinh(d*x+c))/d+1/48*(a-b)*(15*a^2+14*
a*b+15*b^2)*sech(d*x+c)*tanh(d*x+c)/d+5/24*(a^2-b^2)*sech(d*x+c)^3*(a+b*sin
h(d*x+c)^2)*tanh(d*x+c)/d+1/6*(a-b)*sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2*tan
h(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = $\{3269, 424, 540, 393, 209\}$

$$\frac{(a+b)\left(5a^2-2ab+5b^2\right)\text{ArcTan}(\sinh(c+dx))}{16d}+\frac{(a-b)\left(15a^2+14ab+15b^2\right)\tanh(c+dx)\text{sech}(c+dx)}{48d}+\frac{5\left(a^2-b^2\right)\tanh(c+dx)\text{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)}{24d}+\frac{(a-b)\tanh(c+dx)\text{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)*(5*a^2 - 2*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((a - b)*(
15*a^2 + 14*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) + (5*(a^2 - b
^2)*Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) + ((a - b
)*Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
```

```
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{(1+x^2)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-b)\operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d} + \frac{\text{S}}{}$$

$$= \frac{5(a^2-b^2)\operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\tanh(c+dx)}{24d} +$$

$$= \frac{(a-b)\left(15a^2+14ab+15b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{48d} + \frac{5(a}{}$$

$$= \frac{(a+b)\left(5a^2-2ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{(a-b)\,(15a^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 13.52, size = 1192, normalized size = 7.74

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c + d*x]^5*(-117228825*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]] - 10926562
5*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 - 274542345*a^2*b*Arc
Tanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 - 17069535*a^3*ArcTanh[Sqrt[-S
inh[c + d*x]^2]]*Sinh[c + d*x]^4 - 260465625*a^2*b*ArcTanh[Sqrt[-Sinh[c + d
*x]^2]]*Sinh[c + d*x]^4 - 215549775*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*S
inh[c + d*x]^4 + 142065*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6
 - 41427855*a^2*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 2071732
95*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 58009455*b^3*Arc
Tanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 210735*a^2*b*ArcTanh[Sqrt[-S
inh[c + d*x]^2]]*Sinh[c + d*x]^8 - 33756345*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*
x]^2]]*Sinh[c + d*x]^8 - 56109375*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[
c + d*x]^8 - 174825*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10
- 9261945*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10 - 48825*b^3*
ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^12 + 117228825*a^3*Sqrt[-Sinh
[c + d*x]^2] + 4093425*a^3*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 1689515
10*a^2*b*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 215549775*a*b^2*Sinh[c +
d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 9514449*a^2*b*Sinh[c + d*x]^6*Sqrt[-Sinh[c
+ d*x]^2] + 135323370*a*b^2*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 580094
55*b^3*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 7808535*a*b^2*Sinh[c + d*x]
^8*Sqrt[-Sinh[c + d*x]^2] + 36772890*b^3*Sinh[c + d*x]^8*Sqrt[-Sinh[c + d*x
]^2] + 2160711*b^3*Sinh[c + d*x]^10*Sqrt[-Sinh[c + d*x]^2] - 70189350*a^3*(
-Sinh[c + d*x]^2)^(3/2) - 274542345*a^2*b*(-Sinh[c + d*x]^2)^(3/2) + 1024*a
^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[
c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2) + 3072*a^2*b*Hypergeom
etricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*
Sinh[c + d*x]^8*(-Sinh[c + d*x]^2)^(3/2) + 3072*a*b^2*HypergeometricPFQ[{3/
2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x
]^10*(-Sinh[c + d*x]^2)^(3/2) + 1024*b^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2
, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12*(-Sinh[c
 + d*x]^2)^(3/2) + 1536*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1
, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2)*(a + b*
Sinh[c + d*x]^2)^2*(9*a + 7*b*Sinh[c + d*x]^2) + 256*HypergeometricPFQ[{3/2
, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c
+ d*x]^2)^(3/2)*(295*a^3 + 741*a^2*b*Sinh[c + d*x]^2 + 621*a*b^2*Sinh[c + d
*x]^4 + 175*b^3*Sinh[c + d*x]^6)))/(725760*d*Sqrt[-Sinh[c + d*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 1.78, size = 495, normalized size = 3.21

method	result
risch	$\mathrm{e}^{dx+c}(15a^3\mathrm{e}^{10dx+10c}+9a^2b\,\mathrm{e}^{10dx+10c}+9a\,b^2\mathrm{e}^{10dx+10c}-33b^3\mathrm{e}^{10dx+10c}+85a^3\mathrm{e}^{8dx+8c}+51a^2b\,\mathrm{e}^{8dx+8c}-141a\,b^2\mathrm{e}^{8dx+8c}+5b^3\mathrm{e}^{8dx+8c}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*exp(d*x+c)*(15*a^3*exp(10*d*x+10*c)+9*a^2*b*exp(10*d*x+10*c)+9*a*b^2*e
xp(10*d*x+10*c)-33*b^3*exp(10*d*x+10*c)+85*a^3*exp(8*d*x+8*c)+51*a^2*b*exp(
8*d*x+8*c)-141*a*b^2*exp(8*d*x+8*c)+5*b^3*exp(8*d*x+8*c)+198*a^3*exp(6*d*x+
6*c)-342*a^2*b*exp(6*d*x+6*c)+234*a*b^2*exp(6*d*x+6*c)-90*b^3*exp(6*d*x+6*c
)-198*a^3*exp(4*d*x+4*c)+342*a^2*b*exp(4*d*x+4*c)-234*a*b^2*exp(4*d*x+4*c)+
90*b^3*exp(4*d*x+4*c)-85*a^3*exp(2*d*x+2*c)-51*a^2*b*exp(2*d*x+2*c)+141*a*b
^2*exp(2*d*x+2*c)-5*b^3*exp(2*d*x+2*c)-15*a^3-9*a^2*b-9*a*b^2+33*b^3)/d/(1+
exp(2*d*x+2*c))^6+5/16*I/d*ln(exp(d*x+c)+I)*a^3+3/16*I/d*ln(exp(d*x+c)+I)*a
^2*b+3/16*I/d*ln(exp(d*x+c)+I)*a*b^2+5/16*I/d*ln(exp(d*x+c)+I)*b^3-5/16*I/d
*ln(exp(d*x+c)-I)*a^3-3/16*I/d*ln(exp(d*x+c)-I)*a^2*b-3/16*I/d*ln(exp(d*x+c
)-I)*a*b^2-5/16*I/d*ln(exp(d*x+c)-I)*b^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(146) = 292$.
time = 0.50, size = 646, normalized size = 4.19

$$-\frac{1}{24}\,b^{3}\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{33\,e^{(-dx-c)}-5\,e^{(-3\,dx-3\,c)}+90\,e^{(-5\,dx-5\,c)}-90\,e^{(-7\,dx-7\,c)}+5\,e^{(-9\,dx-9\,c)}-33\,e^{(-11\,dx-11\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)-\frac{1}{24}\,a^{3}\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15\,e^{(-dx-c)}+85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}-198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9\,dx-9\,c)}-15\,e^{(-11\,dx-11\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)-\frac{1}{8}\,a^{2}b\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+17\,e^{(-3\,dx-3\,c)}-114\,e^{(-5\,dx-5\,c)}+114\,e^{(-7\,dx-7\,c)}-17\,e^{(-9\,dx-9\,c)}-3\,e^{(-11\,dx-11\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)-\frac{1}{8}\,ab^{2}\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}-47\,e^{(-3\,dx-3\,c)}+78\,e^{(-5\,dx-5\,c)}-78\,e^{(-7\,dx-7\,c)}+47\,e^{(-9\,dx-9\,c)}-3\,e^{(-11\,dx-11\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*b^3*(15*arctan(e^(-d*x - c))/d + (33*e^(-d*x - c) - 5*e^(-3*d*x - 3*c
) + 90*e^(-5*d*x - 5*c) - 90*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) - 33*e^(
-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)
+ 1))) - 1/24*a^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*
d*x - 3*c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9
*c) - 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) +
 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*
d*x - 12*c) + 1))) - 1/8*a^2*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c)
+ 17*e^(-3*d*x - 3*c) - 114*e^(-5*d*x - 5*c) + 114*e^(-7*d*x - 7*c) - 17*e^
(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*
x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c)
 + e^(-12*d*x - 12*c) + 1))) - 1/8*a*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(
-d*x - c) - 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78*e^(-7*d*x - 7*c)
 + 47*e^(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*
e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x
 - 10*c) + e^(-12*d*x - 12*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3675 vs. $2(146) = 292$.
time = 0.44, size = 3675, normalized size = 23.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)^11 + 33*(5*a^3 +
 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(5*a^3 + 3*
a^2*b + 3*a*b^2 - 11*b^3)*sinh(d*x + c)^11 + (85*a^3 + 51*a^2*b - 141*a*b^2
 + 5*b^3)*cosh(d*x + c)^9 + (85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3 + 165*(5
*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 9*(55
*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)^3 + (85*a^3 + 51*a^2*b
- 141*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 + 18*(11*a^3 - 19*a^2*b
 + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 18*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 -
11*b^3)*cosh(d*x + c)^4 + 11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3 + 2*(85*a^3
+ 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 42*(33*(
5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)^5 + 2*(85*a^3 + 51*a^2*b
- 141*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*
b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 18*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*
b^3)*cosh(d*x + c)^5 + 18*(77*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x
 + c)^6 + 7*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)^4 - 11*a^
3 + 19*a^2*b - 13*a*b^2 + 5*b^3 + 21*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 18*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*
b^3)*cosh(d*x + c)^7 + 7*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x +
 c)^5 + 35*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^3 - 5*(11*a
^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - (85*a^3
+ 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (495*(5*a^3 + 3*a^2*b + 3
*a*b^2 - 11*b^3)*cosh(d*x + c)^8 + 84*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^
3)*cosh(d*x + c)^6 + 630*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x +
c)^4 - 85*a^3 - 51*a^2*b + 141*a*b^2 - 5*b^3 - 180*(11*a^3 - 19*a^2*b + 13*
a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 3*(55*(5*a^3 + 3*a^2*b +
3*a*b^2 - 11*b^3)*cosh(d*x + c)^9 + 12*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b
^3)*cosh(d*x + c)^7 + 126*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x +
 c)^5 - 60*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^3 - (85*a^3
 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((5*a^3
 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^12 + 12*(5*a^3 + 3*a^2*b + 3*a*
b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a^3 + 3*a^2*b + 3*a*b^2 +
5*b^3)*sinh(d*x + c)^12 + 6*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x +
c)^10 + 6*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 11*(5*a^3 + 3*a^2*b + 3*a*b^
2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(5*a^3 + 3*a^2*b + 3*
a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh
(d*x + c))*sinh(d*x + c)^9 + 15*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*
x + c)^8 + 15*(33*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 5*a
^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 18*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^8 + 24*(33*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*
cosh(d*x + c)^5 + 30*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^3 +
5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(
```

```
5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 4*(231*(5*a^3 + 3*a^2*
b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 315*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b
^3)*cosh(d*x + c)^4 + 25*a^3 + 15*a^2*b + 15*a*b^2 + 25*b^3 + 105*(5*a^3 +
3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(33*(5*a^3
 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 63*(5*a^3 + 3*a^2*b + 3*a*b
^2 + 5*b^3)*cosh(d*x + c)^5 + 35*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d
*x + c)^3 + 5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 15*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 15*(33*(5*
a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 84*(5*a^3 + 3*a^2*b + 3*
a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 70*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cos
h(d*x + c)^4 + 5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 20*(5*a^3 + 3*a^2*b + 3*
a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 20*(11*(5*a^3 + 3*a^2*b +
 3*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 36*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*
cosh(d*x + c)^7 + 42*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^5 +
20*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(5*a^3 + 3*a^2*b
 + 3*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*a^3 + 3*a^2*b + 3*a*
b^2 + 5*b^3 + 6*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 6*(11
*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 45*(5*a^3 + 3*a^2*b
 + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 70*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3
)*cosh(d*x + c)^6 + 50*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^4
+ 5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 15*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3
)*cosh(d*x + c)^11 + 5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^9
+ 10*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 10*(5*a^3 + 3*a^
2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b
^3)*cosh(d*x + c)^3 + (5*a^3 + 3*a^2*b + 3*a*b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(146) = 292.
time = 0.46, size = 383, normalized size = 2.49

$$\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(5\,a^3+3\,a^2b+3\,ab^2+5\,b^3\right)+\frac{4\left(15\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+9\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+9\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5-33\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+160\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+96\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-96\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-160\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+528\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)-144\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-144\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)-240\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(5*a^3 + 3*
a^2*b + 3*a*b^2 + 5*b^3) + 4*(15*a^3*(e^(d*x + c) - e^(-d*x - c))^5 + 9*a^2
*b*(e^(d*x + c) - e^(-d*x - c))^5 + 9*a*b^2*(e^(d*x + c) - e^(-d*x - c))^5
- 33*b^3*(e^(d*x + c) - e^(-d*x - c))^5 + 160*a^3*(e^(d*x + c) - e^(-d*x -
c))^3 + 96*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 - 96*a*b^2*(e^(d*x + c) - e
^(-d*x - c))^3 - 160*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 528*a^3*(e^(d*x +
 c) - e^(-d*x - c)) - 144*a^2*b*(e^(d*x + c) - e^(-d*x - c)) - 144*a*b^2*(e
^(d*x + c) - e^(-d*x - c)) - 240*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x
 + c) - e^(-d*x - c))^2 + 4)^3)/d
```

Mupad [**B**]
time = 0.20, size = 601, normalized size = 3.90

$$\frac{\mathrm{atan}\left(\frac{e^{d\,x}\,e^c\,\left(5\,a^3\,\sqrt{d^2}+5\,b^3\,\sqrt{d^2}+3\,a\,b^2\,\sqrt{d^2}+3\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{25\,a^6+30\,a^5\,b+39\,a^4\,b^2+68\,a^3\,b^3+39\,a^2\,b^4+30\,a\,b^5+25\,b^6}}\right)\,\sqrt{25\,a^6+30\,a^5\,b+39\,a^4\,b^2+68\,a^3\,b^3+39\,a^2\,b^4+30\,a\,b^5+25\,b^6}}{8\,\sqrt{d^2}}-\frac{6\,e^{c+d\,x}\,\left(3\,a^3-11\,a^2\,b+13\,a\,b^2-5\,b^3\right)}{d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,\left(a^3-57\,a^2\,b+111\,a\,b^2-55\,b^3\right)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,\left(5\,a^3+3\,a^2\,b+3\,a\,b^2-11\,b^3\right)}{8\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,\left(5\,a^3+3\,a^2\,b-93\,a\,b^2+85\,b^3\right)}{12\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}+\frac{80\,e^{c+d\,x}\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{3\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}-\frac{32\,e^{c+d\,x}\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{3\,d\,\left(6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^7,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(5*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 3*a*b^2*(d^
2)^(1/2) + 3*a^2*b*(d^2)^(1/2)))/(d*(30*a*b^5 + 30*a^5*b + 25*a^6 + 25*b^6
+ 39*a^2*b^4 + 68*a^3*b^3 + 39*a^4*b^2)^(1/2)))*(30*a*b^5 + 30*a^5*b + 25*a
^6 + 25*b^6 + 39*a^2*b^4 + 68*a^3*b^3 + 39*a^4*b^2)^(1/2))/(8*(d^2)^(1/2))
- (6*exp(c + d*x)*(13*a*b^2 - 11*a^2*b + 3*a^3 - 5*b^3))/(d*(4*exp(2*c + 2*
d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (
exp(c + d*x)*(111*a*b^2 - 57*a^2*b + a^3 - 55*b^3))/(3*d*(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*(3*a*b^2 +
3*a^2*b + 5*a^3 - 11*b^3))/(8*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(3*
a^2*b - 93*a*b^2 + 5*a^3 + 85*b^3))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4
*d*x) + 1)) + (80*exp(c + d*x)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(3*d*(5*exp
(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d
*x) + exp(10*c + 10*d*x) + 1)) - (32*exp(c + d*x)*(3*a*b^2 - 3*a^2*b + a^3
- b^3))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x
) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1))
```

3.315 $\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=80

$$\frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-b)\tanh^3(c+dx)}{d}+\frac{3a(a-b)^2\tanh^5(c+dx)}{5d}-\frac{(a-b)^3\tanh^7(c+dx)}{7d}$$

```
[Out] a^3*tanh(d*x+c)/d-a^2*(a-b)*tanh(d*x+c)^3/d+3/5*a*(a-b)^2*tanh(d*x+c)^5/d-1
/7*(a-b)^3*tanh(d*x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3270, 200}

$$\frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-b)\tanh^3(c+dx)}{d}-\frac{(a-b)^3\tanh^7(c+dx)}{7d}+\frac{3a(a-b)^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tanh[c + d*x])/d - (a^2*(a - b)*Tanh[c + d*x]^3)/d + (3*a*(a - b)^2*Ta
nh[c + d*x]^5)/(5*d) - ((a - b)^3*Tanh[c + d*x]^7)/(7*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int\left(a-(a-b)x^2\right)^3\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int\left(a^3-3a^2(a-b)x^2+3a(a-b)^2x^4-(a-b)^3x^6\right)\,dx,\right.}{d}\\ &= \frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-b)\tanh^3(c+dx)}{d}+\frac{3a(a-b)^2\tanh}{5d}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 163 vs. 2(80) = 160.
time = 0.60, size = 163, normalized size = 2.04

$$\frac{(512a^3-304a^2b+192ab^2-50b^3+(464a^3+232a^2b-246ab^2+75b^3)\cosh(2(c+dx))+2(64a^3+32a^2b+24ab^2-15b^3)\cosh(4(c+dx))+16a^3\cosh(6(c+dx))+8a^2b\cosh(6(c+dx))+6ab^2\cosh(6(c+dx))+5b^3\cosh(6(c+dx)))\,\mathrm{sech}^6(c+dx)\tanh(c+dx)}{1120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((512*a^3 - 304*a^2*b + 192*a*b^2 - 50*b^3 + (464*a^3 + 232*a^2*b - 246*a*b
^2 + 75*b^3)*Cosh[2*(c + d*x)] + 2*(64*a^3 + 32*a^2*b + 24*a*b^2 - 15*b^3)*
Cosh[4*(c + d*x)] + 16*a^3*Cosh[6*(c + d*x)] + 8*a^2*b*Cosh[6*(c + d*x)] +
6*a*b^2*Cosh[6*(c + d*x)] + 5*b^3*Cosh[6*(c + d*x)])*Sech[c + d*x]^6*Tanh[c
 + d*x])/(1120*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(76) = 152.
time = 1.76, size = 261, normalized size = 3.26

method	result
risch	$-\frac{2(35b^3e^{12dx+12c}+210a\,b^2e^{10dx+10c}+560a^2b\,e^{8dx+8c}-210a\,b^2e^{8dx+8c}+175b^3e^{8dx+8c}+560a^3e^{6dx+6c}-280a^2b\,e^{6dx+6c}+420a\,b^2e^{6dx}}{35d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*(35*b^3*exp(12*d*x+12*c)+210*a*b^2*exp(10*d*x+10*c)+560*a^2*b*exp(8*d
*x+8*c)-210*a*b^2*exp(8*d*x+8*c)+175*b^3*exp(8*d*x+8*c)+560*a^3*exp(6*d*x+6
*c)-280*a^2*b*exp(6*d*x+6*c)+420*a*b^2*exp(6*d*x+6*c)+336*a^3*exp(4*d*x+4*c
)+168*a^2*b*exp(4*d*x+4*c)-84*a*b^2*exp(4*d*x+4*c)+105*b^3*exp(4*d*x+4*c)+1
12*a^3*exp(2*d*x+2*c)+56*a^2*b*exp(2*d*x+2*c)+42*a*b^2*exp(2*d*x+2*c)+16*a^
3+8*a^2*b+6*a*b^2+5*b^3)/d/(1+exp(2*d*x+2*c))^7
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1754 vs. 2(76) = 152.
time = 0.30, size = 1754, normalized size = 21.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 32/35*a^3*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(
-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-
```

```
2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8
*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*
e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d
*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4
*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 1
0*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 16/35*a^2*b*(7*e^
(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x
- 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c)
 + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) +
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10
*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-6*d
*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) + 1)) + 70*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^
(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
- 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d
*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c)
 + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)))
 + 12/35*a*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x -
4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) +
7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d*x - 4*c)/(d*(7
*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*
x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c
) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(
-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x - 8*c)/(d*(7*e^(-
2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8
*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 3
5*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12
*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(
-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x -
 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/7*b^3*(21*e^(
-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x -
 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c)
+ e^(-14*d*x - 14*c) + 1)) + 35*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x - 2*c) + 2
1*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*
d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 7*e^(-12*d*
x - 12*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 3
5*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12
*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(76) = 152$.
time = 0.54, size = 814, normalized size = 10.18

$$-\frac{4\,\left(\left(8\,a^{3}+4\,a^{2}b+3\,ab^{2}+20\,b^{3}\right)\cosh\left(dx+c\right)^{6}-6\,\left(8\,a^{3}+4\,a^{2}b+3\,ab^{2}-15\,b^{3}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+\left(8\,a^{3}+4\,a^{2}b+3\,ab^{2}+20\,b^{3}\right)\sinh\left(dx+c\right)^{6}+14\,\left(4\,a^{3}+2\,a^{2}b+9\,ab^{2}\right)\cosh\left(dx+c\right)^{4}+\left(56\,a^{3}+28\,a^{2}b+126\,ab^{2}+15\,\left(8\,a^{3}+4\,a^{2}b+3\,ab^{2}+20\,b^{3}\right)\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)^{4}-4\,\left(5\,\left(8\,a^{3}+4\,a^{2}b+3\,ab^{2}-15\,b^{3}\right)\cosh\left(dx+c\right)^{3}+28\,\left(2\,a^{3}+a^{2}b-3\,ab^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+280\,a^{3}-140\,a^{2}b+210\,ab^{2}+7\,\left(24\,a^{3}+52\,a^{2}b-21\,ab^{2}+20\,b^{3}\right)\cosh\left(dx+c\right)^{2}+\left(15\,\left(8\,a^{3}+4\,a^{2}b+3\,ab^{2}+20\,b^{3}\right)\cosh\left(dx+c\right)^{4}+168\,a^{3}+364\,a^{2}b-147\,ab^{2}+140\,b^{3}+84\,\left(4\,a^{3}+2\,a^{2}b+9\,ab^{2}\right)\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)^{2}-2\,\left(3\,\left(8\,a^{3}+4\,a^{2}b+3\,ab^{2}-15\,b^{3}\right)\cosh\left(dx+c\right)^{5}+56\,\left(2\,a^{3}+a^{2}b-3\,ab^{2}\right)\cosh\left(dx+c\right)^{3}+7\,\left(24\,a^{3}-28\,a^{2}b+9\,ab^{2}-5\,b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)}{35\,\left(d\cosh\left(dx+c\right)^{8}+8\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{7}+d\sinh\left(dx+c\right)^{8}+8\,d\cosh\left(dx+c\right)^{6}+4\,\left(7\,d\cosh\left(dx+c\right)^{2}+2\,d\right)\sinh\left(dx+c\right)^{6}+4\,\left(14\,d\cosh\left(dx+c\right)^{3}+9\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{5}+28\,d\cosh\left(dx+c\right)^{4}+2\,\left(35\,d\cosh\left(dx+c\right)^{4}+60\,d\cosh\left(dx+c\right)^{2}+14\,d\right)\sinh\left(dx+c\right)^{4}+8\,\left(7\,d\cosh\left(dx+c\right)^{5}+15\,d\cosh\left(dx+c\right)^{3}+7\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+56\,d\cosh\left(dx+c\right)^{2}+4\,\left(7\,d\cosh\left(dx+c\right)^{6}+30\,d\cosh\left(dx+c\right)^{4}+42\,d\cosh\left(dx+c\right)^{2}+14\,d\right)\sinh\left(dx+c\right)^{2}+4\,\left(2\,d\cosh\left(dx+c\right)^{7}+9\,d\cosh\left(dx+c\right)^{5}+14\,d\cosh\left(dx+c\right)^{3}+7\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+35\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/35*((8*a^3 + 4*a^2*b + 3*a*b^2 + 20*b^3)*cosh(d*x + c)^6 - 6*(8*a^3 + 4*
a^2*b + 3*a*b^2 - 15*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (8*a^3 + 4*a^2*b
+ 3*a*b^2 + 20*b^3)*sinh(d*x + c)^6 + 14*(4*a^3 + 2*a^2*b + 9*a*b^2)*cosh(d
*x + c)^4 + (56*a^3 + 28*a^2*b + 126*a*b^2 + 15*(8*a^3 + 4*a^2*b + 3*a*b^2
+ 20*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5*(8*a^3 + 4*a^2*b + 3*a*b^
2 - 15*b^3)*cosh(d*x + c)^3 + 28*(2*a^3 + a^2*b - 3*a*b^2)*cosh(d*x + c))*s
inh(d*x + c)^3 + 280*a^3 - 140*a^2*b + 210*a*b^2 + 7*(24*a^3 + 52*a^2*b - 2
1*a*b^2 + 20*b^3)*cosh(d*x + c)^2 + (15*(8*a^3 + 4*a^2*b + 3*a*b^2 + 20*b^3
)*cosh(d*x + c)^4 + 168*a^3 + 364*a^2*b - 147*a*b^2 + 140*b^3 + 84*(4*a^3 +
 2*a^2*b + 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2*(3*(8*a^3 + 4*a^2*
b + 3*a*b^2 - 15*b^3)*cosh(d*x + c)^5 + 56*(2*a^3 + a^2*b - 3*a*b^2)*cosh(d
*x + c)^3 + 7*(24*a^3 - 28*a^2*b + 9*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x
 + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x
+ c)^8 + 8*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^
6 + 4*(14*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cos
h(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 60*d*cosh(d*x + c)^2 + 14*d)*sinh(
d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 15*d*cosh(d*x + c)^3 + 7*d*cosh(d*x +
 c))*sinh(d*x + c)^3 + 56*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 30*d
*cosh(d*x + c)^4 + 42*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^2 + 4*(2*d*co
sh(d*x + c)^7 + 9*d*cosh(d*x + c)^5 + 14*d*cosh(d*x + c)^3 + 7*d*cosh(d*x +
 c))*sinh(d*x + c) + 35*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**8*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(76) = 152$.
time = 0.48, size = 260, normalized size = 3.25

$$-\frac{2\,\left(35\,b^{3}e^{\left(12\,dx+12\,c\right)}+210\,ab^{2}e^{\left(10\,dx+10\,c\right)}+560\,a^{2}be^{\left(8\,dx+8\,c\right)}-210\,ab^{2}e^{\left(8\,dx+8\,c\right)}+175\,b^{3}e^{\left(8\,dx+8\,c\right)}+560\,a^{3}e^{\left(6\,dx+6\,c\right)}-280\,a^{2}be^{\left(6\,dx+6\,c\right)}+420\,ab^{2}e^{\left(6\,dx+6\,c\right)}+336\,a^{3}e^{\left(4\,dx+4\,c\right)}+168\,a^{2}be^{\left(4\,dx+4\,c\right)}-84\,ab^{2}e^{\left(4\,dx+4\,c\right)}+105\,b^{3}e^{\left(4\,dx+4\,c\right)}+112\,a^{3}e^{\left(2\,dx+2\,c\right)}+56\,a^{2}be^{\left(2\,dx+2\,c\right)}+42\,ab^{2}e^{\left(2\,dx+2\,c\right)}+16\,a^{3}+8\,a^{2}b+6\,ab^{2}+5\,b^{3}\right)}{35\,d{\left(e^{\left(2\,dx+2\,c\right)}+1\right)}^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*b^3*e^(12*d*x + 12*c) + 210*a*b^2*e^(10*d*x + 10*c) + 560*a^2*b*e
^(8*d*x + 8*c) - 210*a*b^2*e^(8*d*x + 8*c) + 175*b^3*e^(8*d*x + 8*c) + 560*
a^3*e^(6*d*x + 6*c) - 280*a^2*b*e^(6*d*x + 6*c) + 420*a*b^2*e^(6*d*x + 6*c)
 + 336*a^3*e^(4*d*x + 4*c) + 168*a^2*b*e^(4*d*x + 4*c) - 84*a*b^2*e^(4*d*x
+ 4*c) + 105*b^3*e^(4*d*x + 4*c) + 112*a^3*e^(2*d*x + 2*c) + 56*a^2*b*e^(2*
d*x + 2*c) + 42*a*b^2*e^(2*d*x + 2*c) + 16*a^3 + 8*a^2*b + 6*a*b^2 + 5*b^3)
/(d*(e^(2*d*x + 2*c) + 1)^7)
```

Mupad [B]
time = 0.87, size = 994, normalized size = 12.42

$$-\frac{\frac{2b^3}{7d}+\frac{8e^{6c+6dx}\left(16a^3-24a^2b+18ab^2-5b^3\right)}{7d}+\frac{2b^3e^{12c+12dx}}{7d}+\frac{6be^{4c+4dx}\left(16a^2-16ab+5b^2\right)}{7d}+\frac{6be^{8c+8dx}\left(16a^2-16ab+5b^2\right)}{7d}+\frac{12b^2e^{2c+2dx}\left(2a-b\right)}{7d}+\frac{12b^2e^{10c+10dx}\left(2a-b\right)}{7d}}{7e^{2c+2dx}+21e^{4c+4dx}+35e^{6c+6dx}+35e^{8c+8dx}+21e^{10c+10dx}+7e^{12c+12dx}+e^{14c+14dx}+1}-\frac{\frac{4e^{4c+4dx}\left(16a^3-24a^2b+18ab^2-5b^3\right)}{7d}+\frac{2b^3e^{10c+10dx}}{7d}+\frac{2b^2\left(2a-b\right)}{7d}+\frac{2be^{2c+2dx}\left(16a^2-16ab+5b^2\right)}{7d}+\frac{4be^{6c+6dx}\left(16a^2-16ab+5b^2\right)}{7d}+\frac{10b^2e^{8c+8dx}\left(2a-b\right)}{7d}}{6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1}-\frac{\frac{2\left(16a^3-24a^2b+18ab^2-5b^3\right)}{35d}+\frac{2b^3e^{6c+6dx}}{7d}+\frac{6be^{2c+2dx}\left(16a^2-16ab+5b^2\right)}{35d}+\frac{6b^2e^{4c+4dx}\left(2a-b\right)}{7d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}-\frac{\frac{2b^3e^{2c+2dx}}{7d}+\frac{2b^2\left(2a-b\right)}{7d}}{2e^{2c+2dx}+e^{4c+4dx}+1}-\frac{\frac{2b\left(16a^2-16ab+5b^2\right)}{35d}+\frac{8e^{2c+2dx}\left(16a^3-24a^2b+18ab^2-5b^3\right)}{35d}+\frac{2b^3e^{8c+8dx}}{7d}+\frac{12be^{4c+4dx}\left(16a^2-16ab+5b^2\right)}{35d}+\frac{8b^2e^{6c+6dx}\left(2a-b\right)}{7d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}-\frac{\frac{2b\left(16a^2-16ab+5b^2\right)}{35d}+\frac{2b^3e^{4c+4dx}}{7d}+\frac{4b^2e^{2c+2dx}\left(2a-b\right)}{7d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}-\frac{2b^3}{7d\left(e^{2c+2dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^8,x)
```

```
[Out] - ((2*b^3)/(7*d) + (8*exp(6*c + 6*d*x)*(18*a*b^2 - 24*a^2*b + 16*a^3 - 5*b^
3))/(7*d) + (2*b^3*exp(12*c + 12*d*x))/(7*d) + (6*b*exp(4*c + 4*d*x)*(16*a^
2 - 16*a*b + 5*b^2))/(7*d) + (6*b*exp(8*c + 8*d*x)*(16*a^2 - 16*a*b + 5*b^2
))/(7*d) + (12*b^2*exp(2*c + 2*d*x)*(2*a - b))/(7*d) + (12*b^2*exp(10*c + 1
0*d*x)*(2*a - b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp
(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c +
12*d*x) + exp(14*c + 14*d*x) + 1) - ((4*exp(4*c + 4*d*x)*(18*a*b^2 - 24*a^2
*b + 16*a^3 - 5*b^3))/(7*d) + (2*b^3*exp(10*c + 10*d*x))/(7*d) + (2*b^2*(2*
a - b))/(7*d) + (2*b*exp(2*c + 2*d*x)*(16*a^2 - 16*a*b + 5*b^2))/(7*d) + (4
*b*exp(6*c + 6*d*x)*(16*a^2 - 16*a*b + 5*b^2))/(7*d) + (10*b^2*exp(8*c + 8*
d*x)*(2*a - b))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6
*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*
x) + 1) - ((2*(18*a*b^2 - 24*a^2*b + 16*a^3 - 5*b^3))/(35*d) + (2*b^3*exp(6
*c + 6*d*x))/(7*d) + (6*b*exp(2*c + 2*d*x)*(16*a^2 - 16*a*b + 5*b^2))/(35*d
) + (6*b^2*exp(4*c + 4*d*x)*(2*a - b))/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4
*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*b^3*exp(2*c
+ 2*d*x))/(7*d) + (2*b^2*(2*a - b))/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c +
4*d*x) + 1) - ((2*b*(16*a^2 - 16*a*b + 5*b^2))/(35*d) + (8*exp(2*c + 2*d*x)
*(18*a*b^2 - 24*a^2*b + 16*a^3 - 5*b^3))/(35*d) + (2*b^3*exp(8*c + 8*d*x))/
(7*d) + (12*b*exp(4*c + 4*d*x)*(16*a^2 - 16*a*b + 5*b^2))/(35*d) + (8*b^2*e
xp(6*c + 6*d*x)*(2*a - b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x)
 + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2
*b*(16*a^2 - 16*a*b + 5*b^2))/(35*d) + (2*b^3*exp(4*c + 4*d*x))/(7*d) + (4*
b^2*exp(2*c + 2*d*x)*(2*a - b))/(7*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*
d*x) + exp(6*c + 6*d*x) + 1) - (2*b^3)/(7*d*(exp(2*c + 2*d*x) + 1))
```

3.316 $\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=108

$$-\frac{(a-b)^3\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}d}+\frac{(a^2-3ab+3b^2)\sinh(c+dx)}{b^3d}-\frac{(a-3b)\sinh^3(c+dx)}{3b^2d}+\frac{\sinh^5(c+dx)}{5bd}$$

```
[Out] (a^2-3*a*b+3*b^2)*sinh(d*x+c)/b^3/d-1/3*(a-3*b)*sinh(d*x+c)^3/b^2/d+1/5*sin
h(d*x+c)^5/b/d-(a-b)^3*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/b^(7/2)/d/a^(1/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3269, 398, 211}

$$\frac{(a^2-3ab+3b^2)\sinh(c+dx)}{b^3d}-\frac{(a-b)^3\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}d}-\frac{(a-3b)\sinh^3(c+dx)}{3b^2d}+\frac{\sinh^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -(((a - b)^3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)*d))
+ ((a^2 - 3*a*b + 3*b^2)*Sinh[c + d*x])/(b^3*d) - ((a - 3*b)*Sinh[c + d*x]^
3)/(3*b^2*d) + Sinh[c + d*x]^5/(5*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^3}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{b^3}-\frac{(a-3b)x^2}{b^2}+\frac{x^4}{b}+\frac{-a^3+3a^2b-3ab^2+b^3}{b^3\left(a+bx^2\right)}\right)dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\left(a^2-3ab+3b^2\right)\sinh(c+dx)}{b^3d}-\frac{(a-3b)\sinh^3(c+dx)}{3b^2d}+\frac{\sinh^5(c+dx)}{5bd}-\ (a\ -\\
&= -\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}d}+\frac{\left(a^2-3ab+3b^2\right)\sinh(c+dx)}{b^3d}-\ (a-3b)\,\mathrm{s}
\end{aligned}
$$

Mathematica [A]
time = 0.36, size = 117, normalized size = 1.08

$$
\frac{30\sqrt{b}\left(8a^2-22ab+19b^2\right)\sinh(c+dx)+5b^{3/2}(-4a+9b)\sinh(3(c+dx))+\frac{3\left(80(a-b)^3\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+\sqrt{a}\,b^{5/2}\sinh(5(c+dx))\right)}{\sqrt{a}}}{240b^{7/2}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (30*Sqrt[b]*(8*a^2 - 22*a*b + 19*b^2)*Sinh[c + d*x] + 5*b^(3/2)*(-4*a + 9*b
)*Sinh[3*(c + d*x)] + (3*(80*(a - b)^3*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[
b]] + Sqrt[a]*b^(5/2)*Sinh[5*(c + d*x)]))/Sqrt[a])/(240*b^(7/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(96) = 192.
time = 1.79, size = 444, normalized size = 4.11

method	result
derivativedivides	$-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{1}{5b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{11b-4a}{8b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{15b-4a}{12b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a^2-3ab+3b^2}{b^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+$

default	$-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{1}{5b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}-\frac{11b-4a}{8b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{15b-4a}{12b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a^{2}-3ab+3b^{2}}{b^{3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+$
risch	$\frac{e^{5dx+5c}}{160bd}+\frac{3\,e^{3dx+3c}}{32bd}-\frac{e^{3dx+3c}a}{24b^{2}d}+\frac{e^{dx+c}a^{2}}{2b^{3}d}-\frac{11a\,e^{dx+c}}{8b^{2}d}+\frac{19\,e^{dx+c}}{16bd}-\frac{e^{-dx-c}a^{2}}{2b^{3}d}+\frac{11\,e^{-dx-c}a}{8b^{2}d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/b/(tanh(1/2*d*x+1/2*c)-1)^4-1/5/b/(tanh(1/2*d*x+1/2*c)-1)^5-1/8*(
11*b-4*a)/b^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/12*(15*b-4*a)/b^2/(tanh(1/2*d*x+1
/2*c)-1)^3-(a^2-3*a*b+3*b^2)/b^3/(tanh(1/2*d*x+1/2*c)-1)+2/b^3*a*(a^3-3*a^2
*b+3*a*b^2-b^3)*(1/2*(a-(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-
a+2*b)*a)^(1/2))-1/2*(-a-(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2
)+a-2*b)*a)^(1/2)))-1/5/b/(tanh(1/2*d*x+1/2*c)+1)^5+1/2/b/(tanh(1/2*d*x+1/2
*c)+1)^4-1/8*(-11*b+4*a)/b^2/(tanh(1/2*d*x+1/2*c)+1)^2-1/12*(15*b-4*a)/b^2/
(tanh(1/2*d*x+1/2*c)+1)^3-(a^2-3*a*b+3*b^2)/b^3/(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(3*b^2*e^(10*d*x + 10*c) - 3*b^2 - 5*(4*a*b*e^(8*c) - 9*b^2*e^(8*c))*
e^(8*d*x) + 30*(8*a^2*e^(6*c) - 22*a*b*e^(6*c) + 19*b^2*e^(6*c))*e^(6*d*x)
- 30*(8*a^2*e^(4*c) - 22*a*b*e^(4*c) + 19*b^2*e^(4*c))*e^(4*d*x) + 5*(4*a*b
*e^(2*c) - 9*b^2*e^(2*c))*e^(2*d*x))*e^(-5*d*x - 5*c)/(b^3*d) - 1/128*integ
rate(256*((a^3*e^(3*c) - 3*a^2*b*e^(3*c) + 3*a*b^2*e^(3*c) - b^3*e^(3*c))*e
^(3*d*x) + (a^3*e^c - 3*a^2*b*e^c + 3*a*b^2*e^c - b^3*e^c)*e^(d*x))/(b^4*e^
(4*d*x + 4*c) + b^4 + 2*(2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1548 vs. 2(96) = 192.

time = 0.47, size = 3066, normalized size = 28.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] [1/480*(3*a*b^3*cosh(d*x + c)^10 + 30*a*b^3*cosh(d*x + c)*sinh(d*x + c)^9 +
 3*a*b^3*sinh(d*x + c)^10 - 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^8 + 5*(27
*a*b^3*cosh(d*x + c)^2 - 4*a^2*b^2 + 9*a*b^3)*sinh(d*x + c)^8 + 40*(9*a*b^3
*cosh(d*x + c)^3 - (4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 3
0*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^6 + 10*(63*a*b^3*cosh(d*x
 + c)^4 + 24*a^3*b - 66*a^2*b^2 + 57*a*b^3 - 14*(4*a^2*b^2 - 9*a*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 4*(189*a*b^3*cosh(d*x + c)^5 - 70*(4*a^2*b^2
- 9*a*b^3)*cosh(d*x + c)^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^5 - 30*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^
4 + 10*(63*a*b^3*cosh(d*x + c)^6 - 35*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^4
 - 24*a^3*b + 66*a^2*b^2 - 57*a*b^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^4 - 3*a*b^3 + 40*(9*a*b^3*cosh(d*x + c)^7 -
7*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^5 + 15*(8*a^3*b - 22*a^2*b^2 + 19*a*b
^3)*cosh(d*x + c)^3 - 3*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^3 + 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2 + 5*(27*a*b^3*cosh(
d*x + c)^8 - 28*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^6 + 90*(8*a^3*b - 22*a^
2*b^2 + 19*a*b^3)*cosh(d*x + c)^4 + 4*a^2*b^2 - 9*a*b^3 - 36*(8*a^3*b - 22*
a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 240*((a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d
*x + c)^4*sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^
3*sinh(d*x + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^2*sinh
(d*x + c)^3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)
^4 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sinh(d*x + c)^5)*sqrt(-a*b)*log((b*cos
h(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2
*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2
 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(
d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*
x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x +
c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*
cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*
cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 10*(3*a*b^
3*cosh(d*x + c)^9 - 4*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^7 + 18*(8*a^3*b -
 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^5 - 12*(8*a^3*b - 22*a^2*b^2 + 19*a*b
^3)*cosh(d*x + c)^3 + (4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/(
a*b^4*d*cosh(d*x + c)^5 + 5*a*b^4*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*a*b^
4*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a*b^4*d*cosh(d*x + c)^2*sinh(d*x +
 c)^3 + 5*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^4 + a*b^4*d*sinh(d*x + c)^5),
 1/480*(3*a*b^3*cosh(d*x + c)^10 + 30*a*b^3*cosh(d*x + c)*sinh(d*x + c)^9 +
```

```
 3*a*b^3*sinh(d*x + c)^10 - 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^8 + 5*(27
*a*b^3*cosh(d*x + c)^2 - 4*a^2*b^2 + 9*a*b^3)*sinh(d*x + c)^8 + 40*(9*a*b^3
*cosh(d*x + c)^3 - (4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 3
0*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^6 + 10*(63*a*b^3*cosh(d*x
 + c)^4 + 24*a^3*b - 66*a^2*b^2 + 57*a*b^3 - 14*(4*a^2*b^2 - 9*a*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 4*(189*a*b^3*cosh(d*x + c)^5 - 70*(4*a^2*b^2
- 9*a*b^3)*cosh(d*x + c)^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^5 - 30*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^
4 + 10*(63*a*b^3*cosh(d*x + c)^6 - 35*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^4
 - 24*a^3*b + 66*a^2*b^2 - 57*a*b^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^4 - 3*a*b^3 + 40*(9*a*b^3*cosh(d*x + c)^7 -
7*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^5 + 15*(8*a^3*b - 22*a^2*b^2 + 19*a*b
^3)*cosh(d*x + c)^3 - 3*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^3 + 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2 + 5*(27*a*b^3*cosh(
d*x + c)^8 - 28*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^6 + 90*(8*a^3*b - 22*a^
2*b^2 + 19*a*b^3)*cosh(d*x + c)^4 + 4*a^2*b^2 - 9*a*b^3 - 36*(8*a^3*b - 22*
a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 480*((a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d
*x + c)^4*sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^
3*sinh(d*x + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^2*sinh
(d*x + c)^3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)
^4 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sinh(d*x + c)^5)*sqrt(a*b)*arctan(1/2*
sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - 480*((a^3 - 3*a^2*b + 3*a*b^
2 - b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^
4*sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^3*sinh(d
*x + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^2*sinh(d*x + c
)^3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*sinh(d*x + c)^5)*sqrt(a*b)*arctan(1/2*(b*cosh(
d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**7/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.51, size = 954, normalized size = 8.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^7/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] exp(5*c + 5*d*x)/(160*b*d) - exp(- 5*c - 5*d*x)/(160*b*d) - ((2*atan((exp(d
*x)*exp(c)*(a - b)^3*(a*b^7*d^2)^(1/2))/(2*a*b^3*d*((a - b)^6)^(1/2))) + 2*
atan((a*b^8*exp(d*x)*exp(c)*((4*(12*a^3*b^5*d*(a^6 - 6*a^5*b - 6*a*b^5 + b^
6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2) - 8*a^2*b^6*d*(a^6 - 6*a^5*
b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2) - 8*a^4*b^4
*d*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(
1/2) + 2*a^5*b^3*d*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3
 + 15*a^4*b^2)^(1/2) + 2*a*b^7*d*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^
4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2)))/(a^2*b^15*d^2*(a - b)^3) - (2*(a^7*(a*
b^7*d^2)^(1/2) - b^7*(a*b^7*d^2)^(1/2) + 7*a*b^6*(a*b^7*d^2)^(1/2) - 7*a^6*
b*(a*b^7*d^2)^(1/2) - 21*a^2*b^5*(a*b^7*d^2)^(1/2) + 35*a^3*b^4*(a*b^7*d^2)
^(1/2) - 35*a^4*b^3*(a*b^7*d^2)^(1/2) + 21*a^5*b^2*(a*b^7*d^2)^(1/2)))/(a^2
*b^11*d*((a - b)^6)^(1/2)*(a*b^7*d^2)^(1/2)))*(a*b^7*d^2)^(1/2))/(4*a^4 - 1
6*a^3*b - 16*a*b^3 + 4*b^4 + 24*a^2*b^2) + (2*exp(3*c)*exp(3*d*x)*(a^7*(a*b
^7*d^2)^(1/2) - b^7*(a*b^7*d^2)^(1/2) + 7*a*b^6*(a*b^7*d^2)^(1/2) - 7*a^6*b
*(a*b^7*d^2)^(1/2) - 21*a^2*b^5*(a*b^7*d^2)^(1/2) + 35*a^3*b^4*(a*b^7*d^2)^
(1/2) - 35*a^4*b^3*(a*b^7*d^2)^(1/2) + 21*a^5*b^2*(a*b^7*d^2)^(1/2)))/(a*b^
3*d*((a - b)^6)^(1/2)*(4*a^4 - 16*a^3*b - 16*a*b^3 + 4*b^4 + 24*a^2*b^2)))
*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/
2))/(2*(a*b^7*d^2)^(1/2)) + (exp(c + d*x)*(8*a^2 - 22*a*b + 19*b^2))/(16*b^
3*d) - (exp(- c - d*x)*(8*a^2 - 22*a*b + 19*b^2))/(16*b^3*d) + (exp(- 3*c -
 3*d*x)*(4*a - 9*b))/(96*b^2*d) - (exp(3*c + 3*d*x)*(4*a - 9*b))/(96*b^2*d)
```

3.317 $\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{(8a^2-20ab+15b^2)\,x}{8b^3}-\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3d}-\frac{(4a-7b)\cosh(c+dx)\sinh(c+dx)}{8b^2d}+\cosh^3$$

```
[Out] 1/8*(8*a^2-20*a*b+15*b^2)*x/b^3-1/8*(4*a-7*b)*cosh(d*x+c)*sinh(d*x+c)/b^2/d
+1/4*cosh(d*x+c)^3*sinh(d*x+c)/b/d-(a-b)^(5/2)*arctanh((a-b)^(1/2)*tanh(d*x
+c)/a^(1/2))/b^3/d/a^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3270, 425, 541, 536, 212, 214}

$$\frac{x(8a^2-20ab+15b^2)}{8b^3}-\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3d}-\frac{(4a-7b)\sinh(c+dx)\cosh(c+dx)}{8b^2d}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((8*a^2 - 20*a*b + 15*b^2)*x)/(8*b^3) - ((a - b)^(5/2)*ArcTanh[(Sqrt[a - b]
*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^3*d) - ((4*a - 7*b)*Cosh[c + d*x]*Sinh
[c + d*x])/(8*b^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd} + \frac{\text{Subst}\left(\int \frac{-a+4b-3(a-b)x^2}{(1-x^2)^2(a+(-a+b)x^2)}\,dx, x, \tanh(c+dx)\right)}{4bd} \\
&= -\frac{(4a-7b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd} + \text{Subst}\Big(\\
&= -\frac{(4a-7b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd} - (a-b)^3 \\
&= \frac{(8a^2-20ab+15b^2)\,x}{8b^3} - \frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3d} - (4a-7b)\,\mathrm{co}
\end{aligned}
$$

Mathematica [A]

time = 0.24, size = 106, normalized size = 0.88

$$\frac{-32(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\,\left(4(8a^2-20ab+15b^2)\,(c+dx)-8(a-2b)b\sinh(2(c+dx))+b^2\sinh(4(c+dx))\right)}{32\sqrt{a}\,b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-32*(a - b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*(
4*(8*a^2 - 20*a*b + 15*b^2)*(c + d*x) - 8*(a - 2*b)*b*Sinh[2*(c + d*x)] + b
^2*Sinh[4*(c + d*x)]))/(32*Sqrt[a]*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(107) = 214$.

time = 1.82, size = 438, normalized size = 3.62

method	result
risch	$\frac{x\,a^2}{b^3}-\frac{5ax}{2b^2}+\frac{15x}{8b}+\frac{e^{4dx+4c}}{64bd}+\frac{e^{2dx+2c}}{4bd}-\frac{a\,e^{2dx+2c}}{8b^2d}-\frac{e^{-2dx-2c}}{4bd}+\frac{a\,e^{-2dx-2c}}{8b^2d}-\frac{e^{-4dx-4c}}{64bd}+\frac{a\sqrt{a\,(a}{}$
derivativedivides	$\frac{\frac{2a\left(a^3-3a^2b+3a\,b^2-b^3\right)\left(\frac{\left(\sqrt{-b\,(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b\,(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b\,(a-b)}\sqrt{\left(2\sqrt{-b\,(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{-b\,(a-}{2a\sqrt{-}}\right.}{b^3}}{}$
default	$\frac{\frac{2a\left(a^3-3a^2b+3a\,b^2-b^3\right)\left(\frac{\left(\sqrt{-b\,(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b\,(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b\,(a-b)}\sqrt{\left(2\sqrt{-b\,(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{-b\,(a-}{2a\sqrt{-}}\right.}{b^3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^3*a*(a^3-3*a^2*b+3*a*b^2-b^3)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
```

```
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b
))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)))-1/4/b/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/
b/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(-9*b+4*a)/b^2/(tanh(1/2*d*x+1/2*c)+1)-1/8*
(11*b-4*a)/b^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/8*(8*a^2-20*a*b+15*b^2)/b^3*ln(t
anh(1/2*d*x+1/2*c)+1)+1/4/b/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/b/(tanh(1/2*d*x+1
/2*c)-1)^3-1/8*(-11*b+4*a)/b^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/8*(-9*b+4*a)/b^2
/(tanh(1/2*d*x+1/2*c)-1)+1/8/b^3*(-8*a^2+20*a*b-15*b^2)*ln(tanh(1/2*d*x+1/2
*c)-1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(107) = 214$.
time = 0.45, size = 1817, normalized size = 15.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh
(d*x + c)^8 + 8*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b - 2*
b^2)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b + 4*b^2)*sinh(d*x +
 c)^6 + 8*(7*b^2*cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(35*b^2*cosh(d*x + c)^4 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x - 60*(
a*b - 2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 +
4*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c) - 20*(a*b - 2*b^2)*cosh(d*x +
 c)^3)*sinh(d*x + c)^3 + 8*(a*b - 2*b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*
x + c)^6 + 12*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b - 2*b
^2)*cosh(d*x + c)^4 + 2*a*b - 4*b^2)*sinh(d*x + c)^2 + 32*((a^2 - 2*a*b + b
^2)*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^3*sinh(d*x + c) +
 6*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^2 - 2*a*b + b
^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^4)*sq
rt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
```

```
3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d
*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cos
h(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x
 + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 -
 a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x +
 c))*sinh(d*x + c) + b)) - b^2 + 8*(b^2*cosh(d*x + c)^7 + 4*(8*a^2 - 20*a*b
 + 15*b^2)*d*x*cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d*x + c)^5 + 2*(a*b -
 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cosh
(d*x + c)^3*sinh(d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^3
*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4), 1/64*(b^2*cosh(d
*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 8*(
8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b - 2*b^2)*cosh(d*x + c
)^6 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b + 4*b^2)*sinh(d*x + c)^6 + 8*(7*b^2*
cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^
2*cosh(d*x + c)^4 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x - 60*(a*b - 2*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8*a^2 - 20*a*b
 + 15*b^2)*d*x*cosh(d*x + c) - 20*(a*b - 2*b^2)*cosh(d*x + c)^3)*sinh(d*x +
 c)^3 + 8*(a*b - 2*b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 12*(8*
a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b - 2*b^2)*cosh(d*x + c)
^4 + 2*a*b - 4*b^2)*sinh(d*x + c)^2 + 64*((a^2 - 2*a*b + b^2)*cosh(d*x + c)
^4 + 4*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^2 - 2*a*b +
 b^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^2 - 2*a*b + b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^4)*sqrt(-(a - b)/a)*ar
ctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)) - b^2 + 8*(b^2*cosh(d*x + c)^7
 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d
*x + c)^5 + 2*(a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d*cosh(d*x +
 c)^4 + 4*b^3*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 4*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^
4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(107) = 214.

time = 2.42, size = 226, normalized size = 1.87

$$\frac{\frac{8\,(8\,a^2-20\,ab+15\,b^2)(dx+c)}{b^3}+\frac{be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}+16\,be^{(2\,dx+2\,c)}}{b^2}-\frac{(48\,a^2e^{(4\,dx+4\,c)}-120\,abe^{(4\,dx+4\,c)}+90\,b^2e^{(4\,dx+4\,c)}-8\,abe^{(2\,dx+2\,c)}+16\,b^2e^{(2\,dx+2\,c)}+b^2)e^{(-4\,dx-4\,c)}}{b^3}-\frac{64\,(a^3-3\,a^2b+3\,ab^2-b^3)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b^3}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(8*(8*a^2 - 20*a*b + 15*b^2)*(d*x + c)/b^3 + (b*e^(4*d*x + 4*c) - 8*a*
e^(2*d*x + 2*c) + 16*b*e^(2*d*x + 2*c))/b^2 - (48*a^2*e^(4*d*x + 4*c) - 120
*a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) - 8*a*b*e^(2*d*x + 2*c) + 16*
b^2*e^(2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c)/b^3 - 64*(a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(
-a^2 + a*b)*b^3))/d
```

Mupad [B]

time = 1.37, size = 264, normalized size = 2.18

$$\frac{x\,(8\,a^2-20\,a\,b+15\,b^2)}{8\,b^3}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}}{64\,b\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}}{64\,b\,d}+\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a-2\,b)}{8\,b^2\,d}-\frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a-2\,b)}{8\,b^2\,d}+\frac{\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a-b)^3}{b^4}-\frac{2\,(a-b)^{5/2}\,(b+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x})}{\sqrt{a}\,b^4}\right)\,(a-b)^{5/2}}{2\,\sqrt{a}\,b^3\,d}-\frac{\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a-b)^3}{b^4}+\frac{2\,(a-b)^{5/2}\,(b+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x})}{\sqrt{a}\,b^4}\right)\,(a-b)^{5/2}}{2\,\sqrt{a}\,b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (x*(8*a^2 - 20*a*b + 15*b^2))/(8*b^3) - exp(- 4*c - 4*d*x)/(64*b*d) + exp(4
*c + 4*d*x)/(64*b*d) + (exp(- 2*c - 2*d*x)*(a - 2*b))/(8*b^2*d) - (exp(2*c
+ 2*d*x)*(a - 2*b))/(8*b^2*d) + (log((4*exp(2*c + 2*d*x)*(a - b)^3)/b^4 - (
2*(a - b)^(5/2)*(b + 2*a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/(a^(1/2)*b
^4))*(a - b)^(5/2))/(2*a^(1/2)*b^3*d) - (log((4*exp(2*c + 2*d*x)*(a - b)^3)
/b^4 + (2*(a - b)^(5/2)*(b + 2*a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/(a
^(1/2)*b^4))*(a - b)^(5/2))/(2*a^(1/2)*b^3*d)
```

3.318 $\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{(a-b)^2\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a-2b)\sinh(c+dx)}{b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

```
[Out] -(a-2*b)*sinh(d*x+c)/b^2/d+1/3*sinh(d*x+c)^3/b/d+(a-b)^2*arctan(sinh(d*x+c)
*b^(1/2)/a^(1/2))/b^(5/2)/d/a^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3269, 398, 211}

$$\frac{(a-b)^2\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a-2b)\sinh(c+dx)}{b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - b)^2*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - (
(a - 2*b)*Sinh[c + d*x])/(b^2*d) + Sinh[c + d*x]^3/(3*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a-2b}{b^2}+\frac{x^2}{b}+\frac{a^2-2ab+b^2}{b^2(a+bx^2)}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{(a-2b)\sinh(c+dx)}{b^2d}+\frac{\sinh^3(c+dx)}{3bd}+\frac{(a-b)^2\text{Subst}(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c}{b^2d}$$

$$= \frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a-2b)\sinh(c+dx)}{b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

Mathematica [**A**]
time = 0.19, size = 79, normalized size = 1.03

$$\frac{-\frac{12(a-b)^2\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}}+3\sqrt{b}\,(-4a+7b)\sinh(c+dx)+b^{3/2}\sinh(3(c+dx))}{12b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((-12*(a - b)^2*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + 3*Sqrt[b
]*(-4*a + 7*b)*Sinh[c + d*x] + b^(3/2)*Sinh[3*(c + d*x)])/(12*b^(5/2)*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(67) = 134.
time = 1.78, size = 318, normalized size = 4.13

method	result
derivativedivides	$\frac{2a\left(a^2-2ab+b^2\right)\left(\frac{\left(-a+\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(a+\sqrt{-b(a-\right.}{2a\sqrt{-}}\right.}{b^2}$

default	$\frac{\frac{2a\left(a^{2}-2ab+b^{2}\right)\left(\frac{\left(-a+\sqrt{-b\left(a-b\right)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}\right)}{2a\sqrt{-b\left(a-b\right)}\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}-\frac{\left(a+\sqrt{-b\left(a-\right.}}{2a\sqrt{-b}}\right.}{b^{2}}}{}$
risch	$\frac{e^{3dx+3c}}{24bd}-\frac{a\,e^{dx+c}}{2b^{2}d}+\frac{7\,e^{dx+c}}{8bd}+\frac{e^{-dx-c}a}{2b^{2}d}-\frac{7\,e^{-dx-c}}{8bd}-\frac{e^{-3dx-3c}}{24bd}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)a^{2}}{2\sqrt{-ab}\,d\,b^{2}}+\frac{\ln\left(e^{2c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2*a*(a^2-2*a*b+b^2)*(1/2*(-a+(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2
)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b
*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*(a+(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2)))-1/3/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/b/(tan
h(1/2*d*x+1/2*c)+1)^2-1/b^2*(2*b-a)/(tanh(1/2*d*x+1/2*c)+1)-1/3/b/(tanh(1/2
*d*x+1/2*c)-1)^3-1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/b^2*(2*b-a)/(tanh(1/2*d*
x+1/2*c)-1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(4*a*e^(4*c) - 7*b*e^(4*c))*e^(4*d*x) - 3*(4*a*e^(2*c) - 7*b*e^(2*
c))*e^(2*d*x) - b*e^(6*d*x + 6*c) + b)*e^(-3*d*x - 3*c)/(b^2*d) + 1/32*inte
grate(64*((a^2*e^(3*c) - 2*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (a^2*e^c
- 2*a*b*e^c + b^2*e^c)*e^(d*x))/(b^3*e^(4*d*x + 4*c) + b^3 + 2*(2*a*b^2*e^(
2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 701 vs. $2(67) = 134$.
time = 0.45, size = 1490, normalized size = 19.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
[Out] [1/24*(a*b^2*cosh(d*x + c)^6 + 6*a*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^
2*sinh(d*x + c)^6 - 3*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^4 + 3*(5*a*b^2*cosh
(d*x + c)^2 - 4*a^2*b + 7*a*b^2)*sinh(d*x + c)^4 + 4*(5*a*b^2*cosh(d*x + c)
^3 - 3*(4*a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a*b^2 + 3*(4*a^
2*b - 7*a*b^2)*cosh(d*x + c)^2 + 3*(5*a*b^2*cosh(d*x + c)^4 + 4*a^2*b - 7*a
*b^2 - 6*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 12*((a^2 -
2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d
*x + c) + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2 - 2*a*
b + b^2)*sinh(d*x + c)^3)*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x
+ c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*
(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2
*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)
*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
- cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sin
h(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cos
h(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*
cosh(d*x + c))*sinh(d*x + c) + b)) + 6*(a*b^2*cosh(d*x + c)^5 - 2*(4*a^2*b
- 7*a*b^2)*cosh(d*x + c)^3 + (4*a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x +
c))/(a*b^3*d*cosh(d*x + c)^3 + 3*a*b^3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*
a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + a*b^3*d*sinh(d*x + c)^3), 1/24*(a*b
^2*cosh(d*x + c)^6 + 6*a*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^2*sinh(d*x
 + c)^6 - 3*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^4 + 3*(5*a*b^2*cosh(d*x + c)^
2 - 4*a^2*b + 7*a*b^2)*sinh(d*x + c)^4 + 4*(5*a*b^2*cosh(d*x + c)^3 - 3*(4*
a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a*b^2 + 3*(4*a^2*b - 7*a*
b^2)*cosh(d*x + c)^2 + 3*(5*a*b^2*cosh(d*x + c)^4 + 4*a^2*b - 7*a*b^2 - 6*(
4*a^2*b - 7*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*((a^2 - 2*a*b + b^
2)*cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d*x + c) +
3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*s
inh(d*x + c)^3)*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x +
c))/a) + 24*((a^2 - 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*co
sh(d*x + c)^2*sinh(d*x + c) + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^2 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^3)*sqrt(a*b)*arctan(1/2*(b*cosh(
d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a -
 b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b
)/(a*b)) + 6*(a*b^2*cosh(d*x + c)^5 - 2*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^3
 + (4*a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a*b^3*d*cosh(d*x + c)
^3 + 3*a*b^3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*b^3*d*cosh(d*x + c)*sinh
(d*x + c)^2 + a*b^3*d*sinh(d*x + c)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 1.28, size = 668, normalized size = 8.68

$$\frac{\left(2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,{\left(a-b\right)}^2\,\sqrt{a\,b^5\,d^2}}{2\,a\,b^2\,d\,\sqrt{{\left(a-b\right)}^4}}\right)-2\,\mathrm{atan}\left(\frac{a\,b^6\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,\left(\frac{4\,\left(2\,a\,b^5\,d\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}-6\,a^2\,b^4\,d\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}+6\,a^3\,b^3\,d\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}-2\,a^4\,b^2\,d\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}\right)}{a^2\,b^{11}\,d^2\,{\left(a-b\right)}^2}+\frac{2\,\left(a^5\,\sqrt{a\,b^5\,d^2}-b^5\,\sqrt{a\,b^5\,d^2}+5\,a\,b^4\,\sqrt{a\,b^5\,d^2}-5\,a^4\,b\,\sqrt{a\,b^5\,d^2}-10\,a^2\,b^3\,\sqrt{a\,b^5\,d^2}+10\,a^3\,b^2\,\sqrt{a\,b^5\,d^2}\right)}{a^2\,b^8\,d\,\sqrt{{\left(a-b\right)}^4}\,\sqrt{a\,b^5\,d^2}}\right)\,\sqrt{a\,b^5\,d^2}}{4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3}-\frac{2\,{\mathrm{e}}^{3\,c}\,{\mathrm{e}}^{3\,d\,x}\,\left(a^5\,\sqrt{a\,b^5\,d^2}-b^5\,\sqrt{a\,b^5\,d^2}+5\,a\,b^4\,\sqrt{a\,b^5\,d^2}-5\,a^4\,b\,\sqrt{a\,b^5\,d^2}-10\,a^2\,b^3\,\sqrt{a\,b^5\,d^2}+10\,a^3\,b^2\,\sqrt{a\,b^5\,d^2}\right)}{a\,b^2\,d\,\sqrt{{\left(a-b\right)}^4}\,\left(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3\right)}\right)\right)\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}}{2\,\sqrt{a\,b^5\,d^2}}-\frac{{\mathrm{e}}^{-3\,c-3\,d\,x}}{24\,b\,d}+\frac{{\mathrm{e}}^{3\,c+3\,d\,x}}{24\,b\,d}-\frac{{\mathrm{e}}^{c+d\,x}\,\left(4\,a-7\,b\right)}{8\,b^2\,d}+\frac{{\mathrm{e}}^{-c-d\,x}\,\left(4\,a-7\,b\right)}{8\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] ((2*atan((exp(d*x)*exp(c)*(a - b)^2*(a*b^5*d^2)^(1/2))/(2*a*b^2*d*((a - b)^
4)^(1/2))) - 2*atan((a*b^6*exp(d*x)*exp(c)*((4*(2*a*b^5*d*(a^4 - 4*a^3*b -
4*a*b^3 + b^4 + 6*a^2*b^2)^(1/2) - 6*a^2*b^4*d*(a^4 - 4*a^3*b - 4*a*b^3 + b
^4 + 6*a^2*b^2)^(1/2) + 6*a^3*b^3*d*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*
b^2)^(1/2) - 2*a^4*b^2*d*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)^(1/2))
)/(a^2*b^11*d^2*(a - b)^2) + (2*(a^5*(a*b^5*d^2)^(1/2) - b^5*(a*b^5*d^2)^(1
/2) + 5*a*b^4*(a*b^5*d^2)^(1/2) - 5*a^4*b*(a*b^5*d^2)^(1/2) - 10*a^2*b^3*(a
*b^5*d^2)^(1/2) + 10*a^3*b^2*(a*b^5*d^2)^(1/2)))/(a^2*b^8*d*((a - b)^4)^(1/
2)*(a*b^5*d^2)^(1/2)))*(a*b^5*d^2)^(1/2))/(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*
b^3) - (2*exp(3*c)*exp(3*d*x)*(a^5*(a*b^5*d^2)^(1/2) - b^5*(a*b^5*d^2)^(1/2
) + 5*a*b^4*(a*b^5*d^2)^(1/2) - 5*a^4*b*(a*b^5*d^2)^(1/2) - 10*a^2*b^3*(a*b
^5*d^2)^(1/2) + 10*a^3*b^2*(a*b^5*d^2)^(1/2)))/(a*b^2*d*((a - b)^4)^(1/2)*(
12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))))*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*
a^2*b^2)^(1/2))/(2*(a*b^5*d^2)^(1/2)) - exp(- 3*c - 3*d*x)/(24*b*d) + exp(3
*c + 3*d*x)/(24*b*d) - (exp(c + d*x)*(4*a - 7*b))/(8*b^2*d) + (exp(- c - d*
x)*(4*a - 7*b))/(8*b^2*d)
```

3.319 $\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=81

$$-\frac{(2a-3b)x}{2b^2}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

```
[Out] -1/2*(2*a-3*b)*x/b^2+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d+(a-b)^(3/2)*arctanh((a
-b)^(1/2)*tanh(d*x+c)/a^(1/2))/b^2/d/a^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3270, 425, 536, 212, 214}

$$\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d}-\frac{x(2a-3b)}{2b^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/2*((2*a - 3*b)*x)/b^2 + ((a - b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x
])/Sqrt[a]])/(Sqrt[a]*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{-a+2b+(-a+b)x^2}{(1-x^2)(a+(-a+b)x^2)}\,dx, x, \tanh(c+dx)\right)}{2bd}\\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{(2a-3b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2b^2d} + (a -\\
&= -\frac{(2a-3b)x}{2b^2} + \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d} + \frac{\cosh(c+dx)\sinh(c+}{2bd}
\end{aligned}
$$

Mathematica [**A**]
time = 0.12, size = 80, normalized size = 0.99

$$
\frac{4(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\,(-2(2a-3b)(c+dx)+b\sinh(2(c+dx)))}{4\sqrt{a}\,b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (4*(a - b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*(-2
*(2*a - 3*b)*(c + d*x) + b*Sinh[2*(c + d*x)]))/(4*Sqrt[a]*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(69) = 138$.

time = 1.64, size = 316, normalized size = 3.90

method	result
risch	$-\frac{ax}{b^{2}}+\frac{3x}{2b}+\frac{e^{2dx+2c}}{8bd}-\frac{e^{-2dx-2c}}{8bd}+\frac{\sqrt{a\left(a-b\right)}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{a\left(a-b\right)}-2a+b}{b}\right)}{2d\,b^{2}}-\frac{\sqrt{a\left(a-\right.}}{}$
derivativedivides	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(2a-3b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{2}}+\frac{2\left(a^{2}-2ab+b^{2}\right)a\left(\frac{\left(-\sqrt{-b\left(a-b\right)}-b\right)\mathrm{ar}}{2a\sqrt{-b\left(a-b\right)}}\right.}{}}{}$
default	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(2a-3b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{2}}+\frac{2\left(a^{2}-2ab+b^{2}\right)a\left(\frac{\left(-\sqrt{-b\left(a-b\right)}-b\right)\mathrm{ar}}{2a\sqrt{-b\left(a-b\right)}}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/b/(tanh(1/2*d*x+1/2*c)-1)+1/2*(2*a
-3*b)/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/b^2*(a^2-2*a*b+b^2)*a*(1/2*(-(-b*(a-b
))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(
a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*(-(-b*(a-b)
)^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(
a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)))-1/2/b/(tanh(1/
2*d*x+1/2*c)+1)^2+1/2/b/(tanh(1/2*d*x+1/2*c)+1)+1/2/b^2*(-2*a+3*b)*ln(tanh(
1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(69) = 138$.
time = 0.42, size = 875, normalized size = 10.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a - 3*b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x
 + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a - 3*b)*d*x - 3*b*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a - b)*cosh(d*x + c)^2 + 2*(a - b)*cosh(
d*x + c)*sinh(d*x + c) + (a - b)*sinh(d*x + c)^2)*sqrt((a - b)/a)*log((b^2*
cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4
 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2
)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b -
 b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*
x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 - a*b)*sqrt((a - b)/a))/
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
+ 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)
) + 4*(2*(2*a - 3*b)*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*x + c) +
 b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*si
nh(d*x + c)^2), -1/8*(4*(2*a - 3*b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4
 - 4*b*cosh(d*x + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a - 3*b)
*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a - b)*cosh(d*x + c)^2 +
2*(a - b)*cosh(d*x + c)*sinh(d*x + c) + (a - b)*sinh(d*x + c)^2)*sqrt(-(a -
 b)/a)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b
*sinh(d*x + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)) + 4*(2*(2*a - 3*b)*d*
x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*x + c) + b)/(b^2*d*cosh(d*x + c
)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.69, size = 138, normalized size = 1.70

$$-\frac{\frac{4\,(dx+c)(2\,a-3\,b)}{b^2}-\frac{e^{(2\,dx+2\,c)}}{b}-\frac{\left(4\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}-b\right)e^{(-2\,dx-2\,c)}}{b^2}-\frac{8\left(a^2-2\,ab+b^2\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*(d*x + c)*(2*a - 3*b)/b^2 - e^(2*d*x + 2*c)/b - (4*a*e^(2*d*x + 2*c
) - 6*b*e^(2*d*x + 2*c) - b)*e^(-2*d*x - 2*c)/b^2 - 8*(a^2 - 2*a*b + b^2)*a
rctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)
*b^2))/d
```

Mupad [B]
time = 1.61, size = 300, normalized size = 3.70

$$\frac{\mathrm{e}^{2\,c+2\,d\,x}}{8\,b\,d}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b\,d}-\frac{x\,(2\,a-3\,b)}{2\,b^2}-\frac{\ln\left(\frac{4\,(a-b)^3\,\left(2\,a\,b-b^2+8\,a^2\,\mathrm{e}^{2\,c+2\,d\,x}+b^2\,\mathrm{e}^{2\,c+2\,d\,x}-8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a\,b^6}-\frac{8\,(a-b)^{7/2}\,\left(b+4\,a\,\mathrm{e}^{2\,c+2\,d\,x}-2\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b^6}\right)\,(a-b)^{3/2}}{2\,\sqrt{a}\,b^2\,d}+\frac{\ln\left(\frac{4\,(a-b)^3\,\left(2\,a\,b-b^2+8\,a^2\,\mathrm{e}^{2\,c+2\,d\,x}+b^2\,\mathrm{e}^{2\,c+2\,d\,x}-8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a\,b^6}+\frac{8\,(a-b)^{7/2}\,\left(b+4\,a\,\mathrm{e}^{2\,c+2\,d\,x}-2\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b^6}\right)\,(a-b)^{3/2}}{2\,\sqrt{a}\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] exp(2*c + 2*d*x)/(8*b*d) - exp(- 2*c - 2*d*x)/(8*b*d) - (x*(2*a - 3*b))/(2*
b^2) - (log((4*(a - b)^3*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) + b^2*exp(2*
c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a*b^6) - (8*(a - b)^(7/2)*(b + 4*a*e
xp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(1/2)*b^6))*(a - b)^(3/2))/(2*a
^(1/2)*b^2*d) + (log((4*(a - b)^3*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) + b
^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a*b^6) + (8*(a - b)^(7/2)*(
b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(1/2)*b^6))*(a - b)^(3
/2))/(2*a^(1/2)*b^2*d)
```

3.320 $\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}+\frac{\sinh(c+dx)}{bd}$$

```
[Out] sinh(d*x+c)/b/d-(a-b)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/b^(3/2)/d/a^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3269, 396, 211}

$$\frac{\sinh(c+dx)}{bd}-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -(((a - b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) +
Sinh[c + d*x]/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\sinh(c+dx)}{bd}-\frac{(a-b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{bd}\\ &= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}+\frac{\sinh(c+dx)}{bd}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 0.96

$$\frac{-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{\sinh(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-(((a - b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2))) + S
inh[c + d*x]/b)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(44) = 88$.
time = 1.49, size = 230, normalized size = 4.42

method	result
risch	$\frac{e^{dx+c}}{2bd}-\frac{e^{-dx-c}}{2bd}-\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)a}{2\sqrt{-ab}\,db}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right.}{2\sqrt{-ab}\,db}$
derivativedivides	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2a(a-b)\left(\frac{\left(a-\sqrt{-b(a-b)}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a\right.}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2\right.}}\right.}{}}{d}$

default	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2a(a-b)\left(\frac{\left(a-\sqrt{-b(a-b)}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a\right.}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right.}}\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b/(tanh(1/2*d*x+1/2*c)-1)-1/b/(tanh(1/2*d*x+1/2*c)+1)+2/b*a*(a-b)*(
1/2*(a-(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-
1/2*(-a-(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*
a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)/(b*d) - 1/8*integrate(16*((a*e^(3*c)
 - b*e^(3*c))*e^(3*d*x) + (a*e^c - b*e^c)*e^(d*x))/(b^2*e^(4*d*x + 4*c) + b
^2 + 2*(2*a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(44) = 88$.
time = 0.40, size = 659, normalized size = 12.67

$$\left[\frac{ab\cosh(dx+c)^2+2ab\cosh(dx+c)\sinh(dx+c)+ab\sinh(dx+c)^2+\sqrt{-ab}\left((a-b)\cosh(dx+c)+(a-b)\sinh(dx+c)\right)\log\left(\frac{b\cosh(dx+c)^4+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2(2a+b)\cosh(dx+c)^2+2\left(3b\cosh(dx+c)^2-2a-b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3-(2a+b)\cosh(dx+c)\right)\sinh(dx+c)-4\left(\cosh(dx+c)^3+3\cosh(dx+c)\sinh(dx+c)^2+\sinh(dx+c)^3+\left(3\cosh(dx+c)^2-1\right)\sinh(dx+c)-\cosh(dx+c)\right)\sqrt{-ab}+b}{b\cosh(dx+c)^4+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2(2a-b)\cosh(dx+c)^2+2\left(3b\cosh(dx+c)^2+2a-b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3+(2a-b)\cosh(dx+c)\right)\sinh(dx+c)+b}\right)-ab}{2\left(ab^2d\cosh(dx+c)+ab^2d\sinh(dx+c)\right)}, \frac{ab\cosh(dx+c)^2+2ab\cosh(dx+c)\sinh(dx+c)+ab\sinh(dx+c)^2-2\sqrt{ab}\left((a-b)\cosh(dx+c)+(a-b)\sinh(dx+c)\right)\arctan\left(\frac{\sqrt{ab}\left(\cosh(dx+c)+\sinh(dx+c)\right)}{2a}\right)-2\sqrt{ab}\left((a-b)\cosh(dx+c)+(a-b)\sinh(dx+c)\right)\arctan\left(\frac{\left(b\cosh(dx+c)^3+3b\cosh(dx+c)\sinh(dx+c)^2+b\sinh(dx+c)^3+(4a-b)\cosh(dx+c)+\left(3b\cosh(dx+c)^2+4a-b\right)\sinh(dx+c)\right)\sqrt{ab}}{2ab}\right)-ab}{2\left(ab^2d\cosh(dx+c)+ab^2d\sinh(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*
x + c)^2 + sqrt(-a*b)*((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*log((
b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 -
```

```
 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x +
 c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(
cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*co
sh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d
*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a
- b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 +
4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) - a*b)/
(a*b^2*d*cosh(d*x + c) + a*b^2*d*sinh(d*x + c)), 1/2*(a*b*cosh(d*x + c)^2 +
 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 - 2*sqrt(a*b)*((a
- b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*arctan(1/2*sqrt(a*b)*(cosh(d*x
+ c) + sinh(d*x + c))/a) - 2*sqrt(a*b)*((a - b)*cosh(d*x + c) + (a - b)*sin
h(d*x + c))*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)
^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4
*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)) - a*b)/(a*b^2*d*cosh(d*x + c) + a*b
^2*d*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: TypeError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 1.14, size = 426, normalized size = 8.19

$$\frac{\mathrm{e}^{c+d\,x}}{2\,b\,d}-\frac{\left(2\,\mathrm{atan}\left(\frac{a\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(\frac{4\,\left(2\,a\,b^3\,d\,\sqrt{a^2-2\,a\,b+b^2}+2\,a^3\,b\,d\,\sqrt{a^2-2\,a\,b+b^2}-4\,a^2\,b^2\,d\,\sqrt{a^2-2\,a\,b+b^2}\right)}{a^2\,b^7\,d^2\,\left(a-b\right)}-\frac{2\,\left(a^3\,\sqrt{a\,b^3\,d^2}-b^3\,\sqrt{a\,b^3\,d^2}+3\,a\,b^2\,\sqrt{a\,b^3\,d^2}-3\,a^2\,b\,\sqrt{a\,b^3\,d^2}\right)}{a^2\,b^5\,d\,\sqrt{{\left(a-b\right)}^2}\,\sqrt{a\,b^3\,d^2}}\right)\,\sqrt{a\,b^3\,d^2}}{4\,a^2-8\,a\,b+4\,b^2}+\frac{2\,\mathrm{e}^{3\,c}\,\mathrm{e}^{3\,d\,x}\,\left(a^3\,\sqrt{a\,b^3\,d^2}-b^3\,\sqrt{a\,b^3\,d^2}+3\,a\,b^2\,\sqrt{a\,b^3\,d^2}-3\,a^2\,b\,\sqrt{a\,b^3\,d^2}\right)}{a\,b\,d\,\sqrt{{\left(a-b\right)}^2}\,\left(4\,a^2-8\,a\,b+4\,b^2\right)}\right)+2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a-b\right)\,\sqrt{a\,b^3\,d^2}}{2\,a\,b\,d\,\sqrt{{\left(a-b\right)}^2}}\right)\right)\,\sqrt{a^2-2\,a\,b+b^2}}{2\,\sqrt{a\,b^3\,d^2}}-\frac{\mathrm{e}^{-c-d\,x}}{2\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] exp(c + d*x)/(2*b*d) - ((2*atan((a*b^4*exp(d*x)*exp(c)*((4*(2*a*b^3*d*(a^2
- 2*a*b + b^2)^(1/2) + 2*a^3*b*d*(a^2 - 2*a*b + b^2)^(1/2) - 4*a^2*b^2*d*(a
^2 - 2*a*b + b^2)^(1/2)))/(a^2*b^7*d^2*(a - b)) - (2*(a^3*(a*b^3*d^2)^(1/2)
 - b^3*(a*b^3*d^2)^(1/2) + 3*a*b^2*(a*b^3*d^2)^(1/2) - 3*a^2*b*(a*b^3*d^2)^
(1/2)))/(a^2*b^5*d*((a - b)^2)^(1/2)*(a*b^3*d^2)^(1/2)))*(a*b^3*d^2)^(1/2))
/(4*a^2 - 8*a*b + 4*b^2) + (2*exp(3*c)*exp(3*d*x)*(a^3*(a*b^3*d^2)^(1/2) -
b^3*(a*b^3*d^2)^(1/2) + 3*a*b^2*(a*b^3*d^2)^(1/2) - 3*a^2*b*(a*b^3*d^2)^(1/
2)))/(a*b*d*((a - b)^2)^(1/2)*(4*a^2 - 8*a*b + 4*b^2))) + 2*atan((exp(d*x)*
exp(c)*(a - b)*(a*b^3*d^2)^(1/2))/(2*a*b*d*((a - b)^2)^(1/2))))*(a^2 - 2*a*
b + b^2)^(1/2))/(2*(a*b^3*d^2)^(1/2)) - exp(- c - d*x)/(2*b*d)
```

3.321 $\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{b}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

```
[Out] x/b-arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))*(a-b)^(1/2)/b/d/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3270, 400, 212, 214}

$$\frac{x}{b}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] x/b - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b
*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
```

```
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\Big(\int \frac{1}{(1-x^2)(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\Big)}{d}$$

$$= \frac{\text{Subst}\big(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\big)}{bd} - \frac{(a-b)\text{Subst}\Big(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \tanh(c+}{bd}$$

$$= \frac{x}{b} - \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 1.00

$$\frac{c+dx-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (c + d*x - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[
a])/(b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(42) = 84.
time = 1.51, size = 219, normalized size = 4.38

method	result
risch	$\frac{x}{b} + \frac{\sqrt{a(a-b)}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{a(a-b)}+2a-b}{b}\right)}{2adb} - \frac{\sqrt{a(a-b)}\,\ln\Big(\mathrm{e}^{2dx+2c}-\frac{2\sqrt{a(a-b)}-2a}{b}}{2adb}$

derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{2a(a-b)\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+\right.}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right.}}\right.}{}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{2a(a-b)\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+\right.}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right.}}\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/b*ln(tanh(1/2*d*x+1/2*c)-1)+2/b*a*(a-b
)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*
a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(42) = 84.

time = 0.45, size = 443, normalized size = 8.86

$$\left[\frac{2\,dx+\sqrt{\frac{a-b}{a}}\log\left(\frac{b^{2}\cosh(dx+c)^{4}+4\,b^{2}\cosh(dx+c)\sinh(dx+c)^{3}+b^{2}\sinh(dx+c)^{4}+2\,(2\,ab-b^{2})\cosh(dx+c)^{2}+2\left(3\,b^{2}\cosh(dx+c)^{2}+2\,ab-b^{2}\right)\sinh(dx+c)^{2}+8\,a^{2}-8\,ab+b^{2}+4\left(b^{2}\cosh(dx+c)^{3}+(2\,ab-b^{2})\cosh(dx+c)\right)\sinh(dx+c)+4\left(ab\cosh(dx+c)^{2}+2\,ab\cosh(dx+c)\sinh(dx+c)+ab\sinh(dx+c)^{2}+2\,a^{2}-ab\right)\sqrt{\frac{a-b}{a}}}{b\cosh(dx+c)^{4}+4\,b\cosh(dx+c)\sinh(dx+c)^{3}+b\sinh(dx+c)^{4}+2\,(2\,a-b)\cosh(dx+c)^{2}+2\left(3\,b\cosh(dx+c)^{2}+2\,a-b\right)\sinh(dx+c)^{2}+4\left(b\cosh(dx+c)^{3}+(2\,a-b)\cosh(dx+c)\right)\sinh(dx+c)+b}\right)}{2\,bd},\ \frac{dx+\sqrt{-\frac{a-b}{a}}\arctan\left(-\frac{\left(b\cosh(dx+c)^{2}+2\,b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^{2}+2\,a-b\right)\sqrt{-\frac{a-b}{a}}}{2\,(a-b)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c
)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 +
 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b +
b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) +
 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x
+ c)^2 + 2*a^2 - a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3
*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a
 - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/(b*d), (d*x + sqrt(-(a - b)/a)*ar
ctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)))/(b*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.10, size = 68, normalized size = 1.36

$$-\frac{\frac{(a-b)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{\sqrt{-a^{2}+ab}\,b}-\frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(
-a^2 + a*b)*b) - (d*x + c)/b)/d
```

Mupad [**B**]
time = 0.48, size = 166, normalized size = 3.32

$$\frac{x}{b}+\frac{\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a-b)}{b^{2}}-\frac{2\,\sqrt{a-b}\,\left(b+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b^{2}}\right)\,\sqrt{a-b}}{2\,\sqrt{a}\,b\,d}-\frac{\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a-b)}{b^{2}}+\frac{2\,\sqrt{a-b}\,\left(b+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b^{2}}\right)\,\sqrt{a-b}}{2\,\sqrt{a}\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] x/b + (log((4*exp(2*c + 2*d*x)*(a - b))/b^2 - (2*(a - b)^(1/2)*(b + 2*a*exp
(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/(a^(1/2)*b^2))*(a - b)^(1/2))/(2*a^(1/
2)*b*d) - (log((4*exp(2*c + 2*d*x)*(a - b))/b^2 + (2*(a - b)^(1/2)*(b + 2*a
*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/(a^(1/2)*b^2))*(a - b)^(1/2))/(2*a
^(1/2)*b*d)
```

3.322 $\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

```
[Out] arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{3269, 211\}$

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\ \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\ \sqrt{b}\ d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [A]
time = 0.53, size = 24, normalized size = 0.75

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{d\sqrt{ab}}$	24
default	$\frac{\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{d\sqrt{ab}}$	24
risch	$-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}\ d}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}\ d}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(a*b)^(1/2)*arctan(b*sinh(d*x+c)/(a*b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(24) = 48.
time = 0.48, size = 459, normalized size = 14.34

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2\,(2\,a+b)\cosh(dx+c)^2+2\,\left(3\,b\cosh(dx+c)^2-2\,a-b\right)\sinh(dx+c)^2+4\,\left(b\cosh(dx+c)^3-(2\,a+b)\cosh(dx+c)\right)\sinh(dx+c)-4\,\left(\cosh(dx+c)^3+3\,\cosh(dx+c)\sinh(dx+c)^2+\sinh(dx+c)^3+\left(3\,\cosh(dx+c)^2-1\right)\sinh(dx+c)-\cosh(dx+c)\right)\sqrt{-ab}+b}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\,(2\,a-b)\cosh(dx+c)^2+2\,\left(3\,b\cosh(dx+c)^2+2\,a-b\right)\sinh(dx+c)^2+4\,\left(b\cosh(dx+c)^3+(2\,a-b)\cosh(dx+c)\right)\sinh(dx+c)+b}\right)}{2\,abd},\ \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,(\cosh(dx+c)+\sinh(dx+c))}{2\,a}\right)+\sqrt{ab}\,\arctan\left(\frac{\left(b\cosh(dx+c)^3+3\,b\cosh(dx+c)\sinh(dx+c)^2+b\sinh(dx+c)^3+(4\,a-b)\cosh(dx+c)+\left(3\,b\cosh(dx+c)^2+4\,a-b\right)\sinh(dx+c)\right)\sqrt{ab}}{2\,ab}\right)}{abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3
 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c
))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + s
inh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqr
t(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sin
h(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a -
 b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(
d*x + c) + b))/(a*b*d), (sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + si
nh(d*x + c))/a) + sqrt(a*b)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x +
c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cos
h(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)))/(a*b*d)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(29) = 58$.
time = 0.86, size = 107, normalized size = 3.34

$$\begin{cases} \frac{\tilde{\infty} x \cosh(c)}{\sinh^2(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x \cosh(c)}{a + b \sinh^2(c)} & \text{for } d = 0 \\ -\frac{1}{bd \sinh(c + dx)} & \text{for } a = 0 \\ \frac{\sinh(c + dx)}{ad} & \text{for } b = 0 \\ \frac{\log\left(-\sqrt{-\frac{a}{b}} + \sinh(c + dx)\right)}{2bd\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{-\frac{a}{b}} + \sinh(c + dx)\right)}{2bd\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*cosh(c)/sinh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x*co
sh(c)/(a + b*sinh(c)**2), Eq(d, 0)), (-1/(b*d*sinh(c + d*x)), Eq(a, 0)), (s
inh(c + d*x)/(a*d), Eq(b, 0)), (log(-sqrt(-a/b) + sinh(c + d*x))/(2*b*d*sqr
t(-a/b)) - log(sqrt(-a/b) + sinh(c + d*x))/(2*b*d*sqrt(-a/b)), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 0.87, size = 23, normalized size = 0.72

$$\frac{\operatorname{atan}\left(\frac{b\sinh(c+d\,x)}{\sqrt{a\,b}}\right)}{d\,\sqrt{a\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] atan((b*sinh(c + d*x))/(a*b)^(1/2))/(d*(a*b)^(1/2))
```

3.323 $\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{\operatorname{ArcTan}(\sinh(c+dx))}{(a-b)d}-\frac{\sqrt{b}\operatorname{ArcTan}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)d}$$

```
[Out] arctan(sinh(d*x+c))/(a-b)/d-arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/(a-
b)/d/a^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3269, 400, 209, 211}

$$\frac{\operatorname{ArcTan}(\sinh(c+dx))}{d(a-b)}-\frac{\sqrt{b}\operatorname{ArcTan}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/((a - b)*d) - (Sqrt[b]*ArcTan[(Sqrt[b]*Sinh[c + d*x])
/Sqrt[a]])/(Sqrt[a]*(a - b)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{(a-b)d} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{(a-b)d}\\ &= \frac{\tan^{-1}(\sinh(c+dx))}{(a-b)d} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)d}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 0.92

$$\frac{\frac{\sqrt{b}\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}} + 2\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad-bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + 2*ArcTan[Tanh[
(c + d*x)/2]])/(a*d - b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(51) = 102.
time = 1.65, size = 209, normalized size = 3.54

method	result
risch	$\frac{i\ln(\mathrm{e}^{dx+c}+i)}{(a-b)d} - \frac{i\ln(\mathrm{e}^{dx+c}-i)}{(a-b)d} + \frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}-\frac{2\sqrt{-ab}\,\mathrm{e}^{dx+c}}{b}-1\right)}{2a(a-b)d} - \frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}+2\sqrt{-}\right.}{2a(a-b)d}$

derivativedivides	$-\frac{\frac{2ba\left(\frac{\left(-a+\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(a+\sqrt{-b(a-b)}-b\right)\mathrm{ar}}{2a\sqrt{-b(a-b)}}\right.}{a-b}}{d}$
default	$-\frac{\frac{2ba\left(\frac{\left(-a+\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(a+\sqrt{-b(a-b)}-b\right)\mathrm{ar}}{2a\sqrt{-b(a-b)}}\right.}{a-b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b/(a-b)*a*(1/2*(-a+(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(
a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/
2)-a+2*b)*a)^(1/2))-1/2*(a+(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(
a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)))+2/(a-b)*arctan(tanh(1/2*d*x+1/2*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(d*x + c))/(a*d - b*d) - 2*integrate((b*e^(3*d*x + 3*c) + b*e^(d
*x + c))/(a*b - b^2 + (a*b*e^(4*c) - b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^2*e^(2
*c) - 3*a*b*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(51) = 102$.
time = 0.40, size = 511, normalized size = 8.66

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{b\cosh(dx+c)^4+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2(2a+b)\cosh(dx+c)^2+2\left(3b\cosh(dx+c)^2-2a-b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3-(2a+b)\cosh(dx+c)\right)\sinh(dx+c)+4\left(a\cosh(dx+c)^3+3a\cosh(dx+c)\sinh(dx+c)^2+a\sinh(dx+c)^3-a\cosh(dx+c)+\left(3a\cosh(dx+c)^2-a\right)\sinh(dx+c)\right)\sqrt{-\frac{b}{a}}+b}{b\cosh(dx+c)^4+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2(2a-b)\cosh(dx+c)^2+2\left(3b\cosh(dx+c)^2+2a-b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3+(2a-b)\cosh(dx+c)\right)\sinh(dx+c)+b}\right)-4\arctan\left(\cosh(dx+c)+\sinh(dx+c)\right)}{2(a-b)d}, -\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{1}{2}\sqrt{\frac{b}{a}}\left(\cosh(dx+c)+\sinh(dx+c)\right)\right)+\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\cosh(dx+c)^3+3b\cosh(dx+c)\sinh(dx+c)^2+b\sinh(dx+c)^3+(4a-b)\cosh(dx+c)+\left(3b\cosh(dx+c)^2+4a-b\right)\sinh(dx+c)\right)\sqrt{\frac{b}{a}}}{2b}\right)-2\arctan\left(\cosh(dx+c)+\sinh(dx+c)\right)}{(a-b)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^
3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^
2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x +
c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^
2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*
x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c
)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
+ c))*sinh(d*x + c) + b)) - 4*arctan(cosh(d*x + c) + sinh(d*x + c)))/((a -
b)*d), -(sqrt(b/a)*arctan(1/2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) +
sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2
 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a
 - b)*sinh(d*x + c))*sqrt(b/a)/b) - 2*arctan(cosh(d*x + c) + sinh(d*x + c))
)/((a - b)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(c+dx\right)}{a+b\sinh^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sinh(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.36, size = 648, normalized size = 10.98

$$\frac{2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,\left(16\,a^2\,\sqrt{a^2\,d^2-2\,a\,b\,d^2+b^2\,d^2}+b^2\,\sqrt{a^2\,d^2-2\,a\,b\,d^2+b^2\,d^2}-8\,a\,b\,\sqrt{a^2\,d^2-2\,a\,b\,d^2+b^2\,d^2}\right)}{16\,d\,a^3-24\,d\,a^2\,b+9\,d\,a\,b^2-d\,b^3}\right)}{\sqrt{a^2\,d^2-2\,a\,b\,d^2+b^2\,d^2}}-\frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{b}\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,\sqrt{a\,d^2\,{\left(a-b\right)}^2}}{2\,a\,d\,\left(a-b\right)}\right)-2\,\mathrm{atan}\left(\frac{\left(a^3\,b^{5/2}\,\sqrt{a^3\,d^2-2\,a^2\,b\,d^2+a\,b^2\,d^2}-a^2\,b^{7/2}\,\sqrt{a^3\,d^2-2\,a^2\,b\,d^2+a\,b^2\,d^2}\right)\,\left({\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,\left(\frac{64\,\left(8\,d\,a^3-10\,d\,a^2\,b+2\,d\,a\,b^2\right)}{a\,b^3\,\left(a\,b-a^2\right)\,\sqrt{a\,d^2\,{\left(a-b\right)}^2}\,\sqrt{a^3\,d^2-2\,a^2\,b\,d^2+a\,b^2\,d^2}}+\frac{32\,\left(b^{3/2}\,\sqrt{a^3\,d^2-2\,a^2\,b\,d^2+a\,b^2\,d^2}-4\,a\,\sqrt{b}\,\sqrt{a^3\,d^2-2\,a^2\,b\,d^2+a\,b^2\,d^2}\right)}{a^2\,b^{5/2}\,d\,\left(a-b\right)\,\left(a\,b-a^2\right)\,\sqrt{a^3\,d^2-2\,a^2\,b\,d^2+a\,b^2\,d^2}}\right)-\frac{32\,{\mathrm{e}}^{3\,c}\,{\mathrm{e}}^{3\,d\,x}\,\left(b^{3/2}\,\sqrt{a^3\,d^2-2\,a^2\,b\,d^2+a\,b^2\,d^2}-4\,a\,\sqrt{b}\,\sqrt{a^3\,d^2-2\,a^2\,b\,d^2+a\,b^2\,d^2}\right)}{a^2\,b^{5/2}\,d\,\left(a-b\right)\,\left(a\,b-a^2\right)\,\sqrt{a^3\,d^2-2\,a^2\,b\,d^2+a\,b^2\,d^2}}\right)}{256\,a-64\,b}\right)\right)}{2\,\sqrt{a^3\,d^2-2\,a^2\,b\,d^2+a\,b^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (2*atan((exp(d*x)*exp(c)*(16*a^2*(a^2*d^2 + b^2*d^2 - 2*a*b*d^2)^(1/2) + b^
2*(a^2*d^2 + b^2*d^2 - 2*a*b*d^2)^(1/2) - 8*a*b*(a^2*d^2 + b^2*d^2 - 2*a*b*
d^2)^(1/2)))/(16*a^3*d - b^3*d + 9*a*b^2*d - 24*a^2*b*d)))/(a^2*d^2 + b^2*d
^2 - 2*a*b*d^2)^(1/2) - (b^(1/2)*(2*atan((b^(1/2)*exp(d*x)*exp(c)*(a*d^2*(a
 - b)^2)^(1/2))/(2*a*d*(a - b))) - 2*atan(((a^3*b^(5/2)*(a^3*d^2 + a*b^2*d^
2 - 2*a^2*b*d^2)^(1/2) - a^2*b^(7/2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1
/2))*(exp(d*x)*exp(c)*((64*(8*a^3*d + 2*a*b^2*d - 10*a^2*b*d))/(a*b^3*(a*b
- a^2)*(a*d^2*(a - b)^2)^(1/2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2)) +
 (32*(b^(3/2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2) - 4*a*b^(1/2)*(a^3*
d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2)))/(a^2*b^(5/2)*d*(a - b)*(a*b - a^2)*(
a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2))) - (32*exp(3*c)*exp(3*d*x)*(b^(3/
2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2) - 4*a*b^(1/2)*(a^3*d^2 + a*b^2
*d^2 - 2*a^2*b*d^2)^(1/2)))/(a^2*b^(5/2)*d*(a - b)*(a*b - a^2)*(a^3*d^2 + a
*b^2*d^2 - 2*a^2*b*d^2)^(1/2))))/(256*a - 64*b))))/(2*(a^3*d^2 + a*b^2*d^2
- 2*a^2*b*d^2)^(1/2))
```

3.324 $\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

<u>**Optimal**</u>. Leaf size=60

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{3/2}d}+\frac{\tanh(c+dx)}{(a-b)d}$$

```
[Out] -b*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/(a-b)^(3/2)/d/a^(1/2)+tanh(d*x+
c)/(a-b)/d
```

<u>**Rubi**</u> [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3270, 396, 214}

$$\frac{\tanh(c+dx)}{d(a-b)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d
)) + Tanh[c + d*x]/((a - b)*d)
```

<u>Rule 214</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

<u>Rule 396</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

<u>Rule 3270</u>

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{(a-b)d} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \tanh(c+dx)\right)}{(a-b)d}$$

$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{3/2}d} + \frac{\tanh(c+dx)}{(a-b)d}$$

Mathematica [**A**]
time = 0.13, size = 60, normalized size = 1.00

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{3/2}d} + \frac{\tanh(c+dx)}{(a-b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d
)) + Tanh[c + d*x]/((a - b)*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(52) = 104$.
time = 1.52, size = 219, normalized size = 3.65

method	result
risch	$-\frac{2}{d(a-b)\left(1+e^{2dx+2c}\right)} + \frac{b\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}+2a^2-2ab}{b\sqrt{a^2-ab}}\right)}{2\sqrt{a^2-ab}\,(a-b)d} - \frac{b\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-a}}{}\right)}{2\sqrt{a^2-}}$
derivativedivides	$\frac{\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a-b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} + \frac{2ba\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}} - \frac{\left(\sqrt{-b}\right.}{2a}\right)}{a-b}}{d}$

default	$\frac{\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a-b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2ba\left(\frac{\left(\sqrt{-b\,(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b\,(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b\,(a-b)}\sqrt{\left(2\sqrt{-b\,(a-b)}-a+2b\right)a}}-\right.}{a-b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a-b)*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)+2*b/(a-b)*a*(-1/
2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2
*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(52) = 104$.
time = 0.41, size = 709, normalized size = 11.82

$$\left[-\frac{\left(b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}+b\right)\sqrt{a^{2}-ab}\log\left(\frac{b^{2}\cosh(dx+c)^{4}+4\,b^{2}\cosh(dx+c)\sinh(dx+c)^{3}+b^{2}\sinh(dx+c)^{4}+2\,(2\,ab-b^{2})\cosh(dx+c)^{2}+2\left(3\,b^{2}\cosh(dx+c)^{2}+2\,ab-b^{2}\right)\sinh(dx+c)^{2}+8\,a^{2}-8\,ab+b^{2}+4\left(b^{2}\cosh(dx+c)^{3}+(2\,ab-b^{2})\cosh(dx+c)\right)\sinh(dx+c)-4\left(b\cosh(dx+c)^{2}+2\,b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^{2}+2\,a-b\right)\sqrt{a^{2}-ab}}{b\cosh(dx+c)^{4}+4\,b\cosh(dx+c)\sinh(dx+c)^{3}+b\sinh(dx+c)^{4}+2\,(2\,a-b)\cosh(dx+c)^{2}+2\left(3\,b\cosh(dx+c)^{2}+2\,a-b\right)\sinh(dx+c)^{2}+4\left(b\cosh(dx+c)^{3}+(2\,a-b)\cosh(dx+c)\right)\sinh(dx+c)+b}\right)+4\,a^{2}-4\,ab}{2\left((a^{3}-2\,a^{2}b+ab^{2})d\cosh\left(dx+c\right)^{2}+2\,(a^{3}-2\,a^{2}b+ab^{2})d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a^{3}-2\,a^{2}b+ab^{2})d\sinh\left(dx+c\right)^{2}+(a^{3}-2\,a^{2}b+ab^{2})d\right)},\ \frac{\left(b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}+b\right)\sqrt{-a^{2}+ab}\arctan\left(-\frac{\left(b\cosh(dx+c)^{2}+2\,b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^{2}+2\,a-b\right)\sqrt{-a^{2}+ab}}{2\,(a^{2}-ab)}\right)-2\,a^{2}+2\,ab}{(a^{3}-2\,a^{2}b+ab^{2})d\cosh\left(dx+c\right)^{2}+2\,(a^{3}-2\,a^{2}b+ab^{2})d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a^{3}-2\,a^{2}b+ab^{2})d\sinh\left(dx+c\right)^{2}+(a^{3}-2\,a^{2}b+ab^{2})d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x +
c)^2 + b)*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*si
nh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(
```

```
3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2
+ 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(
b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2
*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x +
c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
 + c))*sinh(d*x + c) + b)) + 4*a^2 - 4*a*b)/((a^3 - 2*a^2*b + a*b^2)*d*cosh
(d*x + c)^2 + 2*(a^3 - 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^
3 - 2*a^2*b + a*b^2)*d*sinh(d*x + c)^2 + (a^3 - 2*a^2*b + a*b^2)*d), ((b*co
sh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sq
rt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
+ c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) - 2*a^2 +
 2*a*b)/((a^3 - 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 + 2*(a^3 - 2*a^2*b + a*b
^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - 2*a^2*b + a*b^2)*d*sinh(d*x + c)
^2 + (a^3 - 2*a^2*b + a*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
[Out] Integral(sech(c + d*x)**2/(a + b*sinh(c + d*x)**2), x)
```

Giac [A]
time = 0.64, size = 80, normalized size = 1.33

$$-\frac{\frac{b\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,(a-b)}+\frac{2}{(a-b)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
[Out] -(b*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
 a*b)*(a - b)) + 2/((a - b)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B]
time = 1.47, size = 265, normalized size = 4.42

$$\frac{b\ln\left(\frac{4\,(2\,a\,b-b^2+8\,a^2\,e^{2\,c+2\,d\,x}+b^2\,e^{2\,c+2\,d\,x}-8\,a\,b\,e^{2\,c+2\,d\,x})}{a\,(a-b)^3}-\frac{8\,b+32\,a\,e^{2\,c+2\,d\,x}-16\,b\,e^{2\,c+2\,d\,x}}{\sqrt{a}\,(a-b)^{5/2}}\right)}{2\,\sqrt{a}\,d\,(a-b)^{3/2}}-\frac{b\ln\left(\frac{8\,b+32\,a\,e^{2\,c+2\,d\,x}-16\,b\,e^{2\,c+2\,d\,x}}{\sqrt{a}\,(a-b)^{5/2}}+\frac{4\,(2\,a\,b-b^2+8\,a^2\,e^{2\,c+2\,d\,x}+b^2\,e^{2\,c+2\,d\,x}-8\,a\,b\,e^{2\,c+2\,d\,x})}{a\,(a-b)^3}\right)}{2\,\sqrt{a}\,d\,(a-b)^{3/2}}-\frac{2}{(e^{2\,c+2\,d\,x}+1)\,(a\,d-b\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (b*log((4*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) + b^2*exp(2*c + 2*d*x) - 8*
a*b*exp(2*c + 2*d*x)))/(a*(a - b)^3) - (8*b + 32*a*exp(2*c + 2*d*x) - 16*b*
exp(2*c + 2*d*x))/(a^(1/2)*(a - b)^(5/2))))/(2*a^(1/2)*d*(a - b)^(3/2)) - (
b*log((8*b + 32*a*exp(2*c + 2*d*x) - 16*b*exp(2*c + 2*d*x))/(a^(1/2)*(a - b
)^(5/2)) + (4*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) + b^2*exp(2*c + 2*d*x)
- 8*a*b*exp(2*c + 2*d*x)))/(a*(a - b)^3)))/(2*a^(1/2)*d*(a - b)^(3/2)) - 2/
((exp(2*c + 2*d*x) + 1)*(a*d - b*d))
```

3.325 $\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=92

$$\frac{(a-3b)\operatorname{ArcTan}(\sinh(c+dx))}{2(a-b)^2d}+\frac{b^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^2d}+\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d}$$

```
[Out] 1/2*(a-3*b)*arctan(sinh(d*x+c))/(a-b)^2/d+b^(3/2)*arctan(sinh(d*x+c)*b^(1/2
)/a^(1/2))/(a-b)^2/d/a^(1/2)+1/2*sech(d*x+c)*tanh(d*x+c)/(a-b)/d
```

Rubi [A]

time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3269, 425, 536, 209, 211}

$$\frac{b^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^2}+\frac{(a-3b)\operatorname{ArcTan}(\sinh(c+dx))}{2d(a-b)^2}+\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - 3*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^2*d) + (b^(3/2)*ArcTan[(Sqrt[b
]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^2*d) + (Sech[c + d*x]*Tanh[c +
d*x])/(2*(a - b)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{2(a-b)d}\\
&= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d} + \frac{(a-3b)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{2(a-b)^2d} + b^2\mathrm{Su}\\
&= \frac{(a-3b)\tan^{-1}(\sinh(c+dx))}{2(a-b)^2d} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^2d} + \frac{\operatorname{sech}(c+dx)\,\mathrm{tan}}{2(a-b)}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 91, normalized size = 0.99

$$
\frac{-2b^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+2\sqrt{a}(a-3b)\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{a}(a-b)\operatorname{sech}(c+dx)\tanh(c+dx)}{2\sqrt{a}(a-b)^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-2*b^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*Sqrt[a]*(a - 3*b)*A
rcTan[Tanh[(c + d*x)/2]] + Sqrt[a]*(a - b)*Sech[c + d*x]*Tanh[c + d*x])/(2*
Sqrt[a]*(a - b)^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(80) = 160$.

time = 2.02, size = 272, normalized size = 2.96

method	result
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{d(a-b)\left(1+e^{2dx+2c}\right)^2}+\frac{i\ln\left(e^{dx+c}+i\right)a}{2(a-b)^2d}-\frac{3i\ln\left(e^{dx+c}+i\right)b}{2(a-b)^2d}-\frac{i\ln\left(e^{dx+c}-i\right)a}{2(a-b)^2d}+\frac{3i\ln\left(e^{dx+c}-i\right)b}{2(a-b)^2d}+\frac{\sqrt{-ab}\,b\ln}{}$
derivativedivides	$\frac{\frac{2\left(\left(\frac{b}{2}-\frac{a}{2}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{b}{2}+\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+(a-3b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)^2}+\frac{2b^2a\left(\frac{\left(-a+\sqrt{-b(a-b)}+b\right)\mathrm{arct}}{2a\sqrt{-b(a-b)}\sqrt{}}\right.}{}$
default	$\frac{\frac{2\left(\left(\frac{b}{2}-\frac{a}{2}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{b}{2}+\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+(a-3b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)^2}+\frac{2b^2a\left(\frac{\left(-a+\sqrt{-b(a-b)}+b\right)\mathrm{arct}}{2a\sqrt{-b(a-b)}\sqrt{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a-b)^2*(((1/2*b-1/2*a)*tanh(1/2*d*x+1/2*c)^3+(-1/2*b+1/2*a)*tanh(1/
2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^2+1/2*(a-3*b)*arctan(tanh(1/2*d*x+1
/2*c)))+2*b^2/(a-b)^2*a*(1/2*(-a+(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2))-1/2*(a+(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (a*e^c - 3*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^2*d - 2*a*b*d + b^2*d) + (e
^(3*d*x + 3*c) - e^(d*x + c))/(a*d - b*d + (a*d*e^(4*c) - b*d*e^(4*c))*e^(4
*d*x) + 2*(a*d*e^(2*c) - b*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/4*(b^2*e^(
3*d*x + 3*c) + b^2*e^(d*x + c))/(a^2*b - 2*a*b^2 + b^3 + (a^2*b*e^(4*c) - 2
*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^3*e^(2*c) - 5*a^2*b*e^(2*c
) + 4*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(80) = 160$.
time = 0.48, size = 1644, normalized size = 17.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a - b)*cosh(d*x + c)^3 + 6*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 +
 2*(a - b)*sinh(d*x + c)^3 + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*
x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x +
c) + b)*sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)
^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d
*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x +
c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
 + c))*sinh(d*x + c) + b)) + 2*((a - 3*b)*cosh(d*x + c)^4 + 4*(a - 3*b)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a - 3*b)*sinh(d*x + c)^4 + 2*(a - 3*b)*cosh(d
*x + c)^2 + 2*(3*(a - 3*b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c)^2 + 4*(
(a - 3*b)*cosh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + a - 3*
b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a - b)*cosh(d*x + c) + 2*(3*(
a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c))/((a^2 - 2*a*b + b^2)*d*cosh(
d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 -
 2*a*b + b^2)*d*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^2 +
 2*(3*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*d)*sinh(d
*x + c)^2 + (a^2 - 2*a*b + b^2)*d + 4*((a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^
3 + (a^2 - 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*x + c)), ((a - b)*cosh(d*x
+ c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3
+ (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^
4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(
b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(b/a)*arctan(1/
2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) + (b*cosh(d*x + c)^4 + 4*b*cos
```

```
h(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3
*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x
 + c))*sinh(d*x + c) + b)*sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cos
h(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) +
(3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(b/a)/b) + ((a - 3*b)*co
sh(d*x + c)^4 + 4*(a - 3*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a - 3*b)*sinh(
d*x + c)^4 + 2*(a - 3*b)*cosh(d*x + c)^2 + 2*(3*(a - 3*b)*cosh(d*x + c)^2 +
 a - 3*b)*sinh(d*x + c)^2 + 4*((a - 3*b)*cosh(d*x + c)^3 + (a - 3*b)*cosh(d
*x + c))*sinh(d*x + c) + a - 3*b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (
a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c))/(
(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*d*cosh(d*x +
c)*sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*d*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b
 + b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a
^2 - 2*a*b + b^2)*d)*sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*d + 4*((a^2 - 2*
a*b + b^2)*d*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*
x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sinh(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 6.14, size = 2797, normalized size = 30.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)),x)

[Out] exp(c + d*x)/((exp(2*c + 2*d*x) + 1)*(a*d - b*d)) - (2*exp(c + d*x))/((a*d
- b*d)*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + ((2*atan((b^2*exp(d*x
)*exp(c)*(a*d^2*(a - b)^4)^(1/2))/(2*a*d*(a - b)^2*(b^3)^(1/2))) - 2*atan((
exp(d*x)*exp(c)*((64*(20*a^3*d*(b^3)^(5/2) - 232*a^6*d*(b^3)^(3/2) + 2*a^9*
d*(b^3)^(1/2) + 2*a*b^5*d*(b^3)^(3/2) + 10*a^5*b*d*(b^3)^(3/2) - 22*a^8*b*d
*(b^3)^(1/2) - 10*a^2*b^4*d*(b^3)^(3/2) - 20*a^4*b^2*d*(b^3)^(3/2) - 18*a^2
*b^7*d*(b^3)^(1/2) + 102*a^3*b^6*d*(b^3)^(1/2) - 242*a^4*b^5*d*(b^3)^(1/2)
+ 310*a^5*b^4*d*(b^3)^(1/2) + 98*a^7*b^2*d*(b^3)^(1/2)))/(a*b^4*(a - b)^5*(
a*b - a^2)*(a^2 - 2*a*b + b^2)*(a*d^2*(a - b)^4)^(1/2)*(3*a*b^2 - 3*a^2*b +
 a^3 - b^3)*(9*a*b^2 - 6*a^2*b + a^3 - b^3)*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*
d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)) - (32*(b^8*(a^5*d^2 + a*b^4*d^2
 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 36*a^2*b^6*(a^5*d^2
 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) - 47*a^3*
b^5*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/
2) + 30*a^4*b^4*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*
b^2*d^2)^(1/2) - 9*a^5*b^3*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d
^2 + 6*a^3*b^2*d^2)^(1/2) + a^6*b^2*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*
a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) - 12*a*b^7*(a^5*d^2 + a*b^4*d^2 - 4*a^4*
b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))/(a^2*b^2*d*(a - b)^7*(a*b -
a^2)*(b^3)^(1/2)*(a^2 - 2*a*b + b^2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(9*a*b
^2 - 6*a^2*b + a^3 - b^3)*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^
2 + 6*a^3*b^2*d^2)^(1/2))) + (32*exp(3*c)*exp(3*d*x)*(b^8*(a^5*d^2 + a*b^4*
d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 36*a^2*b^6*(a^5*
d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) - 47*a
^3*b^5*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^
(1/2) + 30*a^4*b^4*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a
^3*b^2*d^2)^(1/2) - 9*a^5*b^3*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^
3*d^2 + 6*a^3*b^2*d^2)^(1/2) + a^6*b^2*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 -
 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) - 12*a*b^7*(a^5*d^2 + a*b^4*d^2 - 4*a
^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))/(a^2*b^2*d*(a - b)^7*(a*b
 - a^2)*(b^3)^(1/2)*(a^2 - 2*a*b + b^2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(9*
a*b^2 - 6*a^2*b + a^3 - b^3)*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3
*d^2 + 6*a^3*b^2*d^2)^(1/2)))*((a^2*b^10*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2
 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/64 - (a^3*b^9*(a^5*d^2 + a*b^4*d^2
 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/8 + (7*a^4*b^8*(a^5*
d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/16 -
(7*a^5*b^7*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d
^2)^(1/2))/8 + (35*a^6*b^6*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d
^2 + 6*a^3*b^2*d^2)^(1/2))/32 - (7*a^7*b^5*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d
^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/8 + (7*a^8*b^4*(a^5*d^2 + a*b^4*
d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/16 - (a^9*b^3*(a^
5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/8 +
 (a^10*b^2*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d
```

```
^2)^(1/2))/64)))*(b^3)^(1/2))/(2*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2
*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)) + (atan((exp(d*x)*exp(c)*(a^7*(a^4*d^2 + b
^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) - 3*b^7*(a^4*d^2
+ b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + 55*a*b^6*(a^
4*d^2 + b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) - 15*a^6
*b*(a^4*d^2 + b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) -
297*a^2*b^5*(a^4*d^2 + b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2*b^2*d^2)
^(1/2) + 423*a^3*b^4*(a^4*d^2 + b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2
*b^2*d^2)^(1/2) - 272*a^4*b^3*(a^4*d^2 + b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^
2 + 6*a^2*b^2*d^2)^(1/2) + 90*a^5*b^2*(a^4*d^2 + b^4*d^2 - 4*a*b^3*d^2 - 4*
a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)))/(a^8*d*(a^2 - 6*a*b + 9*b^2)^(1/2) + b^8
*d*(a^2 - 6*a*b + 9*b^2)^(1/2) + 130*a^2*b^6*d*(a^2 - 6*a*b + 9*b^2)^(1/2)
- 314*a^3*b^5*d*(a^2 - 6*a*b + 9*b^2)^(1/2) + 367*a^4*b^4*d*(a^2 - 6*a*b +
9*b^2)^(1/2) - 230*a^5*b^3*d*(a^2 - 6*a*b + 9*b^2)^(1/2) + 79*a^6*b^2*d*(a^
2 - 6*a*b + 9*b^2)^(1/2) - 20*a*b^7*d*(a^2 - 6*a*b + 9*b^2)^(1/2) - 14*a^7*
b*d*(a^2 - 6*a*b + 9*b^2)^(1/2)))*(a^2 - 6*a*b + 9*b^2)^(1/2))/(a^4*d^2 + b
^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)
```

3.326 $\int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{5/2}d}+\frac{(a-2b)\tanh(c+dx)}{(a-b)^2d}-\frac{\tanh^3(c+dx)}{3(a-b)d}$$

```
[Out] b^2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/(a-b)^(5/2)/d/a^(1/2)+(a-2*b)*
tanh(d*x+c)/(a-b)^2/d-1/3*tanh(d*x+c)^3/(a-b)/d
```

Rubi [A]

time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3270, 398, 214}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{5/2}}-\frac{\tanh^3(c+dx)}{3d(a-b)}+\frac{(a-2b)\tanh(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)*d
) + ((a - 2*b)*Tanh[c + d*x])/((a - b)^2*d) - Tanh[c + d*x]^3/(3*(a - b)*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a-2b}{(a-b)^2}-\frac{x^2}{a-b}+\frac{b^2}{(a-b)^2\left(a-(a-b)x^2\right)}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{(a-2b)\tanh(c+dx)}{(a-b)^2d}-\frac{\tanh^3(c+dx)}{3(a-b)d}+\frac{b^2\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \tanh(c+d\right.}{(a-b)^2d}\\
&= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{5/2}d}+\frac{(a-2b)\tanh(c+dx)}{(a-b)^2d}-\frac{\tanh^3(c+dx)}{3(a-b)d}\end{aligned}$$

Mathematica [A]
time = 0.51, size = 84, normalized size = 0.95

$$\frac{\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{5/2}}+\frac{\left(2a-5b+(a-b)\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{(a-b)^2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((3*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2
)) + ((2*a - 5*b + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b)^2)/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(78) = 156$.
time = 1.84, size = 268, normalized size = 3.05

method	result
risch	$-\frac{2\left(-3b\,e^{4dx+4c}+6a\,e^{2dx+2c}-12b\,e^{2dx+2c}+2a-5b\right)}{3d(a-b)^2\left(1+e^{2dx+2c}\right)^3}+\frac{b^2\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}-2a^2+2ab}{b\sqrt{a^2-ab}}\right)}{2\sqrt{a^2-ab}\,(a-b)^2d}-$

derivativedivides	$-\frac{2b^2a\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{-b(a-b)}-b\right)\arctan}{2a\sqrt{-b(a-b)}}\right.}{(a-b)^2}$
default	$-\frac{2b^2a\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}-\frac{\left(\sqrt{-b(a-b)}-b\right)\arctan}{2a\sqrt{-b(a-b)}}\right.}{(a-b)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^2/(a-b)^2*a*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*
(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2)))-2/(a-b)^2*((2*b-a)*tanh(1/2*d*x+1/2*c)^5+(8/3*b-2/3*a
)*tanh(1/2*d*x+1/2*c)^3+(2*b-a)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2
+1)^3)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1094 vs. $2(78) = 156$.
time = 0.40, size = 2444, normalized size = 27.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b - a*b^2)*sinh(d*x + c)^4 - 8*a^3 + 28*a^2*b -
20*a*b^2 - 24*(a^3 - 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^2 - 24*(a^3 - 3*a^2*b
 + 2*a*b^2 - 3*(a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(b^2*co
sh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 +
 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^4 +
3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sin
h(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 + 6*b^2*cosh(d*x + c)^2 + b^2)*sinh
(d*x + c)^2 + b^2 + 6*(b^2*cosh(d*x + c)^5 + 2*b^2*cosh(d*x + c)^3 + b^2*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^
2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cos
h(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*
a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*
x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 +
2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 +
(2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 48*((a^2*b - a*b^2)*cosh(d*x
 + c)^3 - (a^3 - 3*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 - 3
*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^6 + 6*(a^4 - 3*a^3*b + 3*a^2*b^
2 - a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a
*b^3)*d*sinh(d*x + c)^6 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x
+ c)^4 + 3*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + (a^4
- 3*a^3*b + 3*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^4 + 3*(a^4 - 3*a^3*b + 3*a^
2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)
*d*cosh(d*x + c)^3 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c))
*sinh(d*x + c)^3 + 3*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)
^4 + 6*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + (a^4 - 3*a^3
*b + 3*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a
*b^3)*d + 6*((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^5 + 2*(a^4
 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^3 + (a^4 - 3*a^3*b + 3*a^2*
b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b - a*b^2)*cosh(d
*x + c)^4 + 24*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b - a
*b^2)*sinh(d*x + c)^4 - 4*a^3 + 14*a^2*b - 10*a*b^2 - 12*(a^3 - 3*a^2*b + 2
*a*b^2)*cosh(d*x + c)^2 - 12*(a^3 - 3*a^2*b + 2*a*b^2 - 3*(a^2*b - a*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x +
```

```
 c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 + 3*b^2*cosh(d*x + c)^4 + 3*(5*b^
2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2
*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x
 + c)^4 + 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 + b^2 + 6*(b^2*cosh(
d*x + c)^5 + 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt
(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 24*((a^2*
b - a*b^2)*cosh(d*x + c)^3 - (a^3 - 3*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(
d*x + c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^6 + 6*(a^4 -
 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - 3*a^
3*b + 3*a^2*b^2 - a*b^3)*d*sinh(d*x + c)^6 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 -
 a*b^3)*d*cosh(d*x + c)^4 + 3*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh
(d*x + c)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^4 + 3*(a
^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 - 3*a^3*b +
 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^3 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^
3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b
^3)*d*cosh(d*x + c)^4 + 6*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x +
c)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^4 - 3*a^
3*b + 3*a^2*b^2 - a*b^3)*d + 6*((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(
d*x + c)^5 + 2*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^3 + (a^4
 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*sinh(c + d*x)**2), x)
```

Giac [**A**]
time = 0.71, size = 138, normalized size = 1.57

$$\frac{\frac{3\,b^2\arctan\left(\frac{b e^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{-a^2+ab}}+\frac{2\left(3\,b e^{(4\,dx+4\,c)}-6\,a e^{(2\,dx+2\,c)}+12\,b e^{(2\,dx+2\,c)}-2\,a+5\,b\right)}{(a^2-2\,ab+b^2)\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^2
 - 2*a*b + b^2)*sqrt(-a^2 + a*b)) + 2*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x +
```

```
 2*c) + 12*b*e^(2*d*x + 2*c) - 2*a + 5*b)/((a^2 - 2*a*b + b^2)*(e^(2*d*x +
2*c) + 1)^3))/d
```

Mupad [B]
time = 2.46, size = 710, normalized size = 8.07

$$\frac{8}{3\,(a\,d-b\,d)\,(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1)}-\frac{4}{(a\,d-b\,d)\,(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1)}+\frac{\mathrm{atan}\left(\left(e^{2\,c}\,e^{2\,d\,x}\,\left(\frac{4}{d\,(a-b)^2\,\sqrt{b^4}\,(a^2-2\,a\,b+b^2)}+\frac{(2\,a-b)\,\left(2\,a^3\,d\,\sqrt{b^4}-b^3\,d\,\sqrt{b^4}+4\,a\,b^2\,d\,\sqrt{b^4}-5\,a^2\,b\,d\,\sqrt{b^4}\right)}{b^4\,(a^2-2\,a\,b+b^2)\,\sqrt{-a\,d^2\,(a-b)^5}\,\sqrt{-a^6\,d^2+5\,a^5\,b\,d^2-10\,a^4\,b^2\,d^2+10\,a^3\,b^3\,d^2-5\,a^2\,b^4\,d^2+a\,b^5\,d^2}}\right)+\frac{(2\,a-b)\,\left(b^3\,d\,\sqrt{b^4}-2\,a\,b^2\,d\,\sqrt{b^4}+a^2\,b\,d\,\sqrt{b^4}\right)}{b^4\,(a^2-2\,a\,b+b^2)\,\sqrt{-a\,d^2\,(a-b)^5}\,\sqrt{-a^6\,d^2+5\,a^5\,b\,d^2-10\,a^4\,b^2\,d^2+10\,a^3\,b^3\,d^2-5\,a^2\,b^4\,d^2+a\,b^5\,d^2}}\right)\,\left(\frac{b^3\,\sqrt{-a^6\,d^2+5\,a^5\,b\,d^2-10\,a^4\,b^2\,d^2+10\,a^3\,b^3\,d^2-5\,a^2\,b^4\,d^2+a\,b^5\,d^2}}{2}-a\,b^2\,\sqrt{-a^6\,d^2+5\,a^5\,b\,d^2-10\,a^4\,b^2\,d^2+10\,a^3\,b^3\,d^2-5\,a^2\,b^4\,d^2+a\,b^5\,d^2}+\frac{a^2\,b\,\sqrt{-a^6\,d^2+5\,a^5\,b\,d^2-10\,a^4\,b^2\,d^2+10\,a^3\,b^3\,d^2-5\,a^2\,b^4\,d^2+a\,b^5\,d^2}}{2}\right)\right)\,\sqrt{b^4}}{\sqrt{-a^6\,d^2+5\,a^5\,b\,d^2-10\,a^4\,b^2\,d^2+10\,a^3\,b^3\,d^2-5\,a^2\,b^4\,d^2+a\,b^5\,d^2}}+\frac{2\,b}{(e^{2\,c+2\,d\,x}+1)\,(a-b)\,(a\,d-b\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] 8/(3*(a*d - b*d)*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x
) + 1)) - 4/((a*d - b*d)*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (at
an((exp(2*c)*exp(2*d*x)*(4/(d*(a - b)^2*(b^4)^(1/2)*(a^2 - 2*a*b + b^2)) +
((2*a - b)*(2*a^3*d*(b^4)^(1/2) - b^3*d*(b^4)^(1/2) + 4*a*b^2*d*(b^4)^(1/2)
 - 5*a^2*b*d*(b^4)^(1/2)))/(b^4*(a^2 - 2*a*b + b^2)*(-a*d^2*(a - b)^5)^(1/2
)*(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 - 10*
a^4*b^2*d^2)^(1/2))) + ((2*a - b)*(b^3*d*(b^4)^(1/2) - 2*a*b^2*d*(b^4)^(1/2
) + a^2*b*d*(b^4)^(1/2)))/(b^4*(a^2 - 2*a*b + b^2)*(-a*d^2*(a - b)^5)^(1/2)
*(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 - 10*a
^4*b^2*d^2)^(1/2)))*((b^3*(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2 - 5*a^2*b^4*d^
2 + 10*a^3*b^3*d^2 - 10*a^4*b^2*d^2)^(1/2))/2 - a*b^2*(a*b^5*d^2 - a^6*d^2
+ 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 - 10*a^4*b^2*d^2)^(1/2) + (a
^2*b*(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 -
10*a^4*b^2*d^2)^(1/2))/2))*(b^4)^(1/2))/(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2
- 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 - 10*a^4*b^2*d^2)^(1/2) + (2*b)/((exp(2*c
+ 2*d*x) + 1)*(a - b)*(a*d - b*d))
```

3.327 $\int \frac{\mathbf{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{(3a^2-10ab+15b^2)\,\mathrm{ArcTan}(\sinh(c+dx))}{8(a-b)^3d}-\frac{b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^3d}+\frac{(3a-7b)\mathrm{sech}(c+dx)\tanh(c+}{8(a-b)^2d}$$

```
[Out] 1/8*(3*a^2-10*a*b+15*b^2)*arctan(sinh(d*x+c))/(a-b)^3/d-b^(5/2)*arctan(sinh
(d*x+c)*b^(1/2)/a^(1/2))/(a-b)^3/d/a^(1/2)+1/8*(3*a-7*b)*sech(d*x+c)*tanh(d
*x+c)/(a-b)^2/d+1/4*sech(d*x+c)^3*tanh(d*x+c)/(a-b)/d
```

Rubi [A]
time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3269, 425, 541, 536, 209, 211}

$$\frac{(3a^2-10ab+15b^2)\,\mathrm{ArcTan}(\sinh(c+dx))}{8d(a-b)^3}-\frac{b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^3}+\frac{\tanh(c+dx)\mathrm{sech}^3(c+dx)}{4d(a-b)}+\frac{(3a-7b)\tanh(c+dx)\mathrm{sech}(c+dx)}{8d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((3*a^2 - 10*a*b + 15*b^2)*ArcTan[Sinh[c + d*x]])/(8*(a - b)^3*d) - (b^(5/2
)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^3*d) + ((3*a -
7*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*(a - b)^2*d) + (Sech[c + d*x]^3*Tanh[c
 + d*x])/(4*(a - b)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d} - \frac{\text{Subst}\left(\int \frac{-3a+4b-3bx^2}{(1+x^2)^2(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{4(a-b)d}\\
&= \frac{(3a-7b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8(a-b)^2d} + \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d} + \text{Subst}\Big(\int 3a\\
&= \frac{(3a-7b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8(a-b)^2d} + \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d} - b^3\text{Subst}\Big(\int\\
&= \frac{(3a^2-10ab+15b^2)\tan^{-1}(\sinh(c+dx))}{8(a-b)^3d} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^3d} + (3a -
\end{aligned}
$$

Mathematica [A]

time = 0.56, size = 139, normalized size = 1.01

$$\frac{8b^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{b}}\right)+2\sqrt{a}\,(3a^2-10ab+15b^2)\,\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{a}\,(3a^2-10ab+7b^2)\,\text{sech}(c+dx)\tanh(c+dx)+2\sqrt{a}\,(a-b)^2\text{sech}^3(c+dx)\tanh(c+dx)}{8\sqrt{a}\,(a-b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (8*b^(5/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*Sqrt[a]*(3*a^2 - 10*
a*b + 15*b^2)*ArcTan[Tanh[(c + d*x)/2]] + Sqrt[a]*(3*a^2 - 10*a*b + 7*b^2)*
Sech[c + d*x]*Tanh[c + d*x] + 2*Sqrt[a]*(a - b)^2*Sech[c + d*x]^3*Tanh[c +
d*x])/(8*Sqrt[a]*(a - b)^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(124) = 248$.

time = 2.12, size = 352, normalized size = 2.55

method	result
derivativedivides	$-\frac{2b^3a\Bigg(\frac{(-a+\sqrt{-b(a-b)}+b)\arctan\left(\frac{a\tanh(\frac{dx}{2}+\frac{c}{2})}{\sqrt{(2\sqrt{-b(a-b)}-a+2b)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{(2\sqrt{-b(a-b)}-a+2b)a}}-\frac{(a+\sqrt{-b(a-b)}-b}{2a\sqrt{-b(a-}}{(a-b)^3}$
default	$-\frac{2b^3a\Bigg(\frac{(-a+\sqrt{-b(a-b)}+b)\arctan\left(\frac{a\tanh(\frac{dx}{2}+\frac{c}{2})}{\sqrt{(2\sqrt{-b(a-b)}-a+2b)a}}\right)}{2a\sqrt{-b(a-b)}\sqrt{(2\sqrt{-b(a-b)}-a+2b)a}}-\frac{(a+\sqrt{-b(a-b)}-b}{2a\sqrt{-b(a-}}{(a-b)^3}$
risch	$\frac{e^{dx+c}(3a\,e^{6dx+6c}-7b\,e^{6dx+6c}+11a\,e^{4dx+4c}-15b\,e^{4dx+4c}-11a\,e^{2dx+2c}+15b\,e^{2dx+2c}-3a+7b)}{4d(a-b)^2(1+e^{2dx+2c})^4}+\frac{3i\ln(e^{dx+c}+i)a^2}{8(a-b)^3d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^3/(a-b)^3*a*(1/2*(-a+(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2))-1/2*(a+(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(
-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)+a-2*b)*a)^(1/2)))+2/(a-b)^3*(((-5/8*a^2+7/4*a*b-9/8*b^2)*tanh(1/2*d
*x+1/2*c)^7+(3/8*a^2-1/4*a*b-1/8*b^2)*tanh(1/2*d*x+1/2*c)^5+(-3/8*a^2+1/4*a
*b+1/8*b^2)*tanh(1/2*d*x+1/2*c)^3+(5/8*a^2-7/4*a*b+9/8*b^2)*tanh(1/2*d*x+1/
2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^4+1/8*(3*a^2-10*a*b+15*b^2)*arctan(tanh(1/2
*d*x+1/2*c))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*a^2*e^c - 10*a*b*e^c + 15*b^2*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^3*d
 - 3*a^2*b*d + 3*a*b^2*d - b^3*d) + 1/4*((3*a*e^(7*c) - 7*b*e^(7*c))*e^(7*d
*x) + (11*a*e^(5*c) - 15*b*e^(5*c))*e^(5*d*x) - (11*a*e^(3*c) - 15*b*e^(3*c
))*e^(3*d*x) - (3*a*e^c - 7*b*e^c)*e^(d*x))/(a^2*d - 2*a*b*d + b^2*d + (a^2
*d*e^(8*c) - 2*a*b*d*e^(8*c) + b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^2*d*e^(6*c)
- 2*a*b*d*e^(6*c) + b^2*d*e^(6*c))*e^(6*d*x) + 6*(a^2*d*e^(4*c) - 2*a*b*d*e
^(4*c) + b^2*d*e^(4*c))*e^(4*d*x) + 4*(a^2*d*e^(2*c) - 2*a*b*d*e^(2*c) + b^
2*d*e^(2*c))*e^(2*d*x)) - 32*integrate(1/16*(b^3*e^(3*d*x + 3*c) + b^3*e^(d
*x + c))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4 + (a^3*b*e^(4*c) - 3*a^2*b^2*e^
(4*c) + 3*a*b^3*e^(4*c) - b^4*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 7*a^3
*b*e^(2*c) + 9*a^2*b^2*e^(2*c) - 5*a*b^3*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)),
 x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2827 vs. $2(124) = 248$.
time = 0.47, size = 5500, normalized size = 39.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*((3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 - 10*a*b + 7*b^2)
*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 - 10*a*b + 7*b^2)*sinh(d*x + c)^7 +
 (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 - 10*a*b + 7*b^2)*
cosh(d*x + c)^2 + 11*a^2 - 26*a*b + 15*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 -
 10*a*b + 7*b^2)*cosh(d*x + c)^3 + (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^4 - (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^3 + (35*(3*a^2
```

```
 - 10*a*b + 7*b^2)*cosh(d*x + c)^4 + 10*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x
 + c)^2 - 11*a^2 + 26*a*b - 15*b^2)*sinh(d*x + c)^3 + (21*(3*a^2 - 10*a*b +
 7*b^2)*cosh(d*x + c)^5 + 10*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^3 - 3
*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^2*cosh(d*
x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*b^
2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^6 + 6*b^2
*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x
 + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 + 30*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(
d*x + c)^4 + 4*b^2*cosh(d*x + c)^2 + 8*(7*b^2*cosh(d*x + c)^5 + 10*b^2*cosh
(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^2*cosh(d*x + c)
^6 + 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2
+ b^2 + 8*(b^2*cosh(d*x + c)^7 + 3*b^2*cosh(d*x + c)^5 + 3*b^2*cosh(d*x + c
)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log((b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d
*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d
*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 +
 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) +
(3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4
 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh
(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh
(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + ((3*a^2 - 10*a
*b + 15*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)*si
nh(d*x + c)^7 + (3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^8 + 4*(3*a^2 - 10*a
*b + 15*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)
^2 + 3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 - 10*a*b + 15*b
^2)*cosh(d*x + c)^3 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 6*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 - 10*a*b
 + 15*b^2)*cosh(d*x + c)^4 + 30*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 +
 9*a^2 - 30*a*b + 45*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 - 10*a*b + 15*b^2)*
cosh(d*x + c)^5 + 10*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 -
 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^2 - 10*a*b + 15*b
^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^6 + 15*(
3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^4 + 9*(3*a^2 - 10*a*b + 15*b^2)*cosh
(d*x + c)^2 + 3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^2 + 3*a^2 - 10*a*b + 1
5*b^2 + 8*((3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^7 + 3*(3*a^2 - 10*a*b +
15*b^2)*cosh(d*x + c)^5 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + (3*
a^2 - 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) +
 sinh(d*x + c)) - (3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c) + (7*(3*a^2 - 10*a
*b + 7*b^2)*cosh(d*x + c)^6 + 5*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^4
- 3*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^2 - 3*a^2 + 10*a*b - 7*b^2)*si
nh(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a^3 -
3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 - 3*a^2*b +
 3*a*b^2 - b^3)*d*sinh(d*x + c)^8 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos
h(d*x + c)^6 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + (a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*sinh(d*x + c)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^
```

```
2 - b^3)*d*cosh(d*x + c)^4 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*
x + c)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(35*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^4 + 30*(a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*d)*sinh(d*x + c)^4 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2
+ 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^5 + 10*(a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos
h(d*x + c)^6 + 15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^4 + 9*(a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d)*sinh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d + 8*((a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*d*cosh(d*x + c)^5 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^3 +
 (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/4*((3*a
^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x
+ c)*sinh(d*x + c)^6 + (3*a^2 - 10*a*b + 7*b^2)...
```

Sympy **[F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*sinh(c + d*x)**2), x)
```

Giac **[F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad **[B]**

time = 12.05, size = 2500, normalized size = 18.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^5*(a + b*sinh(c + d*x)^2)),x)

[Out] (4*exp(c + d*x))/((a*d - b*d)*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*
exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (6*exp(c + d*x))/((a*d - b*d)*(
3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (atan((e
xp(d*x)*exp(c)*(243*a^12*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 1
5*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) + 3840*b^12*(a^6*d^2
 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 +
15*a^4*b^2*d^2)^(1/2) - 110560*a*b^11*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*
a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) - 4050*
a^11*b*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20
*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) + 976143*a^2*b^10*(a^6*d^2 + b^6*d^2 -
 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d
^2)^(1/2) - 2740050*a^3*b^9*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2
+ 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) + 4252775*a^4*b^8
*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b
^3*d^2 + 15*a^4*b^2*d^2)^(1/2) - 4316760*a^5*b^7*(a^6*d^2 + b^6*d^2 - 6*a*b
^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1
/2) + 3087390*a^6*b^6*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a
^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) - 1608364*a^7*b^5*(a^6*
d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2
 + 15*a^4*b^2*d^2)^(1/2) + 615750*a^8*b^4*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2
- 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) - 1
71000*a^9*b^3*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d
^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) + 33075*a^10*b^2*(a^6*d^2 + b^6
*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4
*b^2*d^2)^(1/2)))/(81*a^13*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*
a^2*b^2)^(1/2) - 256*b^13*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a
^2*b^2)^(1/2) - 82593*a^2*b^11*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 +
190*a^2*b^2)^(1/2) + 343611*a^3*b^10*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*
b^4 + 190*a^2*b^2)^(1/2) - 788535*a^4*b^9*d*(9*a^4 - 60*a^3*b - 300*a*b^3 +
 225*b^4 + 190*a^2*b^2)^(1/2) + 1157013*a^5*b^8*d*(9*a^4 - 60*a^3*b - 300*a
*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) - 1173354*a^6*b^7*d*(9*a^4 - 60*a^3*b -
 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) + 857934*a^7*b^6*d*(9*a^4 - 60*a^
3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) - 461358*a^8*b^5*d*(9*a^4 -
60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) + 182890*a^9*b^4*d*(9*a
^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) - 52581*a^10*b^3*d
*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) + 10503*a^11*
b^2*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) + 7968*a
*b^12*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) - 1323
*a^12*b*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2)))*(9
*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2))/(4*(a^6*d^2 + b
^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a
^4*b^2*d^2)^(1/2)) - ((2*atan((b^3*exp(d*x)*exp(c)*(a*d^2*(a - b)^6)^(1/2))
/(2*a*d*(a - b)^3*(b^5)^(1/2))) - 2*atan((exp(d*x)*exp(c)*((4*(4032*a^5*d*(
```

```
b^5)^(5/2) - 74990*a^10*d*(b^5)^(3/2) + 18*a^15*d*(b^5)^(1/2) + 32*a*b^9*d*
(b^5)^(3/2) + 288*a^9*b*d*(b^5)^(3/2) - 282*a^14*b*d*(b^5)^(1/2) - 288*a^2*
b^8*d*(b^5)^(3/2) + 1152*a^3*b^7*d*(b^5)^(3/2) - 2688*a^4*b^6*d*(b^5)^(3/2)
 - 4032*a^6*b^4*d*(b^5)^(3/2) + 2688*a^7*b^3*d*(b^5)^(3/2) - 1152*a^8*b^2*d
*(b^5)^(3/2) - 450*a^2*b^13*d*(b^5)^(1/2) + 4650*a^3*b^12*d*(b^5)^(1/2) - 2
1980*a^4*b^11*d*(b^5)^(1/2) + 62940*a^5*b^10*d*(b^5)^(1/2) - 121878*a^6*b^9
*d*(b^5)^(1/2) + 168702*a^7*b^8*d*(b^5)^(1/2) - 172008*a^8*b^7*d*(b^5)^(1/2
) + 131112*a^9*b^6*d*(b^5)^(1/2) + 31878*a^11*b^4*d*(b^5)^(1/2) - 9852*a^12
*b^3*d*(b^5)^(1/2) + 2108*a^13*b^2*d*(b^5)^(1/2)))/(a*b^4*(a - b)^7*(a*b -
a^2)*(a*d^2*(a - b)^6)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a^4 - 4*a^3*b
 - 4*a*b^3 + b^4 + 6*a^2*b^2)*(225*a*b^4 - 60*a^4*b + 9*a^5 - 16*b^5 - 300*
a^2*b^3 + 190*a^3*b^2)*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 +
 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)*(a^6 - 6*a^5*b - 6
*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (2*(16*b^14*(a^7*d^
2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d
^2 + 15*a^5*b^2*d^2)^(1/2) - 321*a*b^13*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2
- 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) +
 1890*a^2*b^12*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*
b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) - 5685*a^3*b^11*(a^7*d^2 +
 a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2
+ 15*a^5*b^2*d^2)^(1/2) + 10440*a^4*b^10*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2
 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)
- 12690*a^5*b^9*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3
*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/...
```

3.328 $\int \frac{\mathbf{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=126

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{7/2}d}+\frac{(a^2-3ab+3b^2)\tanh(c+dx)}{(a-b)^3d}-\frac{(2a-3b)\tanh^3(c+dx)}{3(a-b)^2d}+\frac{\tanh^5(c+dx)}{5(a-b)d}$$

```
[Out] -b^3*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/(a-b)^(7/2)/d/a^(1/2)+(a^2-3*
a*b+3*b^2)*tanh(d*x+c)/(a-b)^3/d-1/3*(2*a-3*b)*tanh(d*x+c)^3/(a-b)^2/d+1/5*
tanh(d*x+c)^5/(a-b)/d
```

Rubi [A]
time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3270, 398, 214}

$$\frac{(a^2-3ab+3b^2)\tanh(c+dx)}{d(a-b)^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{7/2}}+\frac{\tanh^5(c+dx)}{5d(a-b)}-\frac{(2a-3b)\tanh^3(c+dx)}{3d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7/2)
*d)) + ((a^2 - 3*a*b + 3*b^2)*Tanh[c + d*x])/((a - b)^3*d) - ((2*a - 3*b)*T
anh[c + d*x]^3)/(3*(a - b)^2*d) + Tanh[c + d*x]^5/(5*(a - b)*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
```

```
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{(a-b)^3} - \frac{(2a-3b)x^2}{(a-b)^2} + \frac{x^4}{a-b} - \frac{b^3}{(a-b)^3(a-(a-b)x^2)}\right)\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a^2-3ab+3b^2)\tanh(c+dx)}{(a-b)^3d} - \frac{(2a-3b)\tanh^3(c+dx)}{3(a-b)^2d} + \frac{\tanh^5(c+dx)}{5(a-b)d} - b^{3}$$

$$= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{7/2}d} + \frac{(a^2-3ab+3b^2)\tanh(c+dx)}{(a-b)^3d} - \frac{(2a-3b)\,\mathrm{t}}{3(a}$$

Mathematica [A]
time = 0.68, size = 119, normalized size = 0.94

$$\frac{-\frac{15b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{7/2}} + \frac{\left(8a^2-26ab+33b^2+(4a^2-13ab+9b^2)\operatorname{sech}^2(c+dx)+3(a-b)^2\operatorname{sech}^4(c+dx)\right)\tanh(c+dx)}{(a-b)^3}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((-15*b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7
/2)) + ((8*a^2 - 26*a*b + 33*b^2 + (4*a^2 - 13*a*b + 9*b^2)*Sech[c + d*x]^2
 + 3*(a - b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(a - b)^3)/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(114) = 228$.
time = 1.84, size = 346, normalized size = 2.75

method	result
risch	$-\frac{2\left(15b^2\mathrm{e}^{8dx+8c}-30ab\,\mathrm{e}^{6dx+6c}+90b^2\mathrm{e}^{6dx+6c}+80a^2\mathrm{e}^{4dx+4c}-230ab\,\mathrm{e}^{4dx+4c}+240b^2\mathrm{e}^{4dx+4c}+40a^2\mathrm{e}^{2dx+2c}-130ab\,\mathrm{e}^{2dx+}\right.}{15d(a-b)^3\left(1+\mathrm{e}^{2dx+2c}\right)^5}$

derivativedivides	$-\frac{2\left(\left(-a^2+3ab-3b^2\right)\left(\tanh^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{4}{3}a^2+\frac{16}{3}ab-8b^2\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{58}{15}a^2+\frac{166}{15}ab-\frac{66}{5}b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{4}{3}a\right.\right.}{(a-b)^3\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
default	$-\frac{2\left(\left(-a^2+3ab-3b^2\right)\left(\tanh^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{4}{3}a^2+\frac{16}{3}ab-8b^2\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{58}{15}a^2+\frac{166}{15}ab-\frac{66}{5}b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{4}{3}a\right.\right.}{(a-b)^3\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/(a-b)^3*((-a^2+3*a*b-3*b^2)*tanh(1/2*d*x+1/2*c)^9+(-4/3*a^2+16/3*a*
b-8*b^2)*tanh(1/2*d*x+1/2*c)^7+(-58/15*a^2+166/15*a*b-66/5*b^2)*tanh(1/2*d*
x+1/2*c)^5+(-4/3*a^2+16/3*a*b-8*b^2)*tanh(1/2*d*x+1/2*c)^3+(-a^2+3*a*b-3*b^
2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^5+2*b^3/(a-b)^3*a*(-1/2*(
(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2*((
-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2895 vs. $2(114) = 228$.
time = 0.45, size = 6046, normalized size = 47.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(60*(a^2*b^2 - a*b^3)*cosh(d*x + c)^8 + 480*(a^2*b^2 - a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^7 + 60*(a^2*b^2 - a*b^3)*sinh(d*x + c)^8 - 120*(a^3*b
 - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^6 - 120*(a^3*b - 4*a^2*b^2 + 3*a*b^3
- 14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 240*(14*(a^2*b^2
- a*b^3)*cosh(d*x + c)^3 - 3*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c))*s
inh(d*x + c)^5 + 40*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c
)^4 + 40*(105*(a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 8*a^4 - 31*a^3*b + 47*a^2
*b^2 - 24*a*b^3 - 45*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 32*a^4 - 136*a^3*b + 236*a^2*b^2 - 132*a*b^3 + 160*(21*(a^2*b^2
- a*b^3)*cosh(d*x + c)^5 - 15*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3
 + (8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^
3 + 40*(4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3)*cosh(d*x + c)^2 + 40*(42*
(a^2*b^2 - a*b^3)*cosh(d*x + c)^6 - 45*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d
*x + c)^4 + 4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3 + 6*(8*a^4 - 31*a^3*b
+ 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 15*(b^3*cosh(d*
x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*sinh(d*x + c)^10 + 5
*b^3*cosh(d*x + c)^8 + 10*b^3*cosh(d*x + c)^6 + 5*(9*b^3*cosh(d*x + c)^2 +
b^3)*sinh(d*x + c)^8 + 40*(3*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(
d*x + c)^7 + 10*b^3*cosh(d*x + c)^4 + 10*(21*b^3*cosh(d*x + c)^4 + 14*b^3*c
osh(d*x + c)^2 + b^3)*sinh(d*x + c)^6 + 4*(63*b^3*cosh(d*x + c)^5 + 70*b^3*
cosh(d*x + c)^3 + 15*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 5*b^3*cosh(d*x +
c)^2 + 10*(21*b^3*cosh(d*x + c)^6 + 35*b^3*cosh(d*x + c)^4 + 15*b^3*cosh(d*
x + c)^2 + b^3)*sinh(d*x + c)^4 + 40*(3*b^3*cosh(d*x + c)^7 + 7*b^3*cosh(d*
x + c)^5 + 5*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c)^3 + b^3
 + 5*(9*b^3*cosh(d*x + c)^8 + 28*b^3*cosh(d*x + c)^6 + 30*b^3*cosh(d*x + c)
^4 + 12*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 10*(b^3*cosh(d*x + c)^
9 + 4*b^3*cosh(d*x + c)^7 + 6*b^3*cosh(d*x + c)^5 + 4*b^3*cosh(d*x + c)^3 +
 b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4
 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b
^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)
^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x
+ c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c
) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*
cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x +
c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x +
c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 80*(6*(a^2*b^2 - a*b^
```

```
3)*cosh(d*x + c)^7 - 9*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^5 + 2*(8
*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 17*a^3*
b + 28*a^2*b^2 - 15*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 - 4*a^4*b +
6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^10 + 10*(a^5 - 4*a^4*b + 6*a
^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 - 4*a^4*
b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^10 + 5*(a^5 - 4*a^4*b +
6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^8 + 5*(9*(a^5 - 4*a^4*b + 6*
a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2
 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^8 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 -
4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 40*(3*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4
*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^
3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*(a^5 - 4*a^4*b + 6*a^3
*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 14*(a^5 - 4*a^4*b + 6*a^3*b^2
 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^
2*b^3 + a*b^4)*d)*sinh(d*x + c)^6 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b
^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(63*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^
3 + a*b^4)*d*cosh(d*x + c)^5 + 70*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 +
a*b^4)*d*cosh(d*x + c)^3 + 15*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^
4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4
*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 35*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2
*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 15*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^
4)*d)*sinh(d*x + c)^4 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d
*cosh(d*x + c)^2 + 40*(3*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*
cosh(d*x + c)^7 + 7*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(
d*x + c)^5 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x +
 c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 5*(9*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh
(d*x + c)^8 + 28*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x
 + c)^6 + 30*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c
)^4 + 12*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(114) = 228.

time = 0.69, size = 253, normalized size = 2.01

$$-\frac{\frac{15\,b^3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{-a^2+ab}}+\frac{2\,(15\,b^2e^{(8\,dx+8\,c)}-30\,abe^{(6\,dx+6\,c)}+90\,b^2e^{(6\,dx+6\,c)}+80\,a^2e^{(4\,dx+4\,c)}-230\,abe^{(4\,dx+4\,c)}+240\,b^2e^{(4\,dx+4\,c)}+40\,a^2e^{(2\,dx+2\,c)}-130\,abe^{(2\,dx+2\,c)}+150\,b^2e^{(2\,dx+2\,c)}+8\,a^2-26\,ab+33\,b^2)}{(a^3-3\,a^2b+3\,ab^2-b^3)\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*b^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-a^2 + a*b)) + 2*(15*b^2*e^(8*d*x + 8*c
) - 30*a*b*e^(6*d*x + 6*c) + 90*b^2*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x + 4*c
) - 230*a*b*e^(4*d*x + 4*c) + 240*b^2*e^(4*d*x + 4*c) + 40*a^2*e^(2*d*x + 2
*c) - 130*a*b*e^(2*d*x + 2*c) + 150*b^2*e^(2*d*x + 2*c) + 8*a^2 - 26*a*b +
33*b^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(e^(2*d*x + 2*c) + 1)^5))/d
```

Mupad [B]
time = 2.89, size = 1152, normalized size = 9.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^6*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] 16/((a*d - b*d)*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*
x) + exp(8*c + 8*d*x) + 1)) - 32/(5*(a*d - b*d)*(5*exp(2*c + 2*d*x) + 10*ex
p(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d
*x) + 1)) + (atan((exp(2*c)*exp(2*d*x)*((4*b)/(d*(a - b)^3*(b^6)^(1/2)*(3*a
*b^2 - 3*a^2*b + a^3 - b^3)) + ((2*a - b)*(2*a^4*d*(b^6)^(1/2) + b^4*d*(b^6
)^(1/2) - 5*a*b^3*d*(b^6)^(1/2) - 7*a^3*b*d*(b^6)^(1/2) + 9*a^2*b^2*d*(b^6)
^(1/2)))/(b^5*(-a*d^2*(a - b)^7)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a*b
^7*d^2 - a^8*d^2 + 7*a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^
4*d^2 + 35*a^5*b^3*d^2 - 21*a^6*b^2*d^2)^(1/2))) - ((2*a - b)*(b^4*d*(b^6)^
(1/2) - 3*a*b^3*d*(b^6)^(1/2) - a^3*b*d*(b^6)^(1/2) + 3*a^2*b^2*d*(b^6)^(1/
2)))/(b^5*(-a*d^2*(a - b)^7)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a*b^7*d
^2 - a^8*d^2 + 7*a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^4*d^
2 + 35*a^5*b^3*d^2 - 21*a^6*b^2*d^2)^(1/2)))*((b^4*(a*b^7*d^2 - a^8*d^2 + 7
*a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^4*d^2 + 35*a^5*b^3*d
^2 - 21*a^6*b^2*d^2)^(1/2))/2 + (3*a^2*b^2*(a*b^7*d^2 - a^8*d^2 + 7*a^7*b*d
^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^4*d^2 + 35*a^5*b^3*d^2 - 21*
a^6*b^2*d^2)^(1/2))/2 - (3*a*b^3*(a*b^7*d^2 - a^8*d^2 + 7*a^7*b*d^2 - 7*a^2
*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^4*d^2 + 35*a^5*b^3*d^2 - 21*a^6*b^2*d^
2)^(1/2))/2 - (a^3*b*(a*b^7*d^2 - a^8*d^2 + 7*a^7*b*d^2 - 7*a^2*b^6*d^2 + 2
1*a^3*b^5*d^2 - 35*a^4*b^4*d^2 + 35*a^5*b^3*d^2 - 21*a^6*b^2*d^2)^(1/2))/2)
)*(b^6)^(1/2))/(a*b^7*d^2 - a^8*d^2 + 7*a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*
b^5*d^2 - 35*a^4*b^4*d^2 + 35*a^5*b^3*d^2 - 21*a^6*b^2*d^2)^(1/2) - (2*b^2)
```

```
/((exp(2*c + 2*d*x) + 1)*(a - b)^2*(a*d - b*d)) + (4*(a*b - b^2))/((a - b)^
2*(a*d - b*d)*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (8*(4*a - 3*b)
)/(3*(a - b)*(a*d - b*d)*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) + 1))
```

3.329 $\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=158

$$-\frac{(4a-5b)x}{2b^3}+\frac{(a-b)^{3/2}(4a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{(a-b)(}{2ab^2d\,(a-}$$

```
[Out] -1/2*(4*a-5*b)*x/b^3+1/2*(a-b)^(3/2)*(4*a+b)*arctanh((a-b)^(1/2)*tanh(d*x+c
)/a^(1/2))/a^(3/2)/b^3/d+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d/(a-(a-b)*tanh(d*x+
c)^2)+1/2*(a-b)*(2*a-b)*tanh(d*x+c)/a/b^2/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.19, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3270, 425, 541, 536, 212, 214}

$$\frac{(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}-\frac{x(4a-5b)}{2b^3}+\frac{(2a-b)(a-b)\tanh(c+dx)}{2ab^2d\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((4*a - 5*b)*x)/b^3 + ((a - b)^(3/2)*(4*a + b)*ArcTanh[(Sqrt[a - b]*Ta
nh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2
*b*d*(a - (a - b)*Tanh[c + d*x]^2)) + ((a - b)*(2*a - b)*Tanh[c + d*x])/(2*
a*b^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a-(a-b)x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\Big(\int \frac{-a+2b-3(a-b)x^2}{\left(1-x^2\right)\left(a+(-a+b)x^2\right)^2}\,dx,x,\tanh(c}{2bd}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{(a-b)(2a-b)\tanh(c+dx)}{2ab^2d\left(a-(a-b)\tanh^2(c+dx)\right)} - \text{Sub}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{(a-b)(2a-b)\tanh(c+dx)}{2ab^2d\left(a-(a-b)\tanh^2(c+dx)\right)} - (4a$$

$$= -\frac{(4a-5b)x}{2b^3} + \frac{(a-b)^{3/2}(4a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{\cosh(c}{2bd\,(a-}$$

Mathematica [A]
time = 0.42, size = 118, normalized size = 0.75

$$\frac{2(-4a+5b)(c+dx)+\frac{2(a-b)^{3/2}(4a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+b\sinh(2(c+dx))+\frac{2(a-b)^2b\sinh(2(c+dx))}{a(2a-b+b\cosh(2(c+dx)))}}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*(-4*a + 5*b)*(c + d*x) + (2*(a - b)^(3/2)*(4*a + b)*ArcTanh[(Sqrt[a - b]
*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + b*Sinh[2*(c + d*x)] + (2*(a - b)^2*b*Si
nh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)])))/(4*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(142) = 284$.
time = 2.00, size = 425, normalized size = 2.69

method	result
derivativedivides	$-\frac{2\left(\frac{-\frac{b\left(a^{2}-2ab+b^{2}\right)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b\left(a^{2}-2ab+b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{\left(4a^{3}-7a^{2}b+2a\,b^{2}+b^{3}\right)\left(\frac{\left(\sqrt{-b\left(a-b\right)}+b\right)\mathrm{arcta}}{2a\sqrt{-b\left(a-b\right)}}\right.}{}\right.}{}$

default	$-\frac{2\left(\frac{-\frac{b\left(a^{2}-2ab+b^{2}\right)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b\left(a^{2}-2ab+b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{\left(4a^{3}-7a^{2}b+2a\,b^{2}+b^{3}\right)\left(\frac{\left(\sqrt{-b\left(a-b\right)}+b\right)\mathrm{arc}}{2a\sqrt{-b\left(a-b\right)}}\right.}{}\right.}{}$
risch	$-\frac{2ax}{b^{3}}+\frac{5x}{2b^{2}}+\frac{\mathrm{e}^{2dx+2c}}{8b^{2}d}-\frac{\mathrm{e}^{-2dx-2c}}{8b^{2}d}-\frac{2a^{3}\mathrm{e}^{2dx+2c}-5a^{2}b\,\mathrm{e}^{2dx+2c}+4a\,b^{2}\mathrm{e}^{2dx+2c}-b^{3}\mathrm{e}^{2dx+2c}+a^{2}b-2a\,b^{2}+b^{3}}{b^{3}ad\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^3*((-1/2*b*(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*a
*b+b^2)/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/
2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(4*a^3-7*a^2*b+2*a*b^2+b^3)*(-1/2*(
(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2*((
-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))))+1/2/b^
2/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/b^2/(tanh(1/2*d*x+1/2*c)-1)+1/2*(4*a-5*b)/b
^3*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/b^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/b^2/(tan
h(1/2*d*x+1/2*c)+1)+1/2/b^3*(-4*a+5*b)*ln(tanh(1/2*d*x+1/2*c)+1))
```

<u>Maxima</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1682 vs. 2(144) = 288.
time = 0.42, size = 3629, normalized size = 22.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(a*b^2*cosh(d*x + c)^8 + 8*a*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + a*b^2
*sinh(d*x + c)^8 + 2*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x
 + c)^6 + 2*(14*a*b^2*cosh(d*x + c)^2 + 2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*
b^2)*d*x)*sinh(d*x + c)^6 + 4*(14*a*b^2*cosh(d*x + c)^3 + 3*(2*a^2*b - a*b^
2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(2*a^3 -
5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^4
 + 2*(35*a*b^2*cosh(d*x + c)^4 - 8*a^3 + 20*a^2*b - 16*a*b^2 + 4*b^3 - 4*(8
*a^3 - 14*a^2*b + 5*a*b^2)*d*x + 15*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a*b^2*cosh(d*x + c)^5 + 5*(2
*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^3 - 4*(2*a^3 - 5*
a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^3 - a*b^2 - 2*(6*a^2*b - 9*a*b^2 + 4*b^3 + 2*(4*a^2*b - 5*a*b^2
)*d*x)*cosh(d*x + c)^2 + 2*(14*a*b^2*cosh(d*x + c)^6 + 15*(2*a^2*b - a*b^2
- 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 - 6*a^2*b + 9*a*b^2 - 4*b^3 -
2*(4*a^2*b - 5*a*b^2)*d*x - 24*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 -
14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2*((4*a^2*b - 3
*a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)*s
inh(d*x + c)^5 + (4*a^2*b - 3*a*b^2 - b^3)*sinh(d*x + c)^6 + 2*(8*a^3 - 10*
a^2*b + a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^3 - 20*a^2*b + 2*a*b^2 + 2*b^3
 + 15*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(4*
a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(8*a^3 - 10*a^2*b + a*b^2 + b^3)
*cosh(d*x + c))*sinh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2
 + (15*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^4 + 4*a^2*b - 3*a*b^2 - b^3
+ 12*(8*a^3 - 10*a^2*b + a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*
(3*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 4*(8*a^3 - 10*a^2*b + a*b^2
+ b^3)*cosh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d
*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^
2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*
(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*
cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 +
```

```
 2*a^2 - a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(
d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*co
sh(d*x + c))*sinh(d*x + c) + b)) + 4*(2*a*b^2*cosh(d*x + c)^7 + 3*(2*a^2*b
- a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^5 - 8*(2*a^3 - 5*a^2*b +
 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^3 - (6*a^2
*b - 9*a*b^2 + 4*b^3 + 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x +
 c))/(a*b^4*d*cosh(d*x + c)^6 + 6*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a
*b^4*d*sinh(d*x + c)^6 + a*b^4*d*cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d*
cosh(d*x + c)^4 + (15*a*b^4*d*cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d)*si
nh(d*x + c)^4 + 4*(5*a*b^4*d*cosh(d*x + c)^3 + 2*(2*a^2*b^3 - a*b^4)*d*cosh
(d*x + c))*sinh(d*x + c)^3 + (15*a*b^4*d*cosh(d*x + c)^4 + a*b^4*d + 12*(2*
a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*a*b^4*d*cosh(d*x
 + c)^5 + a*b^4*d*cosh(d*x + c) + 4*(2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3)*
sinh(d*x + c)), 1/8*(a*b^2*cosh(d*x + c)^8 + 8*a*b^2*cosh(d*x + c)*sinh(d*x
 + c)^7 + a*b^2*sinh(d*x + c)^8 + 2*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2
)*d*x)*cosh(d*x + c)^6 + 2*(14*a*b^2*cosh(d*x + c)^2 + 2*a^2*b - a*b^2 - 2*
(4*a^2*b - 5*a*b^2)*d*x)*sinh(d*x + c)^6 + 4*(14*a*b^2*cosh(d*x + c)^3 + 3*
(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^
5 - 8*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*
cosh(d*x + c)^4 + 2*(35*a*b^2*cosh(d*x + c)^4 - 8*a^3 + 20*a^2*b - 16*a*b^2
 + 4*b^3 - 4*(8*a^3 - 14*a^2*b + 5*a*b^2)*d*x + 15*(2*a^2*b - a*b^2 - 2*(4*
a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a*b^2*cosh(d*
x + c)^5 + 5*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^3
- 4*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^3 - a*b^2 - 2*(6*a^2*b - 9*a*b^2 + 4*b^3 + 2*(4*
a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*a*b^2*cosh(d*x + c)^6 + 15*(2
*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 - 6*a^2*b + 9*a
*b^2 - 4*b^3 - 2*(4*a^2*b - 5*a*b^2)*d*x - 24*(2*a^3 - 5*a^2*b + 4*a*b^2 -
b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 -
4*((4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(4*a^2*b - 3*a*b^2 - b^3)*
cosh(d*x + c)*sinh(d*x + c)^5 + (4*a^2*b - 3*a*b^2 - b^3)*sinh(d*x + c)^6 +
 2*(8*a^3 - 10*a^2*b + a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^3 - 20*a^2*b +
2*a*b^2 + 2*b^3 + 15*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 4*(5*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(8*a^3 - 10*a^2*b
 + a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 ...
```

Sympy **[F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2)**2,x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(144) = 288.
time = 1.65, size = 305, normalized size = 1.93

$$-\frac{\frac{12\,(dx+c)(4\,a-5\,b)}{b^3}-\frac{3\,e^{(2\,dx+2\,c)}}{b^2}-\frac{12\,\left(4\,a^3-7\,a^2b+2\,ab^2+b^3\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,ab^3}-\frac{8\,a^2be^{(6\,dx+6\,c)}-10\,ab^2e^{(6\,dx+6\,c)}-16\,a^3e^{(4\,dx+4\,c)}+64\,a^2be^{(4\,dx+4\,c)}-79\,ab^2e^{(4\,dx+4\,c)}+24\,b^3e^{(4\,dx+4\,c)}-28\,a^2be^{(2\,dx+2\,c)}+44\,ab^2e^{(2\,dx+2\,c)}-24\,b^3e^{(2\,dx+2\,c)}-3\,ab^2}{\left(be^{(6\,dx+6\,c)}+4\,ae^{(4\,dx+4\,c)}-2\,be^{(4\,dx+4\,c)}+be^{(2\,dx+2\,c)}\right)ab^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(d*x + c)*(4*a - 5*b)/b^3 - 3*e^(2*d*x + 2*c)/b^2 - 12*(4*a^3 - 7
*a^2*b + 2*a*b^2 + b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2
+ a*b))/(sqrt(-a^2 + a*b)*a*b^3) - (8*a^2*b*e^(6*d*x + 6*c) - 10*a*b^2*e^(6
*d*x + 6*c) - 16*a^3*e^(4*d*x + 4*c) + 64*a^2*b*e^(4*d*x + 4*c) - 79*a*b^2*
e^(4*d*x + 4*c) + 24*b^3*e^(4*d*x + 4*c) - 28*a^2*b*e^(2*d*x + 2*c) + 44*a*
b^2*e^(2*d*x + 2*c) - 24*b^3*e^(2*d*x + 2*c) - 3*a*b^2)/((b*e^(6*d*x + 6*c)
 + 4*a*e^(4*d*x + 4*c) - 2*b*e^(4*d*x + 4*c) + b*e^(2*d*x + 2*c))*a*b^3))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2)^2, x)
```

3.330 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=104

$$-\frac{(3a^2-2ab-b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{\sinh(c+dx)}{b^2d}+\frac{(a-b)^2\sinh(c+dx)}{2ab^2d\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] -1/2*(3*a^2-2*a*b-b^2)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)/
d+sinh(d*x+c)/b^2/d+1/2*(a-b)^2*sinh(d*x+c)/a/b^2/d/(a+b*sinh(d*x+c)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3269, 398, 393, 211}

$$-\frac{(3a^2-2ab-b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a-b)^2\sinh(c+dx)}{2ab^2d\left(a+b\sinh^2(c+dx)\right)}+\frac{\sinh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((3*a^2 - 2*a*b - b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/
2)*b^(5/2)*d) + Sinh[c + d*x]/(b^2*d) + ((a - b)^2*Sinh[c + d*x])/(2*a*b^2*
d*(a + b*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a^2-b^2+2(a-b)bx^2}{b^2(a+bx^2)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{b^2d}-\frac{\text{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{b^2d}$$

$$= \frac{\sinh(c+dx)}{b^2d}+\frac{(a-b)^2\sinh(c+dx)}{2ab^2d\left(a+b\sinh^2(c+dx)\right)}-\frac{((a-b)(3a+b))\text{Subst}\left(\int \frac{1}{a+bx}\right.}{2ab^2d}$$

$$= -\frac{(a-b)(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{\sinh(c+dx)}{b^2d}+\frac{(a-b)^2\sinh(c}{2ab^2d\left(a+b\sinh^2\right.}$$

Mathematica [A]
time = 0.22, size = 106, normalized size = 1.02

$$\frac{-\frac{(-3a^2+2ab+b^2)\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{b}}\right)}{a^{3/2}}+2\sqrt{b}\,\sinh(c+dx)+\frac{2(a-b)^2\sqrt{b}\,\sinh(c+dx)}{a(2a-b+b\cosh(2(c+dx)))}}{2b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-(((-3*a^2 + 2*a*b + b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/a^(3/2)
) + 2*Sqrt[b]*Sinh[c + d*x] + (2*(a - b)^2*Sqrt[b]*Sinh[c + d*x])/(a*(2*a -
 b + b*Cosh[2*(c + d*x)])))/(2*b^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(92) = 184$.
time = 2.00, size = 337, normalized size = 3.24

method	result
derivativedivides	$\frac{-\frac{1}{b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2\left(\frac{\frac{\left(a^{2}-2ab+b^{2}\right)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{\left(a^{2}-2ab+b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{\left(3a^{2}-2ab\right.}{}\right.}{}}{}$
default	$\frac{-\frac{1}{b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2\left(\frac{\frac{\left(a^{2}-2ab+b^{2}\right)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{\left(a^{2}-2ab+b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{\left(3a^{2}-2ab\right.}{}\right.}{}}{}$
risch	$\frac{\mathrm{e}^{dx+c}}{2b^{2}d}-\frac{\mathrm{e}^{-dx-c}}{2b^{2}d}+\frac{\left(a^{2}-2ab+b^{2}\right)\mathrm{e}^{dx+c}\left(\mathrm{e}^{2dx+2c}-1\right)}{a\,b^{2}d\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)}-\frac{3a\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\,\mathrm{e}^{dx+c}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,d\,b^{2}}+\frac{\ln\left(\mathrm{e}^{2dx+2\right.}{2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^2/(tanh(1/2*d*x+1/2*c)+1)-1/b^2/(tanh(1/2*d*x+1/2*c)-1)-2/b^2*((1
/2*(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^3-1/2*(a^2-2*a*b+b^2)/a*tanh(1/2*d
*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*
d*x+1/2*c)^2+a)+1/2*(3*a^2-2*a*b-b^2)*(1/2*(-a+(-b*(a-b))^(1/2)+b)/a/(-b*(a
-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*(a+(-b*(a-b))^(1/2)-b)/a/(-b*(a
-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b*e^(6*d*x + 6*c) - a*b + (6*a^2*e^(4*c) - 7*a*b*e^(4*c) + 2*b^2*e^(
4*c))*e^(4*d*x) - (6*a^2*e^(2*c) - 7*a*b*e^(2*c) + 2*b^2*e^(2*c))*e^(2*d*x)
)/(a*b^3*d*e^(5*d*x + 5*c) + a*b^3*d*e^(d*x + c) + 2*(2*a^2*b^2*d*e^(3*c) -
 a*b^3*d*e^(3*c))*e^(3*d*x)) - 1/32*integrate(32*((3*a^2*e^(3*c) - 2*a*b*e^
(3*c) - b^2*e^(3*c))*e^(3*d*x) + (3*a^2*e^c - 2*a*b*e^c - b^2*e^c)*e^(d*x))
/(a*b^3*e^(4*d*x + 4*c) + a*b^3 + 2*(2*a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(
2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1442 vs. 2(92) = 184.
time = 0.55, size = 2739, normalized size = 26.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a^2*b^2*cosh(d*x + c)^6 + 12*a^2*b^2*cosh(d*x + c)*sinh(d*x + c)^5
+ 2*a^2*b^2*sinh(d*x + c)^6 + 2*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x +
c)^4 + 2*(15*a^2*b^2*cosh(d*x + c)^2 + 6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*sinh(
d*x + c)^4 - 2*a^2*b^2 + 8*(5*a^2*b^2*cosh(d*x + c)^3 + (6*a^3*b - 7*a^2*b^
2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(6*a^3*b - 7*a^2*b^2 + 2*a*
b^3)*cosh(d*x + c)^2 + 2*(15*a^2*b^2*cosh(d*x + c)^4 - 6*a^3*b + 7*a^2*b^2
- 2*a*b^3 + 6*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + ((3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(3*a^2*b - 2*a*b^2 - b
^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (3*a^2*b - 2*a*b^2 - b^3)*sinh(d*x + c)
^5 + 2*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^3 + 2*(6*a^3 - 7*a^2*b + b^3 +
 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(3*a^2
*b - 2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x +
```

```
c))*sinh(d*x + c)^2 + (3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c) + (5*(3*a^2*b
 - 2*a*b^2 - b^3)*cosh(d*x + c)^4 + 3*a^2*b - 2*a*b^2 - b^3 + 6*(6*a^3 - 7*
a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a*b)*log((b*cosh(d*x + c
)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*c
osh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*c
osh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^
3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2
- 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*
b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x
+ c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x
+ c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(3*a^2*b^2*cosh(d
*x + c)^5 + 2*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^3 - (6*a^3*b -
7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b^4*d*cosh(d*x + c)
^5 + 5*a^2*b^4*d*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*b^4*d*sinh(d*x + c)^5
+ a^2*b^4*d*cosh(d*x + c) + 2*(2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + 2*(
5*a^2*b^4*d*cosh(d*x + c)^2 + (2*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^3 + 2*
(5*a^2*b^4*d*cosh(d*x + c)^3 + 3*(2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sin
h(d*x + c)^2 + (5*a^2*b^4*d*cosh(d*x + c)^4 + a^2*b^4*d + 6*(2*a^3*b^3 - a^
2*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/2*(a^2*b^2*cosh(d*x + c)^6 + 6*
a^2*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*b^2*sinh(d*x + c)^6 + (6*a^3*b
- 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 + (15*a^2*b^2*cosh(d*x + c)^2 + 6*a^
3*b - 7*a^2*b^2 + 2*a*b^3)*sinh(d*x + c)^4 - a^2*b^2 + 4*(5*a^2*b^2*cosh(d*
x + c)^3 + (6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 -
 (6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2 + (15*a^2*b^2*cosh(d*x + c
)^4 - 6*a^3*b + 7*a^2*b^2 - 2*a*b^3 + 6*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^5
+ 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (3*a^2*b - 2*
a*b^2 - b^3)*sinh(d*x + c)^5 + 2*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^3 +
2*(6*a^3 - 7*a^2*b + b^3 + 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^3 + 2*(5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(6*a^3 -
7*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (3*a^2*b - 2*a*b^2 - b^3)*c
osh(d*x + c) + (5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^4 + 3*a^2*b - 2*a
*b^2 - b^3 + 6*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt
(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - ((3*a^2*b -
 2*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)
*sinh(d*x + c)^4 + (3*a^2*b - 2*a*b^2 - b^3)*sinh(d*x + c)^5 + 2*(6*a^3 - 7
*a^2*b + b^3)*cosh(d*x + c)^3 + 2*(6*a^3 - 7*a^2*b + b^3 + 5*(3*a^2*b - 2*a
*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(3*a^2*b - 2*a*b^2 - b^
3)*cosh(d*x + c)^3 + 3*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)
^2 + (3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c) + (5*(3*a^2*b - 2*a*b^2 - b^3)
*cosh(d*x + c)^4 + 3*a^2*b - 2*a*b^2 - b^3 + 6*(6*a^3 - 7*a^2*b + b^3)*cosh
(d*x + c)^2)*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*c
osh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c)
+ (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)) + 2*(3*a^
2*b^2*cosh(d*x + c)^5 + 2*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^3 -
```

```
 (6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b^4*d*c
osh(d*x + c)^5 + 5*a^2*b^4*d*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*b^4*d*sinh
(d*x + c)^5 + a^2*b^4*d*cosh(d*x + c) + 2*(2*a^3*b^3 - a^2*b^4)*d*cosh(d*x
+ c)^3 + 2*(5*a^2*b^4*d*cosh(d*x + c)^2 + (2*a^3*b^3 - a^2*b^4)*d)*sinh(d*x
 + c)^3 + 2*(5*a^2*b^4*d*cosh(d*x + c)^3 + 3*(2*a^3*b^3 - a^2*b^4)*d*cosh(d
*x + c))*sinh(d*x + c)^2 + (5*a^2*b^4*d*cosh(d*x + c)^4 + a^2*b^4*d + 6*(2*
a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^5}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2)^2, x)
```

3.331 $\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=100

$$\frac{x}{b^2}-\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}-\frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)}$$

```
[Out] x/b^2-1/2*(2*a+b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))*(a-b)^(1/2)/a^(3
/2)/b^2/d-1/2*(a-b)*tanh(d*x+c)/a/b/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3270, 425, 536, 212, 214}

$$-\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}-\frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] x/b^2 - (Sqrt[a - b]*(2*a + b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]
)/(2*a^(3/2)*b^2*d) - ((a - b)*Tanh[c + d*x])/(2*a*b*d*(a - (a - b)*Tanh[c
+ d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-(a-b)x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= -\frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-a-b+(-a+b)x^2}{(1-x^2)(a+(-a+b)x^2)}\,dx, x, \tanh(c\right.}{2abd}\\
&= -\frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{b^2d} - \frac{((}{}\\
&= \frac{x}{b^2} - \frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} - \frac{(a-b)\tanh(c+}{2abd\left(a-(a-b)\tanh^2\right.}
\end{aligned}
$$

Mathematica [A]
time = 0.48, size = 108, normalized size = 1.08

$$
\frac{2(c+dx) - \frac{(2a^2-ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}} + \frac{b(-a+b)\sinh(2(c+dx))}{a(2a-b+b\cosh(2(c+dx)))}}{2b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*(c + d*x) - ((2*a^2 - a*b - b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqr
t[a]])/(a^(3/2)*Sqrt[a - b]) + (b*(-a + b)*Sinh[2*(c + d*x)])/(a*(2*a - b +
 b*Cosh[2*(c + d*x)])))/(2*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(88) = 176$.

time = 1.84, size = 320, normalized size = 3.20

method	result
risch	$\frac{x}{b^{2}}+\frac{2 a^{2} {\mathrm e}^{2 d x +2 c}-3 a b \,{\mathrm e}^{2 d x +2 c}+b^{2} {\mathrm e}^{2 d x +2 c}+a b -b^{2}}{a \,b^{2} d \left(b \,{\mathrm e}^{4 d x +4 c}+4 a \,{\mathrm e}^{2 d x +2 c}-2 b \,{\mathrm e}^{2 d x +2 c}+b \right)}+\frac{\sqrt{a \left(a -b \right)}\, \ln \left({\mathrm e}^{2 d x +2 c}+\frac{2 \sqrt{a \left(a -b \right)}+2 a -b}{b}\right)}{2 a d \,b^{2}}+$
derivativedivides	$\frac{\ln \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}{b^{2}}+\frac{\frac{2 \left(-\frac{b \left(a -b \right) \left(\tanh^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2 a}-\frac{b \left(a -b \right) \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)}{2 a}\right)}{a \left(\tanh^{4}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)-2 a \left(\tanh^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+4 b \left(\tanh^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+a}+\left(2 a^{2}-a b -b^{2}\right) (\frac{(\sqrt{-b \left(a -b \right)}+}{2 a \sqrt{-b (a -}}}{}$
default	$\frac{\ln \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}{b^{2}}+\frac{\frac{2 \left(-\frac{b \left(a -b \right) \left(\tanh^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2 a}-\frac{b \left(a -b \right) \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)}{2 a}\right)}{a \left(\tanh^{4}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)-2 a \left(\tanh^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+4 b \left(\tanh^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+a}+\left(2 a^{2}-a b -b^{2}\right) (\frac{(\sqrt{-b \left(a -b \right)}+}{2 a \sqrt{-b (a -}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+2/b^2*((-1/2*b*(a-b)/a*tanh(1/2*d*x+1/
2*c)^3-1/2*b*(a-b)/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh
(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(2*a^2-a*b-b^2)*(-1/2*((
-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*
arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2*((-
b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))))-1/b^2*l
n(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. 2(89) = 178.
time = 0.44, size = 1527, normalized size = 15.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b*d*x*cosh(d*x + c)^4 + 16*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3
+ 4*a*b*d*x*sinh(d*x + c)^4 + 4*a*b*d*x + 4*(2*(2*a^2 - a*b)*d*x + 2*a^2 -
3*a*b + b^2)*cosh(d*x + c)^2 + 4*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - a*
b)*d*x + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh(d*x + c
)^4 + 4*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*
x + c)^4 + 2*(4*a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x +
c)^2 + 4*a^2 - b^2)*sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d
*x + c)^3 + (4*a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a - b)/a)*log
((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x
+ c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b
 - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2
*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*c
osh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 - a*b)*sqrt((a - b
)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
 c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sin
h(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c
) + b)) + 4*a*b - 4*b^2 + 8*(2*a*b*d*x*cosh(d*x + c)^3 + (2*(2*a^2 - a*b)*d
*x + 2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(a*b^3*d*cosh(d*x +
 c)^4 + 4*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^3*d*sinh(d*x + c)^4 +
 a*b^3*d + 2*(2*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*a*b^3*d*cosh(d*x
+ c)^2 + (2*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + 4*(a*b^3*d*cosh(d*x + c)^
3 + (2*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*d*x*cos
h(d*x + c)^4 + 8*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a*b*d*x*sinh(d*x
 + c)^4 + 2*a*b*d*x + 2*(2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a*b + b^2)*cosh(d*
```

```
x + c)^2 + 2*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a
*b + b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a*b + b^2
)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*x + c)^4 + 2*(4*a^2
- b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x + c)^2 + 4*a^2 - b^2)*
sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d*x + c)^3 + (4*a^2 -
 b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a - b)/a)*arctan(-1/2*(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*s
qrt(-(a - b)/a)/(a - b)) + 2*a*b - 2*b^2 + 4*(2*a*b*d*x*cosh(d*x + c)^3 + (
2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(a
*b^3*d*cosh(d*x + c)^4 + 4*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^3*d*
sinh(d*x + c)^4 + a*b^3*d + 2*(2*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*
a*b^3*d*cosh(d*x + c)^2 + (2*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + 4*(a*b^3
*d*cosh(d*x + c)^3 + (2*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.89, size = 178, normalized size = 1.78

$$\frac{\frac{2\,(dx+c)}{b^2} - \frac{(2\,a^2-ab-b^2)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,ab^2} + \frac{2\,\left(2\,a^2e^{(2\,dx+2\,c)}-3\,abe^{(2\,dx+2\,c)}+b^2e^{(2\,dx+2\,c)}+ab-b^2\right)}{\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)ab^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/b^2 - (2*a^2 - a*b - b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) +
2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a*b^2) + 2*(2*a^2*e^(2*d*x + 2
*c) - 3*a*b*e^(2*d*x + 2*c) + b^2*e^(2*d*x + 2*c) + a*b - b^2)/((b*e^(4*d*x
 + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)*a*b^2))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(c+d\,x\right)}^4}{{\left(b\,{\mathrm{sinh}\left(c+d\,x\right)}^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^2, x)
```

3.332 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=77

$$\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*(a+b)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)/d-1/2*(a-b)*s
inh(d*x+c)/a/b/d/(a+b*sinh(d*x+c)^2)
```

Rubi [A]

time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3269, 393, 211}

$$\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)*d) - (
(a - b)*Sinh[c + d*x])/(2*a*b*d*(a + b*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+bx^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= -\frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)}+\frac{(a+b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{2abd}\\ &= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 75, normalized size = 0.97

$$\frac{\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{(a-b)\sinh(c+dx)}{2ab\left(a+b\sinh^2(c+dx)\right)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (((a + b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) - ((
a - b)*Sinh[c + d*x])/(2*a*b*(a + b*Sinh[c + d*x]^2)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(65) = 130$.
time = 1.70, size = 280, normalized size = 3.64

method	result
risch	$-\frac{e^{dx+c}(a-b)\left(e^{2dx+2c}-1\right)}{bda\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,db}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,da}+$

derivativedivides	$\frac{\frac{\frac{(a-b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ab}-\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{ab}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+(a+b)\left(\frac{\left(-a+\sqrt{-b\left(a-b\right)}+b\right)\arctan\left(\frac{}{\sqrt{\left(2\sqrt{-b}\right.}}\right.}{2a\sqrt{-b\left(a-b\right)}\sqrt{\left(2\sqrt{-b(a-\right.}}\right.}{d}$
default	$\frac{\frac{\frac{(a-b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ab}-\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{ab}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+(a+b)\left(\frac{\left(-a+\sqrt{-b\left(a-b\right)}+b\right)\arctan\left(\frac{}{\sqrt{\left(2\sqrt{-b}\right.}}\right.}{2a\sqrt{-b\left(a-b\right)}\sqrt{\left(2\sqrt{-b(a-\right.}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(1/2*(a-b)/a/b*tanh(1/2*d*x+1/2*c)^3-1/2*(a-b)/a/b*tanh(1/2*d*x+1/2*
c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2
*c)^2+a)+(a+b)/b*(1/2*(-a+(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2))-1/2*(a+(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/
2)+a-2*b)*a)^(1/2))))
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -((a*e^(3*c) - b*e^(3*c))*e^(3*d*x) - (a*e^c - b*e^c)*e^(d*x))/(a*b^2*d*e^(
4*d*x + 4*c) + a*b^2*d + 2*(2*a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x))
 + 1/8*integrate(8*((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) + (a*e^c + b*e^c)*e^(
d*x))/(a*b^2*e^(4*d*x + 4*c) + a*b^2 + 2*(2*a^2*b*e^(2*c) - a*b^2*e^(2*c))*
e^(2*d*x)), x)
```

Fricas **[B]** Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(65) = 130$.
time $= 0.43$, size $= 1615$, normalized size $= 20.97$

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b - a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^2 + 4*(a^2*b - a*b^2)*sinh(d*x + c)^3 + ((a*b + b^2)*cosh(d*
x + c)^4 + 4*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b + b^2)*sinh(d
*x + c)^4 + 2*(2*a^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(a*b + b^2)*cosh(d
*x + c)^2 + 2*a^2 + a*b - b^2)*sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b + b^2)
*cosh(d*x + c)^3 + (2*a^2 + a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-
a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*
x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*
sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x
+ c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)
^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)
/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
 + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b
)) - 4*(a^2*b - a*b^2)*cosh(d*x + c) - 4*(a^2*b - a*b^2 - 3*(a^2*b - a*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c))/(a^2*b^3*d*cosh(d*x + c)^4 + 4*a^2*b^3*d*c
osh(d*x + c)*sinh(d*x + c)^3 + a^2*b^3*d*sinh(d*x + c)^4 + a^2*b^3*d + 2*(2
*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*a^2*b^3*d*cosh(d*x + c)^2 + (2
*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + 4*(a^2*b^3*d*cosh(d*x + c)^3 + (2*
a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a^2*b - a*b^2)
*cosh(d*x + c)^3 + 6*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2
*b - a*b^2)*sinh(d*x + c)^3 - ((a*b + b^2)*cosh(d*x + c)^4 + 4*(a*b + b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 + a*
b - b^2)*cosh(d*x + c)^2 + 2*(3*(a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + a*b -
 b^2)*sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b + b^2)*cosh(d*x + c)^3 + (2*a^2
 + a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*sqrt(a*b)*
(cosh(d*x + c) + sinh(d*x + c))/a) - ((a*b + b^2)*cosh(d*x + c)^4 + 4*(a*b
+ b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a
^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(a*b + b^2)*cosh(d*x + c)^2 + 2*a^2
+ a*b - b^2)*sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b + b^2)*cosh(d*x + c)^3 +
 (2*a^2 + a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*(b*
cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (
4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqr
t(a*b)/(a*b)) - 2*(a^2*b - a*b^2)*cosh(d*x + c) - 2*(a^2*b - a*b^2 - 3*(a^2
*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/(a^2*b^3*d*cosh(d*x + c)^4 + 4*
a^2*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b^3*d*sinh(d*x + c)^4 + a^2*b
```

```
^3*d + 2*(2*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*a^2*b^3*d*cosh(d*x
+ c)^2 + (2*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + 4*(a^2*b^3*d*cosh(d*x +
 c)^3 + (2*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^3}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^2, x)
```

3.333 $\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}\,d}+\frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

```
[Out] 1/2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/d/(a-b)^(1/2)+1/2*tanh
(d*x+c)/a/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A]

time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3270, 205, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a-b}}+\frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]*d) + Ta
nh[c + d*x]/(2*a*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx=\frac{\text{Subst}\left(\int \frac{1}{(a-(a-b)x^2)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx,x,\tanh(c+dx)\right)}{2ad}$$

$$=\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}\,d}+\frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Mathematica [A]
time = 0.15, size = 78, normalized size = 0.99

$$\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}}+\frac{\sqrt{a}\,\sinh(2(c+dx))}{2a-b+b\cosh(2(c+dx))}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/Sqrt[a - b] + (Sqrt[a]*Sinh[2
*(c + d*x)])/(2*a - b + b*Cosh[2*(c + d*x)]))/(2*a^(3/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(67) = 134$.
time = 1.49, size = 252, normalized size = 3.19

method	result
risch	$-\frac{2a\,e^{2dx+2c}-b\,e^{2dx+2c}+b}{bda\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}-2a^2+2ab}{b\sqrt{a^2-ab}}\right)}{4\sqrt{a^2-ab}\,da}-\ln\Big(e^{2dx}$
derivativedivides	$\frac{-\frac{2\left(-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{\left(\sqrt{-b\,(a-b)}-b\right)\operatorname{arctanh}\Bigg(\frac{a\tanh\left(\frac{dx}{2}\right.}{\sqrt{\left(2\sqrt{-b\,(a-b)}\right.}}}{2a\sqrt{-b\,(a-b)}\,\sqrt{\left(2\sqrt{-b\,(a-b)}+\right.}}}{d}$

default	$\frac{-\frac{2\left(-\frac{\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{\left(\sqrt{-b\left(a-b\right)}-b\right)\operatorname{arctanh}\left(\frac{a\tanh(}{\sqrt{\left(2\sqrt{-b\left(a-\right.}\right.}}\right.}{2a\sqrt{-b\left(a-b\right)}\sqrt{\left(2\sqrt{-b\left(a-b\right)}\right.}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2/a*tanh(1/2*d*x+1/2*c)^3-1/2/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/
2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(
(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2*((
-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 582 vs. 2(68) = 136.
time = 0.41, size = 1421, normalized size = 17.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*
(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b
+ a*b^2)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(
d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b
^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x +
```

```
 c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^
2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)
^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b
^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b
 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(
d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a
 - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 +
 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/((a^3
*b^2 - a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)*s
inh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^4*b - 3*a^3
*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - a^2*b^3)*d*cosh(d*x + c
)^2 + (2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d)*sinh(d*x + c)^2 + (a^3*b^2 - a^2*b
^3)*d + 4*((a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^3 + (2*a^4*b - 3*a^3*b^2 + a
^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b - 2*a*b^2 + 2*(2*a^3
 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x
+ c)*sinh(d*x + c) + 2*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x + c)^2 + (b^2*cos
h(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 +
2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*s
inh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c)
)*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh
(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^
2 - a*b)))/((a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - a^2*b^3)*d
*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*sinh(d*x + c)^4 + 2*
(2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - a^2*b^3
)*d*cosh(d*x + c)^2 + (2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d)*sinh(d*x + c)^2 +
(a^3*b^2 - a^2*b^3)*d + 4*((a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^3 + (2*a^4*b
 - 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.36, size = 126, normalized size = 1.59

$$\frac{\frac{\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,a}-\frac{2\,\left(2\,ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+b\right)}{\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)ab}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2
+ a*b)*a) - 2*(2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)/((b*e^(4*d*x +
4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)*a*b))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^2}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^2, x)
```

3.334 $\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=66

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*sinh(d*x+c)/a/d/(a+b*sinh(d*x+c)^2)+1/2*arctan(sinh(d*x+c)*b^(1/2)/a^(1
/2))/a^(3/2)/d/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3269, 205, 211}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Sinh[c + d*
x]/(2*a*d*(a + b*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)}+\frac{\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{2ad}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 64, normalized size = 0.97

$$\frac{\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sinh(c+dx)}{2a\left(a+b\sinh^2(c+dx)\right)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sinh[c + d*x
]/(2*a*(a + b*Sinh[c + d*x]^2)))/d
```

Maple [A]
time = 0.55, size = 55, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\frac{\sinh(dx+c)}{2a\left(a+b\left(\sinh^2(dx+c)\right)\right)}+\frac{\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{d}$	55
default	$\frac{\frac{\sinh(dx+c)}{2a\left(a+b\left(\sinh^2(dx+c)\right)\right)}+\frac{\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{d}$	55
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{ad\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,da}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,da}$	147

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*sinh(d*x+c)/a/(a+b*sinh(d*x+c)^2)+1/2/a/(a*b)^(1/2)*arctan(b*sinh(
d*x+c)/(a*b)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (e^(3*d*x + 3*c) - e^(d*x + c))/(a*b*d*e^(4*d*x + 4*c) + a*b*d + 2*(2*a^2*d
*e^(2*c) - a*b*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(2*(e^(3*d*x + 3*c) + e
^(d*x + c))/(a*b*e^(4*d*x + 4*c) + a*b + 2*(2*a^2*e^(2*c) - a*b*e^(2*c))*e^
(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(54) = 108.
time = 0.44, size = 1320, normalized size = 20.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b*
sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(
3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*
a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-a*b)*log((b*cosh(d*x + c)^4
+ 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(
d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(
d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 +
3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)
*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*co
sh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)
^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)
^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(3*a*b*cosh(d*x + c)^
2 - a*b)*sinh(d*x + c))/(a^2*b^2*d*cosh(d*x + c)^4 + 4*a^2*b^2*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + a^2*b^2*d*sinh(d*x + c)^4 + a^2*b^2*d + 2*(2*a^3*b -
a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*a^2*b^2*d*cosh(d*x + c)^2 + (2*a^3*b - a^
2*b^2)*d)*sinh(d*x + c)^2 + 4*(a^2*b^2*d*cosh(d*x + c)^3 + (2*a^3*b - a^2*b
^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*cosh(d*x + c)^3 + 6*a*b*cos
h(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d*x + c)^3 - 2*a*b*cosh(d*x + c) +
```

```
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
+ 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)
*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) + (b*cos
h(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2
*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2
 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(
a*b)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*
sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)
*sinh(d*x + c))*sqrt(a*b)/(a*b)) + 2*(3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x
 + c))/(a^2*b^2*d*cosh(d*x + c)^4 + 4*a^2*b^2*d*cosh(d*x + c)*sinh(d*x + c)
^3 + a^2*b^2*d*sinh(d*x + c)^4 + a^2*b^2*d + 2*(2*a^3*b - a^2*b^2)*d*cosh(d
*x + c)^2 + 2*(3*a^2*b^2*d*cosh(d*x + c)^2 + (2*a^3*b - a^2*b^2)*d)*sinh(d*
x + c)^2 + 4*(a^2*b^2*d*cosh(d*x + c)^3 + (2*a^3*b - a^2*b^2)*d*cosh(d*x +
c))*sinh(d*x + c))]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(54) = 108$.
time = 6.52, size = 377, normalized size = 5.71

$$\begin{cases} \frac{\tilde{\infty} x \cosh(c)}{\sinh^4(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ -\frac{1}{3b^2 d \sinh^3(c+dx)} & \text{for } a = 0 \\ \frac{x\cosh(c)}{(a+b\sinh^2(c))^2} & \text{for } d = 0 \\ \frac{\sinh(c+dx)}{a^2 d} & \text{for } b = 0 \\ \frac{a\log\left(-\sqrt{-\frac{a}{b}}+\sinh(c+dx)\right)}{4a^2bd\sqrt{-\frac{a}{b}}+4ab^2d\sqrt{-\frac{a}{b}}\sinh^2(c+dx)} - \frac{a\log\left(\sqrt{-\frac{a}{b}}+\sinh(c+dx)\right)}{4a^2bd\sqrt{-\frac{a}{b}}+4ab^2d\sqrt{-\frac{a}{b}}\sinh^2(c+dx)} + \frac{2b\sqrt{-\frac{a}{b}}\sinh(c+dx)}{4a^2bd\sqrt{-\frac{a}{b}}+4ab^2d\sqrt{-\frac{a}{b}}\sinh^2(c+dx)} + \frac{b\log\left(-\sqrt{-\frac{a}{b}}+\sinh(c+dx)\right)\sinh^2(c+dx)}{4a^2bd\sqrt{-\frac{a}{b}}+4ab^2d\sqrt{-\frac{a}{b}}\sinh^2(c+dx)} - \frac{b\log\left(\sqrt{-\frac{a}{b}}+\sinh(c+dx)\right)\sinh^2(c+dx)}{4a^2bd\sqrt{-\frac{a}{b}}+4ab^2d\sqrt{-\frac{a}{b}}\sinh^2(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((zoo*x*cosh(c)/sinh(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-1/(
3*b**2*d*sinh(c + d*x)**3), Eq(a, 0)), (x*cosh(c)/(a + b*sinh(c)**2)**2, Eq
(d, 0)), (sinh(c + d*x)/(a**2*d), Eq(b, 0)), (a*log(-sqrt(-a/b) + sinh(c +
d*x))/(4*a**2*b*d*sqrt(-a/b) + 4*a*b**2*d*sqrt(-a/b)*sinh(c + d*x)**2) - a*
log(sqrt(-a/b) + sinh(c + d*x))/(4*a**2*b*d*sqrt(-a/b) + 4*a*b**2*d*sqrt(-a
/b)*sinh(c + d*x)**2) + 2*b*sqrt(-a/b)*sinh(c + d*x)/(4*a**2*b*d*sqrt(-a/b)
 + 4*a*b**2*d*sqrt(-a/b)*sinh(c + d*x)**2) + b*log(-sqrt(-a/b) + sinh(c + d
*x))*sinh(c + d*x)**2/(4*a**2*b*d*sqrt(-a/b) + 4*a*b**2*d*sqrt(-a/b)*sinh(c
 + d*x)**2) - b*log(sqrt(-a/b) + sinh(c + d*x))*sinh(c + d*x)**2/(4*a**2*b*
d*sqrt(-a/b) + 4*a*b**2*d*sqrt(-a/b)*sinh(c + d*x)**2), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 0.91, size = 54, normalized size = 0.82

$$\frac{\sinh(c+d\,x)}{2\,a\,\left(b\,d\sinh\left(c+d\,x\right)^2+a\,d\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sinh(c+d\,x)}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] sinh(c + d*x)/(2*a*(a*d + b*d*sinh(c + d*x)^2)) + atan((b^(1/2)*sinh(c + d*
x))/a^(1/2))/(2*a^(3/2)*b^(1/2)*d)
```

3.335 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=106

$$\frac{\operatorname{ArcTan}(\sinh(c+dx))}{(a-b)^2d}-\frac{(3a-b)\sqrt{b}\operatorname{ArcTan}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d}-\frac{b\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] arctan(sinh(d*x+c))/(a-b)^2/d-1/2*b*sinh(d*x+c)/a/(a-b)/d/(a+b*sinh(d*x+c)^
2)-1/2*(3*a-b)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)/(a-b)^2/
d
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3269, 425, 536, 209, 211}

$$-\frac{\sqrt{b}\,(3a-b)\operatorname{ArcTan}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}+\frac{\operatorname{ArcTan}(\sinh(c+dx))}{d(a-b)^2}-\frac{b\sinh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/((a - b)^2*d) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Si
nh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^2*d) - (b*Sinh[c + d*x])/(2*a*(a
- b)*d*(a + b*Sinh[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \sinh(c+dx\right.}{2a(a-b)d}$$

$$= -\frac{b\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{(a-b)^2d} - \frac{((3}{}$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{(a-b)^2d} - \frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d} - \frac{b\sinh}{2a(a-b)d\,(a+}$$

Mathematica [A]

time = 0.29, size = 174, normalized size = 1.64

$$\frac{(2a-b)\left(-\sqrt{b}\,(-3a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{b}}\right)+4a^{3/2}\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+\left(-b^{3/2}(-3a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{b}}\right)+4a^{3/2}b\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)\cosh(2(c+dx))-2\sqrt{a}\,(a-b)b\sinh(c+dx)}{2a^{3/2}(a-b)^2d(2a-b+b\cosh(2(c+dx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((2*a - b)*(-(Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]]) +
 4*a^(3/2)*ArcTan[Tanh[(c + d*x)/2]]) + (-(b^(3/2)*(-3*a + b)*ArcTan[(Sqrt[
```

```
a]*Csch[c + d*x])/Sqrt[b]]) + 4*a^(3/2)*b*ArcTan[Tanh[(c + d*x)/2]])*Cosh[2
*(c + d*x)] - 2*Sqrt[a]*(a - b)*b*Sinh[c + d*x])/(2*a^(3/2)*(a - b)^2*d*(2*
a - b + b*Cosh[2*(c + d*x)]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(94) = 188$.

time = 1.77, size = 305, normalized size = 2.88

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)^{2}}-\frac{2b\left(\frac{-\frac{(a-b)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(3a-b)\left(\frac{\left(-a+\sqrt{-b\left(a-\right.}\right.}{2a\sqrt{-b\left(a\right.}}\right.}{}\right.}{}}{}$
default	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)^{2}}-\frac{2b\left(\frac{-\frac{(a-b)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(3a-b)\left(\frac{\left(-a+\sqrt{-b\left(a-\right.}\right.}{2a\sqrt{-b\left(a\right.}}\right.}{}\right.}{}}{}$

risch	$-\frac{b\,\mathrm{e}^{dx+c}\left(\mathrm{e}^{2dx+2c}-1\right)}{d(a-b)a\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)}+\frac{i\ln\left(\mathrm{e}^{dx+c}+i\right)}{(a-b)^2d}-\frac{i\ln\left(\mathrm{e}^{dx+c}-i\right)}{(a-b)^2d}+\frac{3\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}-\frac{2\sqrt{}}{}\right.}{4a(a-b)^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a-b)^2*arctan(tanh(1/2*d*x+1/2*c))-2*b/(a-b)^2*((-1/2*(a-b)/a*tanh(
1/2*d*x+1/2*c)^3+1/2*(a-b)/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-
2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(3*a-b)*(1/2*(-a
+(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*(a
+(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(4*c)
- a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(2*a^3*d*e^(2*c) - 3*a^2*b*d*e^(2*c) + a*b
^2*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(a^2*d - 2*a*b*d + b^2*d)
- 2*integrate(1/2*((3*a*b*e^(3*c) - b^2*e^(3*c))*e^(3*d*x) + (3*a*b*e^c - b
^2*e^c)*e^(d*x))/(a^3*b - 2*a^2*b^2 + a*b^3 + (a^3*b*e^(4*c) - 2*a^2*b^2*e^
(4*c) + a*b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 5*a^3*b*e^(2*c) + 4*a
^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(94) = 188$.
time = 0.45, size = 2143, normalized size = 20.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b - b^2)*cosh(d*x + c)^3 + 12*(a*b - b^2)*cosh(d*x + c)*sinh(d*
x + c)^2 + 4*(a*b - b^2)*sinh(d*x + c)^3 + ((3*a*b - b^2)*cosh(d*x + c)^4 +
 4*(3*a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - b^2)*sinh(d*x + c
)^4 + 2*(6*a^2 - 5*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - b^2)*cosh(d*x
 + c)^2 + 6*a^2 - 5*a*b + b^2)*sinh(d*x + c)^2 + 3*a*b - b^2 + 4*((3*a*b -
```

```
b^2)*cosh(d*x + c)^3 + (6*a^2 - 5*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*s
inh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a
 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sin
h(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*s
inh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))
*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2
*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*s
inh(d*x + c) + b)) - 8*(a*b*cosh(d*x + c)^4 + 4*a*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*b*sinh(d*x + c)^4 + 2*(2*a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*a*b*c
osh(d*x + c)^2 + 2*a^2 - a*b)*sinh(d*x + c)^2 + a*b + 4*(a*b*cosh(d*x + c)^
3 + (2*a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh
(d*x + c)) - 4*(a*b - b^2)*cosh(d*x + c) + 4*(3*(a*b - b^2)*cosh(d*x + c)^2
 - a*b + b^2)*sinh(d*x + c))/((a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4
 + 4*(a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b -
 2*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*
b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2
 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^3*b - 2*a^
2*b^2 + a*b^3)*d + 4*((a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (2*a^
4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(
a*b - b^2)*cosh(d*x + c)^3 + 6*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^2 +
2*(a*b - b^2)*sinh(d*x + c)^3 + ((3*a*b - b^2)*cosh(d*x + c)^4 + 4*(3*a*b -
 b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - b^2)*sinh(d*x + c)^4 + 2*(6*
a^2 - 5*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - b^2)*cosh(d*x + c)^2 + 6
*a^2 - 5*a*b + b^2)*sinh(d*x + c)^2 + 3*a*b - b^2 + 4*((3*a*b - b^2)*cosh(d
*x + c)^3 + (6*a^2 - 5*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*a
rctan(1/2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) + ((3*a*b - b^2)*cosh(
d*x + c)^4 + 4*(3*a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - b^2)*
sinh(d*x + c)^4 + 2*(6*a^2 - 5*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - b
^2)*cosh(d*x + c)^2 + 6*a^2 - 5*a*b + b^2)*sinh(d*x + c)^2 + 3*a*b - b^2 +
4*((3*a*b - b^2)*cosh(d*x + c)^3 + (6*a^2 - 5*a*b + b^2)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sin
h(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x
+ c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(b/a)/b) - 4*(a*b*cosh(d*x + c)^4 + 4*
a*b*cosh(d*x + c)*sinh(d*x + c)^3 + a*b*sinh(d*x + c)^4 + 2*(2*a^2 - a*b)*c
osh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c)^2 + 2*a^2 - a*b)*sinh(d*x + c)^2 +
a*b + 4*(a*b*cosh(d*x + c)^3 + (2*a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*
arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a*b - b^2)*cosh(d*x + c) + 2*(3*
(a*b - b^2)*cosh(d*x + c)^2 - a*b + b^2)*sinh(d*x + c))/((a^3*b - 2*a^2*b^2
 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^3*b - 2*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^4
 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b - 2*a^2*b^2
 + a*b^3)*d*cosh(d*x + c)^2 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d)*sinh
(d*x + c)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*d + 4*((a^3*b - 2*a^2*b^2 + a*b^3
```

```
)*d*cosh(d*x + c)^3 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d*cosh(d*x + c)
)*sinh(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(c+dx\right)}{\left(a+b\sinh^{2}\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sinh(c + d*x)**2)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)\,\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^2), x)
```

3.336 $\int \frac{\mathbf{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=114

$$-\frac{(4a-b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}d}+\frac{\tanh(c+dx)}{(a-b)^2d}+\frac{b^2\tanh(c+dx)}{2a(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

```
[Out] -1/2*(4*a-b)*b*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(5/2)
/d+tanh(d*x+c)/(a-b)^2/d+1/2*b^2*tanh(d*x+c)/a/(a-b)^2/d/(a-(a-b)*tanh(d*x+
c)^2)
```

Rubi [A]
time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3270, 398, 393, 214}

$$-\frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{5/2}}+\frac{b^2\tanh(c+dx)}{2ad(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a
 - b)^(5/2)*d) + Tanh[c + d*x]/((a - b)^2*d) + (b^2*Tanh[c + d*x])/(2*a*(a
- b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a-(a-b)x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^2}-\frac{(2a-b)b-2(a-b)bx^2}{(a-b)^2(a+(-a+b)x^2)^2}\right)dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\tanh(c+dx)}{(a-b)^2d}-\frac{\text{Subst}\left(\int \frac{(2a-b)b-2(a-b)bx^2}{(a+(-a+b)x^2)^2}\,dx, x, \tanh(c+dx)\right)}{(a-b)^2d}\\
&= \frac{\tanh(c+dx)}{(a-b)^2d}+\frac{b^2\tanh(c+dx)}{2a(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{((4a-b)b)\text{Subst}}{}\\
&= -\frac{(4a-b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}d}+\frac{\tanh(c+dx)}{(a-b)^2d}+\frac{b^2\tan}{2a(a-b)^2d\,(a-}
\end{aligned}$$

Mathematica [A]
time = 0.67, size = 105, normalized size = 0.92

$$\frac{-\frac{(4a-b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{5/2}}+\frac{\frac{b^2\sinh(2(c+dx))}{a(2a-b+b\cosh(2(c+dx)))}+2\tanh(c+dx)}{(a-b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-(((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a -
 b)^(5/2))) + ((b^2*Sinh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)]))
+ 2*Tanh[c + d*x])/(a - b)^2)/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(102) = 204$.

time = 1.63, size = 307, normalized size = 2.69

method	result
derivativedivides	$2b\left(\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\right.$ $(4a-b)\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{}{\sqrt{\left(2\sqrt{-}\right.}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a}\right.}}\right.$ $(a-$
default	$2b\left(\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\right.$ $(4a-b)\left(\frac{\left(\sqrt{-b(a-b)}+b\right)\arctan\left(\frac{}{\sqrt{\left(2\sqrt{-}\right.}}\right.}{2a\sqrt{-b(a-b)}\sqrt{\left(2\sqrt{-b(a}\right.}}\right.$ $(a-$
risch	$-\frac{4ab\,e^{4dx+4c}-b^2e^{4dx+4c}+8a^2e^{2dx+2c}-2ab\,e^{2dx+2c}+2ab+b^2}{a(a-b)^2d\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)\left(1+e^{2dx+2c}\right)}+\frac{b\ln\left(e^{2dx+2c}+\frac{2a\sqrt{a^2-ab}-b\sqrt{a^2-ab}+}{b\sqrt{a^2-ab}}\right.}{\sqrt{a^2-ab}\,(a-b)^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/(a-b)^2*((1/2/a*b*tanh(1/2*d*x+1/2*c)^3+1/2/a*b*tanh(1/2*d*x+1/2*c
))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*
c)^2+a)+1/2*(4*a-b)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(
a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2))))+2/(a-b)^2*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1
))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1446 vs. $2(103) = 206$.
time = 0.46, size = 3147, normalized size = 27.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^4 + 16*(4*a^3*b - 5*a^
2*b^2 + a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(4*a^3*b - 5*a^2*b^2 + a*b
^3)*sinh(d*x + c)^4 + 8*a^3*b - 4*a^2*b^2 - 4*a*b^3 + 8*(4*a^4 - 5*a^3*b +
a^2*b^2)*cosh(d*x + c)^2 + 8*(4*a^4 - 5*a^3*b + a^2*b^2 + 3*(4*a^3*b - 5*a^
2*b^2 + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a*b^2 - b^3)*cosh(d*x
 + c)^6 + 6*(4*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 - b^3)
*sinh(d*x + c)^6 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^2*b -
 8*a*b^2 + b^3 + 15*(4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5
*(4*a*b^2 - b^3)*cosh(d*x + c)^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)
```

```
)*sinh(d*x + c)^3 + 4*a*b^2 - b^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c
)^2 + (15*(4*a*b^2 - b^3)*cosh(d*x + c)^4 + 16*a^2*b - 8*a*b^2 + b^3 + 6*(1
6*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(4*a*b^2 -
 b^3)*cosh(d*x + c)^5 + 2*(16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^3 + (16*
a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b
^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c
)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b -
b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*
b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh
(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*
a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 16*
((4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 5*a^3*b + a^2*b^2
)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*
d*cosh(d*x + c)^6 + 6*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*sinh(d*x
 + c)^6 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x
+ c)^4 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 +
(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c)^4 +
(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + 4
*(5*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + (4*a^6 -
13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^4 + 6*(4
*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (4*
a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c)^2 + (a^
5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d + 2*(3*(a^5*b - 3*a^4*b^2 + 3*a^3*
b^3 - a^2*b^4)*d*cosh(d*x + c)^5 + 2*(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3
*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*
b^3 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(4*a^3*b - 5*a^2*b^
2 + a*b^3)*cosh(d*x + c)^4 + 8*(4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)*
sinh(d*x + c)^3 + 2*(4*a^3*b - 5*a^2*b^2 + a*b^3)*sinh(d*x + c)^4 + 4*a^3*b
 - 2*a^2*b^2 - 2*a*b^3 + 4*(4*a^4 - 5*a^3*b + a^2*b^2)*cosh(d*x + c)^2 + 4*
(4*a^4 - 5*a^3*b + a^2*b^2 + 3*(4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 - ((4*a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(4*a*b^2 - b^3)*c
osh(d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 - b^3)*sinh(d*x + c)^6 + (16*a^2*b
- 8*a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^2*b - 8*a*b^2 + b^3 + 15*(4*a*b^2
- b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(4*a*b^2 - b^3)*cosh(d*x + c
)^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a*b^2 -
 b^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(4*a*b^2 - b^3)*cos
h(d*x + c)^4 + 16*a^2*b - 8*a*b^2 + b^3 + 6*(16*a^2*b - 8*a*b^2 + b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - b^3)*cosh(d*x + c)^5 + 2*(16
*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d
*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2
*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a
```

```
*b)/(a^2 - a*b)) + 8*((4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^3 + (4*a^
4 - 5*a^3*b + a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 3*a^4*b^2 +
3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^6 + 6*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 -
 a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3
- a^2*b^4)*d*sinh(d*x + c)^6 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 +
 a^2*b^4)*d*cosh(d*x + c)^4 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)
*d*cosh(d*x + c)^2 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*
d)*sinh(d*x + c)^4 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*
d*cosh(d*x + c)^2 + 4*(5*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d
*x + c)^3 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*c
osh(d*x + c)^4 + 6*(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*
cosh(d*x + c)^2 + (4*a^6 - 13*a^5*b + 15*a^4*b^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*sinh(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(103) = 206.
time = 0.76, size = 221, normalized size = 1.94

$$-\frac{\frac{(4ab-b^2)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{(a^3-2a^2b+ab^2)\sqrt{-a^2+ab}}+\frac{2\left(4abe^{(4dx+4c)}-b^2e^{(4dx+4c)}+8a^2e^{(2dx+2c)}-2abe^{(2dx+2c)}+2ab+b^2\right)}{(a^3-2a^2b+ab^2)\left(be^{(6dx+6c)}+4ae^{(4dx+4c)}-be^{(4dx+4c)}+4ae^{(2dx+2c)}-be^{(2dx+2c)}+b\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((4*a*b - b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*
b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(-a^2 + a*b)) + 2*(4*a*b*e^(4*d*x + 4*c) -
 b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 2*a*b*e^(2*d*x + 2*c) + 2*a*
b + b^2)/((a^3 - 2*a^2*b + a*b^2)*(b*e^(6*d*x + 6*c) + 4*a*e^(4*d*x + 4*c)
- b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cosh\left(c+d\,x\right)}^2\,{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2),x)
[Out] int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2), x)
```

3.337 $\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=157

$$\frac{(a-5b)\text{ArcTan}(\sinh(c+dx))}{2(a-b)^3d}+\frac{(5a-b)b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^3d}+\frac{b(a+b)\sinh(c+dx)}{2a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)}+\frac{}{2(}$$

```
[Out] 1/2*(a-5*b)*arctan(sinh(d*x+c))/(a-b)^3/d+1/2*(5*a-b)*b^(3/2)*arctan(sinh(d
*x+c)*b^(1/2)/a^(1/2))/a^(3/2)/(a-b)^3/d+1/2*b*(a+b)*sinh(d*x+c)/a/(a-b)^2/
d/(a+b*sinh(d*x+c)^2)+1/2*sech(d*x+c)*tanh(d*x+c)/(a-b)/d/(a+b*sinh(d*x+c)^
2)
```

Rubi [A]
time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3269, 425, 541, 536, 209, 211}

$$\frac{b^{3/2}(5a-b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^3}+\frac{(a-5b)\text{ArcTan}(\sinh(c+dx))}{2d(a-b)^3}+\frac{b(a+b)\sinh(c+dx)}{2ad(a-b)^2\left(a+b\sinh^2(c+dx)\right)}+\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - 5*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^3*d) + ((5*a - b)*b^(3/2)*ArcT
an[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^3*d) + (b*(a + b)*S
inh[c + d*x])/(2*a*(a - b)^2*d*(a + b*Sinh[c + d*x]^2)) + (Sech[c + d*x]*Ta
nh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{2(a-b)d}\\
&= \frac{b(a+b)\sinh(c+dx)}{2a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\operatorname{Subst}}{}\\
&= \frac{b(a+b)\sinh(c+dx)}{2a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{(a-5}{}\\
&= \frac{(a-5b)\tan^{-1}(\sinh(c+dx))}{2(a-b)^3d} + \frac{(5a-b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^3d} + \frac{b}{2a(a-}\end{aligned}$$

Mathematica [A]
time = 0.79, size = 230, normalized size = 1.46

$$\frac{2\sqrt{a}(a-b)b^2\sinh(c+dx)+(2a-b)\left(b^{3/2}(-5a+b)\text{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+2a^{3/2}(a-5b)\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+a^{3/2}(a-b)\operatorname{sech}(c+dx)\tanh(c+dx)\right)+b\cosh(2(c+dx))\left(b^{3/2}(-5a+b)\text{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+2a^{3/2}(a-5b)\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+a^{3/2}(a-b)\operatorname{sech}(c+dx)\tanh(c+dx)\right)}{2a^{3/2}(a-b)^3d(2a-b+b\cosh(2(c+dx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*Sqrt[a]*(a - b)*b^2*Sinh[c + d*x] + (2*a - b)*(b^(3/2)*(-5*a + b)*ArcTan
[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*a^(3/2)*(a - 5*b)*ArcTan[Tanh[(c + d*
x)/2]] + a^(3/2)*(a - b)*Sech[c + d*x]*Tanh[c + d*x]) + b*Cosh[2*(c + d*x)]
*(b^(3/2)*(-5*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*a^(3/2)*(a
 - 5*b)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*(a - b)*Sech[c + d*x]*Tanh[c +
d*x]))/(2*a^(3/2)*(a - b)^3*d*(2*a - b + b*Cosh[2*(c + d*x)]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(141) = 282$.
time = 2.06, size = 368, normalized size = 2.34

method	result

derivativedivides	$\frac{2b^2\left(\frac{-\frac{(a-b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(5a-b)\left(\frac{\left(-a+\sqrt{-b\,(a-b)}+b\right)\arctan\left(\frac{}{\sqrt{\left(2\right.}}\right.}{2a\sqrt{-b\,(a-b)}\,\sqrt{\left(2\sqrt{-b}\right.}}\right.}{}\right.}{}$
default	$\frac{2b^2\left(\frac{-\frac{(a-b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{(5a-b)\left(\frac{\left(-a+\sqrt{-b\,(a-b)}+b\right)\arctan\left(\frac{}{\sqrt{\left(2\right.}}\right.}{2a\sqrt{-b\,(a-b)}\,\sqrt{\left(2\sqrt{-b}\right.}}\right.}{}\right.}{}$
risch	$\frac{\mathrm{e}^{dx+c}\left(ab\,\mathrm{e}^{6dx+6c}+b^2\mathrm{e}^{6dx+6c}+4a^2\mathrm{e}^{4dx+4c}-3ab\,\mathrm{e}^{4dx+4c}+b^2\mathrm{e}^{4dx+4c}-4a^2\mathrm{e}^{2dx+2c}+3ab\,\mathrm{e}^{2dx+2c}-b^2\mathrm{e}^{2dx+2c}-ab-b^2\right)}{d(a-b)^2\left(1+\mathrm{e}^{2dx+2c}\right)^2a\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^2/(a-b)^3*((-1/2*(a-b)/a*tanh(1/2*d*x+1/2*c)^3+1/2*(a-b)/a*tanh(1/
2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1
```

```
/2*d*x+1/2*c)^2+a)+1/2*(5*a-b)*(1/2*(-a+(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*(a+(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))))+2/(a-b)^3*(((1/2*b-1/2*a)*tanh(1/2*d*x+
1/2*c)^3+(-1/2*b+1/2*a)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^2+1/
2*(a-5*b)*arctan(tanh(1/2*d*x+1/2*c))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (a*e^c - 5*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^3*d - 3*a^2*b*d + 3*a*b^2*d
 - b^3*d) + ((a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) - 3*a*b
*e^(5*c) + b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) - 3*a*b*e^(3*c) + b^2*e^
(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x))/(a^3*b*d - 2*a^2*b^2*d + a*
b^3*d + (a^3*b*d*e^(8*c) - 2*a^2*b^2*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x)
 + 4*(a^4*d*e^(6*c) - 2*a^3*b*d*e^(6*c) + a^2*b^2*d*e^(6*c))*e^(6*d*x) + 2*
(4*a^4*d*e^(4*c) - 9*a^3*b*d*e^(4*c) + 6*a^2*b^2*d*e^(4*c) - a*b^3*d*e^(4*c
))*e^(4*d*x) + 4*(a^4*d*e^(2*c) - 2*a^3*b*d*e^(2*c) + a^2*b^2*d*e^(2*c))*e^
(2*d*x)) + 8*integrate(1/8*((5*a*b^2*e^(3*c) - b^3*e^(3*c))*e^(3*d*x) + (5*
a*b^2*e^c - b^3*e^c)*e^(d*x))/(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4 + (a^4
*b*e^(4*c) - 3*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c) - a*b^4*e^(4*c))*e^(4*d*
x) + 2*(2*a^5*e^(2*c) - 7*a^4*b*e^(2*c) + 9*a^3*b^2*e^(2*c) - 5*a^2*b^3*e^(
2*c) + a*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3474 vs. $2(141) = 282$.
time = 0.70, size = 6548, normalized size = 41.71

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*b - b^3)*cosh(d*x + c)^7 + 28*(a^2*b - b^3)*cosh(d*x + c)*sinh
(d*x + c)^6 + 4*(a^2*b - b^3)*sinh(d*x + c)^7 + 4*(4*a^3 - 7*a^2*b + 4*a*b^
2 - b^3)*cosh(d*x + c)^5 + 4*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3 + 21*(a^2*b -
 b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*b - b^3)*cosh(d*x + c)^
3 + (4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(4
*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^3 + 4*(35*(a^2*b - b^3)*cosh(
d*x + c)^4 - 4*a^3 + 7*a^2*b - 4*a*b^2 + b^3 + 10*(4*a^3 - 7*a^2*b + 4*a*b^
```

```
2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(a^2*b - b^3)*cosh(d*x +
c)^5 + 10*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^3 - 3*(4*a^3 - 7*
a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((5*a*b^2 - b^3)*co
sh(d*x + c)^8 + 8*(5*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2
- b^3)*sinh(d*x + c)^8 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^6 + 4*(5*a^2*b -
 a*b^2 + 7*(5*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(5*a*b^2
 - b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(5*a*b^2 - b^3)*co
sh(d*x + c)^4 + 20*a^2*b - 9*a*b^2 + b^3 + 30*(5*a^2*b - a*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 + 8*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b
- a*b^2)*cosh(d*x + c)^3 + (20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c))*sinh(d
*x + c)^3 + 5*a*b^2 - b^3 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(7*(5*a
*b^2 - b^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - a*b^2)*cosh(d*x + c)^4 + 5*a^2*
b - a*b^2 + 3*(20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 8*((5*a*b^2 - b^3)*cosh(d*x + c)^7 + 3*(5*a^2*b - a*b^2)*cosh(d*x + c)^5 +
 (20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^3 + (5*a^2*b - a*b^2)*cosh(d*x +
c))*sinh(d*x + c))*sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*si
nh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*co
sh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)
*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*si
nh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2
- a)*sinh(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*
sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*
cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a -
b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*((a^2*b - 5*a*b^2)*cosh(d*x + c)^
8 + 8*(a^2*b - 5*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - 5*a*b^2)*s
inh(d*x + c)^8 + 4*(a^3 - 5*a^2*b)*cosh(d*x + c)^6 + 4*(a^3 - 5*a^2*b + 7*(
a^2*b - 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b - 5*a*b^2)*
cosh(d*x + c)^3 + 3*(a^3 - 5*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(4*a
^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2*b - 5*a*b^2)*cosh(d*x
 + c)^4 + 4*a^3 - 21*a^2*b + 5*a*b^2 + 30*(a^3 - 5*a^2*b)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 8*(7*(a^2*b - 5*a*b^2)*cosh(d*x + c)^5 + 10*(a^3 - 5*a^2*
b)*cosh(d*x + c)^3 + (4*a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c))*sinh(d*x +
 c)^3 + a^2*b - 5*a*b^2 + 4*(a^3 - 5*a^2*b)*cosh(d*x + c)^2 + 4*(7*(a^2*b -
 5*a*b^2)*cosh(d*x + c)^6 + 15*(a^3 - 5*a^2*b)*cosh(d*x + c)^4 + a^3 - 5*a^
2*b + 3*(4*a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(
(a^2*b - 5*a*b^2)*cosh(d*x + c)^7 + 3*(a^3 - 5*a^2*b)*cosh(d*x + c)^5 + (4*
a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c)^3 + (a^3 - 5*a^2*b)*cosh(d*x + c))*
sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 4*(a^2*b - b^3)*cosh
(d*x + c) + 4*(7*(a^2*b - b^3)*cosh(d*x + c)^6 + 5*(4*a^3 - 7*a^2*b + 4*a*b
^2 - b^3)*cosh(d*x + c)^4 - a^2*b + b^3 - 3*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^
3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)
*d*cosh(d*x + c)^8 + 8*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x +
 c)*sinh(d*x + c)^7 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*sinh(d*x +
c)^8 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^
```

```
4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 3*a^4*b + 3
*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^6 + 2*(4*a^5 - 13*a^4*b + 15*a^3*b^2 -
 7*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d*cosh(d*x + c)^3 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)
*d*cosh(d*x + c)^4 + 30*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x +
c)^2 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)
^4 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^4*
b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^5 + 10*(a^5 - 3*a^4*b +
3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^3 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7
*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b - 3*a^3*b^
2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^6 + 15*(a^5 - 3*a^4*b + 3*a^3*b^2 -
a^2*b^3)*d*cosh(d*x + c)^4 + 3*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 +
 a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d)*sinh(d
*x + c)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*...
```

Sympy **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sinh(c + d*x)**2)**2, x)
```

Giac **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad **[F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cosh\left(c+d\,x\right)}^3\,{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2),x)
[Out] int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2), x)
```

3.338 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=143

$$\frac{(6a-b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2}d}+\frac{(a-3b)\tanh(c+dx)}{(a-b)^3d}-\frac{\tanh^3(c+dx)}{3(a-b)^2d}-\frac{b^3\tanh(c+dx}{2a(a-b)^3d\,(a-(a-b)\,\mathrm{ta}}$$

```
[Out] 1/2*(6*a-b)*b^2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(7/2
)/d+(a-3*b)*tanh(d*x+c)/(a-b)^3/d-1/3*tanh(d*x+c)^3/(a-b)^2/d-1/2*b^3*tanh(
d*x+c)/a/(a-b)^3/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$,
Rules used = {3270, 398, 393, 214}

$$\frac{b^2(6a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{7/2}}-\frac{b^3\tanh(c+dx)}{2ad(a-b)^3\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\tanh^3(c+dx)}{3d(a-b)^2}+\frac{(a-3b)\tanh(c+dx)}{d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a
- b)^(7/2)*d) + ((a - 3*b)*Tanh[c + d*x])/((a - b)^3*d) - Tanh[c + d*x]^3/(
3*(a - b)^2*d) - (b^3*Tanh[c + d*x])/(2*a*(a - b)^3*d*(a - (a - b)*Tanh[c +
 d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\text{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a-3b}{(a-b)^3}-\frac{x^2}{(a-b)^2}+\frac{(3a-b)b^2-3(a-b)b^2x^2}{(a-b)^3\left(a+(-a+b)x^2\right)^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a-3b)\tanh(c+dx)}{(a-b)^3d}-\frac{\tanh^3(c+dx)}{3(a-b)^2d}+\frac{\text{Subst}\left(\int \frac{(3a-b)b^2-3(a-b)b^2x^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x,\right.}{(a-b)^3d}$$

$$= \frac{(a-3b)\tanh(c+dx)}{(a-b)^3d}-\frac{\tanh^3(c+dx)}{3(a-b)^2d}-\frac{b^3\tanh(c+dx)}{2a(a-b)^3d\left(a-(a-b)\tanh^2(c\right.}$$

$$= \frac{(6a-b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2}d}+\frac{(a-3b)\tanh(c+dx)}{(a-b)^3d}-\frac{\tanh^3(c}{3(a-}$$

Mathematica [A]
time = 1.47, size = 130, normalized size = 0.91

$$\frac{\frac{3(6a-b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{7/2}}+\frac{-\frac{3b^3\sinh(2(c+dx))}{a(2a-b+b\cosh(2(c+dx)))}+2\left(2(a-4b)+(a-b)\text{sech}^2(c+dx)\right)\tanh(c+dx)}{(a-b)^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((3*(6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a
 - b)^(7/2)) + ((-3*b^3*Sinh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)
])) + 2*(2*(a - 4*b) + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b)^3)/(
6*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(129) = 258$.

time = 1.97, size = 356, normalized size = 2.49

method	result
derivativedivides	$-\frac{2b^2\left(\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+(6a-b)\left(\frac{\left(\sqrt{-b\left(a-b\right)}+b\right)\arctan\left(\frac{}{\sqrt{\left(2\sqrt{-}\right.}}\right.}{2a\sqrt{-b\left(a-b\right)}\sqrt{\left(2\sqrt{-b\left(\right.}\right.}}\right.\right.}{(a-}$
default	$-\frac{2b^2\left(\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+(6a-b)\left(\frac{\left(\sqrt{-b\left(a-b\right)}+b\right)\arctan\left(\frac{}{\sqrt{\left(2\sqrt{-}\right.}}\right.}{2a\sqrt{-b\left(a-b\right)}\sqrt{\left(2\sqrt{-b\left(\right.}\right.}}\right.\right.}{(a-}$
risch	$-\frac{-18ab^2e^{8dx+8c}+3b^3e^{8dx+8c}-36a^2be^{6dx+6c}-30ab^2e^{6dx+6c}+6b^3e^{6dx+6c}+48a^3e^{4dx+4c}-164a^2be^{4dx+4c}+26ab^2e^{4dx+}}{3d(a-b)^3\left(1+e^{2dx+2c}\right)^3a\left(be^{4dx+4c}+4ae^{2dx+2c}-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^2/(a-b)^3*((1/2/a*b*tanh(1/2*d*x+1/2*c)^3+1/2/a*b*tanh(1/2*d*x+1/
2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1
/2*c)^2+a)+1/2*(6*a-b)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b
*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2))))-2/(a-b)^3*((3*b-a)*tanh(1/2*d*x+1/2*c)^5+(14/3*b-2/
3*a)*tanh(1/2*d*x+1/2*c)^3+(3*b-a)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c
)^2+1)^3)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3819 vs. $2(130) = 260$.
time = 0.48, size = 7894, normalized size = 55.20

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^8 + 96*(6*a^3*b^2 -
 7*a^2*b^3 + a*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + 12*(6*a^3*b^2 - 7*a^2*b
^3 + a*b^4)*sinh(d*x + c)^8 + 24*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*co
sh(d*x + c)^6 + 24*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4 + 14*(6*a^3*b^2 -
 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 48*(14*(6*a^3*b^2 -
7*a^2*b^3 + a*b^4)*cosh(d*x + c)^3 + 3*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b
^4)*cosh(d*x + c))*sinh(d*x + c)^5 - 16*a^4*b + 80*a^3*b^2 - 52*a^2*b^3 - 1
2*a*b^4 - 8*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^4
```

```
- 8*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3 - 105*(6*a^3*b^2 - 7*a^2*
b^3 + a*b^4)*cosh(d*x + c)^4 - 45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(21*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*co
sh(d*x + c)^5 + 15*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^3
- (24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c))*sinh(d*x +
c)^3 - 8*(8*a^5 - 38*a^4*b + 25*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4)*cosh(d*x + c
)^2 + 8*(42*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 - 8*a^5 + 38*a^
4*b - 25*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 + 45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3
+ a*b^4)*cosh(d*x + c)^4 - 6*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((6*a*b^3 - b^4)*cosh(d*x + c)^10 + 1
0*(6*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (6*a*b^3 - b^4)*sinh(d*x
+ c)^10 + (24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^8 + (24*a^2*b^2 + 2*a*
b^3 - b^4 + 45*(6*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(6*
a*b^3 - b^4)*cosh(d*x + c)^3 + (24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c))*
sinh(d*x + c)^7 + 2*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^6 + 2*(105*
(6*a*b^3 - b^4)*cosh(d*x + c)^4 + 36*a^2*b^2 - 12*a*b^3 + b^4 + 14*(24*a^2*
b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(6*a*b^3 - b^
4)*cosh(d*x + c)^5 + 14*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(3
6*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(36*a^2*b^2
- 12*a*b^3 + b^4)*cosh(d*x + c)^4 + 2*(105*(6*a*b^3 - b^4)*cosh(d*x + c)^6
+ 35*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^4 + 36*a^2*b^2 - 12*a*b^3 +
 b^4 + 15*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 +
6*a*b^3 - b^4 + 8*(15*(6*a*b^3 - b^4)*cosh(d*x + c)^7 + 7*(24*a^2*b^2 + 2*a
*b^3 - b^4)*cosh(d*x + c)^5 + 5*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)
^3 + (36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (24*a^2
*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^2 + (45*(6*a*b^3 - b^4)*cosh(d*x + c)^8
 + 28*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^6 + 30*(36*a^2*b^2 - 12*a*
b^3 + b^4)*cosh(d*x + c)^4 + 24*a^2*b^2 + 2*a*b^3 - b^4 + 12*(36*a^2*b^2 -
12*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(6*a*b^3 - b^4)*cos
h(d*x + c)^9 + 4*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^7 + 6*(36*a^2*b
^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^5 + 4*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh
(d*x + c)^3 + (24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d
*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cos
h(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x +
 c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt
(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sin
h(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a -
 b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(
d*x + c) + b)) + 16*(6*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^7 + 9*
(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^5 - 2*(24*a^5 - 106*a
^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^3 - (8*a^5 - 38*a^4*b + 25*a^
3*b^2 + 2*a^2*b^3 + 3*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b - 4*a^5*
b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^10 + 10*(a^6*b - 4*a
```

```
^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^9 +
 (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*sinh(d*x + c)^10 +
 (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^8 +
 (45*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^
2 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c)
^8 + 2*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*
d*cosh(d*x + c)^6 + 8*(15*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*
b^5)*d*cosh(d*x + c)^3 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*
b^5)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6*b - 4*a^5*b^2 + 6*a^4*b
^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 14*(4*a^7 - 15*a^6*b + 20*a^5
*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^2 + (6*a^7 - 25*a^6*b + 40*a^5
*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(6*a^7 - 2
5*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4
 + 4*(63*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3...
```

Sympy **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
[Out] Integral(sech(c + d*x)**4/(a + b*sinh(c + d*x)**2)**2, x)
```

Giac **[B]** Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(130) = 260.
time = 0.73, size = 270, normalized size = 1.89

$$\frac{\frac{3\,(6\,ab^2-b^3)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^4-3\,a^3b+3\,a^2b^2-ab^3)\sqrt{-a^2+ab}}+\frac{6\,\left(2\,ab^2e^{(2\,dx+2\,c)}-b^3e^{(2\,dx+2\,c)}+b^3\right)}{(a^4-3\,a^3b+3\,a^2b^2-ab^3)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)}+\frac{8\,\left(3\,be^{(4\,dx+4\,c)}-3\,ae^{(2\,dx+2\,c)}+9\,be^{(2\,dx+2\,c)}-a+4\,b\right)}{(a^3-3\,a^2b+3\,ab^2-b^3)\left(e^{(2\,dx+2\,c)}+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
[Out] 1/6*(3*(6*a*b^2 - b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 +
 a*b))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(-a^2 + a*b)) + 6*(2*a*b^2*
e^(2*d*x + 2*c) - b^3*e^(2*d*x + 2*c) + b^3)/((a^4 - 3*a^3*b + 3*a^2*b^2 -
a*b^3)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b))
 + 8*(3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) + 9*b*e^(2*d*x + 2*c) - a +
 4*b)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(e^(2*d*x + 2*c) + 1)^3))/d
```

Mupad **[F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+d\,x)^4\,\left(b\,\sinh(c+d\,x)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2),x)
[Out] int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2), x)
```

3.339 $\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=160

$$\frac{x}{b^3}-\frac{\sqrt{a-b}\,(8a^2+4ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d}-\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+dx)\right)^2}-\frac{(a-b)}{8a^2b^2d\,(a}$$

```
[Out] x/b^3-1/8*(8*a^2+4*a*b+3*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))*(a-b
)^(1/2)/a^(5/2)/b^3/d-1/4*(a-b)*tanh(d*x+c)/a/b/d/(a-(a-b)*tanh(d*x+c)^2)^2
-1/8*(a-b)*(4*a+3*b)*tanh(d*x+c)/a^2/b^2/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3270, 425, 541, 536, 212, 214}

$$-\frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\sqrt{a-b}\,(8a^2+4ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d}-\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+dx)\right)^2}+\frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] x/b^3 - (Sqrt[a - b]*(8*a^2 + 4*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c +
d*x])/Sqrt[a]])/(8*a^(5/2)*b^3*d) - ((a - b)*Tanh[c + d*x])/(4*a*b*d*(a - (
a - b)*Tanh[c + d*x]^2)^2) - ((a - b)*(4*a + 3*b)*Tanh[c + d*x])/(8*a^2*b^2
*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-(a-b)x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\Big(\int \frac{-a-3b-3(a-b)x^2}{(1-x^2)(a+(-a+b)x^2)^2}\,dx, x, \tan}{4abd}$$

$$= -\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d\left(a-(a-b)\tanh^2(c+dx)\right)} -$$

$$= -\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d\left(a-(a-b)\tanh^2(c+dx)\right)} -$$

$$= \frac{x}{b^3} - \frac{\sqrt{a-b}\,(8a^2+4ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} - \frac{(a-}{4abd\,(a-(a}$$

Mathematica [A]
time = 1.33, size = 164, normalized size = 1.02

$$\frac{8(c+dx) - \frac{(8a^3-4a^2b-ab^2-3b^3)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}} + \frac{4(a-b)^2b\sinh(2(c+dx))}{a(2a-b+b\cosh(2(c+dx)))^2} + \frac{3b(-2a^2+ab+b^2)\sinh(2(c+dx))}{a^2(2a-b+b\cosh(2(c+dx)))}}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) - ((8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*ArcTanh[(Sqrt[a - b]*Tanh
[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b]) + (4*(a - b)^2*b*Sinh[2*(c + d*x
)])/(a*(2*a - b + b*Cosh[2*(c + d*x)])^2) + (3*b*(-2*a^2 + a*b + b^2)*Sinh[
2*(c + d*x)])/(a^2*(2*a - b + b*Cosh[2*(c + d*x)])))/(8*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(146) = 292$.
time = 2.05, size = 429, normalized size = 2.68

method	result

derivativedivides	$\frac{2\left(-\frac{b\left(4a^{2}+ab-5b^{2}\right)\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{\left(4a^{3}-23a^{2}b+7a\,b^{2}+12b^{3}\right)b\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}}+\frac{\left(4a^{3}-23a^{2}b+7a\,b^{2}+12b^{3}\right)b\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{8a^{2}}\right.}{\left(a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^{2}}$
default	$\frac{2\left(-\frac{b\left(4a^{2}+ab-5b^{2}\right)\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{\left(4a^{3}-23a^{2}b+7a\,b^{2}+12b^{3}\right)b\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}}+\frac{\left(4a^{3}-23a^{2}b+7a\,b^{2}+12b^{3}\right)b\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{8a^{2}}\right.}{\left(a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^{2}}$
risch	$\frac{x}{b^{3}}+\frac{16a^{3}b\,\mathrm{e}^{6dx+6c}-20a^{2}b^{2}\mathrm{e}^{6dx+6c}+a\,b^{3}\mathrm{e}^{6dx+6c}+3b^{4}\mathrm{e}^{6dx+6c}+48a^{4}\mathrm{e}^{4dx+4c}-72a^{3}b\,\mathrm{e}^{4dx+4c}+18a^{2}b^{2}\mathrm{e}^{4dx+4c}+15a\,b^{3}\mathrm{e}^{4}}{4b^{3}a^{2}d\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^3*((-1/8*b*(4*a^2+a*b-5*b^2)/a*tanh(1/2*d*x+1/2*c)^7+1/8*(4*a^3-23
*a^2*b+7*a*b^2+12*b^3)/a^2*b*tanh(1/2*d*x+1/2*c)^5+1/8*(4*a^3-23*a^2*b+7*a*
b^2+12*b^3)/a^2*b*tanh(1/2*d*x+1/2*c)^3-1/8*b*(4*a^2+a*b-5*b^2)/a*tanh(1/2*
d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2
*d*x+1/2*c)^2+a)^2+1/8/a*(8*a^3-4*a^2*b-a*b^2-3*b^3)*(-1/2*((-b*(a-b))^(1/2
)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh
(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)
+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))))-1/b^3*ln(tanh(1/2*d*x
+1/2*c)-1)+1/b^3*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2623 vs. $2(148) = 296$.
time = 0.46, size = 5511, normalized size = 34.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^2*b^2*d*x*cosh(d*x + c)^8 + 128*a^2*b^2*d*x*cosh(d*x + c)*sinh(
d*x + c)^7 + 16*a^2*b^2*d*x*sinh(d*x + c)^8 + 4*(16*a^3*b - 20*a^2*b^2 + a*
b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^6 + 4*(112*a^2*b^2*
d*x*cosh(d*x + c)^2 + 16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b -
 a^2*b^2)*d*x)*sinh(d*x + c)^6 + 16*a^2*b^2*d*x + 8*(112*a^2*b^2*d*x*cosh(d
*x + c)^3 + 3*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^
2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(48*a^4 - 72*a^3*b + 18*a^2*b^2
+ 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^4 +
 4*(280*a^2*b^2*d*x*cosh(d*x + c)^4 + 48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a
*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x + 15*(16*a^3*b - 20*a^2*
b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + 24*a^2*b^2 - 12*a*b^3 - 12*b^4 + 16*(56*a^2*b^2*d*x*cosh(d*x + c)
^5 + 5*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)
*cosh(d*x + c)^3 + (48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(
8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(32*a^
3*b - 28*a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x
+ c)^2 + 4*(112*a^2*b^2*d*x*cosh(d*x + c)^6 + 15*(16*a^3*b - 20*a^2*b^2 + a
*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^4 + 32*a^3*b - 28*
a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x + 6*(48*a^4 - 72*a^
3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x
+ c)^8 + 8*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8
*a^2*b^2 + 4*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4
)*cosh(d*x + c)^6 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4 + 7*(8*a^2*b^2 + 4*a*b^3
+ 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2*b^2 + 4*a*b^3 + 3*b
^4)*cosh(d*x + c)^3 + 3*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x +
 c)^4 + 2*(35*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 64*a^4 - 32*a
^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4 + 30*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 + 4*a*b^3 + 3*b^4 + 8*(7*(8*a^2*b^
```

```
2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh
(d*x + c)^3 + (64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*
(7*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(16*a^3*b + 2*a*b^3 -
 3*b^4)*cosh(d*x + c)^4 + 16*a^3*b + 2*a*b^3 - 3*b^4 + 3*(64*a^4 - 32*a^3*b
 + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*
a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(16*a^3*b + 2*a*b^3 - 3*b^4)
*cosh(d*x + c)^5 + (64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh
(d*x + c)^3 + (16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sq
rt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d
*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cos
h(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x
 + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 -
 a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x +
 c))*sinh(d*x + c) + b)) + 8*(16*a^2*b^2*d*x*cosh(d*x + c)^7 + 3*(16*a^3*b
- 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^5
+ 2*(48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b
 + 3*a^2*b^2)*d*x)*cosh(d*x + c)^3 + (32*a^3*b - 28*a^2*b^2 - 13*a*b^3 + 9*
b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b^5*d*
cosh(d*x + c)^8 + 8*a^2*b^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*b^5*d*sin
h(d*x + c)^8 + a^2*b^5*d + 4*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7
*a^2*b^5*d*cosh(d*x + c)^2 + (2*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(
8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*a^2*b^5*d*cosh(
d*x + c)^3 + 3*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
35*a^2*b^5*d*cosh(d*x + c)^4 + 30*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 +
 (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^3*b^4 - a^
2*b^5)*d*cosh(d*x + c)^2 + 8*(7*a^2*b^5*d*cosh(d*x + c)^5 + 10*(2*a^3*b^4 -
 a^2*b^5)*d*cosh(d*x + c)^3 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(7*a^2*b^5*d*cosh(d*x + c)^6 + 15*(2*a^3*b^4 -
a^2*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d
*x + c)^2 + (2*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^2 + 8*(a^2*b^5*d*cosh(d*
x + c)^7 + 3*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5 + (8*a^4*b^3 - 8*a^3*b
^4 + 3*a^2*b^5)*d*cosh(d*x + c)^3 + (2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*
sinh(d*x + c)), 1/8*(8*a^2*b^2*d*x*cosh(d*x + c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(148) = 296.
time = 4.08, size = 353, normalized size = 2.21

$$\frac{\frac{8\,(dx+c)}{b^3}-\frac{(8\,a^3-4\,a^2b-ab^2-3\,b^3)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,a^2b^3}+\frac{2\,(16\,a^3be^{(6\,dx+6\,c)}-20\,a^2b^2e^{(6\,dx+6\,c)}+ab^3e^{(6\,dx+6\,c)}+3\,b^4e^{(6\,dx+6\,c)}+48\,a^4e^{(4\,dx+4\,c)}-72\,a^3be^{(4\,dx+4\,c)}+18\,a^2b^2e^{(4\,dx+4\,c)}+15\,ab^3e^{(4\,dx+4\,c)}-9\,b^4e^{(4\,dx+4\,c)}+32\,a^3be^{(2\,dx+2\,c)}-28\,a^2b^2e^{(2\,dx+2\,c)}-13\,ab^3e^{(2\,dx+2\,c)}+9\,b^4e^{(2\,dx+2\,c)}+6\,a^2b^2-3\,ab^3-3\,b^4)}{\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)^2a^2b^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)/b^3 - (8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*arctan(1/2*(b*e^(2
*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a^2*b^3) + 2*(16
*a^3*b*e^(6*d*x + 6*c) - 20*a^2*b^2*e^(6*d*x + 6*c) + a*b^3*e^(6*d*x + 6*c)
 + 3*b^4*e^(6*d*x + 6*c) + 48*a^4*e^(4*d*x + 4*c) - 72*a^3*b*e^(4*d*x + 4*c
) + 18*a^2*b^2*e^(4*d*x + 4*c) + 15*a*b^3*e^(4*d*x + 4*c) - 9*b^4*e^(4*d*x
+ 4*c) + 32*a^3*b*e^(2*d*x + 2*c) - 28*a^2*b^2*e^(2*d*x + 2*c) - 13*a*b^3*e
^(2*d*x + 2*c) + 9*b^4*e^(2*d*x + 2*c) + 6*a^2*b^2 - 3*a*b^3 - 3*b^4)/((b*e
^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2*a^2*b^3))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2)^3, x)
```

3.340 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=133

$$\frac{(3a^2+2ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d}-\frac{(a-b)\cosh^2(c+dx)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}+\frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\sinh(c+dx)}{8d\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] 1/8*(3*a^2+2*a*b+3*b^2)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/b^(5/2)
/d-1/4*(a-b)*cosh(d*x+c)^2*sinh(d*x+c)/a/b/d/(a+b*sinh(d*x+c)^2)^2+3/8*(1/a
^2-1/b^2)*sinh(d*x+c)/d/(a+b*sinh(d*x+c)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 424, 393, 211}

$$\frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\sinh(c+dx)}{8d\left(a+b\sinh^2(c+dx)\right)}+\frac{(3a^2+2ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d}-\frac{(a-b)\sinh(c+dx)\cosh^2(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*a^2 + 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2
)*b^(5/2)*d) - ((a - b)*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*b*d*(a + b*Sinh
[c + d*x]^2)^2) + (3*(a^(-2) - b^(-2))*Sinh[c + d*x])/(8*d*(a + b*Sinh[c +
d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+bx^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\cosh^2(c+dx)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} + \frac{\text{Subst}\Big(\int \frac{a+3b+(3a+b)x^2}{\left(a+bx^2\right)^2}\,dx, x, \sinh(c+}{4abd}$$

$$= -\frac{(a-b)\cosh^2(c+dx)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3\left(a^2-b^2\right)\sinh(c+dx)}{8a^2b^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{(3a^2}{}$$

$$= \frac{\left(3a^2+2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} - \frac{(a-b)\cosh^2(c+dx)\sinh(c+d}{4abd\left(a+b\sinh^2(c+dx)\right)^2}$$

Mathematica [A]
time = 0.26, size = 149, normalized size = 1.12

$$\frac{-\left(\left(3a^2+2ab+3b^2\right)\text{ArcTan}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{b}}\right)\right) + \frac{8a^{3/2}(a-b)^2\sqrt{b}\,\sinh(c+dx)}{(2a-b+b\cosh(2(c+dx)))^2} - \frac{2\sqrt{a}\,\sqrt{b}\,\left(5a^2-2ab-3b^2\right)\sinh(c+dx)}{2a-b+b\cosh(2(c+dx))}}{8a^{5/2}b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-((3*a^2 + 2*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]]) + (8*a^
(3/2)*(a - b)^2*Sqrt[b]*Sinh[c + d*x])/(2*a - b + b*Cosh[2*(c + d*x)])^2 -
(2*Sqrt[a]*Sqrt[b]*(5*a^2 - 2*a*b - 3*b^2)*Sinh[c + d*x])/(2*a - b + b*Cosh
[2*(c + d*x)]))/(8*a^(5/2)*b^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(119) = 238$.

time = 1.90, size = 400, normalized size = 3.01

method	result
derivativedivides	$\dfrac{\dfrac{\left(3a^2+2ab-5b^2\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a\,b^2}-\dfrac{\left(9a^3-14a^2b-7a\,b^2+12b^3\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2b^2}+\dfrac{\left(9a^3-14a^2b-7a\,b^2+12b^3\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2b^2}-}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}$
default	$\dfrac{\dfrac{\left(3a^2+2ab-5b^2\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a\,b^2}-\dfrac{\left(9a^3-14a^2b-7a\,b^2+12b^3\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2b^2}+\dfrac{\left(9a^3-14a^2b-7a\,b^2+12b^3\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2b^2}-}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}$
risch	$-\dfrac{\mathrm{e}^{dx+c}(5a^2b\,\mathrm{e}^{6dx+6c}-2a\,b^2\mathrm{e}^{6dx+6c}-3b^3\mathrm{e}^{6dx+6c}+12a^3\mathrm{e}^{4dx+4c}-7a^2b\,\mathrm{e}^{4dx+4c}-14a\,b^2\mathrm{e}^{4dx+4c}+9b^3\mathrm{e}^{4dx+4c}-12a^3\mathrm{e}^{2dx+}}{4b^2a^2d\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(1/8*(3*a^2+2*a*b-5*b^2)/a/b^2*tanh(1/2*d*x+1/2*c)^7-1/8*(9*a^3-14*a
^2*b-7*a*b^2+12*b^3)/a^2/b^2*tanh(1/2*d*x+1/2*c)^5+1/8*(9*a^3-14*a^2*b-7*a*
b^2+12*b^3)/a^2/b^2*tanh(1/2*d*x+1/2*c)^3-1/8*(3*a^2+2*a*b-5*b^2)/a/b^2*tan
h(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*ta
nh(1/2*d*x+1/2*c)^2+a)^2+1/4/a*(3*a^2+2*a*b+3*b^2)/b^2*(1/2*(-a+(-b*(a-b))^
(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*(a+(-b*(a-b))^
(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*
tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((5*a^2*b*e^(7*c) - 2*a*b^2*e^(7*c) - 3*b^3*e^(7*c))*e^(7*d*x) + (12*a
^3*e^(5*c) - 7*a^2*b*e^(5*c) - 14*a*b^2*e^(5*c) + 9*b^3*e^(5*c))*e^(5*d*x)
- (12*a^3*e^(3*c) - 7*a^2*b*e^(3*c) - 14*a*b^2*e^(3*c) + 9*b^3*e^(3*c))*e^(
3*d*x) - (5*a^2*b*e^c - 2*a*b^2*e^c - 3*b^3*e^c)*e^(d*x))/(a^2*b^4*d*e^(8*d
*x + 8*c) + a^2*b^4*d + 4*(2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*
x) + 2*(8*a^4*b^2*d*e^(4*c) - 8*a^3*b^3*d*e^(4*c) + 3*a^2*b^4*d*e^(4*c))*e^
(4*d*x) + 4*(2*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 1/32*int
egrate(8*((3*a^2*e^(3*c) + 2*a*b*e^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x) + (3*a^
2*e^c + 2*a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^2*b^3*e^(4*d*x + 4*c) + a^2*b^3
+ 2*(2*a^3*b^2*e^(2*c) - a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3266 vs. $2(119) = 238$.
time = 0.43, size = 5844, normalized size = 43.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^7 + 28*(5*a^3*b^2
 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(5*a^3*b^2 - 2*a^
2*b^3 - 3*a*b^4)*sinh(d*x + c)^7 + 4*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9
*a*b^4)*cosh(d*x + c)^5 + 4*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4 +
21*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*
(7*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (12*a^4*b - 7*a^3*b^
2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(12*a^4*b - 7*
a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - 4*(12*a^4*b - 7*a^3*b^2 -
 14*a^2*b^3 + 9*a*b^4 - 35*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^
4 - 10*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(
d*x + c)^3 + 4*(21*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^5 + 10*(
12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - 3*(12*a^4*b
- 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^
2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*
cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b^2 + 2*a*b^3 + 3*b^4)*sinh(d*x + c)
^8 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(6*a^3*b +
 a^2*b^2 + 4*a*b^3 - 3*b^4 + 7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3
*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(
3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24*a^4 - 8*a^3*b + 17*a^2*b^
2 - 18*a*b^3 + 9*b^4 + 30*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x +
```

```
c)^2)*sinh(d*x + c)^4 + 3*a^2*b^2 + 2*a*b^3 + 3*b^4 + 8*(7*(3*a^2*b^2 + 2*a
*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*co
sh(d*x + c)^3 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x
 + c))*sinh(d*x + c)^3 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x +
 c)^2 + 4*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(6*a^3*b +
a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 6*a^3*b + a^2*b^2 + 4*a*b^3 -
3*b^4 + 3*(24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 8*((3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(
6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (24*a^4 - 8*a^3*b +
17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (6*a^3*b + a^2*b^2 + 4*a*b
^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log((b*cosh(d*x + c)^4
 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh
(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh
(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 +
 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1
)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*c
osh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c
)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c
)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) - 4*(5*a^3*b^2 - 2*a^2*b
^3 - 3*a*b^4)*cosh(d*x + c) + 4*(7*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d
*x + c)^6 - 5*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4 + 5*(12*a^4*b - 7*a^3*b^2 - 14*
a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^4 - 3*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 +
 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/(a^3*b^5*d*cosh(d*x + c)^8 + 8*a^
3*b^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*b^5*d*sinh(d*x + c)^8 + a^3*b^5
*d + 4*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*a^3*b^5*d*cosh(d*x +
c)^2 + (2*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b^3 - 8*a^4*b^4
+ 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*a^3*b^5*d*cosh(d*x + c)^3 + 3*(2*a^4*
b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^3*b^5*d*cosh(d*x
+ c)^4 + 30*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^5*b^3 - 8*a^4*b^
4 + 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)
^2 + 8*(7*a^3*b^5*d*cosh(d*x + c)^5 + 10*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x +
 c)^3 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*(7*a^3*b^5*d*cosh(d*x + c)^6 + 15*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x +
c)^4 + 3*(8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^4*b^4
 - a^3*b^5)*d)*sinh(d*x + c)^2 + 8*(a^3*b^5*d*cosh(d*x + c)^7 + 3*(2*a^4*b^
4 - a^3*b^5)*d*cosh(d*x + c)^5 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh
(d*x + c)^3 + (2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(
2*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^7 + 14*(5*a^3*b^2 - 2*a^2
*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(5*a^3*b^2 - 2*a^2*b^3 -
3*a*b^4)*sinh(d*x + c)^7 + 2*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*
cosh(d*x + c)^5 + 2*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4 + 21*(5*a^
3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(5*a^
3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (12*a^4*b - 7*a^3*b^2 - 14*a
^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(12*a^4*b - 7*a^3*b^2
- 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - 2*(12...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^5}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2)^3, x)
```

3.341 $\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=114

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a-b}\,d}+\frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2}+\frac{3\tanh(c+dx)}{8a^2d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

```
[Out] 3/8*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/d/(a-b)^(1/2)+1/4*tanh
(d*x+c)/a/d/(a-(a-b)*tanh(d*x+c)^2)^2+3/8*tanh(d*x+c)/a^2/d/(a-(a-b)*tanh(d
*x+c)^2)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$,
Rules used = {3270, 205, 214}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a-b}}+\frac{3\tanh(c+dx)}{8a^2d\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[a - b]*d)
+ Tanh[c + d*x]/(4*a*d*(a - (a - b)*Tanh[c + d*x]^2)^2) + (3*Tanh[c + d*x])
/(8*a^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
```

```
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a-(a-b)x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(a+(-a+b)x^2)^2}\,dx, x, \tanh(c+\right.}{4ad}$$

$$= \frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8a^2d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{3\text{Su}}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a-b}\,d} + \frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{}{8a^2d\,(}$$

Mathematica [A]
time = 0.49, size = 102, normalized size = 0.89

$$\frac{\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a}\,(8a-3b+(2a+3b)\cosh(2(c+dx)))\sinh(2(c+dx))}{(2a-b+b\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a - b] + (Sqrt[a]*(8
*a - 3*b + (2*a + 3*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(2*a - b + b*C
osh[2*(c + d*x)])^2)/(8*a^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(100) = 200$.
time = 1.53, size = 300, normalized size = 2.63

method	result

derivativedivides	$-\frac{-\frac{2\left(-\frac{5\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{3(a+4b)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}}-\frac{3(a+4b)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^{2}}-3\left(\frac{\left(\sqrt{-b\left(a-b\right)}+b\right.}{2a\sqrt{-b\left(a-\right.}}\right.}{}$
default	$-\frac{-\frac{2\left(-\frac{5\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{3(a+4b)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}}-\frac{3(a+4b)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^{2}}-3\left(\frac{\left(\sqrt{-b\left(a-b\right)}+b\right.}{2a\sqrt{-b\left(a-\right.}}\right.}{}$
risch	$-\frac{8a^{2}b\,\mathrm{e}^{6dx+6c}-3b^{3}\mathrm{e}^{6dx+6c}+16a^{3}\mathrm{e}^{4dx+4c}+8a^{2}b\,\mathrm{e}^{4dx+4c}-18a\,b^{2}\mathrm{e}^{4dx+4c}+9b^{3}\mathrm{e}^{4dx+4c}+8a^{2}b\,\mathrm{e}^{2dx+2c}+16a\,b^{2}\mathrm{e}^{2dx+2c}-9}{4b^{2}a^{2}d\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-5/8/a*tanh(1/2*d*x+1/2*c)^7-3/8*(a+4*b)/a^2*tanh(1/2*d*x+1/2*c)^5
-3/8*(a+4*b)/a^2*tanh(1/2*d*x+1/2*c)^3-5/8/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1
/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-3/
4/a*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2115 vs. $2(102) = 204$.
time = 0.43, size = 4486, normalized size = 39.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 24*
(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 +
 4*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*sinh(d*x + c)^6 + 8*a^3*b^2
+ 4*a^2*b^3 - 12*a*b^4 + 4*(16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*
a*b^4)*cosh(d*x + c)^4 + 4*(16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*
a*b^4 + 15*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + 16*(5*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x +
 c)^3 + (16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(8*a^4*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4)*cosh(d*
x + c)^2 + 4*(8*a^4*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4 + 15*(8*a^4*b - 8*
a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 8*a^4*b - 26*a
^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(b^4*co
sh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x + c)^7 + b^4*sinh(d*x + c)^8 +
 4*(2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(7*b^4*cosh(d*x + c)^2 + 2*a*b^3 - b
^4)*sinh(d*x + c)^6 + 8*(7*b^4*cosh(d*x + c)^3 + 3*(2*a*b^3 - b^4)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^4 +
2*(35*b^4*cosh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b^3 + 3*b^4 + 30*(2*a*b^3 - b^4
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + b^4 + 8*(7*b^4*cosh(d*x + c)^5 + 10*(2
*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 4*(2*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*b^4*cosh(d*x +
 c)^6 + 15*(2*a*b^3 - b^4)*cosh(d*x + c)^4 + 2*a*b^3 - b^4 + 3*(8*a^2*b^2 -
 8*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(b^4*cosh(d*x + c)^7
 + 3*(2*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d
*x + c)^3 + (2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*l
og((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*
x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a
*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 +
(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cos
h(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b
*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 +
2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x +
```

```
c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b))
+ 8*(3*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 + 2*(16*
a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^3 + (8*a^4
*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^4*
b^4 - a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)*si
nh(d*x + c)^7 + (a^4*b^4 - a^3*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^5*b^3 - 3*a^
4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b^4 - a^3*b^5)*d*cosh(d*x +
c)^2 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^6*b^2
- 16*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b^4 -
a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 30*(
2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^6*b^2 - 16*a^5*b^
3 + 11*a^4*b^4 - 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^5*b^3 - 3*a^4*b^4 +
 a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 +
10*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^6*b^2 - 16*a^
5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^
4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 15*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)^4 + 3*(8*a^6*b^2 - 16*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*co
sh(d*x + c)^2 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^2 + (a^4
*b^4 - a^3*b^5)*d + 8*((a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^7 + 3*(2*a^5*b^3
 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + (8*a^6*b^2 - 16*a^5*b^3 + 11*a^
4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^3 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*
cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3
*a*b^4)*cosh(d*x + c)^6 + 12*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*co
sh(d*x + c)*sinh(d*x + c)^5 + 2*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)
*sinh(d*x + c)^6 + 4*a^3*b^2 + 2*a^2*b^3 - 6*a*b^4 + 2*(16*a^5 - 8*a^4*b -
26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^4 + 2*(16*a^5 - 8*a^4*b -
26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4 + 15*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3
*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(8*a^4*b - 8*a^3*b^2 - 3*a^
2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + (16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*
b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^4*b + 8*a^3*b^2 - 25
*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2 + 2*(8*a^4*b + 8*a^3*b^2 - 25*a^2*b^3 +
 9*a*b^4 + 15*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 +
 6*(16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 3*(b^4*cosh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x + c
)^7 + b^4*sinh(d*x + c)^8 + 4*(2*a*b^3 - b^4)*c...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(102) = 204.

time = 1.75, size = 244, normalized size = 2.14

$$\frac{\frac{3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,a^2}-\frac{2\,\left(8\,a^2be^{(6\,dx+6\,c)}-3\,b^3e^{(6\,dx+6\,c)}+16\,a^3e^{(4\,dx+4\,c)}+8\,a^2be^{(4\,dx+4\,c)}-18\,ab^2e^{(4\,dx+4\,c)}+9\,b^3e^{(4\,dx+4\,c)}+8\,a^2be^{(2\,dx+2\,c)}+16\,ab^2e^{(2\,dx+2\,c)}-9\,b^3e^{(2\,dx+2\,c)}+2\,ab^2+3\,b^3\right)}{\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)^2a^2b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^
2 + a*b)*a^2) - 2*(8*a^2*b*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*c) + 16*a^3
*e^(4*d*x + 4*c) + 8*a^2*b*e^(4*d*x + 4*c) - 18*a*b^2*e^(4*d*x + 4*c) + 9*b
^3*e^(4*d*x + 4*c) + 8*a^2*b*e^(2*d*x + 2*c) + 16*a*b^2*e^(2*d*x + 2*c) - 9
*b^3*e^(2*d*x + 2*c) + 2*a*b^2 + 3*b^3)/((b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x
+ 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2*a^2*b^2))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4}{{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^3, x)
```

3.342 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=117

$$\frac{(a+3b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d}-\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}+\frac{(a+3b)\sinh(c+dx)}{8a^2bd\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] 1/8*(a+3*b)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)/d-1/4*(a-b)
*sinh(d*x+c)/a/b/d/(a+b*sinh(d*x+c)^2)^2+1/8*(a+3*b)*sinh(d*x+c)/a^2/b/d/(a
+b*sinh(d*x+c)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 393, 205, 211}

$$\frac{(a+3b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d}+\frac{(a+3b)\sinh(c+dx)}{8a^2bd\left(a+b\sinh^2(c+dx)\right)}-\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a + 3*b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*d) -
 ((a - b)*Sinh[c + d*x])/(4*a*b*d*(a + b*Sinh[c + d*x]^2)^2) + ((a + 3*b)*S
inh[c + d*x])/(8*a^2*b*d*(a + b*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^2)^3}\,dx,x,\sinh(c+dx)\right)}{d}\\
&= -\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}+\frac{(a+3b)\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx,x,\sinh(c+dx\right.}{4abd}\\
&= -\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}+\frac{(a+3b)\sinh(c+dx)}{8a^2bd\left(a+b\sinh^2(c+dx)\right)}+\frac{(a+3b)\text{Subs}}{}\\
&= \frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d}-\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}+\frac{(a+3b}{8a^2bd\,(a+}
\end{aligned}
$$

Mathematica [A]
time = 0.51, size = 114, normalized size = 0.97

$$
\frac{-\frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}+(a+3b)\left(\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{\sinh(c+dx)\left(5a+3b\sinh^2(c+dx)\right)}{8a^2\left(a+b\sinh^2(c+dx)\right)^2}\right)}{3bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-(Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2) + (a + 3*b)*((3*ArcTan[(Sqrt[b]
*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]) + (Sinh[c + d*x]*(5*a + 3*b*S
inh[c + d*x]^2))/(8*a^2*(a + b*Sinh[c + d*x]^2)^2)))/(3*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(103) = 206$.
time = 1.71, size = 354, normalized size = 3.03

method	result
risch	$-\frac{e^{dx+c}(-ab\,e^{6dx+6c}-3b^2e^{6dx+6c}+4a^2e^{4dx+4c}-17ab\,e^{4dx+4c}+9b^2e^{4dx+4c}-4a^2e^{2dx+2c}+17ab\,e^{2dx+2c}-9b^2e^{2dx+2c}+a}{4b\,a^2d\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)^2}$
derivativedivides	$(a+$ $\frac{\frac{\frac{(a-5b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ab}-\frac{\left(3a^2-11ab+12b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2b}+\frac{\left(3a^2-11ab+12b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2b}-\frac{(a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ab}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}+}{}$
default	$(a+$ $\frac{\frac{\frac{(a-5b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ab}-\frac{\left(3a^2-11ab+12b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2b}+\frac{\left(3a^2-11ab+12b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2b}-\frac{(a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ab}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^2}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(1/8*(a-5*b)/a/b*tanh(1/2*d*x+1/2*c)^7-1/8*(3*a^2-11*a*b+12*b^2)/a^2
/b*tanh(1/2*d*x+1/2*c)^5+1/8*(3*a^2-11*a*b+12*b^2)/a^2/b*tanh(1/2*d*x+1/2*c
)^3-1/8*(a-5*b)/a/b*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(
1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/4/a*(a+3*b)/b*(1/2*(-a+(-
b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*(a+(-
b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*a
rctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((a*b*e^(7*c) + 3*b^2*e^(7*c))*e^(7*d*x) - (4*a^2*e^(5*c) - 17*a*b*e^(5
*c) + 9*b^2*e^(5*c))*e^(5*d*x) + (4*a^2*e^(3*c) - 17*a*b*e^(3*c) + 9*b^2*e^
(3*c))*e^(3*d*x) - (a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^2*b^3*d*e^(8*d*x + 8*c
) + a^2*b^3*d + 4*(2*a^3*b^2*d*e^(6*c) - a^2*b^3*d*e^(6*c))*e^(6*d*x) + 2*(
8*a^4*b*d*e^(4*c) - 8*a^3*b^2*d*e^(4*c) + 3*a^2*b^3*d*e^(4*c))*e^(4*d*x) +
4*(2*a^3*b^2*d*e^(2*c) - a^2*b^3*d*e^(2*c))*e^(2*d*x)) + 1/8*integrate(2*((
a*e^(3*c) + 3*b*e^(3*c))*e^(3*d*x) + (a*e^c + 3*b*e^c)*e^(d*x))/(a^2*b^2*e^
(4*d*x + 4*c) + a^2*b^2 + 2*(2*a^3*b*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)),
 x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2696 vs. $2(103) = 206$.
time = 0.42, size = 4907, normalized size = 41.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^7 + 28*(a^2*b^2 + 3*a*b^3)*cosh(
d*x + c)*sinh(d*x + c)^6 + 4*(a^2*b^2 + 3*a*b^3)*sinh(d*x + c)^7 - 4*(4*a^3
*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^5 - 4*(4*a^3*b - 17*a^2*b^2 + 9*a*
b^3 - 21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*
b^2 + 3*a*b^3)*cosh(d*x + c)^3 - (4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^4 + 4*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^3
+ 4*(35*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 4*a^3*b - 17*a^2*b^2 + 9*a*b^
3 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
4*(21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^5 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*
b^3)*cosh(d*x + c)^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^2 - ((a*b^2 + 3*b^3)*cosh(d*x + c)^8 + 8*(a*b^2 + 3*b^3)*cosh(d
*x + c)*sinh(d*x + c)^7 + (a*b^2 + 3*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b + 5*
a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3 + 7*(a*b^2 +
3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a*b^2 + 3*b^3)*cosh(d*x + c
)^3 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a
^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + 2*(35*(a*b^2 + 3*b^3)*c
osh(d*x + c)^4 + 8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3 + 30*(2*a^2*b + 5*a*b^
2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a*b^2 + 3*b^3)*cosh(d*x
 + c)^5 + 10*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^3 + (8*a^3 + 16*a^2*
b - 21*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 + 3*b^3 + 4*(2
*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^2 + 4*(7*(a*b^2 + 3*b^3)*cosh(d*x +
 c)^6 + 15*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^4 + 2*a^2*b + 5*a*b^2
- 3*b^3 + 3*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*sinh(d*x
```

```
 + c)^2 + 8*((a*b^2 + 3*b^3)*cosh(d*x + c)^7 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3
)*cosh(d*x + c)^5 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^3 +
 (2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log((
b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 -
 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x +
 c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(
cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*co
sh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d
*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a
 - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 +
4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) - 4*(a^
2*b^2 + 3*a*b^3)*cosh(d*x + c) + 4*(7*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^6 -
 5*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^4 - a^2*b^2 - 3*a*b^3 + 3
*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(a^3*b^4*
d*cosh(d*x + c)^8 + 8*a^3*b^4*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*b^4*d*s
inh(d*x + c)^8 + a^3*b^4*d + 4*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^6 + 4*
(7*a^3*b^4*d*cosh(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^6 + 2
*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*a^3*b^4*d*cos
h(d*x + c)^3 + 3*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(35*a^3*b^4*d*cosh(d*x + c)^4 + 30*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2
 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^3 -
a^3*b^4)*d*cosh(d*x + c)^2 + 8*(7*a^3*b^4*d*cosh(d*x + c)^5 + 10*(2*a^4*b^3
 - a^3*b^4)*d*cosh(d*x + c)^3 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*cosh(
d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*b^4*d*cosh(d*x + c)^6 + 15*(2*a^4*b^3
- a^3*b^4)*d*cosh(d*x + c)^4 + 3*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*cosh
(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^2 + 8*(a^3*b^4*d*cosh(
d*x + c)^7 + 3*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^5 + (8*a^5*b^2 - 8*a^4
*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^3 + (2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)
)*sinh(d*x + c)), 1/8*(2*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^7 + 14*(a^2*b^2
+ 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(a^2*b^2 + 3*a*b^3)*sinh(d*x +
 c)^7 - 2*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^5 - 2*(4*a^3*b - 1
7*a^2*b^2 + 9*a*b^3 - 21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^5 + 10*(7*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 - (4*a^3*b - 17*a^2*b^2 + 9*
a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*
cosh(d*x + c)^3 + 2*(35*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 4*a^3*b - 17*
a^2*b^2 + 9*a*b^3 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^3 + 2*(21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^5 - 10*(4*a^3*b - 1
7*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*c
osh(d*x + c))*sinh(d*x + c)^2 + ((a*b^2 + 3*b^3)*cosh(d*x + c)^8 + 8*(a*b^2
 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 + 3*b^3)*sinh(d*x + c)^8 +
 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 4*(2*a^2*b + 5*a*b^2 - 3*b
^3 + 7*(a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a*b^2 + 3*b
^3)*cosh(d*x + c)^3 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 2*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^3}{{\left(b\,{\mathrm{sinh}\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^3, x)
```

3.343 $\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=143

$$\frac{(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{3/2}d}-\frac{b\tanh(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2}+\frac{(4a-3b)\tanh(c}{8a^2(a-b)d\left(a-(a-b)\,\mathrm{t}\right.}$$

```
[Out] 1/8*(4*a-3*b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a-b)^(3/2)/
d-1/4*b*tanh(d*x+c)/a/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)^2+1/8*(4*a-3*b)*tanh(
d*x+c)/a^2/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3270, 393, 205, 214}

$$\frac{(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{3/2}}+\frac{(4a-3b)\tanh(c+dx)}{8a^2d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{b\tanh(c+dx)}{4ad(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((4*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a -
b)^(3/2)*d) - (b*Tanh[c + d*x])/(4*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2
)^2) + ((4*a - 3*b)*Tanh[c + d*x])/(8*a^2*(a - b)*d*(a - (a - b)*Tanh[c + d
*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{(4a-3b)\text{Subst}\bigl(\int \frac{1}{(a+(-a+b)x^2)^2}}{4a(a-b)}$$

$$= -\frac{b\tanh(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{(4a-3b)\tanh(c+dx}{8a^2(a-b)d\bigl(a-(a-b)\tanh^2}$$

$$= \frac{(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{3/2}d} - \frac{b\tanh(c+dx)}{4a(a-b)d\bigl(a-(a-b)\tanh^2(c+}$$

Mathematica [A]
time = 0.99, size = 124, normalized size = 0.87

$$\frac{\frac{(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}} + \frac{\sqrt{a}\,\left(8a^2-12ab+3b^2+(2a-3b)b\cosh(2(c+dx))\right)\sinh(2(c+dx))}{(a-b)(2a-b+b\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (((4*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a - b)^(3/2) +
 (Sqrt[a]*(8*a^2 - 12*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*(c + d*x)])*Sinh[2
*(c + d*x)])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*a^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(129) = 258$.
time = 1.60, size = 376, normalized size = 2.63

method	result
derivativedivides	$-\frac{2\left(-\frac{(4a-5b)\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a(a-b)}+\frac{\left(4a^{2}-13ab+12b^{2}\right)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}(a-b)}+\frac{\left(4a^{2}-13ab+12b^{2}\right)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}(a-b)}-\frac{(4a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{8a(a-b)}\right.}{\left(a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^{2}}$
default	$-\frac{2\left(-\frac{(4a-5b)\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a(a-b)}+\frac{\left(4a^{2}-13ab+12b^{2}\right)\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}(a-b)}+\frac{\left(4a^{2}-13ab+12b^{2}\right)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^{2}(a-b)}-\frac{(4a-5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{8a(a-b)}\right.}{\left(a\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\right)^{2}}$
risch	$-\frac{-4a\,b^{2}\mathrm{e}^{6dx+6c}+3b^{3}\mathrm{e}^{6dx+6c}+16a^{3}\mathrm{e}^{4dx+4c}-40a^{2}b\,\mathrm{e}^{4dx+4c}+30a\,b^{2}\mathrm{e}^{4dx+4c}-9b^{3}\mathrm{e}^{4dx+4c}+16a^{2}b\,\mathrm{e}^{2dx+2c}-28a\,b^{2}\mathrm{e}^{2dx+2}}{4b\,a^{2}d\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)^{2}(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/8*(4*a-5*b)/a/(a-b)*tanh(1/2*d*x+1/2*c)^7+1/8*(4*a^2-13*a*b+12*
b^2)/a^2/(a-b)*tanh(1/2*d*x+1/2*c)^5+1/8*(4*a^2-13*a*b+12*b^2)/a^2/(a-b)*ta
nh(1/2*d*x+1/2*c)^3-1/8*(4*a-5*b)/a/(a-b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*
d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-1/4/a
*(4*a-3*b)/(a-b)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2464 vs. $2(131) = 262$.
time = 0.47, size = 5183, normalized size = 36.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 24*(4*a^3*b^2
- 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(4*a^3*b^2 - 7*a^2
*b^3 + 3*a*b^4)*sinh(d*x + c)^6 - 8*a^3*b^2 + 20*a^2*b^3 - 12*a*b^4 - 4*(16
*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^4 - 4*(1
6*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4 - 15*(4*a^3*b^2 - 7*a^
2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(4*a^3*b^2 - 7*a^
2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 - (16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2
*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(16*a^4*b - 44*a^3*b^2 +
 37*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 - 4*(16*a^4*b - 44*a^3*b^2 + 37*a^2*
b^3 - 9*a*b^4 - 15*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 6*(1
6*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + ((4*a*b^3 - 3*b^4)*cosh(d*x + c)^8 + 8*(4*a*b^3 - 3*b^4)*cosh
(d*x + c)*sinh(d*x + c)^7 + (4*a*b^3 - 3*b^4)*sinh(d*x + c)^8 + 4*(8*a^2*b^
2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4 + 7
*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a*b^3 - 3*b^4
)*cosh(d*x + c)^3 + 3*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c)^4 + 2
*(35*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 32*a^3*b - 56*a^2*b^2 + 36*a*b^3 -
 9*b^4 + 30*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4
 + 4*a*b^3 - 3*b^4 + 8*(7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + 10*(8*a^2*b^2
 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (32*a^3*b - 56*a^2*b^2 + 36*a*b^3 -
9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*co
sh(d*x + c)^2 + 4*(7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 15*(8*a^2*b^2 - 10
*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 8*a^2*b^2 - 10*a*b^3 + 3*b^4 + 3*(32*a^3*
b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4
*a*b^3 - 3*b^4)*cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x
 + c)^5 + (32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c)^3 + (8*a
^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*lo
g((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x
 + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*
```

```
b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (
2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh
(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*
cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2
*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c
)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) +
 8*(3*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 - 2*(16*a^5 - 56*a^
4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - (16*a^4*b - 44*a
^3*b^2 + 37*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^3 - 2*
a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*
cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*sinh(d*x
+ c)^8 + 4*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6
+ 4*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b^2 - 5*a
^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^7*b - 24*a^6*b^2
+ 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 -
2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b
^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^3 - 2*a^4*b^4
 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3
*b^5)*d*cosh(d*x + c)^2 + (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 +
 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3
*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x +
 c)^5 + 10*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3
+ (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x +
 c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)
^6 + 15*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3
*(8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x +
c)^2 + (2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^2 + (
a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d + 8*((a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cos
h(d*x + c)^7 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x +
 c)^5 + (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh
(d*x + c)^3 + (2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)
)*sinh(d*x + c)), 1/8*(2*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6
+ 12*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(4
*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*sinh(d*x + c)^6 - 4*a^3*b^2 + 10*a^2*b^3 -
6*a*b^4 - 2*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*
x + c)^4 - 2*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4 - 15*(4
*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(131) = 262.
time = 1.72, size = 269, normalized size = 1.88

$$\frac{\frac{(4\,a-3\,b)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^3-a^2b)\sqrt{-a^2+ab}}+\frac{2\,(4\,ab^2e^{(6\,dx+6\,c)}-3\,b^3e^{(6\,dx+6\,c)}-16\,a^3e^{(4\,dx+4\,c)}+40\,a^2be^{(4\,dx+4\,c)}-30\,ab^2e^{(4\,dx+4\,c)}+9\,b^3e^{(4\,dx+4\,c)}-16\,a^2be^{(2\,dx+2\,c)}+28\,ab^2e^{(2\,dx+2\,c)}-9\,b^3e^{(2\,dx+2\,c)}-2\,ab^2+3\,b^3)}{(a^3b-a^2b^2)(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((4*a - 3*b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))
/((a^3 - a^2*b)*sqrt(-a^2 + a*b)) + 2*(4*a*b^2*e^(6*d*x + 6*c) - 3*b^3*e^(6
*d*x + 6*c) - 16*a^3*e^(4*d*x + 4*c) + 40*a^2*b*e^(4*d*x + 4*c) - 30*a*b^2*
e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) - 16*a^2*b*e^(2*d*x + 2*c) + 28*a*b
^2*e^(2*d*x + 2*c) - 9*b^3*e^(2*d*x + 2*c) - 2*a*b^2 + 3*b^3)/((a^3*b - a^2
*b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2
))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^2}{{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^3, x)
```

3.344 $\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=96

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}+\frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2}+\frac{3\sinh(c+dx)}{8a^2d\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] 1/4*sinh(d*x+c)/a/d/(a+b*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/a^2/d/(a+b*sinh(d
*x+c)^2)+3/8*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/d/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3269, 205, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}+\frac{3\sinh(c+dx)}{8a^2d\left(a+b\sinh^2(c+dx)\right)}+\frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d) + Sinh[c
+ d*x]/(4*a*d*(a + b*Sinh[c + d*x]^2)^2) + (3*Sinh[c + d*x])/(8*a^2*d*(a +
b*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
```

```
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{4ad}\\
&= \frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8a^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{3\text{Subst}\left(\int \frac{1}{a+bx^2}\right.}{8}\\
&= \frac{3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d} + \frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+d}{8a^2d\left(a+b\sinh^2(\right.}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 79, normalized size = 0.82

$$
\frac{\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{a}\sinh(c+dx)\left(5a+3b\sinh^2(c+dx)\right)}{\left(a+b\sinh^2(c+dx)\right)^2}}{8a^{5/2}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/Sqrt[b] + (Sqrt[a]*Sinh[c + d*
x]*(5*a + 3*b*Sinh[c + d*x]^2))/(a + b*Sinh[c + d*x]^2)^2)/(8*a^(5/2)*d)
```

Maple [A]
time = 0.56, size = 86, normalized size = 0.90

method	result
derivativedivides	$\frac{\frac{\sinh(dx+c)}{4a\left(a+b\left(\sinh^2(dx+c)\right)\right)^2}+\frac{\frac{3\sinh(dx+c)}{8a\left(a+b\left(\sinh^2(dx+c)\right)\right)}+\frac{3\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{a}}{d}$

default	$\frac{\frac{\sinh(dx+c)}{4a\left(a+b\left(\sinh^2(dx+c)\right)\right)^2}+\frac{\frac{3\sinh(dx+c)}{8a\left(a+b\left(\sinh^2(dx+c)\right)\right)}+\frac{3\arctan\left(\frac{b\sinh(dx+c)}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{a}}{d}$
risch	$\frac{\left(3b\,\mathrm{e}^{6dx+6c}+20a\,\mathrm{e}^{4dx+4c}-9b\,\mathrm{e}^{4dx+4c}-20a\,\mathrm{e}^{2dx+2c}+9b\,\mathrm{e}^{2dx+2c}-3b\right)\mathrm{e}^{dx+c}}{4\left(b\,\mathrm{e}^{4dx+4c}+4a\,\mathrm{e}^{2dx+2c}-2b\,\mathrm{e}^{2dx+2c}+b\right)^2a^2d}-\frac{3\ln\left(\mathrm{e}^{2dx+2c}-\frac{2a\,\mathrm{e}^{dx+c}}{\sqrt{-ab}}-1\right)}{16\sqrt{-ab}\,d\,a^2}+3\ln(\mathrm{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*sinh(d*x+c)/a/(a+b*sinh(d*x+c)^2)^2+3/4/a*(1/2*sinh(d*x+c)/a/(a+b*
sinh(d*x+c)^2)+1/2/a/(a*b)^(1/2)*arctan(b*sinh(d*x+c)/(a*b)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((20*a*e^(5*c) - 9*b*e^(5*c))*e^(5*d*x) - (20*a*e^(3*c) - 9*b*e^(3*c))*
e^(3*d*x) + 3*b*e^(7*d*x + 7*c) - 3*b*e^(d*x + c))/(a^2*b^2*d*e^(8*d*x + 8*
c) + a^2*b^2*d + 4*(2*a^3*b*d*e^(6*c) - a^2*b^2*d*e^(6*c))*e^(6*d*x) + 2*(8
*a^4*d*e^(4*c) - 8*a^3*b*d*e^(4*c) + 3*a^2*b^2*d*e^(4*c))*e^(4*d*x) + 4*(2*
a^3*b*d*e^(2*c) - a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(3/2*(e^(3*d
*x + 3*c) + e^(d*x + c))/(a^2*b*e^(4*d*x + 4*c) + a^2*b + 2*(2*a^3*e^(2*c)
- a^2*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2111 vs. $2(82) = 164$.
time = 0.63, size = 3934, normalized size = 40.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*a*b^2*cosh(d*x + c)^7 + 84*a*b^2*cosh(d*x + c)*sinh(d*x + c)^6 +
12*a*b^2*sinh(d*x + c)^7 + 4*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^5 + 4*(63*a
*b^2*cosh(d*x + c)^2 + 20*a^2*b - 9*a*b^2)*sinh(d*x + c)^5 + 20*(21*a*b^2*c
osh(d*x + c)^3 + (20*a^2*b - 9*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - 12*a
*b^2*cosh(d*x + c) - 4*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^3 + 4*(105*a*b^2*
```

```
cosh(d*x + c)^4 - 20*a^2*b + 9*a*b^2 + 10*(20*a^2*b - 9*a*b^2)*cosh(d*x + c
)^2)*sinh(d*x + c)^3 + 4*(63*a*b^2*cosh(d*x + c)^5 + 10*(20*a^2*b - 9*a*b^2
)*cosh(d*x + c)^3 - 3*(20*a^2*b - 9*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 -
 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*
x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a
*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*cosh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh
(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*
(35*b^2*cosh(d*x + c)^4 + 30*(2*a*b - b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b
+ 3*b^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 10*(2*a*b - b^2)*cosh
(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2
*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 15*(2*a*b - b^2)*c
osh(d*x + c)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*s
inh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 + 3*(2*a*b - b^2)*cosh(d*x +
c)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sin
h(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cos
h(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*
cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*
x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d
*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x +
c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
 + c))*sinh(d*x + c) + b)) + 4*(21*a*b^2*cosh(d*x + c)^6 + 5*(20*a^2*b - 9*
a*b^2)*cosh(d*x + c)^4 - 3*a*b^2 - 3*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c))/(a^3*b^3*d*cosh(d*x + c)^8 + 8*a^3*b^3*d*cosh(d*x + c)*sinh(
d*x + c)^7 + a^3*b^3*d*sinh(d*x + c)^8 + 4*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x
 + c)^6 + a^3*b^3*d + 4*(7*a^3*b^3*d*cosh(d*x + c)^2 + (2*a^4*b^2 - a^3*b^3
)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^
4 + 8*(7*a^3*b^3*d*cosh(d*x + c)^3 + 3*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c
))*sinh(d*x + c)^5 + 2*(35*a^3*b^3*d*cosh(d*x + c)^4 + 30*(2*a^4*b^2 - a^3*
b^3)*d*cosh(d*x + c)^2 + (8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d)*sinh(d*x + c)
^4 + 4*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^2 + 8*(7*a^3*b^3*d*cosh(d*x +
c)^5 + 10*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^3 + (8*a^5*b - 8*a^4*b^2 +
3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*b^3*d*cosh(d*x + c)^
6 + 15*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^4 + 3*(8*a^5*b - 8*a^4*b^2 + 3
*a^3*b^3)*d*cosh(d*x + c)^2 + (2*a^4*b^2 - a^3*b^3)*d)*sinh(d*x + c)^2 + 8*
(a^3*b^3*d*cosh(d*x + c)^7 + 3*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^5 + (8
*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^3 + (2*a^4*b^2 - a^3*b^3)*d
*cosh(d*x + c))*sinh(d*x + c)), 1/8*(6*a*b^2*cosh(d*x + c)^7 + 42*a*b^2*cos
h(d*x + c)*sinh(d*x + c)^6 + 6*a*b^2*sinh(d*x + c)^7 + 2*(20*a^2*b - 9*a*b^
2)*cosh(d*x + c)^5 + 2*(63*a*b^2*cosh(d*x + c)^2 + 20*a^2*b - 9*a*b^2)*sinh
(d*x + c)^5 + 10*(21*a*b^2*cosh(d*x + c)^3 + (20*a^2*b - 9*a*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^4 - 6*a*b^2*cosh(d*x + c) - 2*(20*a^2*b - 9*a*b^2)*cosh
(d*x + c)^3 + 2*(105*a*b^2*cosh(d*x + c)^4 - 20*a^2*b + 9*a*b^2 + 10*(20*a^
2*b - 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(63*a*b^2*cosh(d*x + c)
```

```
^5 + 10*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^3 - 3*(20*a^2*b - 9*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*si
nh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x + c)^6 + 4*(
7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*cosh(d*x +
c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2 - 8*a*b +
3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + c)^4 + 30*(2*a*b - b^2)*cosh(
d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c
)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x
+ c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(
d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 +
3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^3 +
 (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*sqrt(a*b)
*(cosh(d*x + c) + sinh(d*x + c))/a) + 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d
*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x +
c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*c
osh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(85) = 170$.

time = 30.10, size = 835, normalized size = 8.70

$$\begin{cases} \frac{\tilde{\infty} x \cosh{\left(c \right)}}{\sinh^{6}{\left(c \right)}} & \text{for}\: a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\sinh{\left(c + d x \right)}}{a^{3} d} & \text{for}\: b = 0 \\ - \frac{1}{5 b^{3} d \sinh^{5}{\left(c + d x \right)}} & \text{for}\: a = 0 \\ \frac{x \cosh{\left(c \right)}}{\left(a + b \sinh^{2}{\left(c \right)}\right)^{3}} & \text{for}\: d = 0 \\ \frac{3 a^{2} \log{\left(- \sqrt{- \frac{a}{b}} + \sinh{\left(c + d x \right)} \right)}}{16 a^{4} b d \sqrt{- \frac{a}{b}} + 32 a^{3} b^{2} d \sqrt{- \frac{a}{b}} \sinh^{2}{\left(c + d x \right)} + 16 a^{2} b^{3} d \sqrt{- \frac{a}{b}} \sinh^{4}{\left(c + d x \right)}} - \frac{3 a^{2} \log{\left(\sqrt{- \frac{a}{b}} + \sinh{\left(c + d x \right)} \right)}}{16 a^{4} b d \sqrt{- \frac{a}{b}} + 32 a^{3} b^{2} d \sqrt{- \frac{a}{b}} \sinh^{2}{\left(c + d x \right)} + 16 a^{2} b^{3} d \sqrt{- \frac{a}{b}} \sinh^{4}{\left(c + d x \right)}} + \frac{10 a b \sqrt{- \frac{a}{b}} \sinh{\left(c + d x \right)}}{16 a^{4} b d \sqrt{- \frac{a}{b}} + 32 a^{3} b^{2} d \sqrt{- \frac{a}{b}} \sinh^{2}{\left(c + d x \right)} + 16 a^{2} b^{3} d \sqrt{- \frac{a}{b}} \sinh^{4}{\left(c + d x \right)}} + \frac{6 a b \log{\left(- \sqrt{- \frac{a}{b}} + \sinh{\left(c + d x \right)} \right)} \sinh^{2}{\left(c + d x \right)}}{16 a^{4} b d \sqrt{- \frac{a}{b}} + 32 a^{3} b^{2} d \sqrt{- \frac{a}{b}} \sinh^{2}{\left(c + d x \right)} + 16 a^{2} b^{3} d \sqrt{- \frac{a}{b}} \sinh^{4}{\left(c + d x \right)}} - \frac{6 a b \log{\left(\sqrt{- \frac{a}{b}} + \sinh{\left(c + d x \right)} \right)} \sinh^{2}{\left(c + d x \right)}}{16 a^{4} b d \sqrt{- \frac{a}{b}} + 32 a^{3} b^{2} d \sqrt{- \frac{a}{b}} \sinh^{2}{\left(c + d x \right)} + 16 a^{2} b^{3} d \sqrt{- \frac{a}{b}} \sinh^{4}{\left(c + d x \right)}} + \frac{6 b^{2} \sqrt{- \frac{a}{b}} \sinh^{3}{\left(c + d x \right)}}{16 a^{4} b d \sqrt{- \frac{a}{b}} + 32 a^{3} b^{2} d \sqrt{- \frac{a}{b}} \sinh^{2}{\left(c + d x \right)} + 16 a^{2} b^{3} d \sqrt{- \frac{a}{b}} \sinh^{4}{\left(c + d x \right)}} + \frac{3 b^{2} \log{\left(- \sqrt{- \frac{a}{b}} + \sinh{\left(c + d x \right)} \right)} \sinh^{4}{\left(c + d x \right)}}{16 a^{4} b d \sqrt{- \frac{a}{b}} + 32 a^{3} b^{2} d \sqrt{- \frac{a}{b}} \sinh^{2}{\left(c + d x \right)} + 16 a^{2} b^{3} d \sqrt{- \frac{a}{b}} \sinh^{4}{\left(c + d x \right)}} - \frac{3 b^{2} \log{\left(\sqrt{- \frac{a}{b}} + \sinh{\left(c + d x \right)} \right)} \sinh^{4}{\left(c + d x \right)}}{16 a^{4} b d \sqrt{- \frac{a}{b}} + 32 a^{3} b^{2} d \sqrt{- \frac{a}{b}} \sinh^{2}{\left(c + d x \right)} + 16 a^{2} b^{3} d \sqrt{- \frac{a}{b}} \sinh^{4}{\left(c + d x \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((zoo*x*cosh(c)/sinh(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sinh
(c + d*x)/(a**3*d), Eq(b, 0)), (-1/(5*b**3*d*sinh(c + d*x)**5), Eq(a, 0)),
(x*cosh(c)/(a + b*sinh(c)**2)**3, Eq(d, 0)), (3*a**2*log(-sqrt(-a/b) + sinh
(c + d*x))/(16*a**4*b*d*sqrt(-a/b) + 32*a**3*b**2*d*sqrt(-a/b)*sinh(c + d*x
)**2 + 16*a**2*b**3*d*sqrt(-a/b)*sinh(c + d*x)**4) - 3*a**2*log(sqrt(-a/b)
+ sinh(c + d*x))/(16*a**4*b*d*sqrt(-a/b) + 32*a**3*b**2*d*sqrt(-a/b)*sinh(c
 + d*x)**2 + 16*a**2*b**3*d*sqrt(-a/b)*sinh(c + d*x)**4) + 10*a*b*sqrt(-a/b
)*sinh(c + d*x)/(16*a**4*b*d*sqrt(-a/b) + 32*a**3*b**2*d*sqrt(-a/b)*sinh(c
+ d*x)**2 + 16*a**2*b**3*d*sqrt(-a/b)*sinh(c + d*x)**4) + 6*a*b*log(-sqrt(-
a/b) + sinh(c + d*x))*sinh(c + d*x)**2/(16*a**4*b*d*sqrt(-a/b) + 32*a**3*b*
*2*d*sqrt(-a/b)*sinh(c + d*x)**2 + 16*a**2*b**3*d*sqrt(-a/b)*sinh(c + d*x)*
*4) - 6*a*b*log(sqrt(-a/b) + sinh(c + d*x))*sinh(c + d*x)**2/(16*a**4*b*d*s
qrt(-a/b) + 32*a**3*b**2*d*sqrt(-a/b)*sinh(c + d*x)**2 + 16*a**2*b**3*d*sqr
t(-a/b)*sinh(c + d*x)**4) + 6*b**2*sqrt(-a/b)*sinh(c + d*x)**3/(16*a**4*b*d
*sqrt(-a/b) + 32*a**3*b**2*d*sqrt(-a/b)*sinh(c + d*x)**2 + 16*a**2*b**3*d*s
qrt(-a/b)*sinh(c + d*x)**4) + 3*b**2*log(-sqrt(-a/b) + sinh(c + d*x))*sinh(
c + d*x)**4/(16*a**4*b*d*sqrt(-a/b) + 32*a**3*b**2*d*sqrt(-a/b)*sinh(c + d*
x)**2 + 16*a**2*b**3*d*sqrt(-a/b)*sinh(c + d*x)**4) - 3*b**2*log(sqrt(-a/b)
```

```
 + sinh(c + d*x))*sinh(c + d*x)**4/(16*a**4*b*d*sqrt(-a/b) + 32*a**3*b**2*d
*sqrt(-a/b)*sinh(c + d*x)**2 + 16*a**2*b**3*d*sqrt(-a/b)*sinh(c + d*x)**4),
 True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 0.94, size = 87, normalized size = 0.91

$$\frac{\frac{5\sinh(c+d\,x)}{8\,a}+\frac{3\,b\sinh(c+d\,x)^3}{8\,a^2}}{d\,a^2+2\,d\,a\,b\sinh\left(c+d\,x\right)^2+d\,b^2\sinh\left(c+d\,x\right)^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sinh(c+d\,x)}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] ((5*sinh(c + d*x))/(8*a) + (3*b*sinh(c + d*x)^3)/(8*a^2))/(a^2*d + b^2*d*si
nh(c + d*x)^4 + 2*a*b*d*sinh(c + d*x)^2) + (3*atan((b^(1/2)*sinh(c + d*x))/
a^(1/2)))/(8*a^(5/2)*b^(1/2)*d)
```

3.345 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=159

$$\frac{\operatorname{ArcTan}(\sinh(c+dx))}{(a-b)^3d}-\frac{\sqrt{b}\,(15a^2-10ab+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d}-\frac{b\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2}$$

```
[Out] arctan(sinh(d*x+c))/(a-b)^3/d-1/4*b*sinh(d*x+c)/a/(a-b)/d/(a+b*sinh(d*x+c)^
2)^2-1/8*(7*a-3*b)*b*sinh(d*x+c)/a^2/(a-b)^2/d/(a+b*sinh(d*x+c)^2)-1/8*(15*
a^2-10*a*b+3*b^2)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)/(a-b)
^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3269, 425, 541, 536, 209, 211}

$$-\frac{b(7a-3b)\sinh(c+dx)}{8a^2d(a-b)^2\left(a+b\sinh^2(c+dx)\right)}-\frac{\sqrt{b}\,(15a^2-10ab+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}+\frac{\operatorname{ArcTan}(\sinh(c+dx))}{d(a-b)^3}-\frac{b\sinh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/((a - b)^3*d) - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*Ar
cTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*d) - (b*Sinh[c
+ d*x])/(4*a*(a - b)*d*(a + b*Sinh[c + d*x]^2)^2) - ((7*a - 3*b)*b*Sinh[c +
 d*x])/(8*a^2*(a - b)^2*d*(a + b*Sinh[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\Big(\int \frac{1}{(1+x^2)(a+bx^2)^3}\,dx, x, \sinh(c+dx)\Big)}{d}$$

$$= -\frac{b\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\Big(\int \frac{4a-3b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \sinh(c+d}{4a(a-b)d}$$

$$= -\frac{b\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{(7a-3b)b\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} +$$

$$= -\frac{b\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{(7a-3b)b\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} +$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{(a-b)^3d} - \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d} - \frac{}{4a(}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 321 vs. 2(159) = 318.
time = 0.55, size = 321, normalized size = 2.02

$$\frac{(-2a+b)^2\left(\sqrt{b}\left(15a^2-10ab+3b^2\right)\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+16a^{5/2}\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+\left(b^{5/2}\left(15a^2-10ab+3b^2\right)\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+16a^{5/2}b^2\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)\cosh^2(2(c+dx))-2\sqrt{a}\,b\left(18a^3-35a^2b+20ab^2-3b^3\right)\sinh(c+dx)-2b\cosh(2(c+dx))\left(-\left((2a-b)\left(\sqrt{b}\left(15a^2-10ab+3b^2\right)\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+16a^{5/2}\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)+\sqrt{a}\,b\left(7a^2-10ab+3b^2\right)\sinh(c+dx)\right)}{8a^{5/2}(a-b)^3d(2a-b+b\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((-2*a + b)^2*(Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d
*x])/Sqrt[b]] + 16*a^(5/2)*ArcTan[Tanh[(c + d*x)/2]]) + (b^(5/2)*(15*a^2 -
10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 16*a^(5/2)*b^2*Ar
cTan[Tanh[(c + d*x)/2]])*Cosh[2*(c + d*x)]^2 - 2*Sqrt[a]*b*(18*a^3 - 35*a^2
*b + 20*a*b^2 - 3*b^3)*Sinh[c + d*x] - 2*b*Cosh[2*(c + d*x)]*(-((2*a - b)*(
Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] +
 16*a^(5/2)*ArcTan[Tanh[(c + d*x)/2]])) + Sqrt[a]*b*(7*a^2 - 10*a*b + 3*b^2
)*Sinh[c + d*x]))/(8*a^(5/2)*(a - b)^3*d*(2*a - b + b*Cosh[2*(c + d*x)])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(145) = 290.
time = 1.92, size = 414, normalized size = 2.60

method	result

derivativedivides	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)^3}-\frac{2b\left(\frac{-\frac{\left(9a^2-14ab+5b^2\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{\left(27a^3-70a^2b+55a\,b^2-12b^3\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}-\frac{\left(27a^3-70a\right.}{}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\mathrm{ta}\right.\right.}\right.}{}}{}$
default	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)^3}-\frac{2b\left(\frac{-\frac{\left(9a^2-14ab+5b^2\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{\left(27a^3-70a^2b+55a\,b^2-12b^3\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}-\frac{\left(27a^3-70a\right.}{}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4b\left(\mathrm{ta}\right.\right.}\right.}{}}{}$
risch	$-\frac{e^{dx+c}b\left(7ab\,e^{6dx+6c}-3b^2e^{6dx+6c}+36a^2e^{4dx+4c}-41ab\,e^{4dx+4c}+9b^2e^{4dx+4c}-36a^2e^{2dx+2c}+41ab\,e^{2dx+2c}-9b^2e^{2dx+2}\right.}{4(a-b)^2a^2d\left(b\,e^{4dx+4c}+4a\,e^{2dx+2c}-2b\,e^{2dx+2c}+b\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a-b)^3*arctan(tanh(1/2*d*x+1/2*c))-2*b/(a-b)^3*((-1/8*(9*a^2-14*a*b
+5*b^2)/a*tanh(1/2*d*x+1/2*c)^7+1/8*(27*a^3-70*a^2*b+55*a*b^2-12*b^3)/a^2*t
```

```
anh(1/2*d*x+1/2*c)^5-1/8*(27*a^3-70*a^2*b+55*a*b^2-12*b^3)/a^2*tanh(1/2*d*x
+1/2*c)^3+1/8*(9*a^2-14*a*b+5*b^2)/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1
/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/8/a*(15*
a^2-10*a*b+3*b^2)*(1/2*(-a+(-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(
a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/
2)-a+2*b)*a)^(1/2))-1/2*(a+(-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(
a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((7*a*b^2*e^(7*c) - 3*b^3*e^(7*c))*e^(7*d*x) + (36*a^2*b*e^(5*c) - 41*
a*b^2*e^(5*c) + 9*b^3*e^(5*c))*e^(5*d*x) - (36*a^2*b*e^(3*c) - 41*a*b^2*e^(
3*c) + 9*b^3*e^(3*c))*e^(3*d*x) - (7*a*b^2*e^c - 3*b^3*e^c)*e^(d*x))/(a^4*b
^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) - 2*a^3*b^3*d*e^(8*c) +
 a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(2*a^5*b*d*e^(6*c) - 5*a^4*b^2*d*e^(6*c)
+ 4*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(8*a^6*d*e^(4*c) -
 24*a^5*b*d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) - 14*a^3*b^3*d*e^(4*c) + 3*a^2*b
^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^5*b*d*e^(2*c) - 5*a^4*b^2*d*e^(2*c) + 4*a^
3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(a^
3*d - 3*a^2*b*d + 3*a*b^2*d - b^3*d) - 2*integrate(1/8*((15*a^2*b*e^(3*c) -
 10*a*b^2*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x) + (15*a^2*b*e^c - 10*a*b^2*e^c
 + 3*b^3*e^c)*e^(d*x))/(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4 + (a^5*b*e^
(4*c) - 3*a^4*b^2*e^(4*c) + 3*a^3*b^3*e^(4*c) - a^2*b^4*e^(4*c))*e^(4*d*x)
+ 2*(2*a^6*e^(2*c) - 7*a^5*b*e^(2*c) + 9*a^4*b^2*e^(2*c) - 5*a^3*b^3*e^(2*c
) + a^2*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4384 vs. $2(145) = 290$.
time = 0.49, size = 8083, normalized size = 50.84

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 28*(7*a^2*b^2 -
10*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(7*a^2*b^2 - 10*a*b^3 +
 3*b^4)*sinh(d*x + c)^7 + 4*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh
(d*x + c)^5 + 4*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4 + 21*(7*a^2*b^2 -
```

```
 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(7*a^2*b^2 - 10
*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4
)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*
b^4)*cosh(d*x + c)^3 + 4*(35*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^4
 - 36*a^3*b + 77*a^2*b^2 - 50*a*b^3 + 9*b^4 + 10*(36*a^3*b - 77*a^2*b^2 + 5
0*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(7*a^2*b^2 - 10*a
*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^
4)*cosh(d*x + c)^3 - 3*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^2 + ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^8 +
8*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^2*b
^2 - 10*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^
3 - 3*b^4)*cosh(d*x + c)^6 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 +
7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(
15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(30*a^3*b - 35*a^2*b^2 +
 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(120*a^4 - 200*a^3*b
+ 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 10*
a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b
^3 + 9*b^4 + 30*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 15*a^2*b^2 - 10*a*b^3 + 3*b^4 + 8*(7*(15*a^2*b^2 - 10*a*
b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4
)*cosh(d*x + c)^3 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^
4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 +
 15*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 30*a^3*b -
 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 3*(120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*
a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^2*b^2 - 10*a*b^3
 + 3*b^4)*cosh(d*x + c)^7 + 3*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*co
sh(d*x + c)^5 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh
(d*x + c)^3 + (30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x
+ c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d
*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x
+ c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*si
nh(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*
x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*
x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh
(d*x + c))*sinh(d*x + c) + b)) - 32*(a^2*b^2*cosh(d*x + c)^8 + 8*a^2*b^2*co
sh(d*x + c)*sinh(d*x + c)^7 + a^2*b^2*sinh(d*x + c)^8 + 4*(2*a^3*b - a^2*b^
2)*cosh(d*x + c)^6 + 4*(7*a^2*b^2*cosh(d*x + c)^2 + 2*a^3*b - a^2*b^2)*sinh
(d*x + c)^6 + 8*(7*a^2*b^2*cosh(d*x + c)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4 +
2*(35*a^2*b^2*cosh(d*x + c)^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b -
 a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(d*
x + c)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^3 + (8*a^4 - 8*a^3*b + 3*a^
```

```
2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(d*x + c)
^2 + 4*(7*a^2*b^2*cosh(d*x + c)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^4
+ 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 8*(a^2*b^2*cosh(d*x + c)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(d*x +
 c)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 + (2*a^3*b - a^2*b^2)
*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 4*(7
*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c) + 4*(7*(7*a^2*b^2 - 10*a*b^3 + 3
*b^4)*cosh(d*x + c)^6 + 5*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d
*x + c)^4 - 7*a^2*b^2 + 10*a*b^3 - 3*b^4 - 3*(36*a^3*b - 77*a^2*b^2 + 50*a*
b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*
b^4 - a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2
*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 -
a^2*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4
 + a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2
*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^
2*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 +
 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*
a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^6...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)\,{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^3),x)
[Out] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^3), x)
```

3.346 $\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=172

$$-\frac{3b(8a^2-4ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{7/2}d}+\frac{\tanh(c+dx)}{(a-b)^3d}-\frac{b^3\tanh(c+dx)}{4a(a-b)^3d\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

```
[Out] -3/8*b*(8*a^2-4*a*b+b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(
a-b)^(7/2)/d+tanh(d*x+c)/(a-b)^3/d-1/4*b^3*tanh(d*x+c)/a/(a-b)^3/d/(a-(a-b)
*tanh(d*x+c)^2)^2+3/8*(4*a-b)*b^2*tanh(d*x+c)/a^2/(a-b)^3/d/(a-(a-b)*tanh(d
*x+c)^2)
```

Rubi [A]

time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3270, 398, 1171, 393, 214}

$$\frac{3b^2(4a-b)\tanh(c+dx)}{8a^2d(a-b)^3\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{3b(8a^2-4ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{7/2}}-\frac{b^3\tanh(c+dx)}{4ad(a-b)^3\left(a-(a-b)\tanh^2(c+dx)\right)^2}+\frac{\tanh(c+dx)}{d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(8*a^2 - 4*a*b + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(
8*a^(5/2)*(a - b)^(7/2)*d) + Tanh[c + d*x]/((a - b)^3*d) - (b^3*Tanh[c + d*
x])/(4*a*(a - b)^3*d*(a - (a - b)*Tanh[c + d*x]^2)^2) + (3*(4*a - b)*b^2*Ta
nh[c + d*x])/(8*a^2*(a - b)^3*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\Big(\int \frac{\left(1-x^2\right)^3}{\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\Big)}{d}$$

$$= \frac{\text{Subst}\Big(\int \Big(\frac{1}{(a-b)^3} - \frac{b\left(3a^2-3ab+b^2\right)-3(a-b)(2a-b)bx^2+3(a-b)^2bx^4}{(a-b)^3\left(a+(-a+b)x^2\right)^3}\Big)\,dx, x, \tanh(c+dx}{d}$$

$$= \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{\text{Subst}\Big(\int \frac{b\left(3a^2-3ab+b^2\right)-3(a-b)(2a-b)bx^2+3(a-b)^2bx^4}{\left(a+(-a+b)x^2\right)^3}\,dx, x, \tanh(c}{(a-b)^3d}$$

$$= \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{b^3\tanh(c+dx)}{4a(a-b)^3d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\Big(\int \frac{-3(2a-}{(a}}{}$$

$$= \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{b^3\tanh(c+dx)}{4a(a-b)^3d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{3(4a-}{8a^2(a-b)^3d\,(a}$$

$$= -\frac{3b\left(8a^2-4ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{7/2}d} + \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{}{4a(a-}$$

Mathematica [A]

time = 1.01, size = 165, normalized size = 0.96

$$\frac{-\frac{3b(8a^2-4ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^{7/2}}+\frac{4b^2\sinh(2(c+dx))}{a(a-b)^2(2a-b+b\cosh(2(c+dx)))^2}+\frac{(10a-3b)b^2\sinh(2(c+dx))}{a^2(a-b)^3(2a-b+b\cosh(2(c+dx)))}+\frac{8\tanh(c+dx)}{(a-b)^3}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((-3*b*(8*a^2 - 4*a*b + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/
(a^(5/2)*(a - b)^(7/2)) + (4*b^2*Sinh[2*(c + d*x)])/(a*(a - b)^2*(2*a - b +
 b*Cosh[2*(c + d*x)])^2) + ((10*a - 3*b)*b^2*Sinh[2*(c + d*x)])/(a^2*(a - b
)^3*(2*a - b + b*Cosh[2*(c + d*x)])) + (8*Tanh[c + d*x])/(a - b)^3)/(8*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(158) = 316.
time = 1.69, size = 394, normalized size = 2.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/(a-b)^3*((1/8*b*(12*a-5*b)/a*tanh(1/2*d*x+1/2*c)^7-3/8*(4*a^2-15*a
*b+4*b^2)/a^2*b*tanh(1/2*d*x+1/2*c)^5-3/8*(4*a^2-15*a*b+4*b^2)/a^2*b*tanh(1
/2*d*x+1/2*c)^3+1/8*b*(12*a-5*b)/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2
*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+3/8/a*(8*a^2
-4*a*b+b^2)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/
2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)
*a)^(1/2))))+2/(a-b)^3*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4593 vs. 2(160) = 320.
time = 0.49, size = 9442, normalized size = 54.90

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(12*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^8 + 9
6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)*sinh(d*x + c)^
7 + 12*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*sinh(d*x + c)^8 + 24*(2
4*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^6 + 24
*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5 + 14*(8*a^4*b^2 -
12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 32*a^4*b
^2 + 8*a^3*b^3 - 52*a^2*b^4 + 12*a*b^5 + 48*(14*(8*a^4*b^2 - 12*a^3*b^3 + 5
*a^2*b^4 - a*b^5)*cosh(d*x + c)^3 + 3*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 -
 8*a^2*b^4 + a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(64*a^6 - 88*a^5*b +
 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^4 + 8*(64*a^6 - 88*a^5*b +
 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4 + 105*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4
 - a*b^5)*cosh(d*x + c)^4 + 45*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*
b^4 + a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(21*(8*a^4*b^2 - 12*a^3*
b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^5 + 15*(24*a^5*b - 44*a^4*b^2 + 27*a
^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^3 + (64*a^6 - 88*a^5*b + 28*a^4*b
^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(32*a^5*b - 16
*a^4*b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5)*cosh(d*x + c)^2 + 8*(42*(8*a^
4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^6 + 32*a^5*b - 16*a^4
*b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5 + 45*(24*a^5*b - 44*a^4*b^2 + 27*a
^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^4 + 6*(64*a^6 - 88*a^5*b + 28*a^4
*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((8*a^2*b^
3 - 4*a*b^4 + b^5)*cosh(d*x + c)^10 + 10*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d
*x + c)*sinh(d*x + c)^9 + (8*a^2*b^3 - 4*a*b^4 + b^5)*sinh(d*x + c)^10 + (6
4*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^8 + (64*a^3*b^2 -
56*a^2*b^3 + 20*a*b^4 - 3*b^5 + 45*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c
)^2)*sinh(d*x + c)^8 + 8*(15*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^3 +
(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^7
 + 2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^6 +
 2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5 + 105*(8*a^2*b^3 - 4
*a*b^4 + b^5)*cosh(d*x + c)^4 + 14*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*
b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(8*a^2*b^3 - 4*a*b^4 + b^5)*c
osh(d*x + c)^5 + 14*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x +
 c)^3 + 3*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c
))*sinh(d*x + c)^5 + 8*a^2*b^3 - 4*a*b^4 + b^5 + 2*(64*a^4*b - 64*a^3*b^2 +
 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^4 + 2*(105*(8*a^2*b^3 - 4*a*b^4
+ b^5)*cosh(d*x + c)^6 + 64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5
 + 35*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^4 + 15*(64
*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + 8*(15*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^7 + 7*(64*a^3*b^2
 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^5 + 5*(64*a^4*b - 64*a^3*b^
2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^3 + (64*a^4*b - 64*a^3*b^2 +
```

```
32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + (64*a^3*b^2 -
56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^2 + (45*(8*a^2*b^3 - 4*a*b^4 +
 b^5)*cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cos
h(d*x + c)^6 + 64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5 + 30*(64*a^4*b -
64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^4 + 12*(64*a^4*b - 6
4*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
2*(5*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^9 + 4*(64*a^3*b^2 - 56*a^2*b
^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^7 + 6*(64*a^4*b - 64*a^3*b^2 + 32*a^2*
b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^5 + 4*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^
3 - 8*a*b^4 + b^5)*cosh(d*x + c)^3 + (64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 -
3*b^5)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)
^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b -
 b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x +
c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*
x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*
b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x
+ c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x
+ c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 16*(6*(8*a^4*b^2 -
12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^7 + 9*(24*a^5*b - 44*a^4*b^2
+ 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^5 + 2*(64*a^6 - 88*a^5*b +
28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^3 + (32*a^5*b - 16*a^4*b^2
- 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c))/((a^7*b^
2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^10 + 10*(a
^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)*sinh(
d*x + c)^9 + (a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 -...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(160) = 320.
time = 1.33, size = 367, normalized size = 2.13

$$-\frac{\frac{3\,(8\,a^2b-4\,ab^2+b^3)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sqrt{-a^2+ab}}+\frac{2\,(16\,a^2b^2e^{(6\,dx+6\,c)}-12\,ab^3e^{(6\,dx+6\,c)}+3\,b^4e^{(6\,dx+6\,c)}+80\,a^3be^{(4\,dx+4\,c)}-104\,a^2b^2e^{(4\,dx+4\,c)}+54\,ab^3e^{(4\,dx+4\,c)}-9\,b^4e^{(4\,dx+4\,c)}+64\,a^2b^2e^{(2\,dx+2\,c)}-52\,ab^3e^{(2\,dx+2\,c)}+9\,b^4e^{(2\,dx+2\,c)}+10\,ab^3-3\,b^4)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)^2}+\frac{16}{(a^3-3\,a^2b+3\,ab^2-b^3)\left(e^{(2\,dx+2\,c)}+1\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(8*a^2*b - 4*a*b^2 + b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/
sqrt(-a^2 + a*b))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(-a^2 + a*b))
+ 2*(16*a^2*b^2*e^(6*d*x + 6*c) - 12*a*b^3*e^(6*d*x + 6*c) + 3*b^4*e^(6*d*x
 + 6*c) + 80*a^3*b*e^(4*d*x + 4*c) - 104*a^2*b^2*e^(4*d*x + 4*c) + 54*a*b^3
*e^(4*d*x + 4*c) - 9*b^4*e^(4*d*x + 4*c) + 64*a^2*b^2*e^(2*d*x + 2*c) - 52*
a*b^3*e^(2*d*x + 2*c) + 9*b^4*e^(2*d*x + 2*c) + 10*a*b^3 - 3*b^4)/((a^5 - 3
*a^4*b + 3*a^3*b^2 - a^2*b^3)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*
b*e^(2*d*x + 2*c) + b)^2) + 16/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(e^(2*d*x +
 2*c) + 1)))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^2\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3), x)
```

3.347 $\int \frac{\mathbf{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=217

$$\frac{(a-7b)\mathrm{ArcTan}(\sinh(c+dx))}{2(a-b)^4d}+\frac{b^{3/2}(35a^2-14ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^4d}+\frac{b(2a+b)\sinh(c+}{4a(a-b)^2d\left(a+b\sinh^2(c\right.}$$

```
[Out] 1/2*(a-7*b)*arctan(sinh(d*x+c))/(a-b)^4/d+1/8*b^(3/2)*(35*a^2-14*a*b+3*b^2)
*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/(a-b)^4/d+1/4*b*(2*a+b)*sinh(d
*x+c)/a/(a-b)^2/d/(a+b*sinh(d*x+c)^2)^2+1/8*(4*a-b)*b*(a+3*b)*sinh(d*x+c)/a
^2/(a-b)^3/d/(a+b*sinh(d*x+c)^2)+1/2*sech(d*x+c)*tanh(d*x+c)/(a-b)/d/(a+b*s
inh(d*x+c)^2)^2
```

Rubi [A]

time = 0.21, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = $\{3269, 425, 541, 536, 209, 211\}$

$$\frac{b(4a-b)(a+3b)\sinh(c+dx)}{8a^2d(a-b)^3\left(a+b\sinh^2(c+dx)\right)}+\frac{b^{3/2}(35a^2-14ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^4}+\frac{(a-7b)\mathrm{ArcTan}(\sinh(c+dx))}{2d(a-b)^4}+\frac{b(2a+b)\sinh(c+dx)}{4ad(a-b)^2\left(a+b\sinh^2(c+dx)\right)^2}+\frac{\tanh(c+dx)\mathrm{sech}(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - 7*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^4*d) + (b^(3/2)*(35*a^2 - 14*a
*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^4*d
) + (b*(2*a + b)*Sinh[c + d*x])/(4*a*(a - b)^2*d*(a + b*Sinh[c + d*x]^2)^2)
 + ((4*a - b)*b*(a + 3*b)*Sinh[c + d*x])/(8*a^2*(a - b)^3*d*(a + b*Sinh[c +
 d*x]^2)) + (Sech[c + d*x]*Tanh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]
^2)^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-5bx^2}{(1+x^2)(a+bx^2)^3}\,dx, x, \sinh(c+dx)\right)}{2(a-b)d}$$

$$= \frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \text{Sub}$$

$$= \frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(4a-b)b(a+3b)\sinh(c+dx)}{8a^2(a-b)^3d\left(a+b\sinh^2(c+dx)\right)} + \frac{}{2(}$$

$$= \frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(4a-b)b(a+3b)\sinh(c+dx)}{8a^2(a-b)^3d\left(a+b\sinh^2(c+dx)\right)} + \frac{}{2(}$$

$$= \frac{(a-7b)\tan^{-1}(\sinh(c+dx))}{2(a-b)^4d} + \frac{b^{3/2}(35a^2-14ab+3b^2)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right.}{8a^{5/2}(a-b)^4d}$$

Mathematica [A]
time = 1.30, size = 222, normalized size = 1.02

$$\frac{-\frac{35b^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{14b^{5/2}\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{a^{3/2}} - \frac{3b^{7/2}\operatorname{ArcTan}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{a^{5/2}} + 8a\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - 56b\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2(a-b)b^2(26a^2-21ab+3b^2+(11a-3b)b\cosh(2(c+dx)))\sinh(c+dx)}{a^2(2a-b+b\cosh(2(c+dx)))^2} + 4(a-b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8(a-b)^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((-35*b^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + (14*b^(5/2
)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/a^(3/2) - (3*b^(7/2)*ArcTan[(Sqr
t[a]*Csch[c + d*x])/Sqrt[b]])/a^(5/2) + 8*a*ArcTan[Tanh[(c + d*x)/2]] - 56*
b*ArcTan[Tanh[(c + d*x)/2]] + (2*(a - b)*b^2*(26*a^2 - 21*a*b + 3*b^2 + (11
*a - 3*b)*b*Cosh[2*(c + d*x)])*Sinh[c + d*x])/(a^2*(2*a - b + b*Cosh[2*(c +
 d*x)])^2) + 4*(a - b)*Sech[c + d*x]*Tanh[c + d*x])/(8*(a - b)^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(199) = 398$.
time = 2.29, size = 477, normalized size = 2.20 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a-b)^4*(((1/2*b-1/2*a)*tanh(1/2*d*x+1/2*c)^3+(-1/2*b+1/2*a)*tanh(1/
2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^2+1/2*(a-7*b)*arctan(tanh(1/2*d*x+1
/2*c)))+2*b^2/(a-b)^4*((-1/8*(13*a^2-18*a*b+5*b^2)/a*tanh(1/2*d*x+1/2*c)^7+
1/8*(39*a^3-98*a^2*b+71*a*b^2-12*b^3)/a^2*tanh(1/2*d*x+1/2*c)^5-1/8*(39*a^3
-98*a^2*b+71*a*b^2-12*b^3)/a^2*tanh(1/2*d*x+1/2*c)^3+1/8*(13*a^2-18*a*b+5*b
^2)/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)
^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/8/a*(35*a^2-14*a*b+3*b^2)*(1/2*(-a+(-b*
(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arc
tan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2*(a+(-b*
(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] (a*e^c - 7*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^4*d - 4*a^3*b*d + 6*a^2*b^2
*d - 4*a*b^3*d + b^4*d) + 1/4*((4*a^2*b^2*e^(11*c) + 11*a*b^3*e^(11*c) - 3*
b^4*e^(11*c))*e^(11*d*x) + (32*a^3*b*e^(9*c) + 32*a^2*b^2*e^(9*c) - 31*a*b^
3*e^(9*c) + 3*b^4*e^(9*c))*e^(9*d*x) + 2*(32*a^4*e^(7*c) - 48*a^3*b*e^(7*c)
 + 46*a^2*b^2*e^(7*c) - 21*a*b^3*e^(7*c) + 3*b^4*e^(7*c))*e^(7*d*x) - 2*(32
*a^4*e^(5*c) - 48*a^3*b*e^(5*c) + 46*a^2*b^2*e^(5*c) - 21*a*b^3*e^(5*c) + 3
*b^4*e^(5*c))*e^(5*d*x) - (32*a^3*b*e^(3*c) + 32*a^2*b^2*e^(3*c) - 31*a*b^3
*e^(3*c) + 3*b^4*e^(3*c))*e^(3*d*x) - (4*a^2*b^2*e^c + 11*a*b^3*e^c - 3*b^4
*e^c)*e^(d*x))/(a^5*b^2*d - 3*a^4*b^3*d + 3*a^3*b^4*d - a^2*b^5*d + (a^5*b^
2*d*e^(12*c) - 3*a^4*b^3*d*e^(12*c) + 3*a^3*b^4*d*e^(12*c) - a^2*b^5*d*e^(1
2*c))*e^(12*d*x) + 2*(4*a^6*b*d*e^(10*c) - 13*a^5*b^2*d*e^(10*c) + 15*a^4*b
^3*d*e^(10*c) - 7*a^3*b^4*d*e^(10*c) + a^2*b^5*d*e^(10*c))*e^(10*d*x) + (16
*a^7*d*e^(8*c) - 48*a^6*b*d*e^(8*c) + 47*a^5*b^2*d*e^(8*c) - 13*a^4*b^3*d*e
^(8*c) - 3*a^3*b^4*d*e^(8*c) + a^2*b^5*d*e^(8*c))*e^(8*d*x) + 4*(8*a^7*d*e^
(6*c) - 28*a^6*b*d*e^(6*c) + 37*a^5*b^2*d*e^(6*c) - 23*a^4*b^3*d*e^(6*c) +
7*a^3*b^4*d*e^(6*c) - a^2*b^5*d*e^(6*c))*e^(6*d*x) + (16*a^7*d*e^(4*c) - 48
*a^6*b*d*e^(4*c) + 47*a^5*b^2*d*e^(4*c) - 13*a^4*b^3*d*e^(4*c) - 3*a^3*b^4*
d*e^(4*c) + a^2*b^5*d*e^(4*c))*e^(4*d*x) + 2*(4*a^6*b*d*e^(2*c) - 13*a^5*b^
2*d*e^(2*c) + 15*a^4*b^3*d*e^(2*c) - 7*a^3*b^4*d*e^(2*c) + a^2*b^5*d*e^(2*c
))*e^(2*d*x)) + 8*integrate(1/32*((35*a^2*b^2*e^(3*c) - 14*a*b^3*e^(3*c) +
3*b^4*e^(3*c))*e^(3*d*x) + (35*a^2*b^2*e^c - 14*a*b^3*e^c + 3*b^4*e^c)*e^(d
*x))/(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5 + (a^6*b*e^(4*c)
- 4*a^5*b^2*e^(4*c) + 6*a^4*b^3*e^(4*c) - 4*a^3*b^4*e^(4*c) + a^2*b^5*e^(4*
c))*e^(4*d*x) + 2*(2*a^7*e^(2*c) - 9*a^6*b*e^(2*c) + 16*a^5*b^2*e^(2*c) - 1
4*a^4*b^3*e^(2*c) + 6*a^3*b^4*e^(2*c) - a^2*b^5*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10237 vs. $2(199) = 398$.
time = 0.64, size = 18765, normalized size = 86.47

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(4*a^3*b^2 + 7*a^2*b^3 - 14*a*b^4 + 3*b^5)*cosh(d*x + c)^11 + 44*(
4*a^3*b^2 + 7*a^2*b^3 - 14*a*b^4 + 3*b^5)*cosh(d*x + c)*sinh(d*x + c)^10 +
4*(4*a^3*b^2 + 7*a^2*b^3 - 14*a*b^4 + 3*b^5)*sinh(d*x + c)^11 + 4*(32*a^4*b
 - 63*a^2*b^3 + 34*a*b^4 - 3*b^5)*cosh(d*x + c)^9 + 4*(32*a^4*b - 63*a^2*b^
3 + 34*a*b^4 - 3*b^5 + 55*(4*a^3*b^2 + 7*a^2*b^3 - 14*a*b^4 + 3*b^5)*cosh(d
*x + c)^2)*sinh(d*x + c)^9 + 12*(55*(4*a^3*b^2 + 7*a^2*b^3 - 14*a*b^4 + 3*b
^5)*cosh(d*x + c)^3 + 3*(32*a^4*b - 63*a^2*b^3 + 34*a*b^4 - 3*b^5)*cosh(d*x
 + c))*sinh(d*x + c)^8 + 8*(32*a^5 - 80*a^4*b + 94*a^3*b^2 - 67*a^2*b^3 + 2
4*a*b^4 - 3*b^5)*cosh(d*x + c)^7 + 8*(32*a^5 - 80*a^4*b + 94*a^3*b^2 - 67*a
^2*b^3 + 24*a*b^4 - 3*b^5 + 165*(4*a^3*b^2 + 7*a^2*b^3 - 14*a*b^4 + 3*b^5)*
cosh(d*x + c)^4 + 18*(32*a^4*b - 63*a^2*b^3 + 34*a*b^4 - 3*b^5)*cosh(d*x +
c)^2)*sinh(d*x + c)^7 + 56*(33*(4*a^3*b^2 + 7*a^2*b^3 - 14*a*b^4 + 3*b^5)*c
osh(d*x + c)^5 + 6*(32*a^4*b - 63*a^2*b^3 + 34*a*b^4 - 3*b^5)*cosh(d*x + c)
^3 + (32*a^5 - 80*a^4*b + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh\left(c+d\,x\right)^3\,{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3), x)
```

3.348 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=203

$$\frac{b^2(48a^2-16ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{9/2}d}+\frac{(a-4b)\tanh(c+dx)}{(a-b)^4d}-\frac{\tanh^3(c+dx)}{3(a-b)^3d}+\frac{b^4}{4a(a-b)^4d\,(a}$$

```
[Out] 1/8*b^2*(48*a^2-16*a*b+3*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5
/2)/(a-b)^(9/2)/d+(a-4*b)*tanh(d*x+c)/(a-b)^4/d-1/3*tanh(d*x+c)^3/(a-b)^3/d
+1/4*b^4*tanh(d*x+c)/a/(a-b)^4/d/(a-(a-b)*tanh(d*x+c)^2)^2-1/8*(16*a-3*b)*b
^3*tanh(d*x+c)/a^2/(a-b)^4/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.25, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3270, 398, 1171, 393, 214}

$$-\frac{b^3(16a-3b)\tanh(c+dx)}{8a^2d(a-b)^4\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{b^2(48a^2-16ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{9/2}}+\frac{b^4\tanh(c+dx)}{4ad(a-b)^4\left(a-(a-b)\tanh^2(c+dx)\right)^2}-\frac{\tanh^3(c+dx)}{3d(a-b)^3}+\frac{(a-4b)\tanh(c+dx)}{d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (b^2*(48*a^2 - 16*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]
)/(8*a^(5/2)*(a - b)^(9/2)*d) + ((a - 4*b)*Tanh[c + d*x])/((a - b)^4*d) - T
anh[c + d*x]^3/(3*(a - b)^3*d) + (b^4*Tanh[c + d*x])/(4*a*(a - b)^4*d*(a -
(a - b)*Tanh[c + d*x]^2)^2) - ((16*a - 3*b)*b^3*Tanh[c + d*x])/(8*a^2*(a -
b)^4*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a-4b}{(a-b)^4} - \frac{x^2}{(a-b)^3} + \frac{b^2\left(6a^2-4ab+b^2\right)-4(a-b)(3a-b)b^2x^2+6(a-b)^2b^2x^4}{(a-b)^4\left(a+(-a+b)x^2\right)^3}\right)\,dx, x, \mathrm{t}}{d}$$

$$= \frac{(a-4b)\tanh(c+dx)}{(a-b)^4d} - \frac{\tanh^3(c+dx)}{3(a-b)^3d} + \frac{\operatorname{Subst}\left(\int \frac{b^2\left(6a^2-4ab+b^2\right)-4(a-b)(3a-b}{(a+(-a+b)x^2}\right.}{(a}$$

$$= \frac{(a-4b)\tanh(c+dx)}{(a-b)^4d} - \frac{\tanh^3(c+dx)}{3(a-b)^3d} + \frac{b^4\tanh(c+dx)}{4a(a-b)^4d\left(a-(a-b)\tanh^2(c\right.}$$

$$= \frac{(a-4b)\tanh(c+dx)}{(a-b)^4d} - \frac{\tanh^3(c+dx)}{3(a-b)^3d} + \frac{b^4\tanh(c+dx)}{4a(a-b)^4d\left(a-(a-b)\tanh^2(c\right.}$$

$$= \frac{b^2\left(48a^2-16ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{9/2}d} + \frac{(a-4b)\tanh(c+dx)}{(a-b)^4d}$$

Mathematica [A]
time = 2.02, size = 169, normalized size = 0.83

$$\frac{\frac{3b^2(48a^2-16ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^{9/2}}+\frac{\frac{3b^3\left(-32a^2+24ab-3b^2+b(-14a+3b)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{a^2(2a-b+b\cosh(2(c+dx)))^2}+8\left(2a-11b+(a-b)\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{(a-b)^4}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*b^2*(48*a^2 - 16*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[
a]])/(a^(5/2)*(a - b)^(9/2)) + ((3*b^3*(-32*a^2 + 24*a*b - 3*b^2 + b*(-14*a
 + 3*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a^2*(2*a - b + b*Cosh[2*(c +
 d*x)])^2) + 8*(2*a - 11*b + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b
)^4)/(24*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(187) = 374$.
time = 2.08, size = 445, normalized size = 2.19 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^2/(a-b)^4*((1/8*b*(16*a-5*b)/a*tanh(1/2*d*x+1/2*c)^7-1/8*(16*a^2-
61*a*b+12*b^2)/a^2*b*tanh(1/2*d*x+1/2*c)^5-1/8*(16*a^2-61*a*b+12*b^2)/a^2*b
*tanh(1/2*d*x+1/2*c)^3+1/8*b*(16*a-5*b)/a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*
d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/8/a
*(48*a^2-16*a*b+3*b^2)*(-1/2*((-b*(a-b))^(1/2)-b)/a/(-b*(a-b))^(1/2)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2))+1/2*((-b*(a-b))^(1/2)+b)/a/(-b*(a-b))^(1/2)/((2*(-b
*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2))))-2/(a-b)^4*((4*b-a)*tanh(1/2*d*x+1/2*c)^5+(20/3*b-2/
3*a)*tanh(1/2*d*x+1/2*c)^3+(4*b-a)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c
)^2+1)^3)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9388 vs. $2(189) = 378$.
time = 0.56, size = 19032, normalized size = 93.75

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(12*(48*a^4*b^3 - 64*a^3*b^4 + 19*a^2*b^5 - 3*a*b^6)*cosh(d*x + c)^12
 + 144*(48*a^4*b^3 - 64*a^3*b^4 + 19*a^2*b^5 - 3*a*b^6)*cosh(d*x + c)*sinh(
d*x + c)^11 + 12*(48*a^4*b^3 - 64*a^3*b^4 + 19*a^2*b^5 - 3*a*b^6)*sinh(d*x
+ c)^12 + 72*(48*a^5*b^2 - 64*a^4*b^3 + 19*a^3*b^4 - 3*a^2*b^5)*cosh(d*x +
c)^10 + 72*(48*a^5*b^2 - 64*a^4*b^3 + 19*a^3*b^4 - 3*a^2*b^5 + 11*(48*a^4*b
^3 - 64*a^3*b^4 + 19*a^2*b^5 - 3*a*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^10 +
 240*(11*(48*a^4*b^3 - 64*a^3*b^4 + 19*a^2*b^5 - 3*a*b^6)*cosh(d*x + c)^3 +
 3*(48*a^5*b^2 - 64*a^4*b^3 + 19*a^3*b^4 - 3*a^2*b^5)*cosh(d*x + c))*sinh(d
*x + c)^9 + 4*(768*a^6*b + 800*a^5*b^2 - 2560*a^4*b^3 + 1250*a^3*b^4 - 285*
a^2*b^5 + 27*a*b^6)*cosh(d*x + c)^8 + 4*(768*a^6*b + 800*a^5*b^2 - 2560*a^4
*b^3 + 1250*a^3*b^4 - 285*a^2*b^5 + 27*a*b^6 + 1485*(48*a^4*b^3 - 64*a^3*b^
4 + 19*a^2*b^5 - 3*a*b^6)*cosh(d*x + c)^4 + 810*(48*a^5*b^2 - 64*a^4*b^3 +
19*a^3*b^4 - 3*a^2*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 32*(297*(48*a^4*
b^3 - 64*a^3*b^4 + 19*a^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(189) = 378$.
time = 0.94, size = 436, normalized size = 2.15

$$\frac{\frac{3\,(48\,a^2b^2-16\,ab^3+3\,b^4)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{(a^6-4\,a^5b+6\,a^4b^2-4\,a^3b^3+a^2b^4)\sqrt{-a^2+ab}}+\frac{6\,(24\,a^2b^3e^{(6\,dx+6\,c)}-16\,ab^4e^{(6\,dx+6\,c)}+3\,b^5e^{(6\,dx+6\,c)}+112\,a^3b^2e^{(4\,dx+4\,c)}-136\,a^2b^3e^{(4\,dx+4\,c)}+66\,ab^4e^{(4\,dx+4\,c)}-9\,b^5e^{(4\,dx+4\,c)}+88\,a^2b^3e^{(2\,dx+2\,c)}-64\,ab^4e^{(2\,dx+2\,c)}+9\,b^5e^{(2\,dx+2\,c)}+14\,ab^4-3\,b^5)}{(a^6-4\,a^5b+6\,a^4b^2-4\,a^3b^3+a^2b^4)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)^2}+\frac{16\,(9\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+24\,be^{(2\,dx+2\,c)}-2\,a+11\,b)}{(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)\left(e^{(2\,dx+2\,c)}+1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*(48*a^2*b^2 - 16*a*b^3 + 3*b^4)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a
 - b)/sqrt(-a^2 + a*b))/((a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*
```

```
sqrt(-a^2 + a*b)) + 6*(24*a^2*b^3*e^(6*d*x + 6*c) - 16*a*b^4*e^(6*d*x + 6*c
) + 3*b^5*e^(6*d*x + 6*c) + 112*a^3*b^2*e^(4*d*x + 4*c) - 136*a^2*b^3*e^(4*
d*x + 4*c) + 66*a*b^4*e^(4*d*x + 4*c) - 9*b^5*e^(4*d*x + 4*c) + 88*a^2*b^3*
e^(2*d*x + 2*c) - 64*a*b^4*e^(2*d*x + 2*c) + 9*b^5*e^(2*d*x + 2*c) + 14*a*b
^4 - 3*b^5)/((a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*(b*e^(4*d*x
+ 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2) + 16*(9*b*e^(4*d
*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) - 2*a + 11*b)/((a^4
- 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(e^(2*d*x + 2*c) + 1)^3))/d
```

Mupad **[F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh\left(c+d\,x\right)^4\,{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3), x)
```

3.349 $\int \frac{\cosh^2(x)}{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=19

$$-x+\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)$$

```
[Out] -x+arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3270, 400, 212}

$$\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 - Sinh[x]^2),x]
```

```
[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{1-\sinh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1-2x^2)\,(1-x^2)}\,dx, x, \tanh(x)\right)\\ &= 2\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)\\ &= -x+\sqrt{2}\,\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 24, normalized size = 1.26

$$-2\left(\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 - Sinh[x]^2),x]
```

```
[Out] -2*(x/2 - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.
time = 0.54, size = 54, normalized size = 2.84

method	result
risch	$-x+\frac{\sqrt{2}\,\ln\left(e^{2x}-3+2\sqrt{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(e^{2x}-3-2\sqrt{2}\right)}{2}$
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)-\ln\big(\mathrm{ta}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1-sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(tanh(1/2*x)-1)+2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+2^(1/2)*ar
ctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))-ln(tanh(1/2*x)+1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(15) = 30$.
time = 0.46, size = 64, normalized size = 3.37

$$\frac{1}{2}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{2}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(15) = 30$.
time = 0.44, size = 70, normalized size = 3.68

$$\frac{1}{2}\,\sqrt{2}\,\log\left(-\frac{3\left(2\,\sqrt{2}-3\right)\cosh\left(x\right)^{2}-4\left(3\,\sqrt{2}-4\right)\cosh\left(x\right)\sinh\left(x\right)+3\left(2\,\sqrt{2}-3\right)\sinh\left(x\right)^{2}-2\,\sqrt{2}+3}{\cosh\left(x\right)^{2}+\sinh\left(x\right)^{2}-3}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^
2 - 3)) - x
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(15) = 30$.
time = 3.46, size = 238, normalized size = 12.53

$$-\frac{1331714x}{941664\sqrt{2}+1331714}-\frac{941664\sqrt{2}\,x}{941664\sqrt{2}+1331714}+\frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{941664\sqrt{2}+1331714}+\frac{665857\sqrt{2}\,\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{941664\sqrt{2}+1331714}+\frac{941664\log\left(\tanh\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{941664\sqrt{2}+1331714}+\frac{665857\sqrt{2}\,\log\left(\tanh\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{941664\sqrt{2}+1331714}-\frac{665857\sqrt{2}\,\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{941664\sqrt{2}+1331714}-\frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{941664\sqrt{2}+1331714}-\frac{665857\sqrt{2}\,\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{941664\sqrt{2}+1331714}-\frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{941664\sqrt{2}+1331714}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1-sinh(x)**2),x)
```

```
[Out] -1331714*x/(941664*sqrt(2) + 1331714) - 941664*sqrt(2)*x/(941664*sqrt(2) +
1331714) + 941664*log(tanh(x/2) - 1 + sqrt(2))/(941664*sqrt(2) + 1331714) +
 665857*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))/(941664*sqrt(2) + 1331714) + 9
41664*log(tanh(x/2) + 1 + sqrt(2))/(941664*sqrt(2) + 1331714) + 665857*sqrt
(2)*log(tanh(x/2) + 1 + sqrt(2))/(941664*sqrt(2) + 1331714) - 665857*sqrt(2
)*log(tanh(x/2) - sqrt(2) - 1)/(941664*sqrt(2) + 1331714) - 941664*log(tanh
(x/2) - sqrt(2) - 1)/(941664*sqrt(2) + 1331714) - 665857*sqrt(2)*log(tanh(x
/2) - sqrt(2) + 1)/(941664*sqrt(2) + 1331714) - 941664*log(tanh(x/2) - sqrt
(2) + 1)/(941664*sqrt(2) + 1331714)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.
time = 0.42, size = 41, normalized size = 2.16

$$-\frac{1}{2}\,\sqrt{2}\,\log\left(\frac{\left|-4\,\sqrt{2}+2\,e^{\left(2\,x\right)}-6\right|}{\left|4\,\sqrt{2}+2\,e^{\left(2\,x\right)}-6\right|}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - x
```

Mupad [B]
time = 0.14, size = 56, normalized size = 2.95

$$\frac{\sqrt{2}\,\ln\left(8\,\mathrm{e}^{2\,x}+\frac{\sqrt{2}\,\left(12\,\mathrm{e}^{2\,x}-4\right)}{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(8\,\mathrm{e}^{2\,x}-\frac{\sqrt{2}\,\left(12\,\mathrm{e}^{2\,x}-4\right)}{2}\right)}{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)^2/(sinh(x)^2 - 1),x)
```

```
[Out] (2^(1/2)*log(8*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - (2^(1/2)*log(
8*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - x
```

3.350 $\int \frac{\cosh^3(x)}{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=10

$$2\tanh^{-1}(\sinh(x))-\sinh(x)$$

```
[Out] 2*arctanh(sinh(x))-sinh(x)
```

Rubi [A]

time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 396, 212}

$$2\tanh^{-1}(\sinh(x))-\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 - Sinh[x]^2),x]
```

```
[Out] 2*ArcTanh[Sinh[x]] - Sinh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)}{1-\sinh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{1-x^2}\,dx, x, \sinh(x)\right) \\
&= -\sinh(x) + 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sinh(x)\right) \\
&= 2\tanh^{-1}(\sinh(x)) - \sinh(x)
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 14, normalized size = 1.40

$$-2\left(-\tanh^{-1}(\sinh(x)) + \frac{\sinh(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 - Sinh[x]^2),x]
```

```
[Out] -2*(-ArcTanh[Sinh[x]] + Sinh[x]/2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(10) = 20.
time = 0.52, size = 50, normalized size = 5.00

method	result	size
risch	$-\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\ln\left(e^{2x}+2\,e^{x}-1\right)-\ln\left(e^{2x}-2\,e^{x}-1\right)$	36
default	$\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\ln\left(\tanh^{2}\left(\frac{x}{2}\right)+2\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh^{2}\left(\frac{x}{2}\right)-2\tanh\left(\frac{x}{2}\right)-1\right)$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1-sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(tanh(1/2*x)-1)+1/(tanh(1/2*x)+1)-ln(tanh(1/2*x)^2+2*tanh(1/2*x)-1)+ln(ta
nh(1/2*x)^2-2*tanh(1/2*x)-1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(10) = 20.
time = 0.25, size = 39, normalized size = 3.90

$$\frac{1}{2}\,e^{(-x)} - \frac{1}{2}\,e^{x} - \log\left(2\,e^{(-x)} + e^{(-2\,x)} - 1\right) + \log\left(-2\,e^{(-x)} + e^{(-2\,x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*e^(-x) - 1/2*e^x - log(2*e^(-x) + e^(-2*x) - 1) + log(-2*e^(-x) + e^(-2
*x) - 1)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(10) = 20$.
time = 0.45, size = 71, normalized size = 7.10

$$-\frac{\cosh(x)^{2} - 2(\cosh(x) + \sinh(x))\log\left(\frac{2(\sinh(x)+1)}{\cosh(x)-\sinh(x)}\right) + 2(\cosh(x) + \sinh(x))\log\left(\frac{2(\sinh(x)-1)}{\cosh(x)-\sinh(x)}\right) + 2\cosh(x)\sinh(x) + \sinh(x)^{2} - 1}{2(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh(x)^2 - 2*(cosh(x) + sinh(x))*log(2*(sinh(x) + 1)/(cosh(x) - sinh
(x))) + 2*(cosh(x) + sinh(x))*log(2*(sinh(x) - 1)/(cosh(x) - sinh(x))) + 2*
cosh(x)*sinh(x) + sinh(x)^2 - 1)/(cosh(x) + sinh(x))
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(8) = 16$.
time = 0.62, size = 129, normalized size = 12.90

$$\frac{\log\left(\tanh^{2}\left(\frac{x}{2}\right) - 2\tanh\left(\frac{x}{2}\right) - 1\right)\tanh^{2}\left(\frac{x}{2}\right)}{\tanh^{2}\left(\frac{x}{2}\right) - 1} - \frac{\log\left(\tanh^{2}\left(\frac{x}{2}\right) - 2\tanh\left(\frac{x}{2}\right) - 1\right)}{\tanh^{2}\left(\frac{x}{2}\right) - 1} - \frac{\log\left(\tanh^{2}\left(\frac{x}{2}\right) + 2\tanh\left(\frac{x}{2}\right) - 1\right)\tanh^{2}\left(\frac{x}{2}\right)}{\tanh^{2}\left(\frac{x}{2}\right) - 1} + \frac{\log\left(\tanh^{2}\left(\frac{x}{2}\right) + 2\tanh\left(\frac{x}{2}\right) - 1\right)}{\tanh^{2}\left(\frac{x}{2}\right) - 1} + \frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^{2}\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(1-sinh(x)**2),x)
```

```
[Out] log(tanh(x/2)**2 - 2*tanh(x/2) - 1)*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(t
anh(x/2)**2 - 2*tanh(x/2) - 1)/(tanh(x/2)**2 - 1) - log(tanh(x/2)**2 + 2*ta
nh(x/2) - 1)*tanh(x/2)**2/(tanh(x/2)**2 - 1) + log(tanh(x/2)**2 + 2*tanh(x/
2) - 1)/(tanh(x/2)**2 - 1) + 2*tanh(x/2)/(tanh(x/2)**2 - 1)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.
time = 0.40, size = 37, normalized size = 3.70

$$\frac{1}{2}e^{(-x)} - \frac{1}{2}e^{x} + \log\left(\left|-e^{(-x)} + e^{x} + 2\right|\right) - \log\left(\left|-e^{(-x)} + e^{x} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) - 1/2*e^x + log(abs(-e^(-x) + e^x + 2)) - log(abs(-e^(-x) + e^x
- 2))
```

Mupad [**B**]
time = 0.06, size = 39, normalized size = 3.90

$$\frac{e^{-x}}{2} - \ln\left(32\,e^{2x} - 64\,e^{x} - 32\right) + \ln\left(32\,e^{2x} + 64\,e^{x} - 32\right) - \frac{e^{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)^3/(sinh(x)^2 - 1),x)
```

```
[Out] exp(-x)/2 - log(32*exp(2*x) - 64*exp(x) - 32) + log(32*exp(2*x) + 64*exp(x)
 - 32) - exp(x)/2
```

3.351 $\int \frac{\cosh^4(x)}{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{5x}{2}+2\sqrt{2}\,\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)-\frac{1}{2}\cosh(x)\sinh(x)$$

```
[Out] -5/2*x-1/2*cosh(x)*sinh(x)+2*arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {3270, 425, 536, 212}

$$-\frac{5x}{2}+2\sqrt{2}\,\tanh^{-1}\left(\sqrt{2}\,\tanh(x)\right)-\frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(1 - Sinh[x]^2),x]
```

```
[Out] (-5*x)/2 + 2*Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] - (Cosh[x]*Sinh[x])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^4(x)}{1-\sinh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-2x^2\right)\left(1-x^2\right)^2}\,dx, x, \tanh(x)\right)\\ &= -\frac{1}{2}\cosh(x)\sinh(x)-\frac{1}{2}\text{Subst}\left(\int \frac{-3-2x^2}{\left(1-2x^2\right)\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\ &= -\frac{1}{2}\cosh(x)\sinh(x)-\frac{5}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)+4\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, t\right.\\ &= -\frac{5x}{2}+2\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-\frac{1}{2}\cosh(x)\sinh(x)\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 32, normalized size = 1.07

$$-2\left(\frac{5x}{4}-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)+\frac{1}{8}\sinh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(1 - Sinh[x]^2),x]
[Out] -2*((5*x)/4 - Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + Sinh[2*x]/8)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(22) = 44$.
time = 0.54, size = 98, normalized size = 3.27

method	result
risch	$-\frac{5x}{2}-\frac{e^{2x}}{8}+\frac{e^{-2x}}{8}+\sqrt{2}\,\ln\left(e^{2x}-3+2\sqrt{2}\right)-\sqrt{2}\,\ln\left(e^{2x}-3-2\sqrt{2}\right)$
default	$2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)+2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{1}{2(\tanh(\frac{x}{2})+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(1-sinh(x)^2),x,method=_RETURNVERBOSE)
[Out] 2*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+2*2^(1/2)*arctanh(1/4*(2*t
anh(1/2*x)+2)*2^(1/2))+1/2/(tanh(1/2*x)+1)^2-1/2/(tanh(1/2*x)+1)-5/2*ln(tan
h(1/2*x)+1)-1/2/(tanh(1/2*x)-1)^2-1/2/(tanh(1/2*x)-1)+5/2*ln(tanh(1/2*x)-1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(22) = 44$.
time = 0.48, size = 75, normalized size = 2.50

$$\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-\frac{5}{2}x-\frac{1}{8}e^{(2x)}+\frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - sqrt(2)*log(-
(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - 5/2*x - 1/8*e^(2*x) + 1/8*
e^(-2*x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(22) = 44$.
time = 0.57, size = 163, normalized size = 5.43

$$-\frac{\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+20x\cosh(x)^2+2\left(3\cosh(x)^2+10x\right)\sinh(x)^2-8\left(\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2\right)\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^2-2\sqrt{2}+3}{\cosh(x)^2+\sinh(x)^2-3}\right)+4\left(\cosh(x)^3+10x\cosh(x)\right)\sinh(x)-1}{8\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 20*x*cosh(x)^2 + 2*(3*c
osh(x)^2 + 10*x)*sinh(x)^2 - 8*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(
x) + sqrt(2)*sinh(x)^2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) -
4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^
2 + sinh(x)^2 - 3)) + 4*(cosh(x)^3 + 10*x*cosh(x))*sinh(x) - 1)/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2431 vs. $2(29) = 58$.
time = 8.29, size = 2431, normalized size = 81.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(1-sinh(x)**2),x)
```

```
[Out] -2716698600*sqrt(2)*x*tanh(x/2)**4/(1536796802*tanh(x/2)**4 + 1086679440*sq
rt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)
**2 + 1536796802 + 1086679440*sqrt(2)) - 3841992005*x*tanh(x/2)**4/(1536796
802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**
2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) + 76
83984010*x*tanh(x/2)**2/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(
```

```
x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 15367
96802 + 1086679440*sqrt(2)) + 5433397200*sqrt(2)*x*tanh(x/2)**2/(1536796802
*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 -
 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 27166
98600*sqrt(2)*x/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4
- 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 +
1086679440*sqrt(2)) - 3841992005*x/(1536796802*tanh(x/2)**4 + 1086679440*sq
rt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)
**2 + 1536796802 + 1086679440*sqrt(2)) + 2173358880*log(tanh(x/2) - 1 + sqr
t(2))*tanh(x/2)**4/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)*
*4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802
 + 1086679440*sqrt(2)) + 1536796802*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))*ta
nh(x/2)**4/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 307
3593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 10866
79440*sqrt(2)) - 4346717760*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(1536
796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2
)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) -
 3073593604*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(1536796802*t
anh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2
173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) + 2173358
880*log(tanh(x/2) - 1 + sqrt(2))/(1536796802*tanh(x/2)**4 + 1086679440*sqrt
(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**
2 + 1536796802 + 1086679440*sqrt(2)) + 1536796802*sqrt(2)*log(tanh(x/2) - 1
 + sqrt(2))/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 30
73593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086
679440*sqrt(2)) + 2173358880*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**4/(153
6796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/
2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2))
+ 1536796802*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**4/(1536796802*
tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 -
2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 434671
7760*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1536796802*tanh(x/2)**4 + 1
086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(
2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 3073593604*sqrt(2)*log
(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1536796802*tanh(x/2)**4 + 108667944
0*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(
x/2)**2 + 1536796802 + 1086679440*sqrt(2)) + 2173358880*log(tanh(x/2) + 1 +
 sqrt(2))/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073
593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 108667
9440*sqrt(2)) + 1536796802*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))/(1536796802
*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 -
 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 21733
58880*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**4/(1536796802*tanh(x/2)**4 +
1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt
(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 1536796802*sqrt(2)*lo
```

```
g(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**4/(1536796802*tanh(x/2)**4 + 10866794
40*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh
(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) + 4346717760*log(tanh(x/2) - sq
rt(2) - 1)*tanh(x/2)**2/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(
x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 15367
96802 + 1086679440*sqrt(2)) + 3073593604*sqrt(2)*log(tanh(x/2) - sqrt(2) -
1)*tanh(x/2)**2/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4
- 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 +
1086679440*sqrt(2)) - 2173358880*log(tanh(x/2) - sqrt(2) - 1)/(1536796802*t
anh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2
173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 1536796
802*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(1536796802*tanh(x/2)**4 + 1086679
440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tan
h(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 2173358880*log(tanh(x/2) - s
qrt(2) + 1)*tanh(x/2)**4/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh
(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880...
```

Giac **[B]** Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.
time = 0.41, size = 61, normalized size = 2.03

$$\frac{1}{8}\left(10\,e^{(2\,x)}+1\right)e^{(-2\,x)}-\sqrt{2}\,\log\left(\frac{\left|-4\,\sqrt{2}+2\,e^{(2\,x)}-6\right|}{\left|4\,\sqrt{2}+2\,e^{(2\,x)}-6\right|}\right)-\frac{5}{2}\,x-\frac{1}{8}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] 1/8*(10*e^(2*x) + 1)*e^(-2*x) - sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)
/abs(4*sqrt(2) + 2*e^(2*x) - 6)) - 5/2*x - 1/8*e^(2*x)
```

Mupad **[B]**
time = 0.84, size = 66, normalized size = 2.20

$$\frac{\mathrm{e}^{-2\,x}}{8}-\frac{5\,x}{2}-\frac{\mathrm{e}^{2\,x}}{8}+\sqrt{2}\,\ln\left(16\,\mathrm{e}^{2\,x}+\sqrt{2}\,\left(12\,\mathrm{e}^{2\,x}-4\right)\right)-\sqrt{2}\,\ln\left(16\,\mathrm{e}^{2\,x}-\sqrt{2}\,\left(12\,\mathrm{e}^{2\,x}-4\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)^4/(sinh(x)^2 - 1),x)
```

```
[Out] exp(-2*x)/8 - (5*x)/2 - exp(2*x)/8 + 2^(1/2)*log(16*exp(2*x) + 2^(1/2)*(12*
exp(2*x) - 4)) - 2^(1/2)*log(16*exp(2*x) - 2^(1/2)*(12*exp(2*x) - 4))
```

3.352 $\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$-\frac{a(a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8b^{3/2}f}-\frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf}+\frac{\sinh(e+fx)}{}$$

```
[Out] -1/8*a*(a-4*b)*arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(3/
2)/f+1/4*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/b/f-1/8*(a-4*b)*sinh(f*x+e)*
(a+b*sinh(f*x+e)^2)^(1/2)/b/f
```

Rubi [A]

time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 396, 201, 223, 212}

$$-\frac{a(a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4bf}-\frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/8*(a*(a - 4*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^
2]])/(b^(3/2)*f) - ((a - 4*b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(8
*b*f) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\Big(\int (1+x^2)\sqrt{a+bx^2}\,dx, x, \sinh(e+fx)\Big)}{f}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4bf} - (a-4b)\text{Subst}\Big(\int \sqrt{a$$

$$= -\frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf} + \sinh(e+fx$$

$$= -\frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf} + \sinh(e+fx$$

$$= -\frac{a(a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8b^{3/2}f} - (a-4b)\sin$$

Mathematica [A]
time = 0.51, size = 124, normalized size = 1.06

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(-\sqrt{a}\,(a-4b)\sinh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)+\sqrt{b}\,(a+3b+b\cosh(2(e+fx)))\sinh(e+fx)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\right)}{8b^{3/2}f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b*Sinh[e + f*x]^2]*(-(Sqrt[a]*(a - 4*b)*ArcSinh[(Sqrt[b]*Sinh[e +
 f*x])/Sqrt[a]]) + Sqrt[b]*(a + 3*b + b*Cosh[2*(e + f*x)])*Sinh[e + f*x]*Sq
rt[1 + (b*Sinh[e + f*x]^2)/a]))/(8*b^(3/2)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a
])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.32, size = 52, normalized size = 0.44

method	result	size
default	$\frac{\text{‘}\mathit{int/indef0}\text{‘}\left(\frac{b\left(\cosh^{4}(fx+e)\right)+(a-b)\left(\cosh^{2}(fx+e)\right)}{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘((b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^(1/2
),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. 2(101) = 202.
time = 0.46, size = 3281, normalized size = 28.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(2*((a^2 - 4*a*b)*cosh(f*x + e)^4 + 4*(a^2 - 4*a*b)*cosh(f*x + e)^3*
sinh(f*x + e) + 6*(a^2 - 4*a*b)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 -
4*a*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 4*a*b)*sinh(f*x + e)^4)*sqrt(
b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3
)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 +
```

```
 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5
*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^
6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*
b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*
cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 1
4*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*
a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cos
h(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2
*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2
 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x +
 e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b
+ b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2
- 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*
(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*
sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)
*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*
sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 -
 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e
))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b -
 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(
f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^
3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(
f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x
 + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e
)^5 + sinh(f*x + e)^6)) + 2*((a^2 - 4*a*b)*cosh(f*x + e)^4 + 4*(a^2 - 4*a*b
)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 4*a*b)*cosh(f*x + e)^2*sinh(f*x
+ e)^2 + 4*(a^2 - 4*a*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 4*a*b)*sinh
(f*x + e)^4)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a
)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(2)*(b^
2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*x + e)
^6 + (2*a*b + 5*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 2*a*b + 5*
b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (2*a*b + 5*b^2)*cosh(f*x
+ e))*sinh(f*x + e)^3 - (2*a*b + 5*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x
+ e)^4 + 6*(2*a*b + 5*b^2)*cosh(f*x + e)^2 - 2*a*b - 5*b^2)*sinh(f*x + e)^2
 - b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(2*a*b + 5*b^2)*cosh(f*x + e)^3 - (2*
a*b + 5*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh
(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + s
```

```
inh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)^4 + 4*b^2*f*cosh(f*x + e)^3*sinh(f*x
 + e) + 6*b^2*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b^2*f*cosh(f*x + e)*sin
h(f*x + e)^3 + b^2*f*sinh(f*x + e)^4), 1/64*(4*((a^2 - 4*a*b)*cosh(f*x + e)
^4 + 4*(a^2 - 4*a*b)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 4*a*b)*cosh(f
*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 4*a*b)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a^2 - 4*a*b)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x +
 e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b
)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*co
sh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*s
inh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x
+ e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x +
 e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + 4*((a^2 - 4*a*b)*c
osh(f*x + e)^4 + 4*(a^2 - 4*a*b)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 4
*a*b)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 4*a*b)*cosh(f*x + e)*sinh(
f*x + e)^3 + (a^2 - 4*a*b)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sin...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 871 vs. 2(101) = 202.
time = 0.67, size = 871, normalized size = 7.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b)*((2*a*e^(6*e) + 5*b*e^(6*e))*e^(-2*e)/b + e^(2*f*x + 6*e)) - 8*(a^2*e^(4
*e) - 4*a*b*e^(4*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4
*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))/(sqrt(-b)*b
) + 4*(a^2*sqrt(b)*e^(4*e) - 4*a*b^(3/2)*e^(4*e))*log(abs(-(sqrt(b)*e^(2*f*
x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*
e) + b))*b - 2*a*sqrt(b) + b^(3/2)))/b^2 + 4*(2*(sqrt(b)*e^(2*f*x + 2*e) -
sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*
a^2*e^(4*e) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
```

```
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*b*e^(4*e) - 2*(sqrt(b)*e^(2*f*
x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*
e) + b))^3*b^2*e^(4*e) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(3/2)*e^(4*e) + (
sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*
b*e^(2*f*x + 2*e) + b))^2*b^(5/2)*e^(4*e) + 2*(sqrt(b)*e^(2*f*x + 2*e) - sq
rt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*
b*e^(4*e) - 8*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b^2*e^(4*e) + 4*(sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))*b^3*e^(4*e) - 3*b^(7/2)*e^(4*e))/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b
*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - b)^2
*b))*e^(-4*e)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(e+f\,x)^3\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.353 $\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}\,f}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

```
[Out] 1/2*a*arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f/b^(1/2)+1/2*
sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 201, 223, 212}

$$\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (a*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[b]
*f) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,d}{2f}$$

$$= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x,\right.}{2}$$

$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}\,f} + \frac{\sinh(e+fx)\sqrt{a+b\,\text{si}}}{2f}$$

Mathematica [A]
time = 0.18, size = 96, normalized size = 1.33

$$\frac{\sqrt{b}\,\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)+a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{2\sqrt{b}\,f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2) + a^(3/2)*ArcSinh[(Sqrt[b]*S
inh[e + f*x])/Sqrt[a]]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(2*Sqrt[b]*f*Sqrt[a
 + b*Sinh[e + f*x]^2])
```

Maple [A]
time = 0.51, size = 60, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\frac{\sinh(fx+e)\sqrt{a+b\left(\sinh^2(fx+e)\right)}}{2}+\frac{a\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\left(\sinh^2(fx+e)\right)}\right)}{2\sqrt{b}}}{f}$	60
default	$\frac{\frac{\sinh(fx+e)\sqrt{a+b\left(\sinh^2(fx+e)\right)}}{2}+\frac{a\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b\left(\sinh^2(fx+e)\right)}\right)}{2\sqrt{b}}}{f}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*sin
h(f*x+e)+(a+b*sinh(f*x+e)^2)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 756 vs. $2(60) = 120$.
time = 0.45, size = 2419, normalized size = 33.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e
)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a
*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*
x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4
*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh
(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^
2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2
 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9
*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x +
```

```
 e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10
*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 +
6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x
+ e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b
+ 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^
2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*
cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^
2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 +
3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)
^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(
f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*
a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b
 + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5
- 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*si
nh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(
2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*
b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*
b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x +
e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^
3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*si
nh(f*x + e)^5 + sinh(f*x + e)^6)) + (a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*
sinh(f*x + e) + a*sinh(f*x + e)^2)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cos
h(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3
*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh
(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2)) + sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b
*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*
f*cosh(f*x + e)^2 + 2*b*f*cosh(f*x + e)*sinh(f*x + e) + b*f*sinh(f*x + e)^2
), -1/8*(2*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*
x + e)^2)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh
(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cos
h(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
 e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b
 - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3*a*
b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b^2
)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2
)*cosh(f*x + e))*sinh(f*x + e))) + 2*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)
*sinh(f*x + e) + a*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^
2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
```

```
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x +
 e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(
3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*
a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(2)*(b*cosh(f*x + e)^2 + 2*
b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^2 + 2*b*f*cosh(f*x + e)*si
nh(f*x + e) + b*f*sinh(f*x + e)^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^2\left(e+fx\right)}\,\cosh\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*cosh(e + f*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{16,[4,2,4]%%%}+%%%{%%%{-32,[1]%%%},[4,2,3]%%%}+%%%{%%%{16,[2]
%%%},[
```

Mupad [**B**]
time = 0.96, size = 61, normalized size = 0.85

$$\frac{\sinh(e+f\,x)\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{2\,f}+\frac{a\,\ln\left(\sqrt{b}\,\sinh(e+f\,x)+\sqrt{b\sinh\left(e+f\,x\right)^2+a}\right)}{2\,\sqrt{b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] (sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2))/(2*f) + (a*log(b^(1/2)*sinh(e
 + f*x) + (a + b*sinh(e + f*x)^2)^(1/2)))/(2*b^(1/2)*f)
```

3.354 $\int \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

```
[Out] arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f+arc
tanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3269, 399, 223, 212, 385, 209}

$$\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{(a-b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f} + \frac{b\text{Subst}}{}$$

$$= \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f} + \frac{b\text{S}}{}$$

$$= \frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{}{\sqrt{a}}\right.}{}$$

Mathematica [A]
time = 0.65, size = 130, normalized size = 1.53

$$\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{2a-2b}\sinh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)+\frac{\sqrt{a}\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}}{\sqrt{2a-b+b\cosh(2(e+fx))}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2
*(e + f*x)]]] + (Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]]*S
qrt[(2*a - b + b*Cosh[2*(e + f*x)])/a])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]
)/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.10, size = 51, normalized size = 0.60

method	result	size
default	$\frac{\text{` }\mathit{int/indef0}\text{`}\left(-\frac{-b\left(\sinh^{2}\left(fx+e\right)\right)-a}{\cosh\left(fx+e\right)^{2}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(-(-b*sinh(f*x+e)^2-a)/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),
sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(73) = 146$.
time = 0.50, size = 5139, normalized size = 60.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*
a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f
*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 -
```

```
4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a
^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^
2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 +
9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x
+ e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 1
0*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 +
 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x
 + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b
 + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b
^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)
*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a
^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 +
 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e
)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2
*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*
b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5
 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*s
inh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*
(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2
*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a
*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x +
 e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)
^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*s
inh(f*x + e)^5 + sinh(f*x + e)^6)) + 2*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x
 + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x +
e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a
 + 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x +
 e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x +
 e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cos
h(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + sq
rt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f
*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)
^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*c
osh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*c
osh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/f, 1/4*(4*sqrt(a - b)*arcta
n(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
```

```
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh
(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*
a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2
 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqr
t(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b
^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8
 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b +
 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e
)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*
a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3
)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b -
 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*s
inh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 -
4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*c
osh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 +
 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^
2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^
3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x
 + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*s...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^2\left(e+fx\right)}\,\operatorname{sech}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}}{\cosh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x),x)
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x), x)
```

3.355 $\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=86

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{a-b}\,f}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2f}$$

```
[Out] 1/2*a*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f/(a-b)^(1/
2)+1/2*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3269, 386, 385, 209}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f\sqrt{a-b}}+\frac{\tanh(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (a*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt
[a - b]*f) + (Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(2*f
)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1+x^2)^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2f} + \frac{a\text{Subst}\Big(\int$$

$$= \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2f} + \frac{a\text{Subst}\Bigg(\int$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{a-b}\,f} + \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh}}{2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 175 vs. 2(86) = 172.
time = 1.13, size = 175, normalized size = 2.03

$$\frac{\sinh(e+fx)\left(\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{-\frac{(a-b)\sinh^2(e+fx)}{a}}}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\right)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}} + (2a-b+b\cosh(2(e+fx)))\operatorname{sech}^2(e+fx)\sqrt{-\frac{(a-b)\sinh^2(e+fx)}{a}}\right)}{4f\sqrt{-\frac{(a-b)\sinh^2(e+fx)}{a}}\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sinh[e + f*x]*(Sqrt[2]*a*ArcTanh[Sqrt[-(((a - b)*Sinh[e + f*x]^2)/a)]/Sqrt
[1 + (b*Sinh[e + f*x]^2)/a]]*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a] + (2*a
```

```
 - b + b*Cosh[2*(e + f*x)])*Sech[e + f*x]^2*Sqrt[-(((a - b)*Sinh[e + f*x]^2
)/a)]))/(4*f*Sqrt[-(((a - b)*Sinh[e + f*x]^2)/a)]*Sqrt[a + b*Sinh[e + f*x]^
2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.15, size = 35, normalized size = 0.41

method	result	size
default	$\frac{‘int/indef0‘\left(\frac{\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}}{\cosh\left(fx+e\right)^4},\sinh\left(fx+e\right)\right)}{f}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ‘int/indef0‘(1/cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(74) = 148.
time = 0.44, size = 1327, normalized size = 15.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [-1/4*((a*cosh(f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x + e)^3 + a*sinh(f*x
+ e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2
+ 4*(a*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + a)*sqrt(-a + b)*l
og(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)
*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2
+ 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 -
```

```
(3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*co
sh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*s
inh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*si
nh(f*x + e) + 1)) - 2*sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x
 + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x + e)^4 + 4*(a - b)*f*cosh
(f*x + e)*sinh(f*x + e)^3 + (a - b)*f*sinh(f*x + e)^4 + 2*(a - b)*f*cosh(f*
x + e)^2 + 2*(3*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^2 + (a
 - b)*f + 4*((a - b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh(f*x + e))*sinh(f*x
+ e)), 1/2*((a*cosh(f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x + e)^3 + a*sinh
(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2 + a)*sinh(f*x +
e)^2 + 4*(a*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + a)*sqrt(a -
b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x
 + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b
)) + sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x +
e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)))/((a - b)*f*cosh(f*x + e)^4 + 4*(a - b)*f*cosh(f*x + e)*sinh(f*
x + e)^3 + (a - b)*f*sinh(f*x + e)^4 + 2*(a - b)*f*cosh(f*x + e)^2 + 2*(3*(
a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^2 + (a - b)*f + 4*((a -
 b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^2{(e + fx)}}\,\operatorname{sech}^3{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\cosh\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^3,x)
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^3, x)
```

3.356 $\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=151

$$\frac{a(3a-4b)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8(a-b)^{3/2}f}+\frac{(3a-4b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{8(a-b)f}+$$

```
[Out] 1/8*a*(3*a-4*b)*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/(
a-b)^(3/2)/f+1/4*sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)/(a-b)/
f+1/8*(3*a-4*b)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/f
```

Rubi [A]
time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3269, 390, 386, 385, 209}

$$\frac{a(3a-4b)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8f(a-b)^{3/2}}+\frac{\tanh(e+fx)\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f(a-b)}+\frac{(3a-4b)\tanh(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (a*(3*a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^
2]])/(8*(a - b)^(3/2)*f) + ((3*a - 4*b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2]*Tanh[e + f*x])/(8*(a - b)*f) + (Sech[e + f*x]^3*(a + b*Sinh[e + f*x]
^2)^(3/2)*Tanh[e + f*x])/(4*(a - b)*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1+x^2)^3}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{4(a-b)f} + \frac{(3a-}{}$$

$$= \frac{(3a-4b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{8(a-b)f} +$$

$$= \frac{(3a-4b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{8(a-b)f} +$$

$$= \frac{a(3a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8(a-b)^{3/2}f} + \frac{(3a-4b)\operatorname{sech}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.14, size = 684, normalized size = 4.53

$$-\frac{\operatorname{sech}^3(e+fx)\left(1+\frac{b\sinh^2(e+fx)}{a}\right)\tanh(e+fx)\left(-15a\operatorname{ArcSin}\left(\sqrt{\frac{(a-b)\tanh^2(e+fx)}{a}}\right)-10b\operatorname{ArcSin}\left(\sqrt{\frac{(a-b)\tanh^2(e+fx)}{a}}\right)\sinh^2(e+fx)-30a\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}\left(\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{3/2}-20b\sinh^2(e+fx)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}\left(\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{3/2}-32a\,{}_2F_1\left(2,4;\frac{7}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}\left(\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{5/2}-32b\,{}_2F_1\left(2,4;\frac{7}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\sinh^2(e+fx)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}\left(\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{5/2}+32a\,{}_2F_1\left(2,4;\frac{7}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}\left(\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{7/2}+32b\,{}_2F_1\left(2,4;\frac{7}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\sinh^2(e+fx)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}\left(\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{7/2}+15a\sqrt{\frac{(a-b)\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)\tanh^2(e+fx)}{a^2}}+10b\sinh^2(e+fx)\sqrt{\frac{(a-b)\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)\tanh^2(e+fx)}{a^2}}\right)}{40f\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}\left(\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/40*(Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)*Tanh[e + f*x]*(-15*a*Arc
Sin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]] - 10*b*ArcSin[Sqrt[((a - b)*Tanh[e +
 f*x]^2)/a]]*Sinh[e + f*x]^2 - 30*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f
*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) - 20*b*Sinh[e + f*x]^2*Sqrt[
(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^
(3/2) - 32*a*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)
^(5/2) - 32*b*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sin
h[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Ta
nh[e + f*x]^2)/a)^(5/2) + 32*a*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e
 + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*T
anh[e + f*x]^2)/a)^(7/2) + 32*b*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[
e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2
))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + 15*a*Sqrt[((a - b)*Sech[e + f*x
]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2] + 10*b*Sinh[e + f*x]^2*Sq
rt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2]))
/(f*Sqrt[a + b*Sinh[e + f*x]^2]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^
2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 60.87, size = 35, normalized size = 0.23

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\sqrt{a+b\left(\sinh^2(fx+e)\right)}}{\cosh(fx+e)^6},\sinh(fx+e)\right)}{f}$	35
risch	Expression too large to display	155502204

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(1/cosh(f*x+e)^6*(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^5, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1805 vs. $2(135) = 270$.
time = 0.55, size = 3727, normalized size = 24.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*a^2 - 4*a*b)*cosh(f*x + e)^8 + 8*(3*a^2 - 4*a*b)*cosh(f*x + e)*
sinh(f*x + e)^7 + (3*a^2 - 4*a*b)*sinh(f*x + e)^8 + 4*(3*a^2 - 4*a*b)*cosh(
f*x + e)^6 + 4*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 3*a^2 - 4*a*b)*sinh(f*x
 + e)^6 + 8*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^3 + 3*(3*a^2 - 4*a*b)*cosh(f*x
 + e))*sinh(f*x + e)^5 + 6*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 2*(35*(3*a^2 -
 4*a*b)*cosh(f*x + e)^4 + 30*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 9*a^2 - 12*a
*b)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^5 + 10*(3*a^2 - 4*
a*b)*cosh(f*x + e)^3 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 4
*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 4*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^6 + 1
5*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 9*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 3*a
^2 - 4*a*b)*sinh(f*x + e)^2 + 3*a^2 - 4*a*b + 8*((3*a^2 - 4*a*b)*cosh(f*x +
 e)^7 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e)^5 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e)
^3 + (3*a^2 - 4*a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a - 2
*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)
*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x
+ e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x
+ e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b
)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e
)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x +
e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e
) + 1)) - 2*sqrt(2)*((3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^6 + 6*(3*a^2 - 5
*a*b + 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - 5*a*b + 2*b^2)*sinh(
f*x + e)^6 + (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e)^4 + (15*(3*a^2 - 5*a*
b + 2*b^2)*cosh(f*x + e)^2 + 11*a^2 - 21*a*b + 10*b^2)*sinh(f*x + e)^4 + 4*
(5*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^3 + (11*a^2 - 21*a*b + 10*b^2)*cos
h(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e)^2 +
(15*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^4 + 6*(11*a^2 - 21*a*b + 10*b^2)*
cosh(f*x + e)^2 - 11*a^2 + 21*a*b - 10*b^2)*sinh(f*x + e)^2 - 3*a^2 + 5*a*b
 - 2*b^2 + 2*(3*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^5 + 2*(11*a^2 - 21*a*
```

```
b + 10*b^2)*cosh(f*x + e)^3 - (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b
 + b^2)*f*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x
+ e)^7 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^8 + 4*(a^2 - 2*a*b + b^2)*f*co
sh(f*x + e)^6 + 4*(7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b +
 b^2)*f)*sinh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 8*(7*(
a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e
))*sinh(f*x + e)^5 + 2*(35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 30*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)^2 + 3*(a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^4
 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 8*(7*(a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^5 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2 - 2*a*b + b^2)*f*cosh(f*x
 + e)^6 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 9*(a^2 - 2*a*b + b^2)*
f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^2 - 2*a*b +
 b^2)*f + 8*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 3*(a^2 - 2*a*b + b^2)*
f*cosh(f*x + e)^5 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b
+ b^2)*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(((3*a^2 - 4*a*b)*cosh(f*x + e)
^8 + 8*(3*a^2 - 4*a*b)*cosh(f*x + e)*sinh(f*x + e)^7 + (3*a^2 - 4*a*b)*sinh
(f*x + e)^8 + 4*(3*a^2 - 4*a*b)*cosh(f*x + e)^6 + 4*(7*(3*a^2 - 4*a*b)*cosh
(f*x + e)^2 + 3*a^2 - 4*a*b)*sinh(f*x + e)^6 + 8*(7*(3*a^2 - 4*a*b)*cosh(f*
x + e)^3 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(3*a^2 - 4*
a*b)*cosh(f*x + e)^4 + 2*(35*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 30*(3*a^2 -
4*a*b)*cosh(f*x + e)^2 + 9*a^2 - 12*a*b)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 4*
a*b)*cosh(f*x + e)^5 + 10*(3*a^2 - 4*a*b)*cosh(f*x + e)^3 + 3*(3*a^2 - 4*a*
b)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 4*(
7*(3*a^2 - 4*a*b)*cosh(f*x + e)^6 + 15*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 9*
(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 3*a^2 - 4*a*b)*sinh(f*x + e)^2 + 3*a^2 -
4*a*b + 8*((3*a^2 - 4*a*b)*cosh(f*x + e)^7 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e
)^5 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e)^3 + (3*a^2 - 4*a*b)*cosh(f*x + e))*si
nh(f*x + e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)
^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)
^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^2\left(e+fx\right)}\,\operatorname{sech}^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**5, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\cosh\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^5,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^5, x)
```

3.357 $\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=301

$$-\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf}+\frac{\cosh(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+f\right.}{5bf}$$

```
[Out] 1/5*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/b/f-2/15*(a-3*b)*cosh
(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+1/15*(2*a^2-7*a*b-3*b^2)*
(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/15*(a-9*b)*(1/(1+sinh(
f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+
e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sech(
f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/15*(2*a^2-7*a*b-3*b^2)*(a+b*sinh(f*
x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Rubi [A]
time = 0.20, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3271, 427, 542, 545, 429, 506, 422}

$$\frac{(2a^2-7ab-3b^2)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{15b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(2a^2-7ab-3b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15b^2f}-\frac{(a-9b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\operatorname{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{15bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5bf}-\frac{2(a-3b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-2*(a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*
b*f) + (Cosh[e + f*x]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(5*b*f)
+ ((2*a^2 - 7*a*b - 3*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e
 + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) - ((a - 9*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/
a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2
*(a + b*Sinh[e + f*x]^2))/a]) - ((2*a^2 - 7*a*b - 3*b^2)*Sqrt[a + b*Sinh[e
+ f*x]^2]*Tanh[e + f*x])/(15*b^2*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
```

```
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\left(1+x^2\right)^{3/2}\sqrt{a+bx}}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5bf} + \frac{\left(\sqrt{\cosh}\right.}{}$$

$$= -\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} +$$

$$= -\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} +$$

$$= -\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} +$$

$$= -\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} +$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 1.01, size = 211, normalized size = 0.70

$$\frac{16ia(2a^2-7ab-3b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-32ia(a^2-4ab+3b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,b(8a^2+32ab-15b^2+4b(4a+3b)\cosh(2(e+fx))+3b^2\cosh(4(e+fx)))\sinh(2(e+fx))}{240b^2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((16*I)*a*(2*a^2 - 7*a*b - 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticE[I*(e + f*x), b/a] - (32*I)*a*(a^2 - 4*a*b + 3*b^2)*Sqrt[(2*a - b +
 b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2 + 3
2*a*b - 15*b^2 + 4*b*(4*a + 3*b)*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*x)
])*Sinh[2*(e + f*x)])/(240*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [**A**]
time = 1.75, size = 521, normalized size = 1.73

method	result
default	$\frac{3\sqrt{-\frac{b}{a}}\, b^2 (\cosh^6(fx+e)) \sinh(fx+e)+4\sqrt{-\frac{b}{a}}\, ab(\cosh^4(fx+e)) \sinh(fx+e)+\left(\sqrt{-\frac{b}{a}}\, a^2+2\sqrt{-\frac{b}{a}}\, ab-3\sqrt{-\frac{b}{a}}\, b^2\right)(\mathrm{co}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(3*(-1/a*b)^(1/2)*b^2*cosh(f*x+e)^6*sinh(f*x+e)+4*(-1/a*b)^(1/2)*a*b*c
osh(f*x+e)^4*sinh(f*x+e)+((-1/a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b-3*(-1/a*b
)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2
)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2
*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-2*
(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+7*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+3*(
b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/b/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f
*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)
```

Fricas [F]
time = 0.10, size = 25, normalized size = 0.08

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^2+a}\,\cosh\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(e+f\,x)^4\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.358 $\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=223

$$\frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{(a+b)E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{}}{3bf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+f\right.}{a}}}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*(a+b)*(1/(1+sin
h(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*
x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sec
h(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+2/3*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+
sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^
(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x
+e)^2)/a)^(1/2)+1/3*(a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b/f
```

Rubi [A]
time = 0.14, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3271, 428, 545, 429, 506, 422}

$$\frac{2\,\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3bf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf}+\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((a + b)*
EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e +
 f*x]^2])/(3*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*El
lipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*
Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 428

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^p*((c + d*x^n)^q/(n*(p + q) + 1)), x] + Dist[n/(n*(p
```

```
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\sqrt{1+x^2}\,\sqrt{a+bx}\right.}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(2\sqrt{\cosh}\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(2a\sqrt{\mathrm{co}}\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{2F\left(\tan^{-}\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{(a+b)E}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.83, size = 168, normalized size = 0.75

$$\frac{-2i\sqrt{2}\,a(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+2i\sqrt{2}\,a(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+b(2a-b+b\cosh(2(e+fx)))\sinh(2(e+fx))}{6bf\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE
[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e +
f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])*S
inh[2*(e + f*x)])/(6*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.45, size = 351, normalized size = 1.57

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\,b\left(\cosh^4(fx+e)\right)\sinh(fx+e)+\sqrt{-\frac{b}{a}}\,a\left(\cosh^2(fx+e)\right)\sinh(fx+e)-\sqrt{-\frac{b}{a}}\,b\left(\cosh^2(fx+e)\right)\sinh(fx+e)+a\sqrt{\frac{b\left(\cosh^2\right.}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b*cosh(f*x+e)^4*sinh(f*x+e)+(-1/a*b)^(1/2)*a*cosh(f*x+e
)^2*sinh(f*x+e)-(-1/a*b)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+
e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1
/2),(a/b)^(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*El
lipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/
a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))*a+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*s
inh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)
```

Fricas [F]
time = 0.10, size = 25, normalized size = 0.11

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\cosh\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sinh^{2}\left(e+fx\right)}\,\cosh^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*cosh(e + f*x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(e+f\,x)^2\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.359 $\int \sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(
1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3257, 3256}

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
(b*Sinh[e + f*x]^2)/a])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

$$= -\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

<u>**Mathematica**</u> [A]
time = 0.06, size = 69, normalized size = 1.15

$$-\frac{ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a]
)/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

<u>**Maple**</u> [A]
time = 0.95, size = 140, normalized size = 2.33

method	result
default	$\frac{\sqrt{\frac{a+b\left(\sinh^2(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\left(a\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\right.\right.}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/
cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

<u>**Maxima**</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**F**]
time = 0.13, size = 16, normalized size = 0.27

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sinh^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{b\,\sinh\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.360 $\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{E\left(\operatorname{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] (1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3271, 422}

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
+ f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{\sqrt{a+bx^2}}{(1+x^2)^{3/2}}\,dx,x,\mathrm{si}}{f}$$

$$=\frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+f}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 0.35, size = 148, normalized size = 2.11

$$\frac{2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,|\,\frac{b}{a}\right)-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,|\,\frac{b}{a}\right)+\sqrt{2}\,(2a-b+b\cosh(2(e+fx)))\tanh(e+fx)}{2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a
] - (2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x),
b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(2*f*Sqrt[2*a
 - b + b*Cosh[2*(e + f*x)]])
```

Maple [**A**]
time = 1.46, size = 177, normalized size = 2.53

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\,b(\sinh^3(fx+e))+b\sqrt{\frac{a+b(\sinh^2(fx+e))}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\sqrt{\frac{a+b(\mathrm{si}}{}}}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\left(\sinh^2(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-1/a*b)^(1/2)*b*sinh(f*x+e)^3+b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*((a+b*sinh(f
*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2
),(a/b)^(1/2))+(-1/a*b)^(1/2)*a*sinh(f*x+e))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+
b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(80) = 160$.
time = 0.10, size = 535, normalized size = 7.64

$$-\frac{4\left(b\cosh\left(fx+e\right)^{2}+2\,b\cosh\left(fx+e\right)\sinh\left(fx+e\right)+b\sinh\left(fx+e\right)^{2}+b\right)\sqrt{b}\sqrt{\frac{2\,b\sqrt{\frac{a^{2}-ab}{b^{2}}}-2\,a+b}{b}}\sqrt{\frac{a^{2}-ab}{b^{2}}}F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^{2}-ab}{b^{2}}}-2\,a+b}{b}}\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)\right)\mid\frac{8a^{2}-8ab+b^{2}+4\left(2ab-b^{2}\right)\sqrt{\frac{a^{2}-ab}{b^{2}}}}{b^{2}})+\left(\left(2\,a-b\right)\cosh\left(fx+e\right)^{2}+2\left(2\,a-b\right)\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\left(2\,a-b\right)\sinh\left(fx+e\right)^{2}-2\left(b\cosh\left(fx+e\right)^{2}+2\,b\cosh\left(fx+e\right)\sinh\left(fx+e\right)+b\sinh\left(fx+e\right)^{2}+b\right)\sqrt{\frac{a^{2}-ab}{b^{2}}}+2\,a-b\right)\sqrt{b}\sqrt{\frac{2\,b\sqrt{\frac{a^{2}-ab}{b^{2}}}-2\,a+b}{b}}E(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^{2}-ab}{b^{2}}}-2\,a+b}{b}}\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)\right)\mid\frac{8a^{2}-8ab+b^{2}+4\left(2ab-b^{2}\right)\sqrt{\frac{a^{2}-ab}{b^{2}}}}{b^{2}})-\sqrt{2}\left(b\cosh\left(fx+e\right)+b\sinh\left(fx+e\right)\right)\sqrt{\frac{b\cosh\left(fx+e\right)^{2}+b\sinh\left(fx+e\right)^{2}+2\,a-b}{\cosh\left(fx+e\right)^{2}-2\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^{2}}}}{bf\cosh\left(fx+e\right)^{2}+2\,bf\cosh\left(fx+e\right)\sinh\left(fx+e\right)+bf\sinh\left(fx+e\right)^{2}+bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(4*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^
2 + b)*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*sqrt((a^2 - a*
b)/b^2)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(co
sh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt(
(a^2 - a*b)/b^2))/b^2) + ((2*a - b)*cosh(f*x + e)^2 + 2*(2*a - b)*cosh(f*x
+ e)*sinh(f*x + e) + (2*a - b)*sinh(f*x + e)^2 - 2*(b*cosh(f*x + e)^2 + 2*b
*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 + b)*sqrt((a^2 - a*b)/b^2)
 + 2*a - b)*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_
e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sin
h(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))
/b^2) - sqrt(2)*(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^2 + 2*b*f*cosh(f*x + e)*sinh(
f*x + e) + b*f*sinh(f*x + e)^2 + b*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sinh^{2}\left(e+fx\right)}\,\operatorname{sech}^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\cosh\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^2,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^2, x)
```

3.361 $\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=206

$$\frac{(2a-b)E\left(\operatorname{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{bF\left(\operatorname{ArcTan}(\sinh(e+fx))\,\right|}{3(a-b)f\sqrt{\frac{\operatorname{sech}}{}}}$$

```
[Out] 1/3*(2*a-b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x
+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*b*(1/(
1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+si
nh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-
b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*sech(f*x+e)^2*(a+b*sin
h(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.12, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3271, 423, 539, 429, 422}

$$-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\operatorname{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(2a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])
/(3*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
```

```
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1+x^2)^{5/2}}\,dx, x, \mathrm{si}\right.}{f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3f} - \frac{\left(\sqrt{\cosh^2}\right.}{}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3f} + \frac{\left((2a-b\right.}{}$$

$$= \frac{(2a-b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sin}}{3(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.42, size = 204, normalized size = 0.99

$$\frac{8ia(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\big(i(e+fx)\,|\,\tfrac{b}{a}\big)-16ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\big(i(e+fx)\,|\,\tfrac{b}{a}\big)+\sqrt{2}\,((8a^2-4b^2)\cosh(2(e+fx))+(2a-b)(8a-5b+b\cosh(4(e+fx))))\operatorname{sech}^2(e+fx)\tanh(e+fx)}{24(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((8*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
 f*x), b/a] - (16*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elli
pticF[I*(e + f*x), b/a] + Sqrt[2]*((8*a^2 - 4*b^2)*Cosh[2*(e + f*x)] + (2*a
 - b)*(8*a - 5*b + b*Cosh[4*(e + f*x)]))*Sech[e + f*x]^2*Tanh[e + f*x])/(24
*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 3.15, size = 318, normalized size = 1.54

method	result
default	$\frac{\left(2\sqrt{-\frac{b}{a}}\,ab-\sqrt{-\frac{b}{a}}\,b^{2}\right)\left(\cosh^{4}(fx+e)\right)\sinh(fx+e)+\left(2\sqrt{-\frac{b}{a}}\,a^{2}-2\sqrt{-\frac{b}{a}}\,ab\right)\left(\cosh^{2}(fx+e)\right)\sinh(fx+e)+\sqrt{\frac{b(\cosh^{2}(}{a}}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((2*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(2
*(-1/a*b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(a*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))-2*a*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+b*EllipticE(sin
h(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2+((-1/a*b)^(1/2)*a^2-2*(
-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*sinh(f*x+e))/cosh(f*x+e)^3/(a-b)/(-1/
a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^4, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2257 vs. $2(218) = 436$.
time = 0.12, size = 2257, normalized size = 10.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] -1/3*(((4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^6 + 6*(4*a^2 - 4*a*b + b^2)*cosh
(f*x + e)*sinh(f*x + e)^5 + (4*a^2 - 4*a*b + b^2)*sinh(f*x + e)^6 + 3*(4*a^
2 - 4*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)
^2 + 4*a^2 - 4*a*b + b^2)*sinh(f*x + e)^4 + 4*(5*(4*a^2 - 4*a*b + b^2)*cosh
(f*x + e)^3 + 3*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(4
*a^2 - 4*a*b + b^2)*cosh(f*x + e)^2 + 3*(5*(4*a^2 - 4*a*b + b^2)*cosh(f*x +
 e)^4 + 6*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^2 + 4*a^2 - 4*a*b + b^2)*sinh
(f*x + e)^2 + 4*a^2 - 4*a*b + b^2 + 6*((4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^
5 + 2*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^3 + (4*a^2 - 4*a*b + b^2)*cosh(f*
x + e))*sinh(f*x + e) - 2*((2*a*b - b^2)*cosh(f*x + e)^6 + 6*(2*a*b - b^2)*
cosh(f*x + e)*sinh(f*x + e)^5 + (2*a*b - b^2)*sinh(f*x + e)^6 + 3*(2*a*b -
b^2)*cosh(f*x + e)^4 + 3*(5*(2*a*b - b^2)*cosh(f*x + e)^2 + 2*a*b - b^2)*si
nh(f*x + e)^4 + 4*(5*(2*a*b - b^2)*cosh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f
*x + e))*sinh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f*x + e)^2 + 3*(5*(2*a*b -
b^2)*cosh(f*x + e)^4 + 6*(2*a*b - b^2)*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(
f*x + e)^2 + 2*a*b - b^2 + 6*((2*a*b - b^2)*cosh(f*x + e)^5 + 2*(2*a*b - b^
2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2
- a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic
_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + si
nh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2)
)/b^2) - 2*((2*a^2 - a*b)*cosh(f*x + e)^6 + 6*(2*a^2 - a*b)*cosh(f*x + e)*s
inh(f*x + e)^5 + (2*a^2 - a*b)*sinh(f*x + e)^6 + 3*(2*a^2 - a*b)*cosh(f*x +
 e)^4 + 3*(5*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f*x + e)^4 +
 4*(5*(2*a^2 - a*b)*cosh(f*x + e)^3 + 3*(2*a^2 - a*b)*cosh(f*x + e))*sinh(f
*x + e)^3 + 3*(2*a^2 - a*b)*cosh(f*x + e)^2 + 3*(5*(2*a^2 - a*b)*cosh(f*x +
 e)^4 + 6*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f*x + e)^2 + 2*
a^2 - a*b + 6*((2*a^2 - a*b)*cosh(f*x + e)^5 + 2*(2*a^2 - a*b)*cosh(f*x + e
)^3 + (2*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b - b^2)*cosh(f*x
+ e)^6 + 6*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b - b^2)*sinh(f*x
 + e)^6 + 3*(a*b - b^2)*cosh(f*x + e)^4 + 3*(5*(a*b - b^2)*cosh(f*x + e)^2
+ a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a*b - b^2)*cosh(f*x + e)^3 + 3*(a*b -
b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(a*b - b^2)*cosh(f*x + e)^2 + 3*(5*
(a*b - b^2)*cosh(f*x + e)^4 + 6*(a*b - b^2)*cosh(f*x + e)^2 + a*b - b^2)*si
nh(f*x + e)^2 + a*b - b^2 + 6*((a*b - b^2)*cosh(f*x + e)^5 + 2*(a*b - b^2)*
cosh(f*x + e)^3 + (a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b
)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(ar
```

```
csin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*
x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2
) - sqrt(2)*((2*a*b - b^2)*cosh(f*x + e)^5 + 5*(2*a*b - b^2)*cosh(f*x + e)*
sinh(f*x + e)^4 + (2*a*b - b^2)*sinh(f*x + e)^5 + 2*(3*a*b - 2*b^2)*cosh(f*
x + e)^3 + 2*(5*(2*a*b - b^2)*cosh(f*x + e)^2 + 3*a*b - 2*b^2)*sinh(f*x + e
)^3 + b^2*cosh(f*x + e) + 2*(5*(2*a*b - b^2)*cosh(f*x + e)^3 + 3*(3*a*b - 2
*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (5*(2*a*b - b^2)*cosh(f*x + e)^4 + 6
*(3*a*b - 2*b^2)*cosh(f*x + e)^2 + b^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)))/((a*b - b^2)*f*cosh(f*x + e)^6 + 6*(a*b - b^2
)*f*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b - b^2)*f*sinh(f*x + e)^6 + 3*(a*b
- b^2)*f*cosh(f*x + e)^4 + 3*(5*(a*b - b^2)*f*cosh(f*x + e)^2 + (a*b - b^2)
*f)*sinh(f*x + e)^4 + 3*(a*b - b^2)*f*cosh(f*x + e)^2 + 4*(5*(a*b - b^2)*f*
cosh(f*x + e)^3 + 3*(a*b - b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(5*(a*
b - b^2)*f*cosh(f*x + e)^4 + 6*(a*b - b^2)*f*cosh(f*x + e)^2 + (a*b - b^2)*
f)*sinh(f*x + e)^2 + (a*b - b^2)*f + 6*((a*b - b^2)*f*cosh(f*x + e)^5 + 2*(
a*b - b^2)*f*cosh(f*x + e)^3 + (a*b - b^2)*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^2{(e + fx)}}\,\operatorname{sech}^4{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**4, x)
```

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}}{{\cosh\left(e+f\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^4,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^4, x)
```

3.362 $\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=157

$$-\frac{a^2(a-6b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16b^{3/2}f}-\frac{a(a-6b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{16bf}-(a-6b)\,\mathrm{s}$$

```
[Out] -1/16*a^2*(a-6*b)*arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/b^
(3/2)/f-1/24*(a-6*b)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/b/f+1/6*sinh(f*x
+e)*(a+b*sinh(f*x+e)^2)^(5/2)/b/f-1/16*a*(a-6*b)*sinh(f*x+e)*(a+b*sinh(f*x+
e)^2)^(1/2)/b/f
```

Rubi [A]

time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 396, 201, 223, 212}

$$-\frac{a^2(a-6b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6bf}-\frac{(a-6b)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24bf}-\frac{a(a-6b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{16bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/16*(a^2*(a - 6*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*
x]^2]])/(b^(3/2)*f) - (a*(a - 6*b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2
])/(16*b*f) - ((a - 6*b)*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*b
*f) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(5/2))/(6*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int (1+x^2)\left(a+bx^2\right)^{3/2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6bf} - \frac{(a-6b)\text{Subst}\Big(\int (a}{}$$

$$= -\frac{(a-6b)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24bf} + \frac{\sinh(e+}{}$$

$$= -\frac{a(a-6b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{16bf} - \frac{(a-6b)\,\mathrm{si}}{}$$

$$= -\frac{a(a-6b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{16bf} - \frac{(a-6b)\,\mathrm{si}}{}$$

$$= -\frac{a^2(a-6b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{a(a-6b)}{}$$

Mathematica [**A**]

time = 0.88, size = 149, normalized size = 0.95

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(-3a^{3/2}(a-6b)\sinh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)+\sqrt{b}\,\sinh(e+fx)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,(3a(a+10b)+2b(7a+6b)\sinh^2(e+fx)+8b^2\sinh^4(e+fx))\right)}{48b^{3/2}f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sinh[e + f*x]^2]*(-3*a^(3/2)*(a - 6*b)*ArcSinh[(Sqrt[b]*Sinh[e
+ f*x])/Sqrt[a]] + Sqrt[b]*Sinh[e + f*x]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]*(3
*a*(a + 10*b) + 2*b*(7*a + 6*b)*Sinh[e + f*x]^2 + 8*b^2*Sinh[e + f*x]^4)))/
(48*b^(3/2)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.29, size = 77, normalized size = 0.49

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{b^2\left(\sinh^6(fx+e)\right)+\left(2ab+b^2\right)\left(\sinh^4(fx+e)\right)+\left(a^2+2ab\right)\left(\sinh^2(fx+e)\right)+a^2}{\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^6+(2*a*b+b^2)*sinh(f*x+e)^4+(a^2+2*a*b)*sinh(
f*x+e)^2+a^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1846 vs. 2(137) = 274.
time = 0.50, size = 4603, normalized size = 29.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [-1/384*(6*((a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 6*(a^3 - 6*a^2*b)*cosh(f*x +
e)^5*sinh(f*x + e) + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 2
0*(a^3 - 6*a^2*b)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 - 6*a^2*b)*cosh
(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)*sinh(f*x + e)
^5 + (a^3 - 6*a^2*b)*sinh(f*x + e)^6)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)
*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7
+ (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*
b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a
*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3
)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh
(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2
*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^
2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*
a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*
x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^
3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cos
h(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b
^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^
2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*co
sh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2
- 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 -
 a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e
)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2
)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a
*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^
2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 -
(4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)
^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f
*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7
+ 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2
 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))
/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*si
nh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sin
h(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 6*((a^
3 - 6*a^2*b)*cosh(f*x + e)^6 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)^5*sinh(f*x +
 e) + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 - 6*a^2*
b)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^2*sin
h(f*x + e)^4 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 - 6*a
^2*b)*sinh(f*x + e)^6)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x
+ e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh
(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + s
```

```
inh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) +
b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - s
qrt(2)*(b^3*cosh(f*x + e)^10 + 10*b^3*cosh(f*x + e)*sinh(f*x + e)^9 + b^3*s
inh(f*x + e)^10 + (7*a*b^2 + b^3)*cosh(f*x + e)^8 + (45*b^3*cosh(f*x + e)^2
 + 7*a*b^2 + b^3)*sinh(f*x + e)^8 + 8*(15*b^3*cosh(f*x + e)^3 + (7*a*b^2 +
b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x
 + e)^6 + (210*b^3*cosh(f*x + e)^4 + 6*a^2*b + 39*a*b^2 - 8*b^3 + 28*(7*a*b
^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 2*(126*b^3*cosh(f*x + e)^5 + 2
8*(7*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x
 + e))*sinh(f*x + e)^5 - (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^4 + (21
0*b^3*cosh(f*x + e)^6 + 70*(7*a*b^2 + b^3)*cosh(f*x + e)^4 - 6*a^2*b - 39*a
*b^2 + 8*b^3 + 15*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^2)*sinh(f*x +
e)^4 + 4*(30*b^3*cosh(f*x + e)^7 + 14*(7*a*b^2 + b^3)*cosh(f*x + e)^5 + 5*(
6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^3 - (6*a^2*b + 39*a*b^2 - 8*b^3)*
cosh(f*x + e))*sinh(f*x + e)^3 - b^3 - (7*a*b^2 + b^3)*cosh(f*x + e)^2 + (4
5*b^3*cosh(f*x + e)^8 + 28*(7*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(6*a^2*b +
39*a*b^2 - 8*b^3)*cosh(f*x + e)^4 - 7*a*b^2 - b^3 - 6*(6*a^2*b + 39*a*b^2 -
 8*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*b^3*cosh(f*x + e)^9 + 4*(7*
a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)
^5 - 2*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^3 - (7*a*b^2 + b^3)*cosh(
f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/
(b^2*f*cosh(f*x + e)^6 + 6*b^2*f*cosh(f*x + e)^5*sinh(f*x + e) + 15*b^2*f*c
osh(f*x + e)^4*sinh(f*x + e)^2 + 20*b^2*f*cosh(...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1546 vs. $2(137) = 274$.
time = 1.03, size = 1546, normalized size = 9.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*(((b*e^(2*f*x + 10*e) + (7*a*b^2*e^(14*e) + b^3*e^(14*e))*e^(-6*e)/b^
2)*e^(2*f*x) + (6*a^2*b*e^(12*e) + 39*a*b^2*e^(12*e) - 8*b^3*e^(12*e))*e^(-
```

```
6*e)/b^2)*sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e
) + b) - 24*(a^3*e^(6*e) - 6*a^2*b*e^(6*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)/sqrt(-b))/(sqrt(-b)*b) + 12*(a^3*sqrt(b)*e^(6*e) - 6*a^2*b^(3/2)*e^(6*e))
*log(abs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
+ 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/2)))/b^2 + 2*(12*
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b))^5*a^3*e^(6*e) + 72*(sqrt(b)*e^(2*f*x + 2*e) - sqrt
(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^2*
b*e^(6*e) - 48*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^2*e^(6*e) + 9*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^5*b^3*e^(6*e) + 48*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4
*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2)*e^(6*e)
 + 24*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*
e) - 2*b*e^(2*f*x + 2*e) + b))^4*a*b^(5/2)*e^(6*e) - 9*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))^4*b^(7/2)*e^(6*e) + 32*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*
e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^3*b*e^(6*e) - 192*
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b))^3*a^2*b^2*e^(6*e) + 132*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3
*a*b^3*e^(6*e) - 22*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^4*e^(6*e) - 108*(sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))^2*a*b^(7/2)*e^(6*e) + 30*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(9/2)*
e^(6*e) - 12*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f
*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^3*b^2*e^(6*e) + 72*(sqrt(b)*e^(2*f*
x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*
e) + b))*a^2*b^3*e^(6*e) - 36*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b^4*e^(6*e) - 3*(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b
*e^(2*f*x + 2*e) + b))*b^5*e^(6*e) + 36*a*b^(9/2)*e^(6*e) - 5*b^(11/2)*e^(6
*e))/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - b)^3*b))*e^(-6*e)/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(e+f\,x)^3\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.363 $\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=104

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{b}\,f}+\frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f}+\frac{\sinh(e+fx)\,(a+b\sinh^2(}{4f}$$

```
[Out] 1/4*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f+3/8*a^2*arctanh(sinh(f*x+e)*b^(
1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f/b^(1/2)+3/8*a*sinh(f*x+e)*(a+b*sinh(f*x+e
)^2)^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 201, 223, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{b}\,f}+\frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*Sqr
t[b]*f) + (3*a*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(8*f) + (Sinh[e +
 f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f} + \frac{(3a)\text{Subst}\Big(\int \sqrt{a+b}}{}$$

$$= \frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f} + \frac{\sinh(e+fx)\,(a+b}{4f}$$

$$= \frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f} + \frac{\sinh(e+fx)\,(a+b}{4f}$$

$$= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{b}\,f} + \frac{3a\sinh(e+fx)\sqrt{a}}{8}$$

Mathematica [**A**]
time = 0.38, size = 93, normalized size = 0.89

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(5a\sinh(e+fx)+2b\sinh^3(e+fx)+\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sinh[e + f*x]^2]*(5*a*Sinh[e + f*x] + 2*b*Sinh[e + f*x]^3 + (3*
a^(3/2)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*Sinh
[e + f*x]^2)/a])))/(8*f)
```

Maple [A]
time = 0.50, size = 86, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{\sinh(fx+e)\left(a+b\left(\sinh^{2}(fx+e)\right)\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{\sinh(fx+e)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}{2}+\frac{a\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b}\right.}{2\sqrt{b}}\right.}{4}}{f}$
default	$\frac{\frac{\sinh(fx+e)\left(a+b\left(\sinh^{2}(fx+e)\right)\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{\sinh(fx+e)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}{2}+\frac{a\ln\left(\sqrt{b}\,\sinh(fx+e)+\sqrt{a+b}\right.}{2\sqrt{b}}\right.}{4}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)+3/4*a*(1/2*sinh(f*x+e)*(a+b*
sinh(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*sinh(f*x+e)+(a+b*sinh(f*x+e)^
2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1126 vs. $2(88) = 176$.
time = 0.46, size = 3161, normalized size = 30.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/64*(6*(a^2*cosh(f*x + e)^4 + 4*a^2*cosh(f*x + e)^3*sinh(f*x + e) + 6*a^2
*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*a^2*cosh(f*x + e)*sinh(f*x + e)^3 + a^
2*sinh(f*x + e)^4)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 +
8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2
+ b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^
6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*
x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 +
 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^
2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(
f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f
*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b
 - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*
b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(
a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a
^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 -
 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*
x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh
(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)
*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*
b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 +
 (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x
+ e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*co
sh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cos
h(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e
)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b
+ 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x
+ e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 +
 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*
cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*co
sh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 6*(a^2*cosh(f*x + e)^4 +
4*a^2*cosh(f*x + e)^3*sinh(f*x + e) + 6*a^2*cosh(f*x + e)^2*sinh(f*x + e)^2
 + 4*a^2*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*sinh(f*x + e)^4)*sqrt(b)*log((
b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2
)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*s
qrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e
)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(2)*(b^2*cosh(f*x + e)^6 + 6*b^2*
cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*x + e)^6 + (10*a*b - 3*b^2)*cosh
(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 10*a*b - 3*b^2)*sinh(f*x + e)^4 + 4
*(5*b^2*cosh(f*x + e)^3 + (10*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 -
 (10*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x + e)^4 + 6*(10*a*b - 3
```

```
*b^2)*cosh(f*x + e)^2 - 10*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*b^2*co
sh(f*x + e)^5 + 2*(10*a*b - 3*b^2)*cosh(f*x + e)^3 - (10*a*b - 3*b^2)*cosh(
f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/
(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b*f*cosh(f*x
 + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh(f*x + e)*sinh(f*x + e)^3 + b*f*sinh(f*
x + e)^4), -1/64*(12*(a^2*cosh(f*x + e)^4 + 4*a^2*cosh(f*x + e)^3*sinh(f*x
+ e) + 6*a^2*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*a^2*cosh(f*x + e)*sinh(f*x
 + e)^3 + a^2*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x +
e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)
*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cos
h(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*si
nh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x +
 e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x +
e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + 12*(a^2*cosh(f*x +
e)^4 + 4*a^2*cosh(f*x + e)^3*sinh(f*x + e) + 6*a^2*cosh(f*x + e)^2*sinh(f*x
 + e)^2 + 4*a^2*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*sinh(f*x + e)^4)*sqrt(-
b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
 b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b
*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. 2(88) = 176.
time = 0.78, size = 791, normalized size = 7.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(24*a^2*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))*e^(4*e)/sqrt(-b) -
12*a^2*e^(4*e)*log(abs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/2))
```

```
)/sqrt(b) + sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b)*(b*e^(2*f*x + 6*e) + (10*a*b*e^(6*e) - 3*b^2*e^(6*e))*e^(-2*e)/b)
+ 4*(10*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*e^(4*e) - 8*(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))
^3*a*b*e^(4*e) + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*
e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^2*e^(4*e) + 8*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))^2*a*b^(3/2)*e^(4*e) - 3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(5/2)*e^(4
*e) - 6*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b*e^(4*e) + 4*(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))
*a*b^2*e^(4*e) - 4*a*b^(5/2)*e^(4*e) + b^(7/2)*e^(4*e))/((sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))^2 - b)^2)*e^(-4*e)/f
```

Mupad [B]
time = 1.02, size = 60, normalized size = 0.58

$$\frac{\sinh(e+f\,x)\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\sinh(e+f\,x)^2}{a}\right)}{f\,\left(\frac{b\sinh(e+f\,x)^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] (sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(
b*sinh(e + f*x)^2)/a))/(f*((b*sinh(e + f*x)^2)/a + 1)^(3/2))
```

3.364 $\int \mathbf{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=125

$$\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}+b\sinh(e+$$

```
[Out] (a-b)^(3/2)*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f+1/2
*(3*a-2*b)*arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))*b^(1/2)/f
+1/2*b*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3269, 427, 537, 223, 212, 385, 209}

$$\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^
2]])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Si
nh[e + f*x]^2]])/(2*f) + (b*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f
)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a(2a-b)+(3a-}{(1+x^2)\sqrt{a-}}\right.}{} \\
&= \frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(a-b)^2\text{Subst}\left(\int \frac{}{(1+}\right.}{} \\
&= \frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(a-b)^2\text{Subst}\left(\int \frac{}{1-}\right.}{} \\
&= \frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f} + \frac{(3a-2b)\sqrt{b}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.35, size = 142, normalized size = 1.14

$$
\frac{4(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{2a-2b}\sinh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)+2(3a-2b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)+b\sqrt{4a-2b+2b\cosh(2(e+fx))}\sinh(e+fx)}{4f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*(a - b)^(3/2)*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Co
sh[2*(e + f*x)]]] + 2*(3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e +
 f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + b*Sqrt[4*a - 2*b + 2*b*Cosh[2
*(e + f*x)]]*Sinh[e + f*x])/(4*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.95, size = 63, normalized size = 0.50

method	result	size
default	$\frac{\text{`}int/indef0\text{`}\left(\frac{b^2\left(\sinh^4(fx+e)\right)+2ab\left(\sinh^2(fx+e)\right)+a^2}{\cosh(fx+e)^2\sqrt{a+b\left(\sinh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^2/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(107) = 214$.
time = 0.57, size = 6337, normalized size = 50.70

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x
+ e) + (3*a - 2*b)*sinh(f*x + e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*c
osh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 +
(a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^
3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b
^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*
cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f
*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a
*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*
b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*
b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x
+ e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3
+ 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(
f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2
 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2
- sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh
(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 -
2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a
```

```
^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^
3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*
cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b
 + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2
- 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4
*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 +
3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 +
 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(
cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh
(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(
f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 4*((a -
b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f
*x + e)^2)*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f
*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*
x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*
sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
- 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a
- 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b
)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*
(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x +
 e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + ((3*a - 2*b)*cosh(f*x + e)^2 +
 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*sinh(f*x + e)^2)*s
qrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(
f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e
)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*
cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(2)*(b*cosh(f*x + e)^
2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f
*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)
*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*
sinh(f*x + e) + f*sinh(f*x + e)^2), 1/8*(8*((a - b)*cosh(f*x + e)^2 + 2*(a
- b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2)*sqrt(a - b)*arc
tan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*co
sh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(
2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^
2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - (
(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (
3*a - 2*b)*sinh(f*x + e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x
```

```
+ e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b -
 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(
f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3
)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sinh^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}} \operatorname{sech}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(3/2)*sech(e + f*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}}{\mathrm{cosh}\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x), x)
```

3.365 $\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx$

Optimal. Leaf size=133

$$\frac{\sqrt{a-b}\,(a+2b)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+(a-b)\operatorname{sech}($$

```
[Out] b^(3/2)*arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f+1/2*(a+2*b
)*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f+1
/2*(a-b)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]

time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3269, 424, 537, 223, 212, 385, 209}

$$\frac{\sqrt{a-b}\,(a+2b)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{(a-b)\tanh(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a - b]*(a + 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e
 + f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Si
nh[e + f*x]^2]])/f + ((a - b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tan
h[e + f*x])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{(1+x^2)^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2f} +$$

$$= \frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2f} +$$

$$= \frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2f} +$$

$$= \frac{\sqrt{a-b}\,(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f} + b^{3/2}\,\mathrm{ta}$$

Mathematica [A]
time = 0.78, size = 150, normalized size = 1.13

$$\frac{2\sqrt{a-b}\,(a+2b)\operatorname{ArcTan}\left(\frac{\sqrt{2a-2b}\,\sinh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)+4b^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sinh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)+(a-b)\sqrt{4a-2b+2b\cosh(2(e+fx))}\operatorname{sech}(e+fx)\tanh(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*Sqrt[a - b]*(a + 2*b)*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a -
b + b*Cosh[2*(e + f*x)]]] + 4*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e + f*x
])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + (a - b)*Sqrt[4*a - 2*b + 2*b*Cosh
[2*(e + f*x)]]*Sech[e + f*x]*Tanh[e + f*x])/(4*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.14, size = 63, normalized size = 0.47

method	result	size
default	$\frac{`\mathit{int/indef0}`\left(\frac{b^2\left(\sinh^4(fx+e)\right)+2ab\left(\sinh^2(fx+e)\right)+a^2}{\cosh(fx+e)^4\sqrt{a+b\left(\sinh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^4/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1325 vs. 2(115) = 230.
time = 0.79, size = 7350, normalized size = 55.26

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
 e)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 +
 4*(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log(-((
a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x
 + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 -
4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2
*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*
(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^
3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x +
 e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 +
6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)
^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*
a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)
)*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*
b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*c
osh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x
+ e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*
(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sin
```

```
h(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b +
b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a
*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x +
 e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x
+ e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x +
 e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b
^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 +
 b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5
 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*
x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) +
 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 +
15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh
(f*x + e)^6)) + ((a + 2*b)*cosh(f*x + e)^4 + 4*(a + 2*b)*cosh(f*x + e)*sinh
(f*x + e)^3 + (a + 2*b)*sinh(f*x + e)^4 + 2*(a + 2*b)*cosh(f*x + e)^2 + 2*(
3*(a + 2*b)*cosh(f*x + e)^2 + a + 2*b)*sinh(f*x + e)^2 + 4*((a + 2*b)*cosh(
f*x + e)^3 + (a + 2*b)*cosh(f*x + e))*sinh(f*x + e) + a + 2*b)*sqrt(-a + b)
*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3
 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*
b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^
2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3
- (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*
cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)
*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*
sinh(f*x + e) + 1)) + (b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^
3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*s
inh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*sinh(f*x + e) + b)
*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sin
h(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x +
 e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(
b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 +
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 2*sqrt(2)*((a - b)*cosh
(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)
^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x +
 e)^4 + 4*f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh(f*x + e)^4 + 2*f*cosh(f*
x + e)^2 + 2*(3*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)
^3 + f*cosh(f*x + e))*sinh(f*x + e) + f), 1/4*(2*((a + 2*b)*cosh(f*x + e)^4
 + 4*(a + 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + 2*b)*sinh(f*x + e)^4 +
2*(a + 2*b)*cosh(f*x + e)^2 + 2*(3*(a + 2*b)*cosh(f*x + e)^2 + a + 2*b)*sin
```

```
h(f*x + e)^2 + 4*((a + 2*b)*cosh(f*x + e)^3 + (a + 2*b)*cosh(f*x + e))*sinh
(f*x + e) + a + 2*b)*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x ...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}}{{\mathrm{cosh}\left(e+f\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^3,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^3, x)
```

3.366 $\int \text{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$\frac{3a^2\text{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{a-b}\,f}+\frac{3a\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{8f}+\text{sech}^3(e+f$$

```
[Out] 3/8*a^2*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f/(a-b)^(
1/2)+1/4*sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)/f+3/8*a*sech(f
*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3269, 386, 385, 209}

$$\frac{3a^2\text{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8f\sqrt{a-b}}+\frac{\tanh(e+fx)\text{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f}+\frac{3a\tanh(e+fx)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a^2*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*
Sqrt[a - b]*f) + (3*a*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*
x])/(8*f) + (Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(
4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
```

```
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^3}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{4f} + \frac{(3a)\mathrm{S}}{}$$

$$= \frac{3a\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{8f} + \frac{\operatorname{sech}^3(}{}$$

$$= \frac{3a\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{8f} + \frac{\operatorname{sech}^3(}{}$$

$$= \frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{a-b}\,f} + \frac{3a\operatorname{sech}(e+fx)\sqrt{a-}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 66, normalized size = 0.52

$$\frac{a^2\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};-\frac{(a-b)\sinh^2(e+fx)}{a+b\sinh^2(e+fx)}\right)\sinh(e+fx)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*Hypergeometric2F1[1/2, 3, 3/2, -(((a - b)*Sinh[e + f*x]^2)/(a + b*Sinh
[e + f*x]^2))]*Sinh[e + f*x])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.37, size = 63, normalized size = 0.50

method	result	size
default	$\frac{\text{`}int/indef0\text{`}\left(\frac{b^2\left(\sinh^4(fx+e)\right)+2ab\left(\sinh^2(fx+e)\right)+a^2}{\cosh(fx+e)^6\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^6/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1486 vs. 2(110) = 220.
time = 0.55, size = 3089, normalized size = 24.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(a^2*cosh(f*x + e)^8 + 8*a^2*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*
sinh(f*x + e)^8 + 4*a^2*cosh(f*x + e)^6 + 4*(7*a^2*cosh(f*x + e)^2 + a^2)*s
inh(f*x + e)^6 + 6*a^2*cosh(f*x + e)^4 + 8*(7*a^2*cosh(f*x + e)^3 + 3*a^2*c
osh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a^2*cosh(f*x + e)^4 + 30*a^2*cosh(f*x
 + e)^2 + 3*a^2)*sinh(f*x + e)^4 + 4*a^2*cosh(f*x + e)^2 + 8*(7*a^2*cosh(f*
x + e)^5 + 10*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 +
4*(7*a^2*cosh(f*x + e)^6 + 15*a^2*cosh(f*x + e)^4 + 9*a^2*cosh(f*x + e)^2 +
```

```
 a^2)*sinh(f*x + e)^2 + a^2 + 8*(a^2*cosh(f*x + e)^7 + 3*a^2*cosh(f*x + e)^
5 + 3*a^2*cosh(f*x + e)^3 + a^2*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*
log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3
+ (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b
)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2
 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((
b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(
f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 -
 (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*c
osh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*
sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*s
inh(f*x + e) + 1)) - 2*sqrt(2)*((3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^6 + 6*(
3*a^2 - a*b - 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - a*b - 2*b^2)*
sinh(f*x + e)^6 + (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e)^4 + (15*(3*a^2 -
a*b - 2*b^2)*cosh(f*x + e)^2 + 11*a^2 - 17*a*b + 6*b^2)*sinh(f*x + e)^4 + 4
*(5*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^3 + (11*a^2 - 17*a*b + 6*b^2)*cosh(
f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e)^2 + (15
*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^4 + 6*(11*a^2 - 17*a*b + 6*b^2)*cosh(f
*x + e)^2 - 11*a^2 + 17*a*b - 6*b^2)*sinh(f*x + e)^2 - 3*a^2 + a*b + 2*b^2
+ 2*(3*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^5 + 2*(11*a^2 - 17*a*b + 6*b^2)*
cosh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e))*sinh(f*x + e))*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x + e)^8
 + 8*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a - b)*f*sinh(f*x + e)^8 +
4*(a - b)*f*cosh(f*x + e)^6 + 4*(7*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*s
inh(f*x + e)^6 + 6*(a - b)*f*cosh(f*x + e)^4 + 8*(7*(a - b)*f*cosh(f*x + e)
^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a - b)*f*cosh(f*x
+ e)^4 + 30*(a - b)*f*cosh(f*x + e)^2 + 3*(a - b)*f)*sinh(f*x + e)^4 + 4*(a
 - b)*f*cosh(f*x + e)^2 + 8*(7*(a - b)*f*cosh(f*x + e)^5 + 10*(a - b)*f*cos
h(f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a - b)*f*
cosh(f*x + e)^6 + 15*(a - b)*f*cosh(f*x + e)^4 + 9*(a - b)*f*cosh(f*x + e)^
2 + (a - b)*f)*sinh(f*x + e)^2 + (a - b)*f + 8*((a - b)*f*cosh(f*x + e)^7 +
 3*(a - b)*f*cosh(f*x + e)^5 + 3*(a - b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh
(f*x + e))*sinh(f*x + e)), 1/8*(3*(a^2*cosh(f*x + e)^8 + 8*a^2*cosh(f*x + e
)*sinh(f*x + e)^7 + a^2*sinh(f*x + e)^8 + 4*a^2*cosh(f*x + e)^6 + 4*(7*a^2*
cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^6 + 6*a^2*cosh(f*x + e)^4 + 8*(7*a^2*c
osh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a^2*cosh(f*x
+ e)^4 + 30*a^2*cosh(f*x + e)^2 + 3*a^2)*sinh(f*x + e)^4 + 4*a^2*cosh(f*x +
 e)^2 + 8*(7*a^2*cosh(f*x + e)^5 + 10*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x
+ e))*sinh(f*x + e)^3 + 4*(7*a^2*cosh(f*x + e)^6 + 15*a^2*cosh(f*x + e)^4 +
 9*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^2 + a^2 + 8*(a^2*cosh(f*x + e)^
7 + 3*a^2*cosh(f*x + e)^5 + 3*a^2*cosh(f*x + e)^3 + a^2*cosh(f*x + e))*sinh
(f*x + e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
```

```
 + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3
 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2
 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e
))*sinh(f*x + e) + b)) + sqrt(2)*((3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^6 + 6
*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - a*b - 2*b^2
)*sinh(f*x + e)^6 + (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e)^4 + (15*(3*a^2
- a*b - 2*b^2)*cosh(f*x + e)^2 + 11*a^2 - 17*a*b + 6*b^2)*sinh(f*x + e)^4 +
 4*(5*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^3 + (11*a^2 - 17*a*b + 6*b^2)*cos
h(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e)^2 + (
15*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^4 + 6*(11*a^2 - 17*a*b + 6*b^2)*cosh
(f*x + e)^2 - 11*a^2 + 17*a*b - 6*b^2)*sinh(f*x + e)^2 - 3*a^2 + a*b + 2*b^
2 + 2*(3*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^5 + 2*(11*a^2 - 17*a*b + 6*b^2
)*cosh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e))*sinh(f*x + e))
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2017 vs. $2(110) = 220$.
time = 0.90, size = 2017, normalized size = 16.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*a^2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/sqrt
(a - b) - 2*(3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^2 - 8*(sqrt(b)*e^(2*f*x + 2*e)
- sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^
7*b^2 + 21*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
 + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a^2*sqrt(b) - 64*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))^6*a*b^(3/2) + 8*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*b^(5/2) + 44*(sqrt(b)*e^(2*
f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
```

```
2*e) + b))^5*a^3 - 237*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^2*b + 96*(sqrt(b)*e^(2*
f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
2*e) + b))^5*a*b^2 - 8*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*b^3 - 292*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^4*a^3*sqrt(b) + 141*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2) - 96*(
sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*
b*e^(2*f*x + 2*e) + b))^4*a*b^(5/2) + 72*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*b^(7/2)
 - 176*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^4 - 232*(sqrt(b)*e^(2*f*x + 2*e) - sqrt
(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^3*
b + 129*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*b^2 + 192*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3
*a*b^3 - 88*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*
x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^4 - 528*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2
*a^4*sqrt(b) + 472*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*
e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^3*b^(3/2) - 9*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))^2*a^2*b^(5/2) - 40*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(9/2) - 192*(sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))*a^5 + 48*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^4*b + 188*(sqrt(b
)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b))*a^3*b^2 + 105*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b^3 - 288*(sqr
t(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e
^(2*f*x + 2*e) + b))*a*b^4 + 104*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^5 - 192*a^5*sqr
t(b) + 400*a^4*b^(3/2) - 180*a^3*b^(5/2) - 153*a^2*b^(7/2) + 160*a*b^(9/2)
- 40*b^(11/2))/((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqr
t(b) + 4*a - 3*b)^4)/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^5,x)
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^5, x)
```

3.367 $\int \operatorname{sech}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=205

$$\frac{a^2(5a-6b)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16(a-b)^{3/2}f}+\frac{a(5a-6b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{16(a-b)f}$$

```
[Out] 1/16*a^2*(5*a-6*b)*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)
)/(a-b)^(3/2)/f+1/24*(5*a-6*b)*sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2)*tanh
(f*x+e)/(a-b)/f+1/6*sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(5/2)*tanh(f*x+e)/(a-
b)/f+1/16*a*(5*a-6*b)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-
b)/f
```

Rubi [A]
time = 0.12, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 390, 386, 385, 209}

$$\frac{a^2(5a-6b)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16f(a-b)^{3/2}}+\frac{\tanh(e+fx)\operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6f(a-b)}+\frac{(5a-6b)\tanh(e+fx)\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24f(a-b)}+\frac{a(5a-6b)\tanh(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{16f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*(5*a - 6*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x
]^2]])/(16*(a - b)^(3/2)*f) + (a*(5*a - 6*b)*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2]*Tanh[e + f*x])/(16*(a - b)*f) + ((5*a - 6*b)*Sech[e + f*x]^3*(a
 + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(24*(a - b)*f) + (Sech[e + f*x]^
5*(a + b*Sinh[e + f*x]^2)^(5/2)*Tanh[e + f*x])/(6*(a - b)*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^4}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$=\frac{\operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}\tanh(e+fx)}{6(a-b)f}+\frac{(5a-}{}$$

$$=\frac{(5a-6b)\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx}{24(a-b)f}$$

$$=\frac{a(5a-6b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{16(a-b)f}$$

$$=\frac{a(5a-6b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{16(a-b)f}$$

$$=\frac{a^2(5a-6b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16(a-b)^{3/2}f}+\frac{a(5a-6b)\mathrm{se}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 11.38, size = 959, normalized size = 4.68

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)^2*Tanh[e + f*x]*(45*a*ArcS
in[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]] + 30*b*ArcSin[Sqrt[((a - b)*Tanh[e +
f*x]^2)/a]]*Sinh[e + f*x]^2 + 210*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f
*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) + 140*b*Sinh[e + f*x]^2*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)
^(3/2) - 120*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*
Tanh[e + f*x]^2)/a)^(5/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tan
h[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b
)*Tanh[e + f*x]^2)/a)^(5/2) - 80*b*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a
 + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(5/2) + 256*b*Hyper
```

```
geometric2F1[2, 5, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(
5/2) - 512*a*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)
^(7/2) - 512*b*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Si
nh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*T
anh[e + f*x]^2)/a)^(7/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tanh
[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)
*Tanh[e + f*x]^2)/a)^(9/2) + 256*b*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Ta
nh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(9/2) - 45*a*Sqrt[((a - b)*Sech[e +
f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2] - 30*b*Sinh[e + f*x]^2
*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2
]))/(240*f*(a + b*Sinh[e + f*x]^2)^(3/2)*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[
e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.21, size = 63, normalized size = 0.31

$$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{b^{2}\left(\sinh^{4}\left(fx+e\right)\right)+2ab\left(\sinh^{2}\left(fx+e\right)\right)+a^{2}}{\cosh\left(fx+e\right)^{8}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^8/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^7, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3758 vs. 2(185) = 370.
time = 0.95, size = 7633, normalized size = 37.23

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*((5*a^3 - 6*a^2*b)*cosh(f*x + e)^12 + 12*(5*a^3 - 6*a^2*b)*cosh(f
*x + e)*sinh(f*x + e)^11 + (5*a^3 - 6*a^2*b)*sinh(f*x + e)^12 + 6*(5*a^3 -
6*a^2*b)*cosh(f*x + e)^10 + 6*(5*a^3 - 6*a^2*b + 11*(5*a^3 - 6*a^2*b)*cosh(
f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 3
*(5*a^3 - 6*a^2*b)*cosh(f*x + e))*sinh(f*x + e)^9 + 15*(5*a^3 - 6*a^2*b)*co
sh(f*x + e)^8 + 15*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b
+ 18*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 24*(33*(5*a^3 - 6
*a^2*b)*cosh(f*x + e)^5 + 30*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 5*(5*a^3 -
 6*a^2*b)*cosh(f*x + e))*sinh(f*x + e)^7 + 20*(5*a^3 - 6*a^2*b)*cosh(f*x +
e)^6 + 4*(231*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 315*(5*a^3 - 6*a^2*b)*cos
h(f*x + e)^4 + 25*a^3 - 30*a^2*b + 105*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^2)*s
inh(f*x + e)^6 + 24*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 63*(5*a^3 - 6*a
^2*b)*cosh(f*x + e)^5 + 35*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 5*(5*a^3 - 6
*a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(5*a^3 - 6*a^2*b)*cosh(f*x + e)
^4 + 15*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^8 + 84*(5*a^3 - 6*a^2*b)*cosh(f
*x + e)^6 + 70*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 20*(5*
a^3 - 6*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 20*(11*(5*a^3 - 6*a^2*b)*
cosh(f*x + e)^9 + 36*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 42*(5*a^3 - 6*a^2*
b)*cosh(f*x + e)^5 + 20*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 3*(5*a^3 - 6*a^
2*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 5*a^3 - 6*a^2*b + 6*(5*a^3 - 6*a^2*b)
*cosh(f*x + e)^2 + 6*(11*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^10 + 45*(5*a^3 - 6
*a^2*b)*cosh(f*x + e)^8 + 70*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 50*(5*a^3
- 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 15*(5*a^3 - 6*a^2*b)*cosh(f*
x + e)^2)*sinh(f*x + e)^2 + 12*((5*a^3 - 6*a^2*b)*cosh(f*x + e)^11 + 5*(5*a
^3 - 6*a^2*b)*cosh(f*x + e)^9 + 10*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 10*(
5*a^3 - 6*a^2*b)*cosh(f*x + e)^5 + 5*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + (5
*a^3 - 6*a^2*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a - 2*b)*c
osh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh
(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^
2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cos
h(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sin
h(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2
+ 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1
)) - 2*sqrt(2)*((15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^10 + 10
*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)*sinh(f*x + e)^9 + (15*
a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*sinh(f*x + e)^10 + (85*a^3 - 133*a^2*b +
20*a*b^2 + 28*b^3)*cosh(f*x + e)^8 + (85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^
3 + 45*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)
^8 + 8*(15*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^3 + (85*a^3
- 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(99*a^3
```

```
 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^6 + 2*(105*(15*a^3 - 23*a^
2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^4 + 99*a^3 - 247*a^2*b + 200*a*b^2 - 5
2*b^3 + 14*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^2)*sinh(f
*x + e)^6 + 4*(63*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^5 + 1
4*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^3 + 3*(99*a^3 - 24
7*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(99*a^3 -
247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^4 + 2*(105*(15*a^3 - 23*a^2*b
 + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^6 + 35*(85*a^3 - 133*a^2*b + 20*a*b^2 + 2
8*b^3)*cosh(f*x + e)^4 - 99*a^3 + 247*a^2*b - 200*a*b^2 + 52*b^3 + 15*(99*a
^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(
15*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^7 + 7*(85*a^3 - 133*
a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^5 + 5*(99*a^3 - 247*a^2*b + 200*a*
b^2 - 52*b^3)*cosh(f*x + e)^3 - (99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*c
osh(f*x + e))*sinh(f*x + e)^3 - 15*a^3 + 23*a^2*b - 4*a*b^2 - 4*b^3 - (85*a
^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^2 + (45*(15*a^3 - 23*a^2*
b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^8 + 28*(85*a^3 - 133*a^2*b + 20*a*b^2 +
28*b^3)*cosh(f*x + e)^6 + 30*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh
(f*x + e)^4 - 85*a^3 + 133*a^2*b - 20*a*b^2 - 28*b^3 - 12*(99*a^3 - 247*a^2
*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(15*a^3 -
23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^9 + 4*(85*a^3 - 133*a^2*b + 20*a*
b^2 + 28*b^3)*cosh(f*x + e)^7 + 6*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)
*cosh(f*x + e)^5 - 4*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e
)^3 - (85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e))*sinh(f*x + e)
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**7*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4555 vs. 2(185) = 370.
time = 1.73, size = 4555, normalized size = 22.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(3*(5*a^3 - 6*a^2*b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sq
```

```
rt(a - b))/(a - b)^(3/2) - 2*(15*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^11*a^3 - 18*(sqrt
(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^
(2*f*x + 2*e) + b))^11*a^2*b + 165*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f
*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^10*a^3*sqrt(b)
 - 198*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*
e) - 2*b*e^(2*f*x + 2*e) + b))^10*a^2*b^(3/2) - 192*(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)^10*a*b^(5/2) + 192*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^10*b^(7/2) + 340*(sqrt(b)*e^(
2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b))^9*a^4 + 77*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^9*a^3*b - 2886*(sqrt(b)*e^
(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b))^9*a^2*b^2 + 2944*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^9*a*b^3 - 640*(sqrt
(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^
(2*f*x + 2*e) + b))^9*b^4 + 3060*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^8*a^4*sqrt(b) - 1
6545*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^8*a^3*b^(3/2) + 19902*(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)^8*a^2*b^(5/2) - 7872*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^8*a*b^(7/2) + 960*(sqrt(b)*
e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f
*x + 2*e) + b))^8*b^(9/2) + 3168*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^5 - 32304*(sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))^7*a^4*b + 48390*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^3*b^2 -
26388*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*
e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^2*b^3 + 7680*(sqrt(b)*e^(2*f*x + 2*e) -
sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*
a*b^4 - 1536*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f
*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*b^5 - 26976*(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)^6*a^5*sqrt(b) + 28048*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a^4*b^(3/2) + 8642*(sqrt
(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^
(2*f*x + 2*e) + b))^6*a^3*b^(5/2) - 11100*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b
*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a^2*b^
(7/2) - 1408*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f
*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a*b^(9/2) + 1408*(sqrt(b)*e^(2*f*x
 + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))^6*b^(11/2) - 12672*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
```

```
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^6 - 37536*(sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))^5*a^5*b + 122328*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^4*b^2 - 104502
*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
2*b*e^(2*f*x + 2*e) + b))^5*a^3*b^3 + 38676*(sqrt(b)*e^(2*f*x + 2*e) - sqrt
(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^2*
b^4 - 9984*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
 + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^5 + 2304*(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))
^5*b^6 - 63360*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^6*sqrt(b) + 131232*(sqrt(b)*e^(
2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
+ 2*e) + b))^4*a^5*b^(3/2) - 78888*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f
*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^4*b^(5/2) +
 16974*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^3*b^(7/2) - 23844*(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b))^4*a^2*b^(9/2) + 22656*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + ...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^7,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^7, x)
```

3.368 $\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=357

$$\frac{(a^2+9ab-2b^2)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf}+\frac{2(4a-b)\cosh^3(e+fx)\sinh(e+fx)\sqrt{a}}{35f}$$

```
[Out] 1/35*(a^2+9*a*b-2*b^2)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/
f+2/35*(4*a-b)*cosh(f*x+e)^3*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/7*b*
cosh(f*x+e)^5*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+2/35*(a+b)*(a^2-6*a*b
+b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*
x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)
^(1/2)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/35*(a^2-18*a*b+b
^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+
e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(
1/2)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/35*(a+b)*(a^2-6*a*b+
b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Rubi [A]
time = 0.28, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3271, 427, 542, 545, 429, 506, 422}

$$-\frac{(a^2-18ab+b^2)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\operatorname{ArcTan}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{35bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2(a+b)\left(a^2-6ab+b^2\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{35b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a+b)\left(a^2-6ab+b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35b^2f}+\frac{(a^2+9ab-2b^2)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf}+\frac{b\sinh(e+fx)\cosh^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{7f}+\frac{2(4a-b)\sinh(e+fx)\cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a^2 + 9*a*b - 2*b^2)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]
^2])/(35*b*f) + (2*(4*a - b)*Cosh[e + f*x]^3*Sinh[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(35*f) + (b*Cosh[e + f*x]^5*Sinh[e + f*x]*Sqrt[a + b*Sinh[e +
f*x]^2])/(7*f) + (2*(a + b)*(a^2 - 6*a*b + b^2)*EllipticE[ArcTan[Sinh[e + f
*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b^2*f*Sqrt[(S
ech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a^2 - 18*a*b + b^2)*Ellipti
cF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2
])/(35*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a + b)*
(a^2 - 6*a*b + b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(35*b^2*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
```

```
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^4(e+fx)\,(a+b\sinh^2(e+fx))^{3/2}\,dx = \frac{(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx))\,\text{Subst}(\int(1+x^2)^{3/2}\,(a+b}{f}$$

$$= \frac{b\cosh^5(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{7f} + \frac{(\sqrt{\mathrm{co}}}{}$$

$$= \frac{2(4a-b)\cosh^3(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35f}$$

$$= \frac{(a^2+9ab-2b^2)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{35bf}$$

$$= \frac{(a^2+9ab-2b^2)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{35bf}$$

$$= \frac{(a^2+9ab-2b^2)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{35bf}$$

$$= \frac{(a^2+9ab-2b^2)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{35bf}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.05, size = 256, normalized size = 0.72

$$\frac{128ia(a^3-5a^2b-5ab^2+b^3)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-64ia(2a^3-11a^2b+8ab^2+b^3)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,b(32a^3+400a^2b-212ab^2+30b^3+b(144a^2+192ab-37b^2)\cosh(2(e+fx))+2b^2(26a+b)\cosh(4(e+fx))+5b^3\cosh(6(e+fx)))\sinh(2(e+fx))}{2240b^2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((128*I)*a*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*
x)])/a]*EllipticE[I*(e + f*x), b/a] - (64*I)*a*(2*a^3 - 11*a^2*b + 8*a*b^2
```

```
+ b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a]
+ Sqrt[2]*b*(32*a^3 + 400*a^2*b - 212*a*b^2 + 30*b^3 + b*(144*a^2 + 192*a*b
 - 37*b^2)*Cosh[2*(e + f*x)] + 2*b^2*(26*a + b)*Cosh[4*(e + f*x)] + 5*b^3*C
osh[6*(e + f*x)])*Sinh[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]])
```

Maple [A]
time = 1.78, size = 730, normalized size = 2.04

method	result
default	$5\sqrt{-\frac{b}{a}}\, b^{3} \left(\cosh^{8}\left(f x+e\right)\right) \sinh\left(f x+e\right)+\left(13\sqrt{-\frac{b}{a}}\, a\, b^{2}-7\sqrt{-\frac{b}{a}}\, b^{3}\right) \left(\cosh^{6}\left(f x+e\right)\right) \sinh\left(f x+e\right)+\left(9\sqrt{-\frac{b}{a}}\, a^{2} b-\sqrt{-\frac{b}{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(5*(-1/a*b)^(1/2)*b^3*cosh(f*x+e)^8*sinh(f*x+e)+(13*(-1/a*b)^(1/2)*a*b
^2-7*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^6*sinh(f*x+e)+(9*(-1/a*b)^(1/2)*a^2*b-
(-1/a*b)^(1/2)*a*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+((-1/a*b)^(1/2)*a^3+8*(-1/a
*b)^(1/2)*a^2*b-11*(-1/a*b)^(1/2)*a*b^2+2*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^2
*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3+8*(b/a*cosh(f*x+e)^2+(a-b)/
a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))*a^2*b-11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+2*(b/a*cosh(f*x+e)^2+(a
-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))*b^3-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ell
ipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3+10*(b/a*cosh(f*x+e)^2+(a
-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))*a^2*b+10*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*
EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2-2*(b/a*cosh(f*x+e)^
2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*b^3)/b/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.10, size = 46, normalized size = 0.13

$$\text{integral}\left(\left(b\cosh\left(fx+e\right)^{4}\sinh\left(fx+e\right)^{2}+a\cosh\left(fx+e\right)^{4}\right)\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(f*x + e)^4*sinh(f*x + e)^2 + a*cosh(f*x + e)^4)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.45Error: Bad Argum
ent Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(e+f\,x)^4\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.369 $\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=299

$$\frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}+\frac{b\cosh^3(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+}{5f}$$

```
[Out] 2/15*(3*a-b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/5*b*cosh
(f*x+e)^3*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/15*(3*a^2+7*a*b-2*b^2)*
(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/15*(9*a-b)*(1/(1+sinh(f*
x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)
^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+
e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/15*(3*a^2+7*a*b-2*b^2)*(a+b*sinh(f*x+e)
^2)^(1/2)*tanh(f*x+e)/b/f
```

Rubi [A]
time = 0.20, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3271, 427, 542, 545, 429, 506, 422}

$$-\frac{(3a^2+7ab-2b^2)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{15bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(3a^2+7ab-2b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf}+\frac{(9a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{15f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{b\sinh(e+fx)\cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{5f}+\frac{2(3a-b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(3*a - b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f
) + (b*Cosh[e + f*x]^3*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(5*f) - (
(3*a^2 + 7*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e +
f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((9*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Se
ch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) + ((3*a^2 + 7*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^
2]*Tanh[e + f*x])/(15*b*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
 :> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
```

```
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\Big(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\Big)\operatorname{Subst}\Big(\int \sqrt{1+x^2}\,(a+b}{f}$$

$$= \frac{b\cosh^3(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{5f} + \Big(\sqrt{}$$

$$= \frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}$$

$$= \frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}$$

$$= \frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}$$

$$= \frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.08, size = 213, normalized size = 0.71

$$\frac{-16ia(3a^2+7ab-2b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+16ia(3a^2-2ab-b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,b(48a^2-28ab+5b^2+4(9a-2b)b\cosh(2(e+fx))+3b^2\cosh(4(e+fx)))\sinh(2(e+fx))}{240bf\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-16*I)*a*(3*a^2 + 7*a*b - 2*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*
EllipticE[I*(e + f*x), b/a] + (16*I)*a*(3*a^2 - 2*a*b - b^2)*Sqrt[(2*a - b
+ b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(48*a^2 -
 28*a*b + 5*b^2 + 4*(9*a - 2*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*x
)])*Sinh[2*(e + f*x)])/(240*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.47, size = 535, normalized size = 1.79

method	result
default	$\frac{3\sqrt{-\frac{b}{a}}\,b^{2}\left(\cosh^{6}(fx+e)\right)\sinh(fx+e)+\left(9\sqrt{-\frac{b}{a}}\,ab-5\sqrt{-\frac{b}{a}}\,b^{2}\right)\left(\cosh^{4}(fx+e)\right)\sinh(fx+e)+\left(6\sqrt{-\frac{b}{a}}\,a^{2}-8\sqrt{-\frac{b}{a}}\,ab+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(3*(-1/a*b)^(1/2)*b^2*cosh(f*x+e)^6*sinh(f*x+e)+(9*(-1/a*b)^(1/2)*a*b-
5*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(6*(-1/a*b)^(1/2)*a^2-8*(-1
/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+6*a^2*(b/a*
cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))-8*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+2*(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b^2+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+7*(b/a*cosh(f
*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)
^(1/2),(a/b)^(1/2))*a*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)
^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/(-1/a*b)^(1/2
)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^2, x)
```

Fricas [F]
time = 0.11, size = 46, normalized size = 0.15

$$\text{integral}\left(\left(b\cosh\left(fx+e\right)^{2}\sinh\left(fx+e\right)^{2}+a\cosh\left(fx+e\right)^{2}\right)\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(f*x + e)^2*sinh(f*x + e)^2 + a*cosh(f*x + e)^2)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(e+f\,x)^2\,{\left(b\sinh(e+f\,x)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.370 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=174

$$\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}+\frac{ia(a-}{}$$

```
[Out] 1/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-2/3*I*(2*a-b)*(co
s(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*
(a+b*sinh(f*x+e)^2)^(1/2)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)+1/3*I*a*(a-b)*(cos(
I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))*(1
+b*sinh(f*x+e)^2/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3259, 3251, 3257, 3256, 3262, 3261}

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\;F\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}\;E\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)
/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b
/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\int\frac{a(3a-b)+2(2a}{\sqrt{a+b\,\mathrm{si}}}$$

$$=\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{1}{3}(a(a-b))\int\frac{}{\sqrt{a+}}$$

$$=\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{\Bigl(2(2a-b)\sqrt{a+b\,\mathrm{si}}}{3}$$

$$=\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)E\bigl(ie+ifx}{3f\sqrt{1+\frac{b}{}}}$$

Mathematica [A]
time = 0.53, size = 169, normalized size = 0.97

$$\frac{-4i\sqrt{2}\,a(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+2i\sqrt{2}\,a(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+b(2a-b+b\cosh(2(e+fx)))\sinh(2(e+fx))}{6f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])
*Sinh[2*(e + f*x)])/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.05, size = 428, normalized size = 2.46

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\,b^2\left(\cosh^4(fx+e)\right)\sinh(fx+e)+\sqrt{-\frac{b}{a}}\,ab\left(\cosh^2(fx+e)\right)\sinh(fx+e)-\sqrt{-\frac{b}{a}}\,b^2\left(\cosh^2(fx+e)\right)\sinh(fx+e)+3a^2\sqrt{\frac{b(\mathrm{co}}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*cosh(f*x+e)^4*sinh(f*x+e)+(-1/a*b)^(1/2)*a*b*cosh(f
*x+e)^2*sinh(f*x+e)-(-1/a*b)^(1/2)*b^2*cosh(f*x+e)^2*sinh(f*x+e)+3*a^2*(b/a
*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))-5*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+2*(b/a*cosh
(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*b^2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b^2)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1
/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**F**]
time = 0.09, size = 16, normalized size = 0.09

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.371 $\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=210

$$\frac{(a-2b)E\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{bF\left(\text{ArcTan}(\sinh(e+fx))\right|}{f\sqrt{\operatorname{sech}^2(e}}$$

```
[Out] (a-2*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(
f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^
2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+b*(1/(1+sinh(f*x+e)^
2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(
1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*
(a+b*sinh(f*x+e)^2)/a)^(1/2)-(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/
f+(a-b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.13, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3271, 424, 545, 429, 506, 422}

$$\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(a-2b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{(a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
+ (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sin
h[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a
- 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f + ((a - b)*Sqrt[a + b*S
inh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^{3/2}}\,dx,x,\right.}{f}$$

$$=\frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f}+\frac{\left(\sqrt{\cosh^2(e\right.}{}$$

$$=\frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f}+\frac{\left(ab\sqrt{\cosh^2(\right.}{}$$

$$=\frac{bF\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

$$=\frac{(a-2b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.71, size = 160, normalized size = 0.76

$$\frac{2ia(a-2b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\left|\frac{b}{a}\right.\right)+(a-b)\left(-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\left|\frac{b}{a}\right.\right)+\sqrt{2}\,(2a-b+b\cosh(2(e+fx)))\tanh(e+fx)\right)}{2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
 f*x), b/a] + (a - b)*((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ell
ipticF[I*(e + f*x), b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Tanh[e +
 f*x]))/(2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.39, size = 334, normalized size = 1.59

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\,ab\left(\sinh^3(fx+e)\right)-\sqrt{-\frac{b}{a}}\,b^2\left(\sinh^3(fx+e)\right)+2ab\sqrt{\frac{a+b\left(\sinh^2(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\operatorname{EllipticF}\left(\sinh(fx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-1/a*b)^(1/2)*a*b*sinh(f*x+e)^3-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+2*a*b*((
a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1
/a*b)^(1/2),(a/b)^(1/2))-2*b^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-((a+b*sinh(f*x+e)
^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))*a*b+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+sinh(f*x+e)*(-1/a*b)^(1/2)*a
^2-(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x
+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.09, size = 46, normalized size = 0.22

$$\text{integral}\left(\left(b\,\text{sech}\left(fx+e\right)^{2}\sinh\left(fx+e\right)^{2}+a\,\text{sech}\left(fx+e\right)^{2}\right)\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sech(f*x + e)^2*sinh(f*x + e)^2 + a*sech(f*x + e)^2)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\sinh(e+f\,x)^2+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^2,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^2, x)
```

3.372 $\int \text{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=193

$$\frac{2(a+b)E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{bF\left(\text{ArcTan}(\sinh(e+fx))\,\right|1}{3f\sqrt{\frac{\text{sech}^2(e+}{}}}$$

```
[Out] 2/3*(a+b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sin
h(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e
)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*b*(1/(1+sinh(f
*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e
)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x
+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(a-b)*sech(f*x+e)^2*(a+b*sinh(f*x+e)
^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.13, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3271, 424, 539, 429, 422}

$$-\frac{b\,\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(a-b)\tanh(e+fx)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*S
inh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) +
((a - b)*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
```

```
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{(a+bx^2)^{3/2}}{(1+x^2)^{5/2}}\,dx,x,\right.}{f}$$

$$=\frac{(a-b)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3f}+$$

$$=\frac{(a-b)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3f}-$$

$$=\frac{2(a+b)E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b}}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.46, size = 197, normalized size = 1.02

$$\frac{4ia(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2ia(2a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\frac{(8a^2-3ab+b^2+(4a^2+6ab-2b^2)\cosh(2(e+fx))+b(a+b)\cosh(4(e+fx)))\operatorname{sech}^2(e+fx)\tanh(e+fx)}{\sqrt{2}}}{6f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((4*I)*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f
*x), b/a] - (2*I)*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellip
ticF[I*(e + f*x), b/a] + ((8*a^2 - 3*a*b + b^2 + (4*a^2 + 6*a*b - 2*b^2)*Co
sh[2*(e + f*x)] + b*(a + b)*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh[e + f*x
])/Sqrt[2])/(6*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.70, size = 324, normalized size = 1.68

method	result
default	$\frac{\left(2\sqrt{-\frac{b}{a}}\,ab+2\sqrt{-\frac{b}{a}}\,b^2\right)\left(\cosh^4(fx+e)\right)\sinh(fx+e)+\left(2\sqrt{-\frac{b}{a}}\,a^2+\sqrt{-\frac{b}{a}}\,ab-3\sqrt{-\frac{b}{a}}\,b^2\right)\left(\cosh^2(fx+e)\right)\sinh(fx+e)+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((2*(-1/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+
(2*(-1/a*b)^(1/2)*a^2+(-1/a*b)^(1/2)*a*b-3*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^
2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(a*
EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*b*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))-2*a*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))-2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^
2+((-1/a*b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*sinh(f*x+e))
/cosh(f*x+e)^3/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2071 vs. $2(205) = 410$.
time = 0.15, size = 2071, normalized size = 10.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] -2/3*(((2*a^2 + a*b - b^2)*cosh(f*x + e)^6 + 6*(2*a^2 + a*b - b^2)*cosh(f*x
 + e)*sinh(f*x + e)^5 + (2*a^2 + a*b - b^2)*sinh(f*x + e)^6 + 3*(2*a^2 + a*
b - b^2)*cosh(f*x + e)^4 + 3*(5*(2*a^2 + a*b - b^2)*cosh(f*x + e)^2 + 2*a^2
 + a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(2*a^2 + a*b - b^2)*cosh(f*x + e)^3 +
3*(2*a^2 + a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(2*a^2 + a*b - b^2
)*cosh(f*x + e)^2 + 3*(5*(2*a^2 + a*b - b^2)*cosh(f*x + e)^4 + 6*(2*a^2 + a
*b - b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b - b^2)*sinh(f*x + e)^2 + 2*a^2 + a*
b - b^2 + 6*((2*a^2 + a*b - b^2)*cosh(f*x + e)^5 + 2*(2*a^2 + a*b - b^2)*co
sh(f*x + e)^3 + (2*a^2 + a*b - b^2)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b
+ b^2)*cosh(f*x + e)^6 + 6*(a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b
 + b^2)*sinh(f*x + e)^6 + 3*(a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a*b + b^2)*
cosh(f*x + e)^2 + a*b + b^2)*sinh(f*x + e)^4 + 4*(5*(a*b + b^2)*cosh(f*x +
e)^3 + 3*(a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(a*b + b^2)*cosh(f*
x + e)^2 + 3*(5*(a*b + b^2)*cosh(f*x + e)^4 + 6*(a*b + b^2)*cosh(f*x + e)^2
 + a*b + b^2)*sinh(f*x + e)^2 + a*b + b^2 + 6*((a*b + b^2)*cosh(f*x + e)^5
+ 2*(a*b + b^2)*cosh(f*x + e)^3 + (a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))
*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/
b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*
x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2
- a*b)/b^2))/b^2) - ((2*a^2 - a*b)*cosh(f*x + e)^6 + 6*(2*a^2 - a*b)*cosh(f
*x + e)*sinh(f*x + e)^5 + (2*a^2 - a*b)*sinh(f*x + e)^6 + 3*(2*a^2 - a*b)*c
osh(f*x + e)^4 + 3*(5*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f*x
 + e)^4 + 4*(5*(2*a^2 - a*b)*cosh(f*x + e)^3 + 3*(2*a^2 - a*b)*cosh(f*x + e
))*sinh(f*x + e)^3 + 3*(2*a^2 - a*b)*cosh(f*x + e)^2 + 3*(5*(2*a^2 - a*b)*c
osh(f*x + e)^4 + 6*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f*x +
e)^2 + 2*a^2 - a*b + 6*((2*a^2 - a*b)*cosh(f*x + e)^5 + 2*(2*a^2 - a*b)*cos
h(f*x + e)^3 + (2*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b + 2*b^2
)*cosh(f*x + e)^6 + 6*(a*b + 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b +
2*b^2)*sinh(f*x + e)^6 + 3*(a*b + 2*b^2)*cosh(f*x + e)^4 + 3*(5*(a*b + 2*b^
2)*cosh(f*x + e)^2 + a*b + 2*b^2)*sinh(f*x + e)^4 + 4*(5*(a*b + 2*b^2)*cosh
(f*x + e)^3 + 3*(a*b + 2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(a*b + 2*b
^2)*cosh(f*x + e)^2 + 3*(5*(a*b + 2*b^2)*cosh(f*x + e)^4 + 6*(a*b + 2*b^2)*
cosh(f*x + e)^2 + a*b + 2*b^2)*sinh(f*x + e)^2 + a*b + 2*b^2 + 6*((a*b + 2*
b^2)*cosh(f*x + e)^5 + 2*(a*b + 2*b^2)*cosh(f*x + e)^3 + (a*b + 2*b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^
2 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^
```

```
2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4
*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*((a*b + b^2)*cosh(f*x
+ e)^5 + 5*(a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^4 + (a*b + b^2)*sinh(f*x
 + e)^5 + (3*a*b + b^2)*cosh(f*x + e)^3 + (10*(a*b + b^2)*cosh(f*x + e)^2 +
 3*a*b + b^2)*sinh(f*x + e)^3 + 2*b^2*cosh(f*x + e) + (10*(a*b + b^2)*cosh(
f*x + e)^3 + 3*(3*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (5*(a*b + b^2
)*cosh(f*x + e)^4 + 3*(3*a*b + b^2)*cosh(f*x + e)^2 + 2*b^2)*sinh(f*x + e))
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^6 + 6
*b*f*cosh(f*x + e)*sinh(f*x + e)^5 + b*f*sinh(f*x + e)^6 + 3*b*f*cosh(f*x +
 e)^4 + 3*(5*b*f*cosh(f*x + e)^2 + b*f)*sinh(f*x + e)^4 + 3*b*f*cosh(f*x +
e)^2 + 4*(5*b*f*cosh(f*x + e)^3 + 3*b*f*cosh(f*x + e))*sinh(f*x + e)^3 + 3*
(5*b*f*cosh(f*x + e)^4 + 6*b*f*cosh(f*x + e)^2 + b*f)*sinh(f*x + e)^2 + b*f
 + 6*(b*f*cosh(f*x + e)^5 + 2*b*f*cosh(f*x + e)^3 + b*f*cosh(f*x + e))*sinh
(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1574 vs. 2(205) = 410.
time = 4.65, size = 1574, normalized size = 8.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(6*b^2*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*
e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))*e^e/sqrt(-b) + 3*(a^2
*e^e + a*b*e^e - 2*b^2*e^e)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/
sqrt(a - b))/sqrt(a - b) - 2*(3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^2*e^e + 3*(sqr
t(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e
^(2*f*x + 2*e) + b))^5*a*b*e^e - 6*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f
*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*b^2*e^e + 15*
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b))^4*a^2*sqrt(b)*e^e - 9*(sqrt(b)*e^(2*f*x + 2*e) - s
```

```
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a
*b^(3/2)*e^e - 6*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*b^(5/2)*e^e + 32*(sqrt(b)*e^(2*
f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
2*e) + b))^3*a^3*e^e - 66*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*b*e^e + 30*(sqrt(b
)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b))^3*a*b^2*e^e + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^3*e^e - 96*(
sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*
b*e^(2*f*x + 2*e) + b))^2*a^3*sqrt(b)*e^e + 222*(sqrt(b)*e^(2*f*x + 2*e) -
sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*
a^2*b^(3/2)*e^e - 210*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(5/2)*e^e + 84*(sqrt(b
)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b))^2*b^(7/2)*e^e - 48*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4
*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^4*e^e + 144
*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
2*b*e^(2*f*x + 2*e) + b))*a^3*b*e^e - 321*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b
*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b^2*
e^e + 351*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
+ 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b^3*e^e - 126*(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))
*b^4*e^e - 48*a^4*sqrt(b)*e^e + 16*a^3*b^(3/2)*e^e + 147*a^2*b^(5/2)*e^e -
165*a*b^(7/2)*e^e + 50*b^(9/2)*e^e)/((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4
*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b
)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b))*sqrt(b) + 4*a - 3*b)^3)/f^2
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}}{{\mathrm{cosh}\left(e+f\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^4,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^4, x)
```

3.373 $\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=79

$$-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}f}+\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf}$$

```
[Out] -1/2*(a-2*b)*arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(3/2)
/f+1/2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3269, 396, 223, 212}

$$\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf}-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/2*((a - 2*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/(b^(3/2)*f) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*b*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf} - \frac{(a-2b)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \text{s}\right.}{2bf}$$

$$= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf} - \frac{(a-2b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{}{\sqrt{a+}}\right.}{2bf}$$

$$= -\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(}}{2bf}$$

Mathematica [A]
time = 0.07, size = 77, normalized size = 0.97

$$\frac{-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}} + \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2b}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-1/2*((a - 2*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2
]])/b^(3/2) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*b))/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.27, size = 35, normalized size = 0.44

method	result	size
default	$\frac{\text{`}int/indef0\text{`}\left(\frac{\cosh^2(fx+e)}{\sqrt{a+b\left(\sinh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ‘int/indef0‘(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cosh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(67) = 134$.
time = 0.45, size = 2479, normalized size = 31.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [-1/8*(((a - 2*b)*cosh(f*x + e)^2 + 2*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)
 + (a - 2*b)*sinh(f*x + e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*
x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b
 - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos
h(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f
*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e
)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 +
 b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*
a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 +
b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3
 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3
*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b
```

```
^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt
(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x +
 e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b
+ b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2
*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*
(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f
*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2
)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*
b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b -
 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3
 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3
)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f
*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x +
 e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x +
e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a - 2*b)*cos
h(f*x + e)^2 + 2*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (a - 2*b)*sinh(f*x
 + e)^2)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3
 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*si
nh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x +
 e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(2)*(b*cosh
(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt
((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)^2 + 2*b^
2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2), 1/8*(2*((a - 2*b)
*cosh(f*x + e)^2 + 2*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (a - 2*b)*sinh
(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*c
osh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(
a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3
*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*
b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*
b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((a - 2*b)*cosh(f*x + e)^2 + 2*(a -
 2*b)*cosh(f*x + e)*sinh(f*x + e) + (a - 2*b)*sinh(f*x + e)^2)*sqrt(-b)*arc
tan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(
f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a
 - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 +
 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt
```

```
(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^
2 - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x +
e)^2 + 2*b^2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[4,0,0]%%%}+%%%{%%%{-2,[1]%%%},[2,0,0]%%%}+%%%{%%%{1,[2]%%%
},[0,0
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(e+f\,x)^3}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.374 $\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

```
[Out] arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3269, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[b]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}\,f}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 38, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[b]*f)
```

Maple [**A**]
time = 0.42, size = 34, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\ln\left(\sqrt{b}\sinh(fx+e)+\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\right)}{f\sqrt{b}}$	34
default	$\frac{\ln\left(\sqrt{b}\sinh(fx+e)+\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\right)}{f\sqrt{b}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*ln(b^(1/2)*sinh(f*x+e)+(a+b*sinh(f*x+e)^2)^(1/2))/b^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(32) = 64$.
time = 0.51, size = 1990, normalized size = 52.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*
a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f
*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 -
4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a
^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^
2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 +
9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x
+ e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 1
0*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 +
 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x
 + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b
 + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b
^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)
*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a
^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 +
 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e
)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2
*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*
b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5
 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*s
inh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*
(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2
*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a
```

```
*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x +
 e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)
^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*s
inh(f*x + e)^5 + sinh(f*x + e)^6)) + sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*
(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*co
sh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*si
nh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2)))/(b*f), -1/2*(sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 +
 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(
-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x
+ e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x
 + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2
- 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 -
 (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + sqrt(-b)*arctan(sqrt(2)*(
cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt
(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 +
4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*
x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*
x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)))/(b*f)]
```

Sympy **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh{\left(e+fx \right)}}{\sqrt{a+b\sinh^{2}{\left(e+fx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cosh(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B]
time = 1.02, size = 33, normalized size = 0.87

$$\frac{\ln\left(\sqrt{b}\,\sinh(e+f\,x)+\sqrt{b\sinh\left(e+f\,x\right)^2+a}\right)}{\sqrt{b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] log(b^(1/2)*sinh(e + f*x) + (a + b*sinh(e + f*x)^2)^(1/2))/(b^(1/2)*f)
```

3.375 $\int \frac{\mathbf{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{a-b}\,f}$$

```
[Out] arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3269, 385, 209}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[a - b
]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
```

```
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{a-b}\,f}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.00

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{a-b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[a - b
]*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.14, size = 35, normalized size = 0.76

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{1}{\cosh(fx+e)^2\sqrt{a+b\left(\sinh^2(fx+e)\right)}}, \sinh(fx+e)\right)}{f}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(1/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(40) = 80$.
time = 0.45, size = 598, normalized size = 13.00

$$\left[-\frac{\sqrt{-a+b}\log\left(\frac{(a-2b)\cosh(fx+e)^4+4(a-2b)\cosh(fx+e)\sinh(fx+e)^3+(a-2b)\sinh(fx+e)^4-2(3a-2b)\cosh(fx+e)^2+2\left(3(a-2b)\cosh(fx+e)^2-3a+2b\right)\sinh(fx+e)^2-2\sqrt{2}\left(\cosh(fx+e)^2+2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^2-1\right)\sqrt{-a+b}\sqrt{\frac{b\cosh(fx+e)^2+b\sinh(fx+e)^2+2a-b}{\cosh(fx+e)^2-2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^2}}+4\left((a-2b)\cosh(fx+e)^3-(3a-2b)\cosh(fx+e)\right)\sinh(fx+e)+a-2b}{\cosh(fx+e)^4+4\cosh(fx+e)\sinh(fx+e)^3+\sinh(fx+e)^4+2\left(3\cosh(fx+e)^2+1\right)\sinh(fx+e)^2+2\cosh(fx+e)^2+4\left(\cosh(fx+e)^3+\cosh(fx+e)\right)\sinh(fx+e)+1}\right)}{2(a-b)f}, \frac{\arctan\left(\frac{\sqrt{2}\left(\cosh(fx+e)^2+2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^2-1\right)\sqrt{a-b}\sqrt{\frac{b\cosh(fx+e)^2+b\sinh(fx+e)^2+2a-b}{\cosh(fx+e)^2-2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^2}}}{b\cosh(fx+e)^4+4b\cosh(fx+e)\sinh(fx+e)^3+b\sinh(fx+e)^4+2(2a-b)\cosh(fx+e)^2+2\left(3b\cosh(fx+e)^2+2a-b\right)\sinh(fx+e)^2+4\left(b\cosh(fx+e)^3+(2a-b)\cosh(fx+e)\right)\sinh(fx+e)+b}\right)}{\sqrt{a-b}\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x +
e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e
)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(
2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*
sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b
)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(co
sh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*co
sh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3
 + cosh(f*x + e))*sinh(f*x + e) + 1))/((a - b)*f), arctan(sqrt(2)*(cosh(f*x
 + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e
)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e
)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))/(sqrt(a - b)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}{\left(e + f x \right)}}{\sqrt{a + b \sinh^{2}{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [A]
time = 0.47, size = 80, normalized size = 1.74

$$\frac{2\,\arctan\left(-\frac{\sqrt{b}\,e^{(2\,fx+2\,e)}-\sqrt{be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e)}-2\,be^{(2\,fx+2\,e)}+b}+\sqrt{b}}{2\,\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/(sqrt(a - b)*
f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cosh\left(e+f\,x\right)\,\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

3.376 $\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=97

$$\frac{(a-2b)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)^{3/2}f}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2(a-b)f}$$

```
[Out] 1/2*(a-2*b)*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)
^(3/2)/f+1/2*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/f
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3269, 390, 385, 209}

$$\frac{(a-2b)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f(a-b)^{3/2}}+\frac{\tanh(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a - 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])
/(2*(a - b)^(3/2)*f) + (Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e +
f*x])/(2*(a - b)*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2(a-b)f} + \frac{(a-2b)\text{Subst}\left(\int \frac{}{(1+x^2}\right.}{}$$

$$= \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2(a-b)f} + \frac{(a-2b)\text{Subst}\left(\int \frac{}{1-(-}\right.}{}$$

$$= \frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)^{3/2}f} + \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+}}{2(a-b)f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 8.69, size = 443, normalized size = 4.57

$$\frac{\operatorname{sech}^3(e+fx)\left(1+\frac{b\sinh^2(e+fx)}{a}\right)\tanh(e+fx)\left(45a\text{ArcSin}\left(\sqrt{\frac{(a-b)\tanh^2(e+fx)}{a}}\right)+30b\text{ArcSin}\left(\sqrt{\frac{(a-b)\tanh^2(e+fx)}{a}}\right)\sinh^2(e+fx)+16a\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}\left(\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{5/2}+16b\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\sinh^2(e+fx)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}\left(\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{5/2}-45a\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a^2-b^2\sinh^2(e+fx)+ab\left(-1+\sinh^2(e+fx)\right)\right)\tanh^2(e+fx)}{a^2}}-30b\sinh^2(e+fx)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a^2-b^2\sinh^2(e+fx)+ab\left(-1+\sinh^2(e+fx)\right)\right)\tanh^2(e+fx)}{a^2}}\right)}{30af\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}\left(\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)*Tanh[e + f*x]*(45*a*ArcSin[Sqr
t[((a - b)*Tanh[e + f*x]^2)/a]] + 30*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2
```

```
)/a]]*Sinh[e + f*x]^2 + 16*a*Hypergeometric2F1[2, 3, 7/2, ((a - b)*Tanh[e +
 f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tan
h[e + f*x]^2)/a)^(5/2) + 16*b*Hypergeometric2F1[2, 3, 7/2, ((a - b)*Tanh[e
+ f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(5/2) - 45*a*Sqrt[(Sech[e + f*x]^2*(a^2 -
 b^2*Sinh[e + f*x]^2 + a*b*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2] -
30*b*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[e + f*x]^2 + a*b
*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2]))/(30*a*f*Sqrt[a + b*Sinh[e
+ f*x]^2]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[
e + f*x]^2)/a)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.95, size = 35, normalized size = 0.36

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{1}{\cosh(fx+e)^{4}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	35
risch	Expression too large to display	450266

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 'int/indef0'(1/cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 693 vs. 2(85) = 170.
time = 0.48, size = 1503, normalized size = 15.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)
^3 + (a - 2*b)*sinh(f*x + e)^4 + 2*(a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*
b)*cosh(f*x + e)^2 + a - 2*b)*sinh(f*x + e)^2 + 4*((a - 2*b)*cosh(f*x + e)^
3 + (a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)*sqrt(-a + b)*log(((a
- 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2
*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f
*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f
*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)
*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a -
2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x
+ e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x
 + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x
+ e) + 1)) - 2*sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*s
inh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 4*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*sinh(f*x
 + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^2 - 2*a*b + b^2
)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^2 - 2*a*b
 + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*
f*cosh(f*x + e))*sinh(f*x + e)), 1/2*(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2
*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 + 2*(a - 2*b)
*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 + a - 2*b)*sinh(f*x + e)^
2 + 4*((a - 2*b)*cosh(f*x + e)^3 + (a - 2*b)*cosh(f*x + e))*sinh(f*x + e) +
 a - 2*b)*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3
+ b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2
+ 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e)
)*sinh(f*x + e) + b)) + sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f
*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 4*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f
*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^
2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*
b + b^2)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 693 vs. 2(85) = 170.
time = 0.55, size = 693, normalized size = 7.14

$$\frac{\left(\frac{(a-2b)\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}+\sqrt{b}}{2\sqrt{a-b}}\right)}{\left(ae^{(4e)}-be^{(4e)}\right)\sqrt{a-b}}-\frac{2\left(\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{3}a-2\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{3}b-5\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{2}a\sqrt{b}+2\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{2}b^{\frac{3}{2}}-4\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)a^{2}-\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)ab+2\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)b^{2}-4a^{2}\sqrt{b}+5ab^{\frac{3}{2}}-2b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{2}+2\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)\sqrt{b}+4a-3b\right)^{2}\left(ae^{(4e)}-be^{(4e)}\right)}\right)e^{(4e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] ((a - 2*b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/((a*
e^(4*e) - b*e^(4*e))*sqrt(a - b)) - 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^
(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a - 2*(sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))^3*b - 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*sqrt(b) + 2*(sqr
t(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e
^(2*f*x + 2*e) + b))^2*b^(3/2) - 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f
*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2 - (sqrt(b)*
e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f
*x + 2*e) + b))*a*b + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^2 - 4*a^2*sqrt(b) + 5*a*
b^(3/2) - 2*b^(5/2))/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b))*sqrt(b) + 4*a - 3*b)^2*(a*e^(4*e) - b*e^(4*e))))*e^(4*e)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cosh\left(e+f\,x\right)}^3\,\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

3.377 $\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=241

$$\frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf}+\frac{2(a-2b)E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{sech}(e+fx)}{3b^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+\right.}{a}}}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+2/3*(a-2*b)*(1/(1
+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sin
h(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b^2/
f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(a-3*b)*(1/(1+sinh(f*x+e)
^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^
(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/b/f/(sech(f*x+
e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/3*(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tan
h(f*x+e)/b^2/f
```

Rubi [A]

time = 0.15, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3271, 427, 545, 429, 506, 422}

$$\frac{2(a-2b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3b^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(a-3b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3abf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^2f}+\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f) + (2*(a -
 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Si
nh[e + f*x]^2])/(3*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
 - ((a - 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*a*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]
^2))/a]) - (2*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b^2*f
)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{(1+x^2)^{3/2}}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sec}\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{\left(2(a-2b)\sqrt{\cosh^2(e+\right.}}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{(a-3b)F\left(\tan^{-1}(\sinh(\right.}{3abf\sqrt{}}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{2(a-2b)E\left(\tan^{-1}(\sinh\right.}{3b^2f\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.61, size = 179, normalized size = 0.74

$$\frac{4i\sqrt{2}\,a(a-2b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) - 2i\sqrt{2}\,(2a^2-5ab+3b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) + b(2a-b+b\cosh(2(e+fx)))\sinh(2(e+fx))}{6b^2f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((4*I)*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elliptic
E[I*(e + f*x), b/a] - (2*I)*Sqrt[2]*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b +
 b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[
2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*b^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e +
f*x)]])
```

Maple [A]
time = 1.69, size = 356, normalized size = 1.48

method	result

default	$\frac{\sqrt{-\frac{b}{a}}\, b\left(\cosh^4(fx+e)\right)\sinh(fx+e)+\sqrt{-\frac{b}{a}}\, a\left(\cosh^2(fx+e)\right)\sinh(fx+e)-\sqrt{-\frac{b}{a}}\, b\left(\cosh^2(fx+e)\right)\sinh(fx+e)+a\sqrt{\frac{b(\cosh^2(f}{a}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b*cosh(f*x+e)^4*sinh(f*x+e)+(-1/a*b)^(1/2)*a*cosh(f*x+e
)^2*sinh(f*x+e)-(-1/a*b)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+
e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1
/2),(a/b)^(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*El
lipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*(b/a*cosh(f*x+e)^2+(a-b
)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))*a+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*Ellip
ticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/b/(-1/a*b)^(1/2)/cosh(f*x+e)/
(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F]

time = 0.09, size = 25, normalized size = 0.10

$$\text{integral}\left(\frac{\cosh\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(249) = 498.
time = 1.11, size = 806, normalized size = 3.34

$$\frac{\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\,e^{(fx+e)}}{24bf}+\frac{\left(\frac{4\left(a\sqrt{b}e^{(4e)}-2b^{\frac{3}{2}}e^{(4e)}\right)\log\left(\left|-\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)b-2a\sqrt{b}+b^{\frac{3}{2}}\right|\right)}{b}+\frac{2\left(9a^2e^{(4e)}-21abe^{(4e)}+16b^2e^{(4e)}\right)\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}-\frac{3\left(6\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^3a^2e^{(4e)}-14\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^3abe^{(4e)}+5\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^3b^2e^{(4e)}-4\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^2b^{\frac{5}{2}}e^{(4e)}-10\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)a^2be^{(4e)}+18\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)ab^2e^{(4e)}-7\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)b^3e^{(4e)}-4ab^{\frac{5}{2}}e^{(4e)}+6b^{\frac{7}{2}}e^{(4e)}\right)}{\left(\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^2-b\right)^2b}\right)e^{(-3e)}}{24bf^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
)*e^(f*x + e)/(b*f) + 1/24*(4*(a*sqrt(b)*e^(4*e) - 2*b^(3/2)*e^(4*e))*log(a
bs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
 - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/2)))/b + 2*(9*a^2*e^(4*
e) - 21*a*b*e^(4*e) + 16*b^2*e^(4*e))*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sq
rt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt
(-b))/(sqrt(-b)*b) - 3*(6*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*e^(4*e) - 14*(sqrt
(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^
(2*f*x + 2*e) + b))^3*a*b*e^(4*e) + 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^2*e^(4*e
) - 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*
e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(5/2)*e^(4*e) - 10*(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b))*a^2*b*e^(4*e) + 18*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b^2*e^(4*e) - 7*(sqrt(b)*
e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f
*x + 2*e) + b))*b^3*e^(4*e) - 4*a*b^(5/2)*e^(4*e) + 6*b^(7/2)*e^(4*e))/(((s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b
*e^(2*f*x + 2*e) + b))^2 - b)^2*b))*e^(-3*e)/(b*f^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(e+f\,x)^4}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.378 $\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=177

$$-\frac{E\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{bf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{F\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\text{se}}{af\sqrt{\frac{\text{sech}^2(e+fx)\,(a}{}}}$$

```
[Out] -(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/
(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2
)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(1/(1+sinh(f*x+e)^2))^(1/
2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1
-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*
sinh(f*x+e)^2)/a)^(1/2)+(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b/f
```

Rubi [A]

time = 0.11, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3271, 433, 429, 506, 422}

$$\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{bf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])) + (El
lipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a +
 b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(b*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\,\sqrt{a+bx^2}}\,dx, x, \operatorname{sinh}(\right.}{f}$$

$$= \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{\sqrt{a-}}{}$$

$$= -\frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{F(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 95, normalized size = 0.54

$$-\frac{i\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\left(aE\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+(-a+b)F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)\right)}{bf\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*(a*EllipticE[I*(e + f*x), b/a
] + (-a + b)*EllipticF[I*(e + f*x), b/a]))/(b*f*Sqrt[2*a - b + b*Cosh[2*(e
+ f*x)]])
```

Maple [A]
time = 1.08, size = 86, normalized size = 0.49

method	result	size
default	$\frac{\sqrt{\frac{a+b(\sinh^2(fx+e))}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\operatorname{EllipticE}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\left(\sinh^2(fx+e)\right)}\,f}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(
1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F]
time = 0.10, size = 25, normalized size = 0.14

$$\mathrm{integral}\left(\frac{\cosh(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cosh(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{32,[4,
2,4]%%%}+%%%{%%%{-64,[1]%%%},[4,2,3]%%%}+%%%{%%%{32,[2]%%%},[4,2,2]%%%}+%%%
{%%%{-64,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(e+f\,x\right)}^2}{\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.379 $\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(
1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3262, 3261}

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 1.13

$$-\frac{i\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/
(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 0.80, size = 86, normalized size = 1.43

method	result	size
default	$\frac{\sqrt{\frac{a+b\left(\sinh^2(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\left(\sinh^2(fx+e)\right)}\,f}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(70) = 140$.
time = 0.09, size = 147, normalized size = 2.45

$$-\frac{2\left(2\,b\sqrt{\frac{a^2-ab}{b^2}}+2\,a-b\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}\,F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}\,(\cosh\left(fx+e\right)+\sinh\left(fx+e\right))\right)\mid\frac{8\,a^2-8\,ab+b^2+4\,(2\,ab-b^2)\sqrt{\frac{a^2-ab}{b^2}}}{b^2})}{b^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)
*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*s
qrt((a^2 - a*b)/b^2))/b^2)/(b^(3/2)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(1/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.380 $\int \frac{\text{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=160

$$\frac{E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{bF\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{se}}{a(a-b)f\sqrt{\frac{\text{sech}^2(e+fx)}{}}}$$

```
[Out] (1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-b*(1/(1+sinh(f*x+e)^2)
)^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/
2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)/f/(sech(f*x
+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3271, 425, 21, 433, 429, 506, 422}

$$\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{b\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
+ f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
(b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(a*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\Big(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\Big)\operatorname{Subst}\Big(\int \frac{1}{(1+x^2)^{3/2}\sqrt{a+bx^2}}\,dx, x, \sinh(e+}{f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{(a-b)f} + \frac{\Big(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\Big)\operatorname{Su}}{}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{(a-b)f} + \frac{\Big(b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\Big)\operatorname{S}}{(-a}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{(a-b)f} + \frac{\Big(b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\Big)\operatorname{S}}{}$$

$$= -\frac{bF\big(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\big)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} - \frac{\Big(\sqrt{}}{}$$

$$= \frac{E\big(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\big)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} - \frac{bF\big(\tan}{}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 0.72, size = 159, normalized size = 0.99

$$\frac{2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) - 2i(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) + \sqrt{2}\,(2a-b+b\cosh(2(e+fx)))\tanh(e+fx)}{2(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a
] - (2*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e +
f*x), b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(2*(a -
 b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [**A**]
time = 2.08, size = 131, normalized size = 0.82

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, b\left(\sinh^{3}(fx+e)\right)-b\sqrt{\frac{a+b\left(\sinh^{2}(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticE}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)+\sqrt{-\frac{b}{a}}\ a}{(a-b)\sqrt{-\frac{b}{a}}\ \cosh(fx+e)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\ f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-1/a*b)^(1/2)*b*sinh(f*x+e)^3-b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+(-1/a*b)^(1/2)
*a*sinh(f*x+e))/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/
f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(180) = 360$.
time = 0.10, size = 576, normalized size = 3.60

$$-\frac{\left((2a-b)\cosh(fx+e)^{2}+2(2a-b)\cosh(fx+e)\sinh(fx+e)+(2a-b)\sinh(fx+e)^{2}-2\left(b\cosh(fx+e)^{2}+2b\cosh(fx+e)\sinh(fx+e)+b\sinh(fx+e)^{2}+b\right)\sqrt{\frac{a^{2}-ab}{b^{2}}}+2a-b\right)\sqrt{b}\sqrt{\frac{2b\sqrt{\frac{a^{2}-ab}{b^{2}}}-2a+b}{b}}\,E(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^{2}-ab}{b^{2}}}-2a+b}{b}}(\cosh(fx+e)+\sinh(fx+e))\right)\mid\frac{8a^{2}-8ab+b^{2}+4(2ab-b^{2})\sqrt{\frac{a^{2}-ab}{b^{2}}}}{b^{2}})-2\left((2a-b)\cosh(fx+e)^{2}+2(2a-b)\cosh(fx+e)\sinh(fx+e)+(2a-b)\sinh(fx+e)^{2}+2a-b\right)\sqrt{b}\sqrt{\frac{2b\sqrt{\frac{a^{2}-ab}{b^{2}}}-2a+b}{b}}\,F(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^{2}-ab}{b^{2}}}-2a+b}{b}}(\cosh(fx+e)+\sinh(fx+e))\right)\mid\frac{8a^{2}-8ab+b^{2}+4(2ab-b^{2})\sqrt{\frac{a^{2}-ab}{b^{2}}}}{b^{2}})-\sqrt{2}(b\cosh(fx+e)+b\sinh(fx+e))\sqrt{\frac{b\cosh(fx+e)^{2}+b\sinh(fx+e)^{2}+2a-b}{\cosh(fx+e)^{2}-2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^{2}}}}{(ab-b^{2})f\cosh(fx+e)^{2}+2(ab-b^{2})f\cosh(fx+e)\sinh(fx+e)+(ab-b^{2})f\sinh(fx+e)^{2}+(ab-b^{2})f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -((((2*a - b)*cosh(f*x + e)^2 + 2*(2*a - b)*cosh(f*x + e)*sinh(f*x + e) + (2
*a - b)*sinh(f*x + e)^2 - 2*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x
 + e) + b*sinh(f*x + e)^2 + b)*sqrt((a^2 - a*b)/b^2) + 2*a - b)*sqrt(b)*sqr
t((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt
((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 -
8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((2*a - b)*co
sh(f*x + e)^2 + 2*(2*a - b)*cosh(f*x + e)*sinh(f*x + e) + (2*a - b)*sinh(f*
x + e)^2 + 2*a - b)*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*e
lliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x +
e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*
```

```
b)/b^2))/b^2) - sqrt(2)*(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2)))/((a*b - b^2)*f*cosh(f*x + e)^2 + 2*(a*b
- b^2)*f*cosh(f*x + e)*sinh(f*x + e) + (a*b - b^2)*f*sinh(f*x + e)^2 + (a*b
 - b^2)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [A]
time = 0.48, size = 282, normalized size = 1.76

$$\frac{2\left(\frac{\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}+\sqrt{b}}{2\sqrt{a-b}}\right)e^{(-2e)}}{\sqrt{a-b}}-\frac{2\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}-\sqrt{b}\right)e^{(-2e)}}{\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^2+2\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)\sqrt{b}+4a-3b}\right)e^{(3e)}}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))*e^(-2*e)/sqr
t(a - b) - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f
*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))*e^(-2*e)/((sqrt(b)*e^(2*f*x
 + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e
) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3*b))*e^(3*e)/f^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(e+f\,x\right)^2\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

3.381 $\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=219

$$\frac{2(a-2b)E\left(\operatorname{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(a-3b)bF\big(\operatorname{ArcTan}(\sinh(}{3a(a-b)^2f}$$

```
[Out] 2/3*(a-2*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x
+e)^2)^(1/2)/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(a-3
*b)*b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*
x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)
^(1/2)/a/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*sech(f*x
+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/f
```

Rubi [A]

time = 0.14, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3271, 425, 539, 429, 422}

$$-\frac{b(a-3b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\operatorname{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3af(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2(a-2b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (2*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
 + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2))/a]) - ((a - 3*b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech
[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*(a - b)^2*f*Sqrt[(Sech[e + f*x]
^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]
^2]*Tanh[e + f*x])/(3*(a - b)*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\Bigg(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\Bigg)\operatorname{Subst}\Bigg(\int \frac{1}{(1+x^2)^{5/2}\sqrt{a+bx^2}}\,dx, x, \sinh(e}{f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3(a-b)f} - \frac{\Bigg(\sqrt{\cosh^2(e+fx)}\operatorname{se}}{}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3(a-b)f} + \frac{\Bigg(2(a-2b)\sqrt{\cosh^2(e}}{}$$

$$= \frac{2(a-2b)E\big(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\big)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.54, size = 219, normalized size = 1.00

$$\frac{4ia(a-2b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2i(2a^2-5ab+3b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\frac{(8a^2-15ab+4b^2+(4a^2-6ab-2b^2)\cosh(2(e+fx))+(a-2b)b\cosh(4(e+fx)))\operatorname{sech}^2(e+fx)\tanh(e+fx)}{\sqrt{2}}}{6(a-b)^2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((4*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
 f*x), b/a] - (2*I)*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f
*x)])/a]*EllipticF[I*(e + f*x), b/a] + ((8*a^2 - 15*a*b + 4*b^2 + (4*a^2 -
6*a*b - 2*b^2)*Cosh[2*(e + f*x)] + (a - 2*b)*b*Cosh[4*(e + f*x)])*Sech[e +
f*x]^2*Tanh[e + f*x])/Sqrt[2])/(6*(a - b)^2*f*Sqrt[2*a - b + b*Cosh[2*(e +
f*x)]])
```

Maple [A]
time = 2.28, size = 343, normalized size = 1.57

method	result
default	$\frac{\sqrt{\left(a+b\left(\sinh^2(fx+e)\right)\right)\left(\cosh^2(fx+e)\right)}\,\Big(2\left(\cosh^4(fx+e)\right)\sqrt{-\frac{b}{a}}\,b(-2b+a)\sinh(fx+e)+\left(\cosh^2(fx+e)\right)\sqrt{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/cosh(f*x+e)^3/(b*cosh(f*x+e)^
4+(a-b)*cosh(f*x+e)^2)^(1/2)/(-1/a*b)^(1/2)/(a^2-2*a*b+b^2)*(2*cosh(f*x+e)^
4*(-1/a*b)^(1/2)*b*(-2*b+a)*sinh(f*x+e)+cosh(f*x+e)^2*(-1/a*b)^(1/2)*(2*a^2
-5*a*b+3*b^2)*sinh(f*x+e)+(-1/a*b)^(1/2)*(a^2-2*a*b+b^2)*sinh(f*x+e)+(cosh(
f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(a*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))-2*a*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+4*b*EllipticE(
sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2443 vs. $2(231) = 462$.
time = 0.18, size = 2443, normalized size = 11.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(((2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^6 + 6*(2*a^2 - 5*a*b + 2*b^2)*
cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2 - 5*a*b + 2*b^2)*sinh(f*x + e)^6 + 3
*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^4 + 3*(5*(2*a^2 - 5*a*b + 2*b^2)*cos
h(f*x + e)^2 + 2*a^2 - 5*a*b + 2*b^2)*sinh(f*x + e)^4 + 4*(5*(2*a^2 - 5*a*b
 + 2*b^2)*cosh(f*x + e)^3 + 3*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e))*sinh(f
*x + e)^3 + 3*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^2 + 3*(5*(2*a^2 - 5*a*b
 + 2*b^2)*cosh(f*x + e)^4 + 6*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^2 + 2*a
^2 - 5*a*b + 2*b^2)*sinh(f*x + e)^2 + 2*a^2 - 5*a*b + 2*b^2 + 6*((2*a^2 - 5
*a*b + 2*b^2)*cosh(f*x + e)^5 + 2*(2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^3 +
 (2*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b - 2*b^2)*co
sh(f*x + e)^6 + 6*(a*b - 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b - 2*b^
2)*sinh(f*x + e)^6 + 3*(a*b - 2*b^2)*cosh(f*x + e)^4 + 3*(5*(a*b - 2*b^2)*c
osh(f*x + e)^2 + a*b - 2*b^2)*sinh(f*x + e)^4 + 4*(5*(a*b - 2*b^2)*cosh(f*x
 + e)^3 + 3*(a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(a*b - 2*b^2)*
cosh(f*x + e)^2 + 3*(5*(a*b - 2*b^2)*cosh(f*x + e)^4 + 6*(a*b - 2*b^2)*cosh
```

```
(f*x + e)^2 + a*b - 2*b^2)*sinh(f*x + e)^2 + a*b - 2*b^2 + 6*((a*b - 2*b^2)
*cosh(f*x + e)^5 + 2*(a*b - 2*b^2)*cosh(f*x + e)^3 + (a*b - 2*b^2)*cosh(f*x
 + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 -
a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*
a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - ((2*a^2 - 7*a*b + 3*b^2)*cosh(f*x
+ e)^6 + 6*(2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2 -
 7*a*b + 3*b^2)*sinh(f*x + e)^6 + 3*(2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e)^4
 + 3*(5*(2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a^2 - 7*a*b + 3*b^2)*si
nh(f*x + e)^4 + 4*(5*(2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(2*a^2 - 7
*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(2*a^2 - 7*a*b + 3*b^2)*co
sh(f*x + e)^2 + 3*(5*(2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e)^4 + 6*(2*a^2 - 7
*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a^2 - 7*a*b + 3*b^2)*sinh(f*x + e)^2 + 2*
a^2 - 7*a*b + 3*b^2 + 6*((2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e)^5 + 2*(2*a^2
 - 7*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a^2 - 7*a*b + 3*b^2)*cosh(f*x + e))*
sinh(f*x + e) - 2*((a*b - b^2)*cosh(f*x + e)^6 + 6*(a*b - b^2)*cosh(f*x + e
)*sinh(f*x + e)^5 + (a*b - b^2)*sinh(f*x + e)^6 + 3*(a*b - b^2)*cosh(f*x +
e)^4 + 3*(5*(a*b - b^2)*cosh(f*x + e)^2 + a*b - b^2)*sinh(f*x + e)^4 + 4*(5
*(a*b - b^2)*cosh(f*x + e)^3 + 3*(a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3
 + 3*(a*b - b^2)*cosh(f*x + e)^2 + 3*(5*(a*b - b^2)*cosh(f*x + e)^4 + 6*(a*
b - b^2)*cosh(f*x + e)^2 + a*b - b^2)*sinh(f*x + e)^2 + a*b - b^2 + 6*((a*b
 - b^2)*cosh(f*x + e)^5 + 2*(a*b - b^2)*cosh(f*x + e)^3 + (a*b - b^2)*cosh(
f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2
 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2
) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*
(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*((a*b - 2*b^2)*cosh(f*x
 + e)^5 + 5*(a*b - 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^4 + (a*b - 2*b^2)*sin
h(f*x + e)^5 + (3*a*b - 5*b^2)*cosh(f*x + e)^3 + (10*(a*b - 2*b^2)*cosh(f*x
 + e)^2 + 3*a*b - 5*b^2)*sinh(f*x + e)^3 - b^2*cosh(f*x + e) + (10*(a*b - 2
*b^2)*cosh(f*x + e)^3 + 3*(3*a*b - 5*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 +
(5*(a*b - 2*b^2)*cosh(f*x + e)^4 + 3*(3*a*b - 5*b^2)*cosh(f*x + e)^2 - b^2)
*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cos
h(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2*b -
 2*a*b^2 + b^3)*f*cosh(f*x + e)^6 + 6*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x +
e)*sinh(f*x + e)^5 + (a^2*b - 2*a*b^2 + b^3)*f*sinh(f*x + e)^6 + 3*(a^2*b -
 2*a*b^2 + b^3)*f*cosh(f*x + e)^4 + 3*(5*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x
 + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f)*sinh(f*x + e)^4 + 3*(a^2*b - 2*a*b^2 +
 b^3)*f*cosh(f*x + e)^2 + 4*(5*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^3 +
3*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(5*(a^2*b -
2*a*b^2 + b^3)*f*cosh(f*x + e)^4 + 6*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e
)^2 + (a^2*b - 2*a*b^2 + b^3)*f)*sinh(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*
f + 6*((a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^5 + 2*(a^2*b - 2*a*b^2 + b^3
)*f*cosh(f*x + e)^3 + (a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e))*sinh(f*x + e
))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. 2(231) = 462.
time = 1.30, size = 1142, normalized size = 5.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(a - 2*b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4
*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b)
)/((a*e^(4*e) - b*e^(4*e))*sqrt(a - b)) - 2*(3*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a
 - 6*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^5*b + 15*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^
(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a*sqrt(b)
 - 30*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*
e) - 2*b*e^(2*f*x + 2*e) + b))^4*b^(3/2) + 32*(sqrt(b)*e^(2*f*x + 2*e) - sq
rt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^
2 - 130*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*b + 68*(sqrt(b)*e^(2*f*x + 2*e) - sqrt
(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^2
- 96*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^2*a^2*sqrt(b) + 30*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2
*a*b^(3/2) + 36*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(5/2) - 48*(sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
+ b))*a^3 - 96*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b + 255*(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)*a*b^2 - 126*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^3 - 48*a^3*sqrt(b) + 160*a^2*b^(3/
```

```
2) - 173*a*b^(5/2) + 58*b^(7/2))/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f
*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*
e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f
*x + 2*e) + b))*sqrt(b) + 4*a - 3*b)^3*(a*e^(4*e) - b*e^(4*e))))*e^(5*e)/f^
2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh\left(e+f\,x\right)^4\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

3.382 $\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{3/2}f}-\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(3/2)/f-(a-b)*sinh
(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3269, 393, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{3/2}f}-\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(b^(3/2)*f) -
((a - b)*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}\\
&= -\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{bf}\\
&= -\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+f}}\right.}{bf}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{3/2}f} - \frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 89, normalized size = 1.16

$$
\frac{\sqrt{b}\,(-a+b)\sinh(e+fx)+a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{ab^{3/2}f\sqrt{a+b\sinh^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(-a + b)*Sinh[e + f*x] + a^(3/2)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/S
qrt[a]]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a*b^(3/2)*f*Sqrt[a + b*Sinh[e + f
*x]^2])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.14, size = 35, normalized size = 0.45

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\cosh^2(fx+e)}{\left(a+b\left(\sinh^2(fx+e)\right)\right)^{\frac{3}{2}}},\sinh(fx+e)\right)}{f}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1108 vs. $2(69) = 138$.
time = 0.50, size = 3126, normalized size = 40.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh
(f*x + e)^4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 +
2*a^2 - a*b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)
*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f
*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*
b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*co
sh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 +
b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(
f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x +
e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2
```

```
+ b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5
*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 +
 b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^
3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(
3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
 e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*
b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqr
t(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x
+ e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b
 + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 +
2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3
*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(
f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^
2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a
*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b
- 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^
3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^
3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(
f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x
+ e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x +
 e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + (a*b*cosh(f*x
 + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + 2*(2*
a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f
*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)*cosh(f*x + e))*sin
h(f*x + e))*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e
)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)
*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*
x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - 4*sqrt(2)*((
a*b - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e) + (a
*b - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
 + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)))/(a*b^3*f*cosh(f*x + e)^4 + 4*a*b^3*f*cosh(f*x + e)*sinh(f*x +
 e)^3 + a*b^3*f*sinh(f*x + e)^4 + a*b^3*f + 2*(2*a^2*b^2 - a*b^3)*f*cosh(f*
x + e)^2 + 2*(3*a*b^3*f*cosh(f*x + e)^2 + (2*a^2*b^2 - a*b^3)*f)*sinh(f*x +
 e)^2 + 4*(a*b^3*f*cosh(f*x + e)^3 + (2*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*s
inh(f*x + e)), -1/2*((a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x +
e)^3 + a*b*sinh(f*x + e)^4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cos
h(f*x + e)^2 + 2*a^2 - a*b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3
+ (2*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*((a -
 b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(
```

```
f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(
(a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a*b - b^2)*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b -
b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b
^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + (a*b
*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^
4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 + 2*a^2 - a*
b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)*cosh(f*x
+ e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2
+ b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2...
```

Sympy **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad **[F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(e+f\,x)^3}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.383 $\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3269, 197}

$$\frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Sinh[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}\\ &= \frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$\frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Sinh[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [A]
time = 0.43, size = 28, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\sinh(fx+e)}{af\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}}$	28
default	$\frac{\sinh(fx+e)}{af\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(29) = 58.
time = 0.51, size = 246, normalized size = 8.48

$$\frac{b^2e^{(-6\,fx-6\,e)}+2\,ab-b^2+(8\,a^2-8\,ab+3\,b^2)e^{(-2\,fx-2\,e)}+3\,(2\,ab-b^2)e^{(-4\,fx-4\,e)}}{2\,(a^2-ab)(2\,(2\,a-b)e^{(-2\,fx-2\,e)}+be^{(-4\,fx-4\,e)}+b)^{\frac{3}{2}}f}-\frac{b^2+3\,(2\,ab-b^2)e^{(-2\,fx-2\,e)}+(8\,a^2-8\,ab+3\,b^2)e^{(-4\,fx-4\,e)}+(2\,ab-b^2)e^{(-6\,fx-6\,e)}}{2\,(a^2-ab)(2\,(2\,a-b)e^{(-2\,fx-2\,e)}+be^{(-4\,fx-4\,e)}+b)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*e^(-6*f*x - 6*e) + 2*a*b - b^2 + (8*a^2 - 8*a*b + 3*b^2)*e^(-2*f*x
 - 2*e) + 3*(2*a*b - b^2)*e^(-4*f*x - 4*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2
*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(3/2)*f) - 1/2*(b^2 + 3*(2*a*b - b^2)
*e^(-2*f*x - 2*e) + (8*a^2 - 8*a*b + 3*b^2)*e^(-4*f*x - 4*e) + (2*a*b - b^2
)*e^(-6*f*x - 6*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*
x - 4*e) + b)^(3/2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(27) = 54.
time = 0.42, size = 245, normalized size = 8.45

$$\frac{\sqrt{2}\left(\cosh\left(fx+e\right)^2+2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^2-1\right)\sqrt{\frac{b\cosh\left(fx+e\right)^2+b\sinh\left(fx+e\right)^2+2\,a-b}{\cosh\left(fx+e\right)^2-2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^2}}}{abf\cosh\left(fx+e\right)^4+4\,abf\cosh\left(fx+e\right)\sinh\left(fx+e\right)^3+abf\sinh\left(fx+e\right)^4+2\,(2\,a^2-ab)f\cosh\left(fx+e\right)^2+abf+2\left(3\,abf\cosh\left(fx+e\right)^2+(2\,a^2-ab)f\right)\sinh\left(fx+e\right)^2+4\left(abf\cosh\left(fx+e\right)^3+(2\,a^2-ab)f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
- 1)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*f*cosh(f*x + e)^
4 + 4*a*b*f*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*f*sinh(f*x + e)^4 + 2*(2*a^
2 - a*b)*f*cosh(f*x + e)^2 + a*b*f + 2*(3*a*b*f*cosh(f*x + e)^2 + (2*a^2 -
a*b)*f)*sinh(f*x + e)^2 + 4*(a*b*f*cosh(f*x + e)^3 + (2*a^2 - a*b)*f*cosh(f
*x + e))*sinh(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh{\left(e+fx \right)}}{\left(a+b\sinh^{2}{\left(e+fx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cosh(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(27) = 54.
time = 0.60, size = 118, normalized size = 4.07

$$\frac{\frac{\left(ae^{(4\,e)}-be^{(4\,e)}\right)e^{(2\,fx)}}{a^{2}e^{(2\,e)}-abe^{(2\,e)}}-\frac{ae^{(2\,e)}-be^{(2\,e)}}{a^{2}e^{(2\,e)}-abe^{(2\,e)}}}{\sqrt{be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e)}-2\,be^{(2\,fx+2\,e)}+b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((a*e^(4*e) - b*e^(4*e))*e^(2*f*x)/(a^2*e^(2*e) - a*b*e^(2*e)) - (a*e^(2*e)
 - b*e^(2*e))/(a^2*e^(2*e) - a*b*e^(2*e)))/(sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)*f)
```

Mupad [B]
time = 1.13, size = 191, normalized size = 6.59

$$-\frac{\mathrm{e}^{e+f\,x}\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}\,\left(\frac{2\,\cosh\left(e+f\,x\right)\,\mathrm{e}^{e+f\,x}\,\left(b\,\left(2\,a-b\right)-b\,\left(4\,a-2\,b\right)\right)}{f\,\left(a\,b^2-a^2\,b\right)}-\frac{2\,b^2\,\mathrm{e}^{e+f\,x}\,\sinh\left(e+f\,x\right)}{f\,\left(a\,b^2-a^2\,b\right)}+\frac{b\,\mathrm{e}^{2\,e+2\,f\,x}\,\left(4\,a-2\,b\right)}{f\,\left(a\,b^2-a^2\,b\right)}\right)}{4\,a\,\mathrm{e}^{2\,e+2\,f\,x}-2\,b\,\mathrm{e}^{2\,e+2\,f\,x}+2\,b\,\mathrm{e}^{2\,e+2\,f\,x}\,\cosh\left(2\,e+2\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] -(exp(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)*((2*cosh(e + f*x)*exp(e + f*x)
*(b*(2*a - b) - b*(4*a - 2*b)))/(f*(a*b^2 - a^2*b)) - (2*b^2*exp(e + f*x)*s
inh(e + f*x))/(f*(a*b^2 - a^2*b)) + (b*exp(2*e + 2*f*x)*(4*a - 2*b))/(f*(a*
b^2 - a^2*b))))/(4*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) + 2*b*exp(2*e
+ 2*f*x)*cosh(2*e + 2*f*x))
```

3.384 $\int \frac{\mathbf{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{3/2}f}-\frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*s
inh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 390, 385, 209}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\sinh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/((a - b)^(3
/2)*f) - (b*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \sinh(\right.}{(a-b)f}$$

$$= -\frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\sinh(}{\sqrt{a+b\sin}}\right.}{(a-b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 7.51, size = 315, normalized size = 3.71

$$\frac{\operatorname{sech}^7(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)\left(4(a-b)^2\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a^2-b^2\sinh^2(e+fx)+ab\left(-1+\sinh^2(e+fx)\right)\right)\tanh^2(e+fx)}{a^2}}+15a\cosh^2(e+fx)\left(3a+2b\sinh^2(e+fx)\right)\left(-\operatorname{ArcSin}\left(\sqrt{\frac{(a-b)\tanh^2(e+fx)}{a}}\right)\left(a+b\sinh^2(e+fx)\right)+a\cosh^2(e+fx)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a^2-b^2\sinh^2(e+fx)+ab\left(-1+\sinh^2(e+fx)\right)\right)\tanh^2(e+fx)}{a^2}}\right)\right)}{15a^5f\left(\frac{(a-b)\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)\tanh^2(e+fx)}{a^2}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sech[e + f*x]^7*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]*(4*(a - b)^2*Hyp
ergeometric2F1[2, 2, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^4*(a +
```

```
 b*Sinh[e + f*x]^2)*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[e + f*x]^2 + a*b*
(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2] + 15*a*Cosh[e + f*x]^2*(3*a +
 2*b*Sinh[e + f*x]^2)*(-(ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*(a + b*S
inh[e + f*x]^2)) + a*Cosh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[
e + f*x]^2 + a*b*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2])))/(15*a^5*f
*(((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2)^(3
/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.96, size = 101, normalized size = 1.19

method	result
default	$\frac{`int/indef0`\left(\frac{-b\left(\sinh^{2}(fx+e)\right)-a}{\left(-b^{2}\left(\sinh^{6}(fx+e)\right)+\left(-2ab-b^{2}\right)\left(\sinh^{4}(fx+e)\right)+\left(-a^{2}-2ab\right)\left(\sinh^{2}(fx+e)\right)-a^{2}\right)\sqrt{a+b\left(\sinh^{2}(fx+e)\right)}},\sinh(\right.}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`((-b*sinh(f*x+e)^2-a)/(-b^2*sinh(f*x+e)^6+(-2*a*b-b^2)*sinh(f*x
+e)^4+(-a^2-2*a*b)*sinh(f*x+e)^2-a^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e)
)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 800 vs. 2(77) = 154.
time = 0.46, size = 1717, normalized size = 20.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh
(f*x + e)^4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 +
2*a^2 - a*b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)
```

```
*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4
+ 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2
*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*
sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sin
h(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 +
sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x +
e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*
((a*b - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e) +
(a*b - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f
*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sin
h(f*x + e)^2)))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)^4 + 4*(a^3*b -
 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^3*b - 2*a^2*b^2 +
a*b^3)*f*sinh(f*x + e)^4 + 2*(2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f*cosh(f
*x + e)^2 + 2*(3*(a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)^2 + (2*a^4 - 5
*a^3*b + 4*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3
)*f + 4*((a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)^3 + (2*a^4 - 5*a^3*b +
 4*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e)), ((a*b*cosh(f*x + e)^4
+ 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + 2*(2*a^2 - a*
b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f*x + e)^
2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)*cosh(f*x + e))*sinh(f*x +
e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*
x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*si
nh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a
- b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh
(f*x + e) + b)) - sqrt(2)*((a*b - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh
(f*x + e)*sinh(f*x + e) + (a*b - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*co
sh(f*x + e)^4 + 4*(a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)*sinh(f*x + e)
^3 + (a^3*b - 2*a^2*b^2 + a*b^3)*f*sinh(f*x + e)^4 + 2*(2*a^4 - 5*a^3*b + 4
*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^2 + 2*(3*(a^3*b - 2*a^2*b^2 + a*b^3)*f*co
sh(f*x + e)^2 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^2 +
(a^3*b - 2*a^2*b^2 + a*b^3)*f + 4*((a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x +
 e)^3 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e
))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sech(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(77) = 154.
time = 0.54, size = 294, normalized size = 3.46

$$-\frac{\left(\frac{\frac{\left(a^{2}be^{(4\,e)}-2\,ab^{2}e^{(4\,e)}+b^{3}e^{(4\,e)}\right)e^{(2\,fx)}}{a^{4}e^{(6\,e)}-3\,a^{3}be^{(6\,e)}+3\,a^{2}b^{2}e^{(6\,e)}-ab^{3}e^{(6\,e)}}-\frac{a^{2}be^{(2\,e)}-2\,ab^{2}e^{(2\,e)}+b^{3}e^{(2\,e)}}{a^{4}e^{(6\,e)}-3\,a^{3}be^{(6\,e)}+3\,a^{2}b^{2}e^{(6\,e)}-ab^{3}e^{(6\,e)}}}{\sqrt{be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e)}-2\,be^{(2\,fx+2\,e)}+b}}-\frac{2\,\arctan\left(-\frac{\sqrt{b}e^{(2\,fx+2\,e)}-\sqrt{be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e)}-2\,be^{(2\,fx+2\,e)}+b}+\sqrt{b}}{2\,\sqrt{a-b}}\right)}{\left(ae^{(4\,e)}-be^{(4\,e)}\right)\sqrt{a-b}}\right)e^{(4\,e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(((a^2*b*e^(4*e) - 2*a*b^2*e^(4*e) + b^3*e^(4*e))*e^(2*f*x)/(a^4*e^(6*e) -
 3*a^3*b*e^(6*e) + 3*a^2*b^2*e^(6*e) - a*b^3*e^(6*e)) - (a^2*b*e^(2*e) - 2*
a*b^2*e^(2*e) + b^3*e^(2*e))/(a^4*e^(6*e) - 3*a^3*b*e^(6*e) + 3*a^2*b^2*e^(
6*e) - a*b^3*e^(6*e)))/sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e
^(2*f*x + 2*e) + b) - 2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt
(a - b))/((a*e^(4*e) - b*e^(4*e))*sqrt(a - b)))*e^(4*e)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(e+f\,x\right)\,{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

3.385 $\int \frac{\textbf{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{(a-4b)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)^{5/2}f}+\frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\text{sech}(e+fx)\tanh(e+}{2(a-b)f\sqrt{a+b\sinh^2(e-}}$$

```
[Out] 1/2*(a-4*b)*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)
^(5/2)/f+1/2*b*(a+2*b)*sinh(f*x+e)/a/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/
2*sech(f*x+e)*tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3269, 425, 541, 12, 385, 209}

$$\frac{(a-4b)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f(a-b)^{5/2}}+\frac{b(a+2b)\sinh(e+fx)}{2af(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\tanh(e+fx)\text{sech}(e+fx)}{2f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])
/(2*(a - b)^(5/2)*f) + (b*(a + 2*b)*Sinh[e + f*x])/(2*a*(a - b)^2*f*Sqrt[a
+ b*Sinh[e + f*x]^2]) + (Sech[e + f*x]*Tanh[e + f*x])/(2*(a - b)*f*Sqrt[a +
 b*Sinh[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-2bx^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sinh(e+\right.}{2(a-b)f}$$

$$= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\mathrm{S}}{}$$

$$= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(}{}$$

$$= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(}{}$$

$$= \frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)^{5/2}f} + \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.19, size = 231, normalized size = 1.63

$$\frac{\operatorname{sech}^5(e+fx)\left(16(a-b)\,{}_3F_2\left(2,2,3;1,\frac{9}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^2+16(a-b)\,{}_2F_1\left(2,3;\frac{9}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\sinh^2(e+fx)\left(4a^2+7ab\sinh^2(e+fx)+3b^2\sinh^4(e+fx)\right)+7a\cosh^2(e+fx)\,{}_2F_1\left(1,2;\frac{7}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\left(15a^2+20ab\sinh^2(e+fx)+8b^2\sinh^4(e+fx)\right)\right)\tanh(e+fx)}{105a^4f\sqrt{a+b\sinh^2(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sech[e + f*x]^5*(16*(a - b)*HypergeometricPFQ[{2, 2, 3}, {1, 9/2}, ((a - b
)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^2 + 16*(a - b
)*Hypergeometric2F1[2, 3, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2
*(4*a^2 + 7*a*b*Sinh[e + f*x]^2 + 3*b^2*Sinh[e + f*x]^4) + 7*a*Cosh[e + f*x
]^2*Hypergeometric2F1[1, 2, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*(15*a^2 + 20*
a*b*Sinh[e + f*x]^2 + 8*b^2*Sinh[e + f*x]^4))*Tanh[e + f*x])/(105*a^4*f*Sqr
t[a + b*Sinh[e + f*x]^2])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.68, size = 95, normalized size = 0.67

method	result	size
default	$\frac{\text{‘}\mathit{int/indef0}\text{‘}\left(-\frac{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,\left(\cosh^{2}\left(fx+e\right)\right)}{-b^{2}\left(\cosh^{10}\left(fx+e\right)\right)+\left(-2ab+2b^{2}\right)\left(\cosh^{8}\left(fx+e\right)\right)+\left(-a^{2}+2ab-b^{2}\right)\left(\cosh^{6}\left(fx+e\right)\right)},\sinh\left(fx+e\right)\right)}{f}$	95
risch	Expression too large to display	16593815

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(-(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2/(-b^2*cosh(f*x+e)^10+
(-2*a*b+2*b^2)*cosh(f*x+e)^8+(-a^2+2*a*b-b^2)*cosh(f*x+e)^6),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2364 vs. 2(126) = 252.
time = 0.69, size = 4845, normalized size = 34.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2*b - 4*a*b^2)*cosh(f*x + e)^8 + 8*(a^2*b - 4*a*b^2)*cosh(f*x + e
)*sinh(f*x + e)^7 + (a^2*b - 4*a*b^2)*sinh(f*x + e)^8 + 4*(a^3 - 4*a^2*b)*c
osh(f*x + e)^6 + 4*(a^3 - 4*a^2*b + 7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^2)*si
nh(f*x + e)^6 + 8*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b)*
cosh(f*x + e))*sinh(f*x + e)^5 + 2*(4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x +
e)^4 + 2*(35*(a^2*b - 4*a*b^2)*cosh(f*x + e)^4 + 4*a^3 - 17*a^2*b + 4*a*b^2
 + 30*(a^3 - 4*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(7*(a^2*b - 4*a*
b^2)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b)*cosh(f*x + e)^3 + (4*a^3 - 17*a^2
*b + 4*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b - 4*a*b^2 + 4*(a^3 - 4
*a^2*b)*cosh(f*x + e)^2 + 4*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^6 + 15*(a^3
```

```
- 4*a^2*b)*cosh(f*x + e)^4 + a^3 - 4*a^2*b + 3*(4*a^3 - 17*a^2*b + 4*a*b^2)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b - 4*a*b^2)*cosh(f*x + e)^7 +
3*(a^3 - 4*a^2*b)*cosh(f*x + e)^5 + (4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x +
 e)^3 + (a^3 - 4*a^2*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a
- 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2
*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f
*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f
*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)
*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a -
2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x
+ e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x
 + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x
+ e) + 1)) + 2*sqrt(2)*((a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 6*(a^2*b
+ a*b^2 - 2*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2*b + a*b^2 - 2*b^3)*si
nh(f*x + e)^6 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + (4*a^
3 - 7*a^2*b + 5*a*b^2 - 2*b^3 + 15*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^2)
*sinh(f*x + e)^4 + 4*(5*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (4*a^3 -
7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - a^2*b - a*b^2 +
 2*b^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + (15*(a^2*b +
 a*b^2 - 2*b^3)*cosh(f*x + e)^4 - 4*a^3 + 7*a^2*b - 5*a*b^2 + 2*b^3 + 6*(4*
a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a
^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^5 + 2*(4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^
3)*cosh(f*x + e)^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^8 + 8*(a^4*b - 3*a^3*b^2 + 3*a^
2*b^3 - a*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*f*sinh(f*x + e)^8 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f
*cosh(f*x + e)^6 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x
+ e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)*sinh(f*x + e)^6 + 2*(4*a^
5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^
4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^3 + 3*(a^5 - 3*a^4*b +
 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^4 + 30*(a^5 - 3*a^4*b + 3*a^3*b
^2 - a^2*b^3)*f*cosh(f*x + e)^2 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^
3 + a*b^4)*f)*sinh(f*x + e)^4 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*c
osh(f*x + e)^2 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x +
e)^5 + 10*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^3 + (4*a^5
- 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e))*sinh(f*x + e)
^3 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^6 + 15*(a
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^4 + 3*(4*a^5 - 13*a^4*b
 + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a
^3*b^2 - a^2*b^3)*f)*sinh(f*x + e)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b
^4)*f + 8*((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^7 + 3*(a
```

```
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^5 + (4*a^5 - 13*a^4*b +
 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^3 + (a^5 - 3*a^4*b + 3*a^3
*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e)), 1/2*(((a^2*b - 4*a*b^2)*co
sh(f*x + e)^8 + 8*(a^2*b - 4*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b
- 4*a*b^2)*sinh(f*x + e)^8 + 4*(a^3 - 4*a^2*b)*cosh(f*x + e)^6 + 4*(a^3 - 4
*a^2*b + 7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b
 - 4*a*b^2)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b)*cosh(f*x + e))*sinh(f*x + e
)^5 + 2*(4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x + e)^4 + 2*(35*(a^2*b - 4*a*b
^2)*cosh(f*x + e)^4 + 4*a^3 - 17*a^2*b + 4*a*b^2 + 30*(a^3 - 4*a^2*b)*cosh(
f*x + e)^2)*sinh(f*x + e)^4 + 8*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^5 + 10*(
a^3 - 4*a^2*b)*cosh(f*x + e)^3 + (4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x + e)
)*sinh(f*x + e)^3 + a^2*b - 4*a*b^2 + 4*(a^3 - ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sech(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 984 vs. 2(126) = 252.
time = 0.88, size = 984, normalized size = 6.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (((a^3*b^2*e^(6*e) - 3*a^2*b^3*e^(6*e) + 3*a*b^4*e^(6*e) - b^5*e^(6*e))*e^(
2*f*x)/(a^6*e^(10*e) - 5*a^5*b*e^(10*e) + 10*a^4*b^2*e^(10*e) - 10*a^3*b^3*
e^(10*e) + 5*a^2*b^4*e^(10*e) - a*b^5*e^(10*e)) - (a^3*b^2*e^(4*e) - 3*a^2*
b^3*e^(4*e) + 3*a*b^4*e^(4*e) - b^5*e^(4*e))/(a^6*e^(10*e) - 5*a^5*b*e^(10*
e) + 10*a^4*b^2*e^(10*e) - 10*a^3*b^3*e^(10*e) + 5*a^2*b^4*e^(10*e) - a*b^5
*e^(10*e)))/sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b) + (a - 4*b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a
- b))/((a^2*e^(6*e) - 2*a*b*e^(6*e) + b^2*e^(6*e))*sqrt(a - b)) - 2*((sqrt(
b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(
2*f*x + 2*e) + b))^3*a - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b - 5*(sqrt(b)*e^(2*f
```

```
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^2*a*sqrt(b) + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(3/2) - 4*(sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))*a^2 - (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b + 2*(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b))*b^2 - 4*a^2*sqrt(b) + 5*a*b^(3/2) - 2*b^(5/2))/((a^2*e^(6*e) - 2*a*b*e^
(6*e) + b^2*e^(6*e))*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*
e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
))*sqrt(b) + 4*a - 3*b)^2))*e^(6*e)/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(e+f\,x\right)^3\,{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

3.386 $\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=325

$$-\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}+\frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3ab^2f}+\frac{(8a^2-}{}$$

```
[Out] -(a-b)*cosh(f*x+e)^3*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(4*a-3
*b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/b^2/f+1/3*(8*a^2-13
*a*b+3*b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x
+e)^2)^(1/2)/a/b^3/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/3*(2*a-3
*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+
e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(
1/2)/a/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(8*a^2-13*a*b+
3*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a/b^3/f
```

Rubi [A]
time = 0.21, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3271, 424, 542, 545, 429, 506, 422}

$$\frac{(8a^2-13ab+3b^2)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3ab^3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(8a^2-13ab+3b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3ab^3f}-\frac{2(2a-3b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3ab^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(4a-3b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3ab^2f}-\frac{(a-b)\sinh(e+fx)\cosh^3(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)*Cosh[e + f*x]^3*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) + ((4*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(3*a*b^2*f) + ((8*a^2 - 13*a*b + 3*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1
 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b^3*f*Sqrt[(Sech[e
+ f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(2*a - 3*b)*EllipticF[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b^2*f
*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((8*a^2 - 13*a*b + 3*
b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*b^3*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)}{} \\
&= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+f}{3ab^2f} \\
&= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+f}{3ab^2f} \\
&= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+f}{3ab^2f} \\
&= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+f}{3ab^2f}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.10, size = 196, normalized size = 0.60

$$\frac{4ia(8a^2-13ab+3b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-4ia(8a^2-17ab+9b^2)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,b(8a^2-13ab+6b^2+ab\cosh(2(e+fx)))\sinh(2(e+fx))}{12ab^3f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((4*I)*a*(8*a^2 - 13*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticE[I*(e + f*x), b/a] - (4*I)*a*(8*a^2 - 17*a*b + 9*b^2)*Sqrt[(2*a - b
 + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2 -
 13*a*b + 6*b^2 + a*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(12*a*b^3*f*Sqr
t[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.68, size = 498, normalized size = 1.53

method	result
default	$\sqrt{-\frac{b}{a}}\, ab\left(\cosh^4(fx+e)\right)\sinh(fx+e)+\left(4\sqrt{-\frac{b}{a}}\, a^2-7\sqrt{-\frac{b}{a}}\, ab+3\sqrt{-\frac{b}{a}}\, b^2\right)\left(\cosh^2(fx+e)\right)\sinh(fx+e)+4\sqrt{\frac{b(\cosh^2(fx+}{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*a*b*cosh(f*x+e)^4*sinh(f*x+e)+(4*(-1/a*b)^(1/2)*a^2-7*(
-1/a*b)^(1/2)*a*b+3*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+4*(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*a^2-7*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+3*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*b^2-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+13*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a*b-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/b^2/(-1/a*
b)^(1/2)/a/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.10, size = 55, normalized size = 0.17

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\cosh\left(fx+e\right)^6}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^6/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. $2(331) = 662$.
time = 2.83, size = 1201, normalized size = 3.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(((a^4*b^6*e^(17*e) - 2*a^3*b^7*e^(17*e) + a^2*b^8*e^(17*e))*e^(2*f*x)
/(a^4*b^7*e^(12*e) - 2*a^3*b^8*e^(12*e) + a^2*b^9*e^(12*e)) + 2*(14*a^5*b^5
*e^(15*e) - 53*a^4*b^6*e^(15*e) + 76*a^3*b^7*e^(15*e) - 49*a^2*b^8*e^(15*e)
 + 12*a*b^9*e^(15*e))/(a^4*b^7*e^(12*e) - 2*a^3*b^8*e^(12*e) + a^2*b^9*e^(1
2*e)))*e^(2*f*x) + (48*a^6*b^4*e^(13*e) - 216*a^5*b^5*e^(13*e) + 385*a^4*b^
6*e^(13*e) - 338*a^3*b^7*e^(13*e) + 145*a^2*b^8*e^(13*e) - 24*a*b^9*e^(13*e
))/(a^4*b^7*e^(12*e) - 2*a^3*b^8*e^(12*e) + a^2*b^9*e^(12*e)))*e^(f*x)/(sqr
t(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)*f) + 1
/24*(2*(11*a^2*sqrt(b)*e^(4*e) - 19*a*b^(3/2)*e^(4*e) + 6*b^(5/2)*e^(4*e))*
log(abs(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b - 2*a*sqrt(b) + b^(3/2)))/b + 2*(21*a^3
*e^(4*e) - 39*a^2*b*e^(4*e) + 10*a*b^2*e^(4*e) + 12*b^3*e^(4*e))*arctan(-(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b
*e^(2*f*x + 2*e) + b))/sqrt(-b))/(sqrt(-b)*b) - 3*(14*(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b))^3*a^3*e^(4*e) - 26*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*b*e^(4*e) + 9*(sqrt(b
)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b))^3*a*b^2*e^(4*e) + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^2*b^(3/2
)*e^(4*e) - 8*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(5/2)*e^(4*e) - 18*(sqrt(b)*e^
(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b))*a^3*b*e^(4*e) + 30*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
```

```
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b^2*e^(4*e) -
 11*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
 - 2*b*e^(2*f*x + 2*e) + b))*a*b^3*e^(4*e) - 8*a^2*b^(5/2)*e^(4*e) + 10*a*b
^(7/2)*e^(4*e))/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - b)^2*b))*e^(-3*e)/(a*b^2*f^2
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(e+f\,x)^6}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cosh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.387 $\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=244

$$-\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}-\frac{(2a-b)E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(}}{ab^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] -(a-b)*cosh(f*x+e)*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(1/2)-(2*a-b)*(1/(
1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+si
nh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/b
^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(1/(1+sinh(f*x+e)^2))^(1/2
)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-
b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/b/f/(sech(f*x+e)^2*(a+b
*sinh(f*x+e)^2)/a)^(1/2)+(2*a-b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a/b^
2/f
```

Rubi [A]
time = 0.16, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3271, 424, 545, 429, 506, 422}

$$-\frac{(2a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{ab^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{abf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{ab^2f}-\frac{(a-b)\sinh(e+fx)\cosh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)*Cosh[e + f*x]*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(a*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
 + b*Sinh[e + f*x]^2])/(a*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)
)/a]) + ((2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a*b^2*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)}{}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)}{}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}}{abf\sqrt{\frac{\operatorname{sech}^2(e+fx)\,(a}{}}}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(2a-b)E\left(\tan^{-1}(\sinh(e+fx))|1\right.}{ab^2f\sqrt{\frac{\operatorname{sech}^2(e+f}{}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 155, normalized size = 0.64

$$\frac{-2ia(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+(a-b)\left(4ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-\sqrt{2}\,b\sinh(2(e+fx))\right)}{2ab^2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-2*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] + (a - b)*((4*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ell
ipticF[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)]))/(2*a*b^2*f*Sqrt[2*
a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.72, size = 334, normalized size = 1.37

method	result

default	$-\frac{\sqrt{-\frac{b}{a}}\, a\left(\cosh^{2}(fx+e)\right)\sinh(fx+e)-\sqrt{-\frac{b}{a}}\, b\left(\cosh^{2}(fx+e)\right)\sinh(fx+e)+a\sqrt{\frac{b\left(\cosh^{2}(fx+e)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-1/a*b)^(1/2)*a*cosh(f*x+e)^2*sinh(f*x+e)-(-1/a*b)^(1/2)*b*cosh(f*x+e)^2
*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*(b/a*cosh(f*x+e)^2+(a-b)/a)
^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sin
h(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+b*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/b/
(-1/a*b)^(1/2)/a/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.10, size = 55, normalized size = 0.23

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\cosh\left(fx+e\right)^{4}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{32,[4,
2,4]%%%}+%%%{%%%{-64,[1]%%%},[4,2,3]%%%}+%%%{%%%{32,[2]%%%},[4,2,2]%%%}+%%%
{%%%{-64,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(e+f\,x)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.388 $\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\ \sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\ \sqrt{b}\ f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\ \sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/2)*Ell
ipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1/2
))/f/a^(1/2)/b^(1/2)/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(
f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3271, 422}

$$\frac{\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\ \sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\ \sqrt{b}\ f\sqrt{a+b\sinh^2(e+fx)}\ \sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])
/(Sqrt[a]*Sqrt[b]*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
```

```
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+x^2}}{(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\,\sqrt{b}\,f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\,\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 143, normalized size = 1.57

$$\frac{i\sqrt{2}\,a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,|\,\frac{b}{a}\right)-i\sqrt{2}\,a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,|\,\frac{b}{a}\right)+b\sinh(2(e+fx))}{abf\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x),
 b/a] - I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e
+ f*x), b/a] + b*Sinh[2*(e + f*x)])/(a*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e +
 f*x)]])
```

Maple [A]
time = 1.47, size = 181, normalized size = 1.99

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\,(\cosh^2(fx+e))\sinh(fx+e)+\sqrt{\frac{b(\cosh^2(fx+e))}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{}}{\sqrt{-\frac{b}{a}}\,a\cosh(fx+e)\sqrt{a+b\,(\sinh^2 (}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-1/a*b)^(1/2)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(b
```

```
/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/a/cosh(f*x+e)/(a+b*sinh(f*x+e)
^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(97) = 194$.
time = 0.16, size = 1068, normalized size = 11.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(4*(b^2*cosh(f*x + e)^4 + 4*b^2*cosh(f*x + e)*sinh(f*x + e)^3 + b^2*sinh(f
*x + e)^4 + 2*(2*a*b - b^2)*cosh(f*x + e)^2 + 2*(3*b^2*cosh(f*x + e)^2 + 2*
a*b - b^2)*sinh(f*x + e)^2 + b^2 + 4*(b^2*cosh(f*x + e)^3 + (2*a*b - b^2)*c
osh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a
+ b)/b)*sqrt((a^2 - a*b)/b^2)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/
b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 +
 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) + ((2*a*b - b^2)*cosh(f*x + e)
^4 + 4*(2*a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a*b - b^2)*sinh(f*x
 + e)^4 + 2*(4*a^2 - 4*a*b + b^2)*cosh(f*x + e)^2 + 2*(3*(2*a*b - b^2)*cosh
(f*x + e)^2 + 4*a^2 - 4*a*b + b^2)*sinh(f*x + e)^2 + 2*a*b - b^2 + 4*((2*a*
b - b^2)*cosh(f*x + e)^3 + (4*a^2 - 4*a*b + b^2)*cosh(f*x + e))*sinh(f*x +
e) - 2*(b^2*cosh(f*x + e)^4 + 4*b^2*cosh(f*x + e)*sinh(f*x + e)^3 + b^2*sin
h(f*x + e)^4 + 2*(2*a*b - b^2)*cosh(f*x + e)^2 + 2*(3*b^2*cosh(f*x + e)^2 +
 2*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 4*(b^2*cosh(f*x + e)^3 + (2*a*b - b^2
)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sq
rt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a
*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b
^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*(b^2*cosh(f*x +
e)^3 + 3*b^2*cosh(f*x + e)*sinh(f*x + e)^2 + b^2*sinh(f*x + e)^3 + (2*a*b -
 b^2)*cosh(f*x + e) + (3*b^2*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e))*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*b^3*f*cosh(f*x + e)^4
+ 4*a*b^3*f*cosh(f*x + e)*sinh(f*x + e)^3 + a*b^3*f*sinh(f*x + e)^4 + a*b^3
```

```
*f + 2*(2*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^2 + 2*(3*a*b^3*f*cosh(f*x + e)^2
 + (2*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^2 + 4*(a*b^3*f*cosh(f*x + e)^3 + (2
*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(e+f\,x)^2}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.389 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

```
[Out] -b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-I*(cos(I*e+I
*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*si
nh(f*x+e)^2)^(1/2)/a/(a-b)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3263, 21, 3257, 3256}

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*
f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
```

```
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\int \frac{-a-b\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+b\sin}}{a(a-b)\sqrt{1+\frac{b\sinh^2(e+}{a}}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+f}}{a(a-b)f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

Mathematica [A]
time = 0.12, size = 100, normalized size = 0.87

$$\frac{-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-\sqrt{2}\,b\sinh(2(e+fx))}{2a(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]])
```

Maple [A]
time = 1.15, size = 253, normalized size = 2.20

method	result
default	$-\frac{\sqrt{-\frac{b}{a}}\, b(\cosh^2(fx+e))\sinh(fx+e)-a\sqrt{\frac{b(\cosh^2(fx+e))}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-1/a*b)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)-a*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))+b*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh
(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/a/(a-
b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1464 vs. 2(123) = 246.
time = 0.11, size = 1464, normalized size = 12.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (((2*a*b^2 - b^3)*cosh(f*x + e)^4 + 4*(2*a*b^2 - b^3)*cosh(f*x + e)*sinh(f*
x + e)^3 + (2*a*b^2 - b^3)*sinh(f*x + e)^4 + 2*a*b^2 - b^3 + 2*(4*a^2*b - 4
*a*b^2 + b^3)*cosh(f*x + e)^2 + 2*(4*a^2*b - 4*a*b^2 + b^3 + 3*(2*a*b^2 - b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((2*a*b^2 - b^3)*cosh(f*x + e)^3 +
 (4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e) - 2*(b^3*cosh(f*x +
 e)^4 + 4*b^3*cosh(f*x + e)*sinh(f*x + e)^3 + b^3*sinh(f*x + e)^4 + b^3 + 2
*(2*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(3*b^3*cosh(f*x + e)^2 + 2*a*b^2 - b^3
```

```
)*sinh(f*x + e)^2 + 4*(b^3*cosh(f*x + e)^3 + (2*a*b^2 - b^3)*cosh(f*x + e))
*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b
^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
 b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b -
b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((2*a^2*b - a*b^2)*cosh(f*x + e)^4 + 4
*(2*a^2*b - a*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a^2*b - a*b^2)*sinh(f
*x + e)^4 + 2*a^2*b - a*b^2 + 2*(4*a^3 - 4*a^2*b + a*b^2)*cosh(f*x + e)^2 +
 2*(4*a^3 - 4*a^2*b + a*b^2 + 3*(2*a^2*b - a*b^2)*cosh(f*x + e)^2)*sinh(f*x
 + e)^2 + 4*((2*a^2*b - a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 4*a^2*b + a*b^2)*
cosh(f*x + e))*sinh(f*x + e) + 2*((a*b^2 - b^3)*cosh(f*x + e)^4 + 4*(a*b^2
- b^3)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b^2 - b^3)*sinh(f*x + e)^4 + a*b^
2 - b^3 + 2*(2*a^2*b - 3*a*b^2 + b^3)*cosh(f*x + e)^2 + 2*(2*a^2*b - 3*a*b^
2 + b^3 + 3*(a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((a*b^2 - b^
3)*cosh(f*x + e)^3 + (2*a^2*b - 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)
)*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)
/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f
*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2
 - a*b)/b^2))/b^2) - sqrt(2)*(b^3*cosh(f*x + e)^3 + 3*b^3*cosh(f*x + e)*sin
h(f*x + e)^2 + b^3*sinh(f*x + e)^3 + (2*a*b^2 - b^3)*cosh(f*x + e) + (3*b^3
*cosh(f*x + e)^2 + 2*a*b^2 - b^3)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2)))/((a^2*b^3 - a*b^4)*f*cosh(f*x + e)^4 + 4*(a^2*b^3 -
 a*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2*b^3 - a*b^4)*f*sinh(f*x + e)
^4 + 2*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^2 + 2*(3*(a^2*b^3 -
a*b^4)*f*cosh(f*x + e)^2 + (2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*f)*sinh(f*x + e)
^2 + (a^2*b^3 - a*b^4)*f + 4*((a^2*b^3 - a*b^4)*f*cosh(f*x + e)^3 + (2*a^3*
b^2 - 3*a^2*b^3 + a*b^4)*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sinh^{2}\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(-3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.390 $\int \frac{\text{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=217

$$\frac{\sqrt{b}\,(a+b)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\,(a-b)^2f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}-\frac{2bF\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)\text{sech}(e+}{a(a-b)^2f\sqrt{\frac{\text{sech}^2(e+fx)\,(a+}{a}}}$$

```
[Out] (a+b)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/
2)*EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b
)^(1/2))*b^(1/2)/(a-b)^2/f/a^(1/2)/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1
/2)/(a+b*sinh(f*x+e)^2)^(1/2)-2*b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sec
h(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x
+e)^2)/a)^(1/2)+tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3271, 425, 539, 429, 422}

$$\frac{\sqrt{b}\,(a+b)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\,f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{2b\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{af(a-b)^2\sqrt{\frac{\text{sech}^2(e+fx)\,(a+b\sinh^2(e+fx))}{a}}}+\frac{\tanh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqr
t[a]], 1 - a/b])/(Sqrt[a]*(a - b)^2*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[
e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (2*b*EllipticF[ArcTan[Sinh[e +
f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)^2*f*S
qrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + Tanh[e + f*x]/((a - b)*
f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}(a+bx^2)^{3/2}}\,dx, x, \sinh(e}{f}$$

$$= \frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subs}}{(-}$$

$$= \frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(2b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Su}}{(a-}$$

$$= \frac{\sqrt{b}\,(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\,(a-b)^2f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\,\sqrt{a+b\sinh^2(e+fx)}} - \frac{2bF\left(\tan^{-1}\right.}{a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.06, size = 178, normalized size = 0.82

$$\frac{i\sqrt{2}\,a(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,|\,\frac{b}{a}\right)-i\sqrt{2}\,a(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,|\,\frac{b}{a}\right)+\left(2a^2-ab+b^2+b(a+b)\cosh(2(e+fx))\right)\tanh(e+fx)}{a(a-b)^2f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (I*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
 + f*x), b/a] - I*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]
*EllipticF[I*(e + f*x), b/a] + (2*a^2 - a*b + b^2 + b*(a + b)*Cosh[2*(e + f
*x)])*Tanh[e + f*x])/(a*(a - b)^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]]
)
```

Maple [A]
time = 2.34, size = 342, normalized size = 1.58

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\,ab\left(\sinh^3(fx+e)\right)+\sqrt{-\frac{b}{a}}\,b^2\left(\sinh^3(fx+e)\right)-ab\sqrt{\frac{a+b\left(\sinh^2(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\text{EllipticF}\left(\sinh(fx+e\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-1/a*b)^(1/2)*a*b*sinh(f*x+e)^3+(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3-a*b*((a+
b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))+b^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1
/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-((a+b*sinh(f*x+e)^2)/
a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))*a*b-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sin
h(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+sinh(f*x+e)*(-1/a*b)^(1/2)*a^2+(-1
/a*b)^(1/2)*b^2*sinh(f*x+e))/(a-b)^2/a/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh
(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2786 vs. 2(231) = 462.
time = 0.14, size = 2786, normalized size = 12.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(((2*a^2*b + a*b^2 - b^3)*cosh(f*x + e)^6 + 6*(2*a^2*b + a*b^2 - b^3)*cosh
(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b + a*b^2 - b^3)*sinh(f*x + e)^6 + (8*a^
3 + 2*a^2*b - 5*a*b^2 + b^3)*cosh(f*x + e)^4 + (8*a^3 + 2*a^2*b - 5*a*b^2 +
 b^3 + 15*(2*a^2*b + a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(
2*a^2*b + a*b^2 - b^3)*cosh(f*x + e)^3 + (8*a^3 + 2*a^2*b - 5*a*b^2 + b^3)*
cosh(f*x + e))*sinh(f*x + e)^3 + 2*a^2*b + a*b^2 - b^3 + (8*a^3 + 2*a^2*b -
 5*a*b^2 + b^3)*cosh(f*x + e)^2 + (15*(2*a^2*b + a*b^2 - b^3)*cosh(f*x + e)
^4 + 8*a^3 + 2*a^2*b - 5*a*b^2 + b^3 + 6*(8*a^3 + 2*a^2*b - 5*a*b^2 + b^3)*
cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(2*a^2*b + a*b^2 - b^3)*cosh(f*x +
e)^5 + 2*(8*a^3 + 2*a^2*b - 5*a*b^2 + b^3)*cosh(f*x + e)^3 + (8*a^3 + 2*a^2
*b - 5*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b^2 + b^3)*cosh(f*
```

```
x + e)^6 + 6*(a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b^2 + b^3)*si
nh(f*x + e)^6 + (4*a^2*b + 3*a*b^2 - b^3)*cosh(f*x + e)^4 + (4*a^2*b + 3*a*
b^2 - b^3 + 15*(a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(a*b^2
 + b^3)*cosh(f*x + e)^3 + (4*a^2*b + 3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x
 + e)^3 + a*b^2 + b^3 + (4*a^2*b + 3*a*b^2 - b^3)*cosh(f*x + e)^2 + (15*(a*
b^2 + b^3)*cosh(f*x + e)^4 + 4*a^2*b + 3*a*b^2 - b^3 + 6*(4*a^2*b + 3*a*b^2
 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a*b^2 + b^3)*cosh(f*x + e)
^5 + 2*(4*a^2*b + 3*a*b^2 - b^3)*cosh(f*x + e)^3 + (4*a^2*b + 3*a*b^2 - b^3
)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sq
rt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a
*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b
^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 4*((2*a^2*b - a*b^2)*cos
h(f*x + e)^6 + 6*(2*a^2*b - a*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b
 - a*b^2)*sinh(f*x + e)^6 + (8*a^3 - 6*a^2*b + a*b^2)*cosh(f*x + e)^4 + (8*
a^3 - 6*a^2*b + a*b^2 + 15*(2*a^2*b - a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)
^4 + 4*(5*(2*a^2*b - a*b^2)*cosh(f*x + e)^3 + (8*a^3 - 6*a^2*b + a*b^2)*cos
h(f*x + e))*sinh(f*x + e)^3 + 2*a^2*b - a*b^2 + (8*a^3 - 6*a^2*b + a*b^2)*c
osh(f*x + e)^2 + (15*(2*a^2*b - a*b^2)*cosh(f*x + e)^4 + 8*a^3 - 6*a^2*b +
a*b^2 + 6*(8*a^3 - 6*a^2*b + a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3
*(2*a^2*b - a*b^2)*cosh(f*x + e)^5 + 2*(8*a^3 - 6*a^2*b + a*b^2)*cosh(f*x +
 e)^3 + (8*a^3 - 6*a^2*b + a*b^2)*cosh(f*x + e))*sinh(f*x + e) + ((a*b^2 -
b^3)*cosh(f*x + e)^6 + 6*(a*b^2 - b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b
^2 - b^3)*sinh(f*x + e)^6 + (4*a^2*b - 5*a*b^2 + b^3)*cosh(f*x + e)^4 + (4*
a^2*b - 5*a*b^2 + b^3 + 15*(a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 +
 4*(5*(a*b^2 - b^3)*cosh(f*x + e)^3 + (4*a^2*b - 5*a*b^2 + b^3)*cosh(f*x +
e))*sinh(f*x + e)^3 + a*b^2 - b^3 + (4*a^2*b - 5*a*b^2 + b^3)*cosh(f*x + e)
^2 + (15*(a*b^2 - b^3)*cosh(f*x + e)^4 + 4*a^2*b - 5*a*b^2 + b^3 + 6*(4*a^2
*b - 5*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a*b^2 - b^3)*c
osh(f*x + e)^5 + 2*(4*a^2*b - 5*a*b^2 + b^3)*cosh(f*x + e)^3 + (4*a^2*b - 5
*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*
sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*s
qrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2
 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*(4*a
^2*b*cosh(f*x + e)^3 + (a*b^2 + b^3)*cosh(f*x + e)^5 + 5*(a*b^2 + b^3)*cosh
(f*x + e)*sinh(f*x + e)^4 + (a*b^2 + b^3)*sinh(f*x + e)^5 + 2*(2*a^2*b + 5*
(a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^3 + 2*(6*a^2*b*cosh(f*x + e) +
 5*(a*b^2 + b^3)*cosh(f*x + e)^3)*sinh(f*x + e)^2 + (3*a*b^2 - b^3)*cosh(f*
x + e) + (12*a^2*b*cosh(f*x + e)^2 + 5*(a*b^2 + b^3)*cosh(f*x + e)^4 + 3*a*
b^2 - b^3)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))
/((a^3*b^2 - 2*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^6 + 6*(a^3*b^2 - 2*a^2*b^3
+ a*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f*
sinh(f*x + e)^6 + (4*a^4*b - 9*a^3*b^2 + 6*a^2*b^3 - a*b^4)*f*cosh(f*x + e)
^4 + (15*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^2 + (4*a^4*b - 9*a^3
*b^2 + 6*a^2*b^3 - a*b^4)*f)*sinh(f*x + e)^4 + (4*a^4*b - 9*a^3*b^2 + 6*a^2
```

```
*b^3 - a*b^4)*f*cosh(f*x + e)^2 + 4*(5*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*f*cosh
(f*x + e)^3 + (4*a^4*b - 9*a^3*b^2 + 6*a^2*b^3 - a*b^4)*f*cosh(f*x + e))*si
nh(f*x + e)^3 + (15*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^4 + 6*(4*
a^4*b - 9*a^3*b^2 + 6*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^2 + (4*a^4*b - 9*a^3
*b^2 + 6*a^2*b^3 - a*b^4)*f)*sinh(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4
)*f + 2*(3*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^5 + 2*(4*a^4*b - 9
*a^3*b^2 + 6*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^3 + (4*a^4*b - 9*a^3*b^2 + 6*
a^2*b^3 - a*b^4)*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sech(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.5Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh\left(e+f\,x\right)^2\,{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

3.391 $\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{5/2}f}-\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(5/2)/f-1/3*(a-b)*
cosh(f*x+e)^2*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/3*(a-b)*(3*a+2*
b)*sinh(f*x+e)/a^2/b^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 424, 393, 223, 212}

$$-\frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{5/2}f}-\frac{(a-b)\sinh(e+fx)\cosh^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(b^(5/2)*f) -
((a - b)*Cosh[e + f*x]^2*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3
/2)) - ((a - b)*(3*a + 2*b)*Sinh[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b*Sinh[e +
 f*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+bx^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+2b+3ax^2}{(a+bx^2)^{3/2}}\,dx, x, \sinh(e+\right.}{3abf}$$

$$= -\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} +$$

$$= -\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} +$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{5/2}f} - \frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Mathematica [A]
time = 0.64, size = 126, normalized size = 0.94

$$\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(e+fx)}{\sqrt{2a-b+b\cosh(2(e+fx))}}\right)}{b^{5/2}} + \frac{2\sqrt{2}(-a+b)\left(3a^2+ab-b^2+b(2a+b)\cosh(2(e+fx))\right)\sinh(e+fx)}{3a^2b^2(2a-b+b\cosh(2(e+fx)))^{3/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]
]]/b^(5/2) + (2*Sqrt[2]*(-a + b)*(3*a^2 + a*b - b^2 + b*(2*a + b)*Cosh[2*(e
 + f*x)])*Sinh[e + f*x])/(3*a^2*b^2*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2)))
/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.40, size = 65, normalized size = 0.49

method	result	size

default	$\frac{\text{‘}\mathit{int/indef0}\text{‘}\left(\frac{\cosh^4(fx+e)}{\left(b^2\left(\sinh^4(fx+e)\right)+2ab\left(\sinh^2(fx+e)\right)+a^2\right)\sqrt{a+b\left(\sinh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(cosh(f*x+e)^4/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2932 vs. $2(120) = 240$.
time = 0.69, size = 6774, normalized size = 50.55

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2*cosh(f*x + e)*sinh(f*x + e)^7
 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a
^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*
cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*
(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4 + 2*(35*a^2*b^2*cosh(f*x + e)
^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2)*
sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(f*x + e)^5 + 10*(2*a^3*b - a^
2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e))*sinh(
f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2 + 4*(7*a^2*b^2*cosh(f*x
+ e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^4 + 2*a^3*b - a^2*b^2 + 3*(8*
a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*(a^2*b^2*co
sh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^5 + (8*a^4 - 8*a^3*b +
3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x +
e))*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*
```

```
b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x
 + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*
a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(
f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2
*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2
 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*
a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x +
e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*
(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6
*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x +
 e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b +
 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2
 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*c
osh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2
 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3
*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^
4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f
*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a
*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b
+ 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 -
 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sin
h(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2
*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b
^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b
^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e
)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3
*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sin
h(f*x + e)^5 + sinh(f*x + e)^6)) + 3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2*c
osh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2*b
^2)*cosh(f*x + e)^6 + 4*(7*a^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*sin
h(f*x + e)^6 + 8*(7*a^2*b^2*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(f*
x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4 +
 2*(35*a^2*b^2*cosh(f*x + e)^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b
- a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(f
*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3*a
^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e
)^2 + 4*(7*a^2*b^2*cosh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^4
 + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^2)*sin
h(f*x + e)^2 + 8*(a^2*b^2*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x
+ e)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2
)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f
*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*
cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2
```

```
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*
x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)) - 8*sqrt(2)*((2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^6 + 6*(2*a^2*b^2
 - a*b^3 - b^4)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b^2 - a*b^3 - b^4)*s
inh(f*x + e)^6 + 3*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^4 + 3*
(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4 + 5*(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x +
 e)^2)*sinh(f*x + e)^4 - 2*a^2*b^2 + a*b^3 + b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.97Error: Bad Argum
ent Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(e+f\,x)^5}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cosh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.392 $\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\cosh^2(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] 1/3*cosh(f*x+e)^2*sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(3/2)+2/3*sinh(f*x+e)
/a^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3269, 386, 197}

$$\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)\cosh^2(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cosh[e + f*x]^2*Sinh[e + f*x])/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*
Sinh[e + f*x])/(3*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}\\
&= \frac{\cosh^2(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{3af}\\
&= \frac{\cosh^2(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 50, normalized size = 0.68

$$\frac{3a\sinh(e+fx)+(a+2b)\sinh^3(e+fx)}{3a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*a*Sinh[e + f*x] + (a + 2*b)*Sinh[e + f*x]^3)/(3*a^2*f*(a + b*Sinh[e + f*
x]^2)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.36, size = 65, normalized size = 0.89

method	result	size
default	$\frac{\text{`}int/indef0\text{`}\left(\frac{\cosh^2(fx+e)}{\left(b^2\left(\sinh^4(fx+e)\right)+2ab\left(\sinh^2(fx+e)\right)+a^2\right)\sqrt{a+b\left(\sinh^2(fx+e)\right)}}, \sinh(fx+e)\right)}{f}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 'int/indef0'(cosh(f*x+e)^2/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 955 vs. 2(70) = 140.
time = 0.52, size = 955, normalized size = 13.08

$$-\frac{b^4e^{(-10fx-10e)}-4a^3b+6a^2b^2-b^4-(16a^4-32a^3b+6a^2b^2+10ab^3-5b^4)e^{(-2fx-2e)}+10(2a^3b-3a^2b^2+3ab^3-b^4)e^{(-4fx-4e)}+10(3a^2b^2-3ab^3+b^4)e^{(-6fx-6e)}+5(2ab^3-b^4)e^{(-8fx-8e)}}{12(a^4-2a^3b+a^2b^2)(2(2a-b)e^{(-2fx-2e)}+be^{(-4fx-4e)}+b)^{\frac{3}{2}}f}+\frac{2a^2b^2-2ab^3+b^4+5(4a^3b-6a^2b^2+4ab^3-b^4)e^{(-2fx-2e)}+2(24a^4-48a^3b+49a^2b^2-25ab^3+5b^4)e^{(-4fx-4e)}+10(6a^3b-9a^2b^2+5ab^3-b^4)e^{(-6fx-6e)}+5(4a^2b^2-4ab^3+b^4)e^{(-8fx-8e)}+(2ab^3-b^4)e^{(-10fx-10e)}}{4(a^4-2a^3b+a^2b^2)(2(2a-b)e^{(-2fx-2e)}+be^{(-4fx-4e)}+b)^{\frac{3}{2}}f}-\frac{2ab^3-b^4+5(4a^2b^2-4ab^3+b^4)e^{(-2fx-2e)}+10(6a^3b-9a^2b^2+5ab^3-b^4)e^{(-4fx-4e)}+2(24a^4-48a^3b+49a^2b^2-25ab^3+5b^4)e^{(-6fx-6e)}+5(4a^3b-6a^2b^2+4ab^3-b^4)e^{(-8fx-8e)}+(2a^2b^2-2ab^3+b^4)e^{(-10fx-10e)}}{4(a^4-2a^3b+a^2b^2)(2(2a-b)e^{(-2fx-2e)}+be^{(-4fx-4e)}+b)^{\frac{3}{2}}f}+\frac{b^4+5(2ab^3-b^4)e^{(-2fx-2e)}+10(3a^2b^2-3ab^3+b^4)e^{(-4fx-4e)}+10(2a^3b-3a^2b^2+3ab^3-b^4)e^{(-6fx-6e)}-(16a^4-32a^3b+6a^2b^2+10ab^3-5b^4)e^{(-8fx-8e)}-(4a^3b-6a^2b^2+b^4)e^{(-10fx-10e)}}{12(a^4-2a^3b+a^2b^2)(2(2a-b)e^{(-2fx-2e)}+be^{(-4fx-4e)}+b)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b^4*e^(-10*f*x - 10*e) - 4*a^3*b + 6*a^2*b^2 - b^4 - (16*a^4 - 32*a^
3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-2*f*x - 2*e) + 10*(2*a^3*b - 3*a^2*
b^2 + 3*a*b^3 - b^4)*e^(-4*f*x - 4*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^(-
6*f*x - 6*e) + 5*(2*a*b^3 - b^4)*e^(-8*f*x - 8*e))/((a^4 - 2*a^3*b + a^2*b^
2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) + 1/4*(
2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-2*f
*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-4*f*x
 - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e) + 5*(4*
a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10*f*x - 10
*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x
 - 4*e) + b)^(5/2)*f) - 1/4*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*
e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-4*f*x - 4*e
) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-6*f*x - 6*e)
+ 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) + (2*a^2*b^2 - 2
*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e
^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) + 1/12*(b^4 + 5*(2*a*b^3
 - b^4)*e^(-2*f*x - 2*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^(-4*f*x - 4*e)
+ 10*(2*a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*e^(-6*f*x - 6*e) - (16*a^4 - 32*
a^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-8*f*x - 8*e) - (4*a^3*b - 6*a^2*b
^2 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2
*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(65) = 130$.
time = 0.49, size = 945, normalized size = 12.95

$$\frac{\sqrt{2}\left((a+2b)\cosh(fx+e)^6+6(a+2b)\cosh(fx+e)\sinh(fx+e)^5+(a+2b)\sinh(fx+e)^6+3(3a-2b)\cosh(fx+e)^4+3\left(5(a+2b)\cosh(fx+e)^2+3a-2b\right)\sinh(fx+e)^4+4\left(5(a+2b)\cosh(fx+e)^3+3(3a-2b)\cosh(fx+e)\right)\sinh(fx+e)^3-3(3a-2b)\cosh(fx+e)^2+3\left(5(a+2b)\cosh(fx+e)^4+6(3a-2b)\cosh(fx+e)^2-3a+2b\right)\sinh(fx+e)^2+6\left((a+2b)\cosh(fx+e)^5+2(3a-2b)\cosh(fx+e)^3-(3a-2b)\cosh(fx+e)\right)\sinh(fx+e)-a-2b\right)\sqrt{\frac{b\cosh(fx+e)^2+b\sinh(fx+e)^2+2a-b}{\cosh(fx+e)^2-2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^2}}}{3\left(a^2b^2f\cosh(fx+e)^8+8a^2b^2f\cosh(fx+e)\sinh(fx+e)^7+a^2b^2f\sinh(fx+e)^8+4(2a^3b-a^2b^2)f\cosh(fx+e)^6+4\left(7a^2b^2f\cosh(fx+e)^2+(2a^3b-a^2b^2)f\right)\sinh(fx+e)^6+2(8a^4-8a^3b+3a^2b^2)f\cosh(fx+e)^4+8\left(7a^2b^2f\cosh(fx+e)^3+3(2a^3b-a^2b^2)f\cosh(fx+e)\right)\sinh(fx+e)^5+a^2b^2f+2\left(35a^2b^2f\cosh(fx+e)^4+30(2a^3b-a^2b^2)f\cosh(fx+e)^2+(8a^4-8a^3b+3a^2b^2)f\right)\sinh(fx+e)^4+4(2a^3b-a^2b^2)f\cosh(fx+e)^2+8\left(7a^2b^2f\cosh(fx+e)^5+10(2a^3b-a^2b^2)f\cosh(fx+e)^3+(8a^4-8a^3b+3a^2b^2)f\cosh(fx+e)\right)\sinh(fx+e)^3+4\left(7a^2b^2f\cosh(fx+e)^6+15(2a^3b-a^2b^2)f\cosh(fx+e)^4+3(8a^4-8a^3b+3a^2b^2)f\cosh(fx+e)^2+(2a^3b-a^2b^2)f\right)\sinh(fx+e)^2+8\left(a^2b^2f\cosh(fx+e)^7+3(2a^3b-a^2b^2)f\cosh(fx+e)^5+(8a^4-8a^3b+3a^2b^2)f\cosh(fx+e)^3+(2a^3b-a^2b^2)f\cosh(fx+e)\right)\sinh(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*((a + 2*b)*cosh(f*x + e)^6 + 6*(a + 2*b)*cosh(f*x + e)*sinh(f*x
 + e)^5 + (a + 2*b)*sinh(f*x + e)^6 + 3*(3*a - 2*b)*cosh(f*x + e)^4 + 3*(5*
(a + 2*b)*cosh(f*x + e)^2 + 3*a - 2*b)*sinh(f*x + e)^4 + 4*(5*(a + 2*b)*cos
h(f*x + e)^3 + 3*(3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e)^3 - 3*(3*a - 2*b)
*cosh(f*x + e)^2 + 3*(5*(a + 2*b)*cosh(f*x + e)^4 + 6*(3*a - 2*b)*cosh(f*x
+ e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 6*((a + 2*b)*cosh(f*x + e)^5 + 2*(3*a
 - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) - a - 2*
b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a^2*b^2*f*cosh(f*x + e
)^8 + 8*a^2*b^2*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*f*sinh(f*x + e)^8
 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^6 + 4*(7*a^2*b^2*f*cosh(f*x + e)^2
```

```
 + (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)
*f*cosh(f*x + e)^4 + 8*(7*a^2*b^2*f*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)
*f*cosh(f*x + e))*sinh(f*x + e)^5 + a^2*b^2*f + 2*(35*a^2*b^2*f*cosh(f*x +
e)^4 + 30*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (8*a^4 - 8*a^3*b + 3*a^2*
b^2)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + 8*(7*a^
2*b^2*f*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (8*a^4
 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^2*b^2*f*c
osh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 3*(8*a^4 - 8*a^
3*b + 3*a^2*b^2)*f*cosh(f*x + e)^2 + (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^2
 + 8*(a^2*b^2*f*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^5 +
 (8*a^4 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*f*co
sh(f*x + e))*sinh(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(65) = 130.
time = 1.13, size = 333, normalized size = 4.56

$$\frac{\left(\left(\frac{\left(a^{3}e^{(12\,e)}-3\,ab^{2}e^{(12\,e)}+2\,b^{3}e^{(12\,e)}\right)e^{(2\,fx)}}{a^{4}e^{(6\,e)}-2\,a^{3}be^{(6\,e)}+a^{2}b^{2}e^{(6\,e)}}+\frac{3\left(3\,a^{3}e^{(10\,e)}-8\,a^{2}be^{(10\,e)}+7\,ab^{2}e^{(10\,e)}-2\,b^{3}e^{(10\,e)}\right)}{a^{4}e^{(6\,e)}-2\,a^{3}be^{(6\,e)}+a^{2}b^{2}e^{(6\,e)}}\right)e^{(2\,fx)}-\frac{3\left(3\,a^{3}e^{(8\,e)}-8\,a^{2}be^{(8\,e)}+7\,ab^{2}e^{(8\,e)}-2\,b^{3}e^{(8\,e)}\right)}{a^{4}e^{(6\,e)}-2\,a^{3}be^{(6\,e)}+a^{2}b^{2}e^{(6\,e)}}\right)e^{(2\,fx)}-\frac{a^{3}e^{(6\,e)}-3\,ab^{2}e^{(6\,e)}+2\,b^{3}e^{(6\,e)}}{a^{4}e^{(6\,e)}-2\,a^{3}be^{(6\,e)}+a^{2}b^{2}e^{(6\,e)}}}{3\left(be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e)}-2\,be^{(2\,fx+2\,e)}+b\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((((a^3*e^(12*e) - 3*a*b^2*e^(12*e) + 2*b^3*e^(12*e))*e^(2*f*x)/(a^4*e^
(6*e) - 2*a^3*b*e^(6*e) + a^2*b^2*e^(6*e)) + 3*(3*a^3*e^(10*e) - 8*a^2*b*e^
(10*e) + 7*a*b^2*e^(10*e) - 2*b^3*e^(10*e))/(a^4*e^(6*e) - 2*a^3*b*e^(6*e)
+ a^2*b^2*e^(6*e)))*e^(2*f*x) - 3*(3*a^3*e^(8*e) - 8*a^2*b*e^(8*e) + 7*a*b^
2*e^(8*e) - 2*b^3*e^(8*e))/(a^4*e^(6*e) - 2*a^3*b*e^(6*e) + a^2*b^2*e^(6*e)
))*e^(2*f*x) - (a^3*e^(6*e) - 3*a*b^2*e^(6*e) + 2*b^3*e^(6*e))/(a^4*e^(6*e)
 - 2*a^3*b*e^(6*e) + a^2*b^2*e^(6*e)))/((b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
 2*e) - 2*b*e^(2*f*x + 2*e) + b)^(3/2)*f)
```

Mupad [**B**]
time = 1.83, size = 144, normalized size = 1.97

$$\frac{2\,\mathrm{e}^{e+f\,x}\,\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)\,\sqrt{a+b\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}\,\left(a+2\,b+10\,a\,\mathrm{e}^{2\,e+2\,f\,x}+a\,\mathrm{e}^{4\,e+4\,f\,x}-4\,b\,\mathrm{e}^{2\,e+2\,f\,x}+2\,b\,\mathrm{e}^{4\,e+4\,f\,x}\right)}{3\,a^2\,f\,{\left(b+4\,a\,\mathrm{e}^{2\,e+2\,f\,x}-2\,b\,\mathrm{e}^{2\,e+2\,f\,x}+b\,\mathrm{e}^{4\,e+4\,f\,x}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] (2*exp(e + f*x)*(exp(2*e + 2*f*x) - 1)*(a + b*(exp(e + f*x)/2 - exp(- e - f
*x)/2)^2)^(1/2)*(a + 2*b + 10*a*exp(2*e + 2*f*x) + a*exp(4*e + 4*f*x) - 4*b
*exp(2*e + 2*f*x) + 2*b*exp(4*e + 4*f*x)))/(3*a^2*f*(b + 4*a*exp(2*e + 2*f*
x) - 2*b*exp(2*e + 2*f*x) + b*exp(4*e + 4*f*x))^2)
```

3.393 $\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] 1/3*sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(3/2)+2/3*sinh(f*x+e)/a^2/f/(a+b*si
nh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3269, 198, 197}

$$\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Sinh[e + f*x]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*Sinh[e + f*x])/(3*
a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{3af}$$

$$= \frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.72

$$\frac{\sinh(e+fx)\left(3a+2b\sinh^2(e+fx)\right)}{3a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sinh[e + f*x]*(3*a + 2*b*Sinh[e + f*x]^2))/(3*a^2*f*(a + b*Sinh[e + f*x]^2
)^(3/2))
```

Maple [A]
time = 1.12, size = 56, normalized size = 0.86

method	result	size
derivativedivides	$\frac{\frac{\sinh(fx+e)}{3a\left(a+b\left(\sinh^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sinh(fx+e)}{3a^2\sqrt{a+b\left(\sinh^2(fx+e)\right)}}}{f}$	56
default	$\frac{\frac{\sinh(fx+e)}{3a\left(a+b\left(\sinh^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sinh(fx+e)}{3a^2\sqrt{a+b\left(\sinh^2(fx+e)\right)}}}{f}$	56
risch	Expression too large to display	316751

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*sinh(f*x+e)/a/(a+b*sinh(f*x+e)^2)^(3/2)+2/3/a^2*sinh(f*x+e)/(a+b*s
inh(f*x+e)^2)^(1/2))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(61) = 122$.
time = 0.50, size = 499, normalized size = 7.68

$$\frac{2\,a^2b^2 - 2\,ab^3 + b^4 + 5\,(4\,a^3b - 6\,a^2b^2 + 4\,ab^3 - b^4)e^{(-2\,fx - 2\,e)} + 2\,(24\,a^4 - 48\,a^3b + 49\,a^2b^2 - 25\,ab^3 + 5\,b^4)e^{(-4\,fx - 4\,e)} + 10\,(6\,a^3b - 9\,a^2b^2 + 5\,ab^3 - b^4)e^{(-6\,fx - 6\,e)} + 5\,(4\,a^2b^2 - 4\,ab^3 + b^4)e^{(-8\,fx - 8\,e)} + (2\,ab^3 - b^4)e^{(-10\,fx - 10\,e)}}{3\,(a^4 - 2\,a^3b + a^2b^2)(2\,(2\,a - b)e^{(-2\,fx - 2\,e)} + be^{(-4\,fx - 4\,e)} + b)^{\frac{5}{2}}f} - \frac{2\,ab^3 - b^4 + 5\,(4\,a^2b^2 - 4\,ab^3 + b^4)e^{(-2\,fx - 2\,e)} + 10\,(6\,a^3b - 9\,a^2b^2 + 5\,ab^3 - b^4)e^{(-4\,fx - 4\,e)} + 2\,(24\,a^4 - 48\,a^3b + 49\,a^2b^2 - 25\,ab^3 + 5\,b^4)e^{(-6\,fx - 6\,e)} + 5\,(4\,a^3b - 6\,a^2b^2 + 4\,ab^3 - b^4)e^{(-8\,fx - 8\,e)} + (2\,a^2b^2 - 2\,ab^3 + b^4)e^{(-10\,fx - 10\,e)}}{3\,(a^4 - 2\,a^3b + a^2b^2)(2\,(2\,a - b)e^{(-2\,fx - 2\,e)} + be^{(-4\,fx - 4\,e)} + b)^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^
(-2*f*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-
4*f*x - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e) +
5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10*f*x
 - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-
4*f*x - 4*e) + b)^(5/2)*f) - 1/3*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 +
b^4)*e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-4*f*x
- 4*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-6*f*x -
6*e) + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) + (2*a^2*b^
2 - 2*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a -
 b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(57) = 114$.
time = 0.50, size = 912, normalized size = 14.03

$$\frac{2\,\sqrt{2}\,\left(b\cosh\left(fx+e\right)^{6} + 6\,b\cosh\left(fx+e\right)\sinh\left(fx+e\right)^{5} + b\sinh\left(fx+e\right)^{6} + 3\,(2\,a-b)\cosh\left(fx+e\right)^{4} + 3\,\left(5\,b\cosh\left(fx+e\right)^{2} + 2\,a - b\right)\sinh\left(fx+e\right)^{4} + 4\,\left(5\,b\cosh\left(fx+e\right)^{3} + 3\,(2\,a-b)\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^{3} - 3\,(2\,a-b)\cosh\left(fx+e\right)^{2} + 3\,\left(5\,b\cosh\left(fx+e\right)^{4} + 6\,(2\,a-b)\cosh\left(fx+e\right)^{2} - 2\,a + b\right)\sinh\left(fx+e\right)^{2} + 6\,\left(b\cosh\left(fx+e\right)^{5} + 2\,(2\,a-b)\cosh\left(fx+e\right)^{3} - (2\,a-b)\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right) - b\right)\sqrt{\frac{b\cosh\left(fx+e\right)^{2} + b\sinh\left(fx+e\right)^{2} + 2\,a - b}{\cosh\left(fx+e\right)^{2} - 2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right) + \sinh\left(fx+e\right)^{2}}}}{3\,\left(a^{2}b^{2}f\cosh\left(fx+e\right)^{8} + 8\,a^{2}b^{2}f\cosh\left(fx+e\right)\sinh\left(fx+e\right)^{7} + a^{2}b^{2}f\sinh\left(fx+e\right)^{8} + 4\,(2\,a^{3}b - a^{2}b^{2})f\cosh\left(fx+e\right)^{6} + 4\,\left(7\,a^{2}b^{2}f\cosh\left(fx+e\right)^{2} + (2\,a^{3}b - a^{2}b^{2})f\right)\sinh\left(fx+e\right)^{6} + 2\,(8\,a^{4} - 8\,a^{3}b + 3\,a^{2}b^{2})f\cosh\left(fx+e\right)^{4} + 8\,\left(7\,a^{2}b^{2}f\cosh\left(fx+e\right)^{3} + 3\,(2\,a^{3}b - a^{2}b^{2})f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^{5} + a^{2}b^{2}f + 2\,\left(35\,a^{2}b^{2}f\cosh\left(fx+e\right)^{4} + 30\,(2\,a^{3}b - a^{2}b^{2})f\cosh\left(fx+e\right)^{2} + (8\,a^{4} - 8\,a^{3}b + 3\,a^{2}b^{2})f\right)\sinh\left(fx+e\right)^{4} + 4\,(2\,a^{3}b - a^{2}b^{2})f\cosh\left(fx+e\right)^{2} + 8\,\left(7\,a^{2}b^{2}f\cosh\left(fx+e\right)^{5} + 10\,(2\,a^{3}b - a^{2}b^{2})f\cosh\left(fx+e\right)^{3} + (8\,a^{4} - 8\,a^{3}b + 3\,a^{2}b^{2})f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^{3} + 4\,\left(7\,a^{2}b^{2}f\cosh\left(fx+e\right)^{6} + 15\,(2\,a^{3}b - a^{2}b^{2})f\cosh\left(fx+e\right)^{4} + 3\,(8\,a^{4} - 8\,a^{3}b + 3\,a^{2}b^{2})f\cosh\left(fx+e\right)^{2} + (2\,a^{3}b - a^{2}b^{2})f\right)\sinh\left(fx+e\right)^{2} + 8\,\left(a^{2}b^{2}f\cosh\left(fx+e\right)^{7} + 3\,(2\,a^{3}b - a^{2}b^{2})f\cosh\left(fx+e\right)^{5} + (8\,a^{4} - 8\,a^{3}b + 3\,a^{2}b^{2})f\cosh\left(fx+e\right)^{3} + (2\,a^{3}b - a^{2}b^{2})f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(2)*(b*cosh(f*x + e)^6 + 6*b*cosh(f*x + e)*sinh(f*x + e)^5 + b*sinh
(f*x + e)^6 + 3*(2*a - b)*cosh(f*x + e)^4 + 3*(5*b*cosh(f*x + e)^2 + 2*a -
b)*sinh(f*x + e)^4 + 4*(5*b*cosh(f*x + e)^3 + 3*(2*a - b)*cosh(f*x + e))*si
nh(f*x + e)^3 - 3*(2*a - b)*cosh(f*x + e)^2 + 3*(5*b*cosh(f*x + e)^4 + 6*(2
*a - b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 6*(b*cosh(f*x + e)^5 +
 2*(2*a - b)*cosh(f*x + e)^3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e) - b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a^2*b^2*f*cosh(f*x + e)^8
 + 8*a^2*b^2*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*f*sinh(f*x + e)^8 +
4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^6 + 4*(7*a^2*b^2*f*cosh(f*x + e)^2 +
(2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*f*
cosh(f*x + e)^4 + 8*(7*a^2*b^2*f*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*f*
cosh(f*x + e))*sinh(f*x + e)^5 + a^2*b^2*f + 2*(35*a^2*b^2*f*cosh(f*x + e)^
4 + 30*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (8*a^4 - 8*a^3*b + 3*a^2*b^2
)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + 8*(7*a^2*b
```

```
^2*f*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (8*a^4 -
8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^2*b^2*f*cosh
(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 3*(8*a^4 - 8*a^3*b
 + 3*a^2*b^2)*f*cosh(f*x + e)^2 + (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^2 +
8*(a^2*b^2*f*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^5 + (8
*a^4 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*f*cosh(
f*x + e))*sinh(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(57) = 114.
time = 0.81, size = 333, normalized size = 5.12

$$\frac{2\left(\left(\left(\frac{\left(a^{2}be^{(12\,e)}-2\,ab^{2}e^{(12\,e)}+b^{3}e^{(12\,e)}\right)e^{(2\,fx)}}{a^{4}e^{(6\,e)}-2\,a^{3}be^{(6\,e)}+a^{2}b^{2}e^{(6\,e)}}+\frac{3\left(2\,a^{3}e^{(10\,e)}-5\,a^{2}be^{(10\,e)}+4\,ab^{2}e^{(10\,e)}-b^{3}e^{(10\,e)}\right)}{a^{4}e^{(6\,e)}-2\,a^{3}be^{(6\,e)}+a^{2}b^{2}e^{(6\,e)}}\right)e^{(2\,fx)}-\frac{3\left(2\,a^{3}e^{(8\,e)}-5\,a^{2}be^{(8\,e)}+4\,ab^{2}e^{(8\,e)}-b^{3}e^{(8\,e)}\right)}{a^{4}e^{(6\,e)}-2\,a^{3}be^{(6\,e)}+a^{2}b^{2}e^{(6\,e)}}\right)e^{(2\,fx)}-\frac{a^{2}be^{(6\,e)}-2\,ab^{2}e^{(6\,e)}+b^{3}e^{(6\,e)}}{a^{4}e^{(6\,e)}-2\,a^{3}be^{(6\,e)}+a^{2}b^{2}e^{(6\,e)}}\right)}{3\left(be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e)}-2\,be^{(2\,fx+2\,e)}+b\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*((((a^2*b*e^(12*e) - 2*a*b^2*e^(12*e) + b^3*e^(12*e))*e^(2*f*x)/(a^4*e^
(6*e) - 2*a^3*b*e^(6*e) + a^2*b^2*e^(6*e)) + 3*(2*a^3*e^(10*e) - 5*a^2*b*e^
(10*e) + 4*a*b^2*e^(10*e) - b^3*e^(10*e))/(a^4*e^(6*e) - 2*a^3*b*e^(6*e) +
a^2*b^2*e^(6*e)))*e^(2*f*x) - 3*(2*a^3*e^(8*e) - 5*a^2*b*e^(8*e) + 4*a*b^2*
e^(8*e) - b^3*e^(8*e))/(a^4*e^(6*e) - 2*a^3*b*e^(6*e) + a^2*b^2*e^(6*e)))*e
^(2*f*x) - (a^2*b*e^(6*e) - 2*a*b^2*e^(6*e) + b^3*e^(6*e))/(a^4*e^(6*e) - 2
*a^3*b*e^(6*e) + a^2*b^2*e^(6*e)))/((b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b)^(3/2)*f)
```

Mupad [**B**]
time = 1.49, size = 129, normalized size = 1.98

$$\frac{4\,\mathrm{e}^{e+f\,x}\,\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)\,\sqrt{a+b\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}\,\left(b+6\,a\,\mathrm{e}^{2\,e+2\,f\,x}-2\,b\,\mathrm{e}^{2\,e+2\,f\,x}+b\,\mathrm{e}^{4\,e+4\,f\,x}\right)}{3\,a^2\,f\,{\left(b+4\,a\,\mathrm{e}^{2\,e+2\,f\,x}-2\,b\,\mathrm{e}^{2\,e+2\,f\,x}+b\,\mathrm{e}^{4\,e+4\,f\,x}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] (4*exp(e + f*x)*(exp(2*e + 2*f*x) - 1)*(a + b*(exp(e + f*x)/2 - exp(- e - f
*x)/2)^2)^(1/2)*(b + 6*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) + b*exp(4*
e + 4*f*x)))/(3*a^2*f*(b + 4*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) + b*
exp(4*e + 4*f*x))^2)
```

3.394 $\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{5/2}f}-\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

```
[Out] arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3
*b*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/3*(5*a-2*b)*b*sinh(f*x
+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3269, 425, 541, 12, 385, 209}

$$-\frac{b(5a-2b)\sinh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{b\sinh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/((a - b)^(5
/2)*f) - (b*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) -
((5*a - 2*b)*b*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-2b-2bx^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \text{sin}\right.}{3a(a-b)f}$$

$$= -\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$= -\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$= -\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 8.50, size = 1331, normalized size = 9.93

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sech[e + f*x]*Tanh[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]
] + (2100*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2)/a +
(840*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^4)/a^2 - (
3150*(a - b)*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Tanh[e + f*x]^2)/a -
 (4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2*
Tanh[e + f*x]^2)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]
^2)/a]]*Sinh[e + f*x]^4*Tanh[e + f*x]^2)/a^3 + (1575*(a - b)^2*ArcSin[Sqrt[
```

```
((a - b)*Tanh[e + f*x]^2)/a]]*Tanh[e + f*x]^4)/a^2 + (2100*(a - b)^2*b*ArcS
in[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2*Tanh[e + f*x]^4)/a^3
 + (840*(a - b)^2*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x
]^4*Tanh[e + f*x]^4)/a^4 + 2100*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^
2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) + (2800*b*Sinh[e + f*x]^2*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(
3/2))/a + (1120*b^2*Sinh[e + f*x]^4*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f
*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))/a^2 + 96*Hypergeometric2F1[
2, 2, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
 + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + 24*HypergeometricPFQ[{
2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a +
 b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + (168*b*Hyperg
eometric2F1[2, 2, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(S
ech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7
/2))/a + (48*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x
]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(
((a - b)*Tanh[e + f*x]^2)/a)^(7/2))/a + (72*b^2*Hypergeometric2F1[2, 2, 9/2
, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^4*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2))/a^2 + (24*b^2*Hy
pergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e +
f*x]^4*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e +
 f*x]^2)/a)^(7/2))/a^2 - 1575*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2)*Tanh[e + f*x]^2)/a^2] - (2100*b*Sinh[e + f*x]^2*Sqrt[((a - b)*Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2])/a - (840*b^2*Sin
h[e + f*x]^4*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e +
 f*x]^2)/a^2])/a^2))/(315*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]*Sqrt[(Sech[e +
f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(1 + (b*Sinh[e + f*x]^2)/a)*(((a - b)*Ta
nh[e + f*x]^2)/a)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.93, size = 169, normalized size = 1.26

method	result
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{-b^{2}\left(\sinh^{4}(f x+e)\right)-2 a b\left(\sinh^{2}(f x+e)\right)-a^{2}}{\left(-b^{4}\left(\sinh^{10}(f x+e)\right)+\left(-4 a\, b^{3}-b^{4}\right)\left(\sinh^{8}(f x+e)\right)+\left(-6 a^{2} b^{2}-4 a\, b^{3}\right)\left(\sinh^{6}(f x+e)\right)+\left(-4 a^{3} b-6 a^{2} b^{2}\right)\left(\sinh^{4}(f x+e)\right)+\left(-a^{4}-4 a^{3}\right.\right.}\right.}{f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`((-b^2*sinh(f*x+e)^4-2*a*b*sinh(f*x+e)^2-a^2)/(-b^4*sinh(f*x+e)
^10+(-4*a*b^3-b^4)*sinh(f*x+e)^8+(-6*a^2*b^2-4*a*b^3)*sinh(f*x+e)^6+(-4*a^3
```

```
*b-6*a^2*b^2)*sinh(f*x+e)^4+(-a^4-4*a^3*b)*sinh(f*x+e)^2-a^4)/(a+b*sinh(f*x
+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2640 vs. $2(120) = 240$.
time = 0.64, size = 5396, normalized size = 40.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2*cosh(f*x + e)*sinh(f*x + e)^7
 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a
^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*
cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*
(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4 + 2*(35*a^2*b^2*cosh(f*x + e)
^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2)*
sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(f*x + e)^5 + 10*(2*a^3*b - a^
2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e))*sinh(
f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2 + 4*(7*a^2*b^2*cosh(f*x
+ e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^4 + 2*a^3*b - a^2*b^2 + 3*(8*
a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*(a^2*b^2*co
sh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^5 + (8*a^4 - 8*a^3*b +
3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x +
e))*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)
*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^
2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)
*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sq
rt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*
cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh
(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh
(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 +
 cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*((5*a^2*b^2 - 7*a*b^3 + 2*b
^4)*cosh(f*x + e)^6 + 6*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)*sinh(f*
```

```
x + e)^5 + (5*a^2*b^2 - 7*a*b^3 + 2*b^4)*sinh(f*x + e)^6 + 3*(8*a^3*b - 17*
a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)^4 + 3*(8*a^3*b - 17*a^2*b^2 + 11*
a*b^3 - 2*b^4 + 5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^2)*sinh(f*x +
 e)^4 - 5*a^2*b^2 + 7*a*b^3 - 2*b^4 + 4*(5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*co
sh(f*x + e)^3 + 3*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e))*
sinh(f*x + e)^3 - 3*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)
^2 + 3*(5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^4 - 8*a^3*b + 17*a^2*
b^2 - 11*a*b^3 + 2*b^4 + 6*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f
*x + e)^2)*sinh(f*x + e)^2 + 6*((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)
^5 + 2*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)^3 - (8*a^3*b
 - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cos
h(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
 e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 -
a^2*b^5)*f*cosh(f*x + e)^8 + 8*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*
f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^
5)*f*sinh(f*x + e)^8 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2
*b^5)*f*cosh(f*x + e)^6 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*
f*cosh(f*x + e)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)
*f)*sinh(f*x + e)^6 + 2*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^
3*b^4 - 3*a^2*b^5)*f*cosh(f*x + e)^4 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^
4 - a^2*b^5)*f*cosh(f*x + e)^3 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3
*b^4 + a^2*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^5*b^2 - 3*a^4*b
^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^4 + 30*(2*a^6*b - 7*a^5*b^2 + 9*a
^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^2 + (8*a^7 - 32*a^6*b + 51*a^
5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f)*sinh(f*x + e)^4 + 4*(2*a^6*
b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^2 + 8*(7*(
a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^5 + 10*(2*a^6*b
 - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^3 + (8*a^7 -
 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f*cosh(f*x +
e))*sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*co
sh(f*x + e)^6 + 15*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*
f*cosh(f*x + e)^4 + 3*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*
b^4 - 3*a^2*b^5)*f*cosh(f*x + e)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a
^3*b^4 + a^2*b^5)*f)*sinh(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*f + 8*((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^
7 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x +
e)^5 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5
)*f*cosh(f*x + e)^3 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^
5)*f*cosh(f*x + e))*sinh(f*x + e)), 1/3*(3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2
*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b -
 a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^
2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*c
osh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 8*a^...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sech(e + f*x)/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1177 vs. 2(120) = 240.
time = 0.73, size = 1177, normalized size = 8.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((((5*a^9*b^2*e^(12*e) - 42*a^8*b^3*e^(12*e) + 156*a^7*b^4*e^(12*e) -
 336*a^6*b^5*e^(12*e) + 462*a^5*b^6*e^(12*e) - 420*a^4*b^7*e^(12*e) + 252*a
^3*b^8*e^(12*e) - 96*a^2*b^9*e^(12*e) + 21*a*b^10*e^(12*e) - 2*b^11*e^(12*e
))*e^(2*f*x)/(a^12*e^(12*e) - 10*a^11*b*e^(12*e) + 45*a^10*b^2*e^(12*e) - 1
20*a^9*b^3*e^(12*e) + 210*a^8*b^4*e^(12*e) - 252*a^7*b^5*e^(12*e) + 210*a^6
*b^6*e^(12*e) - 120*a^5*b^7*e^(12*e) + 45*a^4*b^8*e^(12*e) - 10*a^3*b^9*e^(
12*e) + a^2*b^10*e^(12*e)) + 3*(8*a^10*b*e^(10*e) - 73*a^9*b^2*e^(10*e) + 2
98*a^8*b^3*e^(10*e) - 716*a^7*b^4*e^(10*e) + 1120*a^6*b^5*e^(10*e) - 1190*a
^5*b^6*e^(10*e) + 868*a^4*b^7*e^(10*e) - 428*a^3*b^8*e^(10*e) + 136*a^2*b^9
*e^(10*e) - 25*a*b^10*e^(10*e) + 2*b^11*e^(10*e))/(a^12*e^(12*e) - 10*a^11*
b*e^(12*e) + 45*a^10*b^2*e^(12*e) - 120*a^9*b^3*e^(12*e) + 210*a^8*b^4*e^(1
2*e) - 252*a^7*b^5*e^(12*e) + 210*a^6*b^6*e^(12*e) - 120*a^5*b^7*e^(12*e) +
 45*a^4*b^8*e^(12*e) - 10*a^3*b^9*e^(12*e) + a^2*b^10*e^(12*e)))*e^(2*f*x)
- 3*(8*a^10*b*e^(8*e) - 73*a^9*b^2*e^(8*e) + 298*a^8*b^3*e^(8*e) - 716*a^7*
b^4*e^(8*e) + 1120*a^6*b^5*e^(8*e) - 1190*a^5*b^6*e^(8*e) + 868*a^4*b^7*e^(
8*e) - 428*a^3*b^8*e^(8*e) + 136*a^2*b^9*e^(8*e) - 25*a*b^10*e^(8*e) + 2*b^
11*e^(8*e))/(a^12*e^(12*e) - 10*a^11*b*e^(12*e) + 45*a^10*b^2*e^(12*e) - 12
0*a^9*b^3*e^(12*e) + 210*a^8*b^4*e^(12*e) - 252*a^7*b^5*e^(12*e) + 210*a^6*
b^6*e^(12*e) - 120*a^5*b^7*e^(12*e) + 45*a^4*b^8*e^(12*e) - 10*a^3*b^9*e^(1
2*e) + a^2*b^10*e^(12*e)))*e^(2*f*x) - (5*a^9*b^2*e^(6*e) - 42*a^8*b^3*e^(6
*e) + 156*a^7*b^4*e^(6*e) - 336*a^6*b^5*e^(6*e) + 462*a^5*b^6*e^(6*e) - 420
*a^4*b^7*e^(6*e) + 252*a^3*b^8*e^(6*e) - 96*a^2*b^9*e^(6*e) + 21*a*b^10*e^(
6*e) - 2*b^11*e^(6*e))/(a^12*e^(12*e) - 10*a^11*b*e^(12*e) + 45*a^10*b^2*e^
(12*e) - 120*a^9*b^3*e^(12*e) + 210*a^8*b^4*e^(12*e) - 252*a^7*b^5*e^(12*e)
 + 210*a^6*b^6*e^(12*e) - 120*a^5*b^7*e^(12*e) + 45*a^4*b^8*e^(12*e) - 10*a
^3*b^9*e^(12*e) + a^2*b^10*e^(12*e)))/(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
```

```
*e) - 2*b*e^(2*f*x + 2*e) + b)^(3/2) - 6*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*
e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
) + sqrt(b))/sqrt(a - b))/((a^2*e^(6*e) - 2*a*b*e^(6*e) + b^2*e^(6*e))*sqrt
(a - b)))*e^(6*e)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(e+f\,x\right)\,\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(5/2)), x)
```

3.395 $\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=330

$$-\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(8a^2-3ab-2b^2)\,E(\mathrm{A}}{3a}$$

```
[Out] -1/3*(a-b)*cosh(f*x+e)^3*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(a
-b)*(2*a+b)*cosh(f*x+e)*sinh(f*x+e)/a^2/b^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/3
*(8*a^2-3*a*b-2*b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*El
lipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b
*sinh(f*x+e)^2)^(1/2)/a^2/b^3/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
+1/3*(4*a-b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(
sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/a^2/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(8*
a^2-3*a*b-2*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a^2/b^3/f
```

Rubi [A]
time = 0.23, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3271, 424, 540, 545, 429, 506, 422}

$$\frac{(4a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3a^2b^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(8a^2-3ab-2b^2)\,\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3a^2b^3f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a-b)(2a+b)\sinh(e+fx)\cosh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{(8a^2-3ab-2b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2b^3f}-\frac{(a-b)\sinh(e+fx)\cosh^3(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*((a - b)*Cosh[e + f*x]^3*Sinh[e + f*x])/(a*b*f*(a + b*Sinh[e + f*x]^2)
^(3/2)) - (2*(a - b)*(2*a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*b^2*f*Sq
rt[a + b*Sinh[e + f*x]^2]) - ((8*a^2 - 3*a*b - 2*b^2)*EllipticE[ArcTan[Sinh
[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b^3*
f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((4*a - b)*EllipticF
[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(3*a^2*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((8*a^2
- 3*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*b^3*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{(1+x^2)^{5/2}}{(a+bx^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)}{}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx}}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx}}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx}}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.60, size = 206, normalized size = 0.62

$$\frac{-2ia^2(8a^2-3ab-2b^2)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\frac{1}{2}(a-b)\left(4ia^2(8a+b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2\sqrt{2}\,b(8a^2+ab-2b^2+b(5a+2b)\cosh(2(e+fx)))\sinh(2(e+fx))\right)}{6a^2b^3f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-2*I)*a^2*(8*a^2 - 3*a*b - 2*b^2)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/
2)*EllipticE[I*(e + f*x), b/a] + ((a - b)*((4*I)*a^2*(8*a + b)*((2*a - b +
b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] - 2*Sqrt[2]*b*(8*
a^2 + a*b - 2*b^2 + b*(5*a + 2*b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)]))/2)
/(6*a^2*b^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(386) = 772$.
time = 1.85, size = 812, normalized size = 2.46

method	result
default	$-\left(5\sqrt{-\frac{b}{a}}\,a^{2}b-3\sqrt{-\frac{b}{a}}\,a\,b^{2}-2\sqrt{-\frac{b}{a}}\,b^{3}\right)\left(\cosh^{4}(fx+e)\right)\sinh(fx+e)+\left(4\sqrt{-\frac{b}{a}}\,a^{3}-6\sqrt{-\frac{b}{a}}\,a^{2}b+2\sqrt{-\frac{b}{a}}\,b^{3}\right)(\cosh^{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((5*(-1/a*b)^(1/2)*a^2*b-3*(-1/a*b)^(1/2)*a*b^2-2*(-1/a*b)^(1/2)*b^3)*
cosh(f*x+e)^4*sinh(f*x+e)+(4*(-1/a*b)^(1/2)*a^3-6*(-1/a*b)^(1/2)*a^2*b+2*(-
1/a*b)^(1/2)*b^3)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*b*(4*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*a^2-2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*Elliptic
F(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-8*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a^2+3*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*b+2*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e
)^2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sin
h(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3-6*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2
)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a
^2*b+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a^3+11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a*b^2-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)/a^2/(a+b*sinh(f*x+e)^2)^(3/2)/(
-1/a*b)^(1/2)/b^2/cosh(f*x+e)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**F**]
time = 0.10, size = 71, normalized size = 0.22

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\cosh\left(fx+e\right)^{6}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^6/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.72Unable to divide, perhaps due to rou
nding error%%%{32,[4,2,4]%%%}+%%%{%%%{-64,[1]%%%},[4,2,3]%%%}+%%%{%%%{32,[2
]%%%},[4,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(e+f\,x)^{6}}{{\left(b\sinh\left(e+f\,x\right)^{2}+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cosh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.396 $\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{2(a+b)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}-F(\text{ArcTan}$$

```
[Out] -1/3*(a-b)*cosh(f*x+e)*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(3/2)+2/3*(a+b
)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/2)*E
llipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1
/2))/a^(3/2)/b^(3/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sin
h(f*x+e)^2)^(1/2)-1/3*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*E
llipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+
b*sinh(f*x+e)^2)^(1/2)/a^2/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3271, 424, 539, 429, 422}

$$\frac{2(a+b)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}b^{3/2}f\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{3a^2bf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(a-b)\sinh(e+fx)\cosh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*((a - b)*Cosh[e + f*x]*Sinh[e + f*x])/(a*b*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) + (2*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/S
qrt[a]], 1 - a/b])/(3*a^(3/2)*b^(3/2)*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sin
h[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (EllipticF[ArcTan[Sinh[e + f*
x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b*f*Sqrt[(S
ech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx &= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx,x,\sinh(e+fx)\right)}{f}\\
&= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\mathrm{Su}}{}\\
&= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\mathrm{S}}{}\\
&= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{2(a+b)\cosh(e+fx)E\Big(\tan^{-1}\Big(\frac{\sqrt{}}{}}{3a^{3/2}b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\ \sqrt{}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.20, size = 178, normalized size = 0.80

$$\frac{2ia^2(a+b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-ia^2(2a+b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,b(a^2+2ab-b^2+b(a+b)\cosh(2(e+fx)))\sinh(2(e+fx))}{3a^2b^2f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(e
 + f*x), b/a] - I*a^2*(2*a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*E
llipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(a^2 + 2*a*b - b^2 + b*(a + b)*Cosh[
2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*b^2*f*(2*a - b + b*Cosh[2*(e + f*x)
])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(293) = 586.
time = 1.75, size = 597, normalized size = 2.68

method	result
default	$\frac{\left(2\sqrt{-\frac{b}{a}}\,ab+2\sqrt{-\frac{b}{a}}\,b^2\right)(\cosh^4(fx+e))\sinh(fx+e)+\left(\sqrt{-\frac{b}{a}}\,a^2+\sqrt{-\frac{b}{a}}\,ab-2\sqrt{-\frac{b}{a}}\,b^2\right)(\cosh^2(fx+e))\sinh(fx+e)+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((2*(-1/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+
((-1/a*b)^(1/2)*a^2+(-1/a*b)^(1/2)*a*b-2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*
sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(a*El
lipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*b*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))-2*a*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))-2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2+
(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f
*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+2*(b/a
*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/a^2/(a+b*sinh(f*x+e)^2)^(3/2)/(-1/a*b)^(1/2
)/b/cosh(f*x+e)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4355 vs. $2(231) = 462$.
time = 0.16, size = 4355, normalized size = 19.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(((2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^8 + 8*(2*a^2*b^2 + a*b^3 - b
^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (2*a^2*b^2 + a*b^3 - b^4)*sinh(f*x + e)
^8 + 4*(4*a^3*b - 3*a*b^3 + b^4)*cosh(f*x + e)^6 + 4*(4*a^3*b - 3*a*b^3 + b
^4 + 7*(2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(2
*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^3 + 3*(4*a^3*b - 3*a*b^3 + b^4)*cosh(
f*x + e))*sinh(f*x + e)^5 + 2*(16*a^4 - 8*a^3*b - 10*a^2*b^2 + 11*a*b^3 - 3
*b^4)*cosh(f*x + e)^4 + 2*(35*(2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^4 + 1
6*a^4 - 8*a^3*b - 10*a^2*b^2 + 11*a*b^3 - 3*b^4 + 30*(4*a^3*b - 3*a*b^3 + b
^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 2*a^2*b^2 + a*b^3 - b^4 + 8*(7*(2*a^
```

```
2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^5 + 10*(4*a^3*b - 3*a*b^3 + b^4)*cosh(f*
x + e)^3 + (16*a^4 - 8*a^3*b - 10*a^2*b^2 + 11*a*b^3 - 3*b^4)*cosh(f*x + e)
)*sinh(f*x + e)^3 + 4*(4*a^3*b - 3*a*b^3 + b^4)*cosh(f*x + e)^2 + 4*(7*(2*a
^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^6 + 15*(4*a^3*b - 3*a*b^3 + b^4)*cosh(f
*x + e)^4 + 4*a^3*b - 3*a*b^3 + b^4 + 3*(16*a^4 - 8*a^3*b - 10*a^2*b^2 + 11
*a*b^3 - 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((2*a^2*b^2 + a*b^3 -
b^4)*cosh(f*x + e)^7 + 3*(4*a^3*b - 3*a*b^3 + b^4)*cosh(f*x + e)^5 + (16*a^
4 - 8*a^3*b - 10*a^2*b^2 + 11*a*b^3 - 3*b^4)*cosh(f*x + e)^3 + (4*a^3*b - 3
*a*b^3 + b^4)*cosh(f*x + e))*sinh(f*x + e) - 2*((a*b^3 + b^4)*cosh(f*x + e)
^8 + 8*(a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a*b^3 + b^4)*sinh(f*x
 + e)^8 + 4*(2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^2*b^2 + a*b^
3 - b^4 + 7*(a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a*b^3 +
b^4)*cosh(f*x + e)^3 + 3*(2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x
+ e)^5 + 2*(8*a^3*b - 5*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a*b^3 + b^4
)*cosh(f*x + e)^4 + 8*a^3*b - 5*a*b^3 + 3*b^4 + 30*(2*a^2*b^2 + a*b^3 - b^4
)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a*b^3 + b^4 + 8*(7*(a*b^3 + b^4)*cosh(
f*x + e)^5 + 10*(2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^3 + (8*a^3*b - 5*a*
b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2*b^2 + a*b^3 - b^4)*c
osh(f*x + e)^2 + 4*(7*(a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^2*b^2 + a*b^3
 - b^4)*cosh(f*x + e)^4 + 2*a^2*b^2 + a*b^3 - b^4 + 3*(8*a^3*b - 5*a*b^3 +
3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a*b^3 + b^4)*cosh(f*x + e)^7
+ 3*(2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e)^5 + (8*a^3*b - 5*a*b^3 + 3*b^4)
*cosh(f*x + e)^3 + (2*a^2*b^2 + a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e))*
sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b
)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x
 + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 -
 a*b)/b^2))/b^2) - ((2*a^2*b^2 - a*b^3)*cosh(f*x + e)^8 + 8*(2*a^2*b^2 - a*
b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (2*a^2*b^2 - a*b^3)*sinh(f*x + e)^8 +
4*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^6 + 4*(4*a^3*b - 4*a^2*b^2 +
a*b^3 + 7*(2*a^2*b^2 - a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(2*a^
2*b^2 - a*b^3)*cosh(f*x + e)^3 + 3*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x +
 e))*sinh(f*x + e)^5 + 2*(16*a^4 - 24*a^3*b + 14*a^2*b^2 - 3*a*b^3)*cosh(f*
x + e)^4 + 2*(35*(2*a^2*b^2 - a*b^3)*cosh(f*x + e)^4 + 16*a^4 - 24*a^3*b +
14*a^2*b^2 - 3*a*b^3 + 30*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^2)*si
nh(f*x + e)^4 + 2*a^2*b^2 - a*b^3 + 8*(7*(2*a^2*b^2 - a*b^3)*cosh(f*x + e)^
5 + 10*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^3 + (16*a^4 - 24*a^3*b +
 14*a^2*b^2 - 3*a*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(4*a^3*b - 4*a^2*
b^2 + a*b^3)*cosh(f*x + e)^2 + 4*(7*(2*a^2*b^2 - a*b^3)*cosh(f*x + e)^6 + 1
5*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^4 + 4*a^3*b - 4*a^2*b^2 + a*b
^3 + 3*(16*a^4 - 24*a^3*b + 14*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^2)*sinh(f*x
 + e)^2 + 8*((2*a^2*b^2 - a*b^3)*cosh(f*x + e)^7 + 3*(4*a^3*b - 4*a^2*b^2 +
 a*b^3)*cosh(f*x + e)^5 + (16*a^4 - 24*a^3*b + 14*a^2*b^2 - 3*a*b^3)*cosh(f
*x + e)^3 + (4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e) - 2*
((a*b^3 + 2*b^4)*cosh(f*x + e)^8 + 8*(a*b^3 + 2*b^4)*cosh(f*x + e)*sinh(f*x
 + e)^7 + (a*b^3 + 2*b^4)*sinh(f*x + e)^8 + 4*(2*a^2*b^2 + 3*a*b^3 - 2*b^4)
```

```
*cosh(f*x + e)^6 + 4*(2*a^2*b^2 + 3*a*b^3 - 2*b^4 + 7*(a*b^3 + 2*b^4)*cosh(
f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a*b^3 + 2*b^4)*cosh(f*x + e)^3 + 3*(2*a
^2*b^2 + 3*a*b^3 - 2*b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^3*b + 8*a
^2*b^2 - 13*a*b^3 + 6*b^4)*cosh(f*x + e)^4 + 2*(35*(a*b^3 + 2*b^4)*cosh(f*x
 + e)^4 + 8*a^3*b + 8*a^2*b^2 - 13*a*b^3 + 6*b^4 + 30*(2*a^2*b^2 + 3*a*b^3
- 2*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a*b^3 + 2*b^4 + 8*(7*(a*b^3 + 2
*b^4)*cosh(f*x + e)^5 + 10*(2*a^2*b^2 + 3*a*b^3 - 2*b^4)*cosh(f*x + e)^3 +
(8*a^3*b + 8*a^2*b^2 - 13*a*b^3 + 6*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4
*(2*a^2*b^2 + 3*a*b^3 - 2*b^4)*cosh(f*x + e)^2 + 4*(7*(a*b^3 + 2*b^4)*cosh(
f*x + e)^6 + 15*(2*a^2*b^2 + 3*a*b^3 - 2*b^4)*cosh(f*x + e)^4 + 2*a^2*b^2 +
 3*a*b^3 - 2*b^4 + 3*(8*a^3*b + 8*a^2*b^2 - 13*a*b^3 + 6*b^4)*cosh(f*x + e)
^2)*sinh(f*x + e)^2 + 8*((a*b^3 + 2*b^4)*cosh(f*x + e)^7 + 3*(2*a^2*b^2 + 3
*a*b^3 - 2*b^4)*cosh(f*x + e)^5 + (8*a^3*b + 8*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.53Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(e+f\,x)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.397 $\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

<u>**Optimal**</u>. Leaf size=228

$$\frac{\cosh(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{(a-2b)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}(a-b)\sqrt{b}\,f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}+\frac{F(\text{ArcTan}(\text{s}}{3a^2}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/3*(a-2*b)*cosh(
f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/2)*Elliptic
E(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/a^
(3/2)/(a-b)/f/b^(1/2)/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh
(f*x+e)^2)^(1/2)+1/3*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*El
lipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b
*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/
2)
```

<u>**Rubi**</u> [A]
time = 0.14, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3271, 423, 539, 429, 422}

$$\frac{(a-2b)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}\sqrt{b}\,f(a-b)\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{3a^2f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\sinh(e+fx)\cosh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((a -
 2*b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 -
a/b])/(3*a^(3/2)*(a - b)*Sqrt[b]*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e +
 f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + (EllipticF[ArcTan[Sinh[e + f*x]],
1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*(a - b)*f*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

<u>Rule 422</u>

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

<u>Rule 423</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{(a+bx^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\Big(\int}{3af}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\Big(\int}{3a(a -}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a-2b)\cosh(e+fx)E\Big(\tan^{-1}\Big(\frac{\sqrt{b}\,\sin}{\sqrt{}}}{3a^{3/2}(a-b)\sqrt{b}\,f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\,\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.39, size = 193, normalized size = 0.85

$$\frac{2ia^2(a-2b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2ia^2(a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-\sqrt{2}\,b(-4a^2+7ab-2b^2-(a-2b)b\cosh(2(e+fx)))\sinh(2(e+fx))}{6a^2(a-b)bf(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(a - 2*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*
(e + f*x), b/a] - (2*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/
2)*EllipticF[I*(e + f*x), b/a] - Sqrt[2]*b*(-4*a^2 + 7*a*b - 2*b^2 - (a - 2
*b)*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*a^2*(a - b)*b*f*(2*a - b + b
*Cosh[2*(e + f*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(298) = 596.
time = 1.85, size = 662, normalized size = 2.90

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\,ab(\sinh^5(fx+e))-2\sqrt{-\frac{b}{a}}\,b^2(\sinh^5(fx+e))+2\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)\sqrt{\frac{a+b}{}}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5-2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+2*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*((a
+b*sinh(f*x+e)^2)/a)^(1/2)*a*b*sinh(f*x+e)^2-2*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*((a+b*sinh(f*x+e)^2)/a)^(1/2)*b
^2*sinh(f*x+e)^2-(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*((a+b*sinh(f*x+e)^2)/a)^(1/2)*a*b*sinh(f*x+e)^2+2*(cosh(f*x+e
)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*((a+b*sinh(f*x
+e)^2)/a)^(1/2)*b^2*sinh(f*x+e)^2+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-2*(-1/
a*b)^(1/2)*a*b*sinh(f*x+e)^3-2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+2*(cosh(f*x
+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*((a+b*sinh(f
*x+e)^2)/a)^(1/2)*a^2-2*a*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(
1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(cosh(f*x+e)^2)^(1/2
)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*((a+b*sinh(f*x+e)^2)/a)
^(1/2)*a^2+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(
sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+2*sinh(f*x+e)*(-1/a*b)^(1/2)*a^
2-3*(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/a^2/(a-b)/(a+b*sinh(f*x+e)^2)^(3/2)/(-1
/a*b)^(1/2)/cosh(f*x+e)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4770 vs. $2(236) = 472$.
time = 0.17, size = 4770, normalized size = 20.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(((2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^8 + 8*(2*a^2*b^2 - 5*a*b
^3 + 2*b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (2*a^2*b^2 - 5*a*b^3 + 2*b^4)*s
inh(f*x + e)^8 + 4*(4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*b^4)*cosh(f*x + e)^6
```

```
 + 4*(4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*b^4 + 7*(2*a^2*b^2 - 5*a*b^3 + 2*b
^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*c
osh(f*x + e)^3 + 3*(4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*b^4)*cosh(f*x + e))*
sinh(f*x + e)^5 + 2*(16*a^4 - 56*a^3*b + 62*a^2*b^2 - 31*a*b^3 + 6*b^4)*cos
h(f*x + e)^4 + 2*(35*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^4 + 16*a^4
 - 56*a^3*b + 62*a^2*b^2 - 31*a*b^3 + 6*b^4 + 30*(4*a^3*b - 12*a^2*b^2 + 9*
a*b^3 - 2*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 2*a^2*b^2 - 5*a*b^3 + 2*b
^4 + 8*(7*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^5 + 10*(4*a^3*b - 12*
a^2*b^2 + 9*a*b^3 - 2*b^4)*cosh(f*x + e)^3 + (16*a^4 - 56*a^3*b + 62*a^2*b^
2 - 31*a*b^3 + 6*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(4*a^3*b - 12*a^2*
b^2 + 9*a*b^3 - 2*b^4)*cosh(f*x + e)^2 + 4*(7*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)
*cosh(f*x + e)^6 + 15*(4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*b^4)*cosh(f*x + e
)^4 + 4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*b^4 + 3*(16*a^4 - 56*a^3*b + 62*a^
2*b^2 - 31*a*b^3 + 6*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((2*a^2*b^2
 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^7 + 3*(4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*
b^4)*cosh(f*x + e)^5 + (16*a^4 - 56*a^3*b + 62*a^2*b^2 - 31*a*b^3 + 6*b^4)*
cosh(f*x + e)^3 + (4*a^3*b - 12*a^2*b^2 + 9*a*b^3 - 2*b^4)*cosh(f*x + e))*s
inh(f*x + e) - 2*((a*b^3 - 2*b^4)*cosh(f*x + e)^8 + 8*(a*b^3 - 2*b^4)*cosh(
f*x + e)*sinh(f*x + e)^7 + (a*b^3 - 2*b^4)*sinh(f*x + e)^8 + 4*(2*a^2*b^2 -
 5*a*b^3 + 2*b^4)*cosh(f*x + e)^6 + 4*(2*a^2*b^2 - 5*a*b^3 + 2*b^4 + 7*(a*b
^3 - 2*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a*b^3 - 2*b^4)*cosh(f*
x + e)^3 + 3*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e))*sinh(f*x + e)^5 +
 2*(8*a^3*b - 24*a^2*b^2 + 19*a*b^3 - 6*b^4)*cosh(f*x + e)^4 + 2*(35*(a*b^3
 - 2*b^4)*cosh(f*x + e)^4 + 8*a^3*b - 24*a^2*b^2 + 19*a*b^3 - 6*b^4 + 30*(2
*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a*b^3 - 2*b^
4 + 8*(7*(a*b^3 - 2*b^4)*cosh(f*x + e)^5 + 10*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)
*cosh(f*x + e)^3 + (8*a^3*b - 24*a^2*b^2 + 19*a*b^3 - 6*b^4)*cosh(f*x + e))
*sinh(f*x + e)^3 + 4*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^2 + 4*(7*(
a*b^3 - 2*b^4)*cosh(f*x + e)^6 + 15*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x
+ e)^4 + 2*a^2*b^2 - 5*a*b^3 + 2*b^4 + 3*(8*a^3*b - 24*a^2*b^2 + 19*a*b^3 -
 6*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a*b^3 - 2*b^4)*cosh(f*x + e)
^7 + 3*(2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(f*x + e)^5 + (8*a^3*b - 24*a^2*b^
2 + 19*a*b^3 - 6*b^4)*cosh(f*x + e)^3 + (2*a^2*b^2 - 5*a*b^3 + 2*b^4)*cosh(
f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2
 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2
) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*
(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) + 2*((2*a^2*b^2 - a*b^3)*cosh(f*x
 + e)^8 + 8*(2*a^2*b^2 - a*b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (2*a^2*b^2
 - a*b^3)*sinh(f*x + e)^8 + 4*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^6
 + 4*(4*a^3*b - 4*a^2*b^2 + a*b^3 + 7*(2*a^2*b^2 - a*b^3)*cosh(f*x + e)^2)*s
inh(f*x + e)^6 + 8*(7*(2*a^2*b^2 - a*b^3)*cosh(f*x + e)^3 + 3*(4*a^3*b - 4*
a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(16*a^4 - 24*a^3*b + 14
*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^4 + 2*(35*(2*a^2*b^2 - a*b^3)*cosh(f*x +
e)^4 + 16*a^4 - 24*a^3*b + 14*a^2*b^2 - 3*a*b^3 + 30*(4*a^3*b - 4*a^2*b^2 +
 a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 2*a^2*b^2 - a*b^3 + 8*(7*(2*a^2*
```

```
b^2 - a*b^3)*cosh(f*x + e)^5 + 10*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x +
e)^3 + (16*a^4 - 24*a^3*b + 14*a^2*b^2 - 3*a*b^3)*cosh(f*x + e))*sinh(f*x +
 e)^3 + 4*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^2 + 4*(7*(2*a^2*b^2 -
 a*b^3)*cosh(f*x + e)^6 + 15*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^4
+ 4*a^3*b - 4*a^2*b^2 + a*b^3 + 3*(16*a^4 - 24*a^3*b + 14*a^2*b^2 - 3*a*b^3
)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((2*a^2*b^2 - a*b^3)*cosh(f*x + e)^7
 + 3*(4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^5 + (16*a^4 - 24*a^3*b + 1
4*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^3 + (4*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f
*x + e))*sinh(f*x + e) + 4*((a*b^3 - b^4)*cosh(f*x + e)^8 + 8*(a*b^3 - b^4)
*cosh(f*x + e)*sinh(f*x + e)^7 + (a*b^3 - b^4)*sinh(f*x + e)^8 + 4*(2*a^2*b
^2 - 3*a*b^3 + b^4)*cosh(f*x + e)^6 + 4*(2*a^2*b^2 - 3*a*b^3 + b^4 + 7*(a*b
^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a*b^3 - b^4)*cosh(f*x +
e)^3 + 3*(2*a^2*b^2 - 3*a*b^3 + b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*
a^3*b - 16*a^2*b^2 + 11*a*b^3 - 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a*b^3 - b^4
)*cosh(f*x + e)^4 + 8*a^3*b - 16*a^2*b^2 + 11*a*b^3 - 3*b^4 + 30*(2*a^2*b^2
 - 3*a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a*b^3 - b^4 + 8*(7*(a*
b^3 - b^4)*cosh(f*x + e)^5 + 10*(2*a^2*b^2 - 3*a*b^3 + b^4)*cosh(f*x + e)^3
 + (8*a^3*b - 16*a^2*b^2 + 11*a*b^3 - 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3
 + 4*(2*a^2*b^2 - 3*a*b^3 + b^4)*cosh(f*x + e)^...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.51Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{cosh}\left(e+f\,x\right)}^2}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.398 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

<u>**Optimal**</u>. Leaf size=251

$$-\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{}}{3a^2(a-b)^2f\sqrt{1+\frac{b\,\mathrm{s}}{}}}$$

```
[Out] -1/3*b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(2*a
-b)*b*cosh(f*x+e)*sinh(f*x+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-2/3*I
*(2*a-b)*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(
b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)^2/f/(1+b*sinh(f*x+e)^2/a)^(
1/2)+1/3*I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x)
,(b/a)^(1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/
2)
```

<u>**Rubi**</u> [A]
time = 0.20, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3263, 3252, 3251, 3257, 3256, 3262, 3261}

$$-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] -1/3*(b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt
[a + b*Sinh[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]
*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)^2*f*Sqrt[1 + (b*Sinh[e + f*x]^2)
/a]) + ((I/3)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/
(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

<u>Rule 3251</u>

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

<u>Rule 3252</u>

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=-\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\int \frac{-3a+2b+b\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx}{3a(a-b)}$$

$$=-\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{2(2a-b)b\cosh(e+fx)\sinh(e+}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$=-\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{2(2a-b)b\cosh(e+fx)\sinh(e+}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$=-\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{2(2a-b)b\cosh(e+fx)\sinh(e+}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

$$=-\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{2(2a-b)b\cosh(e+fx)\sinh(e+}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+f}}$$

Mathematica [A]
time = 1.08, size = 190, normalized size = 0.76

$$\frac{-2ia^2(2a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+ia^2(a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,b(-5a^2+5ab-b^2+b(-2a+b)\cosh(2(e+fx)))\sinh(2(e+fx))}{3a^2(a-b)^2f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] ((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*
EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a + b
)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b*Cos
h[2*(e + f*x)])^(3/2))
```

Maple [A]
time = 1.76, size = 406, normalized size = 1.62

method	result
default	$\dfrac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(-\frac{\sinh(fx+e)\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right.\right.}}{3ab(a-b)\left(\sinh^2(fx+e)+\frac{a}{b}\right)^2}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/a/b/(a-b)*sinh(f*x+e)*((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+a/b)^2-2/3*b*cosh(f*x+e
)^2/a^2/(a-b)^2*sinh(f*x+e)*(-b+2*a)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1
/2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*Ellip
ticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2/3*b*(-b+2*a)/a^2/(a-b)^2/(-1
/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(
f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))))/cosh(f*x+e)/(a+b
*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5442 vs. $2(259) = 518$.
time = 0.19, size = 5442, normalized size = 21.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(((4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^8 + 8*(4*a^2*b^3 - 4*a*b^4
+ b^5)*cosh(f*x + e)*sinh(f*x + e)^7 + (4*a^2*b^3 - 4*a*b^4 + b^5)*sinh(f*x
```

```
 + e)^8 + 4*(8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e)^6 + 4*(8
*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5 + 7*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(
f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e
)^3 + 3*(8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e))*sinh(f*x +
e)^5 + 4*a^2*b^3 - 4*a*b^4 + b^5 + 2*(32*a^4*b - 64*a^3*b^2 + 52*a^2*b^3 -
20*a*b^4 + 3*b^5)*cosh(f*x + e)^4 + 2*(32*a^4*b - 64*a^3*b^2 + 52*a^2*b^3 -
 20*a*b^4 + 3*b^5 + 35*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^4 + 30*(8*
a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*
(7*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^5 + 10*(8*a^3*b^2 - 12*a^2*b^3
 + 6*a*b^4 - b^5)*cosh(f*x + e)^3 + (32*a^4*b - 64*a^3*b^2 + 52*a^2*b^3 - 2
0*a*b^4 + 3*b^5)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(8*a^3*b^2 - 12*a^2*b^3
 + 6*a*b^4 - b^5)*cosh(f*x + e)^2 + 4*(7*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f
*x + e)^6 + 8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5 + 15*(8*a^3*b^2 - 12*a^2
*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e)^4 + 3*(32*a^4*b - 64*a^3*b^2 + 52*a^2*b
^3 - 20*a*b^4 + 3*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((4*a^2*b^3 - 4
*a*b^4 + b^5)*cosh(f*x + e)^7 + 3*(8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*
cosh(f*x + e)^5 + (32*a^4*b - 64*a^3*b^2 + 52*a^2*b^3 - 20*a*b^4 + 3*b^5)*c
osh(f*x + e)^3 + (8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e))*si
nh(f*x + e) - 2*((2*a*b^4 - b^5)*cosh(f*x + e)^8 + 8*(2*a*b^4 - b^5)*cosh(f
*x + e)*sinh(f*x + e)^7 + (2*a*b^4 - b^5)*sinh(f*x + e)^8 + 4*(4*a^2*b^3 -
4*a*b^4 + b^5)*cosh(f*x + e)^6 + 4*(4*a^2*b^3 - 4*a*b^4 + b^5 + 7*(2*a*b^4
- b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(2*a*b^4 - b^5)*cosh(f*x + e
)^3 + 3*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*a*b^
4 - b^5 + 2*(16*a^3*b^2 - 24*a^2*b^3 + 14*a*b^4 - 3*b^5)*cosh(f*x + e)^4 +
2*(16*a^3*b^2 - 24*a^2*b^3 + 14*a*b^4 - 3*b^5 + 35*(2*a*b^4 - b^5)*cosh(f*x
 + e)^4 + 30*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^4 +
 8*(7*(2*a*b^4 - b^5)*cosh(f*x + e)^5 + 10*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh
(f*x + e)^3 + (16*a^3*b^2 - 24*a^2*b^3 + 14*a*b^4 - 3*b^5)*cosh(f*x + e))*s
inh(f*x + e)^3 + 4*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^2 + 4*(7*(2*a*
b^4 - b^5)*cosh(f*x + e)^6 + 4*a^2*b^3 - 4*a*b^4 + b^5 + 15*(4*a^2*b^3 - 4*
a*b^4 + b^5)*cosh(f*x + e)^4 + 3*(16*a^3*b^2 - 24*a^2*b^3 + 14*a*b^4 - 3*b^
5)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((2*a*b^4 - b^5)*cosh(f*x + e)^7 +
3*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^5 + (16*a^3*b^2 - 24*a^2*b^3 +
14*a*b^4 - 3*b^5)*cosh(f*x + e)^3 + (4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x +
e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b
)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*
a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b
 - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - ((6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh
(f*x + e)^8 + 8*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)*sinh(f*x + e)
^7 + (6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*sinh(f*x + e)^8 + 4*(12*a^4*b - 16*a^3
*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^6 + 4*(12*a^4*b - 16*a^3*b^2 + 7*a^
2*b^3 - a*b^4 + 7*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^2)*sinh(f*x
 + e)^6 + 8*(7*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^3 + 3*(12*a^4*
b - 16*a^3*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*a^3*
b^2 - 5*a^2*b^3 + a*b^4 + 2*(48*a^5 - 88*a^4*b + 66*a^3*b^2 - 23*a^2*b^3 +
```

```
3*a*b^4)*cosh(f*x + e)^4 + 2*(48*a^5 - 88*a^4*b + 66*a^3*b^2 - 23*a^2*b^3 +
 3*a*b^4 + 35*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^4 + 30*(12*a^4*
b - 16*a^3*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(7
*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^5 + 10*(12*a^4*b - 16*a^3*b^
2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^3 + (48*a^5 - 88*a^4*b + 66*a^3*b^2 -
23*a^2*b^3 + 3*a*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(12*a^4*b - 16*a^3
*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^2 + 4*(7*(6*a^3*b^2 - 5*a^2*b^3 + a
*b^4)*cosh(f*x + e)^6 + 12*a^4*b - 16*a^3*b^2 + 7*a^2*b^3 - a*b^4 + 15*(12*
a^4*b - 16*a^3*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^4 + 3*(48*a^5 - 88*a^
4*b + 66*a^3*b^2 - 23*a^2*b^3 + 3*a*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 +
 8*((6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^7 + 3*(12*a^4*b - 16*a^3*
b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^5 + (48*a^5 - 88*a^4*b + 66*a^3*b^2
- 23*a^2*b^3 + 3*a*b^4)*cosh(f*x + e)^3 + (12*a^4*b - 16*a^3*b^2 + 7*a^2*b^
3 - a*b^4)*cosh(f*x + e))*sinh(f*x + e) + 2*((3*a^2*b^3 - 5*a*b^4 + 2*b^5)*
cosh(f*x + e)^8 + 8*(3*a^2*b^3 - 5*a*b^4 + 2*b^5)*cosh(f*x + e)*sinh(f*x +
e)^7 + (3*a^2*b^3 - 5*a*b^4 + 2*b^5)*sinh(f*x + e)^8 + 4*(6*a^3*b^2 - 13*a^
2*b^3 + 9*a*b^4 - 2*b^5)*cosh(f*x + e)^6 + 4*(6*a^3*b^2 - 13*a^2*b^3 + 9*a*
b^4 - 2*b^5 + 7*(3*a^2*b^3 - 5*a*b^4 + 2*b^5)*cosh(f*x + e)^2)*sinh(f*x + e
)^6 + 8*(7*(3*a^2*b^3 - 5*a*b^4 + 2*b^5)*cosh(f*x + e)^3 + 3*(6*a^3*b^2 - 1
3*a^2*b^3 + 9*a*b^4 - 2*b^5)*cosh(f*x + e))*sin...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sinh^{2}\left(e + fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(-5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.48Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(1/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.399 $\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=292

$$\frac{b(3a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\sqrt{b}\,(3a^2+7ab-2b^2)\cosh(e+fx)E\Big(\text{ArcTan}\Big(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\Big)|1-\frac{a}{b}}{3a^{3/2}(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] 1/3*b*(3*a+b)*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(3/2)
+1/3*(3*a^2+7*a*b-2*b^2)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*s
inh(f*x+e)^2/a)^(1/2)*EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e
)^2/a)^(1/2),(1-a/b)^(1/2))*b^(1/2)/a^(3/2)/(a-b)^3/f/(a*cosh(f*x+e)^2/(a+b
*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*(9*a-b)*b*(1/(1+sinh(f
*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e
)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)^3
/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+tanh(f*x+e)/(a-b)/f/(a+b*sin
h(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.22, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3271, 425, 541, 539, 429, 422}

$$-\frac{b(9a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{3a^2f(a-b)^3\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\sqrt{b}\,(3a^2+7ab-2b^2)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{\tanh(e+fx)}{f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{b(3a+b)\sinh(e+fx)\cosh(e+fx)}{3af(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (b*(3*a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)^2*f*(a + b*Sinh[e +
f*x]^2)^(3/2)) + (Sqrt[b]*(3*a^2 + 7*a*b - 2*b^2)*Cosh[e + f*x]*EllipticE[A
rcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*a^(3/2)*(a - b)^3*f*Sq
rt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]
) - ((9*a - b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sq
rt[a + b*Sinh[e + f*x]^2])/(3*a^2*(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*
Sinh[e + f*x]^2))/a]) + Tanh[e + f*x]/((a - b)*f*(a + b*Sinh[e + f*x]^2)^(3
/2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\Big(\int \frac{1}{(1+x^2)^{3/2}(a+bx^2)^{5/2}}\,dx, x, \sinh(e+}{f}$$

$$= \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subs}}{(-}$$

$$= \frac{b(3a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} +$$

$$= \frac{b(3a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} +$$

$$= \frac{b(3a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\sqrt{b}\,(3a^2+7ab-2b^2)\cosh(e+fx)}{3a^{3/2}(a-b)^3f\sqrt{\frac{a\cosh^2(e+}{a+b\sinh^2(e}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.76, size = 260, normalized size = 0.89

$$\frac{2ia^2(3a^2+7ab-2b^2)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2ia^2(3a^2-2ab-b^2)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\frac{(24a^4-24a^3b+41a^2b^2-19ab^3+2b^4+4ab(6a^2+5ab-3b^2)\cosh(2(e+fx))+b^2(3a^2+7ab-2b^2)\cosh(4(e+fx)))\tanh(e+fx)}{\sqrt{2}}}{6a^2(a-b)^3f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(3*a^2 + 7*a*b - 2*b^2)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2
)*EllipticE[I*(e + f*x), b/a] - (2*I)*a^2*(3*a^2 - 2*a*b - b^2)*((2*a - b +
 b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + ((24*a^4 - 24*
a^3*b + 41*a^2*b^2 - 19*a*b^3 + 2*b^4 + 4*a*b*(6*a^2 + 5*a*b - 3*b^2)*Cosh[
2*(e + f*x)] + b^2*(3*a^2 + 7*a*b - 2*b^2)*Cosh[4*(e + f*x)])*Tanh[e + f*x]
)/Sqrt[2])/(6*a^2*(a - b)^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. 2(360) = 720.
time = 2.02, size = 1002, normalized size = 3.43

method	result	size

default	Expression too large to display	1002
risch	Expression too large to display	88750

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(3*(-1/a*b)^(1/2)*a^2*b^2*sinh(f*x+e)^5+7*(-1/a*b)^(1/2)*a*b^3*sinh(f*x
+e)^5-2*(-1/a*b)^(1/2)*b^4*sinh(f*x+e)^5-6*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(c
osh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b
^2*sinh(f*x+e)^2+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^3*sinh(f*x+e)^2-2*((a+b*s
inh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)
^(1/2),(a/b)^(1/2))*b^4*sinh(f*x+e)^2-3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh
(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b^2*
sinh(f*x+e)^2-7*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^3*sinh(f*x+e)^2+2*((a+b*sinh
(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1
/2),(a/b)^(1/2))*b^4*sinh(f*x+e)^2+6*(-1/a*b)^(1/2)*a^3*b*sinh(f*x+e)^3+8*(
-1/a*b)^(1/2)*a^2*b^2*sinh(f*x+e)^3+4*(-1/a*b)^(1/2)*a*b^3*sinh(f*x+e)^3-2*
(-1/a*b)^(1/2)*b^4*sinh(f*x+e)^3-6*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3*b+8*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*a^2*b^2-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^3-3*((a+b*si
nh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^
(1/2),(a/b)^(1/2))*a^3*b-7*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1
/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b^2+2*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*a*b^3+3*(-1/a*b)^(1/2)*a^4*sinh(f*x+e)+8*(-1/a*b)^(1/2)*a^2
*b^2*sinh(f*x+e)-3*(-1/a*b)^(1/2)*a*b^3*sinh(f*x+e))/(-1/a*b)^(1/2)/(a+b*si
nh(f*x+e)^2)^(3/2)/a^2/(a-b)^3/cosh(f*x+e)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8928 vs. 2(298) = 596.

time = 0.29, size = 8928, normalized size = 30.58

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(((6*a^3*b^2 + 11*a^2*b^3 - 11*a*b^4 + 2*b^5)*cosh(f*x + e)^10 + 10*(6
*a^3*b^2 + 11*a^2*b^3 - 11*a*b^4 + 2*b^5)*cosh(f*x + e)*sinh(f*x + e)^9 + (
6*a^3*b^2 + 11*a^2*b^3 - 11*a*b^4 + 2*b^5)*sinh(f*x + e)^10 + (48*a^4*b + 7
0*a^3*b^2 - 121*a^2*b^3 + 49*a*b^4 - 6*b^5)*cosh(f*x + e)^8 + (48*a^4*b + 7
0*a^3*b^2 - 121*a^2*b^3 + 49*a*b^4 - 6*b^5 + 45*(6*a^3*b^2 + 11*a^2*b^3 - 1
1*a*b^4 + 2*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(6*a^3*b^2 + 11*a
^2*b^3 - 11*a*b^4 + 2*b^5)*cosh(f*x + e)^3 + (48*a^4*b + 70*a^3*b^2 - 121*a
^2*b^3 + 49*a*b^4 - 6*b^5)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(48*a^5 + 64*
a^4*b - 126*a^3*b^2 + 71*a^2*b^3 - 19*a*b^4 + 2*b^5)*cosh(f*x + e)^6 + 2*(4
8*a^5 + 64*a^4*b - 126*a^3*b^2 + 71*a^2*b^3 - 19*a*b^4 + 2*b^5 + 105*(6*a^3
*b^2 + 11*a^2*b^3 - 11*a*b^4 + 2*b^5)*cosh(f*x + e)^4 + 14*(48*a^4*b + 70*a
^3*b^2 - 121*a^2*b^3 + 49*a*b^4 - 6*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^6 +
 4*(63*(6*a^3*b^2 + 11*a^2*b^3 - 11*a*b^4 + 2*b^5)*cosh(f*x + e)^5 + 14*(48
*a^4*b + 70*a^3*b^2 - 121*a^2*b^3 + 49*a*b^4 - 6*b^5)*cosh(f*x + e)^3 + 3*(
48*a^5 + 64*a^4*b - 126*a^3*b^2 + 71*a^2*b^3 - 19*a*b^4 + 2*b^5)*cosh(f*x +
 e))*sinh(f*x + e)^5 + 6*a^3*b^2 + 11*a^2*b^3 - 11*a*b^4 + 2*b^5 + 2*(48*a^
5 + 64*a^4*b - 126*a^3*b^2 + 71*a^2*b^3 - 19*a*b^4 + 2*b^5)*cosh(f*x + e)^4
 + 2*(105*(6*a^3*b^2 + 11*a^2*b^3 - 11*a*b^4 + 2*b^5)*cosh(f*x + e)^6 + 48*
a^5 + 64*a^4*b - 126*a^3*b^2 + 71*a^2*b^3 - 19*a*b^4 + 2*b^5 + 35*(48*a^4*b
 + 70*a^3*b^2 - 121*a^2*b^3 + 49*a*b^4 - 6*b^5)*cosh(f*x + e)^4 + 15*(48*a^
5 + 64*a^4*b - 126*a^3*b^2 + 71*a^2*b^3 - 19*a*b^4 + 2*b^5)*cosh(f*x + e)^2
)*sinh(f*x + e)^4 + 8*(15*(6*a^3*b^2 + 11*a^2*b^3 - 11*a*b^4 + 2*b^5)*cosh(
f*x + e)^7 + 7*(48*a^4*b + 70*a^3*b^2 - 121*a^2*b^3 + 49*a*b^4 - 6*b^5)*cos
h(f*x + e)^5 + 5*(48*a^5 + 64*a^4*b - 126*a^3*b^2 + 71*a^2*b^3 - 19*a*b^4 +
 2*b^5)*cosh(f*x + e)^3 + (48*a^5 + 64*a^4*b - 126*a^3*b^2 + 71*a^2*b^3 - 1
9*a*b^4 + 2*b^5)*cosh(f*x + e))*sinh(f*x + e)^3 + (48*a^4*b + 70*a^3*b^2 -
121*a^2*b^3 + 49*a*b^4 - 6*b^5)*cosh(f*x + e)^2 + (45*(6*a^3*b^2 + 11*a^2*b
^3 - 11*a*b^4 + 2*b^5)*cosh(f*x + e)^8 + 28*(48*a^4*b + 70*a^3*b^2 - 121*a^
2*b^3 + 49*a*b^4 - 6*b^5)*cosh(f*x + e)^6 + 48*a^4*b + 70*a^3*b^2 - 121*a^2
*b^3 + 49*a*b^4 - 6*b^5 + 30*(48*a^5 + 64*a^4*b - 126*a^3*b^2 + 71*a^2*b^3
- 19*a*b^4 + 2*b^5)*cosh(f*x + e)^4 + 12*(48*a^5 + 64*a^4*b - 126*a^3*b^2 +
 71*a^2*b^3 - 19*a*b^4 + 2*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(6*
a^3*b^2 + 11*a^2*b^3 - 11*a*b^4 + 2*b^5)*cosh(f*x + e)^9 + 4*(48*a^4*b + 70
*a^3*b^2 - 121*a^2*b^3 + 49*a*b^4 - 6*b^5)*cosh(f*x + e)^7 + 6*(48*a^5 + 64
*a^4*b - 126*a^3*b^2 + 71*a^2*b^3 - 19*a*b^4 + 2*b^5)*cosh(f*x + e)^5 + 4*(
48*a^5 + 64*a^4*b - 126*a^3*b^2 + 71*a^2*b^3 - 19*a*b^4 + 2*b^5)*cosh(f*x +
 e)^3 + (48*a^4*b + 70*a^3*b^2 - 121*a^2*b^3 + 49*a*b^4 - 6*b^5)*cosh(f*x +
 e))*sinh(f*x + e) - 2*((3*a^2*b^3 + 7*a*b^4 - 2*b^5)*cosh(f*x + e)^10 + 10
```

```
*(3*a^2*b^3 + 7*a*b^4 - 2*b^5)*cosh(f*x + e)*sinh(f*x + e)^9 + (3*a^2*b^3 +
 7*a*b^4 - 2*b^5)*sinh(f*x + e)^10 + (24*a^3*b^2 + 47*a^2*b^3 - 37*a*b^4 +
6*b^5)*cosh(f*x + e)^8 + (24*a^3*b^2 + 47*a^2*b^3 - 37*a*b^4 + 6*b^5 + 45*(
3*a^2*b^3 + 7*a*b^4 - 2*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(3*a^
2*b^3 + 7*a*b^4 - 2*b^5)*cosh(f*x + e)^3 + (24*a^3*b^2 + 47*a^2*b^3 - 37*a*
b^4 + 6*b^5)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(24*a^4*b + 44*a^3*b^2 - 41
*a^2*b^3 + 15*a*b^4 - 2*b^5)*cosh(f*x + e)^6 + 2*(24*a^4*b + 44*a^3*b^2 - 4
1*a^2*b^3 + 15*a*b^4 - 2*b^5 + 105*(3*a^2*b^3 + 7*a*b^4 - 2*b^5)*cosh(f*x +
 e)^4 + 14*(24*a^3*b^2 + 47*a^2*b^3 - 37*a*b^4 + 6*b^5)*cosh(f*x + e)^2)*si
nh(f*x + e)^6 + 4*(63*(3*a^2*b^3 + 7*a*b^4 - 2*b^5)*cosh(f*x + e)^5 + 14*(2
4*a^3*b^2 + 47*a^2*b^3 - 37*a*b^4 + 6*b^5)*cosh(f*x + e)^3 + 3*(24*a^4*b +
44*a^3*b^2 - 41*a^2*b^3 + 15*a*b^4 - 2*b^5)*cosh(f*x + e))*sinh(f*x + e)^5
+ 3*a^2*b^3 + 7*a*b^4 - 2*b^5 + 2*(24*a^4*b + 44*a^3*b^2 - 41*a^2*b^3 + 15*
a*b^4 - 2*b^5)*cosh(f*x + e)^4 + 2*(105*(3*a^2*b^3 + 7*a*b^4 - 2*b^5)*cosh(
f*x + e)^6 + 24*a^4*b + 44*a^3*b^2 - 41*a^2*b^3 + 15*a*b^4 - 2*b^5 + 35*(24
*a^3*b^2 + 47*a^2*b^3 - 37*a*b^4 + 6*b^5)*cosh(f*x + e)^4 + 15*(24*a^4*b +
44*a^3*b^2 - 41*a^2*b^3 + 15*a*b^4 - 2*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^
4 + 8*(15*(3*a^2*b^3 + 7*a*b^4 - 2*b^5)*cosh(f*x + e)^7 + 7*(24*a^3*b^2 + 4
7*a^2*b^3 - 37*a*b^4 + 6*b^5)*cosh(f*x + e)^5 + 5*(24*a^4*b + 44*a^3*b^2 -
41*a^2*b^3 + 15*a*b^4 - 2*b^5)*cosh(f*x + e)^3 + (24*a^4*b + 44*a^3*b^2 - 4
1*a^2*b^3 + 15*a*b^4 - 2*b^5)*cosh(f*x + e))*sinh(f*x + e)^3 + (24*a^3*b^2
+ 47*a^2*b^3 - 37*a*b^4 + 6*b^5)*cosh(f*x + e)^2 + (45*(3*a^2*b^3 + 7*a*b^4
 - 2*b^5)*cosh(f*x + e)^8 + 28*(24*a^3*b^2 + 47*a^2*b^3 - 37*a*b^4 + 6*b^5)
*cosh(f*x + e)^6 + 24*a^3*b^2 + 47*a^2*b^3 - 37*a*b^4 + 6*b^5 + 30*(24*a^4*
b + 44*a^3*b^2 - 41*a^2*b^3 + 15*a*b^4 - 2*b^5)*cosh(f*x + e)^4 + 12*(24*a^
4*b + 44*a^3*b^2 - 41*a^2*b^3 + 15*a*b^4 - 2*b^5)*cosh(f*x + e)^2)*sinh(f*x
 + e)^2 + 2*(5*(3*a^2*b^3 + 7*a*b^4 - 2*b^5)*cosh(f*x + e)^9 + 4*(24*a^3*b^
2 + 47*a^2*b^3 - 37*a*b^4 + 6*b^5)*cosh(f*x + e)^7 + 6*(24*a^4*b + 44*a^3*b
^2 - 41*a^2*b^3 + 15*a*b^4 - 2*b^5)*cosh(f*x + ...
```

Sympy **[F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sech(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Giac **[F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 2.05Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh\left(e+f\,x\right)^2\,{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(5/2)), x)
```

3.400 $\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=117

$$\frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)(d\cosh(e+fx))^{-1+m}\cosh^2(e+fx)^{\frac{1-m}{2}}\sinh(e+fx)\,(a}{f}$$

```
[Out] d*AppellF1(1/2,1/2-1/2*m,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(d*cosh(
f*x+e))^(-1+m)*(cosh(f*x+e)^2)^(1/2-1/2*m)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^
p/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A]

time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3272, 441, 440}

$$\frac{d\sinh(e+fx)\cosh^2(e+fx)^{\frac{1-m}{2}}(d\cosh(e+fx))^{m-1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2
)/a)]*(d*Cosh[e + f*x])^(-1 + m)*(Cosh[e + f*x]^2)^((1 - m)/2)*Sinh[e + f*x
]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3272

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[f
f*d^(2*IntPart[(m - 1)/2] + 1)*((d*Cos[e + f*x])^(2*FracPart[(m - 1)/2])/(f
*(Cos[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/
```

```
2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f
, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx &= \frac{\left(d(d\cosh(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cosh^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\operatorname{Subst}\left(\int (\right.}{f}\\ &= \frac{\left(d(d\cosh(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cosh^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\left(a+b\sinh^2\right.\right.}{}\\ &= \frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)(d\cosh(}{}\end{aligned}$$

Mathematica [F]
time = 6.71, size = 0, normalized size = 0.00

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F]
time = 1.57, size = 0, normalized size = 0.00

$$\int \left(d\cosh\left(fx+e\right)\right)^m \left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)
```

Fricas [**F**]
time = 0.45, size = 27, normalized size = 0.23

$$\text{integral}\Big(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\left(d\cosh\left(fx+e\right)\right)^{m},x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cosh(f*x+e))**m*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\mathrm{cosh}\left(e+f\,x\right)\right)}^m\,{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cosh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int((d*cosh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p, x)
```

3.401 $\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=214

$$-\frac{(3a-b(7+2p))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}+\frac{\cosh^2(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1}}{bf(5+2p)}$$

```
[Out] -(3*a-b*(7+2*p))*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1+p)/b^2/f/(4*p^2+16*p+15
)+cosh(f*x+e)^2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1+p)/b/f/(5+2*p)+(3*a^2-2*
a*b*(5+2*p)+b^2*(4*p^2+16*p+15))*hypergeom([1/2, -p],[3/2],-b*sinh(f*x+e)^2
/a)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p/b^2/f/(4*p^2+16*p+15)/((1+b*sinh(f*x+
e)^2/a)^p)
```

Rubi [A]
time = 0.15, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3269, 427, 396, 252, 251}

$$\frac{\left(3a^2-2ab(2p+5)+b^2(4p^2+16p+15)\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)}{b^2f(2p+3)(2p+5)}-\frac{(3a-b(2p+7))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{b^2f(2p+3)(2p+5)}+\frac{\sinh(e+fx)\cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{bf(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -(((3*a - b*(7 + 2*p))*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b^2*
f*(3 + 2*p)*(5 + 2*p))) + (Cosh[e + f*x]^2*Sinh[e + f*x]*(a + b*Sinh[e + f*
x]^2)^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 - 2*a*b*(5 + 2*p) + b^2*(15 + 16*p
 + 4*p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e
 + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Sinh[
e + f*x]^2)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\left(a+bx^2\right)^p\,dx,x,\sinh(e+fx)\right)}{f}\\ &= \frac{\cosh^2(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \text{Subs}\\ &= -\frac{(3a-b(7+2p))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + c\\ &= -\frac{(3a-b(7+2p))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + c\\ &= -\frac{(3a-b(7+2p))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + c\end{aligned}$$

Mathematica [F]
time = 8.00, size = 0, normalized size = 0.00

$$\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [**F**]
time = 2.03, size = 0, normalized size = 0.00

$$\int \left(\cosh^{5}\left(fx+e\right)\right)\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```

Fricas [**F**]
time = 0.41, size = 25, normalized size = 0.12

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\cosh\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(e+f\,x)^5\,\left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int(cosh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^p, x)
```

3.402 $\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=125

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(3+2p)}-\frac{(a-b(3+2p))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\,(a+b\sinh}{bf(3+2p)}$$

```
[Out] sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1+p)/b/f/(3+2*p)-(a-b*(3+2*p))*hypergeom([
1/2, -p],[3/2],-b*sinh(f*x+e)^2/a)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p/b/f/(3
+2*p)/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 396, 252, 251}

$$\frac{\left(1-\frac{a}{2bp+3b}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)}{f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) + ((1 - a/(
3*b + 2*b*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sin
h[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\mathrm{Subst}\left(\int\left(1+x^2\right)\left(a+bx^2\right)^p\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(1-\frac{a}{3b+2bp}\right)\mathrm{Subst}(}{}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(\left(1-\frac{a}{3b+2bp}\right)(a+}{}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(1-\frac{a}{3b+2bp}\right){}_2F_1\left(\frac{1}{2}\right.}{}$$

Mathematica [**A**]
time = 0.23, size = 120, normalized size = 0.96

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\left(\left(-a+b(3+2p)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)+\left(a+b\sinh^2(e+fx)\right)\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^p\right)}{bf(3+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p*((-a + b*(3 + 2*p))*Hypergeometric
2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)] + (a + b*Sinh[e + f*x]^2)*(1 +
(b*Sinh[e + f*x]^2)/a)^p))/(b*f*(3 + 2*p)*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Maple [**F**]
time = 2.17, size = 0, normalized size = 0.00

$$\int\left(\cosh^3(fx+e)\right)\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.40, size = 25, normalized size = 0.20

$$\mathrm{integral}\Big(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\cosh\left(fx+e\right)^{3},x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(e+f\,x)^3\,\left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^p,x)
[Out] int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^p, x)
```

3.403 $\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] hypergeom([1/2, -p],[3/2],-b*sinh(f*x+e)^2/a)*sinh(f*x+e)*(a+b*sinh(f*x+e)^
2)^p/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3269, 252, 251}

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a
 + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \left(a+bx^2\right)^p\,dx, x, \sinh(e+fx)\right)}{f}\\
&= \frac{\left(\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\frac{bx}{a}\right.\right.}{f}\\
&= \frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)}{f}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 67, normalized size = 1.00

$$\frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a
 + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Maple [**F**]
time = 0.76, size = 0, normalized size = 0.00

$$\int \cosh\left(fx+e\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```

Fricas [**F**]
time = 0.50, size = 23, normalized size = 0.34

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\cosh\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```

Mupad [**B**]
time = 1.50, size = 64, normalized size = 0.96

$$\frac{\sinh(e+f\,x)\ \left(b\sinh(e+f\,x)^2+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\sinh(e+f\,x)^2}{a}\right)}{f\,\left(\frac{b\sinh(e+f\,x)^2}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] (sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^p*hypergeom([1/2, -p], 3/2, -(b*sinh
(e + f*x)^2)/a))/(f*((b*sinh(e + f*x)^2)/a + 1)^p)
```

3.404 $\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*sinh(f*x+e)*(a+b*s
inh(f*x+e)^2)^p/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A]

time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3269, 441, 440}

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sinh
[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{1+x^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)\operatorname{Subst}\Bigg(\int \frac{\left(1+\frac{bx^2}{a}\right.}{1+x^{2}}}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\,(a}{f}$$

Mathematica [F]
time = 2.48, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F]
time = 1.11, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(fx+e)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

Fricas [F]
time = 0.43, size = 23, normalized size = 0.29

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\operatorname{sech}\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^p}{\mathrm{cosh}\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x), x)
```

3.405 $\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*sinh(f*x+e)*(a+b*s
inh(f*x+e)^2)^p/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3269, 441, 440}

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sinh
[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^p}{(1+x^2)^2}\,dx, x, \sinh(e+fx)\right)}{f}\\ &= \frac{\left(\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right.}{(1+x^2)}\right.}{f}\\ &= \frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\,(a}{f}\end{aligned}$$

Mathematica [**F**]
time = 4.45, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [**F**]
time = 1.17, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(fx+e)^3\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```

Fricas [F]
time = 0.47, size = 25, normalized size = 0.32

$$\text{integral}\Big(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\operatorname{sech}\left(fx+e\right)^{3},x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,\mathrm{sinh}(e+f\,x)^2+a\right)^p}{{\mathrm{cosh}\left(e+f\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^3,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^3, x)
```

3.406 $\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\,\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,-3/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(a+b*sinh(f*x+e
)^2)^p*(cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3271, 441, 440}

$$\frac{\sqrt{\cosh^2(e+fx)}\,\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*S
qrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Si
nh[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
```

```
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\left(1+x^2\right)^{3/2}\left(a+b\right.\right.}{f}$$

$$=\frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\right.\right.}{}$$

$$=\frac{F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+}}{}$$

Mathematica [F]
time = 7.31, size = 0, normalized size = 0.00

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F]
time = 1.68, size = 0, normalized size = 0.00

$$\int \left(\cosh^4\left(fx+e\right)\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.46, size = 25, normalized size = 0.27

$$\text{integral}\Big(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\cosh\left(fx+e\right)^{4},x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(e+f\,x)^4\,\left(b\sinh(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^p, x)
```

3.407 $\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\,\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,-1/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(a+b*sinh(f*x+e
)^2)^p*(cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{3271, 441, 440\}$

$$\frac{\sqrt{\cosh^2(e+fx)}\,\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*S
qrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Si
nh[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
```

```
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\sqrt{1+x^2}\,(a+bx^2)\right.}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+}}{f}$$

Mathematica [**F**]
time = 7.14, size = 0, normalized size = 0.00

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [**F**]
time = 1.87, size = 0, normalized size = 0.00

$$\int \left(\cosh^2(fx+e)\right)\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.46, size = 25, normalized size = 0.27

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\cosh\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(e+f\,x)^2\,{\left(b\,\sinh(e+f\,x)^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^p, x)
```

3.408 $\int \left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\,\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\mathrm{t}}{f}$$

```
[Out] AppellF1(1/2,1/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(a+b*sinh(f*x+e)
^2)^p*(cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3264, 441, 440}

$$\frac{\sqrt{\cosh^2(e+fx)}\,\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sin
h[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3264

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*
x])), Subst[Int[(a + b*ff^2*x^2)^p/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (a+b\sinh^2(e+fx))^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+bx^2)^p}{\sqrt{1+x^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\,(a+b\sinh^2(e+fx))^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\,(a+b\sinh}{f}$$

Mathematica [F]
time = 0.86, size = 0, normalized size = 0.00

$$\int (a+b\sinh^2(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F]
time = 0.97, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p, x)
```

Fricas [**F**]
time = 0.41, size = 16, normalized size = 0.17

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p, x)
```

3.409 $\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\,\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,3/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(a+b*sinh(f*x+e)
^2)^p*(cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3271, 441, 440}

$$\frac{\sqrt{\cosh^2(e+fx)}\,\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sin
h[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
```

```
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{(a+bx^2)^p}{(1+x^2)^{3/2}}\,dx, x, \operatorname{sinh}\right.}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{bs}{}\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx}}{f}$$

Mathematica [**F**]
time = 3.22, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [**F**]
time = 1.01, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(fx+e)^2\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.47, size = 25, normalized size = 0.27

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\operatorname{sech}\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh(e+f\,x)^{2}+a\right)^{p}}{\cosh\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^2,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^2, x)
```

3.410 $\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}t}{f}$$

```
[Out] AppellF1(1/2,5/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(a+b*sinh(f*x+e)
^2)^p*(cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3271, 441, 440}

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 5/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sin
h[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
```

```
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{(a+bx^2)^p}{(1+x^2)^{5/2}}\,dx,x,\sin}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+f}}{f}$$

Mathematica [F]
time = 6.59, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F]
time = 1.12, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(fx+e)^4\left(a+b\left(\sinh^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.42, size = 25, normalized size = 0.27

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\operatorname{sech}\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^p}{{\mathrm{cosh}\left(e+f\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^4,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^4, x)
```

3.411 $\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal. Leaf size=259

$$-\frac{2a(a^4+b^4)^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d}+\frac{2(a^4+b^4)^2\sqrt{\sinh(c+dx)}}{b^9d}-\frac{a^3(a^4+2b^4)\sinh(c+dx)}{b^8d}+2a^2(a$$

```
[Out] -2*a*(a^4+b^4)^2*ln(a+b*sinh(d*x+c)^(1/2))/b^10/d-a^3*(a^4+2*b^4)*sinh(d*x+
c)/b^8/d+2/3*a^2*(a^4+2*b^4)*sinh(d*x+c)^(3/2)/b^7/d-1/2*a*(a^4+2*b^4)*sinh
(d*x+c)^2/b^6/d+2/5*(a^4+2*b^4)*sinh(d*x+c)^(5/2)/b^5/d-1/3*a^3*sinh(d*x+c)
^3/b^4/d+2/7*a^2*sinh(d*x+c)^(7/2)/b^3/d-1/4*a*sinh(d*x+c)^4/b^2/d+2/9*sinh
(d*x+c)^(9/2)/b/d+2*(a^4+b^4)^2*sinh(d*x+c)^(1/2)/b^9/d
```

Rubi [A]
time = 0.20, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3302, 1904, 1634}

$$-\frac{2a(a^4+b^4)^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d}+\frac{2(a^4+b^4)^2\sqrt{\sinh(c+dx)}}{b^9d}-\frac{a(a^4+2b^4)\sinh^2(c+dx)}{2b^6d}+\frac{2(a^4+2b^4)\sinh^{\frac{5}{2}}(c+dx)}{5b^5d}-\frac{a^3\sinh^3(c+dx)}{3b^4d}+\frac{2a^2\sinh^{\frac{7}{2}}(c+dx)}{7b^3d}-\frac{a^3(a^4+2b^4)\sinh(c+dx)}{b^8d}+\frac{2a^2(a^4+2b^4)\sinh^{\frac{3}{2}}(c+dx)}{3b^7d}-\frac{a\sinh^4(c+dx)}{4b^2d}+\frac{2\sinh^{\frac{9}{2}}(c+dx)}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2*a*(a^4 + b^4)^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^10*d) + (2*(a^4 + b^
4)^2*Sqrt[Sinh[c + d*x]])/(b^9*d) - (a^3*(a^4 + 2*b^4)*Sinh[c + d*x])/(b^8*
d) + (2*a^2*(a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7*d) - (a*(a^4 + 2*b^4)
*Sinh[c + d*x]^2)/(2*b^6*d) + (2*(a^4 + 2*b^4)*Sinh[c + d*x]^(5/2))/(5*b^5*
d) - (a^3*Sinh[c + d*x]^3)/(3*b^4*d) + (2*a^2*Sinh[c + d*x]^(7/2))/(7*b^3*d
) - (a*Sinh[c + d*x]^4)/(4*b^2*d) + (2*Sinh[c + d*x]^(9/2))/(9*b*d)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1904

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x
, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{a+b\sqrt{x}}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \frac{x\left(1+x^4\right)^2}{a+bx}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{\left(a^4+b^4\right)^2}{b^9} - \frac{a^3\left(a^4+2b^4\right)x}{b^8} + \frac{a^2\left(a^4+2b^4\right)x^2}{b^7} - \frac{a\left(a^4+2b^4\right)x^3}{b^6} + \frac{\left(a^4+2b^4\right)x^4}{b^5} - \frac{a^3x}{b^4}\right.\right.}{d}$$

$$= -\frac{2a\left(a^4+b^4\right)^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d} + \frac{2\left(a^4+b^4\right)^2\sqrt{\sinh(c+dx)}}{b^9d} - \frac{a^3}{}$$

Mathematica [A]
time = 0.41, size = 220, normalized size = 0.85

$$\frac{-2520a\left(a^4+b^4\right)^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)+2520b\left(a^4+b^4\right)^2\sqrt{\sinh(c+dx)}-1260a^3b^2\left(a^4+2b^4\right)\sinh(c+dx)+840a^2b^3\left(a^4+2b^4\right)\sinh^{\frac{3}{2}}(c+dx)-630ab^4\left(a^4+2b^4\right)\sinh^2(c+dx)+504b^5\left(a^4+2b^4\right)\sinh^{\frac{5}{2}}(c+dx)-420a^3b^6\sinh^3(c+dx)+360a^2b^7\sinh^{\frac{7}{2}}(c+dx)-315ab^8\sinh^4(c+dx)+280b^9\sinh^{\frac{9}{2}}(c+dx)}{1260b^{10}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2520*a*(a^4 + b^4)^2*Log[a + b*Sqrt[Sinh[c + d*x]]] + 2520*b*(a^4 + b^4)^
2*Sqrt[Sinh[c + d*x]] - 1260*a^3*b^2*(a^4 + 2*b^4)*Sinh[c + d*x] + 840*a^2*
b^3*(a^4 + 2*b^4)*Sinh[c + d*x]^(3/2) - 630*a*b^4*(a^4 + 2*b^4)*Sinh[c + d*
x]^2 + 504*b^5*(a^4 + 2*b^4)*Sinh[c + d*x]^(5/2) - 420*a^3*b^6*Sinh[c + d*x
]^3 + 360*a^2*b^7*Sinh[c + d*x]^(7/2) - 315*a*b^8*Sinh[c + d*x]^4 + 280*b^9
*Sinh[c + d*x]^(9/2))/(1260*b^10*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.51, size = 435, normalized size = 1.68

method	result

default	$a\Bigg(\frac{2\left(-\frac{1}{2}a^8-b^4a^4-\frac{1}{2}b^8\right)\ln\left(a^2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^2\right)}{b^{10}}-\frac{1}{4b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{-2a^2+3b^2}{6b^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{4a^4-4a^2b^2}{8b^6(\tanh(\frac{dx}{2}+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(2/b^10*(-1/2*a^8-b^4*a^4-1/2*b^8)*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*t
anh(1/2*d*x+1/2*c)-a^2)-1/4/b^2/(tanh(1/2*d*x+1/2*c)-1)^4-1/6*(-2*a^2+3*b^2
)/b^4/(tanh(1/2*d*x+1/2*c)-1)^3-1/8*(4*a^4-4*a^2*b^2+9*b^4)/b^6/(tanh(1/2*d
*x+1/2*c)-1)^2+(a^8+2*a^4*b^4+b^8)/b^10*ln(tanh(1/2*d*x+1/2*c)-1)-1/8*(-8*a
^6+4*a^4*b^2-16*a^2*b^4+7*b^6)/b^8/(tanh(1/2*d*x+1/2*c)-1)-1/4/b^2/(tanh(1/
2*d*x+1/2*c)+1)^4-1/6*(-2*a^2-3*b^2)/b^4/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(4*a
^4+4*a^2*b^2+9*b^4)/b^6/(tanh(1/2*d*x+1/2*c)+1)^2+(a^8+2*a^4*b^4+b^8)/b^10*
ln(tanh(1/2*d*x+1/2*c)+1)-1/8*(-8*a^6-4*a^4*b^2-16*a^2*b^4-7*b^6)/b^8/(tanh
(1/2*d*x+1/2*c)+1))+‘int/indef0‘(-cosh(d*x+c)^4*b*sinh(d*x+c)^(1/2)/(-b^2*s
inh(d*x+c)+a^2),sinh(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sqrt(sinh(d*x + c)) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2595 vs. 2(233) = 466.
time = 1.32, size = 2595, normalized size = 10.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/20160*(315*a*b^8*cosh(d*x + c)^8 + 315*a*b^8*sinh(d*x + c)^8 + 840*a^3*b
^6*cosh(d*x + c)^7 - 840*a^3*b^6*cosh(d*x + c) + 315*a*b^8 + 840*(3*a*b^8*c
osh(d*x + c) + a^3*b^6)*sinh(d*x + c)^7 + 1260*(2*a^5*b^4 + 3*a*b^8)*cosh(d
*x + c)^6 + 420*(21*a*b^8*cosh(d*x + c)^2 + 14*a^3*b^6*cosh(d*x + c) + 6*a^
5*b^4 + 9*a*b^8)*sinh(d*x + c)^6 + 2520*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x +
c)^5 + 2520*(7*a*b^8*cosh(d*x + c)^3 + 7*a^3*b^6*cosh(d*x + c)^2 + 4*a^7*b^
2 + 7*a^3*b^6 + 3*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c))*sinh(d*x + c)^5 - 20
```

```
160*((a^9 + 2*a^5*b^4 + a*b^8)*d*x + (a^9 + 2*a^5*b^4 + a*b^8)*c)*cosh(d*x
+ c)^4 + 210*(105*a*b^8*cosh(d*x + c)^4 + 140*a^3*b^6*cosh(d*x + c)^3 - 96*
(a^9 + 2*a^5*b^4 + a*b^8)*d*x + 90*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^2 -
96*(a^9 + 2*a^5*b^4 + a*b^8)*c + 60*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c))*
sinh(d*x + c)^4 - 2520*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c)^3 + 840*(21*a*
b^8*cosh(d*x + c)^5 + 35*a^3*b^6*cosh(d*x + c)^4 - 12*a^7*b^2 - 21*a^3*b^6
+ 30*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^3 + 30*(4*a^7*b^2 + 7*a^3*b^6)*cos
h(d*x + c)^2 - 96*((a^9 + 2*a^5*b^4 + a*b^8)*d*x + (a^9 + 2*a^5*b^4 + a*b^8
)*c)*cosh(d*x + c))*sinh(d*x + c)^3 + 1260*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x +
 c)^2 + 1260*(7*a*b^8*cosh(d*x + c)^6 + 14*a^3*b^6*cosh(d*x + c)^5 + 2*a^5*
b^4 + 3*a*b^8 + 15*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^4 + 20*(4*a^7*b^2 +
7*a^3*b^6)*cosh(d*x + c)^3 - 96*((a^9 + 2*a^5*b^4 + a*b^8)*d*x + (a^9 + 2*a
^5*b^4 + a*b^8)*c)*cosh(d*x + c)^2 - 6*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c
))*sinh(d*x + c)^2 - 20160*((a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x + c)^4 + 4*(
a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^9 + 2*a^5*b^4
 + a*b^8)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^9 + 2*a^5*b^4 + a*b^8)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a^9 + 2*a^5*b^4 + a*b^8)*sinh(d*x + c)^4)*log
((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2*cosh(d*x + c) - b^2 + 2
*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) - 4*(a*b*cosh(d*x + c) + a*b*sinh(
d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 -
 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) - a^2)*sinh(d*x + c))) +
20160*((a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x + c)^4 + 4*(a^9 + 2*a^5*b^4 + a*b
^8)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x +
c)^2*sinh(d*x + c)^2 + 4*(a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a^9 + 2*a^5*b^4 + a*b^8)*sinh(d*x + c)^4)*log(2*(b^2*sinh(d*x + c)
 - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 840*(3*a*b^8*cosh(d*x + c)^7 + 7
*a^3*b^6*cosh(d*x + c)^6 - a^3*b^6 + 9*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^
5 + 15*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c)^4 - 96*((a^9 + 2*a^5*b^4 + a*b
^8)*d*x + (a^9 + 2*a^5*b^4 + a*b^8)*c)*cosh(d*x + c)^3 - 9*(4*a^7*b^2 + 7*a
^3*b^6)*cosh(d*x + c)^2 + 3*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c))*sinh(d*x +
 c) - 8*(35*b^9*cosh(d*x + c)^8 + 35*b^9*sinh(d*x + c)^8 + 90*a^2*b^7*cosh(
d*x + c)^7 - 90*a^2*b^7*cosh(d*x + c) + 35*b^9 + 10*(28*b^9*cosh(d*x + c) +
 9*a^2*b^7)*sinh(d*x + c)^7 + 28*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c)^6 + 14*
(70*b^9*cosh(d*x + c)^2 + 45*a^2*b^7*cosh(d*x + c) + 18*a^4*b^5 + 26*b^9)*s
inh(d*x + c)^6 + 30*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x + c)^5 + 2*(980*b^9*
cosh(d*x + c)^3 + 945*a^2*b^7*cosh(d*x + c)^2 + 420*a^6*b^3 + 705*a^2*b^7 +
 84*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c))*sinh(d*x + c)^5 + 42*(120*a^8*b + 2
28*a^4*b^5 + 101*b^9)*cosh(d*x + c)^4 + 2*(1225*b^9*cosh(d*x + c)^4 + 1575*
a^2*b^7*cosh(d*x + c)^3 + 2520*a^8*b + 4788*a^4*b^5 + 2121*b^9 + 210*(9*a^4
*b^5 + 13*b^9)*cosh(d*x + c)^2 + 75*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x + c)
)*sinh(d*x + c)^4 - 30*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x + c)^3 + 2*(980*b
^9*cosh(d*x + c)^5 + 1575*a^2*b^7*cosh(d*x + c)^4 - 420*a^6*b^3 - 705*a^2*b
^7 + 280*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c)^3 + 150*(28*a^6*b^3 + 47*a^2*b^
7)*cosh(d*x + c)^2 + 84*(120*a^8*b + 228*a^4*b^5 + 101*b^9)*cosh(d*x + c))*
sinh(d*x + c)^3 + 28*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c)^2 + 2*(490*b^9*cosh
```

```
(d*x + c)^6 + 945*a^2*b^7*cosh(d*x + c)^5 + 126*a^4*b^5 + 182*b^9 + 210*(9*
a^4*b^5 + 13*b^9)*cosh(d*x + c)^4 + 150*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x
+ c)^3 + 126*(120*a^8*b + 228*a^4*b^5 + 101*b^9)*cosh(d*x + c)^2 - 45*(28*a
^6*b^3 + 47*a^2*b^7)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(140*b^9*cosh(d*x +
 c)^7 + 315*a^2*b^7*cosh(d*x + c)^6 - 45*a^2*b^7 + 84*(9*a^4*b^5 + 13*b^9)*
cosh(d*x + c)^5 + 75*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x + c)^4 + 84*(120*a^
8*b + 228*a^4*b^5 + 101*b^9)*cosh(d*x + c)^3 - 45*(28*a^6*b^3 + 47*a^2*b^7)
*cosh(d*x + c)^2 + 28*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(sinh(d*x + c)))/(b^10*d*cosh(d*x + c)^4 + 4*b^10*d*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^10*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^10*d*cosh(d*x + c
)*sinh(d*x + c)^3 + b^10*d*sinh(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sqrt(sinh(d*x + c)) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^5}{a+b\,\sqrt{\sinh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^(1/2)),x)
```

```
[Out] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^(1/2)), x)
```

3.412 $\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal. Leaf size=136

$$-\frac{2a(a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}+\frac{2(a^4+b^4)\sqrt{\sinh(c+dx)}}{b^5d}-\frac{a^3\sinh(c+dx)}{b^4d}+\frac{2a^2\sinh^{\frac{3}{2}}(c+dx)}{3b^3d}-$$

```
[Out] -2*a*(a^4+b^4)*ln(a+b*sinh(d*x+c)^(1/2))/b^6/d-a^3*sinh(d*x+c)/b^4/d+2/3*a^
2*sinh(d*x+c)^(3/2)/b^3/d-1/2*a*sinh(d*x+c)^2/b^2/d+2/5*sinh(d*x+c)^(5/2)/b
/d+2*(a^4+b^4)*sinh(d*x+c)^(1/2)/b^5/d
```

Rubi [A]
time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3302, 1904, 1634}

$$-\frac{2a(a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}+\frac{2(a^4+b^4)\sqrt{\sinh(c+dx)}}{b^5d}-\frac{a^3\sinh(c+dx)}{b^4d}+\frac{2a^2\sinh^{\frac{3}{2}}(c+dx)}{3b^3d}-\frac{a\sinh^2(c+dx)}{2b^2d}+\frac{2\sinh^{\frac{5}{2}}(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2*a*(a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^6*d) + (2*(a^4 + b^4)*
Sqrt[Sinh[c + d*x]])/(b^5*d) - (a^3*Sinh[c + d*x])/(b^4*d) + (2*a^2*Sinh[c
 + d*x]^(3/2))/(3*b^3*d) - (a*Sinh[c + d*x]^2)/(2*b^2*d) + (2*Sinh[c + d*x]^
(5/2))/(5*b*d)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1904

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x
, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
```

```
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+b\sqrt{x}}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{2\text{Subst}\left(\int \frac{x(1+x^4)}{a+bx}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{a^4+b^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a(a^4+b^4)}{b^5(a+bx)}\right)dx, x, \sqrt{\sinh(c+dx)}\right.}{d}\\
&= -\frac{2a(a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}+\frac{2(a^4+b^4)\sqrt{\sinh(c+dx)}}{b^5d}-\frac{a^3}{}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 117, normalized size = 0.86

$$\frac{-60a(a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)+60b(a^4+b^4)\sqrt{\sinh(c+dx)}-30a^3b^2\sinh(c+dx)+20a^2b^3\sinh^{\frac{3}{2}}(c+dx)-15ab^4\sinh^2(c+dx)+12b^5\sinh^{\frac{5}{2}}(c+dx)}{30b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-60*a*(a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]] + 60*b*(a^4 + b^4)*Sqrt[S
inh[c + d*x]] - 30*a^3*b^2*Sinh[c + d*x] + 20*a^2*b^3*Sinh[c + d*x]^(3/2) -
 15*a*b^4*Sinh[c + d*x]^2 + 12*b^5*Sinh[c + d*x]^(5/2))/(30*b^6*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.22, size = 245, normalized size = 1.80

method	result
default	$\frac{a\left(\frac{2\left(-\frac{a^4}{2}-\frac{b^4}{2}\right)\ln\left(a^2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^2\right)}{b^6}-\frac{1}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-2a^2+b^2}{2b^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(a^4+b^4\right)\ln\left(\tanh\left(\frac{dx}{2}+\right.\right.}{b^6}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(2/b^6*(-1/2*a^4-1/2*b^4)*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d
*x+1/2*c)-a^2)-1/2/b^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/2*(-2*a^2+b^2)/b^4/(tanh
```

```
(1/2*d*x+1/2*c)-1)+(a^4+b^4)/b^6*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/b^2/(tanh(1/
2*d*x+1/2*c)+1)^2-1/2*(-2*a^2-b^2)/b^4/(tanh(1/2*d*x+1/2*c)+1)+(a^4+b^4)/b^
6*ln(tanh(1/2*d*x+1/2*c)+1))+‘int/indef0‘(-cosh(d*x+c)^2*b*sinh(d*x+c)^(1/2
)/(-b^2*sinh(d*x+c)+a^2),sinh(d*x+c))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 879 vs. 2(122) = 244.
time = 0.92, size = 879, normalized size = 6.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/120*(15*a*b^4*cosh(d*x + c)^4 + 15*a*b^4*sinh(d*x + c)^4 + 60*a^3*b^2*co
sh(d*x + c)^3 - 60*a^3*b^2*cosh(d*x + c) + 15*a*b^4 + 60*(a*b^4*cosh(d*x +
c) + a^3*b^2)*sinh(d*x + c)^3 - 120*((a^5 + a*b^4)*d*x + (a^5 + a*b^4)*c)*c
osh(d*x + c)^2 + 30*(3*a*b^4*cosh(d*x + c)^2 + 6*a^3*b^2*cosh(d*x + c) - 4*
(a^5 + a*b^4)*d*x - 4*(a^5 + a*b^4)*c)*sinh(d*x + c)^2 - 120*((a^5 + a*b^4)
*cosh(d*x + c)^2 + 2*(a^5 + a*b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^5 + a*b
^4)*sinh(d*x + c)^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2
*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) - 4*(a*b*c
osh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2
 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) -
 a^2)*sinh(d*x + c))) + 120*((a^5 + a*b^4)*cosh(d*x + c)^2 + 2*(a^5 + a*b^4
)*cosh(d*x + c)*sinh(d*x + c) + (a^5 + a*b^4)*sinh(d*x + c)^2)*log(2*(b^2*s
inh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 60*(a*b^4*cosh(d*x +
 c)^3 + 3*a^3*b^2*cosh(d*x + c)^2 - a^3*b^2 - 4*((a^5 + a*b^4)*d*x + (a^5 +
 a*b^4)*c)*cosh(d*x + c))*sinh(d*x + c) - 4*(3*b^5*cosh(d*x + c)^4 + 3*b^5*
sinh(d*x + c)^4 + 10*a^2*b^3*cosh(d*x + c)^3 - 10*a^2*b^3*cosh(d*x + c) + 3
*b^5 + 2*(6*b^5*cosh(d*x + c) + 5*a^2*b^3)*sinh(d*x + c)^3 + 6*(10*a^4*b +
9*b^5)*cosh(d*x + c)^2 + 6*(3*b^5*cosh(d*x + c)^2 + 5*a^2*b^3*cosh(d*x + c)
 + 10*a^4*b + 9*b^5)*sinh(d*x + c)^2 + 2*(6*b^5*cosh(d*x + c)^3 + 15*a^2*b^
3*cosh(d*x + c)^2 - 5*a^2*b^3 + 6*(10*a^4*b + 9*b^5)*cosh(d*x + c))*sinh(d*
x + c))*sqrt(sinh(d*x + c)))/(b^6*d*cosh(d*x + c)^2 + 2*b^6*d*cosh(d*x + c)
*sinh(d*x + c) + b^6*d*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^3}{a+b\,\sqrt{\sinh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^(1/2)),x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^(1/2)), x)
```

3.413 $\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal. Leaf size=43

$$-\frac{2a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}+\frac{2\sqrt{\sinh(c+dx)}}{bd}$$

```
[Out] -2*a*ln(a+b*sinh(d*x+c)^(1/2))/b^2/d+2*sinh(d*x+c)^(1/2)/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3302, 196, 45}

$$\frac{2\sqrt{\sinh(c+dx)}}{bd}-\frac{2a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2*a*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^2*d) + (2*Sqrt[Sinh[c + d*x]])/(b*
d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+b\sqrt{x}}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{2\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}\\ &= \frac{2\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}\\ &= -\frac{2a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}+\frac{2\sqrt{\sinh(c+dx)}}{bd}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.95

$$\frac{2\left(-\frac{a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2}+\frac{\sqrt{\sinh(c+dx)}}{b}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (2*(-((a*Log[a + b*Sqrt[Sinh[c + d*x]]])/b^2) + Sqrt[Sinh[c + d*x]]/b))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(39) = 78$.
time = 1.00, size = 80, normalized size = 1.86

method	result	size
derivativedivides	$\frac{\frac{2\left(\sqrt{\sinh}\,(dx+c)\right)}{b}+\frac{a\ln\left(-b\left(\sqrt{\sinh}\,(dx+c)\right)+a\right)}{b^2}-\frac{a\ln\left(a+b\left(\sqrt{\sinh}\,(dx+c)\right)\right)}{b^2}-\frac{a\ln\left(b^2\sinh(dx+c)-a^2\right)}{b^2}}{d}$	80
default	$\frac{\frac{2\left(\sqrt{\sinh}\,(dx+c)\right)}{b}+\frac{a\ln\left(-b\left(\sqrt{\sinh}\,(dx+c)\right)+a\right)}{b^2}-\frac{a\ln\left(a+b\left(\sqrt{\sinh}\,(dx+c)\right)\right)}{b^2}-\frac{a\ln\left(b^2\sinh(dx+c)-a^2\right)}{b^2}}{d}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b*sinh(d*x+c)^(1/2)+1/b^2*a*ln(-b*sinh(d*x+c)^(1/2)+a)-1/b^2*a*ln(a+
b*sinh(d*x+c)^(1/2))-a*ln(b^2*sinh(d*x+c)-a^2)/b^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(39) = 78.
time = 0.86, size = 225, normalized size = 5.23

$$\frac{adx + a\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,a^2\cosh(dx+c)-b^2+2\,(b^2\cosh(dx+c)+a^2)\sinh(dx+c)-4\,(ab\cosh(dx+c)+ab\sinh(dx+c))\sqrt{\sinh(dx+c)}}{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2-2\,a^2\cosh(dx+c)-b^2+2\,(b^2\cosh(dx+c)-a^2)\sinh(dx+c)}\right) - a\log\left(\frac{2\,(b^2\sinh(dx+c)-a^2)}{\cosh(dx+c)-\sinh(dx+c)}\right) + 2\,b\sqrt{\sinh(dx+c)}}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] (a*d*x + a*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2*cosh(d*x
+ c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) - 4*(a*b*cosh(d*x +
c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2 + b^2*sin
h(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) - a^2)*sinh
(d*x + c))) - a*log(2*(b^2*sinh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x +
 c))) + 2*b*sqrt(sinh(d*x + c)))/(b^2*d)
```

Sympy [**A**]
time = 1.11, size = 68, normalized size = 1.58

$$\begin{cases} \frac{x\cosh(c)}{a} & \text{for}\: b = 0 \wedge d = 0 \\ \frac{\sinh(c+dx)}{ad} & \text{for}\: b = 0 \\ \frac{x\cosh(c)}{a+b\sqrt{\sinh(c)}} & \text{for}\: d = 0 \\ -\frac{2a\log\left(\frac{a}{b}+\sqrt{\sinh(c+dx)}\right)}{b^2d} + \frac{2\sqrt{\sinh(c+dx)}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a*d), Eq(b, 0
)), (x*cosh(c)/(a + b*sqrt(sinh(c))), Eq(d, 0)), (-2*a*log(a/b + sqrt(sinh(
c + d*x)))/(b**2*d) + 2*sqrt(sinh(c + d*x))/(b*d), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

Mupad [B]
time = 1.01, size = 39, normalized size = 0.91

$$\frac{2\,\sqrt{\sinh\left(c+d\,x\right)}}{b\,d}-\frac{2\,a\,\ln\left(a+b\,\sqrt{\sinh\left(c+d\,x\right)}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^(1/2)),x)
```

```
[Out] (2*sinh(c + d*x)^(1/2))/(b*d) - (2*a*log(a + b*sinh(c + d*x)^(1/2)))/(b^2*d
)
```

3.414 $\int \frac{\text{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal. Leaf size=286

$$\frac{b(a^2-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\sinh(c+dx)}\right)}{\sqrt{2}\,(a^4+b^4)\,d}-\frac{b(a^2-b^2)\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\sinh(c+dx)}\right)}{\sqrt{2}\,(a^4+b^4)\,d}+\frac{a^3\text{ArcTan}(\text{sin}}{(a^4+b}$$

```
[Out] a^3*arctan(sinh(d*x+c))/(a^4+b^4)/d+a*b^2*ln(cosh(d*x+c))/(a^4+b^4)/d-2*a*b
^2*ln(a+b*sinh(d*x+c)^(1/2))/(a^4+b^4)/d-1/2*b*(a^2-b^2)*arctan(-1+2^(1/2)*
sinh(d*x+c)^(1/2))/(a^4+b^4)/d*2^(1/2)-1/2*b*(a^2-b^2)*arctan(1+2^(1/2)*sin
h(d*x+c)^(1/2))/(a^4+b^4)/d*2^(1/2)-1/4*b*(a^2+b^2)*ln(1+sinh(d*x+c)-2^(1/2
)*sinh(d*x+c)^(1/2))/(a^4+b^4)/d*2^(1/2)+1/4*b*(a^2+b^2)*ln(1+sinh(d*x+c)+2
^(1/2)*sinh(d*x+c)^(1/2))/(a^4+b^4)/d*2^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3302, 6857, 1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 209, 266}

$$-\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{d\,(a^4+b^4)}+\frac{ab^2\log(\cosh(c+dx))}{d\,(a^4+b^4)}+\frac{a^3\text{ArcTan}(\sinh(c+dx))}{d\,(a^4+b^4)}+\frac{b(a^2-b^2)\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\sinh(c+dx)}\right)}{\sqrt{2}\,d\,(a^4+b^4)}-\frac{b(a^2-b^2)\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\sinh(c+dx)}+1\right)}{\sqrt{2}\,d\,(a^4+b^4)}-\frac{b(a^2+b^2)\log\left(\sinh(c+dx)-\sqrt{2}\,\sqrt{\sinh(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^4+b^4)}+\frac{b(a^2+b^2)\log\left(\sinh(c+dx)+\sqrt{2}\,\sqrt{\sinh(c+dx)}+1\right)}{2\sqrt{2}\,d\,(a^4+b^4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (b*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^4 + b^4
)*d) - (b*(a^2 - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^
4 + b^4)*d) + (a^3*ArcTan[Sinh[c + d*x]])/((a^4 + b^4)*d) + (a*b^2*Log[Cosh
[c + d*x]])/((a^4 + b^4)*d) - (2*a*b^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/((a^
4 + b^4)*d) - (b*(a^2 + b^2)*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c +
 d*x]])/(2*Sqrt[2]*(a^4 + b^4)*d) + (b*(a^2 + b^2)*Log[1 + Sqrt[2]*Sqrt[Sin
h[c + d*x]] + Sinh[c + d*x]])/(2*Sqrt[2]*(a^4 + b^4)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(1+x^2)}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{2\text{Subst}\left(\int \frac{x}{(a+bx)(1+x^4)}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}\\
&= \frac{2\text{Subst}\left(\int \left(-\frac{ab^3}{(a^4+b^4)(a+bx)}+\frac{b^3+a^3x-a^2bx^2+ab^2x^3}{(a^4+b^4)(1+x^4)}\right)dx, x, \sqrt{\sinh(c+dx)}\right)}{d}\\
&= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)\,d}+\frac{2\text{Subst}\left(\int \frac{b^3+a^3x-a^2bx^2+ab^2x^3}{1+x^4}\,dx, x, \sqrt{\sin}\right.}{(a^4+b^4)\,d}\\
&= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)\,d}+\frac{2\text{Subst}\left(\int \left(\frac{b^3-a^2bx^2}{1+x^4}+\frac{x(a^3+ab^2x^2)}{1+x^4}\right)dx,\right.}{(a^4+b^4)\,d}\\
&= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)\,d}+\frac{2\text{Subst}\left(\int \frac{b^3-a^2bx^2}{1+x^4}\,dx, x, \sqrt{\sinh(c+dx)}\right.}{(a^4+b^4)\,d}\\
&= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)\,d}+\frac{\text{Subst}\left(\int \frac{a^3+ab^2x}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{(a^4+b^4)\,d}-\\
&= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)\,d}+\frac{a^3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{(a^4+b^4)\,d}+\\
&= \frac{a^3\tan^{-1}(\sinh(c+dx))}{(a^4+b^4)\,d}+\frac{ab^2\log(\cosh(c+dx))}{(a^4+b^4)\,d}-\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+}\right.}{(a^4+b^4)\,d}\\
&= \frac{b(a^2-b^2)\tan^{-1}\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)}{\sqrt{2}\,(a^4+b^4)\,d}-\frac{b(a^2-b^2)\tan^{-1}\left(1+\sqrt{2}\sqrt{\text{s}}\right.}{\sqrt{2}\,(a^4+b^4)\,d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.19, size = 229, normalized size = 0.80

$$\frac{3\left(-2\sqrt{2}\,b^3\text{ArcTan}\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)+2\sqrt{2}\,b^3\text{ArcTan}\left(1+\sqrt{2}\sqrt{\sinh(c+dx)}\right)+4a^3\text{ArcTan}(\sinh(c+dx))+4ab^2\log(\cosh(c+dx))-8ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)-\sqrt{2}\,b^3\log\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}+\sinh(c+dx)\right)+\sqrt{2}\,b^3\log\left(1+\sqrt{2}\sqrt{\sinh(c+dx)}+\sinh(c+dx)\right)\right)-8a^2b\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\sinh^2(c+dx)\right)\sinh^{\frac{3}{2}}(c+dx)}{12\,(a^4+b^4)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (3*(-2*Sqrt[2]*b^3*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]] + 2*Sqrt[2]*b^3*
ArcTan[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]]] + 4*a^3*ArcTan[Sinh[c + d*x]] + 4*a
```

```
*b^2*Log[Cosh[c + d*x]] - 8*a*b^2*Log[a + b*Sqrt[Sinh[c + d*x]]] - Sqrt[2]*
b^3*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]] + Sqrt[2]*b^3*Log[
1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]]) - 8*a^2*b*Hypergeometric2
F1[3/4, 1, 7/4, -Sinh[c + d*x]^2]*Sinh[c + d*x]^(3/2))/(12*(a^4 + b^4)*d)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.81, size = 195, normalized size = 0.68

method	result
default	$\frac{a\left(\frac{b^{2}\ln\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2a^{2}\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{4}+b^{4}}-\frac{b^{2}\ln\left(a^{2}\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^{2}\right)}{a^{4}+b^{4}}\right)}{d}+\frac{\text{‘}int/indef0\text{‘}\left(\frac{}{-b^{4}\left(\cosh^{4}(d\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(2/(a^4+b^4)*(1/2*b^2*ln(tanh(1/2*d*x+1/2*c)^2+1)+a^2*arctan(tanh(1/2*d
*x+1/2*c)))-b^2/(a^4+b^4)*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1
/2*c)-a^2))+`int/indef0`(b*sinh(d*x+c)^(1/2)*(-b^2*sinh(d*x+c)+a^2)/(-b^4*c
osh(d*x+c)^4+2*a^2*b^2*cosh(d*x+c)^2*sinh(d*x+c)+(-a^4+b^4)*cosh(d*x+c)^2),
sinh(d*x+c))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 17277 vs. 2(256) = 512.
time = 5.38, size = 17277, normalized size = 60.41

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/8*(4*sqrt(2)*(a^12 + 3*a^8*b^4 + 3*a^4*b^8 + b^12)*d^5*sqrt((a^8 + 2*a^4
*b^4 + b^8 - 2*(a^10 + 2*a^6*b^4 + a^2*b^8)*d^2*sqrt(b^4/((a^8 + 2*a^4*b^4
+ b^8)*d^4)))/(a^8 - 2*a^4*b^4 + b^8))*(b^4/((a^8 + 2*a^4*b^4 + b^8)*d^4))^
(3/4)*sqrt((a^8*b^4 - 2*a^4*b^8 + b^12)/((a^16 + 4*a^12*b^4 + 6*a^8*b^8 + 4
```

```
*a^4*b^12 + b^16)*d^4))*arctan(-1/2*(sqrt(2)*sqrt(1/2)*((((a^20*b^5 + 3*a^1
6*b^9 + 2*a^12*b^13 - 2*a^8*b^17 - 3*a^4*b^21 - b^25)*d^7*e^(10*d*x + 10*c)
 - 6*(a^22*b^3 + 3*a^18*b^7 + 2*a^14*b^11 - 2*a^10*b^15 - 3*a^6*b^19 - a^2*
b^23)*d^7*e^(9*d*x + 9*c) - 21*(a^20*b^5 + 3*a^16*b^9 + 2*a^12*b^13 - 2*a^8
*b^17 - 3*a^4*b^21 - b^25)*d^7*e^(8*d*x + 8*c) + 56*(a^22*b^3 + 3*a^18*b^7
+ 2*a^14*b^11 - 2*a^10*b^15 - 3*a^6*b^19 - a^2*b^23)*d^7*e^(7*d*x + 7*c) +
106*(a^20*b^5 + 3*a^16*b^9 + 2*a^12*b^13 - 2*a^8*b^17 - 3*a^4*b^21 - b^25)*
d^7*e^(6*d*x + 6*c) - 132*(a^22*b^3 + 3*a^18*b^7 + 2*a^14*b^11 - 2*a^10*b^1
5 - 3*a^6*b^19 - a^2*b^23)*d^7*e^(5*d*x + 5*c) - 106*(a^20*b^5 + 3*a^16*b^9
 + 2*a^12*b^13 - 2*a^8*b^ ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sqrt(sinh(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh(c+d\,x)\,\left(a+b\,\sqrt{\sinh(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^(1/2))),x)
```

```
[Out] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^(1/2))), x)
```

3.415 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal. Leaf size=270

$$\frac{2(a^4+b^4)(9a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d}+\frac{2a(a^4+b^4)^2}{b^{10}d\left(a+b\sqrt{\sinh(c+dx)}\right)}-\frac{16a^3(a^4+b^4)\sqrt{\sinh(c+dx)}}{b^9d}$$

```
[Out] 2*(a^4+b^4)*(9*a^4+b^4)*ln(a+b*sinh(d*x+c)^(1/2))/b^10/d+a^2*(7*a^4+6*b^4)*
sinh(d*x+c)/b^8/d-4/3*a*(3*a^4+2*b^4)*sinh(d*x+c)^(3/2)/b^7/d+1/2*(5*a^4+2*
b^4)*sinh(d*x+c)^2/b^6/d-8/5*a^3*sinh(d*x+c)^(5/2)/b^5/d+a^2*sinh(d*x+c)^3/
b^4/d-4/7*a*sinh(d*x+c)^(7/2)/b^3/d+1/4*sinh(d*x+c)^4/b^2/d-16*a^3*(a^4+b^4
)*sinh(d*x+c)^(1/2)/b^9/d+2*a*(a^4+b^4)^2/b^10/d/(a+b*sinh(d*x+c)^(1/2))
```

Rubi [A]
time = 0.23, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3302, 1904, 1634}

$$\frac{2a(a^4+b^4)^2}{b^{10}d\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{2(a^4+b^4)(9a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d}-\frac{4a(3a^4+2b^4)\sinh^{\frac{3}{2}}(c+dx)}{3b^7d}+\frac{(5a^4+2b^4)\sinh^2(c+dx)}{2b^6d}-\frac{8a^3\sinh^{\frac{5}{2}}(c+dx)}{5b^5d}+\frac{a^2\sinh^3(c+dx)}{b^4d}-\frac{16a^3(a^4+b^4)\sqrt{\sinh(c+dx)}}{b^9d}+\frac{a^2(7a^4+6b^4)\sinh(c+dx)}{b^8d}-\frac{4a\sinh^{\frac{7}{2}}(c+dx)}{7b^3d}+\frac{\sinh^4(c+dx)}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (2*(a^4 + b^4)*(9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^10*d) + (2*
a*(a^4 + b^4)^2)/(b^10*d*(a + b*Sqrt[Sinh[c + d*x]])) - (16*a^3*(a^4 + b^4)
*Sqrt[Sinh[c + d*x]])/(b^9*d) + (a^2*(7*a^4 + 6*b^4)*Sinh[c + d*x])/(b^8*d)
 - (4*a*(3*a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7*d) + ((5*a^4 + 2*b^4)*S
inh[c + d*x]^2)/(2*b^6*d) - (8*a^3*Sinh[c + d*x]^(5/2))/(5*b^5*d) + (a^2*Si
nh[c + d*x]^3)/(b^4*d) - (4*a*Sinh[c + d*x]^(7/2))/(7*b^3*d) + Sinh[c + d*x
]^4/(4*b^2*d)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1904

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x
```

```
, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{\left(a+b\sqrt{x}\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \frac{x(1+x^4)^2}{(a+bx)^2}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{8a^3(a^4+b^4)}{b^9} + \frac{a^2(7a^4+6b^4)x}{b^8} - \frac{2a(3a^4+2b^4)x^2}{b^7} + \frac{(5a^4+2b^4)x^3}{b^6} - \frac{4a^3x^4}{b^5}\right.\right.}{d}$$

$$= \frac{2(a^4+b^4)(9a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d} + \frac{2a(a^4+b^4)^2}{b^{10}d\left(a+b\sqrt{\sinh(c\right.}}$$

Mathematica [A]
time = 0.43, size = 288, normalized size = 1.07

$$\frac{840a(a^4+b^4)\left(a^4+b^4+(9a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)\right)+840b(a^4+b^4)\left(-8a^4+(9a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)\right)\sqrt{\sinh(c+dx)}-420a^3b^2(9a^4+10b^4)\sinh(c+dx)+140a^2b^3(9a^4+10b^4)\sinh^{\frac{3}{2}}(c+dx)-70ab^4(9a^4+10b^4)\sinh^2(c+dx)+42b^5(9a^4+10b^4)\sinh^{\frac{5}{2}}(c+dx)-252a^3b^6\sinh^3(c+dx)+180a^2b^7\sinh^{\frac{7}{2}}(c+dx)-135ab^8\sinh^4(c+dx)+105b^9\sinh^{\frac{9}{2}}(c+dx)}{420b^{10}d\left(a+b\sqrt{\sinh(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (840*a*(a^4 + b^4)*(a^4 + b^4 + (9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]
]) + 840*b*(a^4 + b^4)*(-8*a^4 + (9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]
]])*Sqrt[Sinh[c + d*x]] - 420*a^3*b^2*(9*a^4 + 10*b^4)*Sinh[c + d*x] + 140*
a^2*b^3*(9*a^4 + 10*b^4)*Sinh[c + d*x]^(3/2) - 70*a*b^4*(9*a^4 + 10*b^4)*Si
nh[c + d*x]^2 + 42*b^5*(9*a^4 + 10*b^4)*Sinh[c + d*x]^(5/2) - 252*a^3*b^6*S
inh[c + d*x]^3 + 180*a^2*b^7*Sinh[c + d*x]^(7/2) - 135*a*b^8*Sinh[c + d*x]^
4 + 105*b^9*Sinh[c + d*x]^(9/2))/(420*b^10*d*(a + b*Sqrt[Sinh[c + d*x]]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.89, size = 525, normalized size = 1.94

method	result
default	$\frac{\frac{-\frac{4\left(a^{8}+2b^{4}a^{4}+b^{8}\right)b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^{2}\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^{2}}+\left(9a^{8}+10b^{4}a^{4}+b^{8}\right)\ln\left(a^{2}\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^{2}\right)}{b^{10}}+\frac{1}{4b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^10*(-2*(a^8+2*a^4*b^4+b^8)*b^2*tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d
*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)+1/2*(9*a^8+10*a^4*b^4+b^8)*ln(a^
2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2))+1/4/b^2/(tanh(1/2*d
*x+1/2*c)+1)^4-1/8/b^8*(56*a^6+20*a^4*b^2+48*a^2*b^4+7*b^6)/(tanh(1/2*d*x+1
/2*c)+1)-1/6*(6*a^2+3*b^2)/b^4/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(-20*a^4-12*a^
2*b^2-9*b^4)/b^6/(tanh(1/2*d*x+1/2*c)+1)^2+(-9*a^8-10*a^4*b^4-b^8)/b^10*ln(
tanh(1/2*d*x+1/2*c)+1)+1/4/b^2/(tanh(1/2*d*x+1/2*c)-1)^4-1/8/b^8*(56*a^6-20
*a^4*b^2+48*a^2*b^4-7*b^6)/(tanh(1/2*d*x+1/2*c)-1)-1/6*(6*a^2-3*b^2)/b^4/(t
anh(1/2*d*x+1/2*c)-1)^3-1/8*(-20*a^4+12*a^2*b^2-9*b^4)/b^6/(tanh(1/2*d*x+1/
2*c)-1)^2+(-9*a^8-10*a^4*b^4-b^8)/b^10*ln(tanh(1/2*d*x+1/2*c)-1))+‘int/inde
f0‘(-2*cosh(d*x+c)^4*a*b*sinh(d*x+c)^(1/2)/(b^4*sinh(d*x+c)^2-2*a^2*b^2*sin
h(d*x+c)+a^4),sinh(d*x+c))/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5181 vs. 2(248) = 496.
time = 1.03, size = 5181, normalized size = 19.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(105*b^10*cosh(d*x + c)^10 + 105*b^10*sinh(d*x + c)^10 + 630*a^2*b^8
*cosh(d*x + c)^9 + 630*a^2*b^8*cosh(d*x + c) - 105*b^10 + 210*(5*b^10*cosh(
```

```
d*x + c) + 3*a^2*b^8)*sinh(d*x + c)^9 + 105*(24*a^4*b^6 + 11*b^10)*cosh(d*x
 + c)^8 + 105*(45*b^10*cosh(d*x + c)^2 + 54*a^2*b^8*cosh(d*x + c) + 24*a^4*
b^6 + 11*b^10)*sinh(d*x + c)^8 + 840*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c
)^7 + 840*(15*b^10*cosh(d*x + c)^3 + 27*a^2*b^8*cosh(d*x + c)^2 + 18*a^6*b^
4 + 17*a^2*b^8 + (24*a^4*b^6 + 11*b^10)*cosh(d*x + c))*sinh(d*x + c)^7 - 42
0*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d
*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^6 + 210*(105*b^10*
cosh(d*x + c)^4 + 252*a^2*b^8*cosh(d*x + c)^3 - 224*a^8*b^2 - 188*a^4*b^6 -
 6*b^10 - 32*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x + 14*(24*a^4*b^6 + 11*b^10
)*cosh(d*x + c)^2 - 32*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c + 28*(18*a^6*b^4 +
 17*a^2*b^8)*cosh(d*x + c))*sinh(d*x + c)^6 - 1680*(16*a^10 + 60*a^6*b^4 +
37*a^2*b^8 - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*d*x - 8*(9*a^10 + 10*a^6*b^4
 + a^2*b^8)*c)*cosh(d*x + c)^5 + 420*(63*b^10*cosh(d*x + c)^5 + 189*a^2*b^8
*cosh(d*x + c)^4 - 64*a^10 - 240*a^6*b^4 - 148*a^2*b^8 + 14*(24*a^4*b^6 + 1
1*b^10)*cosh(d*x + c)^3 + 32*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*d*x + 42*(18*a
^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^2 + 32*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*c
 - 6*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10
)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c))*sinh(d*x + c)^
5 + 420*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b
^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^4 + 210*(105
*b^10*cosh(d*x + c)^6 + 378*a^2*b^8*cosh(d*x + c)^5 + 224*a^8*b^2 + 188*a^4
*b^6 + 6*b^10 + 35*(24*a^4*b^6 + 11*b^10)*cosh(d*x + c)^4 + 140*(18*a^6*b^4
 + 17*a^2*b^8)*cosh(d*x + c)^3 + 32*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x - 3
0*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d
*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^2 + 32*(9*a^8*b^2
+ 10*a^4*b^6 + b^10)*c - 40*(16*a^10 + 60*a^6*b^4 + 37*a^2*b^8 - 8*(9*a^10
+ 10*a^6*b^4 + a^2*b^8)*d*x - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*c)*cosh(d*x
 + c))*sinh(d*x + c)^4 + 840*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^3 + 84
0*(15*b^10*cosh(d*x + c)^7 + 63*a^2*b^8*cosh(d*x + c)^6 + 18*a^6*b^4 + 17*a
^2*b^8 + 7*(24*a^4*b^6 + 11*b^10)*cosh(d*x + c)^5 + 35*(18*a^6*b^4 + 17*a^2
*b^8)*cosh(d*x + c)^4 - 10*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b
^2 + 10*a^4*b^6 + b^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*
x + c)^3 - 20*(16*a^10 + 60*a^6*b^4 + 37*a^2*b^8 - 8*(9*a^10 + 10*a^6*b^4 +
 a^2*b^8)*d*x - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*c)*cosh(d*x + c)^2 + 2*(1
12*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x +
 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c))*sinh(d*x + c)^3 - 105
*(24*a^4*b^6 + 11*b^10)*cosh(d*x + c)^2 + 105*(45*b^10*cosh(d*x + c)^8 + 21
6*a^2*b^8*cosh(d*x + c)^7 - 24*a^4*b^6 - 11*b^10 + 28*(24*a^4*b^6 + 11*b^10
)*cosh(d*x + c)^6 + 168*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^5 - 60*(112
*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x + 1
6*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^4 - 160*(16*a^10 + 60*a^
6*b^4 + 37*a^2*b^8 - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*d*x - 8*(9*a^10 + 10
*a^6*b^4 + a^2*b^8)*c)*cosh(d*x + c)^3 + 24*(112*a^8*b^2 + 94*a^4*b^6 + 3*b
^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 +
 b^10)*c)*cosh(d*x + c)^2 + 24*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c))*sin
```

```
h(d*x + c)^2 + 6720*((9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^6 + (9*a
^8*b^2 + 10*a^4*b^6 + b^10)*sinh(d*x + c)^6 - 2*(9*a^10 + 10*a^6*b^4 + a^2*
b^8)*cosh(d*x + c)^5 - 2*(9*a^10 + 10*a^6*b^4 + a^2*b^8 - 3*(9*a^8*b^2 + 10
*a^4*b^6 + b^10)*cosh(d*x + c))*sinh(d*x + c)^5 - (9*a^8*b^2 + 10*a^4*b^6 +
 b^10)*cosh(d*x + c)^4 - (9*a^8*b^2 + 10*a^4*b^6 + b^10 - 15*(9*a^8*b^2 + 1
0*a^4*b^6 + b^10)*cosh(d*x + c)^2 + 10*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh
(d*x + c))*sinh(d*x + c)^4 + 4*(5*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x
+ c)^3 - 5*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh(d*x + c)^2 - (9*a^8*b^2 + 1
0*a^4*b^6 + b^10)*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(9*a^8*b^2 + 10*a^4*
b^6 + b^10)*cosh(d*x + c)^4 - 20*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh(d*x +
 c)^3 - 6*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 2*(3*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^5 - 5*(9*a^10 + 10*a^6
*b^4 + a^2*b^8)*cosh(d*x + c)^4 - 2*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*
x + c)^3)*sinh(d*x + c))*log(-(b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 +
2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) + 4*(
a*b*cosh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x +
 c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x +
 c) - a^2)*sinh(d*x + c))) + 6720*((9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x
 + c)^6 + (9*a^8*b^2 + 10*a^4*b^6 + b^10)*sinh(d*x + c)^6 - 2*(9*a^10 + 10*
a^6*b^4 + a^2*b^8)*cosh(d*x + c)^5 - 2*(9*a^10 ...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^5}{{\left(a+b\,\sqrt{\mathrm{sinh}\left(c+d\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^(1/2))^2,x)
[Out] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^(1/2))^2, x)
```

3.416 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal. Leaf size=142

$$\frac{2(5a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}+\frac{2a(a^4+b^4)}{b^6d\left(a+b\sqrt{\sinh(c+dx)}\right)}-\frac{8a^3\sqrt{\sinh(c+dx)}}{b^5d}+\frac{3a^2\sinh(c+dx}{b^4d}$$

```
[Out] 2*(5*a^4+b^4)*ln(a+b*sinh(d*x+c)^(1/2))/b^6/d+3*a^2*sinh(d*x+c)/b^4/d-4/3*a
*sinh(d*x+c)^(3/2)/b^3/d+1/2*sinh(d*x+c)^2/b^2/d-8*a^3*sinh(d*x+c)^(1/2)/b^
5/d+2*a*(a^4+b^4)/b^6/d/(a+b*sinh(d*x+c)^(1/2))
```

Rubi [A]
time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3302, 1904, 1634}

$$\frac{2a(a^4+b^4)}{b^6d\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{2(5a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}-\frac{8a^3\sqrt{\sinh(c+dx)}}{b^5d}+\frac{3a^2\sinh(c+dx)}{b^4d}-\frac{4a\sinh^{\frac{3}{2}}(c+dx)}{3b^3d}+\frac{\sinh^2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (2*(5*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^6*d) + (2*a*(a^4 + b^4)
)/(b^6*d*(a + b*Sqrt[Sinh[c + d*x]])) - (8*a^3*Sqrt[Sinh[c + d*x]])/(b^5*d)
 + (3*a^2*Sinh[c + d*x])/(b^4*d) - (4*a*Sinh[c + d*x]^(3/2))/(3*b^3*d) + Si
nh[c + d*x]^2/(2*b^2*d)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1904

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x
, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
```

```
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b\sqrt{x}\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \frac{x(1+x^4)}{(a+bx)^2}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a(a^4+b^4)}{b^5(a+bx)^2}+\frac{5a^4+b^4}{b^5(a+bx)}\right)dx, x, \sqrt{\sinh(}\right.}{d}$$

$$= \frac{2(5a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}+\frac{2a(a^4+b^4)}{b^6d\left(a+b\sqrt{\sinh(c+dx)}\right)}-$$

Mathematica [A]
time = 0.46, size = 123, normalized size = 0.87

$$\frac{12\left((5a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)+\frac{a(a^4+b^4)}{a+b\sqrt{\sinh(c+dx)}}\right)-48a^3b\sqrt{\sinh(c+dx)}+18a^2b^2\sinh(c+dx)-8ab^3\sinh^{\frac{3}{2}}(c+dx)+3b^4\sinh^2(c+dx)}{6b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (12*((5*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]] + (a*(a^4 + b^4))/(a + b*
Sqrt[Sinh[c + d*x]])) - 48*a^3*b*Sqrt[Sinh[c + d*x]] + 18*a^2*b^2*Sinh[c +
d*x] - 8*a*b^3*Sinh[c + d*x]^(3/2) + 3*b^4*Sinh[c + d*x]^2)/(6*b^6*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.07, size = 327, normalized size = 2.30

method	result
default	$\frac{\frac{1}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{6a^2+b^2}{2b^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(-5a^4-b^4\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^6}+\frac{-\frac{4b^2\left(a^4+b^4\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^2}+(5a^4+b^4}{b^6}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/b^2/(tanh(1/2*d*x+1/2*c)+1)^2-1/2*(6*a^2+b^2)/b^4/(tanh(1/2*d*x+1/
2*c)+1)+(-5*a^4-b^4)/b^6*ln(tanh(1/2*d*x+1/2*c)+1)+2/b^6*(-2*b^2*(a^4+b^4)*
tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^
2)+1/2*(5*a^4+b^4)*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a
^2))+1/2/b^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/2*(6*a^2-b^2)/b^4/(tanh(1/2*d*x+1/
2*c)-1)+(-5*a^4-b^4)/b^6*ln(tanh(1/2*d*x+1/2*c)-1))+‘int/indef0‘(-2*cosh(d*
x+c)^2*a*b*sinh(d*x+c)^(1/2)/(b^4*sinh(d*x+c)^2-2*a^2*b^2*sinh(d*x+c)+a^4),
sinh(d*x+c))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2137 vs. $2(130) = 260$.
time = 0.99, size = 2137, normalized size = 15.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^6*cosh(d*x + c)^6 + 3*b^6*sinh(d*x + c)^6 + 30*a^2*b^4*cosh(d*x +
 c)^5 + 30*a^2*b^4*cosh(d*x + c) - 3*b^6 + 6*(3*b^6*cosh(d*x + c) + 5*a^2*b
^4)*sinh(d*x + c)^5 - 3*(24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*
a^4*b^2 + b^6)*c)*cosh(d*x + c)^4 + 3*(15*b^6*cosh(d*x + c)^2 + 50*a^2*b^4*
cosh(d*x + c) - 24*a^4*b^2 - b^6 - 8*(5*a^4*b^2 + b^6)*d*x - 8*(5*a^4*b^2 +
 b^6)*c)*sinh(d*x + c)^4 - 24*(4*a^6 + 7*a^2*b^4 - 2*(5*a^6 + a^2*b^4)*d*x
- 2*(5*a^6 + a^2*b^4)*c)*cosh(d*x + c)^3 + 12*(5*b^6*cosh(d*x + c)^3 + 25*a
^2*b^4*cosh(d*x + c)^2 - 8*a^6 - 14*a^2*b^4 + 4*(5*a^6 + a^2*b^4)*d*x + 4*(
5*a^6 + a^2*b^4)*c - (24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*a^4
*b^2 + b^6)*c)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(24*a^4*b^2 + b^6 + 8*(5*
a^4*b^2 + b^6)*d*x + 8*(5*a^4*b^2 + b^6)*c)*cosh(d*x + c)^2 + 3*(15*b^6*cos
h(d*x + c)^4 + 100*a^2*b^4*cosh(d*x + c)^3 + 24*a^4*b^2 + b^6 + 8*(5*a^4*b^
```

```
2 + b^6)*d*x - 6*(24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*a^4*b^2
 + b^6)*c)*cosh(d*x + c)^2 + 8*(5*a^4*b^2 + b^6)*c - 24*(4*a^6 + 7*a^2*b^4
- 2*(5*a^6 + a^2*b^4)*d*x - 2*(5*a^6 + a^2*b^4)*c)*cosh(d*x + c))*sinh(d*x
+ c)^2 + 24*((5*a^4*b^2 + b^6)*cosh(d*x + c)^4 + (5*a^4*b^2 + b^6)*sinh(d*x
 + c)^4 - 2*(5*a^6 + a^2*b^4)*cosh(d*x + c)^3 - 2*(5*a^6 + a^2*b^4 - 2*(5*a
^4*b^2 + b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - (5*a^4*b^2 + b^6)*cosh(d*x +
 c)^2 - (5*a^4*b^2 + b^6 - 6*(5*a^4*b^2 + b^6)*cosh(d*x + c)^2 + 6*(5*a^6 +
 a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(2*(5*a^4*b^2 + b^6)*cosh(d*x
+ c)^3 - 3*(5*a^6 + a^2*b^4)*cosh(d*x + c)^2 - (5*a^4*b^2 + b^6)*cosh(d*x +
 c))*sinh(d*x + c))*log(-(b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2
*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) + 4*(a*b*c
osh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2
 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) -
 a^2)*sinh(d*x + c))) + 24*((5*a^4*b^2 + b^6)*cosh(d*x + c)^4 + (5*a^4*b^2
+ b^6)*sinh(d*x + c)^4 - 2*(5*a^6 + a^2*b^4)*cosh(d*x + c)^3 - 2*(5*a^6 + a
^2*b^4 - 2*(5*a^4*b^2 + b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - (5*a^4*b^2 +
b^6)*cosh(d*x + c)^2 - (5*a^4*b^2 + b^6 - 6*(5*a^4*b^2 + b^6)*cosh(d*x + c)
^2 + 6*(5*a^6 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(2*(5*a^4*b^2 +
 b^6)*cosh(d*x + c)^3 - 3*(5*a^6 + a^2*b^4)*cosh(d*x + c)^2 - (5*a^4*b^2 +
b^6)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b^2*sinh(d*x + c) - a^2)/(cosh(d*
x + c) - sinh(d*x + c))) + 6*(3*b^6*cosh(d*x + c)^5 + 25*a^2*b^4*cosh(d*x +
 c)^4 + 5*a^2*b^4 - 2*(24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*a^
4*b^2 + b^6)*c)*cosh(d*x + c)^3 - 12*(4*a^6 + 7*a^2*b^4 - 2*(5*a^6 + a^2*b^
4)*d*x - 2*(5*a^6 + a^2*b^4)*c)*cosh(d*x + c)^2 + (24*a^4*b^2 + b^6 + 8*(5*
a^4*b^2 + b^6)*d*x + 8*(5*a^4*b^2 + b^6)*c)*cosh(d*x + c))*sinh(d*x + c) -
16*(a*b^5*cosh(d*x + c)^5 + a*b^5*sinh(d*x + c)^5 + 10*a^3*b^3*cosh(d*x + c
)^4 - 10*a^3*b^3*cosh(d*x + c)^2 + a*b^5*cosh(d*x + c) + 5*(a*b^5*cosh(d*x
+ c) + 2*a^3*b^3)*sinh(d*x + c)^4 - 2*(15*a^5*b + 4*a*b^5)*cosh(d*x + c)^3
+ 2*(5*a*b^5*cosh(d*x + c)^2 + 20*a^3*b^3*cosh(d*x + c) - 15*a^5*b - 4*a*b^
5)*sinh(d*x + c)^3 + 2*(5*a*b^5*cosh(d*x + c)^3 + 30*a^3*b^3*cosh(d*x + c)^
2 - 5*a^3*b^3 - 3*(15*a^5*b + 4*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^2 + (5*
a*b^5*cosh(d*x + c)^4 + 40*a^3*b^3*cosh(d*x + c)^3 - 20*a^3*b^3*cosh(d*x +
c) + a*b^5 - 6*(15*a^5*b + 4*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(si
nh(d*x + c)))/(b^8*d*cosh(d*x + c)^4 + b^8*d*sinh(d*x + c)^4 - 2*a^2*b^6*d*
cosh(d*x + c)^3 - b^8*d*cosh(d*x + c)^2 + 2*(2*b^8*d*cosh(d*x + c) - a^2*b^
6*d)*sinh(d*x + c)^3 + (6*b^8*d*cosh(d*x + c)^2 - 6*a^2*b^6*d*cosh(d*x + c)
 - b^8*d)*sinh(d*x + c)^2 + 2*(2*b^8*d*cosh(d*x + c)^3 - 3*a^2*b^6*d*cosh(d
*x + c)^2 - b^8*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^3}{{\left(a+b\,\sqrt{\sinh\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^(1/2))^2,x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^(1/2))^2, x)
```

3.417 $\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal. Leaf size=49

$$\frac{2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}+\frac{2a}{b^2d\left(a+b\sqrt{\sinh(c+dx)}\right)}$$

```
[Out] 2*ln(a+b*sinh(d*x+c)^(1/2))/b^2/d+2*a/b^2/d/(a+b*sinh(d*x+c)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3302, 196, 45}

$$\frac{2a}{b^2d\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^2*d) + (2*a)/(b^2*d*(a + b*Sqrt[Sinh[
c + d*x]]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
```

```
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{2\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d} \\
&= \frac{2\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d} \\
&= \frac{2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}+\frac{2a}{b^2d\left(a+b\sqrt{\sinh(c+dx)}\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 42, normalized size = 0.86

$$\frac{2\left(\log\left(a+b\sqrt{\sinh(c+dx)}\right)+\frac{a}{a+b\sqrt{\sinh(c+dx)}}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (2*(Log[a + b*Sqrt[Sinh[c + d*x]]] + a/(a + b*Sqrt[Sinh[c + d*x]])))/(b^2*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(45) = 90.
time = 1.05, size = 127, normalized size = 2.59

method	result
derivativedivides	$\frac{\frac{2a^{2}}{\left(-b^{2}\sinh(dx+c)+a^{2}\right)b^{2}}+\frac{\ln\left(-b^{2}\sinh(dx+c)+a^{2}\right)}{b^{2}}+\frac{a}{b^{2}\left(a+b\left(\sqrt{\sinh}\,(dx+c)\right)\right)}+\frac{\ln\left(a+b\left(\sqrt{\sinh}\,(dx+c)\right)\right)}{b^{2}}-\frac{}{b^{2}(-b(\sqrt{s}}}{d}$

default	$\dfrac{\frac{2a^2}{\left(-b^2\sinh(dx+c)+a^2\right)b^2}+\frac{\ln\left(-b^2\sinh(dx+c)+a^2\right)}{b^2}+\frac{a}{b^2\left(a+b\left(\sqrt{\sinh}\,(dx+c)\right)\right)}+\frac{\ln\left(a+b\left(\sqrt{\sinh}\,(dx+c)\right)\right)}{b^2}-}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2/(-b^2*sinh(d*x+c)+a^2)/b^2+1/b^2*ln(-b^2*sinh(d*x+c)+a^2)+a/b^2/
(a+b*sinh(d*x+c)^(1/2))+1/b^2*ln(a+b*sinh(d*x+c)^(1/2))-a/b^2/(-b*sinh(d*x+
c)^(1/2)+a)-1/b^2*ln(-b*sinh(d*x+c)^(1/2)+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(45) = 90$.
time = 0.51, size = 564, normalized size = 11.51

$$\frac{b^2dx+b^2c-\left(b^2dx+b^2c\right)\cosh\left(dx+c\right)^2-\left(b^2dx+b^2c\right)\sinh\left(dx+c\right)^2+2\left(a^2dx+a^2c-2a^2\right)\cosh\left(dx+c\right)+\left(b^2\cosh\left(dx+c\right)^2+b^2\sinh\left(dx+c\right)^2-2a^2\cosh\left(dx+c\right)-b^2+2\left(b^2\cosh\left(dx+c\right)-a^2\right)\sinh\left(dx+c\right)\right)\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2a^2\cosh(dx+c)-b^2+2\left(b^2\cosh(dx+c)+a^2\right)\sinh(dx+c)+4\left(ab\cosh(dx+c)+ab\sinh(dx+c)\right)\sqrt{\sinh\left(dx+c\right)}}{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2-2a^2\cosh(dx+c)-b^2+2\left(b^2\cosh(dx+c)-a^2\right)\sinh(dx+c)}\right)+\left(b^2\cosh\left(dx+c\right)^2+b^2\sinh\left(dx+c\right)^2-2a^2\cosh\left(dx+c\right)-b^2+2\left(b^2\cosh\left(dx+c\right)-a^2\right)\sinh\left(dx+c\right)\right)\log\left(\frac{2\left(b^2\sinh(dx+c)-a^2\right)}{\cosh(dx+c)-\sinh(dx+c)}\right)+2\left(a^2dx+a^2c-2a^2-\left(b^2dx+b^2c\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+4\left(ab\cosh\left(dx+c\right)+ab\sinh\left(dx+c\right)\right)\sqrt{\sinh\left(dx+c\right)}}{b^4d\cosh\left(dx+c\right)^2+b^4d\sinh\left(dx+c\right)^2-2a^2b^2d\cosh\left(dx+c\right)-b^4d+2\left(b^4d\cosh\left(dx+c\right)-a^2b^2d\right)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cosh(d*x + c)^2 - (b^2*d*x + b^2*c)*si
nh(d*x + c)^2 + 2*(a^2*d*x + a^2*c - 2*a^2)*cosh(d*x + c) + (b^2*cosh(d*x +
 c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x +
 c) - a^2)*sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 +
2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) + 4*(
a*b*cosh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x +
 c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x +
 c) - a^2)*sinh(d*x + c))) + (b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 - 2
*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) - a^2)*sinh(d*x + c))*log(2
*(b^2*sinh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(a^2*d*x +
a^2*c - 2*a^2 - (b^2*d*x + b^2*c)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cos
h(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^2
 + b^4*d*sinh(d*x + c)^2 - 2*a^2*b^2*d*cosh(d*x + c) - b^4*d + 2*(b^4*d*cos
h(d*x + c) - a^2*b^2*d)*sinh(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(42) = 84$.
time = 4.46, size = 151, normalized size = 3.08

$$\begin{cases} \frac{x\cosh(c)}{a^2} & \text{for}\ b=0 \wedge d=0 \\ \frac{\sinh(c+dx)}{a^2 d} & \text{for}\ b=0 \\ \frac{x\cosh(c)}{\left(a+b\sqrt{\sinh(c)}\right)^2} & \text{for}\ d=0 \\ \frac{2a\log\left(\frac{a}{b}+\sqrt{\sinh(c+dx)}\right)}{ab^2d+b^3d\sqrt{\sinh(c+dx)}} + \frac{2a}{ab^2d+b^3d\sqrt{\sinh(c+dx)}} + \frac{2b\log\left(\frac{a}{b}+\sqrt{\sinh(c+dx)}\right)\sqrt{\sinh(c+dx)}}{ab^2d+b^3d\sqrt{\sinh(c+dx)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((x*cosh(c)/a**2, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a**2*d), E
q(b, 0)), (x*cosh(c)/(a + b*sqrt(sinh(c)))**2, Eq(d, 0)), (2*a*log(a/b + sq
rt(sinh(c + d*x)))/(a*b**2*d + b**3*d*sqrt(sinh(c + d*x))) + 2*a/(a*b**2*d
+ b**3*d*sqrt(sinh(c + d*x))) + 2*b*log(a/b + sqrt(sinh(c + d*x)))*sqrt(sin
h(c + d*x))/(a*b**2*d + b**3*d*sqrt(sinh(c + d*x))), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Mupad [B]
time = 1.41, size = 45, normalized size = 0.92

$$\frac{2\,a}{b^2\,\left(a\,d+b\,d\,\sqrt{\sinh\left(c+d\,x\right)}\right)}+\frac{2\,\ln\left(a+b\,\sqrt{\sinh\left(c+d\,x\right)}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^(1/2))^2,x)
```

```
[Out] (2*a)/(b^2*(a*d + b*d*sinh(c + d*x)^(1/2))) + (2*log(a + b*sinh(c + d*x)^(1
/2)))/(b^2*d)
```

3.418 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal. Leaf size=384

$$\frac{\sqrt{2}\,ab(a^4-2a^2b^2-b^4)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2\,d}-\frac{\sqrt{2}\,ab(a^4-2a^2b^2-b^4)\,\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{\mathrm{si}}\right.}{(a^4+b^4)^2\,d}$$

```
[Out] a^2*(a^4-3*b^4)*arctan(sinh(d*x+c))/(a^4+b^4)^2/d+b^2*(3*a^4-b^4)*ln(cosh(d
*x+c))/(a^4+b^4)^2/d-2*b^2*(3*a^4-b^4)*ln(a+b*sinh(d*x+c)^(1/2))/(a^4+b^4)^
2/d-1/2*a*b*(a^4+2*a^2*b^2-b^4)*ln(1+sinh(d*x+c)-2^(1/2)*sinh(d*x+c)^(1/2))
/(a^4+b^4)^2/d*2^(1/2)+1/2*a*b*(a^4+2*a^2*b^2-b^4)*ln(1+sinh(d*x+c)+2^(1/2)
*sinh(d*x+c)^(1/2))/(a^4+b^4)^2/d*2^(1/2)-a*b*(a^4-2*a^2*b^2-b^4)*arctan(-1
+2^(1/2)*sinh(d*x+c)^(1/2))*2^(1/2)/(a^4+b^4)^2/d-a*b*(a^4-2*a^2*b^2-b^4)*a
rctan(1+2^(1/2)*sinh(d*x+c)^(1/2))*2^(1/2)/(a^4+b^4)^2/d+2*a*b^2/(a^4+b^4)/
d/(a+b*sinh(d*x+c)^(1/2))
```

Rubi [A]

time = 0.44, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3302, 6857, 1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 209, 266}

$$\frac{2ab^2}{d\left(a^4+b^4\right)\left(a+b\sqrt{\sinh(c+dx)}\right)}-\frac{2b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{d\left(a^4+b^4\right)^2}+\frac{b^2(3a^4-b^4)\log(\cosh(c+dx))}{d\left(a^4+b^4\right)^2}+\frac{a^2(a^4-3b^4)\,\mathrm{ArcTan}(\sinh(c+dx))}{d\left(a^4+b^4\right)^2}+\frac{\sqrt{2}\,ab(a^4-2a^2b^2-b^4)\,\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\sinh(c+dx)}\right)}{d\left(a^4+b^4\right)^2}-\frac{\sqrt{2}\,ab(a^4-2a^2b^2-b^4)\,\mathrm{ArcTan}\left(\sqrt{2}\,\sqrt{\sinh(c+dx)}+1\right)}{d\left(a^4+b^4\right)^2}-\frac{ab(a^4+2a^2b^2-b^4)\log\left(\sinh(c+dx)-\sqrt{2}\,\sqrt{\sinh(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^4+b^4\right)^2}+\frac{ab(a^4+2a^2b^2-b^4)\log\left(\sinh(c+dx)+\sqrt{2}\,\sqrt{\sinh(c+dx)}+1\right)}{\sqrt{2}\,d\left(a^4+b^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (Sqrt[2]*a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]
])/((a^4 + b^4)^2*d) - (Sqrt[2]*a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 + Sqrt
[2]*Sqrt[Sinh[c + d*x]]])/((a^4 + b^4)^2*d) + (a^2*(a^4 - 3*b^4)*ArcTan[Sin
h[c + d*x]])/((a^4 + b^4)^2*d) + (b^2*(3*a^4 - b^4)*Log[Cosh[c + d*x]])/((a
^4 + b^4)^2*d) - (2*b^2*(3*a^4 - b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/((a^4
 + b^4)^2*d) - (a*b*(a^4 + 2*a^2*b^2 - b^4)*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d
*x]] + Sinh[c + d*x]])/(Sqrt[2]*(a^4 + b^4)^2*d) + (a*b*(a^4 + 2*a^2*b^2 -
b^4)*Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(Sqrt[2]*(a^4 +
b^4)^2*d) + (2*a*b^2)/((a^4 + b^4)*d*(a + b*Sqrt[Sinh[c + d*x]]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(1+x^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \frac{x}{(a+bx)^2(1+x^4)}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{ab^3}{(a^4+b^4)(a+bx)^2} + \frac{-3a^4b^3+b^7}{(a^4+b^4)^2(a+bx)} + \frac{4a^3b^3+a^2(a^4-3b^4)x-2ab(a^4-b^4)x^2+b}{(a^4+b^4)^2(1+x^4)}\right.\right.}{d}$$

$$= -\frac{2b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)\,d\left(a+b\sqrt{\sinh(c+\right.}}$$

$$= -\frac{2b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)\,d\left(a+b\sqrt{\sinh(c+\right.}}$$

$$= -\frac{2b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)\,d\left(a+b\sqrt{\sinh(c+\right.}}$$

$$= -\frac{2b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)\,d\left(a+b\sqrt{\sinh(c+\right.}}$$

$$= -\frac{2b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)\,d\left(a+b\sqrt{\sinh(c+\right.}}$$

$$= \frac{a^2(a^4-3b^4)\tan^{-1}(\sinh(c+dx))}{(a^4+b^4)^2 d} + \frac{b^2(3a^4-b^4)\log(\cosh(c+dx))}{(a^4+b^4)^2 d} - \frac{2b^2(3}{}$$

$$= \frac{\sqrt{2}\,ab(a^4-2a^2b^2-b^4)\tan^{-1}\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2 d} - \frac{\sqrt{2}\,ab(a^4-2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.81, size = 280, normalized size = 0.73

$$\frac{-6\sqrt{2}a^3b^3\left(\text{ArcTan}\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)-\text{ArcTan}\left(1+\sqrt{2}\sqrt{\sinh(c+dx)}\right)\right)+3a^2(a^4-3b^4)\text{ArcTan}(\sinh(c+dx))-3b^2(-3a^4+b^4)\log(\cosh(c+dx))+6b^2(-3a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)-3\sqrt{2}a^3b^3\left(\log\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}+\sinh(c+dx)\right)-\log\left(1+\sqrt{2}\sqrt{\sinh(c+dx)}+\sinh(c+dx)\right)\right)+\frac{6ab^2(a^4+b^4)}{a+b\sqrt{\sinh(c+dx)}}-4ab(a^4-b^4)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\sinh^2(c+dx)\right)\sinh^{\frac{3}{2}}(c+dx)}{3(a^4+b^4)^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (-6*Sqrt[2]*a^3*b^3*(ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]] - ArcTan[1 + S
qrt[2]*Sqrt[Sinh[c + d*x]]]) + 3*a^2*(a^4 - 3*b^4)*ArcTan[Sinh[c + d*x]] -
3*b^2*(-3*a^4 + b^4)*Log[Cosh[c + d*x]] + 6*b^2*(-3*a^4 + b^4)*Log[a + b*Sq
rt[Sinh[c + d*x]]] - 3*Sqrt[2]*a^3*b^3*(Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]
 + Sinh[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]]) +
 (6*a*b^2*(a^4 + b^4))/(a + b*Sqrt[Sinh[c + d*x]]) - 4*a*b*(a^4 - b^4)*Hype
rgeometric2F1[3/4, 1, 7/4, -Sinh[c + d*x]^2]*Sinh[c + d*x]^(3/2))/(3*(a^4 +
 b^4)^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.29, size = 376, normalized size = 0.98

method	result
default	$\frac{-\frac{2b^{2}\left(\frac{2b^{2}\left(a^{4}+b^{4}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^{2}\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^{2}}+\frac{\left(3a^{4}-b^{4}\right)\ln\left(a^{2}\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^{2}\right)}{2}\right)}{\left(a^{4}+b^{4}\right)^{2}}+\left(3a^{4}b^{2}-b^{6}\right)\ln(\tanh^{2}(\frac{dx}{2}+}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^2/(a^4+b^4)^2*(2*b^2*(a^4+b^4)*tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*
d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)+1/2*(3*a^4-b^4)*ln(a^2*tanh(1/2
*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2))+2/(a^8+2*a^4*b^4+b^8)*(1/2*(3
*a^4*b^2-b^6)*ln(tanh(1/2*d*x+1/2*c)^2+1)+(a^6-3*a^2*b^4)*arctan(tanh(1/2*d
*x+1/2*c))))+`int/indef0`(2*a*b*sinh(d*x+c)^(1/2)*(b^4*sinh(d*x+c)^2-2*a^2*
b^2*sinh(d*x+c)+a^4)/(-b^8*cosh(d*x+c)^6+4*a^2*b^6*cosh(d*x+c)^4*sinh(d*x+c
)+(-6*a^4*b^4+2*b^8)*cosh(d*x+c)^4+(4*a^6*b^2-4*a^2*b^6)*cosh(d*x+c)^2*sinh
(d*x+c)+(-a^8+6*a^4*b^4-b^8)*cosh(d*x+c)^2),sinh(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30856 vs. $2(357) = 714$.
time = 21.64, size = 30856, normalized size = 80.35

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*((a^24*b^2 + 6*a^20*b^6 + 15*a^16*b^10 + 20*a^12*b^14 + 15*a
^8*b^18 + 6*a^4*b^22 + b^26)*d^5*e^(2*d*x + 2*c) - 2*(a^26 + 6*a^22*b^4 + 1
5*a^18*b^8 + 20*a^14*b^12 + 15*a^10*b^16 + 6*a^6*b^20 + a^2*b^24)*d^5*e^(d*
x + c) - (a^24*b^2 + 6*a^20*b^6 + 15*a^16*b^10 + 20*a^12*b^14 + 15*a^8*b^18
 + 6*a^4*b^22 + b^26)*d^5)*(a^4*b^4/((a^16 + 4*a^12*b^4 + 6*a^8*b^8 + 4*a^4
*b^12 + b^16)*d^4))^(3/4)*sqrt((a^16 + 4*a^12*b^4 + 6*a^8*b^8 + 4*a^4*b^12
+ b^16 - 4*(a^20 + 3*a^16*b^4 + 2*a^12*b^8 - 2*a^8*b^12 - 3*a^4*b^16 - b^20
)*sqrt(a^4*b^4/((a^16 + 4*a^12*b^4 + 6*a^8*b^8 + 4*a^4*b^12 + b^16)*d^4))*d
^2)/(a^16 - 12*a^12*b^4 + 38*a^8*b^8 - 12*a^4*b^12 + b^16))*sqrt((a^20*b^4
- 12*a^16*b^8 + 38*a^12*b^12 - 12*a^8*b^16 + a^4*b^20)/((a^32 + 8*a^28*b^4
+ 28*a^24*b^8 + 56*a^20*b^12 + 70*a^16*b^16 + 56*a^12*b^20 + 28*a^8*b^24 +
8*a^4*b^28 + b^32)*d^4))*arctan(1/2*(sqrt(2)*sqrt(1/2)*((a^4*b^4/((a^16 + 4
*a^12*b^4 + 6*a^8*b^8 + 4*a^4*b^12 + b^16)*d^4))^(3/4)*(2*((a^45*b^5 + 2*a^
41*b^9 - 19*a^37*b^13 - 1 ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sqrt(sinh(c + d*x)))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh(c+d\,x)\,\left(a+b\,\sqrt{\sinh(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^(1/2))^2),x)
```

```
[Out] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^(1/2))^2), x)
```

3.419 $\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad}+\frac{2\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3ad}+{}_2F_1\left(1,\frac{5}{n};\frac{5+n}{n};\right.$$

```
[Out] hypergeom([1, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a/d+2/3*hypergeo
m([1, 3/n],[(3+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^3/a/d+1/5*hypergeom([1
, 5/n],[(5+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^5/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3302, 1907, 251, 371}

$$\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}+\frac{\sinh^5(c+dx)\,{}_2F_1\left(1,\frac{5}{n};\frac{n+5}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{5ad}+\frac{2\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d) + (2*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^
n)/a)]*Sinh[c + d*x]^3)/(3*a*d) + (Hypergeometric2F1[1, 5/n, (5 + n)/n, -((
b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a*d)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx &= \frac{\text{Subst}\Big(\int \frac{(1+x^2)^2}{a+bx^n}\,dx, x, \sinh(c+dx)\Big)}{d} \\
&= \frac{\text{Subst}\Big(\int \Big(\frac{1}{a+bx^n}+\frac{2x^2}{a+bx^n}+\frac{x^4}{a+bx^n}\Big)\,dx, x, \sinh(c+dx)\Big)}{d} \\
&= \frac{\text{Subst}\big(\int \frac{1}{a+bx^n}\,dx, x, \sinh(c+dx)\big)}{d} + \frac{\text{Subst}\Big(\int \frac{x^4}{a+bx^n}\,dx, x, \sinh(c+dx)\Big)}{d} + 2\text{Su} \\
&= \frac{{}_2F_1\Big(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\Big)\sinh(c+dx)}{ad} + \frac{2\,{}_2F_1\Big(1,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\Big)\,\text{sin}}{3ad}
\end{aligned}$$

Mathematica [**A**]
time = 0.10, size = 119, normalized size = 0.92

$$\frac{15\,{}_2F_1\Big(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\Big)\sinh(c+dx)+10\,{}_2F_1\Big(1,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\Big)\sinh^3(c+dx)+3\,{}_2F_1\Big(1,\frac{5}{n};\frac{5+n}{n};-\frac{b\sinh^n(c+dx)}{a}\Big)\sinh^5(c+dx)}{15ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (15*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh
[c + d*x] + 10*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a
)]*Sinh[c + d*x]^3 + 3*Hypergeometric2F1[1, 5/n, (5 + n)/n, -((b*Sinh[c + d
*x]^n)/a)]*Sinh[c + d*x]^5)/(15*a*d)
```

Maple [**F**]
time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{\cosh^5(dx+c)}{a+b\left(\sinh^n(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x)
```

```
[Out] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)
```

Fricas [**F**]
time = 0.43, size = 25, normalized size = 0.19

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)^{5}}{b\sinh\left(dx+c\right)^{n}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^5}{a+b\,{\mathrm{sinh}\left(c+d\,x\right)}^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^n),x)
```

```
[Out] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^n), x)
```

3.420 $\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad}+\frac{{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3ad}$$

```
[Out] hypergeom([1, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a/d+1/3*hypergeo
m([1, 3/n],[(3+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^3/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3302, 1907, 251, 371}

$$\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}+\frac{\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
 + d*x])/(a*d) + (Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)
/a)]*Sinh[c + d*x]^3)/(3*a*d)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{a+bx^n}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{a+bx^n}+\frac{x^2}{a+bx^n}\right)\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{a+bx^n}\,dx, x, \sinh(c+dx)\right)}{d}+\frac{\operatorname{Subst}\left(\int \frac{x^2}{a+bx^n}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad}+\frac{{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\text{si}}{3ad}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 82, normalized size = 0.98

$$\frac{\frac{{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a}+\frac{{}_2F_1\left(1,\frac{3}{n};1+\frac{3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3a}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] ((Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
 + d*x])/a + (Hypergeometric2F1[1, 3/n, 1 + 3/n, -((b*Sinh[c + d*x]^n)/a)]*
Sinh[c + d*x]^3)/(3*a))/d
```

Maple [F]
time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(dx+c)}{a+b\left(\sinh^n(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x)
```

```
[Out] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)
```

Fricas [**F**]
time = 0.46, size = 25, normalized size = 0.30

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)^{3}}{b\sinh\left(dx+c\right)^{n}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{cosh}(c+d\,x)^3}{a+b\,{\mathrm{sinh}\left(c+d\,x\right)}^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^n),x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^n), x)
```

3.421 $\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad}$$

```
[Out] hypergeom([1, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a/d
```

Rubi [A]

time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3302, 251}

$$\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^n}\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 37, normalized size = 1.00

$$\frac{{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d)
```

Maple [**F**]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(dx+c\right)}{a+b\left(\sinh^{n}\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x)
```

```
[Out] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)
```

Fricas [**F**]
time = 0.45, size = 23, normalized size = 0.62

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)}{b\sinh\left(dx+c\right)^{n}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)
```

Mupad [B]
time = 1.13, size = 38, normalized size = 1.03

$$\frac{\sinh(c+d\,x)\,{}_2\mathrm{F}_1\left(1,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\sinh(c+d\,x)^n}{a}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^n),x)
```

```
[Out] (sinh(c + d*x)*hypergeom([1, 1/n], 1/n + 1, -(b*sinh(c + d*x)^n)/a))/(a*d)
```

3.422 $\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx$

Optimal. Leaf size=130

$$\frac{{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d}+\frac{2\,{}_2F_1\left(2,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3a^2d}+{}_2F_1\left(2,\frac{5}{n};\frac{5+n}{n};-\right.$$

```
[Out] hypergeom([2, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a^2/d+2/3*hyperg
eom([2, 3/n],[(3+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^3/a^2/d+1/5*hypergeo
m([2, 5/n],[(5+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3302, 1907, 251, 371}

$$\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}+\frac{\sinh^5(c+dx)\,{}_2F_1\left(2,\frac{5}{n};\frac{n+5}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{5a^2d}+\frac{2\sinh^3(c+dx)\,{}_2F_1\left(2,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d) + (2*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x
]^n)/a)]*Sinh[c + d*x]^3)/(3*a^2*d) + (Hypergeometric2F1[2, 5/n, (5 + n)/n,
 -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a^2*d)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
```

```
Q[Pq, x^n])
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx &= \frac{\text{Subst}\Big(\int \frac{(1+x^2)^2}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\Big)}{d}\\
&= \frac{\text{Subst}\Big(\int \Big(\frac{1}{(a+bx^n)^2}+\frac{2x^2}{(a+bx^n)^2}+\frac{x^4}{(a+bx^n)^2}\Big)\,dx, x, \sinh(c+dx)\Big)}{d}\\
&= \frac{\text{Subst}\Big(\int \frac{1}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\Big)}{d}+\frac{\text{Subst}\Big(\int \frac{x^4}{(a+bx^n)^2}\,dx, x, \sinh(c+dx}{d}\\
&= \frac{{}_2F_1\Big(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\Big)\sinh(c+dx)}{a^2d}+\frac{2\,{}_2F_1\Big(2,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}}{3a^2d}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 119, normalized size = 0.92

$$\frac{15\,{}_2F_1\Big(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\Big)\sinh(c+dx)+10\,{}_2F_1\Big(2,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\Big)\sinh^3(c+dx)+3\,{}_2F_1\Big(2,\frac{5}{n};\frac{5+n}{n};-\frac{b\sinh^n(c+dx)}{a}\Big)\sinh^5(c+dx)}{15a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (15*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh
[c + d*x] + 10*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a
)]*Sinh[c + d*x]^3 + 3*Hypergeometric2F1[2, 5/n, (5 + n)/n, -((b*Sinh[c + d
*x]^n)/a)]*Sinh[c + d*x]^5)/(15*a^2*d)
```

Maple [F]
time = 2.34, size = 0, normalized size = 0.00

$$\int \frac{\cosh^5(dx+c)}{(a+b(\sinh^n(dx+c)))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x)
```

```
[Out] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(2^n*e^(c*n + 10*d*x + 10*c) + 3*2^n*e^(c*n + 8*d*x + 8*c) + 2^(n + 1)
*e^(c*n + 6*d*x + 6*c) - 2^(n + 1)*e^(c*n + 4*d*x + 4*c) - 3*2^n*e^(c*n + 2
*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n*x + c*n + 5*d*x +
5*c) + a*b*d*n*e^(5*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1) +
 5*c)) + 1/32*integrate((2^n*n*e^(c*n) - 5*2^n*e^(c*n) + (2^n*n*e^(c*n) - 5
*2^n*e^(c*n))*e^(10*d*x + 10*c) + (5*2^n*n*e^(c*n) - 9*2^n*e^(c*n))*e^(8*d*
x + 8*c) + (5*2^(n + 1)*n*e^(c*n) - 2^(n + 1)*e^(c*n))*e^(6*d*x + 6*c) + (5
*2^(n + 1)*n*e^(c*n) - 2^(n + 1)*e^(c*n))*e^(4*d*x + 4*c) + (5*2^n*n*e^(c*n
) - 9*2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*n*e^(d*n*x + c*n + 5
*d*x + 5*c) + a*b*n*e^(5*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) -
 1) + 5*c)), x)
```

Fricas [F]
time = 0.45, size = 43, normalized size = 0.33

$$\mathrm{integral}\left(\frac{\cosh\left(dx+c\right)^{5}}{b^{2}\sinh\left(dx+c\right)^{2\,n}+2\,ab\sinh\left(dx+c\right)^{n}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^5/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n +
 a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**n)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^5}{\left(a+b\sinh\left(c+d\,x\right)^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^n)^2,x)
```

```
[Out] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^n)^2, x)
```

3.423 $\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx$

Optimal. Leaf size=84

$$\frac{{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d}+\frac{{}_2F_1\left(2,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3a^2d}$$

```
[Out] hypergeom([2, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a^2/d+1/3*hyperg
eom([2, 3/n],[(3+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3302, 1907, 251, 371}

$$\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}+\frac{\sinh^3(c+dx)\,{}_2F_1\left(2,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d) + (Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^
n)/a)]*Sinh[c + d*x]^3)/(3*a^2*d)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{1+x^2}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\mathrm{Subst}\left(\int \left(\frac{1}{(a+bx^n)^2}+\frac{x^2}{(a+bx^n)^2}\right)\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d}+\frac{\mathrm{Subst}\left(\int \frac{x^2}{(a+bx^n)^2}\,dx, x, \sinh(c+dx\right.}{d}\\
&= \frac{{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d}+\frac{{}_2F_1\left(2,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2d}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 82, normalized size = 0.98

$$
\frac{\frac{{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2}+\frac{{}_2F_1\left(2,\frac{3}{n};1+\frac{3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3a^2}}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] ((Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
 + d*x])/a^2 + (Hypergeometric2F1[2, 3/n, 1 + 3/n, -((b*Sinh[c + d*x]^n)/a)
]*Sinh[c + d*x]^3)/(3*a^2))/d
```

Maple [F]
time = 1.64, size = 0, normalized size = 0.00

$$
\int \frac{\cosh^3(dx+c)}{\left(a+b\left(\sinh^n(dx+c)\right)\right)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x)
```

```
[Out] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2^n*e^(c*n + 6*d*x + 6*c) + 2^n*e^(c*n + 4*d*x + 4*c) - 2^n*e^(c*n + 2
*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n*x + c*n + 3*d*x +
3*c) + a*b*d*n*e^(3*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1) +
 3*c)) + 1/8*integrate((2^n*n*e^(c*n) - 3*2^n*e^(c*n) + (2^n*n*e^(c*n) - 3*
2^n*e^(c*n))*e^(6*d*x + 6*c) + (3*2^n*n*e^(c*n) - 2^n*e^(c*n))*e^(4*d*x + 4
*c) + (3*2^n*n*e^(c*n) - 2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*n
*e^(d*n*x + c*n + 3*d*x + 3*c) + a*b*n*e^(3*d*x + n*log(e^(d*x + c) + 1) +
n*log(e^(d*x + c) - 1) + 3*c)), x)
```

Fricas [**F**]
time = 0.48, size = 43, normalized size = 0.51

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)^{3}}{b^{2}\sinh\left(dx+c\right)^{2\,n}+2\,ab\sinh\left(dx+c\right)^{n}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^3/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n +
 a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**n)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^3}{\left(a+b\sinh\left(c+d\,x\right)^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^n)^2,x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^n)^2, x)
```

3.424 $\int \frac{\cosh(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx$

Optimal. Leaf size=37

$$\frac{{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d}$$

```
[Out] hypergeom([2, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a^2/d
```

Rubi [A]

time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3302, 251}

$$\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^n)^2}\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 37, normalized size = 1.00

$$\frac{{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d)
```

Maple [**F**]
time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{\left(a+b\left(\sinh^n(dx+c)\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x)
```

```
[Out] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2^n*e^(c*n + 2*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n
*x + c*n + d*x + c) + a*b*d*n*e^(d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*
x + c) - 1) + c)) + 1/2*integrate((2^n*n*e^(c*n) - 2^n*e^(c*n) + (2^n*n*e^(
c*n) - 2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*n*e^(d*n*x + c*n +
d*x + c) + a*b*n*e^(d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1) +
 c)), x)
```

Fricas [**F**]
time = 0.47, size = 41, normalized size = 1.11

$$\mathrm{integral}\left(\frac{\cosh(dx+c)}{b^2\sinh(dx+c)^{2n}+2\,ab\sinh(dx+c)^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n + a
^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**n)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a)^2, x)
```

Mupad [**B**]
time = 0.95, size = 38, normalized size = 1.03

$$\frac{\sinh(c+d\,x)\ {}_2\mathrm{F}_1\left(2,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\sinh(c+d\,x)^n}{a}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^n)^2,x)
```

```
[Out] (sinh(c + d*x)*hypergeom([2, 1/n], 1/n + 1, -(b*sinh(c + d*x)^n)/a))/(a^2*d
)
```

3.425 $\int \frac{\coth(x)}{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=17

$$\log(\sinh(x))-\frac{1}{2}\log\left(1-\sinh^2(x)\right)$$

```
[Out] ln(sinh(x))-1/2*ln(1-sinh(x)^2)
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3273, 36, 31, 29}

$$\log(\sinh(x))-\frac{1}{2}\log\left(1-\sinh^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(1 - Sinh[x]^2),x]
```

```
[Out] Log[Sinh[x]] - Log[1 - Sinh[x]^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{1-\sinh^2(x)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x}\,dx, x, \sinh^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sinh^2(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sinh^2(x)\right)\\ &= \log(\sinh(x)) - \frac{1}{2}\log\left(1-\sinh^2(x)\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.35

$$-2\left(-\frac{1}{2}\log(\sinh(x)) + \frac{1}{4}\log\left(1-\sinh^2(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(1 - Sinh[x]^2),x]
```

```
[Out] -2*(-1/2*Log[Sinh[x]] + Log[1 - Sinh[x]^2]/4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(15) = 30.
time = 0.61, size = 41, normalized size = 2.41

method	result	size
risch	$\ln\left(e^{2x}-1\right) - \frac{\ln\left(e^{4x}-6\,e^{2x}+1\right)}{2}$	24
default	$-\frac{\ln\left(\tanh^2\left(\frac{x}{2}\right)-2\tanh\left(\frac{x}{2}\right)-1\right)}{2} - \frac{\ln\left(\tanh^2\left(\frac{x}{2}\right)+2\tanh\left(\frac{x}{2}\right)-1\right)}{2} + \ln\left(\tanh\left(\frac{x}{2}\right)\right)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1-sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(tanh(1/2*x)^2-2*tanh(1/2*x)-1)-1/2*ln(tanh(1/2*x)^2+2*tanh(1/2*x)-1
)+ln(tanh(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(15) = 30.
time = 0.26, size = 45, normalized size = 2.65

$$-\frac{1}{2}\log\left(2\,e^{(-x)} + e^{(-2\,x)} - 1\right) + \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right) - \frac{1}{2}\log\left(-2\,e^{(-x)} + e^{(-2\,x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(2*e^(-x) + e^(-2*x) - 1) + log(e^(-x) + 1) + log(e^(-x) - 1) - 1/2
*log(-2*e^(-x) + e^(-2*x) - 1)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.
time = 0.48, size = 47, normalized size = 2.76

$$-\frac{1}{2}\log\left(\frac{2\left(\cosh\left(x\right)^{2}+\sinh\left(x\right)^{2}-3\right)}{\cosh\left(x\right)^{2}-2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}}\right)+\log\left(\frac{2\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)) + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\coth\left(x\right)}{\sinh^{2}\left(x\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)**2),x)
```

```
[Out] -Integral(coth(x)/(sinh(x)**2 - 1), x)
```

Giac [**A**]
time = 0.43, size = 25, normalized size = 1.47

$$-\frac{1}{2}\log\left(\left|e^{(4x)}-6e^{(2x)}+1\right|\right)+\log\left(\left|e^{(2x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(e^(4*x) - 6*e^(2*x) + 1)) + log(abs(e^(2*x) - 1))
```

Mupad [**B**]
time = 0.08, size = 27, normalized size = 1.59

$$\ln\left(5184\,\mathrm{e}^{2x}-5184\right)-\frac{\ln\left(9\,\mathrm{e}^{4x}-54\,\mathrm{e}^{2x}+9\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-coth(x)/(sinh(x)^2 - 1),x)
```

```
[Out] log(5184*exp(2*x) - 5184) - log(9*exp(4*x) - 54*exp(2*x) + 9)/2
```

3.426 $\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^5(e+fx)\,dx$

Optimal. Leaf size=63

$$-\frac{a^2}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}+\frac{2a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] -1/3*a^2/f/(a*cosh(f*x+e)^2)^(3/2)+2*a/f/(a*cosh(f*x+e)^2)^(1/2)+(a*cosh(f*
x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3284, 16, 45}

$$-\frac{a^2}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}+\frac{2a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] -1/3*a^2/(f*(a*Cosh[e + f*x]^2)^(3/2)) + (2*a)/(f*Sqrt[a*Cosh[e + f*x]^2])
+ Sqrt[a*Cosh[e + f*x]^2]/f
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^5(e+fx)\,dx = \int \sqrt{a\cosh^2(e+fx)}\,\tanh^5(e+fx)\,dx$$

$$= \frac{\text{Subst}\Big(\int \frac{(1-x)^2\sqrt{ax}}{x^3}\,dx, x, \cosh^2(e+fx)\Big)}{2f}$$

$$= \frac{a^3\text{Subst}\Big(\int \frac{(1-x)^2}{(ax)^{5/2}}\,dx, x, \cosh^2(e+fx)\Big)}{2f}$$

$$= \frac{a^3\text{Subst}\Big(\int \Big(\frac{1}{(ax)^{5/2}} - \frac{2}{a(ax)^{3/2}} + \frac{1}{a^2\sqrt{ax}}\Big)\,dx, x, \cosh^2(e+f}{2f}$$

$$= -\frac{a^2}{3f\left(a\cosh^2(e+fx)\right)^{3/2}} + \frac{2a}{f\sqrt{a\cosh^2(e+fx)}} + \frac{\sqrt{a\cosh}}{}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 0.81

$$\frac{\sqrt{a\cosh^2(e+fx)}\,\left(-1+6\cosh^2(e+fx)+3\cosh^4(e+fx)\right)\operatorname{sech}^4(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*(-1 + 6*Cosh[e + f*x]^2 + 3*Cosh[e + f*x]^4)*Sech[
e + f*x]^4)/(3*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.37, size = 42, normalized size = 0.67

method	result

default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\left(\sinh^{5}(fx+e)\right)a}{\cosh(fx+e)^{4}\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$
risch	$\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\;\mathrm{e}^{2fx+2e}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}+\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}+\frac{4\left(3\,\mathrm{e}^{4fx+4e}+4\,\mathrm{e}^{2fx+2e}+3\right)\sqrt{\left(\mathrm{e}^{2fx+2\right.}}}{3f\left(\mathrm{e}^{2fx+2e}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(sinh(f*x+e)^5*a/cosh(f*x+e)^4/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(58) = 116$.
time = 0.52, size = 316, normalized size = 5.02

$$\frac{6\sqrt{a}e^{(-2fx-2e)}}{f(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)})}+\frac{25\sqrt{a}e^{(-4fx-4e)}}{3f(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)})}+\frac{6\sqrt{a}e^{(-6fx-6e)}}{f(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)})}+\frac{\sqrt{a}e^{(-8fx-8e)}}{2f(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)})}+\frac{\sqrt{a}}{2f(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] 6*sqrt(a)*e^(-2*f*x - 2*e)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*
f*x - 5*e) + e^(-7*f*x - 7*e))) + 25/3*sqrt(a)*e^(-4*f*x - 4*e)/(f*(e^(-f*x
 - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e))) + 6*sq
rt(a)*e^(-6*f*x - 6*e)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x
- 5*e) + e^(-7*f*x - 7*e))) + 1/2*sqrt(a)*e^(-8*f*x - 8*e)/(f*(e^(-f*x - e)
 + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e))) + 1/2*sqrt(
a)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x -
 7*e)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(55) = 110$.
time = 0.47, size = 875, normalized size = 13.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(24*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + 3*e^(f*x + e)*sinh(f*x
+ e)^8 + 12*(7*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^6 + 24*(7*cos
h(f*x + e)^3 + 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 10*(21*cosh(f
```

```
*x + e)^4 + 54*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^4 + 8*(21*cos
h(f*x + e)^5 + 90*cosh(f*x + e)^3 + 25*cosh(f*x + e))*e^(f*x + e)*sinh(f*x
+ e)^3 + 12*(7*cosh(f*x + e)^6 + 45*cosh(f*x + e)^4 + 25*cosh(f*x + e)^2 +
3)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(3*cosh(f*x + e)^7 + 27*cosh(f*x + e)^5
+ 25*cosh(f*x + e)^3 + 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (3*cosh
(f*x + e)^8 + 36*cosh(f*x + e)^6 + 50*cosh(f*x + e)^4 + 36*cosh(f*x + e)^2
+ 3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x
 - e)/(f*cosh(f*x + e)^7 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^7 + 7*(f*c
osh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^6 + 3*f*cosh(
f*x + e)^5 + 3*(7*f*cosh(f*x + e)^2 + (7*f*cosh(f*x + e)^2 + f)*e^(2*f*x +
2*e) + f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (7
*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^4 +
3*f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 + 30*f*cosh(f*x + e)^2 + (35*f*
cosh(f*x + e)^4 + 30*f*cosh(f*x + e)^2 + 3*f)*e^(2*f*x + 2*e) + 3*f)*sinh(f
*x + e)^3 + 3*(7*f*cosh(f*x + e)^5 + 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x +
e) + (7*f*cosh(f*x + e)^5 + 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*
f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^7 + 3*f*co
sh(f*x + e)^5 + 3*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (7
*f*cosh(f*x + e)^6 + 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 + (7*f*cosh
(f*x + e)^6 + 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*
e) + f)*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\,\tanh^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**5, x)
```

Giac [**A**]
time = 0.45, size = 70, normalized size = 1.11

$$\frac{\sqrt{a}\left(\frac{8\left(3\left(e^{(fx+e)}+e^{(-fx-e)}\right)^2-2\right)}{\left(e^{(fx+e)}+e^{(-fx-e)}\right)^3}+3\,e^{(fx+e)}+3\,e^{(-fx-e)}\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(a)*(8*(3*(e^(f*x + e) + e^(-f*x - e))^2 - 2)/(e^(f*x + e) + e^(-f*
x - e))^3 + 3*e^(f*x + e) + 3*e^(-f*x - e))/f
```

Mupad [B]
time = 0.94, size = 252, normalized size = 4.00

$$\frac{\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{f}+\frac{8\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{f\,\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{16\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{3\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^2\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}+\frac{16\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{3\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^3\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] (a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)/f + (8*exp(3*e + 3*f*x)
*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(f*(exp(2*e + 2*f*x)
+ 1)*(exp(e + f*x) + exp(3*e + 3*f*x))) - (16*exp(3*e + 3*f*x)*(a + a*(exp(
e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3*f*(exp(2*e + 2*f*x) + 1)^2*(exp
(e + f*x) + exp(3*e + 3*f*x))) + (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/
2 - exp(- e - f*x)/2)^2)^(1/2))/(3*f*(exp(2*e + 2*f*x) + 1)^3*(exp(e + f*x)
 + exp(3*e + 3*f*x)))
```

3.427 $\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^3(e+fx)\,dx$

Optimal. Leaf size=38

$$\frac{a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] a/f/(a*cosh(f*x+e)^2)^(1/2)+(a*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3284, 16, 45}

$$\frac{a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

```
[Out] a/(f*Sqrt[a*Cosh[e + f*x]^2]) + Sqrt[a*Cosh[e + f*x]^2]/f
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
```

```
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^3(e+fx)\,dx &= \int \sqrt{a\cosh^2(e+fx)}\,\tanh^3(e+fx)\,dx\\
&= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)\sqrt{ax}}{x^2}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= -\frac{a^2\operatorname{Subst}\left(\int \frac{1-x}{(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= -\frac{a^2\operatorname{Subst}\left(\int \left(\frac{1}{(ax)^{3/2}}-\frac{1}{a\sqrt{ax}}\right)dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= \frac{a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 29, normalized size = 0.76

$$\frac{a\left(1+\cosh^2(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

```
[Out] (a*(1 + Cosh[e + f*x]^2))/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.06, size = 42, normalized size = 1.11

method	result
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\left(\sinh^3(fx+e)\right)a}{\cosh(fx+e)^2\sqrt{a\left(\cosh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$

risch	$\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}\;\mathrm{e}^{2fx+2e}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}+\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}+\frac{2\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}}{f\left(\mathrm{e}^{2fx+2e}+1\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3*a/cosh(f*x+e)^2/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(36) = 72$.
time = 0.53, size = 114, normalized size = 3.00

$$\frac{3\sqrt{a}\,e^{(-2\,fx-2\,e)}}{f\big(e^{(-fx-e)}+e^{(-3\,fx-3\,e)}\big)}+\frac{\sqrt{a}\,e^{(-4\,fx-4\,e)}}{2\,f\big(e^{(-fx-e)}+e^{(-3\,fx-3\,e)}\big)}+\frac{\sqrt{a}}{2\,f\big(e^{(-fx-e)}+e^{(-3\,fx-3\,e)}\big)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 3*sqrt(a)*e^(-2*f*x - 2*e)/(f*(e^(-f*x - e) + e^(-3*f*x - 3*e))) + 1/2*sqrt
(a)*e^(-4*f*x - 4*e)/(f*(e^(-f*x - e) + e^(-3*f*x - 3*e))) + 1/2*sqrt(a)/(f
*(e^(-f*x - e) + e^(-3*f*x - 3*e)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(34) = 68$.
time = 0.46, size = 311, normalized size = 8.18

$$\frac{\left(4\,\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^{3}+e^{(fx+e)}\sinh\left(fx+e\right)^{4}+6\left(\cosh\left(fx+e\right)^{2}+1\right)e^{(fx+e)}\sinh\left(fx+e\right)^{2}+4\left(\cosh\left(fx+e\right)^{3}+3\,\cosh\left(fx+e\right)\right)e^{(fx+e)}\sinh\left(fx+e\right)+\left(\cosh\left(fx+e\right)^{4}+6\,\cosh\left(fx+e\right)^{2}+1\right)e^{(fx+e)}\right)\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\,e^{(-fx-e)}}{2\left(f\cosh\left(fx+e\right)^{3}+\left(fe^{(2\,fx+2\,e)}+f\right)\sinh\left(fx+e\right)^{3}+3\left(f\cosh\left(fx+e\right)e^{(2\,fx+2\,e)}+f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^{2}+f\cosh\left(fx+e\right)+\left(f\cosh\left(fx+e\right)^{3}+f\cosh\left(fx+e\right)\right)e^{(2\,fx+2\,e)}+\left(3\,f\cosh\left(fx+e\right)^{2}+\left(3\,f\cosh\left(fx+e\right)^{2}+f\right)e^{(2\,fx+2\,e)}+f\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e
)^4 + 6*(cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e
)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 + 6*cos
h(f*x + e)^2 + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e)
 + a)*e^(-f*x - e)/(f*cosh(f*x + e)^3 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x +
e)^3 + 3*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^
2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e)
 + (3*f*cosh(f*x + e)^2 + (3*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*si
nh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\,\tanh^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**3, x)
```

Giac [A]
time = 0.43, size = 44, normalized size = 1.16

$$\frac{\sqrt{a}\left(\frac{4}{e^{(fx+e)}+e^{(-fx-e)}}+e^{(fx+e)}+e^{(-fx-e)}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*(4/(e^(f*x + e) + e^(-f*x - e)) + e^(f*x + e) + e^(-f*x - e))/f
```

Mupad [B]
time = 0.91, size = 67, normalized size = 1.76

$$\frac{\sqrt{a+a\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)^2}\,\left(6\,\mathrm{e}^{2\,e+2\,f\,x}+\mathrm{e}^{4\,e+4\,f\,x}+1\right)}{f\,\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] ((a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)*(6*exp(2*e + 2*f*x) +
exp(4*e + 4*f*x) + 1))/(f*(exp(2*e + 2*f*x) + 1)^2)
```

3.428 $\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh(e+fx)\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] (a*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.05, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3255, 3284, 16, 32}

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

```
[Out] Sqrt[a*Cosh[e + f*x]^2]/f
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh(e+fx)\,dx &= \int \sqrt{a\cosh^2(e+fx)}\,\tanh(e+fx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{x}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a\cosh^2(e+fx)}}{f}
\end{aligned}
$$

Mathematica [A]

time = 0.03, size = 18, normalized size = 1.00

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

```
[Out] Sqrt[a*Cosh[e + f*x]^2]/f
```

Maple [A]

time = 0.55, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\sqrt{a+a\left(\sinh^2\left(fx+e\right)\right)}}{f}$	19
default	$\frac{\sqrt{a+a\left(\sinh^2\left(fx+e\right)\right)}}{f}$	19
risch	$\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a\,\mathrm{e}^{-2fx-2e}}\;\mathrm{e}^{2fx+2e}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}+\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a\,\mathrm{e}^{-2fx-2e}}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] (a+a*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**A**]
time = 0.50, size = 34, normalized size = 1.89

$$\frac{\sqrt{a}\,e^{(fx+e)}}{2\,f}+\frac{\sqrt{a}\,e^{(-fx-e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*e^(f*x + e)/f + 1/2*sqrt(a)*e^(-f*x - e)/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(16) = 32$.
time = 0.54, size = 139, normalized size = 7.72

$$\frac{\left(2\,\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)+e^{(fx+e)}\sinh\left(fx+e\right)^{2}+\left(\cosh\left(fx+e\right)^{2}+1\right)e^{(fx+e)}\right)\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\,e^{(-fx-e)}}{2\,\left(f\cosh\left(fx+e\right)e^{(2\,fx+2\,e)}+f\cosh\left(fx+e\right)+\left(fe^{(2\,fx+2\,e)}+f\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^
2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*
x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x +
e) + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\,\tanh\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e),x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x), x)
```

Giac [**A**]
time = 0.41, size = 24, normalized size = 1.33

$$\frac{\sqrt{a}\left(e^{(fx+e)}+e^{(-fx-e)}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*(e^(f*x + e) + e^(-f*x - e))/f
```

Mupad [B]
time = 0.92, size = 18, normalized size = 1.00

$$\frac{\sqrt{a\sinh\left(e+f\,x\right)^2+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] (a + a*sinh(e + f*x)^2)^(1/2)/f
```

3.429 $\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] -arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a*cosh(f*x+e)^2)^(1/2)
/f
```

Rubi [A]
time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3255, 3284, 52, 65, 212}

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a*Cosh[e + f
*x]^2]/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\cosh^2(e+fx)}\,\coth(e+fx)\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{1-x}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$

$$= \frac{\sqrt{a\cosh^2(e+fx)}}{f} - \frac{a\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cosh^2(e+fx\right.}{2f}$$

$$= \frac{\sqrt{a\cosh^2(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{f}$$

$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Mathematica [A]
time = 0.05, size = 42, normalized size = 0.84

$$\frac{\sqrt{a\cosh^2(e+fx)}\,\left(\cosh(e+fx)+\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)\right)\operatorname{sech}(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*(Cosh[e + f*x] + Log[Tanh[(e + f*x)/2]])*Sech[e +
f*x])/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.99, size = 42, normalized size = 0.84

method	result
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{a\left(\cosh^{2}(fx+e)\right)}{\sinh(fx+e)\sqrt{a\left(\cosh^{2}(fx+e)\right)}},\sinh(fx+e)\right)}{f}$
risch	$\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}\,\mathrm{e}^{2fx+2e}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}+\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}-\frac{\ln\left(\mathrm{e}^{fx}+\mathrm{e}^{-e}\right)\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a}}{f\left(\mathrm{e}^{2fx+2e}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(a*cosh(f*x+e)^2/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e
))/f
```

Maxima [A]
time = 0.52, size = 72, normalized size = 1.44

$$\frac{\left(\sqrt{a}\,e^{(-2\,fx-2\,e)}+\sqrt{a}\right)e^{(fx+e)}}{2\,f}-\frac{\sqrt{a}\,\log\left(e^{(-fx-e)}+1\right)}{f}+\frac{\sqrt{a}\,\log\left(e^{(-fx-e)}-1\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a))*e^(f*x + e)/f - sqrt(a)*log(e^(-f*
x - e) + 1)/f + sqrt(a)*log(e^(-f*x - e) - 1)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(42) = 84.
time = 0.49, size = 200, normalized size = 4.00

$$\frac{\left(2\,\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)+e^{(fx+e)}\sinh\left(fx+e\right)^{2}+\left(\cosh\left(fx+e\right)^{2}+1\right)e^{(fx+e)}+2\left(\cosh\left(fx+e\right)e^{(fx+e)}+e^{(fx+e)}\sinh\left(fx+e\right)\right)\log\left(\frac{\cosh(fx+e)+\sinh(fx+e)-1}{\cosh(fx+e)+\sinh(fx+e)+1}\right)\right)\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\,e^{(-fx-e)}}{2\left(f\cosh\left(fx+e\right)e^{(2\,fx+2\,e)}+f\cosh\left(fx+e\right)+\left(fe^{(2\,fx+2\,e)}+f\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^
2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e) + 2*(cosh(f*x + e)*e^(f*x + e) + e^(f
*x + e)*sinh(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e) - 1)/(cosh(f*x +
e) + sinh(f*x + e) + 1)))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)
*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e) + (f*e^(2*
f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\,\coth\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x), x)
```

Giac [**A**]
time = 0.42, size = 47, normalized size = 0.94

$$\frac{\sqrt{a}\left(e^{(fx+e)}+e^{(-fx-e)}-2\,\log\left(e^{(fx+e)}+1\right)+2\,\log\left(\left|e^{(fx+e)}-1\right|\right)\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*(e^(f*x + e) + e^(-f*x - e) - 2*log(e^(f*x + e) + 1) + 2*log(ab
s(e^(f*x + e) - 1)))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{coth}(e+f\,x)\,\sqrt{a\,{\sinh\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

3.430 $\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=87

$$-\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2f}+\frac{3\sqrt{a\cosh^2(e+fx)}}{2f}-\frac{\left(a\cosh^2(e+fx)\right)^{3/2}\operatorname{csch}^2(e+fx)}{2af}$$

```
[Out] -1/2*(a*cosh(f*x+e)^2)^(3/2)*csch(f*x+e)^2/a/f-3/2*arctanh((a*cosh(f*x+e)^2
)^(1/2)/a^(1/2))*a^(1/2)/f+3/2*(a*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3255, 3284, 16, 43, 52, 65, 212}

$$\frac{3\sqrt{a\cosh^2(e+fx)}}{2f}-\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{\operatorname{csch}^2(e+fx)\left(a\cosh^2(e+fx)\right)^{3/2}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-3*Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]])/(2*f) + (3*Sqrt[a*Cos
h[e + f*x]^2])/(2*f) - ((a*Cosh[e + f*x]^2)^(3/2)*Csch[e + f*x]^2)/(2*a*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\cosh^2(e+fx)}\coth^3(e+fx)\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x\sqrt{ax}}{(1-x)^2}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{(ax)^{3/2}}{(1-x)^2}\,dx, x, \cosh^2(e+fx)\right)}{2af}$$

$$= -\frac{\left(a\cosh^2(e+fx)\right)^{3/2}\operatorname{csch}^2(e+fx)}{2af} - \frac{3\text{Subst}\left(\int \frac{\sqrt{ax}}{1-x}\,dx,\right.}{4f}$$

$$= \frac{3\sqrt{a\cosh^2(e+fx)}}{2f} - \frac{\left(a\cosh^2(e+fx)\right)^{3/2}\operatorname{csch}^2(e+fx)}{2af} -$$

$$= \frac{3\sqrt{a\cosh^2(e+fx)}}{2f} - \frac{\left(a\cosh^2(e+fx)\right)^{3/2}\operatorname{csch}^2(e+fx)}{2af} -$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2f} + \frac{3\sqrt{a\cosh^2(e+fx}}{2f}$$

Mathematica [A]
time = 0.18, size = 77, normalized size = 0.89

$$\frac{\sqrt{a\cosh^2(e+fx)}\left(8\cosh(e+fx)-\operatorname{csch}^2\left(\frac{1}{2}(e+fx)\right)+12\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)-\operatorname{sech}^2\left(\frac{1}{2}(e+fx)\right)\right)\operatorname{sech}(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*(8*Cosh[e + f*x] - Csch[(e + f*x)/2]^2 + 12*Log[Ta
nh[(e + f*x)/2]] - Sech[(e + f*x)/2]^2)*Sech[e + f*x])/(8*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.10, size = 54, normalized size = 0.62

method	result

default	$\frac{\text{‘}\mathit{int/indef0}\text{‘}\left(\frac{a\left(\cosh^{4}(fx+e)\right)}{\sinh(fx+e)\left(\cosh^{2}(fx+e)-1\right)\sqrt{a\left(\cosh^{2}(fx+e)\right)}},\sinh(fx+e)\right)}{f}$
risch	$\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\,\mathrm{e}^{2fx+2e}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}+\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}-\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\,\mathrm{e}^{2}}{\left(\mathrm{e}^{2fx+2e}-1\right)^{2}f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(a*cosh(f*x+e)^4/sinh(f*x+e)/(cosh(f*x+e)^2-1)/(a*cosh(f*x+e)^2
)^(1/2),sinh(f*x+e))/f
```

Maxima [**A**]
time = 0.51, size = 134, normalized size = 1.54

$$-\frac{3\sqrt{a}\,\log\left(e^{(-fx-e)}+1\right)}{2f}+\frac{3\sqrt{a}\,\log\left(e^{(-fx-e)}-1\right)}{2f}-\frac{3\sqrt{a}\,e^{(-2fx-2e)}+3\sqrt{a}\,e^{(-4fx-4e)}-\sqrt{a}\,e^{(-6fx-6e)}-\sqrt{a}}{2f\left(e^{(-fx-e)}-2\,e^{(-3fx-3e)}+e^{(-5fx-5e)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(a)*log(e^(-f*x - e) + 1)/f + 3/2*sqrt(a)*log(e^(-f*x - e) - 1)/f
- 1/2*(3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e) - sqrt(a)*e^
(-6*f*x - 6*e) - sqrt(a))/(f*(e^(-f*x - e) - 2*e^(-3*f*x - 3*e) + e^(-5*f*x
 - 5*e)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 764 vs. 2(71) = 142.
time = 0.52, size = 764, normalized size = 8.78

$$\frac{\left(6\cosh(fx+e)e^{(fx+e)}\sinh(fx+e)^{5}+e^{(fx+e)}\sinh(fx+e)^{6}+3\left(5\cosh(fx+e)^{2}-1\right)e^{(fx+e)}\sinh(fx+e)^{4}+4\left(5\cosh(fx+e)^{3}-3\cosh(fx+e)\right)e^{(fx+e)}\sinh(fx+e)^{3}+3\left(5\cosh(fx+e)^{4}-6\cosh(fx+e)^{2}-1\right)e^{(fx+e)}\sinh(fx+e)^{2}+6\left(\cosh(fx+e)^{5}-2\cosh(fx+e)^{3}-\cosh(fx+e)\right)e^{(fx+e)}\sinh(fx+e)+\left(\cosh(fx+e)^{6}-3\cosh(fx+e)^{4}-3\cosh(fx+e)^{2}+1\right)e^{(fx+e)}+3\left(5\cosh(fx+e)e^{(fx+e)}\sinh(fx+e)^{4}+e^{(fx+e)}\sinh(fx+e)^{5}+2\left(5\cosh(fx+e)^{2}-1\right)e^{(fx+e)}\sinh(fx+e)^{3}+2\left(5\cosh(fx+e)^{3}-3\cosh(fx+e)\right)e^{(fx+e)}\sinh(fx+e)^{2}+\left(5\cosh(fx+e)^{4}-6\cosh(fx+e)^{2}+1\right)e^{(fx+e)}\sinh(fx+e)+\left(\cosh(fx+e)^{5}-2\cosh(fx+e)^{3}+\cosh(fx+e)\right)e^{(fx+e)}\right)\log\left(\frac{\cosh(fx+e)+\sinh(fx+e)-1}{\cosh(fx+e)+\sinh(fx+e)+1}\right)\right)\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}\,e^{(-fx-e)}}{2\left(f\cosh(fx+e)^{5}+\left(fe^{(2fx+2e)}+f\right)\sinh(fx+e)^{5}+5\left(f\cosh(fx+e)e^{(2fx+2e)}+f\cosh(fx+e)\right)\sinh(fx+e)^{4}-2f\cosh(fx+e)^{3}+2\left(5f\cosh(fx+e)^{2}+\left(5f\cosh(fx+e)^{2}-f\right)e^{(2fx+2e)}-f\right)\sinh(fx+e)^{3}+2\left(5f\cosh(fx+e)^{3}-3f\cosh(fx+e)+\left(5f\cosh(fx+e)^{3}-3f\cosh(fx+e)\right)e^{(2fx+2e)}\right)\sinh(fx+e)^{2}+f\cosh(fx+e)+\left(f\cosh(fx+e)^{5}-2f\cosh(fx+e)^{3}+f\cosh(fx+e)\right)e^{(2fx+2e)}+\left(5f\cosh(fx+e)^{4}-6f\cosh(fx+e)^{2}+\left(5f\cosh(fx+e)^{4}-6f\cosh(fx+e)^{2}+f\right)e^{(2fx+2e)}+f\right)\sinh(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^5 + e^(f*x + e)*sinh(f*x + e
)^6 + 3*(5*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^4 + 4*(5*cosh(f*x
 + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 3*(5*cosh(f*x + e)
^4 - 6*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 6*(cosh(f*x + e)^
5 - 2*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*
x + e)^6 - 3*cosh(f*x + e)^4 - 3*cosh(f*x + e)^2 + 1)*e^(f*x + e) + 3*(5*co
sh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + e^(f*x + e)*sinh(f*x + e)^5 + 2*(
5*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 2*(5*cosh(f*x + e)^3 -
```

```
 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (5*cosh(f*x + e)^4 - 6*cosh
(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^5 - 2*cosh(f*x
+ e)^3 + cosh(f*x + e))*e^(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e) - 1
)/(cosh(f*x + e) + sinh(f*x + e) + 1)))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f
*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)^5 + (f*e^(2*f*x + 2*e) + f)*si
nh(f*x + e)^5 + 5*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(
f*x + e)^4 - 2*f*cosh(f*x + e)^3 + 2*(5*f*cosh(f*x + e)^2 + (5*f*cosh(f*x +
 e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e)^3 + 2*(5*f*cosh(f*x + e)^3 -
3*f*cosh(f*x + e) + (5*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e)^2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^5 - 2*f*cosh(f*x +
 e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (5*f*cosh(f*x + e)^4 - 6*f*cosh(
f*x + e)^2 + (5*f*cosh(f*x + e)^4 - 6*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e
) + f)*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\coth^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x)**3, x)
```

Giac [**A**]
time = 0.43, size = 108, normalized size = 1.24

$$-\frac{\sqrt{a}\left(\frac{4\left(e^{(fx+e)}+e^{(-fx-e)}\right)}{\left(e^{(fx+e)}+e^{(-fx-e)}\right)^{2}-4}-2e^{(fx+e)}-2e^{(-fx-e)}+3\log\left(e^{(fx+e)}+e^{(-fx-e)}+2\right)-3\log\left(e^{(fx+e)}+e^{(-fx-e)}-2\right)\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(a)*(4*(e^(f*x + e) + e^(-f*x - e))/((e^(f*x + e) + e^(-f*x - e))^
2 - 4) - 2*e^(f*x + e) - 2*e^(-f*x - e) + 3*log(e^(f*x + e) + e^(-f*x - e)
+ 2) - 3*log(e^(f*x + e) + e^(-f*x - e) - 2))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coth}(e+f\,x)^3\,\sqrt{a\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^3*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

3.431 $\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^6(e+fx)\,dx$

Optimal. Leaf size=120

$$-\frac{15\text{ArcTan}(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)}{8f}+\frac{15\sqrt{a\cosh^2(e+fx)}\,\tanh(e+fx)}{8f}-\frac{5\sqrt{a\cosh$$

```
[Out] -15/8*arctan(sinh(f*x+e))*sech(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f+15/8*(a*cos
h(f*x+e)^2)^(1/2)*tanh(f*x+e)/f-5/8*(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3/f
-1/4*(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5/f
```

Rubi [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3255, 3286, 2672, 294, 327, 209}

$$-\frac{15\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\,\text{ArcTan}(\sinh(e+fx))}{8f}-\frac{\tanh^5(e+fx)\sqrt{a\cosh^2(e+fx)}}{4f}-\frac{5\tanh^3(e+fx)\sqrt{a\cosh^2(e+fx)}}{8f}+\frac{15\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^6,x]
```

```
[Out] (-15*ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/(8*f) + (
15*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/(8*f) - (5*Sqrt[a*Cosh[e + f*x]^2
]*Tanh[e + f*x]^3)/(8*f) - (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x]^5)/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^6(e+fx)\,dx &= \int \sqrt{a\cosh^2(e+fx)}\,\tanh^6(e+fx)\,dx\\ &= \left(\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\int \sinh(e+fx)\tanh^5(e+\\ &= \frac{\left(\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,dx, x, \text{sinh}}{f}\\ &= -\frac{\sqrt{a\cosh^2(e+fx)}\,\tanh^5(e+fx)}{4f}+\frac{\left(5\sqrt{a\cosh^2(e+fx)}\,\text{s}}{}\\ &= -\frac{5\sqrt{a\cosh^2(e+fx)}\,\tanh^3(e+fx)}{8f}-\frac{\sqrt{a\cosh^2(e+fx)}\,\text{ta}}{4f}\\ &= \frac{15\sqrt{a\cosh^2(e+fx)}\,\tanh(e+fx)}{8f}-\frac{5\sqrt{a\cosh^2(e+fx)}\,\text{ta}}{8f}\\ &= -\frac{15\tan^{-1}(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)}{8f}+\frac{15}{}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 75, normalized size = 0.62

$$-\frac{\sqrt{a\cosh^2(e+fx)}\,\text{sech}^5(e+fx)\,(60\text{ArcTan}(\sinh(e+fx))\cosh^4(e+fx)-5\sinh(e+fx)-15\sinh(3(e+fx))-2\sinh(5(e+fx)))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^6,x]
```

```
[Out] -1/32*(Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]^5*(60*ArcTan[Sinh[e + f*x]]*Co
sh[e + f*x]^4 - 5*Sinh[e + f*x] - 15*Sinh[3*(e + f*x)] - 2*Sinh[5*(e + f*x)
]))/f
```

Maple [A]
time = 1.66, size = 85, normalized size = 0.71

method	result
default	$-\frac{a\left(15\arctan(\sinh(fx+e))\left(\cosh^4(fx+e)\right)-8\left(\cosh^4(fx+e)\right)\sinh(fx+e)-9\left(\cosh^2(fx+e)\right)\sinh(fx+e)+2\sinh(fx+e)\right)}{8\cosh(fx+e)^3\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$

risch	$\frac{\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\;e^{2fx+2e}}{2f\left(e^{2fx+2e}+1\right)}-\frac{\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}}{2f\left(e^{2fx+2e}+1\right)}+\frac{\left(9\,e^{6fx+6e}+e^{4fx+4e}-e^{2fx+2e}-9\right)\sqrt{}}{4f(e^{2f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a*(15*arctan(sinh(f*x+e))*cosh(f*x+e)^4-8*cosh(f*x+e)^4*sinh(f*x+e)-9*
cosh(f*x+e)^2*sinh(f*x+e)+2*sinh(f*x+e))/cosh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1
/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 955 vs. $2(113) = 226$.
time $= 0.51$, size $= 955$, normalized size $= 7.96$

$$\frac{315\sqrt{a}\arctan\left(e^{(-fx-e)}\right)}{128f}+\frac{105\sqrt{a}\arctan\left(e^{(-fx-e)}\right)+\frac{279\sqrt{a}e^{(-fx-e)}+511\sqrt{a}e^{(-3fx-3e)}+385\sqrt{a}e^{(-5fx-5e)}+105\sqrt{a}e^{(-7fx-7e)}}{4e^{(-2fx-2e)}+6e^{(-4fx-4e)}+4e^{(-6fx-6e)}+e^{(-8fx-8e)}+1}}{128f}+\frac{105\sqrt{a}\arctan\left(e^{(-fx-e)}\right)-\frac{105\sqrt{a}e^{(-fx-e)}+385\sqrt{a}e^{(-3fx-3e)}+511\sqrt{a}e^{(-5fx-5e)}+279\sqrt{a}e^{(-7fx-7e)}}{4e^{(-2fx-2e)}+6e^{(-4fx-4e)}+4e^{(-6fx-6e)}+e^{(-8fx-8e)}+1}}{128f}-\frac{5\left(15\sqrt{a}\arctan\left(e^{(-fx-e)}\right)-\frac{15\sqrt{a}e^{(-fx-e)}+55\sqrt{a}e^{(-3fx-3e)}+73\sqrt{a}e^{(-5fx-5e)}-15\sqrt{a}e^{(-7fx-7e)}}{4e^{(-2fx-2e)}+6e^{(-4fx-4e)}+4e^{(-6fx-6e)}+e^{(-8fx-8e)}+1}\right)}{256f}-\frac{5\left(15\sqrt{a}\arctan\left(e^{(-fx-e)}\right)-\frac{15\sqrt{a}e^{(-fx-e)}-73\sqrt{a}e^{(-3fx-3e)}-55\sqrt{a}e^{(-5fx-5e)}-15\sqrt{a}e^{(-7fx-7e)}}{4e^{(-2fx-2e)}+6e^{(-4fx-4e)}+4e^{(-6fx-6e)}+e^{(-8fx-8e)}+1}\right)}{256f}+\frac{5\left(3\sqrt{a}\arctan\left(e^{(-fx-e)}\right)-\frac{3\sqrt{a}e^{(-fx-e)}+11\sqrt{a}e^{(-3fx-3e)}-11\sqrt{a}e^{(-5fx-5e)}-3\sqrt{a}e^{(-7fx-7e)}}{4e^{(-2fx-2e)}+6e^{(-4fx-4e)}+4e^{(-6fx-6e)}+e^{(-8fx-8e)}+1}\right)}{64f}+\frac{837\sqrt{a}e^{(-2fx-2e)}+1533\sqrt{a}e^{(-4fx-4e)}+1155\sqrt{a}e^{(-6fx-6e)}+315\sqrt{a}e^{(-8fx-8e)}+128\sqrt{a}}{256f\left(e^{(-fx-e)}+4e^{(-3fx-3e)}+6e^{(-5fx-5e)}+4e^{(-7fx-7e)}+e^{(-9fx-9e)}\right)}-\frac{315\sqrt{a}e^{(-fx-e)}+1155\sqrt{a}e^{(-3fx-3e)}+1533\sqrt{a}e^{(-5fx-5e)}+837\sqrt{a}e^{(-7fx-7e)}+128\sqrt{a}e^{(-9fx-9e)}}{256f\left(4e^{(-2fx-2e)}+6e^{(-4fx-4e)}+4e^{(-6fx-6e)}+e^{(-8fx-8e)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 315/128*sqrt(a)*arctan(e^(-f*x - e))/f + 1/128*(105*sqrt(a)*arctan(e^(-f*x
 - e)) + (279*sqrt(a)*e^(-f*x - e) + 511*sqrt(a)*e^(-3*f*x - 3*e) + 385*sqrt
(a)*e^(-5*f*x - 5*e) + 105*sqrt(a)*e^(-7*f*x - 7*e))/(4*e^(-2*f*x - 2*e) +
6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))/f + 1/128*
(105*sqrt(a)*arctan(e^(-f*x - e)) - (105*sqrt(a)*e^(-f*x - e) + 385*sqrt(a)
*e^(-3*f*x - 3*e) + 511*sqrt(a)*e^(-5*f*x - 5*e) + 279*sqrt(a)*e^(-7*f*x -
7*e))/(4*e^(-2*f*x - 2*e) + 6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8
*f*x - 8*e) + 1))/f - 5/256*(15*sqrt(a)*arctan(e^(-f*x - e)) - (15*sqrt(a)*
e^(-f*x - e) + 55*sqrt(a)*e^(-3*f*x - 3*e) + 73*sqrt(a)*e^(-5*f*x - 5*e) -
15*sqrt(a)*e^(-7*f*x - 7*e))/(4*e^(-2*f*x - 2*e) + 6*e^(-4*f*x - 4*e) + 4*e
^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))/f - 5/256*(15*sqrt(a)*arctan(e^(-f
*x - e)) - (15*sqrt(a)*e^(-f*x - e) - 73*sqrt(a)*e^(-3*f*x - 3*e) - 55*sqrt
(a)*e^(-5*f*x - 5*e) - 15*sqrt(a)*e^(-7*f*x - 7*e))/(4*e^(-2*f*x - 2*e) + 6
*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))/f + 5/64*(3
*sqrt(a)*arctan(e^(-f*x - e)) - (3*sqrt(a)*e^(-f*x - e) + 11*sqrt(a)*e^(-3*
f*x - 3*e) - 11*sqrt(a)*e^(-5*f*x - 5*e) - 3*sqrt(a)*e^(-7*f*x - 7*e))/(4*e
^(-2*f*x - 2*e) + 6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e
) + 1))/f + 1/256*(837*sqrt(a)*e^(-2*f*x - 2*e) + 1533*sqrt(a)*e^(-4*f*x -
4*e) + 1155*sqrt(a)*e^(-6*f*x - 6*e) + 315*sqrt(a)*e^(-8*f*x - 8*e) + 128*s
qrt(a))/(f*(e^(-f*x - e) + 4*e^(-3*f*x - 3*e) + 6*e^(-5*f*x - 5*e) + 4*e^(-
7*f*x - 7*e) + e^(-9*f*x - 9*e))) - 1/256*(315*sqrt(a)*e^(-f*x - e) + 1155*
sqrt(a)*e^(-3*f*x - 3*e) + 1533*sqrt(a)*e^(-5*f*x - 5*e) + 837*sqrt(a)*e^(-
7*f*x - 7*e) + 128*sqrt(a)*e^(-9*f*x - 9*e))/(f*(4*e^(-2*f*x - 2*e) + 6*e^(
-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1645 vs. $2(104) = 208$.
time = 0.47, size = 1645, normalized size = 13.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] 1/4*(20*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^9 + 2*e^(f*x + e)*sinh(f*x
+ e)^10 + 15*(6*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^8 + 120*(2*c
osh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^7 + 5*(84*cosh(f*
x + e)^4 + 84*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^6 + 6*(84*cosh
(f*x + e)^5 + 140*cosh(f*x + e)^3 + 5*cosh(f*x + e))*e^(f*x + e)*sinh(f*x +
 e)^5 + 5*(84*cosh(f*x + e)^6 + 210*cosh(f*x + e)^4 + 15*cosh(f*x + e)^2 -
1)*e^(f*x + e)*sinh(f*x + e)^4 + 20*(12*cosh(f*x + e)^7 + 42*cosh(f*x + e)^
5 + 5*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 15*(6*
cosh(f*x + e)^8 + 28*cosh(f*x + e)^6 + 5*cosh(f*x + e)^4 - 2*cosh(f*x + e)^
2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 10*(2*cosh(f*x + e)^9 + 12*cosh(f*x +
e)^7 + 3*cosh(f*x + e)^5 - 2*cosh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)
*sinh(f*x + e) - 15*(9*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^8 + e^(f*x +
 e)*sinh(f*x + e)^9 + 4*(9*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^7
 + 28*(3*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^6 + 6*(
21*cosh(f*x + e)^4 + 14*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^5 +
2*(63*cosh(f*x + e)^5 + 70*cosh(f*x + e)^3 + 15*cosh(f*x + e))*e^(f*x + e)*
sinh(f*x + e)^4 + 4*(21*cosh(f*x + e)^6 + 35*cosh(f*x + e)^4 + 15*cosh(f*x
+ e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 12*(3*cosh(f*x + e)^7 + 7*cosh(f*
x + e)^5 + 5*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 +
 (9*cosh(f*x + e)^8 + 28*cosh(f*x + e)^6 + 30*cosh(f*x + e)^4 + 12*cosh(f*x
 + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^9 + 4*cosh(f*x + e)
^7 + 6*cosh(f*x + e)^5 + 4*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e))*ar
ctan(cosh(f*x + e) + sinh(f*x + e)) + (2*cosh(f*x + e)^10 + 15*cosh(f*x + e
)^8 + 5*cosh(f*x + e)^6 - 5*cosh(f*x + e)^4 - 15*cosh(f*x + e)^2 - 2)*e^(f*
x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*c
osh(f*x + e)^9 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^9 + 9*(f*cosh(f*x +
e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^8 + 4*f*cosh(f*x + e)^7
 + 4*(9*f*cosh(f*x + e)^2 + (9*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*
sinh(f*x + e)^7 + 28*(3*f*cosh(f*x + e)^3 + f*cosh(f*x + e) + (3*f*cosh(f*x
 + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 6*f*cosh(f*x
+ e)^5 + 6*(21*f*cosh(f*x + e)^4 + 14*f*cosh(f*x + e)^2 + (21*f*cosh(f*x +
e)^4 + 14*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^5 + 2*(
63*f*cosh(f*x + e)^5 + 70*f*cosh(f*x + e)^3 + 15*f*cosh(f*x + e) + (63*f*co
sh(f*x + e)^5 + 70*f*cosh(f*x + e)^3 + 15*f*cosh(f*x + e))*e^(2*f*x + 2*e))
*sinh(f*x + e)^4 + 4*f*cosh(f*x + e)^3 + 4*(21*f*cosh(f*x + e)^6 + 35*f*cos
h(f*x + e)^4 + 15*f*cosh(f*x + e)^2 + (21*f*cosh(f*x + e)^6 + 35*f*cosh(f*x
```

```
 + e)^4 + 15*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^3 +
12*(3*f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)^5 + 5*f*cosh(f*x + e)^3 + f*cos
h(f*x + e) + (3*f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)^5 + 5*f*cosh(f*x + e)
^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) +
(f*cosh(f*x + e)^9 + 4*f*cosh(f*x + e)^7 + 6*f*cosh(f*x + e)^5 + 4*f*cosh(f
*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (9*f*cosh(f*x + e)^8 + 28*f*
cosh(f*x + e)^6 + 30*f*cosh(f*x + e)^4 + 12*f*cosh(f*x + e)^2 + (9*f*cosh(f
*x + e)^8 + 28*f*cosh(f*x + e)^6 + 30*f*cosh(f*x + e)^4 + 12*f*cosh(f*x + e
)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\,\tanh^{6}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**6,x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**6, x)
```

Giac [**A**]
time = 0.45, size = 124, normalized size = 1.03

$$-\frac{\left(15\,\pi-\frac{4\left(9\left(e^{(fx+e)}-e^{(-fx-e)}\right)^{3}+28\,e^{(fx+e)}-28\,e^{(-fx-e)}\right)}{\left(\left(e^{(fx+e)}-e^{(-fx-e)}\right)^{2}+4\right)^{2}}+30\,\arctan\left(\frac{1}{2}\left(e^{(2\,fx+2\,e)}-1\right)e^{(-fx-e)}\right)-8\,e^{(fx+e)}+8\,e^{(-fx-e)}\right)\sqrt{a}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="giac")
```

```
[Out] -1/16*(15*pi - 4*(9*(e^(f*x + e) - e^(-f*x - e))^3 + 28*e^(f*x + e) - 28*e^
(-f*x - e))/((e^(f*x + e) - e^(-f*x - e))^2 + 4)^2 + 30*arctan(1/2*(e^(2*f*
x + 2*e) - 1)*e^(-f*x - e)) - 8*e^(f*x + e) + 8*e^(-f*x - e))*sqrt(a)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\tanh(e+f\,x)^{6}\,\sqrt{a\sinh\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^6*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^6*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

3.432 $\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^4(e+fx)\,dx$

Optimal. Leaf size=91

$$-\frac{3\text{ArcTan}(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)}{2f}+\frac{3\sqrt{a\cosh^2(e+fx)}\,\tanh(e+fx)}{2f}-$$

```
[Out] -3/2*arctan(sinh(f*x+e))*sech(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f+3/2*(a*cosh(
f*x+e)^2)^(1/2)*tanh(f*x+e)/f-1/2*(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3/f
```

Rubi [A]
time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3255, 3286, 2672, 294, 327, 209}

$$-\frac{3\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\,\text{ArcTan}(\sinh(e+fx))}{2f}-\frac{\tanh^3(e+fx)\sqrt{a\cosh^2(e+fx)}}{2f}+\frac{3\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

```
[Out] (-3*ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/(2*f) + (3
*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/(2*f) - (Sqrt[a*Cosh[e + f*x]^2]*Ta
nh[e + f*x]^3)/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^4(e+fx)\,dx &= \int \sqrt{a\cosh^2(e+fx)}\,\tanh^4(e+fx)\,dx \\
&= \left(\sqrt{a\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\int \sinh(e+fx)\tanh^3(e \\
&= \frac{\left(\sqrt{a\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\text{Subst}\Big(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \sin}{f} \\
&= -\frac{\sqrt{a\cosh^2(e+fx)}\,\tanh^3(e+fx)}{2f} + \frac{\Big(3\sqrt{a\cosh^2(e+fx)}}{} \\
&= \frac{3\sqrt{a\cosh^2(e+fx)}\,\tanh(e+fx)}{2f} - \frac{\sqrt{a\cosh^2(e+fx)}\,\tan}{2f} \\
&= -\frac{3\tan^{-1}(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)}{2f} + \frac{3}{}
\end{aligned}
$$

Mathematica [A]

time = 0.14, size = 55, normalized size = 0.60

$$\frac{a(-3\mathrm{ArcTan}(\sinh(e+fx))\cosh(e+fx)+(2+\cosh(2(e+fx)))\tanh(e+fx))}{2f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

```
[Out] (a*(-3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + (2 + Cosh[2*(e + f*x)])*Tanh[e
 + f*x]))/(2*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [A]
time = 1.24, size = 69, normalized size = 0.76

method	result
default	$-\frac{a\left(3\arctan(\sinh(fx+e))\left(\cosh^2(fx+e)\right)-2\left(\cosh^2(fx+e)\right)\sinh(fx+e)-\sinh(fx+e)\right)}{2\cosh(fx+e)\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$
risch	$\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a\,\mathrm{e}^{-2fx-2e}}\;\mathrm{e}^{2fx+2e}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}-\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a\,\mathrm{e}^{-2fx-2e}}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}+\frac{\left(\mathrm{e}^{2fx+2e}-1\right)\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a\,\mathrm{e}}}{f\left(\mathrm{e}^{2fx+2e}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*(3*arctan(sinh(f*x+e))*cosh(f*x+e)^2-2*cosh(f*x+e)^2*sinh(f*x+e)-sin
h(f*x+e))/cosh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(86) = 172$.
time = 0.53, size = 413, normalized size = 4.54

$$\frac{15\sqrt{a}\arctan\left(e^{(-fx-e)}\right)}{8f}+\frac{3\sqrt{a}\arctan\left(e^{(-fx-e)}\right)+\frac{5\sqrt{a}e^{(-fx-e)}+3\sqrt{a}e^{(-3fx-3e)}}{2e^{(-2fx-2e)}+e^{(-4fx-4e)}+1}}{4f}+\frac{3\sqrt{a}\arctan\left(e^{(-fx-e)}\right)-\frac{3\sqrt{a}e^{(-fx-e)}+5\sqrt{a}e^{(-3fx-3e)}}{2e^{(-2fx-2e)}+e^{(-4fx-4e)}+1}}{4f}-\frac{3\left(\sqrt{a}\arctan\left(e^{(-fx-e)}\right)-\frac{\sqrt{a}e^{(-fx-e)}-\sqrt{a}e^{(-3fx-3e)}}{2e^{(-2fx-2e)}+e^{(-4fx-4e)}+1}\right)}{8f}+\frac{25\sqrt{a}e^{(-2fx-2e)}+15\sqrt{a}e^{(-4fx-4e)}+8\sqrt{a}}{16f(e^{(-fx-e)}+2e^{(-3fx-3e)}+e^{(-5fx-5e)})}-\frac{15\sqrt{a}e^{(-fx-e)}+25\sqrt{a}e^{(-3fx-3e)}+8\sqrt{a}e^{(-5fx-5e)}}{16f(2e^{(-2fx-2e)}+e^{(-4fx-4e)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 15/8*sqrt(a)*arctan(e^(-f*x - e))/f + 1/4*(3*sqrt(a)*arctan(e^(-f*x - e)) +
 (5*sqrt(a)*e^(-f*x - e) + 3*sqrt(a)*e^(-3*f*x - 3*e))/(2*e^(-2*f*x - 2*e)
+ e^(-4*f*x - 4*e) + 1))/f + 1/4*(3*sqrt(a)*arctan(e^(-f*x - e)) - (3*sqrt(
a)*e^(-f*x - e) + 5*sqrt(a)*e^(-3*f*x - 3*e))/(2*e^(-2*f*x - 2*e) + e^(-4*f
*x - 4*e) + 1))/f - 3/8*(sqrt(a)*arctan(e^(-f*x - e)) - (sqrt(a)*e^(-f*x -
e) - sqrt(a)*e^(-3*f*x - 3*e))/(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))
/f + 1/16*(25*sqrt(a)*e^(-2*f*x - 2*e) + 15*sqrt(a)*e^(-4*f*x - 4*e) + 8*sq
rt(a))/(f*(e^(-f*x - e) + 2*e^(-3*f*x - 3*e) + e^(-5*f*x - 5*e))) - 1/16*(1
```

```
5*sqrt(a)*e^(-f*x - e) + 25*sqrt(a)*e^(-3*f*x - 3*e) + 8*sqrt(a)*e^(-5*f*x
- 5*e))/(f*(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 742 vs. $2(79) = 158$.
time = 0.49, size = 742, normalized size = 8.15

$$\frac{\left(6\cosh(fx+e)e^{(fx+e)}\sinh(fx+e)^5 + e^{(fx+e)}\sinh(fx+e)^6 + 3\left(5\cosh(fx+e)^2+1\right)e^{(fx+e)}\sinh(fx+e)^4 + 4\left(5\cosh(fx+e)^3+3\cosh(fx+e)\right)e^{(fx+e)}\sinh(fx+e)^3 + 3\left(5\cosh(fx+e)^4+6\cosh(fx+e)^2-1\right)e^{(fx+e)}\sinh(fx+e)^2 + 6\left(\cosh(fx+e)^5+2\cosh(fx+e)^3-\cosh(fx+e)\right)e^{(fx+e)}\sinh(fx+e) - 6\left(5\cosh(fx+e)e^{(fx+e)}\sinh(fx+e)^4 + e^{(fx+e)}\sinh(fx+e)^5 + 2\left(5\cosh(fx+e)^2+1\right)e^{(fx+e)}\sinh(fx+e)^3 + 2\left(5\cosh(fx+e)^3+3\cosh(fx+e)\right)e^{(fx+e)}\sinh(fx+e)^2 + \left(5\cosh(fx+e)^4+6\cosh(fx+e)^2+1\right)e^{(fx+e)}\sinh(fx+e) + \left(\cosh(fx+e)^5+2\cosh(fx+e)^3+\cosh(fx+e)\right)e^{(fx+e)}\right)\arctan\left(\cosh(fx+e)+\sinh(fx+e)\right) + \left(\cosh(fx+e)^6+3\cosh(fx+e)^4-3\cosh(fx+e)^2-1\right)e^{(fx+e)}\right)\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}\,e^{(-fx-e)}}{2\left(f\cosh(fx+e)^5 + \left(fe^{(2fx+2e)}+f\right)\sinh(fx+e)^5 + 5\left(f\cosh(fx+e)e^{(2fx+2e)}+f\cosh(fx+e)\right)\sinh(fx+e)^4 + 2f\cosh(fx+e)^3 + 2\left(5f\cosh(fx+e)^2+\left(5f\cosh(fx+e)^2+f\right)e^{(2fx+2e)}+f\right)\sinh(fx+e)^3 + 2\left(5f\cosh(fx+e)^3+3f\cosh(fx+e)+\left(5f\cosh(fx+e)^3+3f\cosh(fx+e)\right)e^{(2fx+2e)}\right)\sinh(fx+e)^2 + f\cosh(fx+e) + \left(f\cosh(fx+e)^5+2f\cosh(fx+e)^3+f\cosh(fx+e)\right)e^{(2fx+2e)} + \left(5f\cosh(fx+e)^4+6f\cosh(fx+e)^2+\left(5f\cosh(fx+e)^4+6f\cosh(fx+e)^2+f\right)e^{(2fx+2e)}+f\right)\sinh(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^5 + e^(f*x + e)*sinh(f*x + e
)^6 + 3*(5*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^4 + 4*(5*cosh(f*x
 + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 3*(5*cosh(f*x + e)
^4 + 6*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 6*(cosh(f*x + e)^
5 + 2*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) - 6*(5*cos
h(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + e^(f*x + e)*sinh(f*x + e)^5 + 2*(5
*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 2*(5*cosh(f*x + e)^3 +
3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (5*cosh(f*x + e)^4 + 6*cosh(
f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^5 + 2*cosh(f*x +
 e)^3 + cosh(f*x + e))*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) +
 (cosh(f*x + e)^6 + 3*cosh(f*x + e)^4 - 3*cosh(f*x + e)^2 - 1)*e^(f*x + e))
*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x
 + e)^5 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^5 + 5*(f*cosh(f*x + e)*e^(2
*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^4 + 2*f*cosh(f*x + e)^3 + 2*(5
*f*cosh(f*x + e)^2 + (5*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*
x + e)^3 + 2*(5*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (5*f*cosh(f*x + e)^
3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) +
 (f*cosh(f*x + e)^5 + 2*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e
) + (5*f*cosh(f*x + e)^4 + 6*f*cosh(f*x + e)^2 + (5*f*cosh(f*x + e)^4 + 6*f
*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^2(e+fx)+1\right)}\,\tanh^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**4, x)
```

Giac [**A**]
time = 0.46, size = 100, normalized size = 1.10

$$-\frac{\left(3\pi - \frac{4\left(e^{(fx+e)}-e^{(-fx-e)}\right)}{\left(e^{(fx+e)}-e^{(-fx-e)}\right)^2+4} + 6\arctan\left(\frac{1}{2}\left(e^{(2fx+2e)}-1\right)e^{(-fx-e)}\right) - 2e^{(fx+e)} + 2e^{(-fx-e)}\right)\sqrt{a}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="giac")
```

```
[Out] -1/4*(3*pi - 4*(e^(f*x + e) - e^(-f*x - e))/((e^(f*x + e) - e^(-f*x - e))^2
 + 4) + 6*arctan(1/2*(e^(2*f*x + 2*e) - 1)*e^(-f*x - e)) - 2*e^(f*x + e) +
2*e^(-f*x - e))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(e+f\,x)^4\,\sqrt{a\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^4*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

3.433 $\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^2(e+fx)\,dx$

Optimal. Leaf size=57

$$-\frac{\operatorname{ArcTan}(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)}{f}+\frac{\sqrt{a\cosh^2(e+fx)}\,\tanh(e+fx)}{f}$$

```
[Out] -arctan(sinh(f*x+e))*sech(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f+(a*cosh(f*x+e)^2
)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3255, 3286, 2672, 327, 209}

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\,\operatorname{ArcTan}(\sinh(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

```
[Out] -((ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/f) + (Sqrt[
a*Cosh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^2(e+fx)\,dx &= \int \sqrt{a\cosh^2(e+fx)}\,\tanh^2(e+fx)\,dx\\
&= \Bigg(\sqrt{a\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\Bigg)\int \sinh(e+fx)\tanh(e+\\
&= \frac{\Bigg(\sqrt{a\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\Bigg)\operatorname{Subst}\Big(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(e}{f}\\
&= \frac{\sqrt{a\cosh^2(e+fx)}\,\tanh(e+fx)}{f} - \frac{\Bigg(\sqrt{a\cosh^2(e+fx)}\,\operatorname{sech}(}{}\\
&= -\frac{\tan^{-1}(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)}{f} + \frac{\sqrt{a}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 0.70

$$
\frac{\sqrt{a\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)(-\operatorname{ArcTan}(\sinh(e+fx))+\sinh(e+fx))}{f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]*(-ArcTan[Sinh[e + f*x]] + Sinh[e + f
*x]))/f
```

Maple [A]

time = 1.10, size = 41, normalized size = 0.72

method	result
default	$-\frac{a\cosh(fx+e)(-\sinh(fx+e)+\arctan(\sinh(fx+e)))}{\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}\, f}$
risch	$\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}\;\mathrm{e}^{2fx+2e}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}-\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}+\frac{i\ln\left(\mathrm{e}^{fx}-i\mathrm{e}^{-e}\right)\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}}}{f\left(\mathrm{e}^{2fx+2e}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*cosh(f*x+e)*(-sinh(f*x+e)+arctan(sinh(f*x+e)))/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [A]

time = 0.48, size = 53, normalized size = 0.93

$$\frac{2\sqrt{a}\arctan\left(e^{(-fx-e)}\right)}{f}+\frac{\sqrt{a}\,e^{(fx+e)}}{2f}-\frac{\sqrt{a}\,e^{(-fx-e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*arctan(e^(-f*x - e))/f + 1/2*sqrt(a)*e^(f*x + e)/f - 1/2*sqrt(a)*
e^(-f*x - e)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(53) = 106.

time = 0.47, size = 182, normalized size = 3.19

$$\frac{\left(2\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)+e^{(fx+e)}\sinh\left(fx+e\right)^{2}-4\left(\cosh\left(fx+e\right)e^{(fx+e)}+e^{(fx+e)}\sinh\left(fx+e\right)\right)\arctan\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)+\left(\cosh\left(fx+e\right)^{2}-1\right)e^{(fx+e)}\right)\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}\,e^{(-fx-e)}}{2\left(f\cosh\left(fx+e\right)e^{(2fx+2e)}+f\cosh\left(fx+e\right)+\left(fe^{(2fx+2e)}+f\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^
2 - 4*(cosh(f*x + e)*e^(f*x + e) + e^(f*x + e)*sinh(f*x + e))*arctan(cosh(f
*x + e) + sinh(f*x + e)) + (cosh(f*x + e)^2 - 1)*e^(f*x + e))*sqrt(a*e^(4*f
*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2*f*x
 + 2*e) + f*cosh(f*x + e) + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\tanh^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**2, x)
```

Giac [**A**]
time = 0.42, size = 35, normalized size = 0.61

$$-\frac{\sqrt{a}\left(4\,\arctan\left(e^{(fx+e)}\right)-e^{(fx+e)}+e^{(-fx-e)}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(4*arctan(e^(f*x + e)) - e^(f*x + e) + e^(-f*x - e))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tanh(e+f\,x)^2\,\sqrt{a\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^2*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

3.434 $\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=56

$$-\frac{\sqrt{a\cosh^2(e+fx)}\,\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)}{f}+\frac{\sqrt{a\cosh^2(e+fx)}\,\tanh(e+fx)}{f}$$

```
[Out] -csch(f*x+e)*sech(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f+(a*cosh(f*x+e)^2)^(1/2)*
tanh(f*x+e)/f
```

Rubi [A]
time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3286, 2670, 14}

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f) + (Sqrt[a*Cosh[e
 + f*x]^2]*Tanh[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
```

```
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\cosh^2(e+fx)}\,\coth^2(e+fx)\,dx$$

$$= \left(\sqrt{a\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\int \cosh(e+fx)\coth^2(e+$$

$$= -\frac{\left(i\sqrt{a\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,-i\,\mathrm{s}}{f}$$

$$= -\frac{\left(i\sqrt{a\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,}{f}$$

$$= -\frac{\sqrt{a\cosh^2(e+fx)}\,\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)}{f}+\frac{\sqrt{a\cosh^2(e}}{}$$

Mathematica [A]
time = 0.06, size = 35, normalized size = 0.62

$$-\frac{\sqrt{a\cosh^2(e+fx)}\,\left(-1+\operatorname{csch}^2(e+fx)\right)\tanh(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a*Cosh[e + f*x]^2]*(-1 + Csch[e + f*x]^2)*Tanh[e + f*x])/f)
```

Maple [A]
time = 0.93, size = 42, normalized size = 0.75

method	result
default	$\frac{\cosh(fx+e)a(\sinh^2(fx+e)-1)}{\sinh(fx+e)\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$

risch	$\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}\;\mathrm{e}^{2fx+2e}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}-\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}-\frac{2\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}}{\left(\mathrm{e}^{2fx+2e}-1\right)f\left(\mathrm{e}^{2fx+2e}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] cosh(f*x+e)*a*(sinh(f*x+e)^2-1)/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(57) = 114$.
time = 0.50, size = 133, normalized size = 2.38

$$\frac{\sqrt{a}\,e^{(-fx-e)}}{f\left(e^{(-2\,fx-2\,e)}-1\right)}-\frac{2\,\sqrt{a}\,e^{(-2\,fx-2\,e)}-\sqrt{a}}{2\,f\left(e^{(-fx-e)}-e^{(-3\,fx-3\,e)}\right)}+\frac{2\,\sqrt{a}\,e^{(-fx-e)}-\sqrt{a}\,e^{(-3\,fx-3\,e)}}{2\,f\left(e^{(-2\,fx-2\,e)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*e^(-f*x - e)/(f*(e^(-2*f*x - 2*e) - 1)) - 1/2*(2*sqrt(a)*e^(-2*f*x
- 2*e) - sqrt(a))/(f*(e^(-f*x - e) - e^(-3*f*x - 3*e))) + 1/2*(2*sqrt(a)*e^
(-f*x - e) - sqrt(a)*e^(-3*f*x - 3*e))/(f*(e^(-2*f*x - 2*e) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(52) = 104$.
time = 0.47, size = 317, normalized size = 5.66

$$\frac{\left(4\,\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^{3}+e^{(fx+e)}\sinh\left(fx+e\right)^{4}+6\left(\cosh\left(fx+e\right)^{2}-1\right)e^{(fx+e)}\sinh\left(fx+e\right)^{2}+4\left(\cosh\left(fx+e\right)^{3}-3\,\cosh\left(fx+e\right)\right)e^{(fx+e)}\sinh\left(fx+e\right)+\left(\cosh\left(fx+e\right)^{4}-6\,\cosh\left(fx+e\right)^{2}+1\right)e^{(fx+e)}\right)\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\,e^{(-fx-e)}}{2\left(f\cosh\left(fx+e\right)^{3}+\left(fe^{(2\,fx+2\,e)}+f\right)\sinh\left(fx+e\right)^{3}+3\left(f\cosh\left(fx+e\right)e^{(2\,fx+2\,e)}+f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^{2}-f\cosh\left(fx+e\right)+\left(f\cosh\left(fx+e\right)^{3}-f\cosh\left(fx+e\right)\right)e^{(2\,fx+2\,e)}+\left(3\,f\cosh\left(fx+e\right)^{2}+\left(3\,f\cosh\left(fx+e\right)^{2}-f\right)e^{(2\,fx+2\,e)}-f\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e
)^4 + 6*(cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e
)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 - 6*cos
h(f*x + e)^2 + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e)
 + a)*e^(-f*x - e)/(f*cosh(f*x + e)^3 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x +
e)^3 + 3*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^
2 - f*cosh(f*x + e) + (f*cosh(f*x + e)^3 - f*cosh(f*x + e))*e^(2*f*x + 2*e)
 + (3*f*cosh(f*x + e)^2 + (3*f*cosh(f*x + e)^2 - f)*e^(2*f*x + 2*e) - f)*si
nh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\,\coth^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x)**2, x)
```

Giac [A]
time = 0.43, size = 48, normalized size = 0.86

$$-\frac{\sqrt{a}\left(\frac{4}{e^{(fx+e)}-e^{(-fx-e)}}-e^{(fx+e)}+e^{(-fx-e)}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(4/(e^(f*x + e) - e^(-f*x - e)) - e^(f*x + e) + e^(-f*x - e))/
f
```

Mupad [B]
time = 0.93, size = 67, normalized size = 1.20

$$\frac{\sqrt{a+a\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)^2}\,\left(\mathrm{e}^{4\,e+4\,f\,x}-6\,\mathrm{e}^{2\,e+2\,f\,x}+1\right)}{f\,\left(\mathrm{e}^{4\,e+4\,f\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] ((a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)*(exp(4*e + 4*f*x) - 6*
exp(2*e + 2*f*x) + 1))/(f*(exp(4*e + 4*f*x) - 1))
```

3.435 $\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=91

$$-\frac{2\sqrt{a\cosh^2(e+fx)}\,\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)}{f}-\frac{\sqrt{a\cosh^2(e+fx)}\,\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)}{3f}+\frac{\sqrt{a\cosh}}{}$$

```
[Out] -2*csch(f*x+e)*sech(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f-1/3*csch(f*x+e)^3*sech
(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f+(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3286, 2670, 276}

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{3f}-\frac{2\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-2*Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f - (Sqrt[a*Cosh[e
 + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x])/(3*f) + (Sqrt[a*Cosh[e + f*x]^2]*
Tanh[e + f*x])/f
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
```

```
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx &= \int \sqrt{a\cosh^2(e+fx)}\,\coth^4(e+fx)\,dx\\ &= \left(\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\int \cosh(e+fx)\coth^4(e+\\ &= \frac{\left(i\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, -i\right.}{f}\\ &= \frac{\left(i\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\,dx\right.}{f}\\ &= -\frac{2\sqrt{a\cosh^2(e+fx)}\,\text{csch}(e+fx)\text{sech}(e+fx)}{f}-\frac{\sqrt{a\cosh^2(e}{}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 47, normalized size = 0.52

$$-\frac{\sqrt{a\cosh^2(e+fx)}\,\left(-3+6\text{csch}^2(e+fx)+\text{csch}^4(e+fx)\right)\tanh(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -1/3*(Sqrt[a*Cosh[e + f*x]^2]*(-3 + 6*Csch[e + f*x]^2 + Csch[e + f*x]^4)*Ta
nh[e + f*x])/f
```

Maple [A]
time = 0.95, size = 55, normalized size = 0.60

method	result
default	$\frac{\cosh(fx+e)a\left(3\left(\sinh^4(fx+e)\right)-6\left(\sinh^2(fx+e)\right)-1\right)}{3\sinh(fx+e)^3\sqrt{a\left(\cosh^2(fx+e)\right)}\,f}$

risch	$\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}\;\mathrm{e}^{2fx+2e}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}-\frac{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}}{2f\left(\mathrm{e}^{2fx+2e}+1\right)}-\frac{4\left(3\,\mathrm{e}^{4fx+4e}-4\,\mathrm{e}^{2fx+2e}+3\right)\sqrt{\left(\mathrm{e}^{2fx-\right.}}}{3\left(\mathrm{e}^{2fx+2e}-1\right)^{3} f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*cosh(f*x+e)*a*(3*sinh(f*x+e)^4-6*sinh(f*x+e)^2-1)/sinh(f*x+e)^3/(a*cosh
(f*x+e)^2)^(1/2)/f
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(91) = 182$.
time $= 0.51$, size $= 521$, normalized size $= 5.73$

$$-\frac{3\sqrt{a}\log\left(e^{(-fx-e)}+1\right)-3\sqrt{a}\log\left(e^{(-fx-e)}-1\right)-\frac{2\left(9\sqrt{a}e^{(-fx-e)}-8\sqrt{a}e^{(-3fx-3e)}+3\sqrt{a}e^{(-5fx-5e)}\right)}{3e^{(-2fx-2e)}-3e^{(-4fx-4e)}+e^{(-6fx-6e)}-1}}{12f}+\frac{3\sqrt{a}\log\left(e^{(-fx-e)}+1\right)-3\sqrt{a}\log\left(e^{(-fx-e)}-1\right)+\frac{2\left(3\sqrt{a}e^{(-fx-e)}-8\sqrt{a}e^{(-3fx-3e)}+9\sqrt{a}e^{(-5fx-5e)}\right)}{3e^{(-2fx-2e)}-3e^{(-4fx-4e)}+e^{(-6fx-6e)}-1}}{12f}+\frac{\sqrt{a}e^{(-3fx-3e)}}{f\left(3e^{(-2fx-2e)}-3e^{(-4fx-4e)}+e^{(-6fx-6e)}-1\right)}-\frac{33\sqrt{a}e^{(-2fx-2e)}-40\sqrt{a}e^{(-4fx-4e)}+15\sqrt{a}e^{(-6fx-6e)}-6\sqrt{a}}{12f\left(e^{(-fx-e)}-3e^{(-3fx-3e)}+3e^{(-5fx-5e)}-e^{(-7fx-7e)}\right)}+\frac{15\sqrt{a}e^{(-fx-e)}-40\sqrt{a}e^{(-3fx-3e)}+33\sqrt{a}e^{(-5fx-5e)}-6\sqrt{a}e^{(-7fx-7e)}}{12f\left(3e^{(-2fx-2e)}-3e^{(-4fx-4e)}+e^{(-6fx-6e)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*sqrt(a)*log(e^(-f*x - e) + 1) - 3*sqrt(a)*log(e^(-f*x - e) - 1) -
2*(9*sqrt(a)*e^(-f*x - e) - 8*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-5*f*
x - 5*e))/(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x - 4*e) + e^(-6*f*x - 6*e) - 1))
/f + 1/12*(3*sqrt(a)*log(e^(-f*x - e) + 1) - 3*sqrt(a)*log(e^(-f*x - e) - 1
) + 2*(3*sqrt(a)*e^(-f*x - e) - 8*sqrt(a)*e^(-3*f*x - 3*e) + 9*sqrt(a)*e^(-
5*f*x - 5*e))/(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x - 4*e) + e^(-6*f*x - 6*e) -
 1))/f + sqrt(a)*e^(-3*f*x - 3*e)/(f*(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x - 4*
e) + e^(-6*f*x - 6*e) - 1)) - 1/12*(33*sqrt(a)*e^(-2*f*x - 2*e) - 40*sqrt(a
)*e^(-4*f*x - 4*e) + 15*sqrt(a)*e^(-6*f*x - 6*e) - 6*sqrt(a))/(f*(e^(-f*x -
 e) - 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) - e^(-7*f*x - 7*e))) + 1/12*(
15*sqrt(a)*e^(-f*x - e) - 40*sqrt(a)*e^(-3*f*x - 3*e) + 33*sqrt(a)*e^(-5*f*
x - 5*e) - 6*sqrt(a)*e^(-7*f*x - 7*e))/(f*(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x
 - 4*e) + e^(-6*f*x - 6*e) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(83) = 166$.
time $= 0.48$, size $= 885$, normalized size $= 9.73$

$$\frac{\left(24\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^{7}+3e^{(fx+e)}\sinh\left(fx+e\right)^{8}+12\left(7\cosh\left(fx+e\right)^{2}-3\right)e^{(fx+e)}\sinh\left(fx+e\right)^{6}+24\left(7\cosh\left(fx+e\right)^{3}-9\cosh\left(fx+e\right)\right)e^{(fx+e)}\sinh\left(fx+e\right)^{5}+10\left(21\cosh\left(fx+e\right)^{4}-54\cosh\left(fx+e\right)^{2}+5\right)e^{(fx+e)}\sinh\left(fx+e\right)^{4}+8\left(21\cosh\left(fx+e\right)^{5}-90\cosh\left(fx+e\right)^{3}+25\cosh\left(fx+e\right)\right)e^{(fx+e)}\sinh\left(fx+e\right)^{3}+12\left(7\cosh\left(fx+e\right)^{6}-45\cosh\left(fx+e\right)^{4}+25\cosh\left(fx+e\right)^{2}-3\right)e^{(fx+e)}\sinh\left(fx+e\right)^{2}+8\left(3\cosh\left(fx+e\right)^{7}-27\cosh\left(fx+e\right)^{5}+25\cosh\left(fx+e\right)^{3}-9\cosh\left(fx+e\right)\right)e^{(fx+e)}\sinh\left(fx+e\right)+\left(3\cosh\left(fx+e\right)^{8}-36\cosh\left(fx+e\right)^{6}+50\cosh\left(fx+e\right)^{4}-36\cosh\left(fx+e\right)^{2}+3\right)e^{(fx+e)}\right)\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}\,e^{(-fx-e)}}{6\left(f\cosh\left(fx+e\right)^{7}+\left(fe^{(2fx+2e)}+f\right)\sinh\left(fx+e\right)^{7}+7\left(f\cosh\left(fx+e\right)e^{(2fx+2e)}+f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^{6}-3f\cosh\left(fx+e\right)^{5}+3\left(7f\cosh\left(fx+e\right)^{2}+\left(7f\cosh\left(fx+e\right)^{2}-f\right)e^{(2fx+2e)}-f\right)\sinh\left(fx+e\right)^{5}+5\left(7f\cosh\left(fx+e\right)^{3}-3f\cosh\left(fx+e\right)+\left(7f\cosh\left(fx+e\right)^{3}-3f\cosh\left(fx+e\right)\right)e^{(2fx+2e)}\right)\sinh\left(fx+e\right)^{4}+3f\cosh\left(fx+e\right)^{3}+\left(35f\cosh\left(fx+e\right)^{4}-30f\cosh\left(fx+e\right)^{2}+\left(35f\cosh\left(fx+e\right)^{4}-30f\cosh\left(fx+e\right)^{2}+3f\right)e^{(2fx+2e)}+3f\right)\sinh\left(fx+e\right)^{3}+3\left(7f\cosh\left(fx+e\right)^{5}-10f\cosh\left(fx+e\right)^{3}+3f\cosh\left(fx+e\right)+\left(7f\cosh\left(fx+e\right)^{5}-10f\cosh\left(fx+e\right)^{3}+3f\cosh\left(fx+e\right)\right)e^{(2fx+2e)}\right)\sinh\left(fx+e\right)^{2}-f\cosh\left(fx+e\right)+\left(f\cosh\left(fx+e\right)^{7}-3f\cosh\left(fx+e\right)^{5}+3f\cosh\left(fx+e\right)^{3}-f\cosh\left(fx+e\right)\right)e^{(2fx+2e)}+\left(7f\cosh\left(fx+e\right)^{6}-15f\cosh\left(fx+e\right)^{4}+9f\cosh\left(fx+e\right)^{2}+\left(7f\cosh\left(fx+e\right)^{6}-15f\cosh\left(fx+e\right)^{4}+9f\cosh\left(fx+e\right)^{2}-f\right)e^{(2fx+2e)}-f\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(24*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + 3*e^(f*x + e)*sinh(f*x
+ e)^8 + 12*(7*cosh(f*x + e)^2 - 3)*e^(f*x + e)*sinh(f*x + e)^6 + 24*(7*cos
h(f*x + e)^3 - 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 10*(21*cosh(f
```

```
*x + e)^4 - 54*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^4 + 8*(21*cos
h(f*x + e)^5 - 90*cosh(f*x + e)^3 + 25*cosh(f*x + e))*e^(f*x + e)*sinh(f*x
+ e)^3 + 12*(7*cosh(f*x + e)^6 - 45*cosh(f*x + e)^4 + 25*cosh(f*x + e)^2 -
3)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(3*cosh(f*x + e)^7 - 27*cosh(f*x + e)^5
+ 25*cosh(f*x + e)^3 - 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (3*cosh
(f*x + e)^8 - 36*cosh(f*x + e)^6 + 50*cosh(f*x + e)^4 - 36*cosh(f*x + e)^2
+ 3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x
 - e)/(f*cosh(f*x + e)^7 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^7 + 7*(f*c
osh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^6 - 3*f*cosh(
f*x + e)^5 + 3*(7*f*cosh(f*x + e)^2 + (7*f*cosh(f*x + e)^2 - f)*e^(2*f*x +
2*e) - f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e) + (7
*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^4 +
3*f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 - 30*f*cosh(f*x + e)^2 + (35*f*
cosh(f*x + e)^4 - 30*f*cosh(f*x + e)^2 + 3*f)*e^(2*f*x + 2*e) + 3*f)*sinh(f
*x + e)^3 + 3*(7*f*cosh(f*x + e)^5 - 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x +
e) + (7*f*cosh(f*x + e)^5 - 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*
f*x + 2*e))*sinh(f*x + e)^2 - f*cosh(f*x + e) + (f*cosh(f*x + e)^7 - 3*f*co
sh(f*x + e)^5 + 3*f*cosh(f*x + e)^3 - f*cosh(f*x + e))*e^(2*f*x + 2*e) + (7
*f*cosh(f*x + e)^6 - 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 + (7*f*cosh
(f*x + e)^6 - 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 - f)*e^(2*f*x + 2*
e) - f)*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\,\coth^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x)**4, x)
```

Giac [**A**]
time = 0.42, size = 74, normalized size = 0.81

$$-\frac{\sqrt{a}\left(\frac{8\left(3\left(e^{(fx+e)}-e^{(-fx-e)}\right)^2+2\right)}{\left(e^{(fx+e)}-e^{(-fx-e)}\right)^3}-3\,e^{(fx+e)}+3\,e^{(-fx-e)}\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(a)*(8*(3*(e^(f*x + e) - e^(-f*x - e))^2 + 2)/(e^(f*x + e) - e^(-f
*x - e))^3 - 3*e^(f*x + e) + 3*e^(-f*x - e))/f
```

Mupad [B]
time = 0.94, size = 281, normalized size = 3.09

$$-\frac{\left(\frac{1}{f}-\frac{{\mathrm{e}}^{2\,e+2\,f\,x}}{f}\right)\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{{\mathrm{e}}^{2\,e+2\,f\,x}+1}-\frac{8\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{f\,\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{16\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{3\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^2\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{16\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{3\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^3\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] - ((1/f - exp(2*e + 2*f*x)/f)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)
^(1/2))/(exp(2*e + 2*f*x) + 1) - (8*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2
 - exp(- e - f*x)/2)^2)^(1/2))/(f*(exp(2*e + 2*f*x) - 1)*(exp(e + f*x) + ex
p(3*e + 3*f*x))) - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e -
f*x)/2)^2)^(1/2))/(3*f*(exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3
*f*x))) - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2
)^(1/2))/(3*f*(exp(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

3.436 $\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=124

$$-\frac{3\sqrt{a\cosh^2(e+fx)}\,\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)}{f}-\frac{\sqrt{a\cosh^2(e+fx)}\,\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)}{f}-\sqrt{a\cosh^2(}$$

```
[Out] -3*csch(f*x+e)*sech(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f-csch(f*x+e)^3*sech(f*x
+e)*(a*cosh(f*x+e)^2)^(1/2)/f-1/5*csch(f*x+e)^5*sech(f*x+e)*(a*cosh(f*x+e)^
2)^(1/2)/f+(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3286, 2670, 276}

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}^5(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{5f}-\frac{\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{3\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^6*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-3*Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f - (Sqrt[a*Cosh[e
 + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x])/f - (Sqrt[a*Cosh[e + f*x]^2]*Csch
[e + f*x]^5*Sech[e + f*x])/(5*f) + (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/
f
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\cosh^2(e+fx)}\coth^6(e+fx)\,dx$$

$$= \left(\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\int \cosh(e+fx)\coth^6(e+$$

$$= -\frac{\left(i\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx, x,\right.}{f}$$

$$= -\frac{\left(i\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x}\right.\right.}{f}$$

$$= -\frac{3\sqrt{a\cosh^2(e+fx)}\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)}{f} - \frac{\sqrt{a\cosh^2(}}{}$$

Mathematica [A]

time = 0.17, size = 67, normalized size = 0.54

$$\frac{\sqrt{a\cosh^2(e+fx)}\,(-182+235\cosh(2(e+fx))-90\cosh(4(e+fx))+5\cosh(6(e+fx)))\operatorname{csch}^5(e+fx)\operatorname{sech}(e+fx)}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^6*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*(-182 + 235*Cosh[2*(e + f*x)] - 90*Cosh[4*(e + f*x
)] + 5*Cosh[6*(e + f*x)])*Csch[e + f*x]^5*Sech[e + f*x])/(160*f)
```

Maple [A]

time = 1.32, size = 65, normalized size = 0.52

method	result
default	$\frac{\cosh(fx+e)a\left(5\left(\sinh^6(fx+e)\right)-15\left(\sinh^4(fx+e)\right)-5\left(\sinh^2(fx+e)\right)-1\right)}{5\sinh(fx+e)^5\sqrt{a\left(\cosh^2(fx+e)\right)}\,f}$

risch	$\frac{\sqrt{\left(e^{2fx+2e}+1\right)^{2} a\, e^{-2fx-2e}}\, e^{2fx+2e}}{2f\left(e^{2fx+2e}+1\right)}-\frac{\sqrt{\left(e^{2fx+2e}+1\right)^{2} a\, e^{-2fx-2e}}}{2f\left(e^{2fx+2e}+1\right)}-\frac{2\left(15\, e^{8fx+8e}-40\, e^{6fx+6e}+66\, e^{4fx+4e}-\right.}{5\left(e^{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*cosh(f*x+e)*a*(5*sinh(f*x+e)^6-15*sinh(f*x+e)^4-5*sinh(f*x+e)^2-1)/sinh
(f*x+e)^5/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1126 vs. $2(125) = 250$.
time = 0.55, size = 1126, normalized size = 9.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/320*(105*sqrt(a)*log(e^(-f*x - e) + 1) - 105*sqrt(a)*log(e^(-f*x - e) -
1) - 2*(375*sqrt(a)*e^(-f*x - e) - 790*sqrt(a)*e^(-3*f*x - 3*e) + 896*sqrt(
a)*e^(-5*f*x - 5*e) - 490*sqrt(a)*e^(-7*f*x - 7*e) + 105*sqrt(a)*e^(-9*f*x
- 9*e))/(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5
*e^(-8*f*x - 8*e) + e^(-10*f*x - 10*e) - 1))/f + 1/320*(105*sqrt(a)*log(e^(
-f*x - e) + 1) - 105*sqrt(a)*log(e^(-f*x - e) - 1) + 2*(105*sqrt(a)*e^(-f*x
 - e) - 490*sqrt(a)*e^(-3*f*x - 3*e) + 896*sqrt(a)*e^(-5*f*x - 5*e) - 790*s
qrt(a)*e^(-7*f*x - 7*e) + 375*sqrt(a)*e^(-9*f*x - 9*e))/(5*e^(-2*f*x - 2*e)
 - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5*e^(-8*f*x - 8*e) + e^(-10*
f*x - 10*e) - 1))/f + 1/256*(15*sqrt(a)*log(e^(-f*x - e) + 1) - 15*sqrt(a)*
log(e^(-f*x - e) - 1) + 2*(15*sqrt(a)*e^(-f*x - e) + 250*sqrt(a)*e^(-3*f*x
- 3*e) - 128*sqrt(a)*e^(-5*f*x - 5*e) + 70*sqrt(a)*e^(-7*f*x - 7*e) - 15*sq
rt(a)*e^(-9*f*x - 9*e))/(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-
6*f*x - 6*e) - 5*e^(-8*f*x - 8*e) + e^(-10*f*x - 10*e) - 1))/f - 1/256*(15*
sqrt(a)*log(e^(-f*x - e) + 1) - 15*sqrt(a)*log(e^(-f*x - e) - 1) + 2*(15*sq
rt(a)*e^(-f*x - e) - 70*sqrt(a)*e^(-3*f*x - 3*e) + 128*sqrt(a)*e^(-5*f*x -
5*e) - 250*sqrt(a)*e^(-7*f*x - 7*e) - 15*sqrt(a)*e^(-9*f*x - 9*e))/(5*e^(-2
*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5*e^(-8*f*x - 8*e
) + e^(-10*f*x - 10*e) - 1))/f + 2*sqrt(a)*e^(-5*f*x - 5*e)/(f*(5*e^(-2*f*x
 - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5*e^(-8*f*x - 8*e) +
e^(-10*f*x - 10*e) - 1)) - 1/640*(2895*sqrt(a)*e^(-2*f*x - 2*e) - 7110*sqrt
(a)*e^(-4*f*x - 4*e) + 8064*sqrt(a)*e^(-6*f*x - 6*e) - 4410*sqrt(a)*e^(-8*f
*x - 8*e) + 945*sqrt(a)*e^(-10*f*x - 10*e) - 320*sqrt(a))/(f*(e^(-f*x - e)
- 5*e^(-3*f*x - 3*e) + 10*e^(-5*f*x - 5*e) - 10*e^(-7*f*x - 7*e) + 5*e^(-9*
f*x - 9*e) - e^(-11*f*x - 11*e))) + 1/640*(945*sqrt(a)*e^(-f*x - e) - 4410*
sqrt(a)*e^(-3*f*x - 3*e) + 8064*sqrt(a)*e^(-5*f*x - 5*e) - 7110*sqrt(a)*e^(
```

```
-7*f*x - 7*e) + 2895*sqrt(a)*e^(-9*f*x - 9*e) - 320*sqrt(a)*e^(-11*f*x - 11
*e))/(f*(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5
*e^(-8*f*x - 8*e) + e^(-10*f*x - 10*e) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1696 vs. $2(114) = 228$.
time = 0.50, size = 1696, normalized size = 13.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(60*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^11 + 5*e^(f*x + e)*sinh(f*
x + e)^12 + 30*(11*cosh(f*x + e)^2 - 3)*e^(f*x + e)*sinh(f*x + e)^10 + 100*
(11*cosh(f*x + e)^3 - 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^9 + 5*(495
*cosh(f*x + e)^4 - 810*cosh(f*x + e)^2 + 47)*e^(f*x + e)*sinh(f*x + e)^8 +
40*(99*cosh(f*x + e)^5 - 270*cosh(f*x + e)^3 + 47*cosh(f*x + e))*e^(f*x + e
)*sinh(f*x + e)^7 + 28*(165*cosh(f*x + e)^6 - 675*cosh(f*x + e)^4 + 235*cos
h(f*x + e)^2 - 13)*e^(f*x + e)*sinh(f*x + e)^6 + 8*(495*cosh(f*x + e)^7 - 2
835*cosh(f*x + e)^5 + 1645*cosh(f*x + e)^3 - 273*cosh(f*x + e))*e^(f*x + e)
*sinh(f*x + e)^5 + 5*(495*cosh(f*x + e)^8 - 3780*cosh(f*x + e)^6 + 3290*cos
h(f*x + e)^4 - 1092*cosh(f*x + e)^2 + 47)*e^(f*x + e)*sinh(f*x + e)^4 + 20*
(55*cosh(f*x + e)^9 - 540*cosh(f*x + e)^7 + 658*cosh(f*x + e)^5 - 364*cosh(
f*x + e)^3 + 47*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 10*(33*cosh(f*
x + e)^10 - 405*cosh(f*x + e)^8 + 658*cosh(f*x + e)^6 - 546*cosh(f*x + e)^4
 + 141*cosh(f*x + e)^2 - 9)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(15*cosh(f*x +
e)^11 - 225*cosh(f*x + e)^9 + 470*cosh(f*x + e)^7 - 546*cosh(f*x + e)^5 + 2
35*cosh(f*x + e)^3 - 45*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (5*cosh(
f*x + e)^12 - 90*cosh(f*x + e)^10 + 235*cosh(f*x + e)^8 - 364*cosh(f*x + e)
^6 + 235*cosh(f*x + e)^4 - 90*cosh(f*x + e)^2 + 5)*e^(f*x + e))*sqrt(a*e^(4
*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)^11 + (
f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^11 + 11*(f*cosh(f*x + e)*e^(2*f*x + 2*
e) + f*cosh(f*x + e))*sinh(f*x + e)^10 - 5*f*cosh(f*x + e)^9 + 5*(11*f*cosh
(f*x + e)^2 + (11*f*cosh(f*x + e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e)
^9 + 15*(11*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e) + (11*f*cosh(f*x + e)^3 -
 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*f*cosh(f*x + e)^7
 + 10*(33*f*cosh(f*x + e)^4 - 18*f*cosh(f*x + e)^2 + (33*f*cosh(f*x + e)^4
- 18*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^7 + 14*(33*f
*cosh(f*x + e)^5 - 30*f*cosh(f*x + e)^3 + 5*f*cosh(f*x + e) + (33*f*cosh(f*
x + e)^5 - 30*f*cosh(f*x + e)^3 + 5*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(
f*x + e)^6 - 10*f*cosh(f*x + e)^5 + 2*(231*f*cosh(f*x + e)^6 - 315*f*cosh(f
*x + e)^4 + 105*f*cosh(f*x + e)^2 + (231*f*cosh(f*x + e)^6 - 315*f*cosh(f*x
 + e)^4 + 105*f*cosh(f*x + e)^2 - 5*f)*e^(2*f*x + 2*e) - 5*f)*sinh(f*x + e)
^5 + 10*(33*f*cosh(f*x + e)^7 - 63*f*cosh(f*x + e)^5 + 35*f*cosh(f*x + e)^3
```

```
 - 5*f*cosh(f*x + e) + (33*f*cosh(f*x + e)^7 - 63*f*cosh(f*x + e)^5 + 35*f*
cosh(f*x + e)^3 - 5*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 5*f
*cosh(f*x + e)^3 + 5*(33*f*cosh(f*x + e)^8 - 84*f*cosh(f*x + e)^6 + 70*f*co
sh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + (33*f*cosh(f*x + e)^8 - 84*f*cosh(f*
x + e)^6 + 70*f*cosh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e)
 + f)*sinh(f*x + e)^3 + 5*(11*f*cosh(f*x + e)^9 - 36*f*cosh(f*x + e)^7 + 42
*f*cosh(f*x + e)^5 - 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (11*f*cosh(
f*x + e)^9 - 36*f*cosh(f*x + e)^7 + 42*f*cosh(f*x + e)^5 - 20*f*cosh(f*x +
e)^3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 - f*cosh(f*x + e
) + (f*cosh(f*x + e)^11 - 5*f*cosh(f*x + e)^9 + 10*f*cosh(f*x + e)^7 - 10*f
*cosh(f*x + e)^5 + 5*f*cosh(f*x + e)^3 - f*cosh(f*x + e))*e^(2*f*x + 2*e) +
 (11*f*cosh(f*x + e)^10 - 45*f*cosh(f*x + e)^8 + 70*f*cosh(f*x + e)^6 - 50*
f*cosh(f*x + e)^4 + 15*f*cosh(f*x + e)^2 + (11*f*cosh(f*x + e)^10 - 45*f*co
sh(f*x + e)^8 + 70*f*cosh(f*x + e)^6 - 50*f*cosh(f*x + e)^4 + 15*f*cosh(f*x
 + e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**6*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**A**]
time = 0.45, size = 96, normalized size = 0.77

$$-\frac{\sqrt{a}\left(\frac{4\left(15\left(e^{(fx+e)}-e^{(-fx-e)}\right)^4+20\left(e^{(fx+e)}-e^{(-fx-e)}\right)^2+16\right)}{\left(e^{(fx+e)}-e^{(-fx-e)}\right)^5}-5\,e^{(fx+e)}+5\,e^{(-fx-e)}\right)}{10\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(a)*(4*(15*(e^(f*x + e) - e^(-f*x - e))^4 + 20*(e^(f*x + e) - e^(
-f*x - e))^2 + 16)/(e^(f*x + e) - e^(-f*x - e))^5 - 5*e^(f*x + e) + 5*e^(-f
*x - e))/f
```

Mupad [**B**]
time = 0.91, size = 427, normalized size = 3.44

$$-\frac{\left(\frac{1}{f}-\frac{\mathrm{e}^{2\,e+2\,f\,x}}{f}\right)\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{\mathrm{e}^{2\,e+2\,f\,x}+1}-\frac{12\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{f\,\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{16\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)}^2\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{144\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{5\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)}^3\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{128\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{5\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)}^4\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{64\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{5\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)}^5\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^6*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] - ((1/f - exp(2*e + 2*f*x)/f)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)
^(1/2))/(exp(2*e + 2*f*x) + 1) - (12*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/
2 - exp(- e - f*x)/2)^2)^(1/2))/(f*(exp(2*e + 2*f*x) - 1)*(exp(e + f*x) + e
xp(3*e + 3*f*x))) - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e -
 f*x)/2)^2)^(1/2))/(f*(exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3*
f*x))) - (144*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2
)^(1/2))/(5*f*(exp(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) -
 (128*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))
/(5*f*(exp(2*e + 2*f*x) - 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (64*exp
(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*f*(ex
p(2*e + 2*f*x) - 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

3.437 $\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}+\frac{2a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -1/5*a^2/f/(a*cosh(f*x+e)^2)^(5/2)+2/3*a/f/(a*cosh(f*x+e)^2)^(3/2)-1/f/(a*c
osh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3284, 16, 45}

$$-\frac{a^2}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}+\frac{2a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -1/5*a^2/(f*(a*Cosh[e + f*x]^2)^(5/2)) + (2*a)/(3*f*(a*Cosh[e + f*x]^2)^(3/
2)) - 1/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx &= \int \frac{\tanh^5(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= \frac{a^3\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{7/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}}-\frac{2}{a(ax)^{5/2}}+\frac{1}{a^2(ax)^{3/2}}\right)\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= -\frac{a^2}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}+\frac{2a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 43, normalized size = 0.65

$$\frac{-15+10\text{sech}^2(e+fx)-3\text{sech}^4(e+fx)}{15f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-15 + 10*Sech[e + f*x]^2 - 3*Sech[e + f*x]^4)/(15*f*Sqrt[a*Cosh[e + f*x]^2
])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.26, size = 41, normalized size = 0.62

method	result	size

default	$\frac{\text{`}int/indef0\text{`}\left(\frac{\sinh^{5}(fx+e)}{\cosh(fx+e)^{6}\sqrt{a\left(\cosh^{2}(fx+e)\right)}},\sinh(fx+e)\right)}{f}$	41
risch	$-\frac{2\left(15\,\mathrm{e}^{8fx+8e}+20\,\mathrm{e}^{6fx+6e}+58\,\mathrm{e}^{4fx+4e}+20\,\mathrm{e}^{2fx+2e}+15\right)}{15\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\,\left(\mathrm{e}^{2fx+2e}+1\right)^{4}f}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 'int/indef0'(sinh(f*x+e)^5/cosh(f*x+e)^6/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e
))/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(59) = 118$.
time = 0.54, size = 476, normalized size = 7.21

$$-\frac{2\,e^{(-fx-e)}}{\left(5\sqrt{a}\,e^{(-2fx-2e)}+10\sqrt{a}\,e^{(-4fx-4e)}+10\sqrt{a}\,e^{(-6fx-6e)}+5\sqrt{a}\,e^{(-8fx-8e)}+\sqrt{a}\,e^{(-10fx-10e)}+\sqrt{a}\right)f}-\frac{8\,e^{(-3fx-3e)}}{3\left(5\sqrt{a}\,e^{(-2fx-2e)}+10\sqrt{a}\,e^{(-4fx-4e)}+10\sqrt{a}\,e^{(-6fx-6e)}+5\sqrt{a}\,e^{(-8fx-8e)}+\sqrt{a}\,e^{(-10fx-10e)}+\sqrt{a}\right)f}-\frac{116\,e^{(-5fx-5e)}}{15\left(5\sqrt{a}\,e^{(-2fx-2e)}+10\sqrt{a}\,e^{(-4fx-4e)}+10\sqrt{a}\,e^{(-6fx-6e)}+5\sqrt{a}\,e^{(-8fx-8e)}+\sqrt{a}\,e^{(-10fx-10e)}+\sqrt{a}\right)f}-\frac{8\,e^{(-7fx-7e)}}{3\left(5\sqrt{a}\,e^{(-2fx-2e)}+10\sqrt{a}\,e^{(-4fx-4e)}+10\sqrt{a}\,e^{(-6fx-6e)}+5\sqrt{a}\,e^{(-8fx-8e)}+\sqrt{a}\,e^{(-10fx-10e)}+\sqrt{a}\right)f}-\frac{2\,e^{(-9fx-9e)}}{\left(5\sqrt{a}\,e^{(-2fx-2e)}+10\sqrt{a}\,e^{(-4fx-4e)}+10\sqrt{a}\,e^{(-6fx-6e)}+5\sqrt{a}\,e^{(-8fx-8e)}+\sqrt{a}\,e^{(-10fx-10e)}+\sqrt{a}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^(-f*x - e)/((5*sqrt(a)*e^(-2*f*x - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e)
+ 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10
*f*x - 10*e) + sqrt(a))*f) - 8/3*e^(-3*f*x - 3*e)/((5*sqrt(a)*e^(-2*f*x - 2
*e) + 10*sqrt(a)*e^(-4*f*x - 4*e) + 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)
*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x - 10*e) + sqrt(a))*f) - 116/15*e^(-5
*f*x - 5*e)/((5*sqrt(a)*e^(-2*f*x - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e) + 10
*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x
 - 10*e) + sqrt(a))*f) - 8/3*e^(-7*f*x - 7*e)/((5*sqrt(a)*e^(-2*f*x - 2*e)
+ 10*sqrt(a)*e^(-4*f*x - 4*e) + 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(
-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x - 10*e) + sqrt(a))*f) - 2*e^(-9*f*x - 9*
e)/((5*sqrt(a)*e^(-2*f*x - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e) + 10*sqrt(a)*
e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x - 10*e)
+ sqrt(a))*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. $2(56) = 112$.
time = 0.59, size = 1387, normalized size = 21.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] -2/15*(135*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^8 + 15*e^(f*x + e)*sinh(
f*x + e)^9 + 20*(27*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^7 + 140*
(9*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^6 + 2*(945*co
sh(f*x + e)^4 + 210*cosh(f*x + e)^2 + 29)*e^(f*x + e)*sinh(f*x + e)^5 + 10*
(189*cosh(f*x + e)^5 + 70*cosh(f*x + e)^3 + 29*cosh(f*x + e))*e^(f*x + e)*s
inh(f*x + e)^4 + 20*(63*cosh(f*x + e)^6 + 35*cosh(f*x + e)^4 + 29*cosh(f*x
+ e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 20*(27*cosh(f*x + e)^7 + 21*cosh(
f*x + e)^5 + 29*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(27*cosh(f*x + e)^8 + 28*cosh(f*x + e)^6 + 58*cosh(f*x + e)^4 + 12*
cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e) + (15*cosh(f*x + e)^9 + 20*c
osh(f*x + e)^7 + 58*cosh(f*x + e)^5 + 20*cosh(f*x + e)^3 + 15*cosh(f*x + e)
)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a*f*cosh(f*x + e)^10 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^10 + 5
*a*f*cosh(f*x + e)^8 + 10*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x
 + e))*sinh(f*x + e)^9 + 5*(9*a*f*cosh(f*x + e)^2 + a*f + (9*a*f*cosh(f*x +
 e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 + 40
*(3*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e)^3 + a*f*
cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^7 + 10*(21*a*f*cosh(f*x + e)^
4 + 14*a*f*cosh(f*x + e)^2 + a*f + (21*a*f*cosh(f*x + e)^4 + 14*a*f*cosh(f*
x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 10*a*f*cosh(f*x + e)^4 +
 4*(63*a*f*cosh(f*x + e)^5 + 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e)
+ (63*a*f*cosh(f*x + e)^5 + 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e))*
e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 10*(21*a*f*cosh(f*x + e)^6 + 35*a*f*cosh
(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 + a*f + (21*a*f*cosh(f*x + e)^6 + 35*a
*f*cosh(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*
x + e)^4 + 5*a*f*cosh(f*x + e)^2 + 40*(3*a*f*cosh(f*x + e)^7 + 7*a*f*cosh(f
*x + e)^5 + 5*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e
)^7 + 7*a*f*cosh(f*x + e)^5 + 5*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^
(2*f*x + 2*e))*sinh(f*x + e)^3 + 5*(9*a*f*cosh(f*x + e)^8 + 28*a*f*cosh(f*x
 + e)^6 + 30*a*f*cosh(f*x + e)^4 + 12*a*f*cosh(f*x + e)^2 + a*f + (9*a*f*co
sh(f*x + e)^8 + 28*a*f*cosh(f*x + e)^6 + 30*a*f*cosh(f*x + e)^4 + 12*a*f*co
sh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x
 + e)^10 + 5*a*f*cosh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 + 10*a*f*cosh(f*x
 + e)^4 + 5*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 10*(a*f*cosh(f*x +
 e)^9 + 4*a*f*cosh(f*x + e)^7 + 6*a*f*cosh(f*x + e)^5 + 4*a*f*cosh(f*x + e)
^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^9 + 4*a*f*cosh(f*x + e)^7 + 6*a
*f*cosh(f*x + e)^5 + 4*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{5}\left(e+fx\right)}{\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**5/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**B**]
time = 0.89, size = 381, normalized size = 5.77

$$\frac{32\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{3\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^2\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{4\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{a\,f\,\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{352\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{15\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^3\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}+\frac{128\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{5\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^4\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{64\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{5\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^5\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] (32*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(
3*a*f*(exp(2*e + 2*f*x) + 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (4*exp(
3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(a*f*(exp
(2*e + 2*f*x) + 1)*(exp(e + f*x) + exp(3*e + 3*f*x))) - (352*exp(3*e + 3*f*
x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(15*a*f*(exp(2*e +
2*f*x) + 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) + (128*exp(3*e + 3*f*x)*(a
 + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a*f*(exp(2*e + 2*f*x)
 + 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (64*exp(3*e + 3*f*x)*(a + a*(e
xp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a*f*(exp(2*e + 2*f*x) + 1)^5
*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

3.438 $\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$\frac{a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] 1/3*a/f/(a*cosh(f*x+e)^2)^(3/2)-1/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3284, 16, 45}

$$\frac{a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] a/(3*f*(a*Cosh[e + f*x]^2)^(3/2)) - 1/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
```

```
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx &= \int \frac{\tanh^3(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\ &= -\frac{a^2\text{Subst}\left(\int \frac{1-x}{(ax)^{5/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\ &= -\frac{a^2\text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}}-\frac{1}{a(ax)^{3/2}}\right)dx, x, \cosh^2(e+fx)\right)}{2f}\\ &= \frac{a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 31, normalized size = 0.74

$$\frac{-3+\operatorname{sech}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
[Out] (-3 + Sech[e + f*x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.12, size = 41, normalized size = 0.98

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\sinh^3(fx+e)}{\cosh(fx+e)^4\sqrt{a\left(\cosh^2(fx+e)\right)}}, \sinh(fx+e)\right)}{f}$	41

risch	$-\frac{2\left(3\,\mathrm{e}^{4fx+4e}+2\,\mathrm{e}^{2fx+2e}+3\right)}{3\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\,\left(\mathrm{e}^{2fx+2e}+1\right)^{2}f}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 'int/indef0'(sinh(f*x+e)^3/cosh(f*x+e)^4/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e
))/f
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(38) = 76$.
time = 0.52, size = 196, normalized size = 4.67

$$-\frac{2\,e^{(-fx-e)}}{\left(3\,\sqrt{a}\,e^{(-2\,fx-2\,e)}+3\,\sqrt{a}\,e^{(-4\,fx-4\,e)}+\sqrt{a}\,e^{(-6\,fx-6\,e)}+\sqrt{a}\right)f}-\frac{4\,e^{(-3\,fx-3\,e)}}{3\left(3\,\sqrt{a}\,e^{(-2\,fx-2\,e)}+3\,\sqrt{a}\,e^{(-4\,fx-4\,e)}+\sqrt{a}\,e^{(-6\,fx-6\,e)}+\sqrt{a}\right)f}-\frac{2\,e^{(-5\,fx-5\,e)}}{\left(3\,\sqrt{a}\,e^{(-2\,fx-2\,e)}+3\,\sqrt{a}\,e^{(-4\,fx-4\,e)}+\sqrt{a}\,e^{(-6\,fx-6\,e)}+\sqrt{a}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^(-f*x - e)/((3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e) +
 sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a))*f) - 4/3*e^(-3*f*x - 3*e)/((3*sqrt(a)*
e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a)*e^(-6*f*x - 6*e) +
sqrt(a))*f) - 2*e^(-5*f*x - 5*e)/((3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e
^(-4*f*x - 4*e) + sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a))*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(36) = 72$.
time = 0.47, size = 641, normalized size = 15.26

$$-\frac{2\left(15\,\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^{4}+3\,e^{(fx+e)}\sinh\left(fx+e\right)^{5}+2\left(15\,\cosh\left(fx+e\right)^{2}+1\right)e^{(fx+e)}\sinh\left(fx+e\right)^{3}+6\left(5\,\cosh\left(fx+e\right)^{3}+\cosh\left(fx+e\right)\right)e^{(fx+e)}\sinh\left(fx+e\right)^{2}+3\left(5\,\cosh\left(fx+e\right)^{4}+2\,\cosh\left(fx+e\right)^{2}+1\right)e^{(fx+e)}\sinh\left(fx+e\right)+\left(3\,\cosh\left(fx+e\right)^{5}+2\,\cosh\left(fx+e\right)^{3}+3\,\cosh\left(fx+e\right)\right)e^{(fx+e)}\right)\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\,e^{(-fx-e)}}{3\left(af\cosh\left(fx+e\right)^{6}+\left(afe^{(2\,fx+2\,e)}+af\right)\sinh\left(fx+e\right)^{6}+3\,af\cosh\left(fx+e\right)^{4}+6\left(af\cosh\left(fx+e\right)e^{(2\,fx+2\,e)}+af\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^{5}+3\left(5\,af\cosh\left(fx+e\right)^{2}+af+\left(5\,af\cosh\left(fx+e\right)^{2}+af\right)e^{(2\,fx+2\,e)}\right)\sinh\left(fx+e\right)^{4}+3\,af\cosh\left(fx+e\right)^{2}+4\left(5\,af\cosh\left(fx+e\right)^{3}+3\,af\cosh\left(fx+e\right)+\left(5\,af\cosh\left(fx+e\right)^{3}+3\,af\cosh\left(fx+e\right)\right)e^{(2\,fx+2\,e)}\right)\sinh\left(fx+e\right)^{3}+3\left(5\,af\cosh\left(fx+e\right)^{4}+6\,af\cosh\left(fx+e\right)^{2}+af+\left(5\,af\cosh\left(fx+e\right)^{4}+6\,af\cosh\left(fx+e\right)^{2}+af\right)e^{(2\,fx+2\,e)}\right)\sinh\left(fx+e\right)^{2}+af+\left(af\cosh\left(fx+e\right)^{6}+3\,af\cosh\left(fx+e\right)^{4}+3\,af\cosh\left(fx+e\right)^{2}+af\right)e^{(2\,fx+2\,e)}+6\left(af\cosh\left(fx+e\right)^{5}+2\,af\cosh\left(fx+e\right)^{3}+af\cosh\left(fx+e\right)+\left(af\cosh\left(fx+e\right)^{5}+2\,af\cosh\left(fx+e\right)^{3}+af\cosh\left(fx+e\right)\right)e^{(2\,fx+2\,e)}\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(15*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + 3*e^(f*x + e)*sinh(f*x
 + e)^5 + 2*(15*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 6*(5*cos
h(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 3*(5*cosh(f*x +
 e)^4 + 2*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (3*cosh(f*x + e)
^5 + 2*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*
e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^6 + (a*f*e^(2
*f*x + 2*e) + a*f)*sinh(f*x + e)^6 + 3*a*f*cosh(f*x + e)^4 + 6*(a*f*cosh(f*
x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 3*(5*a*f*cosh
(f*x + e)^2 + a*f + (5*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x
 + e)^4 + 3*a*f*cosh(f*x + e)^2 + 4*(5*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x
 + e) + (5*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh
```

```
(f*x + e)^3 + 3*(5*a*f*cosh(f*x + e)^4 + 6*a*f*cosh(f*x + e)^2 + a*f + (5*a
*f*cosh(f*x + e)^4 + 6*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x
 + e)^2 + a*f + (a*f*cosh(f*x + e)^6 + 3*a*f*cosh(f*x + e)^4 + 3*a*f*cosh(f
*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 6*(a*f*cosh(f*x + e)^5 + 2*a*f*cosh(f*x
+ e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^5 + 2*a*f*cosh(f*x + e)^3 +
 a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a\left(\sinh^2(e+fx)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 0.12, size = 82, normalized size = 1.95

$$-\frac{4\,{\mathrm{e}}^{2\,e+2\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}\,\left(2\,{\mathrm{e}}^{2\,e+2\,f\,x}+3\,{\mathrm{e}}^{4\,e+4\,f\,x}+3\right)}{3\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] -(4*exp(2*e + 2*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)*(2
*exp(2*e + 2*f*x) + 3*exp(4*e + 4*f*x) + 3))/(3*a*f*(exp(2*e + 2*f*x) + 1)^
4)
```

3.439 $\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -1/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3255, 3284, 16, 32}

$$-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(1/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx &= \int \frac{\tanh(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= \frac{a\operatorname{Subst}\left(\int \frac{1}{(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= -\frac{1}{f\sqrt{a\cosh^2(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 1.00

$$-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(1/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Maple [A]
time = 0.60, size = 20, normalized size = 1.05

method	result	size
derivativedivides	$-\frac{1}{\sqrt{a+a\left(\sinh^2\left(fx+e\right)\right)}\, f}$	20
default	$-\frac{1}{\sqrt{a+a\left(\sinh^2\left(fx+e\right)\right)}\, f}$	20
risch	$-\frac{2}{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a\,\mathrm{e}^{-2fx-2e}}\, f}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a+a*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**A**]
time = 0.52, size = 35, normalized size = 1.84

$$-\frac{2\,e^{(-fx-e)}}{\left(\sqrt{a}\,e^{(-2\,fx-2\,e)}+\sqrt{a}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^(-f*x - e)/((sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a))*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(17) = 34$.
time = 0.47, size = 168, normalized size = 8.84

$$-\frac{2\,\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\,\left(\cosh\left(fx+e\right)e^{(fx+e)}+e^{(fx+e)}\sinh\left(fx+e\right)\right)e^{(-fx-e)}}{af\cosh\left(fx+e\right)^{2}+\left(afe^{(2\,fx+2\,e)}+af\right)\sinh\left(fx+e\right)^{2}+af+\left(af\cosh\left(fx+e\right)^{2}+af\right)e^{(2\,fx+2\,e)}+2\left(af\cosh\left(fx+e\right)e^{(2\,fx+2\,e)}+af\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(cosh(f*x + e)*e^(f*x
+ e) + e^(f*x + e)*sinh(f*x + e))*e^(-f*x - e)/(a*f*cosh(f*x + e)^2 + (a*f*
e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x + e)^2 + a*f)*
e^(2*f*x + 2*e) + 2*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))
*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh\left(e+fx\right)}{\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 0.85, size = 30, normalized size = 1.58

$$-\frac{\sqrt{a\sinh\left(e+f\,x\right)^2+a}}{a\,f\cosh\left(e+f\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] -(a + a*sinh(e + f*x)^2)^(1/2)/(a*f*cosh(e + f*x)^2)
```

3.440 $\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3255, 3284, 65, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx &= \int \frac{\coth(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx \\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{af} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}
\end{aligned}
$$

Mathematica [**A**]
time = 0.06, size = 49, normalized size = 1.58

$$
\frac{\cosh(e+fx)\left(-\log\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a\cosh^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*(-Log[Cosh[(e + f*x)/2]] + Log[Sinh[(e + f*x)/2]]))/(f*Sqrt[
a*Cosh[e + f*x]^2])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.95, size = 33, normalized size = 1.06

method	result	size

default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{1}{\sinh(fx+e)\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	33
risch	$-\frac{\ln\left(\mathrm{e}^{fx}+\mathrm{e}^{-e}\right)\left(\mathrm{e}^{2fx+2e}+1\right)\mathrm{e}^{-fx-e}}{f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}}+\frac{\ln\left(\mathrm{e}^{fx}-\mathrm{e}^{-e}\right)\left(\mathrm{e}^{2fx+2e}+1\right)\mathrm{e}^{-fx-e}}{f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(1/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [A]
time = 0.51, size = 42, normalized size = 1.35

$$-\frac{\log\left(e^{(-fx-e)}+1\right)}{\sqrt{a}\,f}+\frac{\log\left(e^{(-fx-e)}-1\right)}{\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(e^(-f*x - e) + 1)/(sqrt(a)*f) + log(e^(-f*x - e) - 1)/(sqrt(a)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(25) = 50$.
time = 0.45, size = 174, normalized size = 5.61

$$\left[\frac{\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\,\log\left(\frac{\cosh(fx+e)+\sinh(fx+e)-1}{\cosh(fx+e)+\sinh(fx+e)+1}\right)}{afe^{(2\,fx+2\,e)}+af},\ \frac{2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\,\sqrt{-a}}{a\cosh(fx+e)e^{(2\,fx+2\,e)}+a\cosh(fx+e)+\left(ae^{(2\,fx+2\,e)}+a\right)\sinh(fx+e)}\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*log((cosh(f*x + e) + sin
h(f*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1))/(a*f*e^(2*f*x + 2*e) +
 a*f), 2*sqrt(-a)*arctan(sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*
sqrt(-a)/(a*cosh(f*x + e)*e^(2*f*x + 2*e) + a*cosh(f*x + e) + (a*e^(2*f*x +
 2*e) + a)*sinh(f*x + e)))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth\left(e+fx\right)}{\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth(e+f\,x)}{\sqrt{a\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)/(a + a*sinh(e + f*x)^2)^(1/2), x)
```

3.441 $\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=66

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\sqrt{a\cosh^2(e+fx)}\,\operatorname{csch}^2(e+fx)}{2af}$$

```
[Out] -1/2*arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-1/2*csch(f*x+e)^2*(
a*cosh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3255, 3284, 16, 43, 65, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a\cosh^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -1/2*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) - (Sqrt[a*Cosh[e
+ f*x]^2]*Csch[e + f*x]^2)/(2*a*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\int \frac{\coth^3(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$

$$=\frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{ax}}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$

$$=\frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{(1-x)^2}\,dx,x,\cosh^2(e+fx)\right)}{2af}$$

$$=-\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2af}-\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx,x,\cosh^2(e+\right.}{4f}$$

$$=-\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2af}-\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a\cosh^2(e+f}\right.}{2af}$$

$$=-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2af}$$

Mathematica [A]
time = 0.15, size = 65, normalized size = 0.98

$$-\frac{\cosh(e+fx)\left(\text{csch}^2\left(\frac{1}{2}(e+fx)\right)-4\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)+\text{sech}^2\left(\frac{1}{2}(e+fx)\right)\right)}{8f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -1/8*(Cosh[e + f*x]*(Csch[(e + f*x)/2]^2 - 4*Log[Tanh[(e + f*x)/2]] + Sech[
(e + f*x)/2]^2))/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.23, size = 42, normalized size = 0.64

method	result
default	$\frac{\text{`}int/indef0\text{`}\left(\frac{\frac{1}{\sinh(fx+e)}+\frac{1}{\sinh(fx+e)^3}}{\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$

risch	$-\frac{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}}{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}\, f\left(\mathrm{e}^{2fx+2e}-1\right)^{2}}-\frac{\ln\left(\mathrm{e}^{fx}+\mathrm{e}^{-e}\right)\left(\mathrm{e}^{2fx+2e}+1\right)\mathrm{e}^{-fx-e}}{2f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}}+\frac{\ln\left(\mathrm{e}^{fx}-\mathrm{e}^{-e}\right)\left(\mathrm{e}^{2fx+2e}+\right.}{2f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`((1/sinh(f*x+e)+1/sinh(f*x+e)^3)/(a*cosh(f*x+e)^2)^(1/2),sinh(f
*x+e))/f
```

Maxima [A]
time = 0.54, size = 106, normalized size = 1.61

$$-\frac{\log\left(e^{(-fx-e)}+1\right)}{2\sqrt{a}\,f}+\frac{\log\left(e^{(-fx-e)}-1\right)}{2\sqrt{a}\,f}+\frac{e^{(-fx-e)}+e^{(-3\,fx-3\,e)}}{\left(2\sqrt{a}\,e^{(-2\,fx-2\,e)}-\sqrt{a}\,e^{(-4\,fx-4\,e)}-\sqrt{a}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-f*x - e) + 1)/(sqrt(a)*f) + 1/2*log(e^(-f*x - e) - 1)/(sqrt(a)
*f) + (e^(-f*x - e) + e^(-3*f*x - 3*e))/((2*sqrt(a)*e^(-2*f*x - 2*e) - sqrt
(a)*e^(-4*f*x - 4*e) - sqrt(a))*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(54) = 108.
time = 0.46, size = 529, normalized size = 8.02

$$-\frac{\left(6\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^{2}+2\,e^{(fx+e)}\sinh\left(fx+e\right)^{3}+2\left(3\cosh\left(fx+e\right)^{2}+1\right)e^{(fx+e)}\sinh\left(fx+e\right)+2\left(\cosh\left(fx+e\right)^{3}+\cosh\left(fx+e\right)\right)e^{(fx+e)}-\left(4\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^{3}+e^{(fx+e)}\sinh\left(fx+e\right)^{4}+2\left(3\cosh\left(fx+e\right)^{2}-1\right)e^{(fx+e)}\sinh\left(fx+e\right)^{2}+4\left(\cosh\left(fx+e\right)^{3}-\cosh\left(fx+e\right)\right)e^{(fx+e)}\sinh\left(fx+e\right)+\left(\cosh\left(fx+e\right)^{4}-2\cosh\left(fx+e\right)^{2}+1\right)e^{(fx+e)}\right)\log\left(\frac{\cosh(fx+e)+\sinh(fx+e)-1}{\cosh(fx+e)+\sinh(fx+e)+1}\right)\right)\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\,e^{(-fx-e)}}{2\left(af\cosh\left(fx+e\right)^{4}+\left(afe^{(2\,fx+2\,e)}+af\right)\sinh\left(fx+e\right)^{4}-2\,af\cosh\left(fx+e\right)^{2}+4\left(af\cosh\left(fx+e\right)e^{(2\,fx+2\,e)}+af\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^{3}+2\left(3\,af\cosh\left(fx+e\right)^{2}-af+\left(3\,af\cosh\left(fx+e\right)^{2}-af\right)e^{(2\,fx+2\,e)}\right)\sinh\left(fx+e\right)^{2}+af+\left(af\cosh\left(fx+e\right)^{4}-2\,af\cosh\left(fx+e\right)^{2}+af\right)e^{(2\,fx+2\,e)}+4\left(af\cosh\left(fx+e\right)^{3}-af\cosh\left(fx+e\right)+\left(af\cosh\left(fx+e\right)^{3}-af\cosh\left(fx+e\right)\right)e^{(2\,fx+2\,e)}\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + 2*e^(f*x + e)*sinh(f*x
+ e)^3 + 2*(3*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + 2*(cosh(f*x
+ e)^3 + cosh(f*x + e))*e^(f*x + e) - (4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x
 + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*e^(f*x +
e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f
*x + e) + (cosh(f*x + e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(
f*x + e) + sinh(f*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1)))*sqrt(a*
e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^
4 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^4 - 2*a*f*cosh(f*x + e)^2 + 4
*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^3 +
2*(3*a*f*cosh(f*x + e)^2 - a*f + (3*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2
*e))*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x + e)^4 - 2*a*f*cosh(f*x + e)^2 +
 a*f)*e^(2*f*x + 2*e) + 4*(a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e) + (a*f*c
osh(f*x + e)^3 - a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3\left(e+fx\right)}{\sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\mathrm{coth}\left(e+f\,x\right)}^3}{\sqrt{a\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(1/2), x)
```

3.442 $\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=91

$$\frac{3\text{ArcTan}(\sinh(e+fx))\cosh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}}-\frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] 3/8*arctan(sinh(f*x+e))*cosh(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)-3/8*tanh(f*x+
e)/f/(a*cosh(f*x+e)^2)^(1/2)-1/4*tanh(f*x+e)^3/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3286, 2691, 3855}

$$\frac{3\cosh(e+fx)\text{ArcTan}(\sinh(e+fx))}{8f\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}}-\frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(8*f*Sqrt[a*Cosh[e + f*x]^2]) - (3*
Tanh[e + f*x])/(8*f*Sqrt[a*Cosh[e + f*x]^2]) - Tanh[e + f*x]^3/(4*f*Sqrt[a*
Cosh[e + f*x]^2])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx &= \int \frac{\tanh^4(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx \\
&= \frac{\cosh(e+fx)\int \operatorname{sech}(e+fx)\tanh^4(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}} + \frac{(3\cosh(e+fx))\int \operatorname{sech}(e+fx)\tanh^2(e+fx)\,d}{4\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}} - \frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}} + \frac{(3\cosh(e+fx))\int \operatorname{sech}}{8\sqrt{a\cosh^2(e+}} \\
&= \frac{3\tan^{-1}(\sinh(e+fx))\cosh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}} - \frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}} - \frac{\tanh^3(e}{4f\sqrt{a\cosh}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 66, normalized size = 0.73

$$
\frac{3\text{ArcTan}(\sinh(e+fx))\cosh(e+fx)+\tanh(e+fx)\left(3-6\operatorname{sech}^2(e+fx)-8\tanh^2(e+fx)\right)}{8f\sqrt{a\cosh^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + Tanh[e + f*x]*(3 - 6*Sech[e + f*x]
^2 - 8*Tanh[e + f*x]^2))/(8*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [A]
time = 1.44, size = 68, normalized size = 0.75

method	result
default	$\frac{3\arctan(\sinh(fx+e))(\cosh^4(fx+e))-5(\cosh^2(fx+e))\sinh(fx+e)+2\sinh(fx+e)}{8\cosh(fx+e)^3\sqrt{a\left(\cosh^2(fx+e)\right)}\,f}$
risch	$-\frac{5\,\mathrm{e}^{6fx+6e}-3\,\mathrm{e}^{4fx+4e}+3\,\mathrm{e}^{2fx+2e}-5}{4\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a\,\mathrm{e}^{-2fx-2e}}\,(\mathrm{e}^{2fx+2e}+1)^3 f}+\frac{3i\ln(\mathrm{e}^{fx}+i\mathrm{e}^{-e})\,(\mathrm{e}^{2fx+2e}+1)\,\mathrm{e}^{-fx-e}}{8f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a\,\mathrm{e}^{-2fx-2e}}}-\frac{3i\ln(\mathrm{e}^{fx}-i\mathrm{e}^{-e})\,(\mathrm{e}^{2fx+2}}{8f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(3*arctan(sinh(f*x+e))*cosh(f*x+e)^4-5*cosh(f*x+e)^2*sinh(f*x+e)+2*sinh
(f*x+e))/cosh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(86) = 172$.
time = 0.55, size = 672, normalized size = 7.38

$$\frac{\frac{15\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}}-\frac{15\,e^{(-fx-e)}+55\,e^{(-3fx-3e)}+73\,e^{(-5fx-5e)}-15\,e^{(-7fx-7e)}}{4\sqrt{a}\,e^{(-2fx-2e)}+6\sqrt{a}\,e^{(-4fx-4e)}+4\sqrt{a}\,e^{(-6fx-6e)}+\sqrt{a}\,e^{(-8fx-8e)}+\sqrt{a}}}{48\,f}+\frac{\frac{15\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}}-\frac{15\,e^{(-fx-e)}-73\,e^{(-3fx-3e)}-55\,e^{(-5fx-5e)}-15\,e^{(-7fx-7e)}}{4\sqrt{a}\,e^{(-2fx-2e)}+6\sqrt{a}\,e^{(-4fx-4e)}+4\sqrt{a}\,e^{(-6fx-6e)}+\sqrt{a}\,e^{(-8fx-8e)}+\sqrt{a}}}{48\,f}-\frac{3\left(\frac{3\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}}-\frac{3\,e^{(-fx-e)}+11\,e^{(-3fx-3e)}-11\,e^{(-5fx-5e)}-3\,e^{(-7fx-7e)}}{4\sqrt{a}\,e^{(-2fx-2e)}+6\sqrt{a}\,e^{(-4fx-4e)}+4\sqrt{a}\,e^{(-6fx-6e)}+\sqrt{a}\,e^{(-8fx-8e)}+\sqrt{a}}\right)}{32\,f}-\frac{35\arctan\left(e^{(-fx-e)}\right)}{32\sqrt{a}\,f}-\frac{279\,e^{(-fx-e)}+511\,e^{(-3fx-3e)}+385\,e^{(-5fx-5e)}+105\,e^{(-7fx-7e)}}{192\left(4\sqrt{a}\,e^{(-2fx-2e)}+6\sqrt{a}\,e^{(-4fx-4e)}+4\sqrt{a}\,e^{(-6fx-6e)}+\sqrt{a}\,e^{(-8fx-8e)}+\sqrt{a}\right)f}+\frac{105\,e^{(-fx-e)}+385\,e^{(-3fx-3e)}+511\,e^{(-5fx-5e)}+279\,e^{(-7fx-7e)}}{192\left(4\sqrt{a}\,e^{(-2fx-2e)}+6\sqrt{a}\,e^{(-4fx-4e)}+4\sqrt{a}\,e^{(-6fx-6e)}+\sqrt{a}\,e^{(-8fx-8e)}+\sqrt{a}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/48*(15*arctan(e^(-f*x - e))/sqrt(a) - (15*e^(-f*x - e) + 55*e^(-3*f*x - 3
*e) + 73*e^(-5*f*x - 5*e) - 15*e^(-7*f*x - 7*e))/(4*sqrt(a)*e^(-2*f*x - 2*e
) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a)*e^(-8
*f*x - 8*e) + sqrt(a)))/f + 1/48*(15*arctan(e^(-f*x - e))/sqrt(a) - (15*e^(
-f*x - e) - 73*e^(-3*f*x - 3*e) - 55*e^(-5*f*x - 5*e) - 15*e^(-7*f*x - 7*e)
)/(4*sqrt(a)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-
6*f*x - 6*e) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)))/f - 3/32*(3*arctan(e^(-
f*x - e))/sqrt(a) - (3*e^(-f*x - e) + 11*e^(-3*f*x - 3*e) - 11*e^(-5*f*x -
5*e) - 3*e^(-7*f*x - 7*e))/(4*sqrt(a)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*
x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a))
)/f - 35/32*arctan(e^(-f*x - e))/(sqrt(a)*f) - 1/192*(279*e^(-f*x - e) + 51
1*e^(-3*f*x - 3*e) + 385*e^(-5*f*x - 5*e) + 105*e^(-7*f*x - 7*e))/((4*sqrt(
a)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*
e) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a))*f) + 1/192*(105*e^(-f*x - e) + 385
*e^(-3*f*x - 3*e) + 511*e^(-5*f*x - 5*e) + 279*e^(-7*f*x - 7*e))/((4*sqrt(a
)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*e
) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a))*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1328 vs. $2(79) = 158$.
time = 0.41, size = 1328, normalized size = 14.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f*x
 + e)^7 + 3*(35*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*co
sh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + (175*cosh(f*
x + e)^4 - 30*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^3 + 3*(35*cosh
(f*x + e)^5 - 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x +
e)^2 + (35*cosh(f*x + e)^6 - 15*cosh(f*x + e)^4 + 9*cosh(f*x + e)^2 - 5)*e^
(f*x + e)*sinh(f*x + e) - 3*(8*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 +
e^(f*x + e)*sinh(f*x + e)^8 + 4*(7*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*
x + e)^6 + 8*(7*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^5 + 2*(35*cosh(f*x + e)^4 + 30*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x
+ e)^4 + 8*(7*cosh(f*x + e)^5 + 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*
x + e)*sinh(f*x + e)^3 + 4*(7*cosh(f*x + e)^6 + 15*cosh(f*x + e)^4 + 9*cosh
(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(cosh(f*x + e)^7 + 3*cosh(
f*x + e)^5 + 3*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) +
 (cosh(f*x + e)^8 + 4*cosh(f*x + e)^6 + 6*cosh(f*x + e)^4 + 4*cosh(f*x + e)
^2 + 1)*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) + (5*cosh(f*x +
e)^7 - 3*cosh(f*x + e)^5 + 3*cosh(f*x + e)^3 - 5*cosh(f*x + e))*e^(f*x + e)
)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(
f*x + e)^8 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^8 + 4*a*f*cosh(f*x +
 e)^6 + 8*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x
+ e)^7 + 4*(7*a*f*cosh(f*x + e)^2 + a*f + (7*a*f*cosh(f*x + e)^2 + a*f)*e^(
2*f*x + 2*e))*sinh(f*x + e)^6 + 6*a*f*cosh(f*x + e)^4 + 8*(7*a*f*cosh(f*x +
 e)^3 + 3*a*f*cosh(f*x + e) + (7*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))
*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 2*(35*a*f*cosh(f*x + e)^4 + 30*a*f*cosh
(f*x + e)^2 + 3*a*f + (35*a*f*cosh(f*x + e)^4 + 30*a*f*cosh(f*x + e)^2 + 3*
a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 4*a*f*cosh(f*x + e)^2 + 8*(7*a*f*co
sh(f*x + e)^5 + 10*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e) + (7*a*f*cosh(
f*x + e)^5 + 10*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*e^(2*f*x + 2*e))
*sinh(f*x + e)^3 + 4*(7*a*f*cosh(f*x + e)^6 + 15*a*f*cosh(f*x + e)^4 + 9*a*
f*cosh(f*x + e)^2 + a*f + (7*a*f*cosh(f*x + e)^6 + 15*a*f*cosh(f*x + e)^4 +
 9*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + a*f + (a*f
*cosh(f*x + e)^8 + 4*a*f*cosh(f*x + e)^6 + 6*a*f*cosh(f*x + e)^4 + 4*a*f*co
sh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 8*(a*f*cosh(f*x + e)^7 + 3*a*f*cosh(
f*x + e)^5 + 3*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)
^7 + 3*a*f*cosh(f*x + e)^5 + 3*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(
2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a\left(\sinh^2(e+fx)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**4/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(e+f\,x)^4}{\sqrt{a\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^4/(a + a*sinh(e + f*x)^2)^(1/2), x)
```

3.443 $\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\operatorname{ArcTan}(\sinh(e+fx))\cosh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] 1/2*arctan(sinh(f*x+e))*cosh(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)-1/2*tanh(f*x+
e)/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3286, 2691, 3855}

$$\frac{\cosh(e+fx)\operatorname{ArcTan}(\sinh(e+fx))}{2f\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(2*f*Sqrt[a*Cosh[e + f*x]^2]) - Tanh[
e + f*x]/(2*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx &= \int \frac{\tanh^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx\\ &= \frac{\cosh(e+fx)\int \operatorname{sech}(e+fx)\tanh^2(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}}\\ &= -\frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}+\frac{\cosh(e+fx)\int \operatorname{sech}(e+fx)\,dx}{2\sqrt{a\cosh^2(e+fx)}}\\ &= \frac{\tan^{-1}(\sinh(e+fx))\cosh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 0.71

$$\frac{\operatorname{ArcTan}(\sinh(e+fx))\cosh(e+fx)-\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] - Tanh[e + f*x])/(2*f*Sqrt[a*Cosh[e +
f*x]^2])
```

Maple [A]
time = 1.32, size = 51, normalized size = 0.82

method	result
default	$\frac{\frac{\arctan(\sinh(fx+e))\left(\cosh^2(fx+e)\right)}{2}-\frac{\sinh(fx+e)}{2}}{\cosh(fx+e)\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$

risch	$-\frac{\mathrm{e}^{2fx+2e}-1}{\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}\,\left(\mathrm{e}^{2fx+2e}+1\right) f}+\frac{i \ln\left(\mathrm{e}^{fx}+i\mathrm{e}^{-e}\right)\left(\mathrm{e}^{2fx+2e}+1\right)\mathrm{e}^{-fx-e}}{2f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a\,\mathrm{e}^{-2fx-2e}}}-\frac{i \ln\left(\mathrm{e}^{fx}-i\mathrm{e}^{-e}\right)\left(\mathrm{e}^{2fx+2e}\right.}{2f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2} a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*arctan(sinh(f*x+e))*cosh(f*x+e)^2-1/2*sinh(f*x+e))/cosh(f*x+e)/(a*cosh
(f*x+e)^2)^(1/2)/f
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(59) = 118$.
time = 0.51, size = 231, normalized size = 3.73

$$\frac{\frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}}-\frac{e^{(-fx-e)}-e^{(-3\,fx-3\,e)}}{2\,\sqrt{a}\,e^{(-2\,fx-2\,e)}+\sqrt{a}\,e^{(-4\,fx-4\,e)}+\sqrt{a}}}{2\,f}-\frac{3\,\arctan\left(e^{(-fx-e)}\right)}{2\,\sqrt{a}\,f}-\frac{5\,e^{(-fx-e)}+3\,e^{(-3\,fx-3\,e)}}{4\,\left(2\,\sqrt{a}\,e^{(-2\,fx-2\,e)}+\sqrt{a}\,e^{(-4\,fx-4\,e)}+\sqrt{a}\right)f}+\frac{3\,e^{(-fx-e)}+5\,e^{(-3\,fx-3\,e)}}{4\,\left(2\,\sqrt{a}\,e^{(-2\,fx-2\,e)}+\sqrt{a}\,e^{(-4\,fx-4\,e)}+\sqrt{a}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(arctan(e^(-f*x - e))/sqrt(a) - (e^(-f*x - e) - e^(-3*f*x - 3*e))/(2*sq
rt(a)*e^(-2*f*x - 2*e) + sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a)))/f - 3/2*arcta
n(e^(-f*x - e))/(sqrt(a)*f) - 1/4*(5*e^(-f*x - e) + 3*e^(-3*f*x - 3*e))/((2
*sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a))*f) + 1/4*(3
*e^(-f*x - e) + 5*e^(-3*f*x - 3*e))/((2*sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a)*
e^(-4*f*x - 4*e) + sqrt(a))*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(54) = 108$.
time = 0.44, size = 504, normalized size = 8.13

$$-\frac{\left(3\,\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^{2}+e^{(fx+e)}\sinh\left(fx+e\right)^{3}+\left(3\,\cosh\left(fx+e\right)^{2}-1\right)e^{(fx+e)}\sinh\left(fx+e\right)-\left(4\,\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^{3}+e^{(fx+e)}\sinh\left(fx+e\right)^{4}+2\left(3\,\cosh\left(fx+e\right)^{2}+1\right)e^{(fx+e)}\sinh\left(fx+e\right)^{2}+4\left(\cosh\left(fx+e\right)^{3}+\cosh\left(fx+e\right)\right)e^{(fx+e)}\sinh\left(fx+e\right)+\left(\cosh\left(fx+e\right)^{4}+2\,\cosh\left(fx+e\right)^{2}+1\right)e^{(fx+e)}\right)\arctan\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)+\left(\cosh\left(fx+e\right)^{3}-\cosh\left(fx+e\right)\right)e^{(fx+e)}\right)\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\,e^{(-fx-e)}}{af\cosh\left(fx+e\right)^{4}+\left(afe^{(2\,fx+2\,e)}+af\right)\sinh\left(fx+e\right)^{4}+2\,af\cosh\left(fx+e\right)^{2}+4\left(af\cosh\left(fx+e\right)e^{(2\,fx+2\,e)}+af\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^{3}+2\left(3\,af\cosh\left(fx+e\right)^{2}+af+\left(3\,af\cosh\left(fx+e\right)^{2}+af\right)e^{(2\,fx+2\,e)}\right)\sinh\left(fx+e\right)^{2}+af+\left(af\cosh\left(fx+e\right)^{4}+2\,af\cosh\left(fx+e\right)^{2}+af\right)e^{(2\,fx+2\,e)}+4\left(af\cosh\left(fx+e\right)^{3}+af\cosh\left(fx+e\right)+\left(af\cosh\left(fx+e\right)^{3}+af\cosh\left(fx+e\right)\right)e^{(2\,fx+2\,e)}\right)\sinh\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x + e)^3
 + (3*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e) - (4*cosh(f*x + e)*e^(
f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)
^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*e
^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 + 2*cosh(f*x + e)^2 + 1)*e^(f*x
 + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) + (cosh(f*x + e)^3 - cosh(f*x
+ e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*
x - e)/(a*f*cosh(f*x + e)^4 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^4 +
 2*a*f*cosh(f*x + e)^2 + 4*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*
x + e))*sinh(f*x + e)^3 + 2*(3*a*f*cosh(f*x + e)^2 + a*f + (3*a*f*cosh(f*x
```

```
+ e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x + e)^4
 + 2*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 4*(a*f*cosh(f*x + e)^3 +
a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{2}\left(e + fx\right)}{\sqrt{a\left(\sinh^{2}\left(e + fx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**2/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\mathrm{tanh}\left(e+f\,x\right)}^2}{\sqrt{a\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(1/2), x)
```

3.444 $\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=25

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -coth(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3286, 2686, 8}

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx &= \int \frac{\coth^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx \\
&= \frac{\cosh(e+fx)\int \coth(e+fx)\text{csch}(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{(i\cosh(e+fx))\text{Subst}\left(\int 1\,dx, x, -i\text{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 1.00

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Maple [A]
time = 0.99, size = 32, normalized size = 1.28

method	result	size
default	$-\frac{\cosh(fx+e)}{\sinh(fx+e)\sqrt{a\left(\cosh^2(fx+e)\right)}\,f}$	32
risch	$-\frac{2\left(e^{2fx+2e}+1\right)}{\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\,f\left(e^{2fx+2e}-1\right)}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -cosh(f*x+e)/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(25) = 50$.
time = 0.50, size = 107, normalized size = 4.28

$$\frac{\frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}} + \frac{\sqrt{a}\,e^{(-fx-e)}}{ae^{(-2\,fx-2\,e)}-a}}{f} - \frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}\,f} + \frac{\sqrt{a}\,e^{(-fx-e)}}{(ae^{(-2\,fx-2\,e)}-a)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (arctan(e^(-f*x - e))/sqrt(a) + sqrt(a)*e^(-f*x - e)/(a*e^(-2*f*x - 2*e) -
a))/f - arctan(e^(-f*x - e))/(sqrt(a)*f) + sqrt(a)*e^(-f*x - e)/((a*e^(-2*f
*x - 2*e) - a)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(23) = 46$.
time = 0.46, size = 170, normalized size = 6.80

$$-\frac{2\,\sqrt{ae^{(4\,fx+4\,e)} + 2\,ae^{(2\,fx+2\,e)} + a}\,\left(\cosh\left(fx+e\right)e^{(fx+e)} + e^{(fx+e)}\sinh\left(fx+e\right)\right)e^{(-fx-e)}}{af\cosh\left(fx+e\right)^{2} + \left(afe^{(2\,fx+2\,e)} + af\right)\sinh\left(fx+e\right)^{2} - af + \left(af\cosh\left(fx+e\right)^{2} - af\right)e^{(2\,fx+2\,e)} + 2\left(af\cosh\left(fx+e\right)e^{(2\,fx+2\,e)} + af\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(cosh(f*x + e)*e^(f*x
+ e) + e^(f*x + e)*sinh(f*x + e))*e^(-f*x - e)/(a*f*cosh(f*x + e)^2 + (a*f*
e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^2 - a*f + (a*f*cosh(f*x + e)^2 - a*f)*
e^(2*f*x + 2*e) + 2*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))
*sinh(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{2}\left(e + fx\right)}{\sqrt{a\left(\sinh^{2}\left(e + fx\right) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**2/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 0.11, size = 76, normalized size = 3.04

$$-\frac{4\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{a\,f\,\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] -(4*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(
a*f*(exp(2*e + 2*f*x) - 1)*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

3.445 $\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=61

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -coth(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)-1/3*coth(f*x+e)*csch(f*x+e)^2/f/(a*c
osh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3255, 3286, 2686}

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2])) - (Coth[e + f*x]*Csch[e + f*x]
^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx &= \int \frac{\coth^4(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx \\
&= \frac{\cosh(e+fx)\int \coth^3(e+fx)\operatorname{csch}(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}} \\
&= \frac{(i\cosh(e+fx))\operatorname{Subst}\left(\int\left(-1+x^2\right)\,dx, x, -i\operatorname{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.05, size = 37, normalized size = 0.61

$$-\frac{\coth(e+fx)\left(3+\operatorname{csch}^2(e+fx)\right)}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
[Out] -1/3*(Coth[e + f*x]*(3 + Csch[e + f*x]^2))/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [**A**]
time = 1.21, size = 44, normalized size = 0.72

method	result	size
default	$-\frac{\cosh(fx+e)\left(3\left(\sinh^2(fx+e)\right)+1\right)}{3\sinh(fx+e)^3\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$	44
risch	$-\frac{2\left(\mathrm{e}^{2fx+2e}+1\right)\left(3\,\mathrm{e}^{4fx+4e}-2\,\mathrm{e}^{2fx+2e}+3\right)}{3\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a\,\mathrm{e}^{-2fx-2e}}\,f\left(\mathrm{e}^{2fx+2e}-1\right)^3}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*cosh(f*x+e)*(3*sinh(f*x+e)^2+1)/sinh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/
f
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(60) = 120$.
time $= 0.53$, size $= 592$, normalized size $= 9.70$

$$\frac{\frac{6\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}}+\frac{3\log\left(e^{(-fx-e)}+1\right)}{\sqrt{a}}-\frac{3\log\left(e^{(-fx-e)}-1\right)}{\sqrt{a}}+\frac{4\left(3\sqrt{a}e^{(-fx-e)}-\sqrt{a}e^{(-3fx-3e)}\right)}{3ae^{(-2fx-2e)}-3ae^{(-4fx-4e)}+ae^{(-6fx-6e)}-a}}{12f}+\frac{\frac{6\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}}-\frac{3\log\left(e^{(-fx-e)}+1\right)}{\sqrt{a}}+\frac{3\log\left(e^{(-fx-e)}-1\right)}{\sqrt{a}}-\frac{4\left(\sqrt{a}e^{(-3fx-3e)}-3\sqrt{a}e^{(-5fx-5e)}\right)}{3ae^{(-2fx-2e)}-3ae^{(-4fx-4e)}+ae^{(-6fx-6e)}-a}}{12f}-\frac{\frac{3\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}}+\frac{3\sqrt{a}e^{(-fx-e)}-10\sqrt{a}e^{(-3fx-3e)}+3\sqrt{a}e^{(-5fx-5e)}}{3ae^{(-2fx-2e)}-3ae^{(-4fx-4e)}+ae^{(-6fx-6e)}-a}}{4f}-\frac{\arctan\left(e^{(-fx-e)}\right)}{4\sqrt{a}f}+\frac{27\sqrt{a}e^{(-fx-e)}-38\sqrt{a}e^{(-3fx-3e)}+15\sqrt{a}e^{(-5fx-5e)}}{24\left(3ae^{(-2fx-2e)}-3ae^{(-4fx-4e)}+ae^{(-6fx-6e)}-a\right)f}+\frac{15\sqrt{a}e^{(-fx-e)}-38\sqrt{a}e^{(-3fx-3e)}+27\sqrt{a}e^{(-5fx-5e)}}{24\left(3ae^{(-2fx-2e)}-3ae^{(-4fx-4e)}+ae^{(-6fx-6e)}-a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(6*arctan(e^(-f*x - e))/sqrt(a) + 3*log(e^(-f*x - e) + 1)/sqrt(a) - 3*
log(e^(-f*x - e) - 1)/sqrt(a) + 4*(3*sqrt(a)*e^(-f*x - e) - sqrt(a)*e^(-3*f
*x - 3*e))/(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e
) - a))/f + 1/12*(6*arctan(e^(-f*x - e))/sqrt(a) - 3*log(e^(-f*x - e) + 1)/
sqrt(a) + 3*log(e^(-f*x - e) - 1)/sqrt(a) - 4*(sqrt(a)*e^(-3*f*x - 3*e) - 3
*sqrt(a)*e^(-5*f*x - 5*e))/(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a
*e^(-6*f*x - 6*e) - a))/f - 1/4*(3*arctan(e^(-f*x - e))/sqrt(a) + (3*sqrt(a
)*e^(-f*x - e) - 10*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-5*f*x - 5*e))/
(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a))/f -
 1/4*arctan(e^(-f*x - e))/(sqrt(a)*f) + 1/24*(27*sqrt(a)*e^(-f*x - e) - 38*
sqrt(a)*e^(-3*f*x - 3*e) + 15*sqrt(a)*e^(-5*f*x - 5*e))/((3*a*e^(-2*f*x - 2
*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a)*f) + 1/24*(15*sqrt(a)*
e^(-f*x - e) - 38*sqrt(a)*e^(-3*f*x - 3*e) + 27*sqrt(a)*e^(-5*f*x - 5*e))/(
(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a)*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(55) = 110$.
time $= 0.44$, size $= 647$, normalized size $= 10.61$

$$-\frac{2\left(15\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^{4}+3e^{(fx+e)}\sinh\left(fx+e\right)^{5}+2\left(15\cosh\left(fx+e\right)^{2}-1\right)e^{(fx+e)}\sinh\left(fx+e\right)^{3}+6\left(5\cosh\left(fx+e\right)^{3}-\cosh\left(fx+e\right)\right)e^{(fx+e)}\sinh\left(fx+e\right)^{2}+3\left(5\cosh\left(fx+e\right)^{4}-2\cosh\left(fx+e\right)^{2}+1\right)e^{(fx+e)}\sinh\left(fx+e\right)+\left(3\cosh\left(fx+e\right)^{5}-2\cosh\left(fx+e\right)^{3}+3\cosh\left(fx+e\right)\right)e^{(fx+e)}\right)\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}\,e^{(-fx-e)}}{3\left(af\cosh\left(fx+e\right)^{6}+\left(afe^{(2fx+2e)}+af\right)\sinh\left(fx+e\right)^{6}-3af\cosh\left(fx+e\right)^{4}+6\left(af\cosh\left(fx+e\right)e^{(2fx+2e)}+af\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^{5}+3\left(5af\cosh\left(fx+e\right)^{2}-af+\left(5af\cosh\left(fx+e\right)^{2}-af\right)e^{(2fx+2e)}\right)\sinh\left(fx+e\right)^{4}+3af\cosh\left(fx+e\right)^{2}+4\left(5af\cosh\left(fx+e\right)^{3}-3af\cosh\left(fx+e\right)+\left(5af\cosh\left(fx+e\right)^{3}-3af\cosh\left(fx+e\right)\right)e^{(2fx+2e)}\right)\sinh\left(fx+e\right)^{3}+3\left(5af\cosh\left(fx+e\right)^{4}-6af\cosh\left(fx+e\right)^{2}+af+\left(5af\cosh\left(fx+e\right)^{4}-6af\cosh\left(fx+e\right)^{2}+af\right)e^{(2fx+2e)}\right)\sinh\left(fx+e\right)^{2}-af+\left(af\cosh\left(fx+e\right)^{6}-3af\cosh\left(fx+e\right)^{4}+3af\cosh\left(fx+e\right)^{2}-af\right)e^{(2fx+2e)}+6\left(af\cosh\left(fx+e\right)^{5}-2af\cosh\left(fx+e\right)^{3}+af\cosh\left(fx+e\right)+\left(af\cosh\left(fx+e\right)^{5}-2af\cosh\left(fx+e\right)^{3}+af\cosh\left(fx+e\right)\right)e^{(2fx+2e)}\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(15*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + 3*e^(f*x + e)*sinh(f*x
 + e)^5 + 2*(15*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 6*(5*cos
h(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 3*(5*cosh(f*x +
 e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (3*cosh(f*x + e)
^5 - 2*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*
e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^6 + (a*f*e^(2
*f*x + 2*e) + a*f)*sinh(f*x + e)^6 - 3*a*f*cosh(f*x + e)^4 + 6*(a*f*cosh(f*
x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 3*(5*a*f*cosh
(f*x + e)^2 - a*f + (5*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*x
 + e)^4 + 3*a*f*cosh(f*x + e)^2 + 4*(5*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x
```

```
 + e) + (5*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh
(f*x + e)^3 + 3*(5*a*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2 + a*f + (5*a
*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x
 + e)^2 - a*f + (a*f*cosh(f*x + e)^6 - 3*a*f*cosh(f*x + e)^4 + 3*a*f*cosh(f
*x + e)^2 - a*f)*e^(2*f*x + 2*e) + 6*(a*f*cosh(f*x + e)^5 - 2*a*f*cosh(f*x
+ e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^5 - 2*a*f*cosh(f*x + e)^3 +
 a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(e+fx)}{\sqrt{a\left(\sinh^2(e+fx)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**4/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**B**]
time = 0.89, size = 95, normalized size = 1.56

$$-\frac{4\,{\mathrm{e}}^{2\,e+2\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}\,\left(3\,{\mathrm{e}}^{4\,e+4\,f\,x}-2\,{\mathrm{e}}^{2\,e+2\,f\,x}+3\right)}{3\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^3\,\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] -(4*exp(2*e + 2*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)*(3
*exp(4*e + 4*f*x) - 2*exp(2*e + 2*f*x) + 3))/(3*a*f*(exp(2*e + 2*f*x) - 1)^
3*(exp(2*e + 2*f*x) + 1))
```

3.446 $\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=96

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -coth(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)-2/3*coth(f*x+e)*csch(f*x+e)^2/f/(a*c
osh(f*x+e)^2)^(1/2)-1/5*coth(f*x+e)*csch(f*x+e)^4/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3286, 2686, 200}

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5f\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^6/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2])) - (2*Coth[e + f*x]*Csch[e + f*
x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x]^4)/(5*f*
Sqrt[a*Cosh[e + f*x]^2])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx &= \int \frac{\coth^6(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx\\
&= \frac{\cosh(e+fx)\int \coth^5(e+fx)\text{csch}(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}}\\
&= -\frac{(i\cosh(e+fx))\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, -i\text{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}}\\
&= -\frac{(i\cosh(e+fx))\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -i\text{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}}\\
&= -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\text{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^4}{5f\sqrt{a\cosh^2(e+}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.51

$$
-\frac{\coth(e+fx)\left(15+10\text{csch}^2(e+fx)+3\text{csch}^4(e+fx)\right)}{15f\sqrt{a\cosh^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^6/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -1/15*(Coth[e + f*x]*(15 + 10*Csch[e + f*x]^2 + 3*Csch[e + f*x]^4))/(f*Sqrt
[a*Cosh[e + f*x]^2])
```

Maple [A]
time = 1.58, size = 54, normalized size = 0.56

method	result	size

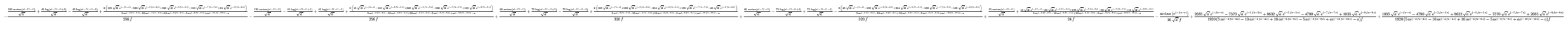
default	$-\frac{\cosh(fx+e)\left(15\left(\sinh^4(fx+e)\right)+10\left(\sinh^2(fx+e)\right)+3\right)}{15\sinh(fx+e)^5\sqrt{a\left(\cosh^2(fx+e)\right)}\, f}$	54
risch	$-\frac{2\left(e^{2fx+2e}+1\right)\left(15\,e^{8fx+8e}-20\,e^{6fx+6e}+58\,e^{4fx+4e}-20\,e^{2fx+2e}+15\right)}{15\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\, f\left(e^{2fx+2e}-1\right)^5}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*cosh(f*x+e)*(15*sinh(f*x+e)^4+10*sinh(f*x+e)^2+3)/sinh(f*x+e)^5/(a*co
sh(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(94) = 188$.
time = 0.57, size = 1315, normalized size = 13.70



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/256*(120*arctan(e^(-f*x - e))/sqrt(a) + 45*log(e^(-f*x - e) + 1)/sqrt(a)
 - 45*log(e^(-f*x - e) - 1)/sqrt(a) + 2*(105*sqrt(a)*e^(-f*x - e) - 530*sqr
t(a)*e^(-3*f*x - 3*e) + 328*sqrt(a)*e^(-5*f*x - 5*e) - 110*sqrt(a)*e^(-7*f*
x - 7*e) + 15*sqrt(a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*
f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x -
10*e) - a))/f - 1/256*(120*arctan(e^(-f*x - e))/sqrt(a) - 45*log(e^(-f*x -
e) + 1)/sqrt(a) + 45*log(e^(-f*x - e) - 1)/sqrt(a) + 2*(15*sqrt(a)*e^(-f*x
- e) - 110*sqrt(a)*e^(-3*f*x - 3*e) + 328*sqrt(a)*e^(-5*f*x - 5*e) - 530*sq
rt(a)*e^(-7*f*x - 7*e) + 105*sqrt(a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*f*x - 2*e
) - 10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) +
a*e^(-10*f*x - 10*e) - a))/f + 1/320*(60*arctan(e^(-f*x - e))/sqrt(a) + 75*
log(e^(-f*x - e) + 1)/sqrt(a) - 75*log(e^(-f*x - e) - 1)/sqrt(a) + 2*(105*s
qrt(a)*e^(-f*x - e) + 130*sqrt(a)*e^(-3*f*x - 3*e) - 284*sqrt(a)*e^(-5*f*x
- 5*e) + 190*sqrt(a)*e^(-7*f*x - 7*e) - 45*sqrt(a)*e^(-9*f*x - 9*e))/(5*a*e
^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8
*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f + 1/320*(60*arctan(e^(-f*x - e))
/sqrt(a) - 75*log(e^(-f*x - e) + 1)/sqrt(a) + 75*log(e^(-f*x - e) - 1)/sqrt
(a) - 2*(45*sqrt(a)*e^(-f*x - e) - 190*sqrt(a)*e^(-3*f*x - 3*e) + 284*sqrt(
a)*e^(-5*f*x - 5*e) - 130*sqrt(a)*e^(-7*f*x - 7*e) - 105*sqrt(a)*e^(-9*f*x
- 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*
e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f + 1/24*(15*arctan(
e^(-f*x - e))/sqrt(a) + (15*sqrt(a)*e^(-f*x - e) - 80*sqrt(a)*e^(-3*f*x - 3
```

```
*e) + 178*sqrt(a)*e^(-5*f*x - 5*e) - 80*sqrt(a)*e^(-7*f*x - 7*e) + 15*sqrt(
a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e
^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f - 1/1
6*arctan(e^(-f*x - e))/(sqrt(a)*f) + 1/1920*(2685*sqrt(a)*e^(-f*x - e) - 73
70*sqrt(a)*e^(-3*f*x - 3*e) + 8632*sqrt(a)*e^(-5*f*x - 5*e) - 4790*sqrt(a)*
e^(-7*f*x - 7*e) + 1035*sqrt(a)*e^(-9*f*x - 9*e))/((5*a*e^(-2*f*x - 2*e) -
10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^
(-10*f*x - 10*e) - a)*f) + 1/1920*(1035*sqrt(a)*e^(-f*x - e) - 4790*sqrt(a)
*e^(-3*f*x - 3*e) + 8632*sqrt(a)*e^(-5*f*x - 5*e) - 7370*sqrt(a)*e^(-7*f*x
- 7*e) + 2685*sqrt(a)*e^(-9*f*x - 9*e))/((5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4
*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x -
 10*e) - a)*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1399 vs. $2(86) = 172$.
time $= 0.45$, size $= 1399$, normalized size $= 14.57$

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(135*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^8 + 15*e^(f*x + e)*sinh(
f*x + e)^9 + 20*(27*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^7 + 140*
(9*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^6 + 2*(945*co
sh(f*x + e)^4 - 210*cosh(f*x + e)^2 + 29)*e^(f*x + e)*sinh(f*x + e)^5 + 10*
(189*cosh(f*x + e)^5 - 70*cosh(f*x + e)^3 + 29*cosh(f*x + e))*e^(f*x + e)*s
inh(f*x + e)^4 + 20*(63*cosh(f*x + e)^6 - 35*cosh(f*x + e)^4 + 29*cosh(f*x
+ e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 20*(27*cosh(f*x + e)^7 - 21*cosh(
f*x + e)^5 + 29*cosh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(27*cosh(f*x + e)^8 - 28*cosh(f*x + e)^6 + 58*cosh(f*x + e)^4 - 12*
cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e) + (15*cosh(f*x + e)^9 - 20*c
osh(f*x + e)^7 + 58*cosh(f*x + e)^5 - 20*cosh(f*x + e)^3 + 15*cosh(f*x + e)
)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a*f*cosh(f*x + e)^10 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^10 - 5
*a*f*cosh(f*x + e)^8 + 10*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x
 + e))*sinh(f*x + e)^9 + 5*(9*a*f*cosh(f*x + e)^2 - a*f + (9*a*f*cosh(f*x +
 e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 + 40
*(3*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e)^3 - a*f*
cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^7 + 10*(21*a*f*cosh(f*x + e)^
4 - 14*a*f*cosh(f*x + e)^2 + a*f + (21*a*f*cosh(f*x + e)^4 - 14*a*f*cosh(f*
x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 - 10*a*f*cosh(f*x + e)^4 +
 4*(63*a*f*cosh(f*x + e)^5 - 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e)
+ (63*a*f*cosh(f*x + e)^5 - 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e))*
e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 10*(21*a*f*cosh(f*x + e)^6 - 35*a*f*cosh
```

```
(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 - a*f + (21*a*f*cosh(f*x + e)^6 - 35*a
*f*cosh(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*
x + e)^4 + 5*a*f*cosh(f*x + e)^2 + 40*(3*a*f*cosh(f*x + e)^7 - 7*a*f*cosh(f
*x + e)^5 + 5*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e
)^7 - 7*a*f*cosh(f*x + e)^5 + 5*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e))*e^
(2*f*x + 2*e))*sinh(f*x + e)^3 + 5*(9*a*f*cosh(f*x + e)^8 - 28*a*f*cosh(f*x
 + e)^6 + 30*a*f*cosh(f*x + e)^4 - 12*a*f*cosh(f*x + e)^2 + a*f + (9*a*f*co
sh(f*x + e)^8 - 28*a*f*cosh(f*x + e)^6 + 30*a*f*cosh(f*x + e)^4 - 12*a*f*co
sh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 - a*f + (a*f*cosh(f*x
 + e)^10 - 5*a*f*cosh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 - 10*a*f*cosh(f*x
 + e)^4 + 5*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e) + 10*(a*f*cosh(f*x +
 e)^9 - 4*a*f*cosh(f*x + e)^7 + 6*a*f*cosh(f*x + e)^5 - 4*a*f*cosh(f*x + e)
^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^9 - 4*a*f*cosh(f*x + e)^7 + 6*a
*f*cosh(f*x + e)^5 - 4*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{6}\left(e+fx\right)}{\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**6/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**6/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**B**]
time = 0.90, size = 381, normalized size = 3.97

$$-\frac{4\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{a\,f\,\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{32\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{3\,a\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)}^2\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{352\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{15\,a\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)}^3\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{128\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{5\,a\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)}^4\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{64\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{5\,a\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)}^5\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^6/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] - (4*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(a*f*(exp(2*e + 2*f*x) - 1)*(exp(e + f*x) + exp(3*e + 3*f*x))) - (32*exp(3*
e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3*a*f*(exp
(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (352*exp(3*e + 3*
f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(15*a*f*(exp(2*e
+ 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (128*exp(3*e + 3*f*x)*
(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a*f*(exp(2*e + 2*f*
x) - 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (64*exp(3*e + 3*f*x)*(a + a*
(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a*f*(exp(2*e + 2*f*x) - 1)
^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

3.447 $\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{a^2}{7f\left(a\cosh^2(e+fx)\right)^{7/2}}+\frac{2a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

```
[Out] -1/7*a^2/f/(a*cosh(f*x+e)^2)^(7/2)+2/5*a/f/(a*cosh(f*x+e)^2)^(5/2)-1/3/f/(a
*cosh(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3284, 16, 45}

$$-\frac{a^2}{7f\left(a\cosh^2(e+fx)\right)^{7/2}}+\frac{2a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/7*a^2/(f*(a*Cosh[e + f*x]^2)^(7/2)) + (2*a)/(5*f*(a*Cosh[e + f*x]^2)^(5/
2)) - 1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
```

```
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\tanh^5(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx\\ &= \frac{\operatorname{Subst}\left(\int \frac{(1-x)^2}{x^3(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\ &= \frac{a^3\operatorname{Subst}\left(\int \frac{(1-x)^2}{(ax)^{9/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\ &= \frac{a^3\operatorname{Subst}\left(\int \left(\frac{1}{(ax)^{9/2}}-\frac{2}{a(ax)^{7/2}}+\frac{1}{a^2(ax)^{5/2}}\right)dx, x, \cosh^2(e+fx)\right)}{2f}\\ &= -\frac{a^2}{7f\left(a\cosh^2(e+fx)\right)^{7/2}}+\frac{2a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\,(a\cosh^2(e+fx}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 51, normalized size = 0.75

$$\frac{\left(-15+42\cosh^2(e+fx)-35\cosh^4(e+fx)\right)\operatorname{sech}^4(e+fx)}{105f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-15 + 42*Cosh[e + f*x]^2 - 35*Cosh[e + f*x]^4)*Sech[e + f*x]^4)/(105*f*(a
*Cosh[e + f*x]^2)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.25, size = 44, normalized size = 0.65

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\sinh^5(fx+e)}{\cosh(fx+e)^8 a\sqrt{a\left(\cosh^2(fx+e)\right)}}, \sinh(fx+e)\right)}{f}$	44

risch	$-\frac{8\left(35\,{\mathrm e}^{8 f x +8 e}-28\,{\mathrm e}^{6 f x +6 e}+114\,{\mathrm e}^{4 f x +4 e}-28\,{\mathrm e}^{2 f x +2 e}+35\right){\mathrm e}^{2 f x +2 e}}{105 f \sqrt{\left({\mathrm e}^{2 f x +2 e}+1\right)^{2} a \,{\mathrm e}^{-2 f x -2 e}}\, \left({\mathrm e}^{2 f x +2 e}+1\right)^{6} a}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 'int/indef0'(sinh(f*x+e)^5/cosh(f*x+e)^8/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(59) = 118$.
time = 0.57, size = 626, normalized size = 9.21

$$-\frac{8 \, e^{\left(-3 \, f x - 3 \, e\right)}}{3 \, {\left(7 \, a^{\frac{3}{2}} e^{\left(-2 \, f x - 2 \, e\right)} + 21 \, a^{\frac{3}{2}} e^{\left(-4 \, f x - 4 \, e\right)} + 35 \, a^{\frac{3}{2}} e^{\left(-6 \, f x - 6 \, e\right)} + 35 \, a^{\frac{3}{2}} e^{\left(-8 \, f x - 8 \, e\right)} + 21 \, a^{\frac{3}{2}} e^{\left(-10 \, f x - 10 \, e\right)} + 7 \, a^{\frac{3}{2}} e^{\left(-12 \, f x - 12 \, e\right)} + a^{\frac{3}{2}} e^{\left(-14 \, f x - 14 \, e\right)} + a^{\frac{3}{2}}\right)} f} + \frac{32 \, e^{\left(-5 \, f x - 5 \, e\right)}}{15 \, {\left(7 \, a^{\frac{3}{2}} e^{\left(-2 \, f x - 2 \, e\right)} + 21 \, a^{\frac{3}{2}} e^{\left(-4 \, f x - 4 \, e\right)} + 35 \, a^{\frac{3}{2}} e^{\left(-6 \, f x - 6 \, e\right)} + 35 \, a^{\frac{3}{2}} e^{\left(-8 \, f x - 8 \, e\right)} + 21 \, a^{\frac{3}{2}} e^{\left(-10 \, f x - 10 \, e\right)} + 7 \, a^{\frac{3}{2}} e^{\left(-12 \, f x - 12 \, e\right)} + a^{\frac{3}{2}} e^{\left(-14 \, f x - 14 \, e\right)} + a^{\frac{3}{2}}\right)} f} - \frac{304 \, e^{\left(-7 \, f x - 7 \, e\right)}}{35 \, {\left(7 \, a^{\frac{3}{2}} e^{\left(-2 \, f x - 2 \, e\right)} + 21 \, a^{\frac{3}{2}} e^{\left(-4 \, f x - 4 \, e\right)} + 35 \, a^{\frac{3}{2}} e^{\left(-6 \, f x - 6 \, e\right)} + 35 \, a^{\frac{3}{2}} e^{\left(-8 \, f x - 8 \, e\right)} + 21 \, a^{\frac{3}{2}} e^{\left(-10 \, f x - 10 \, e\right)} + 7 \, a^{\frac{3}{2}} e^{\left(-12 \, f x - 12 \, e\right)} + a^{\frac{3}{2}} e^{\left(-14 \, f x - 14 \, e\right)} + a^{\frac{3}{2}}\right)} f} + \frac{32 \, e^{\left(-9 \, f x - 9 \, e\right)}}{15 \, {\left(7 \, a^{\frac{3}{2}} e^{\left(-2 \, f x - 2 \, e\right)} + 21 \, a^{\frac{3}{2}} e^{\left(-4 \, f x - 4 \, e\right)} + 35 \, a^{\frac{3}{2}} e^{\left(-6 \, f x - 6 \, e\right)} + 35 \, a^{\frac{3}{2}} e^{\left(-8 \, f x - 8 \, e\right)} + 21 \, a^{\frac{3}{2}} e^{\left(-10 \, f x - 10 \, e\right)} + 7 \, a^{\frac{3}{2}} e^{\left(-12 \, f x - 12 \, e\right)} + a^{\frac{3}{2}} e^{\left(-14 \, f x - 14 \, e\right)} + a^{\frac{3}{2}}\right)} f} - \frac{8 \, e^{\left(-11 \, f x - 11 \, e\right)}}{3 \, {\left(7 \, a^{\frac{3}{2}} e^{\left(-2 \, f x - 2 \, e\right)} + 21 \, a^{\frac{3}{2}} e^{\left(-4 \, f x - 4 \, e\right)} + 35 \, a^{\frac{3}{2}} e^{\left(-6 \, f x - 6 \, e\right)} + 35 \, a^{\frac{3}{2}} e^{\left(-8 \, f x - 8 \, e\right)} + 21 \, a^{\frac{3}{2}} e^{\left(-10 \, f x - 10 \, e\right)} + 7 \, a^{\frac{3}{2}} e^{\left(-12 \, f x - 12 \, e\right)} + a^{\frac{3}{2}} e^{\left(-14 \, f x - 14 \, e\right)} + a^{\frac{3}{2}}\right)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*e^(-3*f*x - 3*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x -
 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3
/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x
- 14*e) + a^(3/2))*f) + 32/15*e^(-5*f*x - 5*e)/((7*a^(3/2)*e^(-2*f*x - 2*e)
 + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e
^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12
*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f) - 304/35*e^(-7*f*x - 7*e)/((
7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6
*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) +
 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f) +
32/15*e^(-9*f*x - 9*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x
- 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(
3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x
 - 14*e) + a^(3/2))*f) - 8/3*e^(-11*f*x - 11*e)/((7*a^(3/2)*e^(-2*f*x - 2*e
) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*
e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 1
2*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2507 vs. $2(56) = 112$.
time = 0.45, size = 2507, normalized size = 36.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/105*(385*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^10 + 35*e^(f*x + e)*sin
h(f*x + e)^11 + 7*(275*cosh(f*x + e)^2 - 4)*e^(f*x + e)*sinh(f*x + e)^9 + 2
1*(275*cosh(f*x + e)^3 - 12*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^8 + 6*
(1925*cosh(f*x + e)^4 - 168*cosh(f*x + e)^2 + 19)*e^(f*x + e)*sinh(f*x + e)
^7 + 42*(385*cosh(f*x + e)^5 - 56*cosh(f*x + e)^3 + 19*cosh(f*x + e))*e^(f*
x + e)*sinh(f*x + e)^6 + 14*(1155*cosh(f*x + e)^6 - 252*cosh(f*x + e)^4 + 1
71*cosh(f*x + e)^2 - 2)*e^(f*x + e)*sinh(f*x + e)^5 + 14*(825*cosh(f*x + e)
^7 - 252*cosh(f*x + e)^5 + 285*cosh(f*x + e)^3 - 10*cosh(f*x + e))*e^(f*x +
 e)*sinh(f*x + e)^4 + 7*(825*cosh(f*x + e)^8 - 336*cosh(f*x + e)^6 + 570*co
sh(f*x + e)^4 - 40*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^3 + 7*(27
5*cosh(f*x + e)^9 - 144*cosh(f*x + e)^7 + 342*cosh(f*x + e)^5 - 40*cosh(f*x
 + e)^3 + 15*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 7*(55*cosh(f*x +
e)^10 - 36*cosh(f*x + e)^8 + 114*cosh(f*x + e)^6 - 20*cosh(f*x + e)^4 + 15*
cosh(f*x + e)^2)*e^(f*x + e)*sinh(f*x + e) + (35*cosh(f*x + e)^11 - 28*cosh
(f*x + e)^9 + 114*cosh(f*x + e)^7 - 28*cosh(f*x + e)^5 + 35*cosh(f*x + e)^3
)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a^2*f*cosh(f*x + e)^14 + 7*a^2*f*cosh(f*x + e)^12 + (a^2*f*e^(2*f*x + 2
*e) + a^2*f)*sinh(f*x + e)^14 + 14*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a
^2*f*cosh(f*x + e))*sinh(f*x + e)^13 + 21*a^2*f*cosh(f*x + e)^10 + 7*(13*a^
2*f*cosh(f*x + e)^2 + a^2*f + (13*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x +
 2*e))*sinh(f*x + e)^12 + 28*(13*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x +
 e) + (13*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*s
inh(f*x + e)^11 + 35*a^2*f*cosh(f*x + e)^8 + 7*(143*a^2*f*cosh(f*x + e)^4 +
 66*a^2*f*cosh(f*x + e)^2 + 3*a^2*f + (143*a^2*f*cosh(f*x + e)^4 + 66*a^2*f
*cosh(f*x + e)^2 + 3*a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^10 + 14*(143*a^2
*f*cosh(f*x + e)^5 + 110*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (
143*a^2*f*cosh(f*x + e)^5 + 110*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x +
 e))*e^(2*f*x + 2*e))*sinh(f*x + e)^9 + 35*a^2*f*cosh(f*x + e)^6 + 7*(429*a
^2*f*cosh(f*x + e)^6 + 495*a^2*f*cosh(f*x + e)^4 + 135*a^2*f*cosh(f*x + e)^
2 + 5*a^2*f + (429*a^2*f*cosh(f*x + e)^6 + 495*a^2*f*cosh(f*x + e)^4 + 135*
a^2*f*cosh(f*x + e)^2 + 5*a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 8*(429*
a^2*f*cosh(f*x + e)^7 + 693*a^2*f*cosh(f*x + e)^5 + 315*a^2*f*cosh(f*x + e)
^3 + 35*a^2*f*cosh(f*x + e) + (429*a^2*f*cosh(f*x + e)^7 + 693*a^2*f*cosh(f
*x + e)^5 + 315*a^2*f*cosh(f*x + e)^3 + 35*a^2*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e)^7 + 21*a^2*f*cosh(f*x + e)^4 + 7*(429*a^2*f*cosh(f*x +
e)^8 + 924*a^2*f*cosh(f*x + e)^6 + 630*a^2*f*cosh(f*x + e)^4 + 140*a^2*f*co
sh(f*x + e)^2 + 5*a^2*f + (429*a^2*f*cosh(f*x + e)^8 + 924*a^2*f*cosh(f*x +
 e)^6 + 630*a^2*f*cosh(f*x + e)^4 + 140*a^2*f*cosh(f*x + e)^2 + 5*a^2*f)*e^
(2*f*x + 2*e))*sinh(f*x + e)^6 + 14*(143*a^2*f*cosh(f*x + e)^9 + 396*a^2*f*
cosh(f*x + e)^7 + 378*a^2*f*cosh(f*x + e)^5 + 140*a^2*f*cosh(f*x + e)^3 + 1
5*a^2*f*cosh(f*x + e) + (143*a^2*f*cosh(f*x + e)^9 + 396*a^2*f*cosh(f*x + e
)^7 + 378*a^2*f*cosh(f*x + e)^5 + 140*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh
(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 7*a^2*f*cosh(f*x + e)^2 + 7*(
143*a^2*f*cosh(f*x + e)^10 + 495*a^2*f*cosh(f*x + e)^8 + 630*a^2*f*cosh(f*x
 + e)^6 + 350*a^2*f*cosh(f*x + e)^4 + 75*a^2*f*cosh(f*x + e)^2 + 3*a^2*f +
```

```
(143*a^2*f*cosh(f*x + e)^10 + 495*a^2*f*cosh(f*x + e)^8 + 630*a^2*f*cosh(f*
x + e)^6 + 350*a^2*f*cosh(f*x + e)^4 + 75*a^2*f*cosh(f*x + e)^2 + 3*a^2*f)*
e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 28*(13*a^2*f*cosh(f*x + e)^11 + 55*a^2*f
*cosh(f*x + e)^9 + 90*a^2*f*cosh(f*x + e)^7 + 70*a^2*f*cosh(f*x + e)^5 + 25
*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e) + (13*a^2*f*cosh(f*x + e)^11
 + 55*a^2*f*cosh(f*x + e)^9 + 90*a^2*f*cosh(f*x + e)^7 + 70*a^2*f*cosh(f*x
+ e)^5 + 25*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))
*sinh(f*x + e)^3 + a^2*f + 7*(13*a^2*f*cosh(f*x + e)^12 + 66*a^2*f*cosh(f*x
 + e)^10 + 135*a^2*f*cosh(f*x + e)^8 + 140*a^2*f*cosh(f*x + e)^6 + 75*a^2*f
*cosh(f*x + e)^4 + 18*a^2*f*cosh(f*x + e)^2 + a^2*f + (13*a^2*f*cosh(f*x +
e)^12 + 66*a^2*f*cosh(f*x + e)^10 + 135*a^2*f*cosh(f*x + e)^8 + 140*a^2*f*c
osh(f*x + e)^6 + 75*a^2*f*cosh(f*x + e)^4 + 18*a^2*f*cosh(f*x + e)^2 + a^2*
f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^14 + 7*a^2*f*cos
h(f*x + e)^12 + 21*a^2*f*cosh(f*x + e)^10 + 35*a^2*f*cosh(f*x + e)^8 + 35*a
^2*f*cosh(f*x + e)^6 + 21*a^2*f*cosh(f*x + e)^4 + 7*a^2*f*cosh(f*x + e)^2 +
 a^2*f)*e^(2*f*x + 2*e) + 14*(a^2*f*cosh(f*x + e)^13 + 6*a^2*f*cosh(f*x + e
)^11 + 15*a^2*f*cosh(f*x + e)^9 + 20*a^2*f*cosh(f*x + e)^7 + 15*a^2*f*cosh(
f*x + e)^5 + 6*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*
x + e)^13 + 6*a^2*f*cosh(f*x + e)^11 + 15*a^2*f*cosh(f*x + e)^9 + 20*a^2*f*
cosh(f*x + e)^7 + 15*a^2*f*cosh(f*x + e)^5 + 6*a^2*f*cosh(f*x + e)^3 + a^2*
f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{5}\left(e + fx\right)}{\left(a\left(\sinh^{2}\left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**5/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 0.16, size = 457, normalized size = 6.72

$$\frac{464\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{15\,a^2\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}^3\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{16\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{3\,a^2\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}^2\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{3072\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{35\,a^2\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}^4\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}+\frac{4736\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{35\,a^2\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}^5\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{768\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{7\,a^2\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}^6\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}+\frac{256\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{7\,a^2\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}^7\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] (464*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(15*a^2*f*(exp(2*e + 2*f*x) + 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (16
*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3*a
^2*f*(exp(2*e + 2*f*x) + 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (3072*ex
p(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(35*a^2
*f*(exp(2*e + 2*f*x) + 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) + (4736*exp(
3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(35*a^2*f
*(exp(2*e + 2*f*x) + 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x))) - (768*exp(3*e
 + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(7*a^2*f*(ex
p(2*e + 2*f*x) + 1)^6*(exp(e + f*x) + exp(3*e + 3*f*x))) + (256*exp(3*e + 3
*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(7*a^2*f*(exp(2*
e + 2*f*x) + 1)^7*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

3.448 $\int \frac{\tanh^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=44

$$\frac{a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/5*a/f/(a*cosh(f*x+e)^2)^(5/2)-1/3/f/(a*cosh(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3284, 16, 45}

$$\frac{a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] a/(5*f*(a*Cosh[e + f*x]^2)^(5/2)) - 1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
```

```
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\tanh^3(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{1-x}{x^2(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= -\frac{a^2\text{Subst}\left(\int \frac{1-x}{(ax)^{7/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= -\frac{a^2\text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}}-\frac{1}{a(ax)^{5/2}}\right)\,dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= \frac{a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.09, size = 34, normalized size = 0.77

$$\frac{a\left(3-5\cosh^2(e+fx)\right)}{15f\left(a\cosh^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*(3 - 5*Cosh[e + f*x]^2))/(15*f*(a*Cosh[e + f*x]^2)^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.18, size = 44, normalized size = 1.00

method	result	size
default	$\frac{\text{`}int/indef0\text{`}\left(\frac{\sinh^3(fx+e)}{\cosh(fx+e)^6 a\sqrt{a\left(\cosh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$	44
risch	$-\frac{8\left(5\,e^{4fx+4e}-2\,e^{2fx+2e}+5\right)e^{2fx+2e}}{15f\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\left(e^{2fx+2e}+1\right)^4 a}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3/cosh(f*x+e)^6/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(38) = 76$.
time = 0.52, size = 286, normalized size = 6.50

$$-\frac{8\,e^{(-3\,fx-3\,e)}}{3\,\left(5\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+10\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+10\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+5\,a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}e^{(-10\,fx-10\,e)}+a^{\frac{3}{2}}\right)f}+\frac{16\,e^{(-5\,fx-5\,e)}}{15\,\left(5\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+10\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+10\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+5\,a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}e^{(-10\,fx-10\,e)}+a^{\frac{3}{2}}\right)f}-\frac{8\,e^{(-7\,fx-7\,e)}}{3\,\left(5\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+10\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+10\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+5\,a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}e^{(-10\,fx-10\,e)}+a^{\frac{3}{2}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*e^(-3*f*x - 3*e)/((5*a^(3/2)*e^(-2*f*x - 2*e) + 10*a^(3/2)*e^(-4*f*x -
 4*e) + 10*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*
e^(-10*f*x - 10*e) + a^(3/2))*f) + 16/15*e^(-5*f*x - 5*e)/((5*a^(3/2)*e^(-2
*f*x - 2*e) + 10*a^(3/2)*e^(-4*f*x - 4*e) + 10*a^(3/2)*e^(-6*f*x - 6*e) + 5
*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) + a^(3/2))*f) - 8/3*
e^(-7*f*x - 7*e)/((5*a^(3/2)*e^(-2*f*x - 2*e) + 10*a^(3/2)*e^(-4*f*x - 4*e)
 + 10*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-1
0*f*x - 10*e) + a^(3/2))*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1400 vs. $2(36) = 72$.
time = 0.43, size = 1400, normalized size = 31.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/15*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f*
x + e)^7 + (105*cosh(f*x + e)^2 - 2)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*co
sh(f*x + e)^3 - 2*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 5*(35*cosh(f
*x + e)^4 - 4*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 5*(21*cosh
(f*x + e)^5 - 4*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(7*cosh(f*x + e)^6 - 2*cosh(f*x + e)^4 + 3*cosh(f*x + e)^2)*e^(f*x
+ e)*sinh(f*x + e) + (5*cosh(f*x + e)^7 - 2*cosh(f*x + e)^5 + 5*cosh(f*x +
e)^3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*
x - e)/(a^2*f*cosh(f*x + e)^10 + 5*a^2*f*cosh(f*x + e)^8 + (a^2*f*e^(2*f*x
+ 2*e) + a^2*f)*sinh(f*x + e)^10 + 10*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e)
```

```
+ a^2*f*cosh(f*x + e))*sinh(f*x + e)^9 + 10*a^2*f*cosh(f*x + e)^6 + 5*(9*a^
2*f*cosh(f*x + e)^2 + a^2*f + (9*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x +
2*e))*sinh(f*x + e)^8 + 40*(3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) +
 (3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x
+ e)^7 + 10*a^2*f*cosh(f*x + e)^4 + 10*(21*a^2*f*cosh(f*x + e)^4 + 14*a^2*f
*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*cosh(f*x + e)^4 + 14*a^2*f*cosh(f*x +
e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 4*(63*a^2*f*cosh(f*x + e)^
5 + 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (63*a^2*f*cosh(f*x
+ e)^5 + 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e)
)*sinh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^2 + 10*(21*a^2*f*cosh(f*x + e)^6
+ 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*c
osh(f*x + e)^6 + 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 + a^2*
f)*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 40*(3*a^2*f*cosh(f*x + e)^7 + 7*a^2*f
*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (3*a^2*f
*cosh(f*x + e)^7 + 7*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 + a^2*
f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 + a^2*f + 5*(9*a^2*f*cosh
(f*x + e)^8 + 28*a^2*f*cosh(f*x + e)^6 + 30*a^2*f*cosh(f*x + e)^4 + 12*a^2*
f*cosh(f*x + e)^2 + a^2*f + (9*a^2*f*cosh(f*x + e)^8 + 28*a^2*f*cosh(f*x +
e)^6 + 30*a^2*f*cosh(f*x + e)^4 + 12*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*
x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^10 + 5*a^2*f*cosh(f*x + e)
^8 + 10*a^2*f*cosh(f*x + e)^6 + 10*a^2*f*cosh(f*x + e)^4 + 5*a^2*f*cosh(f*x
 + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 10*(a^2*f*cosh(f*x + e)^9 + 4*a^2*f*cosh
(f*x + e)^7 + 6*a^2*f*cosh(f*x + e)^5 + 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cos
h(f*x + e) + (a^2*f*cosh(f*x + e)^9 + 4*a^2*f*cosh(f*x + e)^7 + 6*a^2*f*cos
h(f*x + e)^5 + 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{3}\left(e + fx\right)}{\left(a \left(\sinh^{2}\left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 0.93, size = 305, normalized size = 6.93

$$\frac{272\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{15\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^3\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{16\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{3\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^2\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}-\frac{128\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{5\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^4\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}+\frac{64\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{5\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^5\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] (272*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(15*a^2*f*(exp(2*e + 2*f*x) + 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (16
*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3*a
^2*f*(exp(2*e + 2*f*x) + 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (128*exp
(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a^2*f
*(exp(2*e + 2*f*x) + 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) + (64*exp(3*e
+ 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a^2*f*(exp
(2*e + 2*f*x) + 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

3.449 $\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

```
[Out] -1/3/f/(a*cosh(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3255, 3284, 16, 32}

$$-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*1/(f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\tanh(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= \frac{a\text{Subst}\left(\int \frac{1}{(ax)^{5/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= -\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 21, normalized size = 1.00

$$-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*1/(f*(a*Cosh[e + f*x]^2)^(3/2))
```

Maple [A]
time = 0.62, size = 20, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{1}{3\left(a+a\left(\sinh^2(fx+e)\right)\right)^{\frac{3}{2}} f}$	20
default	$-\frac{1}{3\left(a+a\left(\sinh^2(fx+e)\right)\right)^{\frac{3}{2}} f}$	20
risch	$-\frac{8\,\mathrm{e}^{2fx+2e}}{3f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a\,\mathrm{e}^{-2fx-2e}}\,\left(\mathrm{e}^{2fx+2e}+1\right)^2 a}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a+a*sinh(f*x+e)^2)^(3/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(18) = 36$.

time = 0.56, size = 65, normalized size = 3.10

$$-\frac{8\,e^{\left(-3\,fx-3\,e\right)}}{3\,\left(3\,a^{\frac{3}{2}}e^{\left(-2\,fx-2\,e\right)}+3\,a^{\frac{3}{2}}e^{\left(-4\,fx-4\,e\right)}+a^{\frac{3}{2}}e^{\left(-6\,fx-6\,e\right)}+a^{\frac{3}{2}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*e^(-3*f*x - 3*e)/((3*a^(3/2)*e^(-2*f*x - 2*e) + 3*a^(3/2)*e^(-4*f*x -
4*e) + a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2))*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(17) = 34$.
time = 0.41, size = 608, normalized size = 28.95

$$-\frac{8\,\left(\cosh\left(fx+e\right)^{3}e^{\left(fx+e\right)}+3\,\cosh\left(fx+e\right)^{2}e^{\left(fx+e\right)}\sinh\left(fx+e\right)+3\,\cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)^{2}+e^{\left(fx+e\right)}\sinh\left(fx+e\right)^{3}\right)\sqrt{ae^{\left(4\,fx+4\,e\right)}+2\,ae^{\left(2\,fx+2\,e\right)}+a}\,e^{\left(-fx-e\right)}}{3\,\left(a^{2}f\cosh\left(fx+e\right)^{6}+3\,a^{2}f\cosh\left(fx+e\right)^{4}+\left(a^{2}fe^{\left(2\,fx+2\,e\right)}+a^{2}f\right)\sinh\left(fx+e\right)^{6}+6\,\left(a^{2}f\cosh\left(fx+e\right)e^{\left(2\,fx+2\,e\right)}+a^{2}f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^{5}+3\,a^{2}f\cosh\left(fx+e\right)^{2}+3\,\left(5\,a^{2}f\cosh\left(fx+e\right)^{2}+a^{2}f+\left(5\,a^{2}f\cosh\left(fx+e\right)^{2}+a^{2}f\right)e^{\left(2\,fx+2\,e\right)}\right)\sinh\left(fx+e\right)^{4}+4\,\left(5\,a^{2}f\cosh\left(fx+e\right)^{3}+3\,a^{2}f\cosh\left(fx+e\right)+\left(5\,a^{2}f\cosh\left(fx+e\right)^{3}+3\,a^{2}f\cosh\left(fx+e\right)\right)e^{\left(2\,fx+2\,e\right)}\right)\sinh\left(fx+e\right)^{3}+a^{2}f+3\,\left(5\,a^{2}f\cosh\left(fx+e\right)^{4}+6\,a^{2}f\cosh\left(fx+e\right)^{2}+a^{2}f+\left(5\,a^{2}f\cosh\left(fx+e\right)^{4}+6\,a^{2}f\cosh\left(fx+e\right)^{2}+a^{2}f\right)e^{\left(2\,fx+2\,e\right)}\right)\sinh\left(fx+e\right)^{2}+\left(a^{2}f\cosh\left(fx+e\right)^{6}+3\,a^{2}f\cosh\left(fx+e\right)^{4}+3\,a^{2}f\cosh\left(fx+e\right)^{2}+a^{2}f\right)e^{\left(2\,fx+2\,e\right)}+6\,\left(a^{2}f\cosh\left(fx+e\right)^{5}+2\,a^{2}f\cosh\left(fx+e\right)^{3}+a^{2}f\cosh\left(fx+e\right)+\left(a^{2}f\cosh\left(fx+e\right)^{5}+2\,a^{2}f\cosh\left(fx+e\right)^{3}+a^{2}f\cosh\left(fx+e\right)\right)e^{\left(2\,fx+2\,e\right)}\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(f*x + e)^3*e^(f*x + e) + 3*cosh(f*x + e)^2*e^(f*x + e)*sinh(f*x
+ e) + 3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x +
 e)^3)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*
f*cosh(f*x + e)^6 + 3*a^2*f*cosh(f*x + e)^4 + (a^2*f*e^(2*f*x + 2*e) + a^2*
f)*sinh(f*x + e)^6 + 6*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*
x + e))*sinh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^2 + 3*(5*a^2*f*cosh(f*x + e
)^2 + a^2*f + (5*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x +
 e)^4 + 4*(5*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e) + (5*a^2*f*cosh(
f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 + a^2*
f + 3*(5*a^2*f*cosh(f*x + e)^4 + 6*a^2*f*cosh(f*x + e)^2 + a^2*f + (5*a^2*f
*cosh(f*x + e)^4 + 6*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f
*x + e)^2 + (a^2*f*cosh(f*x + e)^6 + 3*a^2*f*cosh(f*x + e)^4 + 3*a^2*f*cosh
(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 6*(a^2*f*cosh(f*x + e)^5 + 2*a^2*f*c
osh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^5 + 2*a^2*f*cos
h(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(e+fx\right)}{\left(a\left(\sinh^{2}\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**B**]
time = 0.88, size = 58, normalized size = 2.76

$$-\frac{16\,{\mathrm{e}}^{4\,e+4\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{3\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] -(16*exp(4*e + 4*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(3*a^2*f*(exp(2*e + 2*f*x) + 1)^4)
```

3.450 $\int \frac{\coth(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{1}{af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a*cosh(f*x+e)^2)
^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3255, 3284, 53, 65, 212}

$$\frac{1}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a*Cos
h[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\coth(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= \frac{1}{af\sqrt{a\cosh^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2af}\\
&= \frac{1}{af\sqrt{a\cosh^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{a^2f}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 41, normalized size = 0.77

$$\frac{1+\cosh(e+fx)\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (1 + Cosh[e + f*x]*Log[Tanh[(e + f*x)/2]])/(a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.03, size = 44, normalized size = 0.83

method	result
default	$\frac{\text{` }int/indef0\text{`}\left(\frac{1}{\cosh(fx+e)^2\sinh(fx+e)a\sqrt{a\left(\cosh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$
risch	$\frac{2}{a\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\,f}-\frac{\ln\left(e^{fx}+e^{-e}\right)\left(e^{2fx+2e}+1\right)e^{-fx-e}}{f\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\,a}+\frac{\ln\left(e^{fx}-e^{-e}\right)\left(e^{2fx+2e}+1\right)e^{-fx-e}}{f\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(1/cosh(f*x+e)^2/sinh(f*x+e)/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [A]
time = 0.53, size = 80, normalized size = 1.51

$$\frac{2\sqrt{a}\,e^{(-fx-e)}}{\left(a^2e^{(-2\,fx-2\,e)}+a^2\right)f}-\frac{\log\left(e^{(-fx-e)}+1\right)}{a^{\frac{3}{2}}f}+\frac{\log\left(e^{(-fx-e)}-1\right)}{a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*e^(-f*x - e)/((a^2*e^(-2*f*x - 2*e) + a^2)*f) - log(e^(-f*x - e)
+ 1)/(a^(3/2)*f) + log(e^(-f*x - e) - 1)/(a^(3/2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(45) = 90$.
time = 0.42, size = 271, normalized size = 5.11

$$\frac{\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\left(2\,\cosh\left(fx+e\right)e^{(fx+e)}+\left(2\,\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)+e^{(fx+e)}\sinh\left(fx+e\right)^2+\left(\cosh\left(fx+e\right)^2+1\right)e^{(fx+e)}\right)\log\left(\frac{\cosh(fx+e)+\sinh(fx+e)-1}{\cosh(fx+e)+\sinh(fx+e)+1}\right)+2\,e^{(fx+e)}\sinh\left(fx+e\right)\right)e^{(-fx-e)}}{a^2f\cosh\left(fx+e\right)^2+a^2f+\left(a^2fe^{(2\,fx+2\,e)}+a^2f\right)\sinh\left(fx+e\right)^2+\left(a^2f\cosh\left(fx+e\right)^2+a^2f\right)e^{(2\,fx+2\,e)}+2\left(a^2f\cosh\left(fx+e\right)e^{(2\,fx+2\,e)}+a^2f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(2*cosh(f*x + e)*e^(f*x +
 e) + (2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e
)^2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e)
 - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1)) + 2*e^(f*x + e)*sinh(f*x + e))*e
^(-f*x - e)/(a^2*f*cosh(f*x + e)^2 + a^2*f + (a^2*f*e^(2*f*x + 2*e) + a^2*f
)*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 2*(a^
2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth{\left(e + fx \right)}}{\left(a \left(\sinh^{2}{\left(e + fx \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{coth}\left(e+f\,x\right)}{{\left(a\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)/(a + a*sinh(e + f*x)^2)^(3/2), x)
```

3.451 $\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\sqrt{a\cosh^2(e+fx)}\,\mathrm{csch}^2(e+fx)}{2a^2f}$$

```
[Out] 1/2*arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/2*csch(f*x+e)^2*(a
*cosh(f*x+e)^2)^(1/2)/a^2/f
```

Rubi [A]

time = 0.10, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3255, 3284, 16, 44, 65, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\mathrm{csch}^2(e+fx)\sqrt{a\cosh^2(e+fx)}}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(2*a^(3/2)*f) - (Sqrt[a*Cosh[e + f
*x]^2]*Csch[e + f*x]^2)/(2*a^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\coth^3(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx$$

$$=\frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(ax)^{3/2}}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$

$$=\frac{\text{Subst}\left(\int \frac{1}{(1-x)^2\sqrt{ax}}\,dx,x,\cosh^2(e+fx)\right)}{2af}$$

$$=-\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2a^2f}+\frac{\text{Subst}\Bigg(\int \frac{1}{(1-x)\sqrt{ax}}\,dx,x,\cosh^2(e+}{4af}$$

$$=-\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2a^2f}+\frac{\text{Subst}\Bigg(\int \frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a\cosh^2(e+f}}{2a^2f}$$

$$=\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2a^2f}$$

Mathematica [A]
time = 0.12, size = 67, normalized size = 1.02

$$-\frac{\cosh^3(e+fx)\left(\text{csch}^2\left(\frac{1}{2}(e+fx)\right)+4\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)+\text{sech}^2\left(\frac{1}{2}(e+fx)\right)\right)}{8f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*(Cosh[e + f*x]^3*(Csch[(e + f*x)/2]^2 + 4*Log[Tanh[(e + f*x)/2]] + Sec
h[(e + f*x)/2]^2))/(f*(a*Cosh[e + f*x]^2)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.20, size = 36, normalized size = 0.55

method	result
default	$\frac{`int/indef0`\left(\frac{1}{\sinh(fx+e)^3 a\sqrt{a\left(\cosh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$

risch	$-\frac{(e^{2fx+2e}+1)^2}{a\sqrt{(e^{2fx+2e}+1)^2 a\, e^{-2fx-2e}}\, f(e^{2fx+2e}-1)^2}+\frac{\ln(e^{fx}+e^{-e})(e^{2fx+2e}+1)e^{-fx-e}}{2f\sqrt{(e^{2fx+2e}+1)^2 a\, e^{-2fx-2e}}\, a}-\frac{\ln(e^{fx}-e^{-e})(e^{2fx}}{2f\sqrt{(e^{2fx+2e}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 'int/indef0'(1/sinh(f*x+e)^3/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**A**]
time = 0.54, size = 106, normalized size = 1.61

$$\frac{e^{(-fx-e)}+e^{(-3\,fx-3\,e)}}{\left(2\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}-a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}-a^{\frac{3}{2}}\right)f}+\frac{\log\left(e^{(-fx-e)}+1\right)}{2\,a^{\frac{3}{2}}f}-\frac{\log\left(e^{(-fx-e)}-1\right)}{2\,a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (e^(-f*x - e) + e^(-3*f*x - 3*e))/((2*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^
(-4*f*x - 4*e) - a^(3/2))*f) + 1/2*log(e^(-f*x - e) + 1)/(a^(3/2)*f) - 1/2*
log(e^(-f*x - e) - 1)/(a^(3/2)*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. 2(54) = 108.
time = 0.60, size = 565, normalized size = 8.56

$$-\frac{\left(6\,\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^2+2\,e^{(fx+e)}\sinh\left(fx+e\right)^3+2\left(3\,\cosh\left(fx+e\right)^2+1\right)e^{(fx+e)}\sinh\left(fx+e\right)+2\left(\cosh\left(fx+e\right)^3+\cosh\left(fx+e\right)\right)e^{(fx+e)}-\left(4\,\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^3+e^{(fx+e)}\sinh\left(fx+e\right)^4+2\left(3\,\cosh\left(fx+e\right)^2-1\right)e^{(fx+e)}\sinh\left(fx+e\right)^2+4\left(\cosh\left(fx+e\right)^3-\cosh\left(fx+e\right)\right)e^{(fx+e)}\sinh\left(fx+e\right)+\left(\cosh\left(fx+e\right)^4-2\,\cosh\left(fx+e\right)^2+1\right)e^{(fx+e)}\right)\log\left(\frac{\cosh(fx+e)+\sinh(fx+e)+1}{\cosh(fx+e)+\sinh(fx+e)-1}\right)\right)\sqrt{ae^{(4\,fx+4\,e)}+2\,ae^{(2\,fx+2\,e)}+a}\,e^{(-fx-e)}}{2\left(a^2f\cosh\left(fx+e\right)^4-2\,a^2f\cosh\left(fx+e\right)^2+\left(a^2fe^{(2\,fx+2\,e)}+a^2f\right)\sinh\left(fx+e\right)^4+4\left(a^2f\cosh\left(fx+e\right)e^{(2\,fx+2\,e)}+a^2f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^3+a^2f+2\left(3\,a^2f\cosh\left(fx+e\right)^2-a^2f+\left(3\,a^2f\cosh\left(fx+e\right)^2-a^2f\right)e^{(2\,fx+2\,e)}\right)\sinh\left(fx+e\right)^2+\left(a^2f\cosh\left(fx+e\right)^4-2\,a^2f\cosh\left(fx+e\right)^2+a^2f\right)e^{(2\,fx+2\,e)}+4\left(a^2f\cosh\left(fx+e\right)^3-a^2f\cosh\left(fx+e\right)+\left(a^2f\cosh\left(fx+e\right)^3-a^2f\cosh\left(fx+e\right)\right)e^{(2\,fx+2\,e)}\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + 2*e^(f*x + e)*sinh(f*x
+ e)^3 + 2*(3*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + 2*(cosh(f*x
+ e)^3 + cosh(f*x + e))*e^(f*x + e) - (4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x
 + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*e^(f*x +
e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f
*x + e) + (cosh(f*x + e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(
f*x + e) + sinh(f*x + e) + 1)/(cosh(f*x + e) + sinh(f*x + e) - 1)))*sqrt(a*
e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e
)^4 - 2*a^2*f*cosh(f*x + e)^2 + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x +
e)^4 + 4*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f
*x + e)^3 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2 - a^2*f + (3*a^2*f*cosh(f*x
+ e)^2 - a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^4 -
 2*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 4*(a^2*f*cosh(f*x + e)^
```

```
3 - a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*e^(
2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(e+fx)}{\left(a\left(\sinh^2(e+fx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\mathrm{coth}\left(e+f\,x\right)}^3}{{\left(a\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(3/2), x)
```

3.452 $\int \frac{\tanh^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{\text{ArcTan}(\sinh(e+fx))\cosh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}+\frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\text{sech}^2(e+fx)\tanh(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] 1/8*arctan(sinh(f*x+e))*cosh(f*x+e)/a/f/(a*cosh(f*x+e)^2)^(1/2)+1/8*tanh(f*
x+e)/a/f/(a*cosh(f*x+e)^2)^(1/2)-1/4*sech(f*x+e)^2*tanh(f*x+e)/a/f/(a*cosh(
f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3255, 3286, 2691, 3853, 3855}

$$\frac{\cosh(e+fx)\text{ArcTan}(\sinh(e+fx))}{8af\sqrt{a\cosh^2(e+fx)}}+\frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)\text{sech}^2(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(8*a*f*Sqrt[a*Cosh[e + f*x]^2]) + Tan
h[e + f*x]/(8*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Sech[e + f*x]^2*Tanh[e + f*x]
)/(4*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
```

```
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\tanh^2(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx\\
&= \frac{\cosh(e+fx)\int \operatorname{sech}^3(e+fx)\tanh^2(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}}\\
&= -\frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}+\frac{\cosh(e+fx)\int \operatorname{sech}^3(e+fx)\,dx}{4a\sqrt{a\cosh^2(e+fx)}}\\
&= \frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}+\frac{\cosh(e+fx)\int \mathrm{se}}{8a\sqrt{a\cosh^2(}}\\
&= \frac{\tan^{-1}(\sinh(e+fx))\cosh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}+\frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\operatorname{sech}^2(e+f}{4af\sqrt{a\,\mathrm{c}}}
\end{aligned}$$

Mathematica [**A**]
time = 0.07, size = 58, normalized size = 0.55

$$\frac{\mathrm{ArcTan}(\sinh(e+fx))\cosh(e+fx)+\left(1-2\operatorname{sech}^2(e+fx)\right)\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + (1 - 2*Sech[e + f*x]^2)*Tanh[e + f*x
])/(8*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [A]
time = 1.27, size = 69, normalized size = 0.65

method	result
default	$\frac{\arctan(\sinh(fx+e))(\cosh^4(fx+e))+(\cosh^2(fx+e))\sinh(fx+e)-2\sinh(fx+e)}{8a\cosh(fx+e)^3\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$
risch	$\frac{\mathrm{e}^{6fx+6e}-7\,\mathrm{e}^{4fx+4e}+7\,\mathrm{e}^{2fx+2e}-1}{4a(\mathrm{e}^{2fx+2e}+1)^3\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2a\,\mathrm{e}^{-2fx-2e}}\,f}+\frac{i\ln(\mathrm{e}^{fx}+i\mathrm{e}^{-e})(\mathrm{e}^{2fx+2e}+1)\mathrm{e}^{-fx-e}}{8f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2a\,\mathrm{e}^{-2fx-2e}}\,a}-\frac{i\ln(\mathrm{e}^{fx}-i\mathrm{e}^{-e})(\mathrm{e}^{2fx}}{8f\sqrt{(\mathrm{e}^{2fx+2e}+1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a*(arctan(sinh(f*x+e))*cosh(f*x+e)^4+cosh(f*x+e)^2*sinh(f*x+e)-2*sinh(f
*x+e))/cosh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(102) = 204.
time = 0.50, size = 395, normalized size = 3.73

$$-\frac{\frac{3\,e^{(-fx-e)}+11\,e^{(-3\,fx-3\,e)}-11\,e^{(-5\,fx-5\,e)}-3\,e^{(-7\,fx-7\,e)}}{4\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+6\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+4\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}}-\frac{3\,\arctan\left(e^{(-fx-e)}\right)}{a^{\frac{3}{2}}}}{8\,f}+\frac{15\,e^{(-fx-e)}+55\,e^{(-3\,fx-3\,e)}+73\,e^{(-5\,fx-5\,e)}-15\,e^{(-7\,fx-7\,e)}}{48\left(4\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+6\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+4\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}\right)f}+\frac{15\,e^{(-fx-e)}-73\,e^{(-3\,fx-3\,e)}-55\,e^{(-5\,fx-5\,e)}-15\,e^{(-7\,fx-7\,e)}}{48\left(4\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+6\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+4\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}\right)f}-\frac{5\,\arctan\left(e^{(-fx-e)}\right)}{8\,a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*((3*e^(-f*x - e) + 11*e^(-3*f*x - 3*e) - 11*e^(-5*f*x - 5*e) - 3*e^(-7
*f*x - 7*e))/(4*a^(3/2)*e^(-2*f*x - 2*e) + 6*a^(3/2)*e^(-4*f*x - 4*e) + 4*a
^(3/2)*e^(-6*f*x - 6*e) + a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)) - 3*arctan(e^
(-f*x - e))/a^(3/2))/f + 1/48*(15*e^(-f*x - e) + 55*e^(-3*f*x - 3*e) + 73*e
^(-5*f*x - 5*e) - 15*e^(-7*f*x - 7*e))/((4*a^(3/2)*e^(-2*f*x - 2*e) + 6*a^(
3/2)*e^(-4*f*x - 4*e) + 4*a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2)*e^(-8*f*x - 8*
e) + a^(3/2))*f) + 1/48*(15*e^(-f*x - e) - 73*e^(-3*f*x - 3*e) - 55*e^(-5*f
*x - 5*e) - 15*e^(-7*f*x - 7*e))/((4*a^(3/2)*e^(-2*f*x - 2*e) + 6*a^(3/2)*e
^(-4*f*x - 4*e) + 4*a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2)*e^(-8*f*x - 8*e) + a
^(3/2))*f) - 5/8*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1423 vs. 2(94) = 188.
time = 0.44, size = 1423, normalized size = 13.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(7*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + e^(f*x + e)*sinh(f*x + e
)^7 + 7*(3*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^5 + 35*(cosh(f*x
+ e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 7*(5*cosh(f*x + e)^4
- 10*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 7*(3*cosh(f*x + e)^
5 - 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (7*
cosh(f*x + e)^6 - 35*cosh(f*x + e)^4 + 21*cosh(f*x + e)^2 - 1)*e^(f*x + e)*
sinh(f*x + e) + (8*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + e^(f*x + e)*
sinh(f*x + e)^8 + 4*(7*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^6 + 8
*(7*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 2*(35*
cosh(f*x + e)^4 + 30*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^4 + 8*(
7*cosh(f*x + e)^5 + 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(
f*x + e)^3 + 4*(7*cosh(f*x + e)^6 + 15*cosh(f*x + e)^4 + 9*cosh(f*x + e)^2
+ 1)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(cosh(f*x + e)^7 + 3*cosh(f*x + e)^5 +
 3*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x +
 e)^8 + 4*cosh(f*x + e)^6 + 6*cosh(f*x + e)^4 + 4*cosh(f*x + e)^2 + 1)*e^(f
*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) + (cosh(f*x + e)^7 - 7*cosh(
f*x + e)^5 + 7*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*
x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^8 + 4
*a^2*f*cosh(f*x + e)^6 + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^8 +
8*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e)
^7 + 6*a^2*f*cosh(f*x + e)^4 + 4*(7*a^2*f*cosh(f*x + e)^2 + a^2*f + (7*a^2*
f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 8*(7*a^2*f*co
sh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e) + (7*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*
cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 4*a^2*f*cosh(f*x + e)^2 +
 2*(35*a^2*f*cosh(f*x + e)^4 + 30*a^2*f*cosh(f*x + e)^2 + 3*a^2*f + (35*a^2
*f*cosh(f*x + e)^4 + 30*a^2*f*cosh(f*x + e)^2 + 3*a^2*f)*e^(2*f*x + 2*e))*s
inh(f*x + e)^4 + 8*(7*a^2*f*cosh(f*x + e)^5 + 10*a^2*f*cosh(f*x + e)^3 + 3*
a^2*f*cosh(f*x + e) + (7*a^2*f*cosh(f*x + e)^5 + 10*a^2*f*cosh(f*x + e)^3 +
 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 + a^2*f + 4*(7*a^2
*f*cosh(f*x + e)^6 + 15*a^2*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f*x + e)^2 + a
^2*f + (7*a^2*f*cosh(f*x + e)^6 + 15*a^2*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f
*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^
8 + 4*a^2*f*cosh(f*x + e)^6 + 6*a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x +
e)^2 + a^2*f)*e^(2*f*x + 2*e) + 8*(a^2*f*cosh(f*x + e)^7 + 3*a^2*f*cosh(f*x
 + e)^5 + 3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x +
 e)^7 + 3*a^2*f*cosh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x
+ e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(e+fx)}{\left(a\left(\sinh^2(e+fx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**2/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(e+f\,x)^2}{\left(a\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(3/2), x)
```

3.453 $\int \frac{\coth^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{\operatorname{ArcTan}(\sinh(e+fx))\cosh(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -arctan(sinh(f*x+e))*cosh(f*x+e)/a/f/(a*cosh(f*x+e)^2)^(1/2)-coth(f*x+e)/a/
f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3255, 3286, 2701, 327, 213}

$$-\frac{\cosh(e+fx)\operatorname{ArcTan}(\sinh(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(a*f*Sqrt[a*Cosh[e + f*x]^2])) - Co
th[e + f*x]/(a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\coth^2(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx\\
&= \frac{\cosh(e+fx)\int \operatorname{csch}^2(e+fx)\operatorname{sech}(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}}\\
&= -\frac{(i\cosh(e+fx))\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}\\
&= -\frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}} - \frac{(i\cosh(e+fx))\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\operatorname{csch}(e\right.}{af\sqrt{a\cosh^2(e+fx)}}\\
&= -\frac{\tan^{-1}(\sinh(e+fx))\cosh(e+fx)}{af\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 46, normalized size = 0.72

$$
-\frac{\coth(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\sinh^2(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Coth[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[e + f*x]^2])/(a*f*Sq
rt[a*Cosh[e + f*x]^2]))
```

Maple [A]
time = 1.36, size = 51, normalized size = 0.80

method	result
default	$-\frac{\cosh(fx+e)(\arctan(\sinh(fx+e))\sinh(fx+e)+1)}{a\sinh(fx+e)\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}\,f}$
risch	$-\frac{2(\mathrm{e}^{2fx+2e}+1)}{a\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\,f(\mathrm{e}^{2fx+2e}-1)}+\frac{i\ln(\mathrm{e}^{fx}-i\mathrm{e}^{-e})(\mathrm{e}^{2fx+2e}+1)\mathrm{e}^{-fx-e}}{f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\,a}-\frac{i\ln(\mathrm{e}^{fx}+i\mathrm{e}^{-e})(\mathrm{e}^{2fx+2e}}{f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*cosh(f*x+e)*(arctan(sinh(f*x+e))*sinh(f*x+e)+1)/sinh(f*x+e)/(a*cosh(f*
x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(65) = 130.
time = 0.49, size = 341, normalized size = 5.33

$$-\frac{\frac{3\sqrt{a}\,e^{(-fx-e)}+2\sqrt{a}\,e^{(-3fx-3e)}+3\sqrt{a}\,e^{(-5fx-5e)}}{a^{2}e^{(-2fx-2e)}-a^{2}e^{(-4fx-4e)}-a^{2}e^{(-6fx-6e)}+a^{2}}-\frac{3\arctan\left(e^{(-fx-e)}\right)}{a^{\frac{3}{2}}}}{2f}-\frac{5\sqrt{a}\,e^{(-fx-e)}+6\sqrt{a}\,e^{(-3fx-3e)}-3\sqrt{a}\,e^{(-5fx-5e)}}{4\left(a^{2}e^{(-2fx-2e)}-a^{2}e^{(-4fx-4e)}-a^{2}e^{(-6fx-6e)}+a^{2}\right)f}+\frac{3\sqrt{a}\,e^{(-fx-e)}-6\sqrt{a}\,e^{(-3fx-3e)}-5\sqrt{a}\,e^{(-5fx-5e)}}{4\left(a^{2}e^{(-2fx-2e)}-a^{2}e^{(-4fx-4e)}-a^{2}e^{(-6fx-6e)}+a^{2}\right)f}+\frac{\arctan\left(e^{(-fx-e)}\right)}{2a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*((3*sqrt(a)*e^(-f*x - e) + 2*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-
5*f*x - 5*e))/(a^2*e^(-2*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*x
- 6*e) + a^2) - 3*arctan(e^(-f*x - e))/a^(3/2))/f - 1/4*(5*sqrt(a)*e^(-f*x
- e) + 6*sqrt(a)*e^(-3*f*x - 3*e) - 3*sqrt(a)*e^(-5*f*x - 5*e))/((a^2*e^(-2
*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*x - 6*e) + a^2)*f) + 1/4*(
3*sqrt(a)*e^(-f*x - e) - 6*sqrt(a)*e^(-3*f*x - 3*e) - 5*sqrt(a)*e^(-5*f*x -
 5*e))/((a^2*e^(-2*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*x - 6*e)
 + a^2)*f) + 1/2*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(60) = 120.
time = 0.45, size = 254, normalized size = 3.97

$$-\frac{2\left(\left(2\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)+e^{(fx+e)}\sinh\left(fx+e\right)^{2}+\left(\cosh\left(fx+e\right)^{2}-1\right)e^{(fx+e)}\right)\arctan\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)+\cosh\left(fx+e\right)e^{(fx+e)}+e^{(fx+e)}\sinh\left(fx+e\right)\right)\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}\,e^{(-fx-e)}}{a^{2}f\cosh\left(fx+e\right)^{2}-a^{2}f+\left(a^{2}fe^{(2fx+2e)}+a^{2}f\right)\sinh\left(fx+e\right)^{2}+\left(a^{2}f\cosh\left(fx+e\right)^{2}-a^{2}f\right)e^{(2fx+2e)}+2\left(a^{2}f\cosh\left(fx+e\right)e^{(2fx+2e)}+a^{2}f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^
2 + (cosh(f*x + e)^2 - 1)*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)
) + cosh(f*x + e)*e^(f*x + e) + e^(f*x + e)*sinh(f*x + e))*sqrt(a*e^(4*f*x
+ 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^2 - a^2
*f + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)
^2 - a^2*f)*e^(2*f*x + 2*e) + 2*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*
f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(e+fx)}{\left(a\left(\sinh^2(e+fx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**2/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\mathrm{coth}\left(e+f\,x\right)}^2}{{\left(a\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(3/2), x)
```

3.454 $\int \frac{\coth^4(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -1/3*coth(f*x+e)*csch(f*x+e)^2/a/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3286, 2686, 30}

$$-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*(Coth[e + f*x]*Csch[e + f*x]^2)/(a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^4(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\coth^4(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx\\ &= \frac{\cosh(e+fx)\int \coth(e+fx)\text{csch}^3(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}}\\ &= \frac{(i\cosh(e+fx))\text{Subst}\left(\int x^2\,dx, x, -i\text{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}\\ &= -\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 0.76

$$-\frac{\coth^3(e+fx)}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/(a + a*Sinh[e + f*x]^2)^(3/2),x]
[Out] -1/3*Coth[e + f*x]^3/(f*(a*Cosh[e + f*x]^2)^(3/2))
```

Maple [A]
time = 1.25, size = 35, normalized size = 0.92

method	result	size
default	$-\frac{\cosh(fx+e)}{3a\sinh(fx+e)^3\sqrt{a\left(\cosh^2(fx+e)\right)}\,f}$	35
risch	$-\frac{8\left(e^{2fx+2e}+1\right)e^{2fx+2e}}{3\left(e^{2fx+2e}-1\right)^3 f\sqrt{\left(e^{2fx+2e}+1\right)^2 a\,e^{-2fx-2e}}\,a}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*cosh(f*x+e)/a/sinh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(37) = 74$.
time = 0.53, size = 881, normalized size = 23.18

$$\frac{\frac{21\,e^{(-fx-e)}-16\,e^{(-3fx-3e)}+34\,e^{(-5fx-5e)}+8\,e^{(-7fx-7e)}-15\,e^{(-9fx-9e)}}{a^{\frac{3}{2}}e^{(-2fx-2e)}+2\,a^{\frac{3}{2}}e^{(-4fx-4e)}-2\,a^{\frac{3}{2}}e^{(-6fx-6e)}-a^{\frac{3}{2}}e^{(-8fx-8e)}+a^{\frac{3}{2}}e^{(-10fx-10e)}-a^{\frac{3}{2}}}+\frac{3\arctan\left(e^{(-fx-e)}\right)}{a^{\frac{3}{2}}}+\frac{9\log\left(e^{(-fx-e)}+1\right)}{a^{\frac{3}{2}}}-\frac{9\log\left(e^{(-fx-e)}-1\right)}{a^{\frac{3}{2}}}}{12\,f}-\frac{\frac{15\,e^{(-fx-e)}-8\,e^{(-3fx-3e)}-34\,e^{(-5fx-5e)}+16\,e^{(-7fx-7e)}-21\,e^{(-9fx-9e)}}{a^{\frac{3}{2}}e^{(-2fx-2e)}+2\,a^{\frac{3}{2}}e^{(-4fx-4e)}-2\,a^{\frac{3}{2}}e^{(-6fx-6e)}-a^{\frac{3}{2}}e^{(-8fx-8e)}+a^{\frac{3}{2}}e^{(-10fx-10e)}-a^{\frac{3}{2}}}-\frac{3\arctan\left(e^{(-fx-e)}\right)}{a^{\frac{3}{2}}}+\frac{9\log\left(e^{(-fx-e)}+1\right)}{a^{\frac{3}{2}}}-\frac{9\log\left(e^{(-fx-e)}-1\right)}{a^{\frac{3}{2}}}}{12\,f}-\frac{\frac{15\,e^{(-fx-e)}-20\,e^{(-3fx-3e)}-22\,e^{(-5fx-5e)}-20\,e^{(-7fx-7e)}+15\,e^{(-9fx-9e)}}{a^{\frac{3}{2}}e^{(-2fx-2e)}+2\,a^{\frac{3}{2}}e^{(-4fx-4e)}-2\,a^{\frac{3}{2}}e^{(-6fx-6e)}-a^{\frac{3}{2}}e^{(-8fx-8e)}+a^{\frac{3}{2}}e^{(-10fx-10e)}-a^{\frac{3}{2}}}+\frac{15\arctan\left(e^{(-fx-e)}\right)}{a^{\frac{3}{2}}}}{8\,f}+\frac{45\,e^{(-fx-e)}-52\,e^{(-3fx-3e)}-74\,e^{(-5fx-5e)}+92\,e^{(-7fx-7e)}+21\,e^{(-9fx-9e)}}{48\left(a^{\frac{3}{2}}e^{(-2fx-2e)}+2\,a^{\frac{3}{2}}e^{(-4fx-4e)}-2\,a^{\frac{3}{2}}e^{(-6fx-6e)}-a^{\frac{3}{2}}e^{(-8fx-8e)}+a^{\frac{3}{2}}e^{(-10fx-10e)}-a^{\frac{3}{2}}\right)f}+\frac{21\,e^{(-fx-e)}+92\,e^{(-3fx-3e)}-74\,e^{(-5fx-5e)}-52\,e^{(-7fx-7e)}+45\,e^{(-9fx-9e)}}{48\left(a^{\frac{3}{2}}e^{(-2fx-2e)}+2\,a^{\frac{3}{2}}e^{(-4fx-4e)}-2\,a^{\frac{3}{2}}e^{(-6fx-6e)}-a^{\frac{3}{2}}e^{(-8fx-8e)}+a^{\frac{3}{2}}e^{(-10fx-10e)}-a^{\frac{3}{2}}\right)f}+\frac{11\arctan\left(e^{(-fx-e)}\right)}{8\,a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*((21*e^(-f*x - e) - 16*e^(-3*f*x - 3*e) + 34*e^(-5*f*x - 5*e) + 8*e^(-
7*f*x - 7*e) - 15*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e
^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a
^(3/2)*e^(-10*f*x - 10*e) - a^(3/2)) + 3*arctan(e^(-f*x - e))/a^(3/2) + 9*l
og(e^(-f*x - e) + 1)/a^(3/2) - 9*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/12*((
15*e^(-f*x - e) - 8*e^(-3*f*x - 3*e) - 34*e^(-5*f*x - 5*e) + 16*e^(-7*f*x -
 7*e) - 21*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*
x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*
e^(-10*f*x - 10*e) - a^(3/2)) - 3*arctan(e^(-f*x - e))/a^(3/2) + 9*log(e^(-
f*x - e) + 1)/a^(3/2) - 9*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/8*((15*e^(-f
*x - e) - 20*e^(-3*f*x - 3*e) - 22*e^(-5*f*x - 5*e) - 20*e^(-7*f*x - 7*e) +
 15*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4*e
) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*
f*x - 10*e) - a^(3/2)) + 15*arctan(e^(-f*x - e))/a^(3/2))/f + 1/48*(45*e^(-
f*x - e) - 52*e^(-3*f*x - 3*e) - 74*e^(-5*f*x - 5*e) + 92*e^(-7*f*x - 7*e)
+ 21*e^(-9*f*x - 9*e))/((a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4
*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-1
0*f*x - 10*e) - a^(3/2))*f) + 1/48*(21*e^(-f*x - e) + 92*e^(-3*f*x - 3*e) -
 74*e^(-5*f*x - 5*e) - 52*e^(-7*f*x - 7*e) + 45*e^(-9*f*x - 9*e))/((a^(3/2)
*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e)
 - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - a^(3/2))*f) + 11
/8*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(34) = 68$.
time = 0.44, size = 612, normalized size = 16.11

$$-\frac{8\left(\cosh\left(fx+e\right)^{3}e^{(fx+e)}+3\cosh\left(fx+e\right)^{2}e^{(fx+e)}\sinh\left(fx+e\right)+3\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^{2}+e^{(fx+e)}\sinh\left(fx+e\right)^{3}\right)\sqrt{ae^{(4fx+4e)}+2\,ae^{(2fx+2e)}+a}\,e^{(-fx-e)}}{3\left(a^{2}f\cosh\left(fx+e\right)^{6}-3\,a^{2}f\cosh\left(fx+e\right)^{4}+\left(a^{2}fe^{(2fx+2e)}+a^{2}f\right)\sinh\left(fx+e\right)^{6}+6\left(a^{2}f\cosh\left(fx+e\right)e^{(2fx+2e)}+a^{2}f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)^{5}+3\,a^{2}f\cosh\left(fx+e\right)^{2}+3\left(5\,a^{2}f\cosh\left(fx+e\right)^{2}-a^{2}f+\left(5\,a^{2}f\cosh\left(fx+e\right)^{2}-a^{2}f\right)e^{(2fx+2e)}\right)\sinh\left(fx+e\right)^{4}+4\left(5\,a^{2}f\cosh\left(fx+e\right)^{3}-3\,a^{2}f\cosh\left(fx+e\right)+\left(5\,a^{2}f\cosh\left(fx+e\right)^{3}-3\,a^{2}f\cosh\left(fx+e\right)\right)e^{(2fx+2e)}\right)\sinh\left(fx+e\right)^{3}-a^{2}f+3\left(5\,a^{2}f\cosh\left(fx+e\right)^{4}-6\,a^{2}f\cosh\left(fx+e\right)^{2}+a^{2}f+\left(5\,a^{2}f\cosh\left(fx+e\right)^{4}-6\,a^{2}f\cosh\left(fx+e\right)^{2}+a^{2}f\right)e^{(2fx+2e)}\right)\sinh\left(fx+e\right)^{2}+\left(a^{2}f\cosh\left(fx+e\right)^{6}-3\,a^{2}f\cosh\left(fx+e\right)^{4}+3\,a^{2}f\cosh\left(fx+e\right)^{2}-a^{2}f\right)e^{(2fx+2e)}+6\left(a^{2}f\cosh\left(fx+e\right)^{5}-2\,a^{2}f\cosh\left(fx+e\right)^{3}+a^{2}f\cosh\left(fx+e\right)+\left(a^{2}f\cosh\left(fx+e\right)^{5}-2\,a^{2}f\cosh\left(fx+e\right)^{3}+a^{2}f\cosh\left(fx+e\right)\right)e^{(2fx+2e)}\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(f*x + e)^3*e^(f*x + e) + 3*cosh(f*x + e)^2*e^(f*x + e)*sinh(f*x
+ e) + 3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x +
 e)^3)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*
```

```
f*cosh(f*x + e)^6 - 3*a^2*f*cosh(f*x + e)^4 + (a^2*f*e^(2*f*x + 2*e) + a^2*
f)*sinh(f*x + e)^6 + 6*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*
x + e))*sinh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^2 + 3*(5*a^2*f*cosh(f*x + e
)^2 - a^2*f + (5*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x + 2*e))*sinh(f*x +
 e)^4 + 4*(5*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e) + (5*a^2*f*cosh(
f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 - a^2*
f + 3*(5*a^2*f*cosh(f*x + e)^4 - 6*a^2*f*cosh(f*x + e)^2 + a^2*f + (5*a^2*f
*cosh(f*x + e)^4 - 6*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f
*x + e)^2 + (a^2*f*cosh(f*x + e)^6 - 3*a^2*f*cosh(f*x + e)^4 + 3*a^2*f*cosh
(f*x + e)^2 - a^2*f)*e^(2*f*x + 2*e) + 6*(a^2*f*cosh(f*x + e)^5 - 2*a^2*f*c
osh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^5 - 2*a^2*f*cos
h(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(e+fx)}{\left(a\left(\sinh^2(e+fx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**4/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**B**]
time = 0.90, size = 71, normalized size = 1.87

$$-\frac{16\,{\mathrm{e}}^{4\,e+4\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{3\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^3\,\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] -(16*exp(4*e + 4*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(3*a^2*f*(exp(2*e + 2*f*x) - 1)^3*(exp(2*e + 2*f*x) + 1))
```

3.455 $\int \frac{\coth^6(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -1/3*coth(f*x+e)*csch(f*x+e)^2/a/f/(a*cosh(f*x+e)^2)^(1/2)-1/5*coth(f*x+e)*
csch(f*x+e)^4/a/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.160$, Rules used = {3255, 3286, 2686, 14}

$$-\frac{\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^6/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*(Coth[e + f*x]*Csch[e + f*x]^2)/(a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[
e + f*x]*Csch[e + f*x]^4)/(5*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^6(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\coth^6(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx\\
&= \frac{\cosh(e+fx)\int \coth^3(e+fx)\text{csch}^3(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}}\\
&= -\frac{(i\cosh(e+fx))\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx, x, -i\text{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}\\
&= -\frac{(i\cosh(e+fx))\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx, x, -i\text{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}\\
&= -\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 41, normalized size = 0.53

$$-\frac{\coth^3(e+fx)\left(5+3\text{csch}^2(e+fx)\right)}{15f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^6/(a + a*Sinh[e + f*x]^2)^(3/2),x]
[Out] -1/15*(Coth[e + f*x]^3*(5 + 3*Csch[e + f*x]^2))/(f*(a*Cosh[e + f*x]^2)^(3/2
))
```

Maple [A]
time = 1.50, size = 67, normalized size = 0.87

method	result	size

default	$-\frac{\cosh(fx+e)\left(5\left(\cosh^{2}(fx+e)\right)-2\right)}{15(\cosh(fx+e)+1)^{2}(\cosh(fx+e)-1)^{2}a\sinh(fx+e)\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}\,f}$	67
risch	$-\frac{8\left(5\,\mathrm{e}^{4fx+4e}+2\,\mathrm{e}^{2fx+2e}+5\right)\left(\mathrm{e}^{2fx+2e}+1\right)\mathrm{e}^{2fx+2e}}{15\left(\mathrm{e}^{2fx+2e}-1\right)^{5}f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^{2}a\,\mathrm{e}^{-2fx-2e}}\,a}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*cosh(f*x+e)*(5*cosh(f*x+e)^2-2)/(cosh(f*x+e)+1)^2/(cosh(f*x+e)-1)^2/a
/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1641 vs. $2(75) = 150$.
time = 0.58, size = 1641, normalized size = 21.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/256*(2*(105*e^(-f*x - e) - 300*e^(-3*f*x - 3*e) + 81*e^(-5*f*x - 5*e) -
248*e^(-7*f*x - 7*e) + 51*e^(-9*f*x - 9*e) + 100*e^(-11*f*x - 11*e) - 45*e^
(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) -
5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*
x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(
3/2)) + 60*arctan(e^(-f*x - e))/a^(3/2) + 75*log(e^(-f*x - e) + 1)/a^(3/2)
 - 75*log(e^(-f*x - e) - 1)/a^(3/2))/f + 1/48*((105*e^(-f*x - e) - 350*e^(-3
*f*x - 3*e) + 231*e^(-5*f*x - 5*e) + 412*e^(-7*f*x - 7*e) + 231*e^(-9*f*x -
 9*e) - 350*e^(-11*f*x - 11*e) + 105*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f
*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/
2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 1
2*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2)) + 105*arctan(e^(-f*x - e))/a^(
3/2))/f + 3/256*(2*(45*e^(-f*x - e) - 100*e^(-3*f*x - 3*e) - 51*e^(-5*f*x -
 5*e) + 248*e^(-7*f*x - 7*e) - 81*e^(-9*f*x - 9*e) + 300*e^(-11*f*x - 11*e)
 - 105*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x
- 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*
e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14
*e) - a^(3/2)) - 60*arctan(e^(-f*x - e))/a^(3/2) + 75*log(e^(-f*x - e) + 1)
/a^(3/2) - 75*log(e^(-f*x - e) - 1)/a^(3/2))/f - 3/320*(4*(45*e^(-f*x - e)
- 135*e^(-3*f*x - 3*e) + 54*e^(-5*f*x - 5*e) + 198*e^(-7*f*x - 7*e) - 211*e
^(-9*f*x - 9*e) - 15*e^(-11*f*x - 11*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3
/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8
```

```
*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e
^(-14*f*x - 14*e) - a^(3/2)) + 90*arctan(e^(-f*x - e))/a^(3/2) + 45*log(e^(
-f*x - e) + 1)/a^(3/2) - 45*log(e^(-f*x - e) - 1)/a^(3/2))/f + 3/320*(4*(15
*e^(-3*f*x - 3*e) + 211*e^(-5*f*x - 5*e) - 198*e^(-7*f*x - 7*e) - 54*e^(-9*
f*x - 9*e) + 135*e^(-11*f*x - 11*e) - 45*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(
-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a
^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x
 - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2)) - 90*arctan(e^(-f*x - e))/
a^(3/2) + 45*log(e^(-f*x - e) + 1)/a^(3/2) - 45*log(e^(-f*x - e) - 1)/a^(3/
2))/f + 1/1920*(1155*e^(-f*x - e) + 1460*e^(-3*f*x - 3*e) - 4173*e^(-5*f*x
- 5*e) + 2024*e^(-7*f*x - 7*e) + 1857*e^(-9*f*x - 9*e) - 2140*e^(-11*f*x -
11*e) + 585*e^(-13*f*x - 13*e))/((3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-
4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^
(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*
x - 14*e) - a^(3/2))*f) + 1/1920*(585*e^(-f*x - e) - 2140*e^(-3*f*x - 3*e)
+ 1857*e^(-5*f*x - 5*e) + 2024*e^(-7*f*x - 7*e) - 4173*e^(-9*f*x - 9*e) + 1
460*e^(-11*f*x - 11*e) + 1155*e^(-13*f*x - 13*e))/((3*a^(3/2)*e^(-2*f*x - 2
*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(
-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) +
 a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2))*f) + 29/32*arctan(e^(-f*x - e))/(a^(
3/2)*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1410 vs. $2(69) = 138$.
time = 0.43, size = 1410, normalized size = 18.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/15*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f*
x + e)^7 + (105*cosh(f*x + e)^2 + 2)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*co
sh(f*x + e)^3 + 2*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 5*(35*cosh(f
*x + e)^4 + 4*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 5*(21*cosh
(f*x + e)^5 + 4*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(7*cosh(f*x + e)^6 + 2*cosh(f*x + e)^4 + 3*cosh(f*x + e)^2)*e^(f*x
+ e)*sinh(f*x + e) + (5*cosh(f*x + e)^7 + 2*cosh(f*x + e)^5 + 5*cosh(f*x +
e)^3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*
x - e)/(a^2*f*cosh(f*x + e)^10 - 5*a^2*f*cosh(f*x + e)^8 + (a^2*f*e^(2*f*x
+ 2*e) + a^2*f)*sinh(f*x + e)^10 + 10*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e)
+ a^2*f*cosh(f*x + e))*sinh(f*x + e)^9 + 10*a^2*f*cosh(f*x + e)^6 + 5*(9*a^
2*f*cosh(f*x + e)^2 - a^2*f + (9*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x +
2*e))*sinh(f*x + e)^8 + 40*(3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e) +
 (3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x
```

```
+ e)^7 - 10*a^2*f*cosh(f*x + e)^4 + 10*(21*a^2*f*cosh(f*x + e)^4 - 14*a^2*f
*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*cosh(f*x + e)^4 - 14*a^2*f*cosh(f*x +
e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 4*(63*a^2*f*cosh(f*x + e)^
5 - 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (63*a^2*f*cosh(f*x
+ e)^5 - 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e)
)*sinh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^2 + 10*(21*a^2*f*cosh(f*x + e)^6
- 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 - a^2*f + (21*a^2*f*c
osh(f*x + e)^6 - 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 - a^2*
f)*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 40*(3*a^2*f*cosh(f*x + e)^7 - 7*a^2*f
*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e) + (3*a^2*f
*cosh(f*x + e)^7 - 7*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 - a^2*
f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 - a^2*f + 5*(9*a^2*f*cosh
(f*x + e)^8 - 28*a^2*f*cosh(f*x + e)^6 + 30*a^2*f*cosh(f*x + e)^4 - 12*a^2*
f*cosh(f*x + e)^2 + a^2*f + (9*a^2*f*cosh(f*x + e)^8 - 28*a^2*f*cosh(f*x +
e)^6 + 30*a^2*f*cosh(f*x + e)^4 - 12*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*
x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^10 - 5*a^2*f*cosh(f*x + e)
^8 + 10*a^2*f*cosh(f*x + e)^6 - 10*a^2*f*cosh(f*x + e)^4 + 5*a^2*f*cosh(f*x
 + e)^2 - a^2*f)*e^(2*f*x + 2*e) + 10*(a^2*f*cosh(f*x + e)^9 - 4*a^2*f*cosh
(f*x + e)^7 + 6*a^2*f*cosh(f*x + e)^5 - 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cos
h(f*x + e) + (a^2*f*cosh(f*x + e)^9 - 4*a^2*f*cosh(f*x + e)^7 + 6*a^2*f*cos
h(f*x + e)^5 - 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**6/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: TypeError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 0.16, size = 305, normalized size = 3.96

$$-\frac{16\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{3\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^2\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{272\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{15\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^3\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{128\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{5\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^4\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{64\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{5\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^5\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^6/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))
/(3*a^2*f*(exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (27
2*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(15
*a^2*f*(exp(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (128*e
xp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a^2
*f*(exp(2*e + 2*f*x) - 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (64*exp(3*
e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a^2*f*(e
xp(2*e + 2*f*x) - 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

3.456 $\int \frac{\coth^8(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^6(e+fx)}{7af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -1/3*coth(f*x+e)*csch(f*x+e)^2/a/f/(a*cosh(f*x+e)^2)^(1/2)-2/5*coth(f*x+e)*
csch(f*x+e)^4/a/f/(a*cosh(f*x+e)^2)^(1/2)-1/7*coth(f*x+e)*csch(f*x+e)^6/a/f
/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3255, 3286, 2686, 276}

$$-\frac{\coth(e+fx)\text{csch}^6(e+fx)}{7af\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^8/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*(Coth[e + f*x]*Csch[e + f*x]^2)/(a*f*Sqrt[a*Cosh[e + f*x]^2]) - (2*Cot
h[e + f*x]*Csch[e + f*x]^4)/(5*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e + f*x
]*Csch[e + f*x]^6)/(7*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^8(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\coth^8(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx \\
&= \frac{\cosh(e+fx)\int \coth^5(e+fx)\text{csch}^3(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}} \\
&= \frac{(i\cosh(e+fx))\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, -i\text{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}} \\
&= \frac{(i\cosh(e+fx))\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, -i\text{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}} - \frac{2\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+}{7af\sqrt{a}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.10, size = 51, normalized size = 0.44

$$-\frac{\coth^3(e+fx)\left(35+42\text{csch}^2(e+fx)+15\text{csch}^4(e+fx)\right)}{105f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^8/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/105*(Coth[e + f*x]^3*(35 + 42*Csch[e + f*x]^2 + 15*Csch[e + f*x]^4))/(f*
(a*Cosh[e + f*x]^2)^(3/2))
```

Maple [**A**]
time = 1.78, size = 57, normalized size = 0.50

method	result	size
default	$-\frac{\cosh(fx+e)\left(35\left(\cosh^4(fx+e)\right)-28\left(\cosh^2(fx+e)\right)+8\right)}{105a\sinh(fx+e)^7\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$	57
risch	$-\frac{8\left(35\,\mathrm{e}^{8fx+8e}+28\,\mathrm{e}^{6fx+6e}+114\,\mathrm{e}^{4fx+4e}+28\,\mathrm{e}^{2fx+2e}+35\right)\left(\mathrm{e}^{2fx+2e}+1\right)\mathrm{e}^{2fx+2e}}{105\left(\mathrm{e}^{2fx+2e}-1\right)^7 f\sqrt{\left(\mathrm{e}^{2fx+2e}+1\right)^2 a\,\mathrm{e}^{-2fx-2e}}\,a}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*cosh(f*x+e)*(35*cosh(f*x+e)^4-28*cosh(f*x+e)^2+8)/a/sinh(f*x+e)^7/(a
*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2378 vs. $2(112) = 224$.
time = 0.66, size = 2378, normalized size = 20.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3840*(2*(4095*e^(-f*x - e) - 20090*e^(-3*f*x - 3*e) + 31654*e^(-5*f*x - 5
*e) - 850*e^(-7*f*x - 7*e) - 51148*e^(-9*f*x - 9*e) + 51090*e^(-11*f*x - 11
*e) - 2646*e^(-13*f*x - 13*e) + 4410*e^(-15*f*x - 15*e) - 1155*e^(-17*f*x -
 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/
2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x
- 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/
2)) + 2940*arctan(e^(-f*x - e))/a^(3/2) + 2625*log(e^(-f*x - e) + 1)/a^(3/2
) - 2625*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/8960*(2*(4095*e^(-f*x - e) -
21630*e^(-3*f*x - 3*e) + 39354*e^(-5*f*x - 5*e) - 13830*e^(-7*f*x - 7*e) -
47848*e^(-9*f*x - 9*e) + 66950*e^(-11*f*x - 11*e) - 22106*e^(-13*f*x - 13*e
) - 18690*e^(-15*f*x - 15*e) + 3465*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*
x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^
(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*
f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/2)) + 7560*arctan(e^(-f*x -
 e))/a^(3/2) + 315*log(e^(-f*x - e) + 1)/a^(3/2) - 315*log(e^(-f*x - e) - 1
)/a^(3/2))/f - 1/8960*(2*(3465*e^(-f*x - e) - 18690*e^(-3*f*x - 3*e) - 2210
6*e^(-5*f*x - 5*e) + 66950*e^(-7*f*x - 7*e) - 47848*e^(-9*f*x - 9*e) - 1383
0*e^(-11*f*x - 11*e) + 39354*e^(-13*f*x - 13*e) - 21630*e^(-15*f*x - 15*e)
+ 4095*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*
x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*
a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*
```

```
f*x - 18*e) - a^(3/2)) + 7560*arctan(e^(-f*x - e))/a^(3/2) - 315*log(e^(-f*
x - e) + 1)/a^(3/2) + 315*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/3840*(2*(115
5*e^(-f*x - e) - 4410*e^(-3*f*x - 3*e) + 2646*e^(-5*f*x - 5*e) - 51090*e^(-
7*f*x - 7*e) + 51148*e^(-9*f*x - 9*e) + 850*e^(-11*f*x - 11*e) - 31654*e^(-
13*f*x - 13*e) + 20090*e^(-15*f*x - 15*e) - 4095*e^(-17*f*x - 17*e))/(5*a^(
3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x -
 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^
(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/2)) - 2940*arc
tan(e^(-f*x - e))/a^(3/2) + 2625*log(e^(-f*x - e) + 1)/a^(3/2) - 2625*log(e
^(-f*x - e) - 1)/a^(3/2))/f + 1/768*(2*(1155*e^(-f*x - e) - 5670*e^(-3*f*x
- 3*e) + 8946*e^(-5*f*x - 5*e) - 270*e^(-7*f*x - 7*e) + 4696*e^(-9*f*x - 9*
e) - 2930*e^(-11*f*x - 11*e) - 658*e^(-13*f*x - 13*e) + 1190*e^(-15*f*x - 1
5*e) - 315*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-
4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e)
+ 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(
-18*f*x - 18*e) - a^(3/2)) + 840*arctan(e^(-f*x - e))/a^(3/2) + 735*log(e^(
-f*x - e) + 1)/a^(3/2) - 735*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/768*(2*(3
15*e^(-f*x - e) - 1190*e^(-3*f*x - 3*e) + 658*e^(-5*f*x - 5*e) + 2930*e^(-7
*f*x - 7*e) - 4696*e^(-9*f*x - 9*e) + 270*e^(-11*f*x - 11*e) - 8946*e^(-13*
f*x - 13*e) + 5670*e^(-15*f*x - 15*e) - 1155*e^(-17*f*x - 17*e))/(5*a^(3/2)
*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e
) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2
)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/2)) - 840*arctan(e
^(-f*x - e))/a^(3/2) + 735*log(e^(-f*x - e) + 1)/a^(3/2) - 735*log(e^(-f*x
- e) - 1)/a^(3/2))/f - 1/128*((315*e^(-f*x - e) - 1680*e^(-3*f*x - 3*e) + 3
108*e^(-5*f*x - 5*e) - 1200*e^(-7*f*x - 7*e) - 3646*e^(-9*f*x - 9*e) - 1200
*e^(-11*f*x - 11*e) + 3108*e^(-13*f*x - 13*e) - 1680*e^(-15*f*x - 15*e) + 3
15*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x -
4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3
/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x
- 18*e) - a^(3/2)) + 315*arctan(e^(-f*x - e))/a^(3/2))/f + 1/2688*(1155*e^(
-f*x - e) + 1393*e^(-3*f*x - 3*e) - 4865*e^(-5*f*x - 5*e) + 3965*e^(-7*f*x
- 7*e) + 825*e^(-9*f*x - 9*e) - 3245*e^(-11*f*x - 11*e) + 1925*e^(-13*f*x -
 13*e) - 385*e^(-15*f*x - 15*e))/((5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e
^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*
e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*
e^(-18*f*x - 18*e) - a^(3/2))*f) - 1/2688*(385*e^(-3*f*x - 3*e) - 1925*e^(-
5*f*x - 5*e) + 3245*e^(-7*f*x - 7*e) - 825*e^(-9*f*x - 9*e) - 3965*e^(-11*f
*x - 11*e) + 4865*e^(-13*f*x - 13*e) - 1393*e^(-15*f*x - 15*e) - 1155*e^(-1
7*f*x - 17*e))/((5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) +
14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(
-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e)
 - a^(3/2))*f) + 55/128*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2511 vs.

$2(103) = 206$.
time = 0.45, size = 2511, normalized size = 21.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/105*(385*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^10 + 35*e^(f*x + e)*sin
h(f*x + e)^11 + 7*(275*cosh(f*x + e)^2 + 4)*e^(f*x + e)*sinh(f*x + e)^9 + 2
1*(275*cosh(f*x + e)^3 + 12*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^8 + 6*
(1925*cosh(f*x + e)^4 + 168*cosh(f*x + e)^2 + 19)*e^(f*x + e)*sinh(f*x + e)
^7 + 42*(385*cosh(f*x + e)^5 + 56*cosh(f*x + e)^3 + 19*cosh(f*x + e))*e^(f*
x + e)*sinh(f*x + e)^6 + 14*(1155*cosh(f*x + e)^6 + 252*cosh(f*x + e)^4 + 1
71*cosh(f*x + e)^2 + 2)*e^(f*x + e)*sinh(f*x + e)^5 + 14*(825*cosh(f*x + e)
^7 + 252*cosh(f*x + e)^5 + 285*cosh(f*x + e)^3 + 10*cosh(f*x + e))*e^(f*x +
 e)*sinh(f*x + e)^4 + 7*(825*cosh(f*x + e)^8 + 336*cosh(f*x + e)^6 + 570*co
sh(f*x + e)^4 + 40*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^3 + 7*(27
5*cosh(f*x + e)^9 + 144*cosh(f*x + e)^7 + 342*cosh(f*x + e)^5 + 40*cosh(f*x
 + e)^3 + 15*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 7*(55*cosh(f*x +
e)^10 + 36*cosh(f*x + e)^8 + 114*cosh(f*x + e)^6 + 20*cosh(f*x + e)^4 + 15*
cosh(f*x + e)^2)*e^(f*x + e)*sinh(f*x + e) + (35*cosh(f*x + e)^11 + 28*cosh
(f*x + e)^9 + 114*cosh(f*x + e)^7 + 28*cosh(f*x + e)^5 + 35*cosh(f*x + e)^3
)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a^2*f*cosh(f*x + e)^14 - 7*a^2*f*cosh(f*x + e)^12 + (a^2*f*e^(2*f*x + 2
*e) + a^2*f)*sinh(f*x + e)^14 + 14*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a
^2*f*cosh(f*x + e))*sinh(f*x + e)^13 + 21*a^2*f*cosh(f*x + e)^10 + 7*(13*a^
2*f*cosh(f*x + e)^2 - a^2*f + (13*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x +
 2*e))*sinh(f*x + e)^12 + 28*(13*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x +
 e) + (13*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*s
inh(f*x + e)^11 - 35*a^2*f*cosh(f*x + e)^8 + 7*(143*a^2*f*cosh(f*x + e)^4 -
 66*a^2*f*cosh(f*x + e)^2 + 3*a^2*f + (143*a^2*f*cosh(f*x + e)^4 - 66*a^2*f
*cosh(f*x + e)^2 + 3*a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^10 + 14*(143*a^2
*f*cosh(f*x + e)^5 - 110*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (
143*a^2*f*cosh(f*x + e)^5 - 110*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x +
 e))*e^(2*f*x + 2*e))*sinh(f*x + e)^9 + 35*a^2*f*cosh(f*x + e)^6 + 7*(429*a
^2*f*cosh(f*x + e)^6 - 495*a^2*f*cosh(f*x + e)^4 + 135*a^2*f*cosh(f*x + e)^
2 - 5*a^2*f + (429*a^2*f*cosh(f*x + e)^6 - 495*a^2*f*cosh(f*x + e)^4 + 135*
a^2*f*cosh(f*x + e)^2 - 5*a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 8*(429*
a^2*f*cosh(f*x + e)^7 - 693*a^2*f*cosh(f*x + e)^5 + 315*a^2*f*cosh(f*x + e)
^3 - 35*a^2*f*cosh(f*x + e) + (429*a^2*f*cosh(f*x + e)^7 - 693*a^2*f*cosh(f
*x + e)^5 + 315*a^2*f*cosh(f*x + e)^3 - 35*a^2*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e)^7 - 21*a^2*f*cosh(f*x + e)^4 + 7*(429*a^2*f*cosh(f*x +
e)^8 - 924*a^2*f*cosh(f*x + e)^6 + 630*a^2*f*cosh(f*x + e)^4 - 140*a^2*f*co
sh(f*x + e)^2 + 5*a^2*f + (429*a^2*f*cosh(f*x + e)^8 - 924*a^2*f*cosh(f*x +
```

```
 e)^6 + 630*a^2*f*cosh(f*x + e)^4 - 140*a^2*f*cosh(f*x + e)^2 + 5*a^2*f)*e^
(2*f*x + 2*e))*sinh(f*x + e)^6 + 14*(143*a^2*f*cosh(f*x + e)^9 - 396*a^2*f*
cosh(f*x + e)^7 + 378*a^2*f*cosh(f*x + e)^5 - 140*a^2*f*cosh(f*x + e)^3 + 1
5*a^2*f*cosh(f*x + e) + (143*a^2*f*cosh(f*x + e)^9 - 396*a^2*f*cosh(f*x + e
)^7 + 378*a^2*f*cosh(f*x + e)^5 - 140*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh
(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 7*a^2*f*cosh(f*x + e)^2 + 7*(
143*a^2*f*cosh(f*x + e)^10 - 495*a^2*f*cosh(f*x + e)^8 + 630*a^2*f*cosh(f*x
 + e)^6 - 350*a^2*f*cosh(f*x + e)^4 + 75*a^2*f*cosh(f*x + e)^2 - 3*a^2*f +
(143*a^2*f*cosh(f*x + e)^10 - 495*a^2*f*cosh(f*x + e)^8 + 630*a^2*f*cosh(f*
x + e)^6 - 350*a^2*f*cosh(f*x + e)^4 + 75*a^2*f*cosh(f*x + e)^2 - 3*a^2*f)*
e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 28*(13*a^2*f*cosh(f*x + e)^11 - 55*a^2*f
*cosh(f*x + e)^9 + 90*a^2*f*cosh(f*x + e)^7 - 70*a^2*f*cosh(f*x + e)^5 + 25
*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e) + (13*a^2*f*cosh(f*x + e)^11
 - 55*a^2*f*cosh(f*x + e)^9 + 90*a^2*f*cosh(f*x + e)^7 - 70*a^2*f*cosh(f*x
+ e)^5 + 25*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))
*sinh(f*x + e)^3 - a^2*f + 7*(13*a^2*f*cosh(f*x + e)^12 - 66*a^2*f*cosh(f*x
 + e)^10 + 135*a^2*f*cosh(f*x + e)^8 - 140*a^2*f*cosh(f*x + e)^6 + 75*a^2*f
*cosh(f*x + e)^4 - 18*a^2*f*cosh(f*x + e)^2 + a^2*f + (13*a^2*f*cosh(f*x +
e)^12 - 66*a^2*f*cosh(f*x + e)^10 + 135*a^2*f*cosh(f*x + e)^8 - 140*a^2*f*c
osh(f*x + e)^6 + 75*a^2*f*cosh(f*x + e)^4 - 18*a^2*f*cosh(f*x + e)^2 + a^2*
f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^14 - 7*a^2*f*cos
h(f*x + e)^12 + 21*a^2*f*cosh(f*x + e)^10 - 35*a^2*f*cosh(f*x + e)^8 + 35*a
^2*f*cosh(f*x + e)^6 - 21*a^2*f*cosh(f*x + e)^4 + 7*a^2*f*cosh(f*x + e)^2 -
 a^2*f)*e^(2*f*x + 2*e) + 14*(a^2*f*cosh(f*x + e)^13 - 6*a^2*f*cosh(f*x + e
)^11 + 15*a^2*f*cosh(f*x + e)^9 - 20*a^2*f*cosh(f*x + e)^7 + 15*a^2*f*cosh(
f*x + e)^5 - 6*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*
x + e)^13 - 6*a^2*f*cosh(f*x + e)^11 + 15*a^2*f*cosh(f*x + e)^9 - 20*a^2*f*
cosh(f*x + e)^7 + 15*a^2*f*cosh(f*x + e)^5 - 6*a^2*f*cosh(f*x + e)^3 + a^2*
f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy **[F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**8/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac **[F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 0.95, size = 457, normalized size = 3.97

$$-\frac{16\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{3\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^2\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{464\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{15\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^3\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{3072\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{35\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^4\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{4736\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{35\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^5\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{768\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{7\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^6\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{256\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{7\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^7\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^8/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))
/(3*a^2*f*(exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (46
4*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(15
*a^2*f*(exp(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (3072*
exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(35*a
^2*f*(exp(2*e + 2*f*x) - 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (4736*ex
p(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(35*a^2
*f*(exp(2*e + 2*f*x) - 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x))) - (768*exp(3
*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(7*a^2*f*(
exp(2*e + 2*f*x) - 1)^6*(exp(e + f*x) + exp(3*e + 3*f*x))) - (256*exp(3*e +
 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(7*a^2*f*(exp(
2*e + 2*f*x) - 1)^7*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

3.457 $\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^5(e+fx)\,dx$

Optimal. Leaf size=187

$$-\frac{(8a^2-24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{3/2}f}+\frac{(8a^2-24ab+15b^2)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f}+(8$$

```
[Out] -1/8*(8*a^2-24*a*b+15*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(
a-b)^(3/2)/f+1/8*(8*a-7*b)*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/
f-1/4*sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)/f+1/8*(8*a^2-24*a*b+15*
b^2)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/f
```

Rubi [A]
time = 0.17, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 52, 65, 214}

$$\frac{(8a^2-24ab+15b^2)\sqrt{a+b\sinh^2(e+fx)}}{8f(a-b)^2}-\frac{(8a^2-24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{3/2}}-\frac{\operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f(a-b)}+\frac{(8a-7b)\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] -1/8*((8*a^2 - 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a
 - b]])/((a - b)^(3/2)*f) + ((8*a^2 - 24*a*b + 15*b^2)*Sqrt[a + b*Sinh[e + f
*x]^2])/(8*(a - b)^2*f) + ((8*a - 7*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]
^2)^(3/2))/(8*(a - b)^2*f) - (Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2)
)/(4*(a - b)*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{(1+x)^3}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4(a-b)f} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(-4a+3\right.}{}\right.}{}$$

$$= \frac{(8a-7b)\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{8(a-b)^2 f} - \frac{\operatorname{sech}^4(e+}{}$$

$$= \frac{(8a^2-24ab+15b^2)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2 f} + \frac{(8a-7b)\operatorname{sech}^2}{}$$

$$= \frac{(8a^2-24ab+15b^2)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2 f} + \frac{(8a-7b)\operatorname{sech}^2}{}$$

$$= -\frac{(8a^2-24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{3/2} f} + \frac{(}{}$$

Mathematica [A]
time = 0.40, size = 151, normalized size = 0.81

$$-\frac{-\left((8a-7b)\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\right) + 2(a-b)\operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2} + (8a^2-24ab+15b^2)\left(\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right) - \sqrt{a+b\sinh^2(e+fx)}\right)}{8(a-b)^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] -1/8*(-((8*a - 7*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2)) + 2*(a -
 b)*Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2) + (8*a^2 - 24*a*b + 15*b^
2)*(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] - Sqrt[a +
 b*Sinh[e + f*x]^2]))/((a - b)^2*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.43, size = 43, normalized size = 0.23

method	result	size

default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\ \left(\sinh^5\left(fx+e\right)\right)}{\cosh\left(fx+e\right)^6},\sinh\left(fx+e\right)\right)}{f}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)^5/cosh(f*x+e)^6,sinh(f*x
+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^5, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2254 vs. $2(167) = 334$.
time = 1.13, size = 4704, normalized size = 25.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^9 + 9*(8*a^2 - 24*a*b + 15
*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^2 - 24*a*b + 15*b^2)*sinh(f*x +
e)^9 + 4*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 - 24*a*b +
 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^7 + 28*(3
*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (8*a^2 - 24*a*b + 15*b^2)*cosh
(f*x + e))*sinh(f*x + e)^6 + 6*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 +
6*(21*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 14*(8*a^2 - 24*a*b + 15*b
^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^5 + 2*(63*(8*a
^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 70*(8*a^2 - 24*a*b + 15*b^2)*cosh(f
*x + e)^3 + 15*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + 4
*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^2 - 24*a*b + 15*b^2
)*cosh(f*x + e)^6 + 35*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 15*(8*a^
2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x +
e)^3 + 12*(3*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^7 + 7*(8*a^2 - 24*a*b
```

```
+ 15*b^2)*cosh(f*x + e)^5 + 5*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (
8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 - 24*a*b +
 15*b^2)*cosh(f*x + e) + (9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^8 + 28*
(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^6 + 30*(8*a^2 - 24*a*b + 15*b^2)*co
sh(f*x + e)^4 + 12*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a
*b + 15*b^2)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f
*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2
 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a -
 b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(
f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)
 + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4
+ 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f
*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*(2*(a^2 - 2*a*b
+ b^2)*cosh(f*x + e)^8 + 16*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)
^7 + 2*(a^2 - 2*a*b + b^2)*sinh(f*x + e)^8 + (16*a^2 - 33*a*b + 17*b^2)*cos
h(f*x + e)^6 + (56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 + 16*a^2 - 33*a*b +
17*b^2)*sinh(f*x + e)^6 + 2*(56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 + 3*(16
*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(10*a^2 - 21*a*b
 + 11*b^2)*cosh(f*x + e)^4 + (140*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 15*
(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^2 + 20*a^2 - 42*a*b + 22*b^2)*sinh
(f*x + e)^4 + 4*(28*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 5*(16*a^2 - 33*a*
b + 17*b^2)*cosh(f*x + e)^3 + 2*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e))*s
inh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^2 + (56*(a^2 - 2*
a*b + b^2)*cosh(f*x + e)^6 + 15*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^4
+ 12*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^2 + 16*a^2 - 33*a*b + 17*b^2)
*sinh(f*x + e)^2 + 2*a^2 - 4*a*b + 2*b^2 + 2*(8*(a^2 - 2*a*b + b^2)*cosh(f*
x + e)^7 + 3*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^5 + 4*(10*a^2 - 21*a*
b + 11*b^2)*cosh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b
 + b^2)*f*cosh(f*x + e)^9 + 9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x
+ e)^8 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^9 + 4*(a^2 - 2*a*b + b^2)*f*co
sh(f*x + e)^7 + 4*(9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b +
 b^2)*f)*sinh(f*x + e)^7 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 28*(3*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e)
)*sinh(f*x + e)^6 + 6*(21*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 14*(a^2 -
 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^5 +
4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 2*(63*(a^2 - 2*a*b + b^2)*f*cosh(
f*x + e)^5 + 70*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 15*(a^2 - 2*a*b + b
^2)*f*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(21*(a^2 - 2*a*b + b^2)*f*cosh(f*x
 + e)^6 + 35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 15*(a^2 - 2*a*b + b^2)
*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^3 + (a^2 - 2*a*b
+ b^2)*f*cosh(f*x + e) + 12*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 7*(a
^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 5*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)
```

```
^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (9*(a^2 - 2*a*b
 + b^2)*f*cosh(f*x + e)^8 + 28*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 30*(
a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 12*(a^2 - 2*a*b + b^2)*f*cosh(f*x +
e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)), -1/8*(((8*a^2 - 24*a*b + 15*b
^2)*cosh(f*x + e)^9 + 9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)*sinh(f*x +
e)^8 + (8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^9 + 4*(8*a^2 - 24*a*b + 15*b
^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^
2 - 24*a*b + 15*b^2)*sinh(f*x + e)^7 + 28*(3*(8...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^2\left(e+fx\right)}\,\tanh^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**5, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.93Unable to divide, perhaps due to rou
nding error%%%{%%{[262144,0]:[1,0,%%%{-1,[1]%%%}]%%},[10,13,13]%%%}+%%%{%%{
[%%%{-157
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tanh}\left(e+f\,x\right)}^5\,\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.458 $\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^3(e+fx)\,dx$

Optimal. Leaf size=126

$$-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2\sqrt{a-b}\,f}+\frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f}+\frac{\operatorname{sech}^2(e+fx)\,(a+b\,\mathrm{sin}}{2(a-b)f}$$

```
[Out] 1/2*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)/f-1/2*(2*a-3*b)*arctanh((
a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)+1/2*(2*a-3*b)*(a+b*sinh
(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A]

time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 52, 65, 214}

$$\frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f\sqrt{a-b}}+\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

```
[Out] -1/2*((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(Sqrt[a
 - b]*f) + ((2*a - 3*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(a - b)*f) + (Sech[
e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(2*(a - b)*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{(1+x)^2}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2(a-b)f} + (2a-3b)\text{Subst}\Bigg(\int \sqrt{}$$

$$= \frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(\right.}{2(a-b)f}$$

$$= \frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(\right.}{2(a-b)f}$$

$$= -\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2\sqrt{a-b}\,f} + \frac{(2a-3b)\sqrt{a}}{2}$$

Mathematica [A]
time = 0.29, size = 88, normalized size = 0.70

$$\frac{-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+(2+\cosh(2(e+fx)))\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

```
[Out] (-(((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a -
b]) + (2 + Cosh[2*(e + f*x)])*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/
(2*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.26, size = 43, normalized size = 0.34

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,\left(\sinh^{3}\left(fx+e\right)\right)}{\cosh\left(fx+e\right)^{4}},\sinh\left(fx+e\right)\right)}{f}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)^3/cosh(f*x+e)^4,sinh(f*x
+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 737 vs. 2(110) = 220.
time = 0.91, size = 1670, normalized size = 13.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a - 3*b)*cosh(f*x + e)^5 + 5*(2*a - 3*b)*cosh(f*x + e)*sinh(f*x
+ e)^4 + (2*a - 3*b)*sinh(f*x + e)^5 + 2*(2*a - 3*b)*cosh(f*x + e)^3 + 2*(5
*(2*a - 3*b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e)^3 + 2*(5*(2*a - 3*b
)*cosh(f*x + e)^3 + 3*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a - 3
*b)*cosh(f*x + e) + (5*(2*a - 3*b)*cosh(f*x + e)^4 + 6*(2*a - 3*b)*cosh(f*x
 + e)^2 + 2*a - 3*b)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*
b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*
x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*
sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e)
 + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(
f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x
 + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4
*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((a - b)
*cosh(f*x + e)^4 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f
*x + e)^4 + 4*(a - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f*x + e)^2 + 2*a
- 2*b)*sinh(f*x + e)^2 + 4*((a - b)*cosh(f*x + e)^3 + 2*(a - b)*cosh(f*x +
e))*sinh(f*x + e) + a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
)/((a - b)*f*cosh(f*x + e)^5 + 5*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^4 +
(a - b)*f*sinh(f*x + e)^5 + 2*(a - b)*f*cosh(f*x + e)^3 + 2*(5*(a - b)*f*co
sh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^3 + (a - b)*f*cosh(f*x + e) + 2*(5
*(a - b)*f*cosh(f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (
5*(a - b)*f*cosh(f*x + e)^4 + 6*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh
(f*x + e)), -1/2*(((2*a - 3*b)*cosh(f*x + e)^5 + 5*(2*a - 3*b)*cosh(f*x + e
)*sinh(f*x + e)^4 + (2*a - 3*b)*sinh(f*x + e)^5 + 2*(2*a - 3*b)*cosh(f*x +
e)^3 + 2*(5*(2*a - 3*b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e)^3 + 2*(5
*(2*a - 3*b)*cosh(f*x + e)^3 + 3*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)^2
 + (2*a - 3*b)*cosh(f*x + e) + (5*(2*a - 3*b)*cosh(f*x + e)^4 + 6*(2*a - 3*
b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqr
t(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)
*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*((a - b)*cosh(f*x + e)^4
 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f*x + e)^4 + 4*(a
 - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f*x + e)^2 + 2*a - 2*b)*sinh(f*x
+ e)^2 + 4*((a - b)*cosh(f*x + e)^3 + 2*(a - b)*cosh(f*x + e))*sinh(f*x + e
) + a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cos
h(f*x + e)^5 + 5*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^4 + (a - b)*f*sinh(f
*x + e)^5 + 2*(a - b)*f*cosh(f*x + e)^3 + 2*(5*(a - b)*f*cosh(f*x + e)^2 +
(a - b)*f)*sinh(f*x + e)^3 + (a - b)*f*cosh(f*x + e) + 2*(5*(a - b)*f*cosh(
```

```
f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (5*(a - b)*f*cosh
(f*x + e)^4 + 6*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^{2}{\left(e + fx \right)}}\tanh^{3}{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.43Unable to divide, perhaps due to rou
nding error%%%{%%{[16384,0]:[1,0,%%%{-1,[1]%%%}]%%},[6,9,9]%%%}+%%%{%%{[%%%
{-65536,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tanh}\left(e+f\,x\right)}^3\,\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}\,\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.459 $\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f+(a+b*sinh(f*x
+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3273, 52, 65, 214}

$$\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[
a + b*Sinh[e + f*x]^2]/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx,\right.}{2f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a\right.}}{bf}$$

$$= -\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{\sqrt{a+b\sinh^2}}{f}$$

Mathematica [A]
time = 0.04, size = 65, normalized size = 1.05

$$\frac{-\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b+b\cosh^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{a-b+b\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

```
[Out] (-(Sqrt[a - b]*ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]]) + Sqrt
[a - b + b*Cosh[e + f*x]^2])/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.96, size = 41, normalized size = 0.66

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,\sinh(fx+e)}{\cosh(fx+e)^{2}},\sinh(fx+e)\right)}{f}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)/cosh(f*x+e)^2,sinh(f*x+e
))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(54) = 108.
time = 0.88, size = 624, normalized size = 10.06

$$\left[\frac{\sqrt{a-b}\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)\log\left(\frac{b\cosh(fx+e)^{4}+4\,b\cosh(fx+e)\sinh(fx+e)^{3}+b\sinh(fx+e)^{4}+2\,(4\,a-3\,b)\cosh(fx+e)^{2}+2\left(3\,b\cosh(fx+e)^{2}+4\,a-3\,b\right)\sinh(fx+e)^{2}-4\,\sqrt{2}\sqrt{a-b}\sqrt{\frac{b\cosh\left(fx+e\right)^{2}+b\sinh\left(fx+e\right)^{2}+2\,a-b}{\cosh\left(fx+e\right)^{2}-2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^{2}}}\,(\cosh(fx+e)+\sinh(fx+e))+4\left(b\cosh(fx+e)^{3}+(4\,a-3\,b)\cosh(fx+e)\right)\sinh(fx+e)+b}{\cosh(fx+e)^{4}+4\,\cosh(fx+e)\sinh(fx+e)^{3}+\sinh(fx+e)^{4}+2\left(3\,\cosh(fx+e)^{2}+1\right)\sinh(fx+e)^{2}+2\,\cosh(fx+e)^{2}+4\left(\cosh(fx+e)^{3}+\cosh(fx+e)\right)\sinh(fx+e)+1}\right)+\sqrt{2}\sqrt{\frac{b\cosh\left(fx+e\right)^{2}+b\sinh\left(fx+e\right)^{2}+2\,a-b}{\cosh\left(fx+e\right)^{2}-2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^{2}}}}{2\left(f\cosh\left(fx+e\right)+f\sinh\left(fx+e\right)\right)},-\frac{2\sqrt{-a+b}\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)\arctan\left(-\frac{\sqrt{2}\sqrt{-a+b}\sqrt{\frac{b\cosh\left(fx+e\right)^{2}+b\sinh\left(fx+e\right)^{2}+2\,a-b}{\cosh\left(fx+e\right)^{2}-2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^{2}}}}{2\left((a-b)\cosh(fx+e)+(a-b)\sinh(fx+e)\right)}\right)-\sqrt{2}\sqrt{\frac{b\cosh\left(fx+e\right)^{2}+b\sinh\left(fx+e\right)^{2}+2\,a-b}{\cosh\left(fx+e\right)^{2}-2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^{2}}}}{2\left(f\cosh\left(fx+e\right)+f\sinh\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*(cosh(f*x + e) + sinh(f*x + e))*log((b*cosh(f*x + e)^4 +
4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(
f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2
)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(
f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x +
e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sin
h(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f
*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 +
 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
```

```
x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e
)), -1/2*(2*sqrt(-a + b)*(cosh(f*x + e) + sinh(f*x + e))*arctan(-1/2*sqrt(2
)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*co
sh(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^2\left(e + fx\right)}\,\tanh\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tanh(e+f\,x)\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.460 $\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=54

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sinh(f*x+e)^2)^(
1/2)/f
```

Rubi [A]

time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3273, 52, 65, 214}

$$\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Si
nh[e + f*x]^2]/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^2(e\right.}{2f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh}\right.}{bf}$$

$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a+b\sinh^2(e+}}{f}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.98

$$-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] - Sqrt[a + b*Sinh[e
 + f*x]^2])/f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.98, size = 46, normalized size = 0.85

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{b\sinh(fx+e)+\frac{a}{\sinh(fx+e)}}{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘((b*sinh(f*x+e)+a/sinh(f*x+e))/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f
*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(46) = 92.
time = 0.64, size = 605, normalized size = 11.20

$$\left[\frac{\sqrt{a}\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)\log\left(\frac{b\cosh(fx+e)^{4}+4b\cosh(fx+e)\sinh(fx+e)^{3}+b\sinh(fx+e)^{4}+2(4a-b)\cosh(fx+e)^{2}+2\left(3b\cosh(fx+e)^{2}+4a-b\right)\sinh(fx+e)^{2}-4\sqrt{2}\sqrt{a}\sqrt{\frac{b\cosh\left(fx+e\right)^{2}+b\sinh\left(fx+e\right)^{2}+2a-b}{\cosh\left(fx+e\right)^{2}-2\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^{2}}}(\cosh(fx+e)+\sinh(fx+e))+4\left(b\cosh(fx+e)^{3}+(4a-b)\cosh(fx+e)\right)\sinh(fx+e)+b}{\cosh(fx+e)^{4}+4\cosh(fx+e)\sinh(fx+e)^{3}+\sinh(fx+e)^{4}+2\left(3\cosh(fx+e)^{2}-1\right)\sinh(fx+e)^{2}-2\cosh(fx+e)^{2}+4\left(\cosh(fx+e)^{3}-\cosh(fx+e)\right)\sinh(fx+e)+1}\right)+\sqrt{2}\sqrt{\frac{b\cosh\left(fx+e\right)^{2}+b\sinh\left(fx+e\right)^{2}+2a-b}{\cosh\left(fx+e\right)^{2}-2\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^{2}}}}{2\left(f\cosh\left(fx+e\right)+f\sinh\left(fx+e\right)\right)},\ \frac{2\sqrt{-a}\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{b\cosh\left(fx+e\right)^{2}+b\sinh\left(fx+e\right)^{2}+2a-b}{\cosh\left(fx+e\right)^{2}-2\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^{2}}}}{2\left(a\cosh(fx+e)+a\sinh(fx+e)\right)}\right)+\sqrt{2}\sqrt{\frac{b\cosh\left(fx+e\right)^{2}+b\sinh\left(fx+e\right)^{2}+2a-b}{\cosh\left(fx+e\right)^{2}-2\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^{2}}}}{2\left(f\cosh\left(fx+e\right)+f\sinh\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*(cosh(f*x + e) + sinh(f*x + e))*log((b*cosh(f*x + e)^4 + 4*b*
cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x +
e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*
x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b
)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*
(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x +
 e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
```

```
x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e)), 1/2*(2*sqr
t(-a)*(cosh(f*x + e) + sinh(f*x + e))*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))
) + sqrt(2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x +
 e) + f*sinh(f*x + e))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^{2}{\left(e + fx \right)}}\coth{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{coth}\left(e+f\,x\right)\,\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.461 $\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=106

$$-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}+\frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af}-\frac{\operatorname{csch}^2(e+fx)\,(a+b\sinh^2(e}{2af}$$

```
[Out] -1/2*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2)/a/f-1/2*(2*a+b)*arctanh((a+b*s
inh(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+1/2*(2*a+b)*(a+b*sinh(f*x+e)^2)^(1/2
)/a/f
```

Rubi [A]
time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 52, 65, 214}

$$\frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/2*((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(Sqrt[a]*f) +
 ((2*a + b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*a*f) - (Csch[e + f*x]^2*(a + b*
Sinh[e + f*x]^2)^(3/2))/(2*a*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)\sqrt{a+bx}}{x^2}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2af} + \frac{(2a+b)\text{Subst}\Big(\int}{}$$

$$= \frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af} - \frac{\text{csch}^2(e+fx)\left(a+b\sinh^2(\right.}{2af}$$

$$= \frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af} - \frac{\text{csch}^2(e+fx)\left(a+b\sinh^2(\right.}{2af}$$

$$= -\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f} + \frac{(2a+b)\sqrt{a}}{}$$

Mathematica [**A**]
time = 0.44, size = 69, normalized size = 0.65

$$-\frac{\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\left(-2+\operatorname{csch}^2(e+fx)\right)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/2*(((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (-
2 + Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2])/f
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.23, size = 58, normalized size = 0.55

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{b\sinh(fx+e)+\frac{a+b}{\sinh(fx+e)}+\frac{a}{\sinh(fx+e)^3}}{\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`((b*sinh(f*x+e)+(a+b)/sinh(f*x+e)+a/sinh(f*x+e)^3)/(a+b*sinh(f*
x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(90) = 180.
time = 0.69, size = 1445, normalized size = 13.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a + b)*cosh(f*x + e)^5 + 5*(2*a + b)*cosh(f*x + e)*sinh(f*x + e)^
4 + (2*a + b)*sinh(f*x + e)^5 - 2*(2*a + b)*cosh(f*x + e)^3 + 2*(5*(2*a + b
)*cosh(f*x + e)^2 - 2*a - b)*sinh(f*x + e)^3 + 2*(5*(2*a + b)*cosh(f*x + e)
^3 - 3*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a + b)*cosh(f*x + e) +
 (5*(2*a + b)*cosh(f*x + e)^4 - 6*(2*a + b)*cosh(f*x + e)^2 + 2*a + b)*sinh
(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)
^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)
^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 +
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e
)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh
(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sin
h(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh
(f*x + e) + 1)) + 2*sqrt(2)*(a*cosh(f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x
 + e)^3 + a*sinh(f*x + e)^4 - 4*a*cosh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2
- 2*a)*sinh(f*x + e)^2 + 4*(a*cosh(f*x + e)^3 - 2*a*cosh(f*x + e))*sinh(f*x
 + e) + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x
+ e)^5 + 5*a*f*cosh(f*x + e)*sinh(f*x + e)^4 + a*f*sinh(f*x + e)^5 - 2*a*f*
cosh(f*x + e)^3 + 2*(5*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^3 + a*f*cos
h(f*x + e) + 2*(5*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*sinh(f*x + e)^
2 + (5*a*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2 + a*f)*sinh(f*x + e)), 1
/2*(((2*a + b)*cosh(f*x + e)^5 + 5*(2*a + b)*cosh(f*x + e)*sinh(f*x + e)^4
+ (2*a + b)*sinh(f*x + e)^5 - 2*(2*a + b)*cosh(f*x + e)^3 + 2*(5*(2*a + b)*
cosh(f*x + e)^2 - 2*a - b)*sinh(f*x + e)^3 + 2*(5*(2*a + b)*cosh(f*x + e)^3
 - 3*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a + b)*cosh(f*x + e) + (
5*(2*a + b)*cosh(f*x + e)^4 - 6*(2*a + b)*cosh(f*x + e)^2 + 2*a + b)*sinh(f
*x + e))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + sqrt(2)*(a*cosh(
f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x + e)^3 + a*sinh(f*x + e)^4 - 4*a*co
sh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2 - 2*a)*sinh(f*x + e)^2 + 4*(a*cosh(f
*x + e)^3 - 2*a*cosh(f*x + e))*sinh(f*x + e) + a)*sqrt((b*cosh(f*x + e)^2 +
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^5 + 5*a*f*cosh(f*x + e)*sinh(f*
x + e)^4 + a*f*sinh(f*x + e)^5 - 2*a*f*cosh(f*x + e)^3 + 2*(5*a*f*cosh(f*x
+ e)^2 - a*f)*sinh(f*x + e)^3 + a*f*cosh(f*x + e) + 2*(5*a*f*cosh(f*x + e)^
3 - 3*a*f*cosh(f*x + e))*sinh(f*x + e)^2 + (5*a*f*cosh(f*x + e)^4 - 6*a*f*c
osh(f*x + e)^2 + a*f)*sinh(f*x + e))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^2(e+fx)}\coth^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{128,[6,12,6]%%%}+%%%{%%%{-384,[1]%%%},[6,12,5]%%%}+%%%{%%%{38
4,[2]%
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(e+f\,x)^3\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.462 $\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=167

$$-\frac{(8a^2+8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}+\frac{(8a^2+8ab-b^2)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{(8a-b)\mathrm{cs}}{}$$

```
[Out] -1/8*(8*a^2+8*a*b-b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f
-1/8*(8*a-b)*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2)/a^2/f-1/4*csch(f*x+e)^
4*(a+b*sinh(f*x+e)^2)^(3/2)/a/f+1/8*(8*a^2+8*a*b-b^2)*(a+b*sinh(f*x+e)^2)^(
1/2)/a^2/f
```

Rubi [A]
time = 0.13, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 52, 65, 214}

$$\frac{(8a^2+8ab-b^2)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{(8a-b)\mathrm{csch}^2(e+fx)\,(a+b\sinh^2(e+fx))^{3/2}}{8a^2f}-\frac{(8a^2+8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}-\frac{\mathrm{csch}^4(e+fx)\,(a+b\sinh^2(e+fx))^{3/2}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/8*((8*a^2 + 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(
a^(3/2)*f) + ((8*a^2 + 8*a*b - b^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a^2*f)
- ((8*a - b)*Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(8*a^2*f) - (Cs
ch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*a*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\mathrm{Subst}\left(\int \frac{(1+x)^2\sqrt{a+bx}}{x^3}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4af} + \mathrm{Subst}\Big(\int \tfrac{(\frac{1}{2}(8a-b)+}{}$$

$$= -\frac{(8a-b)\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{8a^2f} - \operatorname{csch}^4(e+$$

$$= \frac{(8a^2+8ab-b^2)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - (8a-b)\operatorname{csch}^2(e+$$

$$= \frac{(8a^2+8ab-b^2)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - (8a-b)\operatorname{csch}^2(e+$$

$$= -\frac{(8a^2+8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f} + (8a^2$$

Mathematica [A]
time = 0.63, size = 102, normalized size = 0.61

$$\frac{(-8a^2-8ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right) - \sqrt{a}\left(-8a+(8a+b)\operatorname{csch}^2(e+fx)+2a\operatorname{csch}^4(e+fx)\right)\sqrt{a+b\sinh^2(e+fx)}}{8a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 - 8*a*b + b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] - Sqrt
[a]*(-8*a + (8*a + b)*Csch[e + f*x]^2 + 2*a*Csch[e + f*x]^4)*Sqrt[a + b*Sin
h[e + f*x]^2])/(8*a^(3/2)*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.29, size = 80, normalized size = 0.48

method	result	size

default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\left(\cosh^4(fx+e)\right)\left(a-b+b\left(\cosh^2(fx+e)\right)\right)}{\sinh(fx+e)\left(\cosh^4(fx+e)-2\left(\cosh^2(fx+e)\right)+1\right)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)/(cosh(f*x+e)^4-2*cosh(f*x+e)^2+1)*cosh(f*x+e)^4*
(a-b+b*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^5, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1839 vs. $2(147) = 294$.
time = 0.84, size = 3880, normalized size = 23.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^9 + 9*(8*a^2 + 8*a*b - b^2)*co
sh(f*x + e)*sinh(f*x + e)^8 + (8*a^2 + 8*a*b - b^2)*sinh(f*x + e)^9 - 4*(8*
a^2 + 8*a*b - b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 + 8*a*b - b^2)*cosh(f*x +
e)^2 - 8*a^2 - 8*a*b + b^2)*sinh(f*x + e)^7 + 28*(3*(8*a^2 + 8*a*b - b^2)*c
osh(f*x + e)^3 - (8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(
8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^5 + 6*(21*(8*a^2 + 8*a*b - b^2)*cosh(f*x
 + e)^4 - 14*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*s
inh(f*x + e)^5 + 2*(63*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^5 - 70*(8*a^2 +
8*a*b - b^2)*cosh(f*x + e)^3 + 15*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh
(f*x + e)^4 - 4*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^2 + 8*a*
b - b^2)*cosh(f*x + e)^6 - 35*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 + 15*(8
*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 - 8*a^2 - 8*a*b + b^2)*sinh(f*x + e)^3
+ 12*(3*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^7 - 7*(8*a^2 + 8*a*b - b^2)*cos
h(f*x + e)^5 + 5*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 - (8*a^2 + 8*a*b - b
```

```
^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 + 8*a*b - b^2)*cosh(f*x + e) +
(9*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^8 - 28*(8*a^2 + 8*a*b - b^2)*cosh(f*
x + e)^6 + 30*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 - 12*(8*a^2 + 8*a*b - b
^2)*cosh(f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*sinh(f*x + e))*sqrt(a)*log((b*co
sh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(
4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^
2 + 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cos
h(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e
))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 +
sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x +
e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*
(2*a^2*cosh(f*x + e)^8 + 16*a^2*cosh(f*x + e)*sinh(f*x + e)^7 + 2*a^2*sinh(
f*x + e)^8 - (16*a^2 + a*b)*cosh(f*x + e)^6 + (56*a^2*cosh(f*x + e)^2 - 16*
a^2 - a*b)*sinh(f*x + e)^6 + 2*(56*a^2*cosh(f*x + e)^3 - 3*(16*a^2 + a*b)*c
osh(f*x + e))*sinh(f*x + e)^5 + 2*(10*a^2 + a*b)*cosh(f*x + e)^4 + (140*a^2
*cosh(f*x + e)^4 - 15*(16*a^2 + a*b)*cosh(f*x + e)^2 + 20*a^2 + 2*a*b)*sinh
(f*x + e)^4 + 4*(28*a^2*cosh(f*x + e)^5 - 5*(16*a^2 + a*b)*cosh(f*x + e)^3
+ 2*(10*a^2 + a*b)*cosh(f*x + e))*sinh(f*x + e)^3 - (16*a^2 + a*b)*cosh(f*x
 + e)^2 + (56*a^2*cosh(f*x + e)^6 - 15*(16*a^2 + a*b)*cosh(f*x + e)^4 + 12*
(10*a^2 + a*b)*cosh(f*x + e)^2 - 16*a^2 - a*b)*sinh(f*x + e)^2 + 2*a^2 + 2*
(8*a^2*cosh(f*x + e)^7 - 3*(16*a^2 + a*b)*cosh(f*x + e)^5 + 4*(10*a^2 + a*b
)*cosh(f*x + e)^3 - (16*a^2 + a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*co
sh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f*x + e)^9 + 9*a^2*f*co
sh(f*x + e)*sinh(f*x + e)^8 + a^2*f*sinh(f*x + e)^9 - 4*a^2*f*cosh(f*x + e)
^7 + 6*a^2*f*cosh(f*x + e)^5 + 4*(9*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x
 + e)^7 + 28*(3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)^
6 - 4*a^2*f*cosh(f*x + e)^3 + 6*(21*a^2*f*cosh(f*x + e)^4 - 14*a^2*f*cosh(f
*x + e)^2 + a^2*f)*sinh(f*x + e)^5 + 2*(63*a^2*f*cosh(f*x + e)^5 - 70*a^2*f
*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e))*sinh(f*x + e)^4 + a^2*f*cosh(f*x
 + e) + 4*(21*a^2*f*cosh(f*x + e)^6 - 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*c
osh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^3 + 12*(3*a^2*f*cosh(f*x + e)^7 - 7*a
^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(
f*x + e)^2 + (9*a^2*f*cosh(f*x + e)^8 - 28*a^2*f*cosh(f*x + e)^6 + 30*a^2*f
*cosh(f*x + e)^4 - 12*a^2*f*cosh(f*x + e)^2 + a^2*f)*sinh(f*x + e)), 1/8*((
(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^9 + 9*(8*a^2 + 8*a*b - b^2)*cosh(f*x +
e)*sinh(f*x + e)^8 + (8*a^2 + 8*a*b - b^2)*sinh(f*x + e)^9 - 4*(8*a^2 + 8*a
*b - b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 - 8*
a^2 - 8*a*b + b^2)*sinh(f*x + e)^7 + 28*(3*(8*a^2 + 8*a*b - b^2)*cosh(f*x +
 e)^3 - (8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(8*a^2 + 8
*a*b - b^2)*cosh(f*x + e)^5 + 6*(21*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 -
 14*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*sinh(f*x +
 e)^5 + 2*(63*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^5 - 70*(8*a^2 + 8*a*b - b
^2)*cosh(f*x + e)^3 + 15*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)
```

```
^4 - 4*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^2 + 8*a*b - b^2)*
cosh(f*x + e)^6 - 35*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 + 15*(8*a^2 + 8*
a*b - b^2)*cosh(f*x + e)^2 - 8*a^2 - 8*a*b + b^2)*sinh(f*x + e)^3 + 12*(3*(
8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^7 - 7*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e
)^5 + 5*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 - (8*a^2 + 8*a*b - b^2)*cosh(
f*x + e))*sinh(f*x + e)^2 + (8*a^2 + 8*a*b - b^2)*cosh(f*x + e) + (9*(8*a^2
 + 8*a*b - b^2)*cosh(f*x + e)^8 - 28*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^6
+ 30*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 - 12...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.92Unable to divide
, perhaps due to rounding error%%%{2048,[10,20,10]%%%}+%%%{%%%{-10240,[1]%%
%},[10
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(e+f\,x)^5\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.463 $\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^4(e+fx)\,dx$

Optimal. Leaf size=292

$$-\frac{(7a-8b)E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(3a-4b)F\left(\text{ArcTan}(\text{sin}\right.}{3(a-b)f}$$

```
[Out] -1/3*(7*a-8*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*Elliptic
E(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(
f*x+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*
a-4*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2
)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(7*a-8*b)*(
a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/f-1/3*(3*a-4*b)*(a+b*sinh(f*x+e)
^2)^(1/2)*tanh(f*x+e)/(a-b)/f-1/3*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3/f
```

Rubi [A]
time = 0.22, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3275, 478, 592, 545, 429, 506, 422}

$$\frac{(3a-4b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(7a-8b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\tanh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)}-\frac{(3a-4b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

```
[Out] -1/3*((7*a - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*S
qrt[a + b*Sinh[e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sec
h[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*
(a + b*Sinh[e + f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tan
h[e + f*x])/(3*(a - b)*f) - ((3*a - 4*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e
 + f*x])/(3*(a - b)*f) - (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3)/(3*f
)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
```

```
e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^4(e+fx)\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{(1+x^2)^{5/2}}\,dx,\right.}{f}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh^3(e+fx)}{3f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\right.}{}$$

$$= -\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3(a-b)f} - \frac{\sqrt{a+b\,\mathrm{si}}}{}$$

$$= -\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3(a-b)f} - \frac{\sqrt{a+b\,\mathrm{si}}}{}$$

$$= \frac{(3a-4b)F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b}}{3(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

$$= -\frac{(7a-8b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+}}{3(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+f\right.}{a}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.43, size = 214, normalized size = 0.73

$$\frac{-2ia(7a-8b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\left|\frac{b}{a}\right.\right)+8ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\left|\frac{b}{a}\right.\right)-\frac{\left(8a^2-12ab+b^2+4\left(4a^2-6ab+b^2\right)\cosh(2(e+fx))+(4a-5b)b\cosh(4(e+fx))\right)\operatorname{sech}^2(e+fx)\tanh(e+fx)}{2\sqrt{2}}}{6(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

```
[Out] ((-2*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(
e + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*El
lipticF[I*(e + f*x), b/a] - ((8*a^2 - 12*a*b + b^2 + 4*(4*a^2 - 6*a*b + b^2
)*Cosh[2*(e + f*x)] + (4*a - 5*b)*b*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh
[e + f*x])/(2*Sqrt[2]))/(6*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.70, size = 366, normalized size = 1.25

method	result
default	$\frac{\left(-4\sqrt{-\frac{b}{a}}\, ab+5\sqrt{-\frac{b}{a}}\, b^{2}\right)\left(\cosh^{4}\left(fx+e\right)\right)\sinh\left(fx+e\right)+\left(-4\sqrt{-\frac{b}{a}}\, a^{2}+10\sqrt{-\frac{b}{a}}\, ab-6\sqrt{-\frac{b}{a}}\, b^{2}\right)\left(\cosh^{2}\left(fx+e\right)\right)\sinh(f}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-4*(-1/a*b)^(1/2)*a*b+5*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)
+(-4*(-1/a*b)^(1/2)*a^2+10*(-1/a*b)^(1/2)*a*b-6*(-1/a*b)^(1/2)*b^2)*cosh(f*
x+e)^2*sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-11*EllipticF(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*b^2+7*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b
-8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2+((-
1/a*b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*sinh(f*x+e))/cosh
(f*x+e)^3/(a-b)/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)
```

Fricas [F]
time = 0.11, size = 25, normalized size = 0.09

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\tanh\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sinh^{2}\left(e+fx\right)}\,\tanh^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**4, x)
```

Giac **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.62Unable to divide, perhaps due to rou
nding error%%%{65536,[8,11,11]%%%}+%%%{%%%{-327680,[1]%%%},[8,11,10]%%%}+%%
%{%%%{655
```

Mupad **[F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{tanh}(e+f\,x)^4\,\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.464 $\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^2(e+fx)\,dx$

Optimal. Leaf size=168

$$-\frac{2E\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{f\sqrt{\text{sech}^2(e+fx)\,(}}$$

```
[Out] -2*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e
)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1
/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(1/(1+sinh(f*x+e)^2))^(1/
2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1
-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*si
nh(f*x+e)^2)/a)^(1/2)+(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3275, 478, 545, 429, 506, 422}

$$\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

```
[Out] (-2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Elli
pticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x
]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*S
inh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^2(e+fx)\,dx = \frac{\Big(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\Big)\,\mathrm{Subst}\Big(\int \frac{x^2\sqrt{a+bx^2}}{(1+x^2)^{3/2}}\,dx, x,}{f}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{f} + \frac{\Big(\sqrt{\cosh^2(e+fx)}\ \mathrm{se}}{}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{f} + \frac{\Big(a\sqrt{\cosh^2(e+fx)}}{}$$

$$= \frac{F\big(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\big)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\,\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

$$= -\frac{2E\big(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\big)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\,\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 0.38, size = 150, normalized size = 0.89

$$\frac{-2i\sqrt{2}\,a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+i\sqrt{2}\,a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+(-2a+b-b\cosh(2(e+fx)))\tanh(e+fx)}{f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
f*x), b/a] + I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[
I*(e + f*x), b/a] + (-2*a + b - b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(f*Sqrt
[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [**A**]
time = 1.58, size = 233, normalized size = 1.39

method	result
default	$-\frac{\sqrt{-\frac{b}{a}}\,b\left(\sinh^3(fx+e)\right)-a\sqrt{\frac{a+b\left(\sinh^2(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)+2b\sqrt{\frac{a+}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -((-1/a*b)^(1/2)*b*sinh(f*x+e)^3-a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*b*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))-2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*E
llipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+(-1/a*b)^(1/2)*a*sinh(f*x+
e))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima **[F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)
```

Fricas **[F]**
time = 0.09, size = 25, normalized size = 0.15

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\tanh\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)
```

Sympy **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sinh^{2}\left(e+fx\right)}\,\tanh^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**2, x)
```

Giac **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(e+f\,x)^2\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.465 $\int \sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(
1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3257, 3256}

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
(b*Sinh[e + f*x]^2)/a])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\,dx=\frac{\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

$$=-\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

Mathematica [A]
time = 0.11, size = 69, normalized size = 1.15

$$-\frac{ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a]
)/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.03, size = 140, normalized size = 2.33

method	result
default	$\frac{\sqrt{\frac{a+b(\sinh^2(fx+e))}{a}}\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,(a\,\mathrm{EllipticF}(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}})-b\,\mathrm{EllipticF}(\sinh(fx+e)\sqrt{-\frac{b}{a}},}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/
cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**F**]
time = 0.10, size = 16, normalized size = 0.27

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sinh^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{b\,\mathrm{sinh}\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.466 $\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=202

$$-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{2E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx}}{f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] -coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-2*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+s
inh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(
1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+
e)^2)/a)^(1/2)+(a+b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*El
lipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b
*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+2*(a+
b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.14, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3275, 484, 545, 429, 506, 422}

$$\frac{(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - (2*EllipticE[ArcTan[Sinh
[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Se
ch[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*EllipticF[ArcTan[Sinh
[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*Sqrt[a + b*Sinh[e + f*x]^
2]*Tanh[e + f*x])/f
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 484

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^p*((c + d*x^n)^q/(e*(m
+ 1))), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}\sqrt{a+bx^2}}{x^2}\right.}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(2\sqrt{\cosh^2(e+fx)}\,\mathrm{se}\right.}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(2b\sqrt{\cosh^2(e+fx)}\,\mathrm{s}\right.}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(a+b)F\left(\tan^{-1}(\sinh(\right.}{af\sqrt{\frac{\mathrm{s}}{-}}}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{2E\left(\tan^{-1}(\sinh(e+f\right.}{f\sqrt{\frac{\mathrm{sech}}{}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.42, size = 154, normalized size = 0.76

$$\frac{(-2a+b-b\cosh(2(e+fx)))\coth(e+fx)-2i\sqrt{2}\,a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+i\sqrt{2}\,(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*Sqrt[2]*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + I*Sqrt[2]*(a -
 b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(f
*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.46, size = 215, normalized size = 1.06

method	result
default	$-\frac{\sqrt{-\frac{b}{a}}\,b(\cosh^4(fx+e))+\left(\sqrt{-\frac{b}{a}}\,a-\sqrt{-\frac{b}{a}}\,b\right)(\cosh^2(fx+e))-\sinh(fx+e)\sqrt{\frac{b(\cosh^2(fx+e))}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}}}{\sinh(fx+e)\sqrt{-\frac{b}{a}}\,\cosh(fx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-1/a*b)^(1/2)*b*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^(1/2)*b)*cosh(f
*x+e)^2-sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)
*(a*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))))/sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2
)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.11, size = 25, normalized size = 0.12

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\coth\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sinh^{2}\left(e+fx\right)}\,\coth^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{64,[4,8,4]%%%}+%%%{%%%{-128,[1]%%%},[4,8,3]%%%}+%%%{%%%{64,[2
]%%%},
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \coth(e+f\,x)^2\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.467 $\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=270

$$-\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}-\frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{(7a+b)E(\text{ArcTan}(}{3a}$$

```
[Out] -1/3*(3*a+b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-1/3*coth(f*x+e)^3*(a
+b*sinh(f*x+e)^2)^(1/2)/f-1/3*(7*a+b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f
*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))
*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^
2)/a)^(1/2)+1/3*(3*a+5*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/
2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)
*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+
1/3*(7*a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a/f
```

Rubi [A]

time = 0.21, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3275, 484, 594, 545, 429, 506, 422}

$$\frac{(3a+5b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(7a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(7a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}-\frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/3*((3*a + b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f) - (Coth[e
+ f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((7*a + b)*EllipticE[ArcTan[S
inh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f*S
qrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + 5*b)*EllipticF[
ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/
(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a + b)*Sqrt
[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 484

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^p*((c + d*x^n)^q/(e*(m
+ 1))), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}\sqrt{a+bx^2}}{x^4}\right.}{f}$$

$$= -\frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(2\sqrt{\cosh^2(e+fx)}\right.}{}$$

$$= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth^3(e+f}{}$$

$$= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth^3(e+f}{}$$

$$= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth^3(e+f}{}$$

$$= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth^3(e+f}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.38, size = 210, normalized size = 0.78

$$\frac{-\frac{(-8a^2+4ab+3b^2+4(4a^2-2ab-b^2)\cosh(2(e+fx))+b(4a+b)\cosh(4(e+fx)))\coth(e+fx)\operatorname{csch}^2(e+fx)}{2\sqrt{2}} - 2ia(7a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) + 8ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{6af\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-1/2*((-8*a^2 + 4*a*b + 3*b^2 + 4*(4*a^2 - 2*a*b - b^2)*Cosh[2*(e + f*x)]
+ b*(4*a + b)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] - (
2*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f
*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cF[I*(e + f*x), b/a])/(6*a*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.49, size = 519, normalized size = 1.92

method	result
default	$-\frac{4\sqrt{-\frac{b}{a}}\,ab(\sinh^6(fx+e))+\sqrt{-\frac{b}{a}}\,b^2(\sinh^6(fx+e))-3a^2\sqrt{\frac{a+b(\sinh^2(fx+e))}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh(fx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6+(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6-3
*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x
+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*sinh(f*x+e)^3+2*b*((a+b*sinh(f*x+e)^2)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)
)*a*sinh(f*x+e)^3+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3-7*((a+b*sinh
(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1
/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+
e)^3+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4+6*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^4+
(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2+2*(-1/a
*b)^(1/2)*a*b*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^2)/a/sinh(f*x+e)^3/(-1/a*b)^(1
/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)
```

Fricas [F]
time = 0.13, size = 25, normalized size = 0.09

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^2+a}\,\coth\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^{2}(e+fx)}\,\coth^{4}(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x)**4, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.48Unable to divide
, perhaps due to rounding error%%%{1024,[8,16,8]%%%}+%%%{%%%{-4096,[1]%%%},
[8,16,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{coth}(e+f\,x)^4\,\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.468 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx$

Optimal. Leaf size=232

$$-\frac{(8a^2-40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8\sqrt{a-b}\,f}+\frac{(8a^2-40ab+35b^2)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)f}+(8a^2$$

```
[Out] 1/24*(8*a^2-40*a*b+35*b^2)*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/f+1/8*(8*a-9*b
)*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(5/2)/(a-b)^2/f-1/4*sech(f*x+e)^4*(a+b*
sinh(f*x+e)^2)^(5/2)/(a-b)/f-1/8*(8*a^2-40*a*b+35*b^2)*arctanh((a+b*sinh(f*
x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)+1/8*(8*a^2-40*a*b+35*b^2)*(a+b*sin
h(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A]

time = 0.19, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 52, 65, 214}

$$\frac{(8a^2-40ab+35b^2)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24f(a-b)^2}+\frac{(8a^2-40ab+35b^2)\sqrt{a+b\sinh^2(e+fx)}}{8f(a-b)}-\frac{(8a^2-40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f\sqrt{a-b}}-\frac{\operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{4f(a-b)}+\frac{(8a-9b)\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^5,x]
```

```
[Out] -1/8*((8*a^2 - 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a
 - b]])/(Sqrt[a - b]*f) + ((8*a^2 - 40*a*b + 35*b^2)*Sqrt[a + b*Sinh[e + f*x
]^2])/(8*(a - b)*f) + ((8*a^2 - 40*a*b + 35*b^2)*(a + b*Sinh[e + f*x]^2)^(3
/2))/(24*(a - b)^2*f) + ((8*a - 9*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2
)^(5/2))/(8*(a - b)^2*f) - (Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/
(4*(a - b)*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx=\frac{\text{Subst}\left(\int\frac{x^2(a+bx)^{3/2}}{(1+x)^3}\,dx,x,\sinh^2(e+fx)\right)}{2f}$$

$$=-\frac{\text{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{4(a-b)f}+\text{Subst}\Big(\int\ (\tfrac{1}{2}(-4a+5$$

$$=\frac{(8a-9b)\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{8(a-b)^2f}-\text{sech}^4(e+$$

$$=\frac{(8a^2-40ab+35b^2)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24(a-b)^2f}+(8a-9b)\text{sec}$$

$$=\frac{(8a^2-40ab+35b^2)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)f}+(8a^2-40ab+$$

$$=\frac{(8a^2-40ab+35b^2)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)f}+(8a^2-40ab+$$

$$=-\frac{(8a^2-40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8\sqrt{a-b}\,f}+$$

Mathematica [A]
time = 1.53, size = 169, normalized size = 0.73

$$-\frac{-3(8a-9b)\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}+6(a-b)\text{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}-(8a^2-40ab+35b^2)\left(-3(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{a+b\sinh^2(e+fx)}\left(4a-3b+b\sinh^2(e+fx)\right)\right)}{24(a-b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^5,x]
```

```
[Out] -1/24*(-3*(8*a - 9*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2) + 6*(a
- b)*Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2) - (8*a^2 - 40*a*b + 35*b
^2)*(-3*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + Sq
rt[a + b*Sinh[e + f*x]^2]*(4*a - 3*b + b*Sinh[e + f*x]^2)))/((a - b)^2*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.53, size = 71, normalized size = 0.31

method	result	size

default	$\frac{\text{`}int/indef0\text{`}\left(\frac{\left(\sinh^{5}(fx+e)\right)\left(b^{2}\left(\sinh^{4}(fx+e)\right)+2ab\left(\sinh^{2}(fx+e)\right)+a^{2}\right)}{\cosh(fx+e)^{6}\sqrt{a+b\left(\sinh^{2}(fx+e)\right)}},\sinh(fx+e)\right)}{f}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 'int/indef0'(sinh(f*x+e)^5*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh
(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^5, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3092 vs. $2(208) = 416$.
time = 0.94, size = 6380, normalized size = 27.50

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^11 + 11*(8*a^2 - 40*a*b +
 35*b^2)*cosh(f*x + e)*sinh(f*x + e)^10 + (8*a^2 - 40*a*b + 35*b^2)*sinh(f*
x + e)^11 + 4*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^9 + (55*(8*a^2 - 40*a
*b + 35*b^2)*cosh(f*x + e)^2 + 32*a^2 - 160*a*b + 140*b^2)*sinh(f*x + e)^9
+ 3*(55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + 12*(8*a^2 - 40*a*b + 35
*b^2)*cosh(f*x + e))*sinh(f*x + e)^8 + 6*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x
 + e)^7 + 6*(55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 24*(8*a^2 - 40*
a*b + 35*b^2)*cosh(f*x + e)^2 + 8*a^2 - 40*a*b + 35*b^2)*sinh(f*x + e)^7 +
42*(11*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 8*(8*a^2 - 40*a*b + 35*b
^2)*cosh(f*x + e)^3 + (8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e
)^6 + 4*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 2*(231*(8*a^2 - 40*a*b
+ 35*b^2)*cosh(f*x + e)^6 + 252*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 +
 63*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^2 + 16*a^2 - 80*a*b + 70*b^2)*s
inh(f*x + e)^5 + 2*(165*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^7 + 252*(8*
```

```
a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 105*(8*a^2 - 40*a*b + 35*b^2)*cosh
(f*x + e)^3 + 10*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 +
 (8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + (165*(8*a^2 - 40*a*b + 35*b^2)
*cosh(f*x + e)^8 + 336*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^6 + 210*(8*a
^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 40*(8*a^2 - 40*a*b + 35*b^2)*cosh(f
*x + e)^2 + 8*a^2 - 40*a*b + 35*b^2)*sinh(f*x + e)^3 + (55*(8*a^2 - 40*a*b
+ 35*b^2)*cosh(f*x + e)^9 + 144*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^7 +
 126*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 40*(8*a^2 - 40*a*b + 35*b^
2)*cosh(f*x + e)^3 + 3*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x +
e)^2 + (11*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^10 + 36*(8*a^2 - 40*a*b
+ 35*b^2)*cosh(f*x + e)^8 + 42*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^6 +
20*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 40*a*b + 35*b^2)*
cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*co
sh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x +
e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt
(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + s
inh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x
+ e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e
)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(co
sh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*((a*b - b^2)
*cosh(f*x + e)^12 + 12*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^11 + (a*b -
b^2)*sinh(f*x + e)^12 + 2*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^10 + 2*(3
3*(a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 25*a*b + 17*b^2)*sinh(f*x + e)^10 +
 20*(11*(a*b - b^2)*cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^2)*cosh(f*x +
e))*sinh(f*x + e)^9 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^8 + (495*
(a*b - b^2)*cosh(f*x + e)^4 + 90*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^2
+ 112*a^2 - 335*a*b + 223*b^2)*sinh(f*x + e)^8 + 8*(99*(a*b - b^2)*cosh(f*x
 + e)^5 + 30*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^3 + (112*a^2 - 335*a*b
 + 223*b^2)*cosh(f*x + e))*sinh(f*x + e)^7 + 8*(18*a^2 - 59*a*b + 41*b^2)*c
osh(f*x + e)^6 + 4*(231*(a*b - b^2)*cosh(f*x + e)^6 + 105*(8*a^2 - 25*a*b +
 17*b^2)*cosh(f*x + e)^4 + 7*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^2
+ 36*a^2 - 118*a*b + 82*b^2)*sinh(f*x + e)^6 + 8*(99*(a*b - b^2)*cosh(f*x +
 e)^7 + 63*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^5 + 7*(112*a^2 - 335*a*b
 + 223*b^2)*cosh(f*x + e)^3 + 6*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e))*s
inh(f*x + e)^5 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^4 + (495*(a*b
- b^2)*cosh(f*x + e)^8 + 420*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^6 + 70
*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^4 + 120*(18*a^2 - 59*a*b + 41*
b^2)*cosh(f*x + e)^2 + 112*a^2 - 335*a*b + 223*b^2)*sinh(f*x + e)^4 + 4*(55
*(a*b - b^2)*cosh(f*x + e)^9 + 60*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^7
 + 14*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^5 + 40*(18*a^2 - 59*a*b +
 41*b^2)*cosh(f*x + e)^3 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e))*sin
h(f*x + e)^3 + 2*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^2 + 2*(33*(a*b - b
^2)*cosh(f*x + e)^10 + 45*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^8 + 14*(1
12*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^6 + 60*(18*a^2 - 59*a*b + 41*b^2)
```

```
*cosh(f*x + e)^4 + 3*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^2 + 8*a^2
- 25*a*b + 17*b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*(3*(a*b - b^2)*cosh(f*x
+ e)^11 + 5*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^9 + 2*(112*a^2 - 335*a*
b + 223*b^2)*cosh(f*x + e)^7 + 12*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^
5 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^
2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)))/((a - b)*f*cosh(f*x + e)^11 + 11*(a - b...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 12.72Error: Bad Argu
ment Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tanh(e+f\,x)^5\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.469 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)\,dx$

Optimal. Leaf size=156

$$-\frac{(2a-5b)\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f}+\frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{(2a-5b)\left(a+b\,\mathrm{s}\right.}{6(a-}$$

```
[Out] 1/6*(2*a-5*b)*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)/f+1/2*sech(f*x+e)^2*(a+b*sinh
(f*x+e)^2)^(5/2)/(a-b)/f-1/2*(2*a-5*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a
-b)^(1/2))*(a-b)^(1/2)/f+1/2*(2*a-5*b)*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 52, 65, 214}

$$\frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6f(a-b)}+\frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f}-\frac{(2a-5b)\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f}+\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3,x]
```

```
[Out] -1/2*((2*a - 5*b)*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a -
b]])/f + ((2*a - 5*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f) + ((2*a - 5*b)*(a
+ b*Sinh[e + f*x]^2)^(3/2))/(6*(a - b)*f) + (Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2)^(5/2))/(2*(a - b)*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{(1+x)^2}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2(a-b)f} + \frac{(2a-5b)\text{Subst}\Big(\int}{}$$

$$= \frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6(a-b)f} + \frac{\text{sech}^2(e+fx)\left(a+b\sin\right.}{2(a-b)}$$

$$= \frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a-5b)\left(a+b\sinh^2(e+\right.}{6(a-b)f}$$

$$= \frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a-5b)\left(a+b\sinh^2(e+\right.}{6(a-b)f}$$

$$= -\frac{(2a-5b)\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f} + \frac{(2a}{}$$

Mathematica [A]
time = 0.44, size = 122, normalized size = 0.78

$$\frac{3\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}+(2a-5b)\left(-3(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{a+b\sinh^2(e+fx)}\left(4a-3b+b\sinh^2(e+fx)\right)\right)}{6(a-b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3,x]
```

```
[Out] (3*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2) + (2*a - 5*b)*(-3*(a - b)^
(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a + b*Sinh[e
+ f*x]^2]*(4*a - 3*b + b*Sinh[e + f*x]^2)))/(6*(a - b)*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.27, size = 71, normalized size = 0.46

method	result	size

default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\left(\sinh^{3}(fx+e)\right)\left(b^{2}\left(\sinh^{4}(fx+e)\right)+2ab\left(\sinh^{2}(fx+e)\right)+a^{2}\right)}{\cosh(fx+e)^{4}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(sinh(f*x+e)^3*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh
(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. $2(136) = 272$.
time = 1.12, size = 2454, normalized size = 15.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(6*((2*a - 5*b)*cosh(f*x + e)^7 + 7*(2*a - 5*b)*cosh(f*x + e)*sinh(f
*x + e)^6 + (2*a - 5*b)*sinh(f*x + e)^7 + 2*(2*a - 5*b)*cosh(f*x + e)^5 + (
21*(2*a - 5*b)*cosh(f*x + e)^2 + 4*a - 10*b)*sinh(f*x + e)^5 + 5*(7*(2*a -
5*b)*cosh(f*x + e)^3 + 2*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (2*a
- 5*b)*cosh(f*x + e)^3 + (35*(2*a - 5*b)*cosh(f*x + e)^4 + 20*(2*a - 5*b)*c
osh(f*x + e)^2 + 2*a - 5*b)*sinh(f*x + e)^3 + (21*(2*a - 5*b)*cosh(f*x + e)
^5 + 20*(2*a - 5*b)*cosh(f*x + e)^3 + 3*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x
 + e)^2 + (7*(2*a - 5*b)*cosh(f*x + e)^6 + 10*(2*a - 5*b)*cosh(f*x + e)^4 +
 3*(2*a - 5*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x
+ e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3
*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 +
 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(co
sh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x
```

```
+ e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3
 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x
 + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - sqrt(2)
*(b*cosh(f*x + e)^8 + 8*b*cosh(f*x + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)^8
 + 8*(2*a - 3*b)*cosh(f*x + e)^6 + 4*(7*b*cosh(f*x + e)^2 + 4*a - 6*b)*sinh
(f*x + e)^6 + 8*(7*b*cosh(f*x + e)^3 + 6*(2*a - 3*b)*cosh(f*x + e))*sinh(f*
x + e)^5 + 2*(28*a - 37*b)*cosh(f*x + e)^4 + 2*(35*b*cosh(f*x + e)^4 + 60*(
2*a - 3*b)*cosh(f*x + e)^2 + 28*a - 37*b)*sinh(f*x + e)^4 + 8*(7*b*cosh(f*x
 + e)^5 + 20*(2*a - 3*b)*cosh(f*x + e)^3 + (28*a - 37*b)*cosh(f*x + e))*sin
h(f*x + e)^3 + 8*(2*a - 3*b)*cosh(f*x + e)^2 + 4*(7*b*cosh(f*x + e)^6 + 30*
(2*a - 3*b)*cosh(f*x + e)^4 + 3*(28*a - 37*b)*cosh(f*x + e)^2 + 4*a - 6*b)*
sinh(f*x + e)^2 + 8*(b*cosh(f*x + e)^7 + 6*(2*a - 3*b)*cosh(f*x + e)^5 + (2
8*a - 37*b)*cosh(f*x + e)^3 + 2*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) +
b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^7 + 7
*f*cosh(f*x + e)*sinh(f*x + e)^6 + f*sinh(f*x + e)^7 + 2*f*cosh(f*x + e)^5
+ (21*f*cosh(f*x + e)^2 + 2*f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 + 2
*f*cosh(f*x + e))*sinh(f*x + e)^4 + f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)
^4 + 20*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)^3 + (21*f*cosh(f*x + e)^5 + 20
*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*sinh(f*x + e)^2 + (7*f*cosh(f*x + e
)^6 + 10*f*cosh(f*x + e)^4 + 3*f*cosh(f*x + e)^2)*sinh(f*x + e)), -1/24*(12
*((2*a - 5*b)*cosh(f*x + e)^7 + 7*(2*a - 5*b)*cosh(f*x + e)*sinh(f*x + e)^6
 + (2*a - 5*b)*sinh(f*x + e)^7 + 2*(2*a - 5*b)*cosh(f*x + e)^5 + (21*(2*a -
 5*b)*cosh(f*x + e)^2 + 4*a - 10*b)*sinh(f*x + e)^5 + 5*(7*(2*a - 5*b)*cosh
(f*x + e)^3 + 2*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (2*a - 5*b)*co
sh(f*x + e)^3 + (35*(2*a - 5*b)*cosh(f*x + e)^4 + 20*(2*a - 5*b)*cosh(f*x +
 e)^2 + 2*a - 5*b)*sinh(f*x + e)^3 + (21*(2*a - 5*b)*cosh(f*x + e)^5 + 20*(
2*a - 5*b)*cosh(f*x + e)^3 + 3*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x + e)^2 +
 (7*(2*a - 5*b)*cosh(f*x + e)^6 + 10*(2*a - 5*b)*cosh(f*x + e)^4 + 3*(2*a -
 5*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt
(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
 e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x
 + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*(b*cosh(f*x + e)^8 + 8*b*cosh(f*x
 + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)^8 + 8*(2*a - 3*b)*cosh(f*x + e)^6 +
 4*(7*b*cosh(f*x + e)^2 + 4*a - 6*b)*sinh(f*x + e)^6 + 8*(7*b*cosh(f*x + e)
^3 + 6*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(28*a - 37*b)*cosh(f*
x + e)^4 + 2*(35*b*cosh(f*x + e)^4 + 60*(2*a - 3*b)*cosh(f*x + e)^2 + 28*a
- 37*b)*sinh(f*x + e)^4 + 8*(7*b*cosh(f*x + e)^5 + 20*(2*a - 3*b)*cosh(f*x
+ e)^3 + (28*a - 37*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*(2*a - 3*b)*cosh(
f*x + e)^2 + 4*(7*b*cosh(f*x + e)^6 + 30*(2*a - 3*b)*cosh(f*x + e)^4 + 3*(2
8*a - 37*b)*cosh(f*x + e)^2 + 4*a - 6*b)*sinh(f*x + e)^2 + 8*(b*cosh(f*x +
e)^7 + 6*(2*a - 3*b)*cosh(f*x + e)^5 + (28*a - 37*b)*cosh(f*x + e)^3 + 2*(2
*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sin
h(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2)))/(f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)*sinh(f*x + e)^6 +
```

```
 f*sinh(f*x + e)^7 + 2*f*cosh(f*x + e)^5 + (21*f*cosh(f*x + e)^2 + 2*f)*sin
h(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 + 2*f*cosh(f*x + e))*sinh(f*x + e)^4
+ f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 + 20*f*cosh(f*x + e)^2 + f)*sin
h(f*x + e)^3 + (21*f*cosh(f*x + e)^5 + 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x
+ e))*sinh(f*x + e)^2 + (7*f*cosh(f*x + e)^6 + 10*f*cosh(f*x + e)^4 + 3*f*c
osh(f*x + e)^2)*sinh(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.75Error: Bad Argum
ent Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(e+f\,x)^3\,{\left(b\,{\sinh(e+f\,x)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.470 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx$

Optimal. Leaf size=90

$$-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}$$

```
[Out] -(a-b)^(3/2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+1/3*(a+b*sinh
(f*x+e)^2)^(3/2)/f+(a-b)*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3273, 52, 65, 214}

$$\frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x],x]
```

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/f) + ((a
 - b)*Sqrt[a + b*Sinh[e + f*x]^2])/f + (a + b*Sinh[e + f*x]^2)^(3/2)/(3*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x,\right.}{2f}$$

$$= \frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} +$$

$$= \frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} +$$

$$= -\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + (a-b)\sqrt{a}$$

Mathematica [A]
time = 0.11, size = 86, normalized size = 0.96

$$\frac{-3(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b+b\cosh^2(e+fx)}}{\sqrt{a-b}}\right) + \left(4a-4b+b\cosh^2(e+fx)\right)\sqrt{a-b+b\cosh^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x],x]
[Out] (-3*(a - b)^(3/2)*ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]] + (4
*a - 4*b + b*Cosh[e + f*x]^2)*Sqrt[a - b + b*Cosh[e + f*x]^2])/(3*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.88, size = 69, normalized size = 0.77

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\sinh(fx+e)\left(b^2\left(\sinh^4(fx+e)\right)+2ab\left(\sinh^2(fx+e)\right)+a^2\right)}{\cosh(fx+e)^2\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 'int/indef0'(sinh(f*x+e)*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f
*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(78) = 156.
time = 0.86, size = 1052, normalized size = 11.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="fricas")
```

```
[Out] [-1/24*(12*((a - b)*cosh(f*x + e)^3 + 3*(a - b)*cosh(f*x + e)^2*sinh(f*x +
e) + 3*(a - b)*cosh(f*x + e)*sinh(f*x + e)^2 + (a - b)*sinh(f*x + e)^3)*sqr
t(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sin
h(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a
 - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^
3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh
(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sin
h(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh
(f*x + e) + 1)) - sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
```

```
 e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - 7*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x
 + e)^2 + 8*a - 7*b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (8*a - 7*b)*c
osh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f*x + e) + 3*f*cosh(f
*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3), -1/24*(24*((a - b)*cosh(f*x +
 e)^3 + 3*(a - b)*cosh(f*x + e)^2*sinh(f*x + e) + 3*(a - b)*cosh(f*x + e)*s
inh(f*x + e)^2 + (a - b)*sinh(f*x + e)^3)*sqrt(-a + b)*arctan(-1/2*sqrt(2)*
sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh
(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh
(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - 7*b)*cosh(f*x + e)
^2 + 2*(3*b*cosh(f*x + e)^2 + 8*a - 7*b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +
e)^3 + (8*a - 7*b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f
*x + e) + 3*f*cosh(f*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^{2}\left(e + fx\right)\right)^{\frac{3}{2}}\tanh\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(3/2)*tanh(e + f*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(e+f\,x)\,\left(b\,{\sinh(e+f\,x)}^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.471 $\int \coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}$$

```
[Out] -a^(3/2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sinh(f*x+e)^
2)^(3/2)/f+a*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3273, 52, 65, 214}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b
*Sinh[e + f*x]^2])/f + (a + b*Sinh[e + f*x]^2)^(3/2)/(3*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sinh^2(e\right.}{2f}$$

$$= \frac{a\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\text{Subs}}{}$$

$$= \frac{a\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\text{Subs}}{}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b\sinh^2(e}}{f}$$

Mathematica [A]
time = 0.11, size = 69, normalized size = 0.88

$$\frac{-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right) + \sqrt{a+b\sinh^2(e+fx)}\left(4a+b\sinh^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + Sqrt[a + b*Sinh[
e + f*x]^2]*(4*a + b*Sinh[e + f*x]^2))/(3*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.98, size = 62, normalized size = 0.79

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{b^{2}\left(\sinh^{3}\left(fx+e\right)\right)+2ab\sinh\left(fx+e\right)+\frac{a^{2}}{\sinh\left(fx+e\right)}}{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^3+2*a*b*sinh(f*x+e)+a^2/sinh(f*x+e))/(a+b*sin
h(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(66) = 132.
time = 0.64, size = 1000, normalized size = 12.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(12*(a*cosh(f*x + e)^3 + 3*a*cosh(f*x + e)^2*sinh(f*x + e) + 3*a*cosh
(f*x + e)*sinh(f*x + e)^2 + a*sinh(f*x + e)^3)*sqrt(a)*log((b*cosh(f*x + e)
^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*co
sh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(
2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e)
+ sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x
 + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x +
e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(c
```

```
osh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*(b*cosh(f*x +
 e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - b)
*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 8*a - b)*sinh(f*x + e)^2 + 4*(b
*cosh(f*x + e)^3 + (8*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh
(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e
)^2*sinh(f*x + e) + 3*f*cosh(f*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3),
 1/24*(24*(a*cosh(f*x + e)^3 + 3*a*cosh(f*x + e)^2*sinh(f*x + e) + 3*a*cosh
(f*x + e)*sinh(f*x + e)^2 + a*sinh(f*x + e)^3)*sqrt(-a)*arctan(1/2*sqrt(2)*
sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
 e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e)
+ a*sinh(f*x + e))) + sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f
*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f
*x + e)^2 + 8*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (8*a - b)*cos
h(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f*x + e) + 3*f*cosh(f*x
 + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(e+f\,x)\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(coth(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.472 $\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2f}+\frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{(2a+3b)\left(a+b\sinh^2\right.}{6af}$$

```
[Out] 1/6*(2*a+3*b)*(a+b*sinh(f*x+e)^2)^(3/2)/a/f-1/2*csch(f*x+e)^2*(a+b*sinh(f*x
+e)^2)^(5/2)/a/f-1/2*(2*a+3*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))*a
^(1/2)/f+1/2*(2*a+3*b)*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 52, 65, 214}

$$\frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6af}+\frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f}-\frac{\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[a]*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/f +
 ((2*a + 3*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f) + ((2*a + 3*b)*(a + b*Sinh
[e + f*x]^2)^(3/2))/(6*a*f) - (Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2
))/(2*a*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)(a+bx)^{3/2}}{x^2}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2af} + \frac{(2a+3b)\text{Subst}}{}$$

$$= \frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6af} - \frac{\text{csch}^2(e+fx)\left(a+b\,\text{s}\right.}{2af}$$

$$= \frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a+3b)\left(a+b\sinh^2(e\right.}{6af}$$

$$= \frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a+3b)\left(a+b\sinh^2(e\right.}{6af}$$

$$= -\frac{\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2f} + \frac{(2a+}{}$$

Mathematica [**A**]
time = 0.34, size = 90, normalized size = 0.64

$$\frac{-3\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)+\left(8a+5b+b\cosh(2(e+fx))-3a\operatorname{csch}^2(e+fx)\right)\sqrt{a+b\sinh^2(e+fx)}}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*Sqrt[a]*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + (8*a
 + 5*b + b*Cosh[2*(e + f*x)] - 3*a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x
]^2])/(6*f)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.20, size = 84, normalized size = 0.60

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{b^2\left(\sinh^3(fx+e)\right)+\left(2ab+b^2\right)\sinh(fx+e)+\frac{a^2+2ab}{\sinh(fx+e)}+\frac{a^2}{\sinh(fx+e)^3}}{\sqrt{a+b\left(\sinh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^3+(2*a*b+b^2)*sinh(f*x+e)+(a^2+2*a*b)/sinh(f*
x+e)+a^2/sinh(f*x+e)^3)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. 2(120) = 240.
time = 0.73, size = 2406, normalized size = 17.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [1/24*(6*((2*a + 3*b)*cosh(f*x + e)^7 + 7*(2*a + 3*b)*cosh(f*x + e)*sinh(f*
x + e)^6 + (2*a + 3*b)*sinh(f*x + e)^7 - 2*(2*a + 3*b)*cosh(f*x + e)^5 + (2
1*(2*a + 3*b)*cosh(f*x + e)^2 - 4*a - 6*b)*sinh(f*x + e)^5 + 5*(7*(2*a + 3*
b)*cosh(f*x + e)^3 - 2*(2*a + 3*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (2*a +
3*b)*cosh(f*x + e)^3 + (35*(2*a + 3*b)*cosh(f*x + e)^4 - 20*(2*a + 3*b)*cos
h(f*x + e)^2 + 2*a + 3*b)*sinh(f*x + e)^3 + (21*(2*a + 3*b)*cosh(f*x + e)^5
 - 20*(2*a + 3*b)*cosh(f*x + e)^3 + 3*(2*a + 3*b)*cosh(f*x + e))*sinh(f*x +
 e)^2 + (7*(2*a + 3*b)*cosh(f*x + e)^6 - 10*(2*a + 3*b)*cosh(f*x + e)^4 + 3
*(2*a + 3*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4
 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh
(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)
*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
 e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) +
sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x +
 e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)
^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cos
h(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*(b*cosh(f*x + e
)^8 + 8*b*cosh(f*x + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)^8 + 8*(2*a + b)*c
osh(f*x + e)^6 + 4*(7*b*cosh(f*x + e)^2 + 4*a + 2*b)*sinh(f*x + e)^6 + 8*(7
*b*cosh(f*x + e)^3 + 6*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(28*a +
 9*b)*cosh(f*x + e)^4 + 2*(35*b*cosh(f*x + e)^4 + 60*(2*a + b)*cosh(f*x + e
)^2 - 28*a - 9*b)*sinh(f*x + e)^4 + 8*(7*b*cosh(f*x + e)^5 + 20*(2*a + b)*c
osh(f*x + e)^3 - (28*a + 9*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*(2*a + b)*
cosh(f*x + e)^2 + 4*(7*b*cosh(f*x + e)^6 + 30*(2*a + b)*cosh(f*x + e)^4 - 3
*(28*a + 9*b)*cosh(f*x + e)^2 + 4*a + 2*b)*sinh(f*x + e)^2 + 8*(b*cosh(f*x
+ e)^7 + 6*(2*a + b)*cosh(f*x + e)^5 - (28*a + 9*b)*cosh(f*x + e)^3 + 2*(2*
a + b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f
*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sin
h(f*x + e)^2)))/(f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)*sinh(f*x + e)^6 + f*
sinh(f*x + e)^7 - 2*f*cosh(f*x + e)^5 + (21*f*cosh(f*x + e)^2 - 2*f)*sinh(f
*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 - 2*f*cosh(f*x + e))*sinh(f*x + e)^4 + f
*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + f)*sinh(f
*x + e)^3 + (21*f*cosh(f*x + e)^5 - 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e
))*sinh(f*x + e)^2 + (7*f*cosh(f*x + e)^6 - 10*f*cosh(f*x + e)^4 + 3*f*cosh
(f*x + e)^2)*sinh(f*x + e)), 1/24*(12*((2*a + 3*b)*cosh(f*x + e)^7 + 7*(2*a
 + 3*b)*cosh(f*x + e)*sinh(f*x + e)^6 + (2*a + 3*b)*sinh(f*x + e)^7 - 2*(2*
a + 3*b)*cosh(f*x + e)^5 + (21*(2*a + 3*b)*cosh(f*x + e)^2 - 4*a - 6*b)*sin
h(f*x + e)^5 + 5*(7*(2*a + 3*b)*cosh(f*x + e)^3 - 2*(2*a + 3*b)*cosh(f*x +
e))*sinh(f*x + e)^4 + (2*a + 3*b)*cosh(f*x + e)^3 + (35*(2*a + 3*b)*cosh(f*
x + e)^4 - 20*(2*a + 3*b)*cosh(f*x + e)^2 + 2*a + 3*b)*sinh(f*x + e)^3 + (2
1*(2*a + 3*b)*cosh(f*x + e)^5 - 20*(2*a + 3*b)*cosh(f*x + e)^3 + 3*(2*a + 3
*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (7*(2*a + 3*b)*cosh(f*x + e)^6 - 10*(2
*a + 3*b)*cosh(f*x + e)^4 + 3*(2*a + 3*b)*cosh(f*x + e)^2)*sinh(f*x + e))*s
```

```
qrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + sqrt(2)*(b*cosh(f*x + e)^8
 + 8*b*cosh(f*x + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)^8 + 8*(2*a + b)*cosh
(f*x + e)^6 + 4*(7*b*cosh(f*x + e)^2 + 4*a + 2*b)*sinh(f*x + e)^6 + 8*(7*b*
cosh(f*x + e)^3 + 6*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(28*a + 9*
b)*cosh(f*x + e)^4 + 2*(35*b*cosh(f*x + e)^4 + 60*(2*a + b)*cosh(f*x + e)^2
 - 28*a - 9*b)*sinh(f*x + e)^4 + 8*(7*b*cosh(f*x + e)^5 + 20*(2*a + b)*cosh
(f*x + e)^3 - (28*a + 9*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*(2*a + b)*cos
h(f*x + e)^2 + 4*(7*b*cosh(f*x + e)^6 + 30*(2*a + b)*cosh(f*x + e)^4 - 3*(2
8*a + 9*b)*cosh(f*x + e)^2 + 4*a + 2*b)*sinh(f*x + e)^2 + 8*(b*cosh(f*x + e
)^7 + 6*(2*a + b)*cosh(f*x + e)^5 - (28*a + 9*b)*cosh(f*x + e)^3 + 2*(2*a +
 b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)))/(f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)*sinh(f*x + e)^6 + f*sin
h(f*x + e)^7 - 2*f*cosh(f*x + e)^5 + (21*f*cosh(f*x + e)^2 - 2*f)*sinh(f*x
+ e)^5 + 5*(7*f*cosh(f*x + e)^3 - 2*f*cosh(f*x + e))*sinh(f*x + e)^4 + f*co
sh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + f)*sinh(f*x
+ e)^3 + (21*f*cosh(f*x + e)^5 - 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*
sinh(f*x + e)^2 + (7*f*cosh(f*x + e)^6 - 10*f*cosh(f*x + e)^4 + 3*f*cosh(f*
x + e)^2)*sinh(f*x + e))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.61Unable to divide
, perhaps due to rounding error%%%{2,[6,0,8]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,
[1]%%%
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(e+f\,x)^3\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(coth(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.473 $\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=203

$$-\frac{(8a^2+3b(8a+b))\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,f}+\frac{(8a^2+3b(8a+b))\sqrt{a+b\sinh^2(e+fx)}}{8af}+(8a^2+$$

```
[Out] 1/24*(8*a^2+3*b*(8*a+b))*(a+b*sinh(f*x+e)^2)^(3/2)/a^2/f-1/8*(8*a+b)*csch(f
*x+e)^2*(a+b*sinh(f*x+e)^2)^(5/2)/a^2/f-1/4*csch(f*x+e)^4*(a+b*sinh(f*x+e)^
2)^(5/2)/a/f-1/8*(8*a^2+3*b*(8*a+b))*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1
/2))/f/a^(1/2)+1/8*(8*a^2+3*b*(8*a+b))*(a+b*sinh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.16, antiderivative size = 199, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 52, 65, 214}

$$\frac{\left(\frac{3b(8a+b)}{a^2}+8\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24f}+\frac{(8a^2+3b(8a+b))\sqrt{a+b\sinh^2(e+fx)}}{8af}-\frac{(8a^2+3b(8a+b))\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,f}-\frac{(8a+b)\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{8a^2f}-\frac{\text{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*((8*a^2 + 3*b*(8*a + b))*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])
/(Sqrt[a]*f) + ((8*a^2 + 3*b*(8*a + b))*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a*f
) + ((8 + (3*b*(8*a + b))/a^2)*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*f) - ((8*
a + b)*Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2))/(8*a^2*f) - (Csch[e +
 f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/(4*a*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2(a+bx)^{3/2}}{x^3}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{4af} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(8a+b)\right.}{}\right.}{}$$

$$= -\frac{(8a+b)\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{8a^2f} - \frac{\text{csch}^4(e+}{}$$

$$= \frac{(8a^2+3b(8a+b))\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24a^2f} - \frac{(8a+b)\text{csch}^2(}{}$$

$$= \frac{(8a^2+3b(8a+b))\sqrt{a+b\sinh^2(e+fx)}}{8af} + \frac{(8a^2+3b(8a+}{}$$

$$= \frac{(8a^2+3b(8a+b))\sqrt{a+b\sinh^2(e+fx)}}{8af} + \frac{(8a^2+3b(8a+}{}$$

$$= -\frac{(8a^2+3b(8a+b))\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,f} + \frac{(8}{}$$

Mathematica [A]
time = 0.94, size = 123, normalized size = 0.61

$$\frac{-3(8a^2+24ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\,\sqrt{a+b\sinh^2(e+fx)}\,\left(-3(8a+5b)\text{csch}^2(e+fx)-6a\text{csch}^4(e+fx)+8\left(4a+6b+b\sinh^2(e+fx)\right)\right)}{24\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(8*a^2 + 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] +
 Sqrt[a]*Sqrt[a + b*Sinh[e + f*x]^2]*(-3*(8*a + 5*b)*Csch[e + f*x]^2 - 6*a*
Csch[e + f*x]^4 + 8*(4*a + 6*b + b*Sinh[e + f*x]^2)))/(24*Sqrt[a]*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.28, size = 113, normalized size = 0.56

method	result	size

default	$\frac{`int/indef0`\left(\frac{\left(\cosh^4(fx+e)\right)\left(b^2\left(\cosh^4(fx+e)\right)+2ab\left(\cosh^2(fx+e)\right)-2b^2\left(\cosh^2(fx+e)\right)+a^2-2ab+b^2\right)}{\sinh(fx+e)\left(\cosh^4(fx+e)-2\left(\cosh^2(fx+e)\right)+1\right)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)/(cosh(f*x+e)^4-2*cosh(f*x+e)^2+1)*cosh(f*x+e)^4*
(b^2*cosh(f*x+e)^4+2*a*b*cosh(f*x+e)^2-2*b^2*cosh(f*x+e)^2+a^2-2*a*b+b^2)/(
a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^5, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2653 vs. $2(179) = 358$.
time = 0.82, size = 5509, normalized size = 27.14

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^11 + 11*(8*a^2 + 24*a*b +
3*b^2)*cosh(f*x + e)*sinh(f*x + e)^10 + (8*a^2 + 24*a*b + 3*b^2)*sinh(f*x +
 e)^11 - 4*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^9 + (55*(8*a^2 + 24*a*b +
 3*b^2)*cosh(f*x + e)^2 - 32*a^2 - 96*a*b - 12*b^2)*sinh(f*x + e)^9 + 3*(55
*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 - 12*(8*a^2 + 24*a*b + 3*b^2)*cos
h(f*x + e))*sinh(f*x + e)^8 + 6*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 +
6*(55*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^4 - 24*(8*a^2 + 24*a*b + 3*b^2
)*cosh(f*x + e)^2 + 8*a^2 + 24*a*b + 3*b^2)*sinh(f*x + e)^7 + 42*(11*(8*a^2
 + 24*a*b + 3*b^2)*cosh(f*x + e)^5 - 8*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x +
e)^3 + (8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 - 4*(8*a^2 +
 24*a*b + 3*b^2)*cosh(f*x + e)^5 + 2*(231*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x
 + e)^6 - 252*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^4 + 63*(8*a^2 + 24*a*b
 + 3*b^2)*cosh(f*x + e)^2 - 16*a^2 - 48*a*b - 6*b^2)*sinh(f*x + e)^5 + 2*(1
```

```
65*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 - 252*(8*a^2 + 24*a*b + 3*b^2)*
cosh(f*x + e)^5 + 105*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 - 10*(8*a^2
 + 24*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + (8*a^2 + 24*a*b + 3*b^2)
*cosh(f*x + e)^3 + (165*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^8 - 336*(8*a
^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^6 + 210*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*
x + e)^4 - 40*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 + 24*a*b + 3
*b^2)*sinh(f*x + e)^3 + (55*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^9 - 144*
(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 + 126*(8*a^2 + 24*a*b + 3*b^2)*cos
h(f*x + e)^5 - 40*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 + 24*
a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (11*(8*a^2 + 24*a*b + 3*b^2)*
cosh(f*x + e)^10 - 36*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^8 + 42*(8*a^2
 + 24*a*b + 3*b^2)*cosh(f*x + e)^6 - 20*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x +
 e)^4 + 3*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a)*l
og((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)
^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f
*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh
(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x +
 e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cos
h(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*
sqrt(2)*(a*b*cosh(f*x + e)^12 + 12*a*b*cosh(f*x + e)*sinh(f*x + e)^11 + a*b
*sinh(f*x + e)^12 + 2*(8*a^2 + 9*a*b)*cosh(f*x + e)^10 + 2*(33*a*b*cosh(f*x
 + e)^2 + 8*a^2 + 9*a*b)*sinh(f*x + e)^10 + 20*(11*a*b*cosh(f*x + e)^3 + (8
*a^2 + 9*a*b)*cosh(f*x + e))*sinh(f*x + e)^9 - (112*a^2 + 111*a*b)*cosh(f*x
 + e)^8 + (495*a*b*cosh(f*x + e)^4 + 90*(8*a^2 + 9*a*b)*cosh(f*x + e)^2 - 1
12*a^2 - 111*a*b)*sinh(f*x + e)^8 + 8*(99*a*b*cosh(f*x + e)^5 + 30*(8*a^2 +
 9*a*b)*cosh(f*x + e)^3 - (112*a^2 + 111*a*b)*cosh(f*x + e))*sinh(f*x + e)^
7 + 8*(18*a^2 + 23*a*b)*cosh(f*x + e)^6 + 4*(231*a*b*cosh(f*x + e)^6 + 105*
(8*a^2 + 9*a*b)*cosh(f*x + e)^4 - 7*(112*a^2 + 111*a*b)*cosh(f*x + e)^2 + 3
6*a^2 + 46*a*b)*sinh(f*x + e)^6 + 8*(99*a*b*cosh(f*x + e)^7 + 63*(8*a^2 + 9
*a*b)*cosh(f*x + e)^5 - 7*(112*a^2 + 111*a*b)*cosh(f*x + e)^3 + 6*(18*a^2 +
 23*a*b)*cosh(f*x + e))*sinh(f*x + e)^5 - (112*a^2 + 111*a*b)*cosh(f*x + e)
^4 + (495*a*b*cosh(f*x + e)^8 + 420*(8*a^2 + 9*a*b)*cosh(f*x + e)^6 - 70*(1
12*a^2 + 111*a*b)*cosh(f*x + e)^4 + 120*(18*a^2 + 23*a*b)*cosh(f*x + e)^2 -
 112*a^2 - 111*a*b)*sinh(f*x + e)^4 + 4*(55*a*b*cosh(f*x + e)^9 + 60*(8*a^2
 + 9*a*b)*cosh(f*x + e)^7 - 14*(112*a^2 + 111*a*b)*cosh(f*x + e)^5 + 40*(18
*a^2 + 23*a*b)*cosh(f*x + e)^3 - (112*a^2 + 111*a*b)*cosh(f*x + e))*sinh(f*
x + e)^3 + 2*(8*a^2 + 9*a*b)*cosh(f*x + e)^2 + 2*(33*a*b*cosh(f*x + e)^10 +
 45*(8*a^2 + 9*a*b)*cosh(f*x + e)^8 - 14*(112*a^2 + 111*a*b)*cosh(f*x + e)^
6 + 60*(18*a^2 + 23*a*b)*cosh(f*x + e)^4 - 3*(112*a^2 + 111*a*b)*cosh(f*x +
 e)^2 + 8*a^2 + 9*a*b)*sinh(f*x + e)^2 + a*b + 4*(3*a*b*cosh(f*x + e)^11 +
5*(8*a^2 + 9*a*b)*cosh(f*x + e)^9 - 2*(112*a^2 + 111*a*b)*cosh(f*x + e)^7 +
 12*(18*a^2 + 23*a*b)*cosh(f*x + e)^5 - (112*a^2 + 111*a*b)*cosh(f*x + e)^3
 + (8*a^2 + 9*a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 +
```

```
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^11 + 11*a*f*cosh(f*x + e)*sinh(f
*x + e)^10 + a*f*sinh(f*x + e)^11 - 4*a*f*cosh(f*x + e)^9 + (55*a*f*cosh(f*
x + e)^2 - 4*a*f)*sinh(f*x + e)^9 + 6*a*f*cosh(f*x + e)^7 + 3*(55*a*f*cosh(
f*x + e)^3 - 12*a*f*cosh(f*x + e))*sinh(f*x + e)^8 + 6*(55*a*f*cosh(f*x + e
)^4 - 24*a*f*cosh(f*x + e)^2 + a*f)*sinh(f*x + e)^7 - 4*a*f*cosh(f*x + e)^5
 + 42*(11*a*f*cosh(f*x + e)^5 - 8*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*
sinh(f*x + e)^6 + 2*(231*a*f*cosh(f*x + e)^6 - 252*a*f*cosh(f*x + e)^4 + 63
*a*f*cosh(f*x + e)^2 - 2*a*f)*sinh(f*x + e)^5 +...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 3.78Unable to divide
, perhaps due to rounding error%%%{1,[10,0,12]%%%}+%%%{%%{[10,0]:[1,0,%%%{-
1,[1]%
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \coth(e+f\,x)^5\,{\left(b\,{\sinh(e+f\,x)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.474 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx$

Optimal. Leaf size=305

$$-\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{8(a-2b)E\left(\mathrm{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\mathrm{sec}}{3f\sqrt{\frac{\mathrm{sech}^2(e+fx)\,(a+b\,\mathrm{s}}{a}}}$$

```
[Out] -1/3*(3*a-8*b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-8/3*(a-2
*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+
e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(
1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a-8*b)*(1/(1+sinh
(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x
+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f
*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+8/3*(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*
tanh(f*x+e)/f+(a-2*b)*sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
-1/3*(a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3/f
```

Rubi [A]
time = 0.26, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = $\{3275, 478, 591, 596, 545, 429, 506, 422\}$

$$\frac{(3a-8b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\mathrm{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f\sqrt{\frac{\mathrm{sech}^2(e+fx)\,(a+b\sinh^2(e+fx))}{a}}}-\frac{8(a-2b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\mathrm{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f\sqrt{\frac{\mathrm{sech}^2(e+fx)\,(a+b\sinh^2(e+fx))}{a}}}-\frac{\tanh^3(e+fx)\,(a+b\sinh^2(e+fx))^{3/2}}{3f}+\frac{(a-2b)\sinh^2(e+fx)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{8(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{(3a-8b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^4,x]
```

```
[Out] -1/3*((3*a - 8*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/
f - (8*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sq
rt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^
2))/a]) + ((3*a - 8*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f
*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*
f) + ((a - 2*b)*Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/
f - ((a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3)/(3*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
```

```
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^{5/2}}\,dx, x,\right.}{f}$$

$$= -\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)}{3f} + \frac{\left(\sqrt{\cosh^2(e+f}\right.}{}$$

$$= \frac{(a-2b)\sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{f} -$$

$$= -\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

$$= -\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

$$= -\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

$$= -\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.01, size = 224, normalized size = 0.73

$$\frac{-32ia(a-2b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+4ia(5a-8b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-\frac{(32a^2-108ab+18b^2+(64a^2-160ab+17b^2)\cosh(2(e+fx))+2(6a-17b)b\cosh(4(e+fx))-b^2\cosh(6(e+fx)))\operatorname{sech}^2(e+fx)\tanh(e+fx)}{4\sqrt{2}}}{12f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^4,x]
```

```
[Out] ((-32*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
 + f*x), b/a] + (4*I)*a*(5*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]
*EllipticF[I*(e + f*x), b/a] - ((32*a^2 - 108*a*b + 18*b^2 + (64*a^2 - 160*
a*b + 17*b^2)*Cosh[2*(e + f*x)] + 2*(6*a - 17*b)*b*Cosh[4*(e + f*x)] - b^2*
Cosh[6*(e + f*x)])*Sech[e + f*x]^2*Tanh[e + f*x])/(4*Sqrt[2]))/(12*f*Sqrt[2
*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.77, size = 385, normalized size = 1.26

method	result
default	$\sqrt{-\frac{b}{a}}\, b^2 \left(\cosh^6(fx+e)\right)\sinh(fx+e)+\left(-3\sqrt{-\frac{b}{a}}\, ab+7\sqrt{-\frac{b}{a}}\, b^2\right)\left(\cosh^4(fx+e)\right)\sinh(fx+e)+\left(-4\sqrt{-\frac{b}{a}}\, a^2+13\sqrt{-\frac{b}{a}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*cosh(f*x+e)^6*sinh(f*x+e)+(-3*(-1/a*b)^(1/2)*a*b+7*
(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(-4*(-1/a*b)^(1/2)*a^2+13*(-1
/a*b)^(1/2)*a*b-9*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+(cosh(f*x+e
)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(3*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a^2-16*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*a*b+16*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+8*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-16*EllipticE(sinh(f*x+e)*(-1
/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2+((-1/a*b)^(1/2)*a^2-2*(-1/a*b)^
(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*sinh(f*x+e))/(-1/a*b)^(1/2)/cosh(f*x+e)^3/(a+
b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.12, size = 25, normalized size = 0.08

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.73Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tanh(e+f\,x)^4\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.475 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx$

Optimal. Leaf size=260

$$\frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{(7a-8b)E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{sech}(e+f}{3f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+\right.}{a}}}$$

```
[Out] 4/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*(7*a-8*b)*(1/
(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+s
inh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/
(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a-4*b)*(1/(1+sinh(f*x+e)
^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^
(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2
*(a+b*sinh(f*x+e)^2)/a)^(1/2)-(a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)/f+1/3*(
7*a-8*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]

time = 0.17, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3275, 478, 542, 545, 429, 506, 422}

$$\frac{(3a-4b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(7a-8b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{f}+\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{4b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^2,x]
```

```
[Out] (4*b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((7*a
 - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*
Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) +
 ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f) - ((a
+ b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/f
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^2(a+bx^2)^{3/2}}{(1+x^2)^{3/2}}\,dx, x\right.}{f}$$

$$= -\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{f} + \frac{\left(\sqrt{\cosh^2(e+f}\right.}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{(a+}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{(a+}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{(3a}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{(7a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.26, size = 188, normalized size = 0.72

$$\frac{-8ia(7a-8b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+32ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\left(-24a^2+40ab-13b^2-4(2a-3b)b\cosh(2(e+fx))+b^2\cosh(4(e+fx))\right)\tanh(e+fx)}{24f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^2,x]
```

```
[Out] ((-8*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(
e + f*x), b/a] + (32*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticF[I*(e + f*x), b/a] + Sqrt[2]*(-24*a^2 + 40*a*b - 13*b^2 - 4*(2*a -
3*b)*b*Cosh[2*(e + f*x)] + b^2*Cosh[4*(e + f*x)])*Tanh[e + f*x])/(24*f*Sqrt
[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.57, size = 413, normalized size = 1.59

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\, b^2 (\sinh^5(fx+e)) - 2\sqrt{-\frac{b}{a}}\, ab (\sinh^3(fx+e)) + 4\sqrt{-\frac{b}{a}}\, b^2 (\sinh^3(fx+e)) + 3a^2 \sqrt{\frac{a+b(\sinh^2(fx+e))}{a}}\, \sqrt{\frac{\cosh(2fx+2e)}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5-2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^3+4*
(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-11*a*b*((
a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1
/a*b)^(1/2),(a/b)^(1/2))+8*b^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+7*((a+b*sinh(f*x+
e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(
a/b)^(1/2))*a*b-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-3*sinh(f*x+e)*(-1/a*b)^(1/
2)*a^2+4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)
```

Fricas [F]
time = 0.11, size = 25, normalized size = 0.10

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}\tanh^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(3/2)*tanh(e + f*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.58Error: Bad Argum
ent Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tanh(e+f\,x)^2\,{\left(b\,\sinh(e+f\,x)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.476 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=174

$$\frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}+ia(a$$

```
[Out] 1/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-2/3*I*(2*a-b)*(co
s(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*
(a+b*sinh(f*x+e)^2)^(1/2)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)+1/3*I*a*(a-b)*(cos(
I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))*(1
+b*sinh(f*x+e)^2/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3259, 3251, 3257, 3256, 3262, 3261}

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)
/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b
/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a-b)+2(2a}{\sqrt{a+b\,\mathrm{si}}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{1}{3}(a(a-b))\int \frac{}{\sqrt{a+}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{\Big(2(2a-b)\sqrt{a+b\sin}}{3}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)E\big(ie+ifx}{3f\sqrt{1+}}$$

Mathematica [A]
time = 0.53, size = 169, normalized size = 0.97

$$\frac{-4i\sqrt{2}\,a(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+2i\sqrt{2}\,a(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+b(2a-b+b\cosh(2(e+fx)))\sinh(2(e+fx))}{6f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])
*Sinh[2*(e + f*x)])/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.16, size = 428, normalized size = 2.46

method	result
default	$\frac{\sqrt{-\frac{b}{a}}\,b^2\left(\cosh^4(fx+e)\right)\sinh(fx+e)+\sqrt{-\frac{b}{a}}\,ab\left(\cosh^2(fx+e)\right)\sinh(fx+e)-\sqrt{-\frac{b}{a}}\,b^2\left(\cosh^2(fx+e)\right)\sinh(fx+e)+3a^2\sqrt{\frac{b(\mathrm{co}}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*cosh(f*x+e)^4*sinh(f*x+e)+(-1/a*b)^(1/2)*a*b*cosh(f
*x+e)^2*sinh(f*x+e)-(-1/a*b)^(1/2)*b^2*cosh(f*x+e)^2*sinh(f*x+e)+3*a^2*(b/a
*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))-5*a*b*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*
x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*(b/a*cosh
(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*b^2+4*a*b*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x
+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2*(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b^2)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1
/2)/f
```

Maxima **[F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas **[F]**
time = 0.15, size = 16, normalized size = 0.09

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Sympy **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.477 $\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=256

$$\frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{f}-(7a+b)E(\mathrm{A}$$

```
[Out] -coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f+4/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b
*sinh(f*x+e)^2)^(1/2)/f-1/3*(7*a+b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x
+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*s
ech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a
)^(1/2)+1/3*(3*a+5*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*E
llipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+
b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(7
*a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]

time = 0.19, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3275, 484, 542, 545, 429, 506, 422}

$$\frac{(3a+5b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\operatorname{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(7a+b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(7a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{4b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (Coth
[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/f - ((7*a + b)*EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + 5*b)*EllipticF[Arc
Tan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*
f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a + b)*Sqrt[a +
b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 484

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^p*((c + d*x^n)^q/(e*(m
+ 1))), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx &= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}\,(a+bx^2)}{x^2}\right.}{f}\\ &= -\frac{\coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{f}+\frac{\left(2\sqrt{\cosh^2(e+\right.}{}\\ &= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth}{}\\ &= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth}{}\\ &= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth}{}\\ &= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.90, size = 184, normalized size = 0.72

$$\frac{\sqrt{2}\left(-24a^2+8ab+3b^2-4b(2a+b)\cosh(2(e+fx))+b^2\cosh(4(e+fx))\right)\coth(e+fx)-8ia(7a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+32ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{24f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*(-24*a^2 + 8*a*b + 3*b^2 - 4*b*(2*a + b)*Cosh[2*(e + f*x)] + b^2*C
osh[4*(e + f*x)])*Coth[e + f*x] - (8*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[
2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (32*I)*a*(a - b)*Sqrt[(2*a -
 b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(24*f*Sqrt[2*a -
b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.39, size = 327, normalized size = 1.28

method	result

default	$\sqrt{-\frac{b}{a}}\, b^2 (\cosh^6(fx+e)) + \left(-2\sqrt{-\frac{b}{a}}\, ab - 2\sqrt{-\frac{b}{a}}\, b^2\right)(\cosh^4(fx+e)) + \left(-3\sqrt{-\frac{b}{a}}\, a^2 + 2\sqrt{-\frac{b}{a}}\, ab + \sqrt{-\frac{b}{a}}\, b^2\right)(\cosh^2(f$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*cosh(f*x+e)^6+(-2*(-1/a*b)^(1/2)*a*b-2*(-1/a*b)^(1/
2)*b^2)*cosh(f*x+e)^4+(-3*(-1/a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^
(1/2)*b^2)*cosh(f*x+e)^2+sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)
^2+(a-b)/a)^(1/2)*(3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-
2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-EllipticF(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+7*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(
a/b)^(1/2))*a*b+EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2))/sin
h(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.12, size = 46, normalized size = 0.18

$$\text{integral}\left(\left(b\coth\left(fx+e\right)^{2}\sinh\left(fx+e\right)^{2}+a\coth\left(fx+e\right)^{2}\right)\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*coth(f*x + e)^2*sinh(f*x + e)^2 + a*coth(f*x + e)^2)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{64,[4,8,4]%%%}+%%%{%%%{-128,[1]%%%},[4,8,3]%%%}+%%%{%%%{64,[2
]%%%},
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \coth(e+f\,x)^2\,\left(b\sinh(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.478 $\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=306

$$-\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{(3a+5b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sin}}{3f}$$

```
[Out] -1/3*coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2)/f-(a+b)*cosh(f*x+e)^2*coth(f*x
+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/3*(3*a+5*b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*
sinh(f*x+e)^2)^(1/2)/f-8/3*(a+b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)
^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech
(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(
1/2)+1/3*(3*a+b)*(a+3*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2
)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*
(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+8
/3*(a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A]
time = 0.25, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 484, 594, 542, 545, 429, 506, 422}

$$\frac{(3a+b)(a+3b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\operatorname{ArcTan}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{3af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{8(a+b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{8(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{(3a+5b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a + b)*Cosh[e + f*x]^2*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) +
((3*a + 5*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f)
 - (Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2))/(3*f) - (8*(a + b)*Ellip
ticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]
^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + b)*(
a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b
*Sinh[e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
) + (8*(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 484

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^p*((c + d*x^n)^q/(e*(m
+ 1))), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
```

```
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff, x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\Bigg(\int \frac{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}}{x^4}}{f}$$

$$= -\frac{\coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{\Bigg(2\sqrt{\cosh^2(e+}}{}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Mathematica [C] Result contains complex when optimal does not.

time = 3.69, size = 229, normalized size = 0.75

$$\frac{-\frac{\left(-32a^2-44ab+58b^2+\left(64a^2+32ab-79b^2\right)\cosh(2(e+fx))+2b(6a+11b)\cosh(4(e+fx))-b^2\cosh(6(e+fx))\right)\coth(e+fx)\operatorname{csch}^2(e+fx)}{4\sqrt{2}}-32ia(a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+4i\left(5a^2-2ab-3b^2\right)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{12f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-1/4*((-32*a^2 - 44*a*b + 58*b^2 + (64*a^2 + 32*a*b - 79*b^2)*Cosh[2*(e +
f*x)] + 2*b*(6*a + 11*b)*Cosh[4*(e + f*x)] - b^2*Cosh[6*(e + f*x)])*Coth[e
+ f*x]*Csch[e + f*x]^2)/Sqrt[2] - (32*I)*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2
*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (4*I)*(5*a^2 - 2*a*b - 3*b^2)
*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(12*f
*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.52, size = 540, normalized size = 1.76

method	result
default	$\sqrt{-\frac{b}{a}}\,b^2\left(\sinh^8(fx+e)\right)-3\sqrt{-\frac{b}{a}}\,ab\left(\sinh^6(fx+e)\right)-3\sqrt{-\frac{b}{a}}\,b^2\left(\sinh^6(fx+e)\right)+3a^2\sqrt{\frac{a+b\left(\sinh^2(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2}{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(f*x+e)^8-3*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-3*
(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*sinh(f*x+
e)^3+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b*a*sinh(f*x+e)^3-5*((a+b*sinh(f*x+e)^2)/
a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))*b^2*sinh(f*x+e)^3+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/
2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3+8*((
a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1
/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3-4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)
^4-8*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^4-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4-5*
(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2-5*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2-(-1/a*b
)^(1/2)*a^2)/(-1/a*b)^(1/2)/sinh(f*x+e)^3/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(
1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.15, size = 46, normalized size = 0.15

$$\mathrm{integral}\left(\left(b\coth\left(fx+e\right)^{4}\sinh\left(fx+e\right)^{2}+a\coth\left(fx+e\right)^{4}\right)\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*coth(f*x + e)^4*sinh(f*x + e)^2 + a*coth(f*x + e)^4)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.95Unable to divide
, perhaps due to rounding error%%%{1024,[8,16,8]%%%}+%%%{%%%{-4096,[1]%%%},
[8,16,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{coth}(e+f\,x)^4\,{\left(b\,\mathrm{sinh}(e+f\,x)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.479 $\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=142

$$-\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}f}+\frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f}-\text{se}$$

```
[Out] -1/8*(8*a^2-8*a*b+3*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-
b)^(5/2)/f+1/8*(8*a-5*b)*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/f-
1/4*sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A]
time = 0.13, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 91, 79, 65, 214}

$$-\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{5/2}}-\frac{\text{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4f(a-b)}+\frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/8*((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a -
b]])/((a - b)^(5/2)*f) + ((8*a - 5*b)*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f
*x]^2])/(8*(a - b)^2*f) - (Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2])/(4*
(a - b)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
```

```
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)^3\sqrt{a+bx}}\,dx,x,\sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4(a-b)f} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a+b)+2(a-b)x}{(1+x)^2\sqrt{a+bx}}\,dx,x,\mathrm{si}\right.}{4(a-b)f}$$

$$= \frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} - \frac{\text{sech}^4(e+fx)\sqrt{a+b\,\mathrm{sinh}}}{4(a-b)f}$$

$$= \frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} - \frac{\text{sech}^4(e+fx)\sqrt{a+b\,\mathrm{sinh}}}{4(a-b)f}$$

$$= -\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}f} + \frac{(8a-5b)\text{sech}^2(e}{}$$

Mathematica [A]
time = 0.33, size = 116, normalized size = 0.82

$$\frac{(-8a^2+8ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{a-b}\,\text{sech}^2(e+fx)\,(8a-5b-2(a-b)\text{sech}^2(e+fx))\,\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 + 8*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]
+ Sqrt[a - b]*Sech[e + f*x]^2*(8*a - 5*b - 2*(a - b)*Sech[e + f*x]^2)*Sqrt[
a + b*Sinh[e + f*x]^2])/(8*(a - b)^(5/2)*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.45, size = 43, normalized size = 0.30

method	result	size
default	$\frac{\text{`}int/indef0\text{`}\left(\frac{\sinh^5(fx+e)}{\cosh(fx+e)^6\sqrt{a+b\left(\sinh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(sinh(f*x+e)^5/cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1952 vs. $2(126) = 252$.
time = 0.64, size = 4100, normalized size = 28.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)
*cosh(f*x + e)*sinh(f*x + e)^7 + (8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^8 +
4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*co
sh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(
f*x + e)^5 + 6*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 + 30*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 +
24*a^2 - 24*a*b + 9*b^2)*sinh(f*x + e)^4 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cos
h(f*x + e)^5 + 10*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(
f*x + e)^2 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 + 15*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 + 9*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 8
*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2 + 8*((8*a^2 -
 8*a*b + 3*b^2)*cosh(f*x + e)^7 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5
 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x
 + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 +
 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a - b
```

```
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*
x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) +
 b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 +
2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x
 + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*((8*a^2 - 13*a*b +
 5*b^2)*cosh(f*x + e)^5 + 5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)*sinh(f*x
 + e)^4 + (8*a^2 - 13*a*b + 5*b^2)*sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*b + b^2
)*cosh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^2 + 4*a^2 -
 5*a*b + b^2)*sinh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)
^3 + 3*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 - 13*a
*b + 5*b^2)*cosh(f*x + e) + (5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^4 + 6
*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*sinh(f*x +
 e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3 - 3*a^2*b + 3*
a*b^2 - b^3)*f*cosh(f*x + e)^8 + 8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f
*x + e)*sinh(f*x + e)^7 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*sinh(f*x + e)^8
 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^6 + 4*(7*(a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*s
inh(f*x + e)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^4 + 8*(7
*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^3 + 3*(a^3 - 3*a^2*b + 3*a
*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*f*cosh(f*x + e)^4 + 30*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x
 + e)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*sinh(f*x + e)^4 + 4*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^2 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*f*cosh(f*x + e)^5 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e
)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)^3 +
4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^6 + 15*(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*f*cosh(f*x + e)^4 + 9*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*co
sh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*sinh(f*x + e)^2 + (a^3 -
 3*a^2*b + 3*a*b^2 - b^3)*f + 8*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x
 + e)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^5 + 3*(a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*
f*cosh(f*x + e))*sinh(f*x + e)), -1/8*(((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x +
e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (8*a^2 - 8
*a*b + 3*b^2)*sinh(f*x + e)^8 + 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 +
 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh
(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(8*a^2 - 8*a*b + 3*b^2)*cosh
(f*x + e)^4 + 2*(35*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 30*(8*a^2 - 8
*a*b + 3*b^2)*cosh(f*x + e)^2 + 24*a^2 - 24*a*b + 9*b^2)*sinh(f*x + e)^4 +
8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5 + 10*(8*a^2 - 8*a*b + 3*b^2)*c
osh(f*x + e)^3 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 +
 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*c
osh(f*x + e)^6 + 15*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 9*(8*a^2 - 8*
```

```
a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^2 + 8*a
^2 - 8*a*b + 3*b^2 + 8*((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^7 + 3*(8*a^2
- 8*a*b + 3*b^2)*cosh(f*x + e)^5 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^
3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**5/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2265 vs. $2(126) = 252$.
time = 9.72, size = 2265, normalized size = 15.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(24*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))*e^e/sqrt(-b) - 3*(15*a
^2*e^e - 20*a*b*e^e + 8*b^2*e^e)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqr
t(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt
(b))/sqrt(a - b))/((a^2 - 2*a*b + b^2)*sqrt(a - b)) + 2*(21*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^7*a^2*e^e - 12*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a*b*e^e + 243*(sqrt(b)*
e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f
*x + 2*e) + b))^6*a^2*sqrt(b)*e^e - 276*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a*b^(3/2
)*e^e + 96*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
 + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*b^(5/2)*e^e + 436*(sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
+ b))^5*a^3*e^e - 363*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^2*b*e^e + 180*(sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))^5*a*b^2*e^e - 64*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*b^3*e^e + 1796*(
sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*
```

```
b*e^(2*f*x + 2*e) + b))^4*a^3*sqrt(b)*e^e - 2421*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4
*a^2*b^(3/2)*e^e + 1164*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a*b^(5/2)*e^e - 224*(sqr
t(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e
^(2*f*x + 2*e) + b))^4*b^(7/2)*e^e + 1840*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b
*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^4*e^
e - 1176*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^3*b*e^e - 1497*(sqrt(b)*e^(2*f*x + 2*
e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
))^3*a^2*b^2*e^e + 1532*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*b^3*e^e - 384*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))^3*b^4*e^e + 7056*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^4*sqrt(b)*e^
e - 18136*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
+ 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^3*b^(3/2)*e^e + 18561*(sqrt(b)*e^(2*
f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
2*e) + b))^2*a^2*b^(5/2)*e^e - 8988*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(7/2)*e^
e + 1696*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(9/2)*e^e + 4800*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))*a^5*e^e - 17328*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^4*b*e^e + 27364*(sqrt(b)*e^
(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b))*a^3*b^2*e^e - 22737*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b^3*e^e + 95
64*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
- 2*b*e^(2*f*x + 2*e) + b))*a*b^4*e^e - 1600*(sqrt(b)*e^(2*f*x + 2*e) - sqr
t(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^5*e
^e - 1344*a^5*sqrt(b)*e^e + 6128*a^4*b^(3/2)*e^e - 10284*a^3*b^(5/2)*e^e +
8193*a^2*b^(7/2)*e^e - 3164*a*b^(9/2)*e^e + 480*b^(11/2)*e^e)/(((sqrt(b)*e^
(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3*b)^4*(a^2 -
2*a*b + b^2)))/f^2
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(e+f\,x)^5}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(tanh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.480 $\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=89

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}f}+\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f}$$

```
[Out] -1/2*(2*a-b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+1
/2*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A]

time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3273, 79, 65, 214}

$$\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/2*((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/((a - b)^
(3/2)*f) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(a - b)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x)^2\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{(2a-b)\text{Subst}\Bigg(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx,}{4(a-b)f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{(2a-b)\text{Subst}\Bigg(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a-}}{2(a-b)bf}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}f} + \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(}}{2(a-b)f}$$

Mathematica [A]
time = 0.08, size = 85, normalized size = 0.96

$$-\frac{\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a-b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/2*(((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(a - b)^
(3/2) - (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(a - b))/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.31, size = 43, normalized size = 0.48

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\sinh^{3}(fx+e)}{\cosh(fx+e)^{4}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3/cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 562 vs. 2(77) = 154.
time = 0.53, size = 1320, normalized size = 14.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)^
3 + (2*a - b)*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a - b
)*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e)^3
 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*sqrt(a - b)*log((b*cos
h(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4
*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x +
e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
```

```
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cos
h(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x
+ e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*co
sh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4
*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 4*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f
*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^
2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*
b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((2*a - b)*cosh(f*x + e)^4
+ 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a - b)*sinh(f*x + e)^4 + 2
*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a - b)*cosh(f*x + e)^2 + 2*a - b)*sinh
(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(
f*x + e) + 2*a - b)*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*si
nh(f*x + e))) - 2*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2
 - 2*a*b + b^2)*f*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2
 + 2*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh
(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e
)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 702 vs. 2(77) = 154.
time = 2.62, size = 702, normalized size = 7.89

$$-\frac{\frac{2\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}}{\sqrt{-b}}\right)e^{e}}{\sqrt{-b}}-\frac{(3ae^{e}-2be^{e})\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}+\sqrt{b}}{2\sqrt{a-b}}\right)}{(a-b)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{3}ae^{e}+7\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{2}a\sqrt{b}e^{e}-4\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{2}b^{\frac{3}{2}}e^{e}+12\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)a^{2}e^{e}-17\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)abe^{e}+8\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)b^{2}e^{e}-4a^{2}\sqrt{b}e^{e}+9ab^{\frac{3}{2}}e^{e}-4b^{\frac{5}{2}}e^{e}\right)}{\left(\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)^{2}+2\left(\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}\right)\sqrt{b}+4a-3b\right)^{2}(a-b)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*
x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))*e^e/sqrt(-b) - (3*a*e^e - 2*
b*e^e)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*
e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/(a - b)^
(3/2) + 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
 + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*e^e + 7*(sqrt(b)*e^(2*f*x + 2*e) -
sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*
a*sqrt(b)*e^e - 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(3/2)*e^e + 12*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))*a^2*e^e - 17*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b*e^e + 8*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))*b^2*e^e - 4*a^2*sqrt(b)*e^e + 9*a*b^(3/2)*e^e - 4*b^(5/2)*e^e)/
(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3
*b)^2*(a - b)))/f^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(e+f\,x)^3}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.481 $\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3273, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
```

```
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 1.07

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a-b+b\cosh^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
[Out] -(ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.96, size = 41, normalized size = 1.00

method	result	size
default	$\frac{\text{`}int/indef0\text{`}\left(\frac{\sinh(fx+e)}{\cosh(fx+e)^2\sqrt{a+b\left(\sinh^2(fx+e)\right)}}, \sinh(fx+e)\right)}{f}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 'int/indef0'(sinh(f*x+e)/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e
))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(35) = 70$.
time = 0.52, size = 433, normalized size = 10.56

$$\left[\frac{\log\left(\frac{b\cosh(fx+e)^4+4\,b\cosh(fx+e)\sinh(fx+e)^3+b\sinh(fx+e)^4+2\,(4\,a-3\,b)\cosh(fx+e)^2+2\left(3\,b\cosh(fx+e)^2+4\,a-3\,b\right)\sinh(fx+e)^2-4\,\sqrt{2}\,\sqrt{a-b}\,\sqrt{\frac{b\cosh\left(fx+e\right)^2+b\sinh\left(fx+e\right)^2+2\,a-b}{\cosh\left(fx+e\right)^2-2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^2}}\,(\cosh(fx+e)+\sinh(fx+e))+4\left(b\cosh(fx+e)^3+(4\,a-3\,b)\cosh(fx+e)\right)\sinh(fx+e)+b}{\cosh(fx+e)^4+4\,\cosh(fx+e)\sinh(fx+e)^3+\sinh(fx+e)^4+2\left(3\,\cosh(fx+e)^2+1\right)\sinh(fx+e)^2+2\,\cosh(fx+e)^2+4\left(\cosh(fx+e)^3+\cosh(fx+e)\right)\sinh(fx+e)+1}\right)}{2\,\sqrt{a-b}\,f},\,-\frac{\sqrt{-a+b}\,\arctan\left(-\frac{\sqrt{2}\,\sqrt{-a+b}\,\sqrt{\frac{b\cosh\left(fx+e\right)^2+b\sinh\left(fx+e\right)^2+2\,a-b}{\cosh\left(fx+e\right)^2-2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^2}}}{2\,((a-b)\cosh(fx+e)+(a-b)\sinh(fx+e))}\right)}{(a-b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*
x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3
*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sin
h(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 +
(4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x
 + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*
x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x
 + e) + 1))/(sqrt(a - b)*f), -sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) +
(a - b)*sinh(f*x + e)))/((a - b)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(e+f\,x)}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.482 $\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3273, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
```

`rQ[(m - 1)/2]`

Rubi steps

$$\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.00, size = 35, normalized size = 1.06

method	result	size
default	$\frac{\text{`}int/indef0\text{`}\left(\frac{1}{\sinh(fx+e)\sqrt{a+b\left(\sinh^2(fx+e)\right)}}, \sinh(fx+e)\right)}{f}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(1/sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(27) = 54$.
time = 0.49, size = 410, normalized size = 12.42

$$\left[\frac{\log\left(\frac{b\cosh(fx+e)^4+4\,b\cosh(fx+e)\sinh(fx+e)^3+b\sinh(fx+e)^4+2\,(4\,a-b)\cosh(fx+e)^2+2\left(3\,b\cosh(fx+e)^2+4\,a-b\right)\sinh(fx+e)^2-4\,\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{b\cosh\left(fx+e\right)^2+b\sinh\left(fx+e\right)^2+2\,a-b}{\cosh\left(fx+e\right)^2-2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^2}}\,(\cosh(fx+e)+\sinh(fx+e))+4\left(b\cosh(fx+e)^3+(4\,a-b)\cosh(fx+e)\right)\sinh(fx+e)+b}{\cosh(fx+e)^4+4\,\cosh(fx+e)\sinh(fx+e)^3+\sinh(fx+e)^4+2\left(3\,\cosh(fx+e)^2-1\right)\sinh(fx+e)^2-2\,\cosh(fx+e)^2+4\left(\cosh(fx+e)^3-\cosh(fx+e)\right)\sinh(fx+e)+1}\right)}{2\,\sqrt{a}\,f}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{b\cosh\left(fx+e\right)^2+b\sinh\left(fx+e\right)^2+2\,a-b}{\cosh\left(fx+e\right)^2-2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\sinh\left(fx+e\right)^2}}}{2\,(a\cosh(fx+e)+a\sinh(fx+e))}\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*
x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*
sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b
)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh
(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 -
 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)
)/(sqrt(a)*f), sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e)))/(a*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth(e+f\,x)}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.483 $\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af}$$

```
[Out] -1/2*(2*a-b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/2*csch(
f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]

time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3273, 79, 65, 214}

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/2*((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(a^(3/2)*f) -
 (Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(2*a*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x}{x^2\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af} + \frac{(2a-b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,\right.}{4af}$$

$$= -\frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af} + \frac{(2a-b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \right.}{2abf}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh}}{2af}$$

Mathematica [A]
time = 0.14, size = 72, normalized size = 0.94

$$-\frac{\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/2*(((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/a^(3/2) + (C
sch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/a)/f
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.43, size = 44, normalized size = 0.57

method	result	size
default	$\frac{\text{`}int/indef0\text{`}\left(\frac{\frac{1}{\sinh(fx+e)}+\frac{1}{\sinh(fx+e)^{3}}}{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘((1/sinh(f*x+e)+1/sinh(f*x+e)^3)/(a+b*sinh(f*x+e)^2)^(1/2),sinh
(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(65) = 130.
time = 0.51, size = 1144, normalized size = 14.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)
^3 + (2*a - b)*sinh(f*x + e)^4 - 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a -
b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e)^
3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*sqrt(a)*log((b*cosh(f
*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a
- b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 +
4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
```

```
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*
x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*s
inh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh
(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2
 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*(a*c
osh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)))/(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2
*f*sinh(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x
+ e)^2 - a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x
 + e))*sinh(f*x + e)), 1/2*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f
*x + e)*sinh(f*x + e)^3 + (2*a - b)*sinh(f*x + e)^4 - 2*(2*a - b)*cosh(f*x
+ e)^2 + 2*(3*(2*a - b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 4*((2*
a - b)*cosh(f*x + e)^3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*
sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
 e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) - 2*sqrt(2)*(a*cosh(f*x + e
) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2
*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*f*sinh(f*x
 + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2 - a^
2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh
(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
```

```
ding error%%%{256,[6,8,6]%%%}+%%%{%%%{-768,[1]%%%},[6,8,5]%%%}+%%%{%%%{768,
[2]%%%
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(e+f\,x)^3}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.484 $\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=126

$$-\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}-\frac{(8a-3b)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\text{cs}-$$

```
[Out] -1/8*(8*a^2-8*a*b+3*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)
/f-1/8*(8*a-3*b)*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f-1/4*csch(f*x
+e)^4*(a+b*sinh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 91, 79, 65, 214}

$$-\frac{(8a-3b)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}-\frac{\operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/8*((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])
/(a^(5/2)*f) - ((8*a - 3*b)*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(8
*a^2*f) - (Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2])/(4*a*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4af} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(8a-3b)+2ax}{x^2\sqrt{a+bx}}\,dx, x, \sinh^2\right.}{4af}$$

$$= -\frac{(8a-3b)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - \frac{\operatorname{csch}^4(e+fx)\sqrt{a+b\,\mathrm{si}}}{4af}$$

$$= -\frac{(8a-3b)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - \frac{\operatorname{csch}^4(e+fx)\sqrt{a+b\,\mathrm{si}}}{4af}$$

$$= -\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} - \frac{(8a-3b)\operatorname{csch}^2(e}{}$$

Mathematica [A]
time = 0.26, size = 100, normalized size = 0.79

$$\frac{(-8a^2+8ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\operatorname{csch}^2(e+fx)\left(-8a+3b-2a\operatorname{csch}^2(e+fx)\right)\sqrt{a+b\sinh^2(e+fx)}}{8a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 + 8*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + Sq
rt[a]*Csch[e + f*x]^2*(-8*a + 3*b - 2*a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e
+ f*x]^2])/(8*a^(5/2)*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.61, size = 54, normalized size = 0.43

method	result	size
default	$\frac{\text{`}int/indef0\text{`}\left(\frac{\frac{1}{\sinh(fx+e)}+\frac{2}{\sinh(fx+e)^3}+\frac{1}{\sinh(fx+e)^5}}{\sqrt{a+b\left(\sinh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘((1/sinh(f*x+e)+2/sinh(f*x+e)^3+1/sinh(f*x+e)^5)/(a+b*sinh(f*x+
e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1442 vs. $2(110) = 220$.
time = 0.52, size = 3086, normalized size = 24.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)
*cosh(f*x + e)*sinh(f*x + e)^7 + (8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^8 -
4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*co
sh(f*x + e)^2 - 8*a^2 + 8*a*b - 3*b^2)*sinh(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e)^3 - 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(
f*x + e)^5 + 6*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 - 30*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 +
24*a^2 - 24*a*b + 9*b^2)*sinh(f*x + e)^4 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cos
h(f*x + e)^5 - 10*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(
f*x + e)^2 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 - 15*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 + 9*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 - 8
*a^2 + 8*a*b - 3*b^2)*sinh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2 + 8*((8*a^2 -
 8*a*b + 3*b^2)*cosh(f*x + e)^7 - 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5
 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 - (8*a^2 - 8*a*b + 3*b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e
)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*
b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*co
```

```
sh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*
(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x
+ e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x
+ e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh
(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*((8*a^2 - 3*a*b)*cosh(f*x + e)^5
 + 5*(8*a^2 - 3*a*b)*cosh(f*x + e)*sinh(f*x + e)^4 + (8*a^2 - 3*a*b)*sinh(f
*x + e)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(5*(8*a^2 - 3*a*b)*cosh(f
*x + e)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e)^3 + 2*(5*(8*a^2 - 3*a*b)*cosh(f*x
+ e)^3 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 - 3*a*b)
*cosh(f*x + e) + (5*(8*a^2 - 3*a*b)*cosh(f*x + e)^4 - 6*(4*a^2 - 3*a*b)*cos
h(f*x + e)^2 + 8*a^2 - 3*a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2)))/(a^3*f*cosh(f*x + e)^8 + 8*a^3*f*cosh(f*x + e)*sinh(f*x
 + e)^7 + a^3*f*sinh(f*x + e)^8 - 4*a^3*f*cosh(f*x + e)^6 + 6*a^3*f*cosh(f*
x + e)^4 + 4*(7*a^3*f*cosh(f*x + e)^2 - a^3*f)*sinh(f*x + e)^6 - 4*a^3*f*co
sh(f*x + e)^2 + 8*(7*a^3*f*cosh(f*x + e)^3 - 3*a^3*f*cosh(f*x + e))*sinh(f*
x + e)^5 + 2*(35*a^3*f*cosh(f*x + e)^4 - 30*a^3*f*cosh(f*x + e)^2 + 3*a^3*f
)*sinh(f*x + e)^4 + a^3*f + 8*(7*a^3*f*cosh(f*x + e)^5 - 10*a^3*f*cosh(f*x
+ e)^3 + 3*a^3*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*f*cosh(f*x + e)^
6 - 15*a^3*f*cosh(f*x + e)^4 + 9*a^3*f*cosh(f*x + e)^2 - a^3*f)*sinh(f*x +
e)^2 + 8*(a^3*f*cosh(f*x + e)^7 - 3*a^3*f*cosh(f*x + e)^5 + 3*a^3*f*cosh(f*
x + e)^3 - a^3*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(((8*a^2 - 8*a*b + 3*b^
2)*cosh(f*x + e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^
7 + (8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^8 - 4*(8*a^2 - 8*a*b + 3*b^2)*cos
h(f*x + e)^6 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 8*a*b
 - 3*b^2)*sinh(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 -
3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(8*a^2 - 8*a*b
 + 3*b^2)*cosh(f*x + e)^4 + 2*(35*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 -
 30*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 24*a^2 - 24*a*b + 9*b^2)*sinh
(f*x + e)^4 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5 - 10*(8*a^2 - 8*
a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sin
h(f*x + e)^3 - 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(8*a^2 - 8*
a*b + 3*b^2)*cosh(f*x + e)^6 - 15*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 +
 9*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 8*a*b - 3*b^2)*sinh(f*
x + e)^2 + 8*a^2 - 8*a*b + 3*b^2 + 8*((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)
^7 - 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5 + 3*(8*a^2 - 8*a*b + 3*b^2)*
cosh(f*x + e)^3 - (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqr
t(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) - 2*sqrt(2)*((8*a^2 - 3*a*b)*c
osh(f*x + e)^5 + 5*(8*a^2 - 3*a*b)*cosh(f*x + e)*sinh(f*x + e)^4 + (8*a^2 -
 3*a*b)*sinh(f*x + e)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(5*(8*a^2 -
 3*a*b)*cosh(f*x + e)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e)^3 + 2*(5*(8*a^2 - 3*
a*b)*cosh(f*x + e)^3 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (
```

```
8*a^2 - 3*a*b)*cosh(f*x + e) + (5*(8*a^2 - 3*a*b)*cosh(f*x + e)^4 - 6*(4*a^
2 - 3*a*b)*cosh(f*x + e)^2 + 8*a^2 - 3*a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x
 + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2)))/(a^3*f*cosh(f...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**5/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{4096,[10,12,10]%%%}+%%%{%%%{-20480,[1]%%%},[10,12,9]%%%}+%%%{
%%%{40
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{coth}\left(e+f\,x\right)}^5}{\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.485 $\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=219

$$-\frac{2(2a-b)E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(3a-b)F\big(\text{ArcTan}(\sinh}{3(a-b)^2f}$$

```
[Out] -2/3*(2*a-b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(
sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*
a-b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x
+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^
(1/2)/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*sech(f*x+e)
^2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/f
```

Rubi [A]
time = 0.14, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3275, 481, 539, 429, 422}

$$\frac{(3a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f(a-b)^2\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(2a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3f(a-b)^2\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-2*(2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + ((3*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[
e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*
(a + b*Sinh[e + f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]
*Tanh[e + f*x])/(3*(a - b)*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^4}{(1+x^2)^{5/2}\sqrt{a+bx^2}}\,dx, x, \sinh(e}{f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3(a-b)f} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{se}}{}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3(a-b)f} - \frac{\left(2(2a-b)\sqrt{\cosh^2(e}}{}$$

$$= -\frac{2(2a-b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+f}}{3(a-b)^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.82, size = 206, normalized size = 0.94

$$\frac{-4ia(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,|\,\frac{b}{a}\right)+2ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,|\,\frac{b}{a}\right)-\frac{(2(4a^2-3ab+b^2)\cosh(2(e+fx))+(2a-b)(2a+b+b\cosh(4(e+fx))))\operatorname{sech}^2(e+fx)\tanh(e+fx)}{\sqrt{2}}}{6(a-b)^2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-4*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] + (2*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elli
pticF[I*(e + f*x), b/a] - ((2*(4*a^2 - 3*a*b + b^2)*Cosh[2*(e + f*x)] + (2*
a - b)*(2*a + b + b*Cosh[4*(e + f*x)]))*Sech[e + f*x]^2*Tanh[e + f*x])/Sqrt
[2])/(6*(a - b)^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.74, size = 366, normalized size = 1.67

method	result
default	$\frac{\left(-4\sqrt{-\frac{b}{a}}\,ab+2\sqrt{-\frac{b}{a}}\,b^2\right)\left(\cosh^4(fx+e)\right)\sinh(fx+e)+\left(-4\sqrt{-\frac{b}{a}}\,a^2+7\sqrt{-\frac{b}{a}}\,ab-3\sqrt{-\frac{b}{a}}\,b^2\right)\left(\cosh^2(fx+e)\right)\sinh(f}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-4*(-1/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)
+(-4*(-1/a*b)^(1/2)*a^2+7*(-1/a*b)^(1/2)*a*b-3*(-1/a*b)^(1/2)*b^2)*cosh(f*x
+e)^2*sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(
3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-5*EllipticF(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*b^2+4*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2
*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2+((-1/
a*b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*sinh(f*x+e))/cosh(f
*x+e)^3/(a-b)^2/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2755 vs. 2(231) = 462.
time = 0.16, size = 2755, normalized size = 12.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(((4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^6 + 6*(4*a^2*b - 4*a*b^2 + b^
3)*cosh(f*x + e)*sinh(f*x + e)^5 + (4*a^2*b - 4*a*b^2 + b^3)*sinh(f*x + e)^
6 + 3*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + 3*(4*a^2*b - 4*a*b^2 + b^
3 + 5*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(4*
a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f
*x + e))*sinh(f*x + e)^3 + 4*a^2*b - 4*a*b^2 + b^3 + 3*(4*a^2*b - 4*a*b^2 +
 b^3)*cosh(f*x + e)^2 + 3*(5*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + 4*
a^2*b - 4*a*b^2 + b^3 + 6*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f
*x + e)^2 + 6*((4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^5 + 2*(4*a^2*b - 4*a
*b^2 + b^3)*cosh(f*x + e)^3 + (4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh
(f*x + e) - 2*((2*a*b^2 - b^3)*cosh(f*x + e)^6 + 6*(2*a*b^2 - b^3)*cosh(f*x
 + e)*sinh(f*x + e)^5 + (2*a*b^2 - b^3)*sinh(f*x + e)^6 + 3*(2*a*b^2 - b^3)
*cosh(f*x + e)^4 + 3*(2*a*b^2 - b^3 + 5*(2*a*b^2 - b^3)*cosh(f*x + e)^2)*si
nh(f*x + e)^4 + 4*(5*(2*a*b^2 - b^3)*cosh(f*x + e)^3 + 3*(2*a*b^2 - b^3)*co
sh(f*x + e))*sinh(f*x + e)^3 + 2*a*b^2 - b^3 + 3*(2*a*b^2 - b^3)*cosh(f*x +
 e)^2 + 3*(5*(2*a*b^2 - b^3)*cosh(f*x + e)^4 + 2*a*b^2 - b^3 + 6*(2*a*b^2 -
 b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((2*a*b^2 - b^3)*cosh(f*x + e)^5
```

```
 + 2*(2*a*b^2 - b^3)*cosh(f*x + e)^3 + (2*a*b^2 - b^3)*cosh(f*x + e))*sinh(
f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)
*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*s
qrt((a^2 - a*b)/b^2))/b^2) - ((6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)^6 + 6
*(6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (6*a^3 - 5*a^2*b
 + a*b^2)*sinh(f*x + e)^6 + 3*(6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)^4 + 3
*(6*a^3 - 5*a^2*b + a*b^2 + 5*(6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)^2)*si
nh(f*x + e)^4 + 4*(5*(6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)^3 + 3*(6*a^3 -
 5*a^2*b + a*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 6*a^3 - 5*a^2*b + a*b^2
+ 3*(6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)^2 + 3*(5*(6*a^3 - 5*a^2*b + a*b
^2)*cosh(f*x + e)^4 + 6*a^3 - 5*a^2*b + a*b^2 + 6*(6*a^3 - 5*a^2*b + a*b^2)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x +
 e)^5 + 2*(6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)^3 + (6*a^3 - 5*a^2*b + a*
b^2)*cosh(f*x + e))*sinh(f*x + e) + 2*((3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x
 + e)^6 + 6*(3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*
a^2*b - 5*a*b^2 + 2*b^3)*sinh(f*x + e)^6 + 3*(3*a^2*b - 5*a*b^2 + 2*b^3)*co
sh(f*x + e)^4 + 3*(3*a^2*b - 5*a*b^2 + 2*b^3 + 5*(3*a^2*b - 5*a*b^2 + 2*b^3
)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(
f*x + e)^3 + 3*(3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 +
 3*a^2*b - 5*a*b^2 + 2*b^3 + 3*(3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^2
+ 3*(5*(3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^4 + 3*a^2*b - 5*a*b^2 + 2*
b^3 + 6*(3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((
3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^5 + 2*(3*a^2*b - 5*a*b^2 + 2*b^3)*
cosh(f*x + e)^3 + (3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e))*sinh(f*x + e))
*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/
b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*
x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2
- a*b)/b^2))/b^2) - sqrt(2)*((2*a*b^2 - b^3)*cosh(f*x + e)^5 + 5*(2*a*b^2 -
 b^3)*cosh(f*x + e)*sinh(f*x + e)^4 + (2*a*b^2 - b^3)*sinh(f*x + e)^5 + (3*
a*b^2 - b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3 + 10*(2*a*b^2 - b^3)*cosh(f*x
 + e)^2)*sinh(f*x + e)^3 + (10*(2*a*b^2 - b^3)*cosh(f*x + e)^3 + 3*(3*a*b^2
 - b^3)*cosh(f*x + e))*sinh(f*x + e)^2 + (3*a*b^2 - 2*b^3)*cosh(f*x + e) +
(5*(2*a*b^2 - b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(3*a*b^2 - b^3)*co
sh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2)))/((a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^6 + 6*(a^2*b^2 - 2*a*b^3 +
b^4)*f*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2*b^2 - 2*a*b^3 + b^4)*f*sinh(f*x
 + e)^6 + 3*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^4 + 3*(5*(a^2*b^2 - 2
*a*b^3 + b^4)*f*cosh(f*x + e)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*f)*sinh(f*x + e
)^4 + 3*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^2 + 4*(5*(a^2*b^2 - 2*a*b
^3 + b^4)*f*cosh(f*x + e)^3 + 3*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e))*
sinh(f*x + e)^3 + 3*(5*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^4 + 6*(a^2
*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*f)*sinh
(f*x + e)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*f + 6*((a^2*b^2 - 2*a*b^3 + b^4)*f*
```

```
cosh(f*x + e)^5 + 2*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^3 + (a^2*b^2
- 2*a*b^3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. 2(231) = 462.
time = 4.36, size = 1227, normalized size = 5.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(6*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))*e^e/sqrt(-b) - 3*(3*a*e^e
 - 2*b*e^e)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/(a
- b)^(3/2) + 2*(9*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*e^e - 6*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))^5*b*e^e + 21*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a*sqrt(b)*e^e - 6*(sqrt(b)*e^(
2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
+ 2*e) + b))^4*b^(3/2)*e^e + 64*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*e^e - 38*(sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))^3*a*b*e^e + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^2*e^e + 19
2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
 2*b*e^(2*f*x + 2*e) + b))^2*a^2*sqrt(b)*e^e - 246*(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))
^2*a*b^(3/2)*e^e + 84*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(5/2)*e^e + 240*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))*a^3*e^e - 576*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
```

```
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b*e^e + 477*(sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))*a*b^2*e^e - 126*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^3*e^e - 14
4*a^3*sqrt(b)*e^e + 320*a^2*b^(3/2)*e^e - 223*a*b^(5/2)*e^e + 50*b^(7/2)*e^
e)/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a
- 3*b)^3*(a - b)))/f^2
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(e+f\,x)^4}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.486 $\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=156

$$-\frac{E\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\text{sech}^2(e+fx)\,\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)\text{se}}{(a-b)f\sqrt{\frac{\text{sech}^2(e+fx}{}}}$$

```
[Out] -(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/
(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2
)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(1/(1+sinh(f*x+e)^2))
^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2
),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)
^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3275, 482, 433, 429, 506, 422}

$$\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\,\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\,\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]))
 + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^2}{(1+x^2)^{3/2}\sqrt{a+bx^2}}\,dx, x, \sinh(e+\right.}{f}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{(a-b)f} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)}{(-}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{(a-b)f} - \frac{\left(a\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)}{}$$

$$= \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{\left(\sqrt{\mathrm{co}}\right.}{}$$

$$= -\frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{F\left(\mathrm{ta}\right.}{}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 0.32, size = 109, normalized size = 0.70

$$\frac{-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,(-2a+b-b\cosh(2(e+fx)))\tanh(e+fx)}{2(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] + Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(2*(a - b)*f*S
qrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [**A**]
time = 1.51, size = 239, normalized size = 1.53

method	result

default	$-\sqrt{-\frac{b}{a}}\, b\left(\sinh^{3}(fx+e)\right)+a\sqrt{\frac{a+b\left(\sinh^{2}(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right)-b\sqrt{\frac{a+b}{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-1/a*b)^(1/2)*b*sinh(f*x+e)^3+a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))+b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(-1/a*b)^(1/2)*a*sinh(f*x+e))/
(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(176) = 352.
time = 0.12, size = 703, normalized size = 4.51

$$\frac{\left((2ab-b^2)\cosh(fx+e)^2+2(2ab-b^2)\cosh(fx+e)\sinh(fx+e)+(2ab-b^2)\sinh(fx+e)^2+2ab-b^2-2\left(b^2\cosh(fx+e)^2+2b^2\cosh(fx+e)\sinh(fx+e)+b^2\sinh(fx+e)^2+b^2\right)\sqrt{\frac{a^2-ab}{b^2}}\right)\sqrt{b}\sqrt{\frac{2b\sqrt{\frac{a^2-ab}{b^2}}-2a+b}{b}}\,E\left(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^2-ab}{b^2}}-2a+b}{b}}(\cosh(fx+e)+\sinh(fx+e))\right)\,\middle|\,\frac{8a^2-8ab+b^2+4(2ab-b^2)\sqrt{\frac{a^2-ab}{b^2}}}{b^2}\right)-2\left((2a^2-ab)\cosh(fx+e)^2+2(2a^2-ab)\cosh(fx+e)\sinh(fx+e)+(2a^2-ab)\sinh(fx+e)^2+2a^2-ab+2\left((ab-b^2)\cosh(fx+e)^2+2(ab-b^2)\cosh(fx+e)\sinh(fx+e)+(ab-b^2)\sinh(fx+e)^2+ab-b^2\right)\sqrt{\frac{a^2-ab}{b^2}}\right)\sqrt{b}\sqrt{\frac{2b\sqrt{\frac{a^2-ab}{b^2}}-2a+b}{b}}\,F\left(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^2-ab}{b^2}}-2a+b}{b}}(\cosh(fx+e)+\sinh(fx+e))\right)\,\middle|\,\frac{8a^2-8ab+b^2+4(2ab-b^2)\sqrt{\frac{a^2-ab}{b^2}}}{b^2}\right)-\sqrt{2}\left(b^2\cosh(fx+e)+b^2\sinh(fx+e)\right)\sqrt{\frac{b\cosh(fx+e)^2+b\sinh(fx+e)^2+2a-b}{\cosh(fx+e)^2-2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^2}}}{(ab^2-b^3)f\cosh(fx+e)^2+2(ab^2-b^3)f\cosh(fx+e)\sinh(fx+e)+(ab^2-b^3)f\sinh(fx+e)^2+(ab^2-b^3)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (((2*a*b - b^2)*cosh(f*x + e)^2 + 2*(2*a*b - b^2)*cosh(f*x + e)*sinh(f*x +
e) + (2*a*b - b^2)*sinh(f*x + e)^2 + 2*a*b - b^2 - 2*(b^2*cosh(f*x + e)^2 +
 2*b^2*cosh(f*x + e)*sinh(f*x + e) + b^2*sinh(f*x + e)^2 + b^2)*sqrt((a^2 -
 a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_
e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sin
h(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))
/b^2) - 2*((2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(2*a^2 - a*b)*cosh(f*x + e)*si
nh(f*x + e) + (2*a^2 - a*b)*sinh(f*x + e)^2 + 2*a^2 - a*b + 2*((a*b - b^2)*
cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e) + (a*b - b^2)*s
inh(f*x + e)^2 + a*b - b^2)*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((
```

```
a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/
b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 +
 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*(b^2*cosh(f*x + e) +
 b^2*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a*b
^2 - b^3)*f*cosh(f*x + e)^2 + 2*(a*b^2 - b^3)*f*cosh(f*x + e)*sinh(f*x + e)
 + (a*b^2 - b^3)*f*sinh(f*x + e)^2 + (a*b^2 - b^3)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage3:=type(sage2):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{tanh}\left(e+f\,x\right)}^2}{\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.487 $\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(
1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3262, 3261}

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A]
time = 0.06, size = 68, normalized size = 1.13

$$-\frac{i\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/
(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 0.87, size = 86, normalized size = 1.43

method	result	size
default	$\frac{\sqrt{\frac{a+b\left(\sinh^2(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\,\cosh(fx+e)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\,f}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(70) = 140$.
time = 0.10, size = 147, normalized size = 2.45

$$-\frac{2\left(2\,b\sqrt{\frac{a^2-ab}{b^2}}+2\,a-b\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}\,F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2-ab}{b^2}}-2\,a+b}{b}}\,(\cosh\left(fx+e\right)+\sinh\left(fx+e\right))\right)\mid\frac{8\,a^2-8\,ab+b^2+4\,(2\,ab-b^2)\sqrt{\frac{a^2-ab}{b^2}}}{b^2})}{b^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)
*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*s
qrt((a^2 - a*b)/b^2))/b^2)/(b^(3/2)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(1/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.488 $\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=207

$$-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{E\left(\operatorname{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+f}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] -coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-(1/(1+sinh(f*x+e)^2))^(1/2)*(1+s
inh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(
1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*
x+e)^2)/a)^(1/2)+(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sin
h(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(a+b*sinh
(f*x+e)^2)^(1/2)*tanh(f*x+e)/a/f
```

Rubi [A]

time = 0.13, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3275, 486, 433, 429, 506, 422}

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\operatorname{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f)) - (EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e +
 f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tan
h[e + f*x])/(a*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx\right.}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sec}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a\right.}{}}}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} - \frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sec}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a\right.}{}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.33, size = 105, normalized size = 0.51

$$\frac{\sqrt{2}\,(-2a+b-b\cosh(2(e+fx)))\coth(e+fx)-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{2af\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a])/(2*a*f*Sqrt[2*a
 - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.63, size = 216, normalized size = 1.04

method	result

default	$\frac{-\sqrt{-\frac{b}{a}}\, b(\cosh^4(fx+e))+\left(-\sqrt{-\frac{b}{a}}\, a+\sqrt{-\frac{b}{a}}\, b\right)(\cosh^2(fx+e))+\sinh(fx+e)\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{b(\cosh^2(fx+e))}{a}}}{\sqrt{-\frac{b}{a}}\, a\sinh(fx+e)\cosh(fx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+(-(-1/a*b)^(1/2)*a+(-1/a*b)^(1/2)*b)*cosh(
f*x+e)^2+sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2
)*(a*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x
+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b
)^(1/2))))/(-1/a*b)^(1/2)/a/sinh(f*x+e)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/
2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 674 vs. 2(223) = 446.
time = 0.12, size = 674, normalized size = 3.26

$$\frac{\left((2ab-b^2)\cosh(fx+e)^2+2(2ab-b^2)\cosh(fx+e)\sinh(fx+e)+(2ab-b^2)\sinh(fx+e)^2-2ab+b^2-2\left(b^2\cosh(fx+e)^2+2b^2\cosh(fx+e)\sinh(fx+e)+b^2\sinh(fx+e)^2-b^2\right)\sqrt{\frac{a^2-ab}{b^2}}\right)\sqrt{b}\sqrt{\frac{2b\sqrt{\frac{a^2-ab}{b^2}}-2a+b}{b}}\,E\left(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^2-ab}{b^2}}-2a+b}{b}}(\cosh(fx+e)+\sinh(fx+e))\right)\middle|\,\frac{8a^2-8ab+b^2+4(2ab-b^2)\sqrt{\frac{a^2-ab}{b^2}}}{b^2}\right)-2\left((2a^2-ab)\cosh(fx+e)^2+2(2a^2-ab)\cosh(fx+e)\sinh(fx+e)+(2a^2-ab)\sinh(fx+e)^2-2a^2+ab+2\left((ab-b^2)\cosh(fx+e)^2+2(ab-b^2)\cosh(fx+e)\sinh(fx+e)+(ab-b^2)\sinh(fx+e)^2-ab+b^2\right)\sqrt{\frac{a^2-ab}{b^2}}\right)\sqrt{b}\sqrt{\frac{2b\sqrt{\frac{a^2-ab}{b^2}}-2a+b}{b}}\,F\left(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^2-ab}{b^2}}-2a+b}{b}}(\cosh(fx+e)+\sinh(fx+e))\right)\middle|\,\frac{8a^2-8ab+b^2+4(2ab-b^2)\sqrt{\frac{a^2-ab}{b^2}}}{b^2}\right)-\sqrt{2}\left(b^2\cosh(fx+e)+b^2\sinh(fx+e)\right)\sqrt{\frac{b\cosh(fx+e)^2+b\sinh(fx+e)^2+2a-b}{\cosh(fx+e)^2-2\cosh(fx+e)\sinh(fx+e)+\sinh(fx+e)^2}}}{ab^2f\cosh(fx+e)^2+2ab^2f\cosh(fx+e)\sinh(fx+e)+ab^2f\sinh(fx+e)^2-ab^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (((2*a*b - b^2)*cosh(f*x + e)^2 + 2*(2*a*b - b^2)*cosh(f*x + e)*sinh(f*x +
e) + (2*a*b - b^2)*sinh(f*x + e)^2 - 2*a*b + b^2 - 2*(b^2*cosh(f*x + e)^2 +
 2*b^2*cosh(f*x + e)*sinh(f*x + e) + b^2*sinh(f*x + e)^2 - b^2)*sqrt((a^2 -
 a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_
e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sin
h(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))
/b^2) - 2*((2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(2*a^2 - a*b)*cosh(f*x + e)*si
nh(f*x + e) + (2*a^2 - a*b)*sinh(f*x + e)^2 - 2*a^2 + a*b + 2*((a*b - b^2)*
cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e) + (a*b - b^2)*s
inh(f*x + e)^2 - a*b + b^2)*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((
```

```
a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/
b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 +
 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*(b^2*cosh(f*x + e) +
 b^2*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*b^
2*f*cosh(f*x + e)^2 + 2*a*b^2*f*cosh(f*x + e)*sinh(f*x + e) + a*b^2*f*sinh(
f*x + e)^2 - a*b^2*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{64,[4,6,4]%%%}+%%%{%%%{-128,[1]%%%},[4,6,3]%%%}+%%%{%%%{64,[2
]%%%},
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{coth}\left(e+f\,x\right)}^2}{\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.489 $\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=285

$$-\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}-\frac{\coth(e+fx)\mathrm{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}-\frac{2(2a-$$

```
[Out] -2/3*(2*a-b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f-1/3*coth(f*x+e)*cs
ch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-2/3*(2*a-b)*(1/(1+sinh(f*x+e)^2))
^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2
),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2
*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a-b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+si
nh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1
/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b*sinh(f
*x+e)^2)/a)^(1/2)+2/3*(2*a-b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a^2/f
```

Rubi [A]
time = 0.21, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3275, 485, 597, 545, 429, 506, 422}

$$\frac{(3a-b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\mathrm{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3a^2f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(2a-b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\mathrm{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3a^2f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}-\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}-\frac{\coth(e+fx)\mathrm{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-2*(2*a - b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f) - (Coth[
e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (2*(2*a - b
)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (
(3*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
b*Sinh[e + f*x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + (2*(2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
```

```
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right.}{f}$$

$$= -\frac{\coth(e+fx)\mathrm{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\ \mathrm{sec}\right.}{}$$

$$= -\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\mathrm{csch}^2(e+fx}{3a}$$

$$= -\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\mathrm{csch}^2(e+fx}{3a}$$

$$= -\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\mathrm{csch}^2(e+fx}{3a}$$

$$= -\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\mathrm{csch}^2(e+fx}{3a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.88, size = 208, normalized size = 0.73

$$\frac{-\frac{(2(4a^2-5ab+2b^2)\cosh(2(e+fx))-(2a-b)(2a-3b-b\cosh(4(e+fx))))\coth(e+fx)\mathrm{csch}^2(e+fx)}{\sqrt{2}} - 4ia(2a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) + 2ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{6a^2f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-((((2*(4*a^2 - 5*a*b + 2*b^2)*Cosh[2*(e + f*x)] - (2*a - b)*(2*a - 3*b - b
*Cosh[4*(e + f*x)]))*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2]) - (4*I)*a*(2*a
 - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] +
 (2*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f
*x), b/a])/(6*a^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 1.92, size = 522, normalized size = 1.83

method	result
default	$-4\sqrt{-\frac{b}{a}}\,ab\left(\sinh^{6}(fx+e)\right)+2\sqrt{-\frac{b}{a}}\,b^{2}\left(\sinh^{6}(fx+e)\right)+3a^{2}\sqrt{\frac{a+b\left(\sinh^{2}(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh(\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6+2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6
+3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*sinh(f*x+e)^3-5*((a+b*sinh(f*x+e)^2)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)
)*b*a*sinh(f*x+e)^3+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*E
llipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3+4*((a+b*
sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sin
h(f*x+e)^3-4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4-3*(-1/a*b)^(1/2)*a*b*sinh(f*x
+e)^4+2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4-5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2
+(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2)/(-1/a*b)^(1/2)/a^2/si
nh(f*x+e)^3/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2584 vs. 2(289) = 578.
time = 0.13, size = 2584, normalized size = 9.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(((4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^6 + 6*(4*a^2*b - 4*a*b^2 + b^
3)*cosh(f*x + e)*sinh(f*x + e)^5 + (4*a^2*b - 4*a*b^2 + b^3)*sinh(f*x + e)^
6 - 3*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 - 3*(4*a^2*b - 4*a*b^2 + b^
3 - 5*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(4*
a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 - 3*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f
*x + e))*sinh(f*x + e)^3 - 4*a^2*b + 4*a*b^2 - b^3 + 3*(4*a^2*b - 4*a*b^2 +
 b^3)*cosh(f*x + e)^2 + 3*(5*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + 4*
a^2*b - 4*a*b^2 + b^3 - 6*(4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f
*x + e)^2 + 6*((4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^5 - 2*(4*a^2*b - 4*a
*b^2 + b^3)*cosh(f*x + e)^3 + (4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh
(f*x + e) - 2*((2*a*b^2 - b^3)*cosh(f*x + e)^6 + 6*(2*a*b^2 - b^3)*cosh(f*x
 + e)*sinh(f*x + e)^5 + (2*a*b^2 - b^3)*sinh(f*x + e)^6 - 3*(2*a*b^2 - b^3)
*cosh(f*x + e)^4 - 3*(2*a*b^2 - b^3 - 5*(2*a*b^2 - b^3)*cosh(f*x + e)^2)*si
nh(f*x + e)^4 + 4*(5*(2*a*b^2 - b^3)*cosh(f*x + e)^3 - 3*(2*a*b^2 - b^3)*co
sh(f*x + e))*sinh(f*x + e)^3 - 2*a*b^2 + b^3 + 3*(2*a*b^2 - b^3)*cosh(f*x +
 e)^2 + 3*(5*(2*a*b^2 - b^3)*cosh(f*x + e)^4 + 2*a*b^2 - b^3 - 6*(2*a*b^2 -
 b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((2*a*b^2 - b^3)*cosh(f*x + e)^5
 - 2*(2*a*b^2 - b^3)*cosh(f*x + e)^3 + (2*a*b^2 - b^3)*cosh(f*x + e))*sinh(
f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)
*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*s
qrt((a^2 - a*b)/b^2))/b^2) - ((6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)^6 + 6
*(6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (6*a^3 - 5*a^2*b
 + a*b^2)*sinh(f*x + e)^6 - 3*(6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)^4 - 3
*(6*a^3 - 5*a^2*b + a*b^2 - 5*(6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)^2)*si
nh(f*x + e)^4 + 4*(5*(6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)^3 - 3*(6*a^3 -
 5*a^2*b + a*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 6*a^3 + 5*a^2*b - a*b^2
+ 3*(6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)^2 + 3*(5*(6*a^3 - 5*a^2*b + a*b
^2)*cosh(f*x + e)^4 + 6*a^3 - 5*a^2*b + a*b^2 - 6*(6*a^3 - 5*a^2*b + a*b^2)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x +
 e)^5 - 2*(6*a^3 - 5*a^2*b + a*b^2)*cosh(f*x + e)^3 + (6*a^3 - 5*a^2*b + a*
b^2)*cosh(f*x + e))*sinh(f*x + e) + 2*((3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x
 + e)^6 + 6*(3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*
a^2*b - 5*a*b^2 + 2*b^3)*sinh(f*x + e)^6 - 3*(3*a^2*b - 5*a*b^2 + 2*b^3)*co
sh(f*x + e)^4 - 3*(3*a^2*b - 5*a*b^2 + 2*b^3 - 5*(3*a^2*b - 5*a*b^2 + 2*b^3
)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(
f*x + e)^3 - 3*(3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 -
 3*a^2*b + 5*a*b^2 - 2*b^3 + 3*(3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^2
+ 3*(5*(3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^4 + 3*a^2*b - 5*a*b^2 + 2*
b^3 - 6*(3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((
3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e)^5 - 2*(3*a^2*b - 5*a*b^2 + 2*b^3)*
cosh(f*x + e)^3 + (3*a^2*b - 5*a*b^2 + 2*b^3)*cosh(f*x + e))*sinh(f*x + e))
*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/
b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*
x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2
```

```
- a*b)/b^2))/b^2) - sqrt(2)*((2*a*b^2 - b^3)*cosh(f*x + e)^5 + 5*(2*a*b^2 -
 b^3)*cosh(f*x + e)*sinh(f*x + e)^4 + (2*a*b^2 - b^3)*sinh(f*x + e)^5 - (3*
a*b^2 - 2*b^3)*cosh(f*x + e)^3 - (3*a*b^2 - 2*b^3 - 10*(2*a*b^2 - b^3)*cosh
(f*x + e)^2)*sinh(f*x + e)^3 + (10*(2*a*b^2 - b^3)*cosh(f*x + e)^3 - 3*(3*a
*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^2 + (3*a*b^2 - b^3)*cosh(f*x + e
) + (5*(2*a*b^2 - b^3)*cosh(f*x + e)^4 + 3*a*b^2 - b^3 - 3*(3*a*b^2 - 2*b^3
)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2)))/(a^2*b^2*f*cosh(f*x + e)^6 + 6*a^2*b^2*f*cosh(f*x + e)*sinh(f*x +
e)^5 + a^2*b^2*f*sinh(f*x + e)^6 - 3*a^2*b^2*f*cosh(f*x + e)^4 + 3*a^2*b^2*
f*cosh(f*x + e)^2 - a^2*b^2*f + 3*(5*a^2*b^2*f*cosh(f*x + e)^2 - a^2*b^2*f)
*sinh(f*x + e)^4 + 4*(5*a^2*b^2*f*cosh(f*x + e)^3 - 3*a^2*b^2*f*cosh(f*x +
e))*sinh(f*x + e)^3 + 3*(5*a^2*b^2*f*cosh(f*x + e)^4 - 6*a^2*b^2*f*cosh(f*x
 + e)^2 + a^2*b^2*f)*sinh(f*x + e)^2 + 6*(a^2*b^2*f*cosh(f*x + e)^5 - 2*a^2
*b^2*f*cosh(f*x + e)^3 + a^2*b^2*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1024,[8,10,8]%%%}+%%%{%%%{-4096,[1]%%%},[8,10,7]%%%}+%%%{%%%{
6144,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(e+f\,x)^4}{\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

3.490 $\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$-\frac{(8a^2+8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{7/2}f}+\frac{8a^2+8ab-b^2}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}}+\frac{(8a-3b)\mathrm{se}}{8(a-b)^2f\sqrt{a+}}$$

```
[Out] -1/8*(8*a^2+8*a*b-b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)
^(7/2)/f+1/8*(8*a^2+8*a*b-b^2)/(a-b)^3/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/8*(8*a
-3*b)*sech(f*x+e)^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/4*sech(f*x+e)^4/(
a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 53, 65, 214}

$$\frac{8a^2+8ab-b^2}{8f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}-\frac{(8a^2+8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{7/2}}-\frac{\mathrm{sech}^4(e+fx)}{4f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{(8a-3b)\mathrm{sech}^2(e+fx)}{8f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*((8*a^2 + 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]
])/((a - b)^(7/2)*f) + (8*a^2 + 8*a*b - b^2)/(8*(a - b)^3*f*Sqrt[a + b*Sinh
[e + f*x]^2]) + ((8*a - 3*b)*Sech[e + f*x]^2)/(8*(a - b)^2*f*Sqrt[a + b*Sin
h[e + f*x]^2]) - Sech[e + f*x]^4/(4*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)^3(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{sech}^4(e+fx)}{4(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a-b)+2(a-b)x}{(1+x)^2(a+bx)^{3/2}}\,dx, x, \sinh^2}{4(a-b)f}$$

$$= \frac{(8a-3b)\text{sech}^2(e+fx)}{8(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{sech}^4(e+fx)}{4(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(}{}$$

$$= \frac{8a^2+8ab-b^2}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8a-3b)\text{sech}^2(e+fx)}{8(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= \frac{8a^2+8ab-b^2}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8a-3b)\text{sech}^2(e+fx)}{8(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= -\frac{(8a^2+8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{7/2}f} + \frac{8a^2+8}{8(a-b)^3f\sqrt{a+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.35, size = 113, normalized size = 0.60

$$\frac{(8a^2+8ab-b^2)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sinh^2(e+fx)}{a-b}\right)+\frac{1}{2}(a-b)(4a+b+(8a-3b)\cosh(2(e+fx)))\text{sech}^4(e+fx)}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((8*a^2 + 8*a*b - b^2)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sinh[e + f*x]
^2)/(a - b)] + ((a - b)*(4*a + b + (8*a - 3*b)*Cosh[2*(e + f*x)])*Sech[e +
f*x]^4)/2)/(8*(a - b)^3*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 12.44, size = 103, normalized size = 0.55

method	result	size
default	$\frac{\text{‘}\mathit{int/indef0}\text{‘}\left(-\frac{\left(\sinh^{5}(fx+e)\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,\left(\cosh^{4}(fx+e)\right)}{-b^{2}\left(\cosh^{14}(fx+e)\right)+\left(-2ab+2b^{2}\right)\left(\cosh^{12}(fx+e)\right)+\left(-a^{2}+2ab-b^{2}\right)\left(\cosh^{10}(fx+e)\right)},\sinh(fx+e)\right)}{f}$	103
risch	Expression too large to display	2584739

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(-sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^4/(-b^2*c
osh(f*x+e)^14+(-2*a*b+2*b^2)*cosh(f*x+e)^12+(-a^2+2*a*b-b^2)*cosh(f*x+e)^10
),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4986 vs. $2(167) = 334$.
time = 0.80, size = 10168, normalized size = 54.37

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^12 + 12*(8*a^2*b + 8*a*b^2
- b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (8*a^2*b + 8*a*b^2 - b^3)*sinh(f*x
+ e)^12 + 2*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^10 + 2*(16*a^
3 + 24*a^2*b + 6*a*b^2 - b^3 + 33*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^2
)*sinh(f*x + e)^10 + 20*(11*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^3 + (16
*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e)^9 + (128*a^3
+ 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^8 + (495*(8*a^2*b + 8*a*b^2 - b
^3)*cosh(f*x + e)^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 90*(16*a^3 + 2
4*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(8*a^2*b
+ 8*a*b^2 - b^3)*cosh(f*x + e)^5 + 30*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*c
osh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e))*sinh
```

```
(f*x + e)^7 + 4*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^6 + 4*(2
31*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^6 + 105*(16*a^3 + 24*a^2*b + 6*a
*b^2 - b^3)*cosh(f*x + e)^4 + 48*a^3 + 40*a^2*b - 14*a*b^2 + b^3 + 7*(128*a
^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(
8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^7 + 63*(16*a^3 + 24*a^2*b + 6*a*b^2
 - b^3)*cosh(f*x + e)^5 + 7*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x
 + e)^3 + 3*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e
)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^4 + (495*(8*a^2*
b + 8*a*b^2 - b^3)*cosh(f*x + e)^8 + 420*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3
)*cosh(f*x + e)^6 + 70*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)
^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 60*(48*a^3 + 40*a^2*b - 14*a*b^
2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(8*a^2*b + 8*a*b^2 - b^3)
*cosh(f*x + e)^9 + 60*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^7 +
 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^5 + 20*(48*a^3 + 4
0*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2
 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*a^2*b + 8*a*b^2 - b^3 + 2*(16*a^
3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(33*(8*a^2*b + 8*a*b^2 -
b^3)*cosh(f*x + e)^10 + 45*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e
)^8 + 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^6 + 30*(48*a^
3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^4 + 16*a^3 + 24*a^2*b + 6*a*b^
2 - b^3 + 3*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*
x + e)^2 + 4*(3*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^11 + 5*(16*a^3 + 24
*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^9 + 2*(128*a^3 + 120*a^2*b - 24*a*b^2
 + b^3)*cosh(f*x + e)^7 + 6*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x +
 e)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^3 + (16*a^3 +
24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*
cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2
*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x
 + e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*
cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f
*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2
*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1))
+ 4*sqrt(2)*((8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^9 + 9*(8*a^3 - 9*a*b^2 +
 b^3)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^3 - 9*a*b^2 + b^3)*sinh(f*x + e)
^9 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^7 + 4*(16*a^3 -
19*a^2*b + 5*a*b^2 - 2*b^3 + 9*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^7 + 28*(3*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^3 + (16*a^3 - 19
*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(40*a^3 - 28*a
^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*b^2 + b^3)*co
sh(f*x + e)^4 + 40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3 + 42*(16*a^3 - 19*a^2*
b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*
b^2 + b^3)*cosh(f*x + e)^5 + 70*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(
f*x + e)^3 + 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e))*sinh(f
```

```
*x + e)^4 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + 4*(21
*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^6 + 35*(16*a^3 - 19*a^2*b + 5*a*b^2
- 2*b^3)*cosh(f*x + e)^4 + 16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3 + 5*(40*a^3
- 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^3 + 4*(9*(8*a
^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^7 + 21*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b
^3)*cosh(f*x + e)^5 + 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e
)^3 + 3*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^
2 + (8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e) + (9*(8*a^3 - 9*a*b^2 + b^3)*cosh
(f*x + e)^8 + 28*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 10
*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e)^4 + 8*a^3 - 9*a*b^2 +
 b^3 + 12*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**5/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2452 vs. $2(167) = 334$.
time = 15.80, size = 2452, normalized size = 13.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(a^9*e^(5*e) - 5*a^8*b*e^(5*e) + 10*a^7*b^2*e^(5*e) - 10*a^6*b^3*e^(5*e)
+ 5*a^5*b^4*e^(5*e) - a^4*b^5*e^(5*e))*e^(f*x)/((a^10*e^(4*e) - 8*a^9*b*e^(
4*e) + 28*a^8*b^2*e^(4*e) - 56*a^7*b^3*e^(4*e) + 70*a^6*b^4*e^(4*e) - 56*a^
5*b^5*e^(4*e) + 28*a^4*b^6*e^(4*e) - 8*a^3*b^7*e^(4*e) + a^2*b^8*e^(4*e))*s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)*f) +
 1/12*(45*a^2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))*e
^e/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a - b)) - 24*a^2*arctan(-(sqrt(b)*
e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f
*x + 2*e) + b))/sqrt(-b))*e^e/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-b)) -
2*(21*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*
e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^2*e^e + 243*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a
```

```
^2*sqrt(b)*e^e - 144*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a*b^(3/2)*e^e + 48*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))^6*b^(5/2)*e^e + 436*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4
*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^3*e^e - 1
23*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
 - 2*b*e^(2*f*x + 2*e) + b))^5*a^2*b*e^e + 288*(sqrt(b)*e^(2*f*x + 2*e) - sq
rt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*
b^2*e^e - 160*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*b^3*e^e + 1796*(sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))^4*a^3*sqrt(b)*e^e - 1029*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2)*e^e -
 240*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^4*a*b^(5/2)*e^e + 208*(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)^4*b^(7/2)*e^e + 1840*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^4*e^e + 168*(sqrt(b)*e^
(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b))^3*a^3*b*e^e - 2553*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*b^2*e^e + 1
472*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
 - 2*b*e^(2*f*x + 2*e) + b))^3*a*b^3*e^e - 192*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b
^4*e^e + 7056*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^4*sqrt(b)*e^e - 14872*(sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))^2*a^3*b^(3/2)*e^e + 11745*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^2*b^(
5/2)*e^e - 3696*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(7/2)*e^e + 208*(sqrt(b)*e^(
2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
+ 2*e) + b))^2*b^(9/2)*e^e + 4800*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^5*e^e - 15024*
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b))*a^4*b*e^e + 19876*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(
b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^3*b^2
*e^e - 12705*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f
*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b^3*e^e + 3360*(sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))*a*b^4*e^e - 160*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^5*e^e - 1344*a^5*sqrt(b)*
e^e + 5360*a^4*b^(3/2)*e^e - 7404*a^3*b^(5/2)*e^e + 4401*a^2*b^(7/2)*e^e -
1040*a*b^(9/2)*e^e + 48*b^(11/2)*e^e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*((sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
```

```
e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*
e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3*b)^4
))/f^2
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] \text{Hanged}
```

3.491 $\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}f}+\frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\operatorname{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(}}$$

```
[Out] -1/2*(2*a+b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+1
/2*(2*a+b)/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/2*sech(f*x+e)^2/(a-b)/f/(a
+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 53, 65, 214}

$$\frac{2a+b}{2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{5/2}}+\frac{\operatorname{sech}^2(e+fx)}{2f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/((a - b)^
(5/2)*f) + (2*a + b)/(2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) + Sech[e +
 f*x]^2/(2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x)^2(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\operatorname{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a+b)\text{Subst}\Bigl(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \text{si}}{4(a-b)f}$$

$$= \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} +$$

$$= \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} +$$

$$= -\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}f} + \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 79, normalized size = 0.65

$$\frac{(2a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sinh^2(e+fx)}{a-b}\right)+(a-b)\operatorname{sech}^2(e+fx)}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sinh[e + f*x]^2)/(a - b)]
 + (a - b)*Sech[e + f*x]^2)/(2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.07, size = 103, normalized size = 0.84

method	result	size

default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(-\frac{\left(\sinh^3(fx+e)\right)\sqrt{a+b\left(\sinh^2(fx+e)\right)}\left(\cosh^2(fx+e)\right)}{-b^2\left(\cosh^{10}(fx+e)\right)+\left(-2ab+2b^2\right)\left(\cosh^8(fx+e)\right)+\left(-a^2+2ab-b^2\right)\left(\cosh^6(fx+e)\right)},\sinh(fx+e)\right)}{f}$	103
risch	Expression too large to display	289421

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(-sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2/(-b^2*c
osh(f*x+e)^10+(-2*a*b+2*b^2)*cosh(f*x+e)^8+(-a^2+2*a*b-b^2)*cosh(f*x+e)^6),
sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. 2(106) = 212.
time = 0.56, size = 4050, normalized size = 33.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a*b + b^2)*cosh(f*x + e)^8 + 8*(2*a*b + b^2)*cosh(f*x + e)*sinh(f
*x + e)^7 + (2*a*b + b^2)*sinh(f*x + e)^8 + 4*(2*a^2 + a*b)*cosh(f*x + e)^6
 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b)*sinh(f*x + e)^6 + 8*(7
*(2*a*b + b^2)*cosh(f*x + e)^3 + 3*(2*a^2 + a*b)*cosh(f*x + e))*sinh(f*x +
e)^5 + 2*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^4 + 2*(35*(2*a*b + b^2)*cosh(f
*x + e)^4 + 30*(2*a^2 + a*b)*cosh(f*x + e)^2 + 8*a^2 + 2*a*b - b^2)*sinh(f*
x + e)^4 + 8*(7*(2*a*b + b^2)*cosh(f*x + e)^5 + 10*(2*a^2 + a*b)*cosh(f*x +
 e)^3 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2 + a
*b)*cosh(f*x + e)^2 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^6 + 15*(2*a^2 + a*b)
*cosh(f*x + e)^4 + 3*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b)*s
inh(f*x + e)^2 + 2*a*b + b^2 + 8*((2*a*b + b^2)*cosh(f*x + e)^7 + 3*(2*a^2
+ a*b)*cosh(f*x + e)^5 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 + a
```

```
*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*
cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x
+ e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sq
rt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) +
 sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*
x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x +
 e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(
cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*((2*a^2 -
a*b - b^2)*cosh(f*x + e)^5 + 5*(2*a^2 - a*b - b^2)*cosh(f*x + e)*sinh(f*x +
 e)^4 + (2*a^2 - a*b - b^2)*sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*b + b^2)*cosh(
f*x + e)^3 + 2*(5*(2*a^2 - a*b - b^2)*cosh(f*x + e)^2 + 4*a^2 - 5*a*b + b^2
)*sinh(f*x + e)^3 + 2*(5*(2*a^2 - a*b - b^2)*cosh(f*x + e)^3 + 3*(4*a^2 - 5
*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a^2 - a*b - b^2)*cosh(f*x +
 e) + (5*(2*a^2 - a*b - b^2)*cosh(f*x + e)^4 + 6*(4*a^2 - 5*a*b + b^2)*cosh
(f*x + e)^2 + 2*a^2 - a*b - b^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)))/((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)
^8 + 8*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7
+ (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*sinh(f*x + e)^8 + 4*(a^4 - 3*a^3*b
+ 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^6 + 4*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3
- b^4)*f*cosh(f*x + e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*sinh(f*x
+ e)^6 + 2*(4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*x + e)^
4 + 8*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^3 + 3*(a^4 - 3
*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^3*b
 - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^4 + 30*(a^4 - 3*a^3*b + 3*a^2
*b^2 - a*b^3)*f*cosh(f*x + e)^2 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3
+ b^4)*f)*sinh(f*x + e)^4 + 4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*
x + e)^2 + 8*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^5 + 10*
(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^3 + (4*a^4 - 13*a^3*b +
 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^3*b
 - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^6 + 15*(a^4 - 3*a^3*b + 3*a^2
*b^2 - a*b^3)*f*cosh(f*x + e)^4 + 3*(4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^
3 + b^4)*f*cosh(f*x + e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*sinh(f*
x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f + 8*((a^3*b - 3*a^2*b^2 +
3*a*b^3 - b^4)*f*cosh(f*x + e)^7 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*
cosh(f*x + e)^5 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*
x + e)^3 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x +
e)), -1/2*(((2*a*b + b^2)*cosh(f*x + e)^8 + 8*(2*a*b + b^2)*cosh(f*x + e)*s
inh(f*x + e)^7 + (2*a*b + b^2)*sinh(f*x + e)^8 + 4*(2*a^2 + a*b)*cosh(f*x +
 e)^6 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b)*sinh(f*x + e)^6 +
 8*(7*(2*a*b + b^2)*cosh(f*x + e)^3 + 3*(2*a^2 + a*b)*cosh(f*x + e))*sinh(f
*x + e)^5 + 2*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^4 + 2*(35*(2*a*b + b^2)*c
osh(f*x + e)^4 + 30*(2*a^2 + a*b)*cosh(f*x + e)^2 + 8*a^2 + 2*a*b - b^2)*si
nh(f*x + e)^4 + 8*(7*(2*a*b + b^2)*cosh(f*x + e)^5 + 10*(2*a^2 + a*b)*cosh(
```

```
f*x + e)^3 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^
2 + a*b)*cosh(f*x + e)^2 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^6 + 15*(2*a^2 +
 a*b)*cosh(f*x + e)^4 + 3*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 + a
*b)*sinh(f*x + e)^2 + 2*a*b + b^2 + 8*((2*a*b + b^2)*cosh(f*x + e)^7 + 3*(2
*a^2 + a*b)*cosh(f*x + e)^5 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^3 + (2*a^
2 + a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqr
t(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*
x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*((2*a^2 - a*b - b^2)*cosh(f*x
+ e)^5 + 5*(2*a^2 - a*b - b^2)*cosh(f*x + e)*si...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.91Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(e+f\,x)^3}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.492 $\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f}+\frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+1/(a-b)/f/(a+
b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3273, 53, 65, 214}

$$\frac{1}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f)) + 1/(
(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\Bigg(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \sinh^2(e+}{2(a-b)f}$$

$$= \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\Bigg(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+}}{(a-b)bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 58, normalized size = 0.84

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{b\cosh^2(e+fx)}{a-b}\right)}{(-a+b)f\sqrt{a-b+b\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Cosh[e + f*x]^2)/(a - b)]/((-a + b
)*f*Sqrt[a - b + b*Cosh[e + f*x]^2]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.02, size = 93, normalized size = 1.35

method	result	size
default	$\frac{\text{‘}int/indef0\text{‘}\left(-\frac{\sinh(fx+e)\sqrt{a+b\left(\sinh^2(fx+e)\right)}}{-b^2\left(\sinh^6(fx+e)\right)+\left(-2ab-b^2\right)\left(\sinh^4(fx+e)\right)+\left(-a^2-2ab\right)\left(\sinh^2(fx+e)\right)-a^2},\sinh(fx+e)\right)}{f}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(-sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(-b^2*sinh(f*x+e)^6+(-2
*a*b-b^2)*sinh(f*x+e)^4+(-a^2-2*a*b)*sinh(f*x+e)^2-a^2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 587 vs. 2(61) = 122.
time = 0.52, size = 1370, normalized size = 19.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x
+ e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*si
nh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x +
e) + b)*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e
)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x +
 e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x
 + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
```

```
inh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cos
h(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)
^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)
^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x
 + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2*b - 2*a
*b^2 + b^3)*f*cosh(f*x + e)^4 + 4*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)*s
inh(f*x + e)^3 + (a^2*b - 2*a*b^2 + b^3)*f*sinh(f*x + e)^4 + 2*(2*a^3 - 5*a
^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e)^2 + 2*(3*(a^2*b - 2*a*b^2 + b^3)*f*co
sh(f*x + e)^2 + (2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f)*sinh(f*x + e)^2 + (a^2
*b - 2*a*b^2 + b^3)*f + 4*((a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^3 + (2*a
^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)), -((b*cosh(f*
x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a -
 b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4
*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-a +
 b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
 e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*((a
 - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2)))/((a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^4 + 4*(a^2*b -
 2*a*b^2 + b^3)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2*b - 2*a*b^2 + b^3)*f
*sinh(f*x + e)^4 + 2*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e)^2 +
2*(3*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^2 + (2*a^3 - 5*a^2*b + 4*a*b^2
 - b^3)*f)*sinh(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f + 4*((a^2*b - 2*a*b^
2 + b^3)*f*cosh(f*x + e)^3 + (2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x +
 e))*sinh(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh{\left(e + f x \right)}}{\left(a + b \sinh^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(e+f\,x)}{{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.493 $\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{1}{af\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a+b*sinh(f*x+e
)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3273, 53, 65, 214}

$$\frac{1}{af\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
 + b*Sinh[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= \frac{1}{af\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2af} \\
&= \frac{1}{af\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\Bigg(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}}{abf} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 46, normalized size = 0.81

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{b\sinh^2(e+fx)}{a}\right)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sinh[e + f*x]^2)/a]/(a*f*Sqrt[a + b*
Sinh[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.08, size = 35, normalized size = 0.61

method	result	size
default	$\frac{`int/indef0`\left(\frac{1}{\sinh(fx+e)\left(a+b\left(\sinh^{2}(fx+e)\right)\right)^{\frac{3}{2}}},\sinh(fx+e)\right)}{f}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(1/sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(49) = 98$.
time = 0.50, size = 1137, normalized size = 19.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
 e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sin
h(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e
) + b)*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 +
 b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3
+ (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x
```

```
 + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*
x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x
 + e) + 1)) + 4*sqrt(2)*(a*cosh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*b*f*cosh(f*x + e)^4 + 4*a^2*b*f*cos
h(f*x + e)*sinh(f*x + e)^3 + a^2*b*f*sinh(f*x + e)^4 + a^2*b*f + 2*(2*a^3 -
 a^2*b)*f*cosh(f*x + e)^2 + 2*(3*a^2*b*f*cosh(f*x + e)^2 + (2*a^3 - a^2*b)*
f)*sinh(f*x + e)^2 + 4*(a^2*b*f*cosh(f*x + e)^3 + (2*a^3 - a^2*b)*f*cosh(f*
x + e))*sinh(f*x + e)), ((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x +
e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x
 + e))*sinh(f*x + e) + b)*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh
(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) +
 2*sqrt(2)*(a*cosh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2)))/(a^2*b*f*cosh(f*x + e)^4 + 4*a^2*b*f*cosh(f*x + e)*si
nh(f*x + e)^3 + a^2*b*f*sinh(f*x + e)^4 + a^2*b*f + 2*(2*a^3 - a^2*b)*f*cos
h(f*x + e)^2 + 2*(3*a^2*b*f*cosh(f*x + e)^2 + (2*a^3 - a^2*b)*f)*sinh(f*x +
 e)^2 + 4*(a^2*b*f*cosh(f*x + e)^3 + (2*a^3 - a^2*b)*f*cosh(f*x + e))*sinh(
f*x + e))]
```

Sympy **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth (e+fx)}{\left(a+b\sinh ^{2}(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad **[F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth(e+f\,x)}{{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.494 $\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}+\frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -1/2*(2*a-3*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/2*(2*
a-3*b)/a^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/2*csch(f*x+e)^2/a/f/(a+b*sinh(f*x+
e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 53, 65, 214}

$$-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}+\frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(a^(5/2)*f)
 + (2*a - 3*b)/(2*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - Csch[e + f*x]^2/(2*a
*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=\frac{\text{Subst}\left(\int \frac{1+x}{x^2(a+bx)^{3/2}}\,dx,x,\sinh^2(e+fx)\right)}{2f}$$

$$=-\frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}}+\frac{(2a-3b)\text{Subst}\Big(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,\sinh^2(e}{4af}$$

$$=\frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}}+\frac{(2a-3b)\text{Sub}}{}$$

$$=\frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}}+\frac{(2a-3b)\text{Sub}}{}$$

$$=-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}+\frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 69, normalized size = 0.63

$$\frac{-a\operatorname{csch}^2(e+fx)+(2a-3b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{b\sinh^2(e+fx)}{a}\right)}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(a*Csch[e + f*x]^2) + (2*a - 3*b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*
Sinh[e + f*x]^2)/a])/(2*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.23, size = 43, normalized size = 0.39

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\cosh^2(fx+e)}{\sinh(fx+e)^3\left(a+b\left(\sinh^2(fx+e)\right)\right)^{\frac{3}{2}}},\sinh(fx+e)\right)}{f}$	43

risch	Expression too large to display	289430

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(cosh(f*x+e)^2/sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x
+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1513 vs. $2(94) = 188$.
time = 0.58, size = 3228, normalized size = 29.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a*b - 3*b^2)*cosh(f*x + e)^8 + 8*(2*a*b - 3*b^2)*cosh(f*x + e)*s
inh(f*x + e)^7 + (2*a*b - 3*b^2)*sinh(f*x + e)^8 + 4*(2*a^2 - 5*a*b + 3*b^2
)*cosh(f*x + e)^6 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^2 + 2*a^2 - 5*a*b +
3*b^2)*sinh(f*x + e)^6 + 8*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^3 + 3*(2*a^2 -
5*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(8*a^2 - 18*a*b + 9*b^2)*
cosh(f*x + e)^4 + 2*(35*(2*a*b - 3*b^2)*cosh(f*x + e)^4 + 30*(2*a^2 - 5*a*b
 + 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 18*a*b - 9*b^2)*sinh(f*x + e)^4 + 8*(7*
(2*a*b - 3*b^2)*cosh(f*x + e)^5 + 10*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^
3 - (8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2 - 5*
a*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^6 + 15*(2
*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^4 - 3*(8*a^2 - 18*a*b + 9*b^2)*cosh(f*x
 + e)^2 + 2*a^2 - 5*a*b + 3*b^2)*sinh(f*x + e)^2 + 2*a*b - 3*b^2 + 8*((2*a*
b - 3*b^2)*cosh(f*x + e)^7 + 3*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^5 - (8
*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e)^3 + (2*a^2 - 5*a*b + 3*b^2)*cosh(f*x +
 e))*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh
(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh
(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x
 + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
```

```
inh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cos
h(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4
 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2
 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x +
 e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*((2*a^2 - 3*a*b)*cosh(f*x + e)^5 + 5*(
2*a^2 - 3*a*b)*cosh(f*x + e)*sinh(f*x + e)^4 + (2*a^2 - 3*a*b)*sinh(f*x + e
)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e
)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e)^3
 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a^2 - 3*a*b)*cosh(
f*x + e) + (5*(2*a^2 - 3*a*b)*cosh(f*x + e)^4 - 6*(4*a^2 - 3*a*b)*cosh(f*x
+ e)^2 + 2*a^2 - 3*a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
 + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)))/(a^3*b*f*cosh(f*x + e)^8 + 8*a^3*b*f*cosh(f*x + e)*sinh(f*x +
 e)^7 + a^3*b*f*sinh(f*x + e)^8 + 4*(a^4 - a^3*b)*f*cosh(f*x + e)^6 + 4*(7*
a^3*b*f*cosh(f*x + e)^2 + (a^4 - a^3*b)*f)*sinh(f*x + e)^6 - 2*(4*a^4 - 3*a
^3*b)*f*cosh(f*x + e)^4 + 8*(7*a^3*b*f*cosh(f*x + e)^3 + 3*(a^4 - a^3*b)*f*
cosh(f*x + e))*sinh(f*x + e)^5 + a^3*b*f + 2*(35*a^3*b*f*cosh(f*x + e)^4 +
30*(a^4 - a^3*b)*f*cosh(f*x + e)^2 - (4*a^4 - 3*a^3*b)*f)*sinh(f*x + e)^4 +
 4*(a^4 - a^3*b)*f*cosh(f*x + e)^2 + 8*(7*a^3*b*f*cosh(f*x + e)^5 + 10*(a^4
 - a^3*b)*f*cosh(f*x + e)^3 - (4*a^4 - 3*a^3*b)*f*cosh(f*x + e))*sinh(f*x +
 e)^3 + 4*(7*a^3*b*f*cosh(f*x + e)^6 + 15*(a^4 - a^3*b)*f*cosh(f*x + e)^4 -
 3*(4*a^4 - 3*a^3*b)*f*cosh(f*x + e)^2 + (a^4 - a^3*b)*f)*sinh(f*x + e)^2 +
 8*(a^3*b*f*cosh(f*x + e)^7 + 3*(a^4 - a^3*b)*f*cosh(f*x + e)^5 - (4*a^4 -
3*a^3*b)*f*cosh(f*x + e)^3 + (a^4 - a^3*b)*f*cosh(f*x + e))*sinh(f*x + e)),
 1/2*(((2*a*b - 3*b^2)*cosh(f*x + e)^8 + 8*(2*a*b - 3*b^2)*cosh(f*x + e)*si
nh(f*x + e)^7 + (2*a*b - 3*b^2)*sinh(f*x + e)^8 + 4*(2*a^2 - 5*a*b + 3*b^2)
*cosh(f*x + e)^6 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^2 + 2*a^2 - 5*a*b + 3
*b^2)*sinh(f*x + e)^6 + 8*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^3 + 3*(2*a^2 - 5
*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(8*a^2 - 18*a*b + 9*b^2)*c
osh(f*x + e)^4 + 2*(35*(2*a*b - 3*b^2)*cosh(f*x + e)^4 + 30*(2*a^2 - 5*a*b
+ 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 18*a*b - 9*b^2)*sinh(f*x + e)^4 + 8*(7*(
2*a*b - 3*b^2)*cosh(f*x + e)^5 + 10*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^3
 - (8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2 - 5*a
*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^6 + 15*(2*
a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^4 - 3*(8*a^2 - 18*a*b + 9*b^2)*cosh(f*x
+ e)^2 + 2*a^2 - 5*a*b + 3*b^2)*sinh(f*x + e)^2 + 2*a*b - 3*b^2 + 8*((2*a*b
 - 3*b^2)*cosh(f*x + e)^7 + 3*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^5 - (8*
a^2 - 18*a*b + 9*b^2)*cosh(f*x + e)^3 + (2*a^2 - 5*a*b + 3*b^2)*cosh(f*x +
e))*sinh(f*x + e))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*sqrt
(2)*((2*a^2 - 3*a*b)*cosh(f*x + e)^5 + 5*(2*a^2 - 3*a*b)*cosh(f*x + e)*sinh
(f*x + e)^4 + (2*a^2 - 3*a*b)*sinh(f*x + e)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x
+ e)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e
)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e)^3 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e
```

```
))*sinh(f*x + e)^2 + (2*a^2 - 3*a*b)*cosh(f*x + e) + (5*(2*a^2 - 3*a*b)*cos
h(f*x + e)^4 - 6*(4*a^2 - 3*a*b)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b)*sinh(f*x
+ e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.51Error: Bad Argum
ent Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{coth}(e+f\,x)^3}{\left(b\,\mathrm{sinh}\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.495 $\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{(8a^2-24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(8a-5b)\text{csch}^2(}{8a^2f\sqrt{a+b\sinh^2}}$$

```
[Out] -1/8*(8*a^2-24*a*b+15*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/
2)/f+1/8*(8*a^2-24*a*b+15*b^2)/a^3/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/8*(8*a-5*b
)*csch(f*x+e)^2/a^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/4*csch(f*x+e)^4/a/f/(a+b*
sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 53, 65, 214}

$$-\frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(8a^2-24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*((8*a^2 - 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]
])/(a^(7/2)*f) + (8*a^2 - 24*a*b + 15*b^2)/(8*a^3*f*Sqrt[a + b*Sinh[e + f*x
]^2]) - ((8*a - 5*b)*Csch[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
 - Csch[e + f*x]^4/(4*a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(8a-5b)+2ax}{x^2(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{4af}$$

$$= -\frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8a^2-24a}{}$$

$$= \frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^4}{4af\sqrt{a+b}}$$

$$= \frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^4}{4af\sqrt{a+b}}$$

$$= -\frac{(8a^2-24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f} + \frac{8a^2-24}{8a^3f\sqrt{a+b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.23, size = 94, normalized size = 0.56

$$\frac{a\text{csch}^2(e+fx)\left(-8a+5b-2a\text{csch}^2(e+fx)\right)+\left(8a^2-24ab+15b^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{b\sinh^2(e+fx)}{a}\right)}{8a^3f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Csch[e + f*x]^2*(-8*a + 5*b - 2*a*Csch[e + f*x]^2) + (8*a^2 - 24*a*b + 1
5*b^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sinh[e + f*x]^2)/a])/(8*a^3*f
*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 8.83, size = 43, normalized size = 0.26

method	result	size
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(\frac{\cosh^4(fx+e)}{\sinh(fx+e)^5\left(a+b\left(\sinh^2(fx+e)\right)\right)^{\frac{3}{2}}},\sinh(fx+e)\right)}{f}$	43
risch	Expression too large to display	2583800

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(cosh(f*x+e)^4/sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x
+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3680 vs. 2(147) = 294.
time = 0.72, size = 7562, normalized size = 45.28

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^12 + 12*(8*a^2*b - 24*a
*b^2 + 15*b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (8*a^2*b - 24*a*b^2 + 15*b^
3)*sinh(f*x + e)^12 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x +
 e)^10 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3 + 33*(8*a^2*b - 24*a*b^2
 + 15*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(8*a^2*b - 24*a*b^2 +
 15*b^3)*cosh(f*x + e)^3 + (16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*
x + e))*sinh(f*x + e)^9 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(
f*x + e)^8 + (495*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^4 - 128*a^3 +
 504*a^2*b - 600*a*b^2 + 225*b^3 + 90*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(8*a^2*b - 24*a*b^2 + 15*b^3)*
cosh(f*x + e)^5 + 30*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)
^3 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e))*sinh(f*x +
e)^7 + 4*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^6 + 4*(231
```

```
*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^6 + 105*(16*a^3 - 72*a^2*b + 1
02*a*b^2 - 45*b^3)*cosh(f*x + e)^4 + 48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^
3 - 7*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^2)*sinh(f*x
 + e)^6 + 8*(99*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^7 + 63*(16*a^3
- 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^5 - 7*(128*a^3 - 504*a^2*b +
 600*a*b^2 - 225*b^3)*cosh(f*x + e)^3 + 3*(48*a^3 - 184*a^2*b + 210*a*b^2 -
 75*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 - (128*a^3 - 504*a^2*b + 600*a*b^2
- 225*b^3)*cosh(f*x + e)^4 + (495*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x +
e)^8 + 420*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^6 - 70*(1
28*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^4 - 128*a^3 + 504*a
^2*b - 600*a*b^2 + 225*b^3 + 60*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*c
osh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(
f*x + e)^9 + 60*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^7 -
14*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^5 + 20*(48*a^3
 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^3 - (128*a^3 - 504*a^2*b +
 600*a*b^2 - 225*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*a^2*b - 24*a*b^2 +
 15*b^3 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^2 + 2*(3
3*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^10 + 45*(16*a^3 - 72*a^2*b +
102*a*b^2 - 45*b^3)*cosh(f*x + e)^8 - 14*(128*a^3 - 504*a^2*b + 600*a*b^2 -
 225*b^3)*cosh(f*x + e)^6 + 30*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*co
sh(f*x + e)^4 + 16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3 - 3*(128*a^3 - 504*a
^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(8*a^2*
b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^11 + 5*(16*a^3 - 72*a^2*b + 102*a*b^2
- 45*b^3)*cosh(f*x + e)^9 - 2*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*c
osh(f*x + e)^7 + 6*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^
5 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^3 + (16*a^3 -
 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log((
b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x +
 e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*
(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x
 + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^
3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*
x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt
(2)*((8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^9 + 9*(8*a^3 - 24*a^2*b +
15*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^3 - 24*a^2*b + 15*a*b^2)*sin
h(f*x + e)^9 - 4*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^7 - 4*(16*a^3
 - 29*a^2*b + 15*a*b^2 - 9*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*s
inh(f*x + e)^7 + 28*(3*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 - (16*
a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(40*a^3 - 92*
a^2*b + 45*a*b^2)*cosh(f*x + e)^5 + 2*(63*(8*a^3 - 24*a^2*b + 15*a*b^2)*cos
h(f*x + e)^4 + 40*a^3 - 92*a^2*b + 45*a*b^2 - 42*(16*a^3 - 29*a^2*b + 15*a*
b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(63*(8*a^3 - 24*a^2*b + 15*a*b^2)
*cosh(f*x + e)^5 - 70*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 + 5*(4
```

```
0*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 - 4*(16*a^3 - 2
9*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^3 - 24*a^2*b + 15*a*b^2)*c
osh(f*x + e)^6 - 35*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^4 - 16*a^3
 + 29*a^2*b - 15*a*b^2 + 5*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e)^2)*
sinh(f*x + e)^3 + 4*(9*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^7 - 21*(
16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^5 + 5*(40*a^3 - 92*a^2*b + 45*a
*b^2)*cosh(f*x + e)^3 - 3*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e))*sin
h(f*x + e)^2 + (8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e) + (9*(8*a^3 - 24
*a^2*b + 15*a*b^2)*cosh(f*x + e)^8 - 28*(16*a^3...
```

Sympy **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{5}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**5/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 2.63Error: Bad Argum
ent Type
```

Mupad **[F(-1)]**
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] \text{Hanged}
```

3.496 $\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=275

$$-\frac{\sqrt{a}\,\sqrt{b}\,(7a+b)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\,\sqrt{a+b\sinh^2(e+fx)}}+\frac{(3a+5b)F\left(\text{ArcTan}(\sinh(e+fx))\,|1\right.}{3(a-b)^3f\sqrt{\frac{\text{sech}^2(e+}{}}}$$

```
[Out] -1/3*(7*a+b)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2
/a)^(1/2)*EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2)
,(1-a/b)^(1/2))*a^(1/2)*b^(1/2)/(a-b)^3/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)
^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(3*a+5*b)*(1/(1+sinh(f*x+e)^2))^(1
/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(
1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^3/f/(sech(f*x+e)^
2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-4/3*a*tanh(f*x+e)/(a-b)^2/f/(a+b*sinh(f*x+e)
^2)^(1/2)+1/3*sech(f*x+e)^2*tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3275, 481, 541, 539, 429, 422}

$$-\frac{\sqrt{a}\,\sqrt{b}\,(7a+b)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\,\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{(3a+5b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{3f(a-b)^3\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{4a\tanh(e+fx)}{3f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\tanh(e+fx)\text{sech}^2(e+fx)}{3f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*(Sqrt[a]*Sqrt[b]*(7*a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sin
h[e + f*x])/Sqrt[a]], 1 - a/b])/((a - b)^3*f*Sqrt[(a*Cosh[e + f*x]^2)/(a +
b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + 5*b)*EllipticF[A
rcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(
3*(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (4*a*Tan
h[e + f*x])/(3*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) + (Sech[e + f*x]^2*
Tanh[e + f*x])/(3*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\Big(\int \frac{x^4}{(1+x^2)^{5/2}(a+bx^2)^{3/2}}\,dx, x, \sinh(e}{f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Sub}}{3(}$$

$$= -\frac{4a\tanh(e+fx)}{3(a-b)^2 f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= -\frac{4a\tanh(e+fx)}{3(a-b)^2 f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}} +$$

$$= -\frac{\sqrt{a}\sqrt{b}\,(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3(a-b)^3 f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{(3a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.60, size = 212, normalized size = 0.77

$$\frac{-2ia(7a+b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,|\,\frac{b}{a}\right)+8ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,|\,\frac{b}{a}\right)-\frac{(8a^2+21ab-5b^2+4(4a^2+3ab+b^2)\cosh(2(e+fx))+b(7a+b)\cosh(4(e+fx)))\operatorname{sech}^2(e+fx)\tanh(e+fx)}{2\sqrt{2}}}{6(a-b)^3 f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-2*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elli
pticF[I*(e + f*x), b/a] - ((8*a^2 + 21*a*b - 5*b^2 + 4*(4*a^2 + 3*a*b + b^2
)*Cosh[2*(e + f*x)] + b*(7*a + b)*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh[e
 + f*x])/(2*Sqrt[2]))/(6*(a - b)^3*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 6.75, size = 352, normalized size = 1.28

method	result

default	$\frac{\left(-7\sqrt{-\frac{b}{a}}\, ab-\sqrt{-\frac{b}{a}}\, b^{2}\right)(\cosh^{4}(fx+e))\sinh(fx+e)+\left(-4\sqrt{-\frac{b}{a}}\, a^{2}+4\sqrt{-\frac{b}{a}}\, ab\right)(\cosh^{2}(fx+e))\sinh(fx+e)+\sqrt{\frac{b(\cosh}{}}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-7*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(
-4*(-1/a*b)^(1/2)*a^2+4*(-1/a*b)^(1/2)*a*b)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*
cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(7*EllipticE(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))*b^2+3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-2*Ellip
ticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-EllipticF(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2+((-1/a*b)^(1/2)*a^2-2*(-1/a*b)^(
1/2)*a*b+(-1/a*b)^(1/2)*b^2)*sinh(f*x+e))/(-1/a*b)^(1/2)/cosh(f*x+e)^3/(a-b
)^3/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7400 vs. $2(279) = 558$.
time = 0.27, size = 7400, normalized size = 26.91

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(((14*a^2*b^2 - 5*a*b^3 - b^4)*cosh(f*x + e)^10 + 10*(14*a^2*b^2 - 5*a*
b^3 - b^4)*cosh(f*x + e)*sinh(f*x + e)^9 + (14*a^2*b^2 - 5*a*b^3 - b^4)*sin
h(f*x + e)^10 + (56*a^3*b - 6*a^2*b^2 - 9*a*b^3 - b^4)*cosh(f*x + e)^8 + (5
6*a^3*b - 6*a^2*b^2 - 9*a*b^3 - b^4 + 45*(14*a^2*b^2 - 5*a*b^3 - b^4)*cosh(
f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(14*a^2*b^2 - 5*a*b^3 - b^4)*cosh(f*x +
 e)^3 + (56*a^3*b - 6*a^2*b^2 - 9*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)
```

```
^7 + 2*(84*a^3*b - 44*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^6 + 2*(105*(14*a
^2*b^2 - 5*a*b^3 - b^4)*cosh(f*x + e)^4 + 84*a^3*b - 44*a^2*b^2 - a*b^3 + b
^4 + 14*(56*a^3*b - 6*a^2*b^2 - 9*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x +
e)^6 + 4*(63*(14*a^2*b^2 - 5*a*b^3 - b^4)*cosh(f*x + e)^5 + 14*(56*a^3*b -
6*a^2*b^2 - 9*a*b^3 - b^4)*cosh(f*x + e)^3 + 3*(84*a^3*b - 44*a^2*b^2 - a*b
^3 + b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(84*a^3*b - 44*a^2*b^2 - a*b^3
 + b^4)*cosh(f*x + e)^4 + 2*(105*(14*a^2*b^2 - 5*a*b^3 - b^4)*cosh(f*x + e)
^6 + 35*(56*a^3*b - 6*a^2*b^2 - 9*a*b^3 - b^4)*cosh(f*x + e)^4 + 84*a^3*b -
 44*a^2*b^2 - a*b^3 + b^4 + 15*(84*a^3*b - 44*a^2*b^2 - a*b^3 + b^4)*cosh(f
*x + e)^2)*sinh(f*x + e)^4 + 14*a^2*b^2 - 5*a*b^3 - b^4 + 8*(15*(14*a^2*b^2
 - 5*a*b^3 - b^4)*cosh(f*x + e)^7 + 7*(56*a^3*b - 6*a^2*b^2 - 9*a*b^3 - b^4
)*cosh(f*x + e)^5 + 5*(84*a^3*b - 44*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^3
 + (84*a^3*b - 44*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + (
56*a^3*b - 6*a^2*b^2 - 9*a*b^3 - b^4)*cosh(f*x + e)^2 + (45*(14*a^2*b^2 - 5
*a*b^3 - b^4)*cosh(f*x + e)^8 + 28*(56*a^3*b - 6*a^2*b^2 - 9*a*b^3 - b^4)*c
osh(f*x + e)^6 + 30*(84*a^3*b - 44*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^4 +
 56*a^3*b - 6*a^2*b^2 - 9*a*b^3 - b^4 + 12*(84*a^3*b - 44*a^2*b^2 - a*b^3 +
 b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(14*a^2*b^2 - 5*a*b^3 - b^4)*
cosh(f*x + e)^9 + 4*(56*a^3*b - 6*a^2*b^2 - 9*a*b^3 - b^4)*cosh(f*x + e)^7
+ 6*(84*a^3*b - 44*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^5 + 4*(84*a^3*b - 4
4*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^3 + (56*a^3*b - 6*a^2*b^2 - 9*a*b^3
- b^4)*cosh(f*x + e))*sinh(f*x + e) - 2*((7*a*b^3 + b^4)*cosh(f*x + e)^10 +
 10*(7*a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x + e)^9 + (7*a*b^3 + b^4)*sinh(f*
x + e)^10 + (28*a^2*b^2 + 11*a*b^3 + b^4)*cosh(f*x + e)^8 + (28*a^2*b^2 + 1
1*a*b^3 + b^4 + 45*(7*a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15
*(7*a*b^3 + b^4)*cosh(f*x + e)^3 + (28*a^2*b^2 + 11*a*b^3 + b^4)*cosh(f*x +
 e))*sinh(f*x + e)^7 + 2*(42*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^6 + 2*(10
5*(7*a*b^3 + b^4)*cosh(f*x + e)^4 + 42*a^2*b^2 - a*b^3 - b^4 + 14*(28*a^2*b
^2 + 11*a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(7*a*b^3 + b^
4)*cosh(f*x + e)^5 + 14*(28*a^2*b^2 + 11*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(
42*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(42*a^2*b^2 -
a*b^3 - b^4)*cosh(f*x + e)^4 + 2*(105*(7*a*b^3 + b^4)*cosh(f*x + e)^6 + 35*
(28*a^2*b^2 + 11*a*b^3 + b^4)*cosh(f*x + e)^4 + 42*a^2*b^2 - a*b^3 - b^4 +
15*(42*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 7*a*b^3 +
b^4 + 8*(15*(7*a*b^3 + b^4)*cosh(f*x + e)^7 + 7*(28*a^2*b^2 + 11*a*b^3 + b^
4)*cosh(f*x + e)^5 + 5*(42*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^3 + (42*a^2
*b^2 - a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + (28*a^2*b^2 + 11*a*b^3
 + b^4)*cosh(f*x + e)^2 + (45*(7*a*b^3 + b^4)*cosh(f*x + e)^8 + 28*(28*a^2*
b^2 + 11*a*b^3 + b^4)*cosh(f*x + e)^6 + 30*(42*a^2*b^2 - a*b^3 - b^4)*cosh(
f*x + e)^4 + 28*a^2*b^2 + 11*a*b^3 + b^4 + 12*(42*a^2*b^2 - a*b^3 - b^4)*co
sh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(7*a*b^3 + b^4)*cosh(f*x + e)^9 + 4*(
28*a^2*b^2 + 11*a*b^3 + b^4)*cosh(f*x + e)^7 + 6*(42*a^2*b^2 - a*b^3 - b^4)
*cosh(f*x + e)^5 + 4*(42*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^3 + (28*a^2*b
^2 + 11*a*b^3 + b^4)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*s
qrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt
```

```
((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))),
 (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((6
*a^3*b + 7*a^2*b^2 - 5*a*b^3)*cosh(f*x + e)^10 + 10*(6*a^3*b + 7*a^2*b^2 -
5*a*b^3)*cosh(f*x + e)*sinh(f*x + e)^9 + (6*a^3*b + 7*a^2*b^2 - 5*a*b^3)*si
nh(f*x + e)^10 + (24*a^4 + 34*a^3*b - 13*a^2*b^2 - 5*a*b^3)*cosh(f*x + e)^8
 + (24*a^4 + 34*a^3*b - 13*a^2*b^2 - 5*a*b^3 + 45*(6*a^3*b + 7*a^2*b^2 - 5*
a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(6*a^3*b + 7*a^2*b^2 - 5*a*
b^3)*cosh(f*x + e)^3 + (24*a^4 + 34*a^3*b - 13*a^2*b^2 - 5*a*b^3)*cosh(f*x
+ e))*sinh(f*x + e)^7 + 2*(36*a^4 + 36*a^3*b - 37*a^2*b^2 + 5*a*b^3)*cosh(f
*x + e)^6 + 2*(105*(6*a^3*b + 7*a^2*b^2 - 5*a*b^3)*cosh(f*x + e)^4 + 36*a^4
 + 36*a^3*b - 37*a^2*b^2 + 5*a*b^3 + 14*(24*a^4 + 34*a^3*b - 13*a^2*b^2 - 5
*a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(6*a^3*b + 7*a^2*b^2 - 5*a
*b^3)*cosh(f*x + e)^5 + 14*(24*a^4 + 34*a^3*b - 13*a^2*b^2 - 5*a*b^3)*cosh(
f*x + e)^3 + 3*(36*a^4 + 36*a^3*b - 37*a^2*b^2 + 5*a*b^3)*cosh(f*x + e))*si
nh(f*x + e)^5 + 2*(36*a^4 + 36*a^3*b - 37*a^2*b^2 + 5*a*b^3)*cosh(f*x + e)^
4 + 2*(105*(6*a^3*b + 7*a^2*b^2 - 5*a*b^3)*cosh...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4{(e+fx)}}{\left(a+b\sinh^2{(e+fx)}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
[Out] Integral(tanh(e + f*x)**4/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 1.4Error: Bad Argument Type
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{tanh}\left(e+f\,x\right)}^4}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.497 $\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{2\sqrt{a}\sqrt{b}\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{(a-b)^2f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}+\frac{(a+b)F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)\text{sec}}{a(a-b)^2f\sqrt{\frac{\text{sech}^2(e+fx)\,(a}{}}}$$

```
[Out] -2*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/2)*
EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(
1/2))*a^(1/2)*b^(1/2)/(a-b)^2/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)
/(a+b*sinh(f*x+e)^2)^(1/2)+(a+b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)
^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech
(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+
e)^2)/a)^(1/2)-tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3275, 482, 539, 429, 422}

$$-\frac{2\sqrt{a}\sqrt{b}\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{af(a-b)^2\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\tanh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[b]*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/
Sqrt[a]], 1 - a/b])/((a - b)^2*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f
*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((a + b)*EllipticF[ArcTan[Sinh[e + f
*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)^2*f*Sq
rt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - Tanh[e + f*x]/((a - b)*f
*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^2}{(1+x^2)^{3/2}(a+bx^2)^{3/2}}\,dx, x, \sinh(e}{f}$$

$$= -\frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Su}}{(}$$

$$= -\frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(2ab\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)}{(a-b)}$$

$$= -\frac{2\sqrt{a}\sqrt{b}\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{(a-b)^2 f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{(a+b)F(\mathrm{t}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.97, size = 158, normalized size = 0.73

$$\frac{-2i\sqrt{2}\,a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,|\,\frac{b}{a}\right)+i\sqrt{2}\,(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,|\,\frac{b}{a}\right)-2(a+b\cosh(2(e+fx)))\tanh(e+fx)}{(a-b)^2f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
f*x), b/a] + I*Sqrt[2]*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elli
pticF[I*(e + f*x), b/a] - 2*(a + b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/((a -
b)^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 2.01, size = 257, normalized size = 1.18

method	result
default	$\frac{-2\sqrt{-\frac{b}{a}}\,b\left(\sinh^3(fx+e)\right)+a\sqrt{\frac{a+b\left(\sinh^2(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\operatorname{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\sqrt{\frac{a+}{}}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*(-1/a*b)^(1/2)*b*sinh(f*x+e)^3+a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*
x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))+2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*E
llipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(-1/a*b)^(1/2)*a*sinh(f*x+
e)-b*sinh(f*x+e)*(-1/a*b)^(1/2))/(a-b)^2/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2612 vs. $2(231) = 462$.
time = 0.15, size = 2612, normalized size = 12.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(((2*a*b^2 - b^3)*cosh(f*x + e)^6 + 6*(2*a*b^2 - b^3)*cosh(f*x + e)*sinh(
f*x + e)^5 + (2*a*b^2 - b^3)*sinh(f*x + e)^6 + (8*a^2*b - 6*a*b^2 + b^3)*co
sh(f*x + e)^4 + (8*a^2*b - 6*a*b^2 + b^3 + 15*(2*a*b^2 - b^3)*cosh(f*x + e)
^2)*sinh(f*x + e)^4 + 4*(5*(2*a*b^2 - b^3)*cosh(f*x + e)^3 + (8*a^2*b - 6*a
*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 2*a*b^2 - b^3 + (8*a^2*b - 6*a
*b^2 + b^3)*cosh(f*x + e)^2 + (15*(2*a*b^2 - b^3)*cosh(f*x + e)^4 + 8*a^2*b
 - 6*a*b^2 + b^3 + 6*(8*a^2*b - 6*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x +
e)^2 + 2*(3*(2*a*b^2 - b^3)*cosh(f*x + e)^5 + 2*(8*a^2*b - 6*a*b^2 + b^3)*c
osh(f*x + e)^3 + (8*a^2*b - 6*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e) - 2
*(b^3*cosh(f*x + e)^6 + 6*b^3*cosh(f*x + e)*sinh(f*x + e)^5 + b^3*sinh(f*x
+ e)^6 + (4*a*b^2 - b^3)*cosh(f*x + e)^4 + (15*b^3*cosh(f*x + e)^2 + 4*a*b^
2 - b^3)*sinh(f*x + e)^4 + 4*(5*b^3*cosh(f*x + e)^3 + (4*a*b^2 - b^3)*cosh(
f*x + e))*sinh(f*x + e)^3 + b^3 + (4*a*b^2 - b^3)*cosh(f*x + e)^2 + (15*b^3
*cosh(f*x + e)^4 + 4*a*b^2 - b^3 + 6*(4*a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(
f*x + e)^2 + 2*(3*b^3*cosh(f*x + e)^5 + 2*(4*a*b^2 - b^3)*cosh(f*x + e)^3 +
 (4*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(
b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*
```

```
b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*
a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - ((2*a^2*b
 + a*b^2 - b^3)*cosh(f*x + e)^6 + 6*(2*a^2*b + a*b^2 - b^3)*cosh(f*x + e)*s
inh(f*x + e)^5 + (2*a^2*b + a*b^2 - b^3)*sinh(f*x + e)^6 + (8*a^3 + 2*a^2*b
 - 5*a*b^2 + b^3)*cosh(f*x + e)^4 + (8*a^3 + 2*a^2*b - 5*a*b^2 + b^3 + 15*(
2*a^2*b + a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(2*a^2*b + a
*b^2 - b^3)*cosh(f*x + e)^3 + (8*a^3 + 2*a^2*b - 5*a*b^2 + b^3)*cosh(f*x +
e))*sinh(f*x + e)^3 + 2*a^2*b + a*b^2 - b^3 + (8*a^3 + 2*a^2*b - 5*a*b^2 +
b^3)*cosh(f*x + e)^2 + (15*(2*a^2*b + a*b^2 - b^3)*cosh(f*x + e)^4 + 8*a^3
+ 2*a^2*b - 5*a*b^2 + b^3 + 6*(8*a^3 + 2*a^2*b - 5*a*b^2 + b^3)*cosh(f*x +
e)^2)*sinh(f*x + e)^2 + 2*(3*(2*a^2*b + a*b^2 - b^3)*cosh(f*x + e)^5 + 2*(8
*a^3 + 2*a^2*b - 5*a*b^2 + b^3)*cosh(f*x + e)^3 + (8*a^3 + 2*a^2*b - 5*a*b^
2 + b^3)*cosh(f*x + e))*sinh(f*x + e) + 2*((a*b^2 - b^3)*cosh(f*x + e)^6 +
6*(a*b^2 - b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a*b^2 - b^3)*sinh(f*x + e)
^6 + (4*a^2*b - 5*a*b^2 + b^3)*cosh(f*x + e)^4 + (4*a^2*b - 5*a*b^2 + b^3 +
 15*(a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(a*b^2 - b^3)*cos
h(f*x + e)^3 + (4*a^2*b - 5*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + a
*b^2 - b^3 + (4*a^2*b - 5*a*b^2 + b^3)*cosh(f*x + e)^2 + (15*(a*b^2 - b^3)*
cosh(f*x + e)^4 + 4*a^2*b - 5*a*b^2 + b^3 + 6*(4*a^2*b - 5*a*b^2 + b^3)*cos
h(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a*b^2 - b^3)*cosh(f*x + e)^5 + 2*(4*a
^2*b - 5*a*b^2 + b^3)*cosh(f*x + e)^3 + (4*a^2*b - 5*a*b^2 + b^3)*cosh(f*x
+ e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a
*b)/b^2) - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) -
2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a
*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*(b^3*cosh(f*x + e)^5 + 5*b^
3*cosh(f*x + e)*sinh(f*x + e)^4 + b^3*sinh(f*x + e)^5 + a*b^2*cosh(f*x + e)
 + (3*a*b^2 - b^3)*cosh(f*x + e)^3 + (10*b^3*cosh(f*x + e)^2 + 3*a*b^2 - b^
3)*sinh(f*x + e)^3 + (10*b^3*cosh(f*x + e)^3 + 3*(3*a*b^2 - b^3)*cosh(f*x +
 e))*sinh(f*x + e)^2 + (5*b^3*cosh(f*x + e)^4 + a*b^2 + 3*(3*a*b^2 - b^3)*c
osh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/((a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^6 + 6*(a^2*b^3 - 2*a*b^4 +
 b^5)*f*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2*b^3 - 2*a*b^4 + b^5)*f*sinh(f*
x + e)^6 + (4*a^3*b^2 - 9*a^2*b^3 + 6*a*b^4 - b^5)*f*cosh(f*x + e)^4 + (15*
(a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^2 + (4*a^3*b^2 - 9*a^2*b^3 + 6*a*
b^4 - b^5)*f)*sinh(f*x + e)^4 + (4*a^3*b^2 - 9*a^2*b^3 + 6*a*b^4 - b^5)*f*c
osh(f*x + e)^2 + 4*(5*(a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (4*a^3*
b^2 - 9*a^2*b^3 + 6*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x + e)^3 + (15*(a^
2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^4 + 6*(4*a^3*b^2 - 9*a^2*b^3 + 6*a*b
^4 - b^5)*f*cosh(f*x + e)^2 + (4*a^3*b^2 - 9*a^2*b^3 + 6*a*b^4 - b^5)*f)*si
nh(f*x + e)^2 + (a^2*b^3 - 2*a*b^4 + b^5)*f + 2*(3*(a^2*b^3 - 2*a*b^4 + b^5
)*f*cosh(f*x + e)^5 + 2*(4*a^3*b^2 - 9*a^2*b^3 + 6*a*b^4 - b^5)*f*cosh(f*x
+ e)^3 + (4*a^3*b^2 - 9*a^2*b^3 + 6*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x
+ e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.45Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(e+f\,x)^2}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.498 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

```
[Out] -b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-I*(cos(I*e+I
*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*si
nh(f*x+e)^2)^(1/2)/a/(a-b)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3263, 21, 3257, 3256}

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*
f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
```

```
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\int \frac{-a-b\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{a(a-b)}\\
&= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{a(a-b)}\\
&= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sqrt{a+b\sinh^2(e+fx)}\int\sqrt{1+\frac{b\sinh}{}}}{a(a-b)\sqrt{1+\frac{b\sinh^2(e+}{a}}}\\
&= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx}}{a(a-b)f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 100, normalized size = 0.87

$$\frac{-2ia\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-\sqrt{2}\,b\sinh(2(e+fx))}{2a(a-b)f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]])
```

Maple [**A**]
time = 1.18, size = 253, normalized size = 2.20

method	result
default	$-\frac{\sqrt{-\frac{b}{a}}\, b\left(\cosh^{2}(fx+e)\right)\sinh(fx+e)-\sqrt{\frac{b\left(\cosh^{2}(fx+e)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-1/a*b)^(1/2)*b*cosh(f*x+e)^2*sinh(f*x+e)-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1
/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))
*a+b*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh
(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/a/(a-
b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1464 vs. 2(123) = 246.
time = 0.12, size = 1464, normalized size = 12.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (((2*a*b^2 - b^3)*cosh(f*x + e)^4 + 4*(2*a*b^2 - b^3)*cosh(f*x + e)*sinh(f*
x + e)^3 + (2*a*b^2 - b^3)*sinh(f*x + e)^4 + 2*a*b^2 - b^3 + 2*(4*a^2*b - 4
*a*b^2 + b^3)*cosh(f*x + e)^2 + 2*(4*a^2*b - 4*a*b^2 + b^3 + 3*(2*a*b^2 - b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((2*a*b^2 - b^3)*cosh(f*x + e)^3 +
 (4*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e) - 2*(b^3*cosh(f*x +
 e)^4 + 4*b^3*cosh(f*x + e)*sinh(f*x + e)^3 + b^3*sinh(f*x + e)^4 + b^3 + 2
*(2*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(3*b^3*cosh(f*x + e)^2 + 2*a*b^2 - b^3
```

```
)*sinh(f*x + e)^2 + 4*(b^3*cosh(f*x + e)^3 + (2*a*b^2 - b^3)*cosh(f*x + e))
*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b
^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
 b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b -
b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((2*a^2*b - a*b^2)*cosh(f*x + e)^4 + 4
*(2*a^2*b - a*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a^2*b - a*b^2)*sinh(f
*x + e)^4 + 2*a^2*b - a*b^2 + 2*(4*a^3 - 4*a^2*b + a*b^2)*cosh(f*x + e)^2 +
 2*(4*a^3 - 4*a^2*b + a*b^2 + 3*(2*a^2*b - a*b^2)*cosh(f*x + e)^2)*sinh(f*x
 + e)^2 + 4*((2*a^2*b - a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 4*a^2*b + a*b^2)*
cosh(f*x + e))*sinh(f*x + e) + 2*((a*b^2 - b^3)*cosh(f*x + e)^4 + 4*(a*b^2
- b^3)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b^2 - b^3)*sinh(f*x + e)^4 + a*b^
2 - b^3 + 2*(2*a^2*b - 3*a*b^2 + b^3)*cosh(f*x + e)^2 + 2*(2*a^2*b - 3*a*b^
2 + b^3 + 3*(a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((a*b^2 - b^
3)*cosh(f*x + e)^3 + (2*a^2*b - 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)
)*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)
/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f
*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2
 - a*b)/b^2))/b^2) - sqrt(2)*(b^3*cosh(f*x + e)^3 + 3*b^3*cosh(f*x + e)*sin
h(f*x + e)^2 + b^3*sinh(f*x + e)^3 + (2*a*b^2 - b^3)*cosh(f*x + e) + (3*b^3
*cosh(f*x + e)^2 + 2*a*b^2 - b^3)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2)))/((a^2*b^3 - a*b^4)*f*cosh(f*x + e)^4 + 4*(a^2*b^3 -
 a*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2*b^3 - a*b^4)*f*sinh(f*x + e)
^4 + 2*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^2 + 2*(3*(a^2*b^3 -
a*b^4)*f*cosh(f*x + e)^2 + (2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*f)*sinh(f*x + e)
^2 + (a^2*b^3 - a*b^4)*f + 4*((a^2*b^3 - a*b^4)*f*cosh(f*x + e)^3 + (2*a^3*
b^2 - 3*a^2*b^3 + a*b^4)*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \sinh^{2}\left(e + f x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(-3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.499 $\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f}-\frac{2E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)\text{sech}(e}{a^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\right.}{a}}}$$

```
[Out] coth(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(1/2)-2*coth(f*x+e)*(a+b*sinh(f*x+e)^2)
^(1/2)/a^2/f-2*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*Elliptic
E(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(
f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(1/(1+sin
h(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*
x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f/(s
ech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(
f*x+e)/a^2/f
```

Rubi [A]

time = 0.18, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3275, 480, 597, 545, 429, 506, 422}

$$\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{a^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{a^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f}-\frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f}+\frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Coth[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2]) - (2*Coth[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(a^2*f) - (2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]
*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*f*Sqrt[(Sech[e + f*x]^2*(a
 + b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sec
h[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) + (2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a^2
*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
```

```
e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2(a+bx^2)^{3/2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\right.}{af}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} + \left(\sqrt{\text{c}}\right.$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} + \left(\sqrt{\text{c}}\right.$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} + F(\text{ta}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} - 2E(\text{t}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.60, size = 153, normalized size = 0.65

$$\frac{-2(a-b+b\cosh(2(e+fx)))\coth(e+fx)-2i\sqrt{2}\,a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+i\sqrt{2}\,a\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{a^2f\sqrt{4a-2b+2b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a - b + b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*Sqrt[2]*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + I*Sqrt[2]*a*Sq
rt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(a^2*f*S
qrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A]
time = 2.11, size = 218, normalized size = 0.92

method	result
default	$\frac{-2\sqrt{-\frac{b}{a}}\, b(\cosh^4(fx+e))+\left(-\sqrt{-\frac{b}{a}}\, a+2\sqrt{-\frac{b}{a}}\, b\right)(\cosh^2(fx+e))+\sinh(fx+e)\sqrt{\frac{b(\cosh^2(fx+e))}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh(2f}{2}}}{a^2\sinh(fx+e)\sqrt{-\frac{b}{a}}\,\cos}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+(-(-1/a*b)^(1/2)*a+2*(-1/a*b)^(1/2)*b)*c
osh(f*x+e)^2+sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^
(1/2)*(a*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2*b*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1
/2),(a/b)^(1/2))))/a^2/sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x
+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2436 vs. $2(251) = 502$.
time = 0.14, size = 2436, normalized size = 10.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(((2*a*b^2 - b^3)*cosh(f*x + e)^6 + 6*(2*a*b^2 - b^3)*cosh(f*x + e)*sinh(
f*x + e)^5 + (2*a*b^2 - b^3)*sinh(f*x + e)^6 + (8*a^2*b - 10*a*b^2 + 3*b^3)
*cosh(f*x + e)^4 + (8*a^2*b - 10*a*b^2 + 3*b^3 + 15*(2*a*b^2 - b^3)*cosh(f*
x + e)^2)*sinh(f*x + e)^4 + 4*(5*(2*a*b^2 - b^3)*cosh(f*x + e)^3 + (8*a^2*b
 - 10*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - 2*a*b^2 + b^3 - (8*a^
2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^2 + (15*(2*a*b^2 - b^3)*cosh(f*x + e)
```

```
^4 - 8*a^2*b + 10*a*b^2 - 3*b^3 + 6*(8*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x +
 e)^2)*sinh(f*x + e)^2 + 2*(3*(2*a*b^2 - b^3)*cosh(f*x + e)^5 + 2*(8*a^2*b
- 10*a*b^2 + 3*b^3)*cosh(f*x + e)^3 - (8*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x
 + e))*sinh(f*x + e) - 2*(b^3*cosh(f*x + e)^6 + 6*b^3*cosh(f*x + e)*sinh(f*
x + e)^5 + b^3*sinh(f*x + e)^6 + (4*a*b^2 - 3*b^3)*cosh(f*x + e)^4 + (15*b^
3*cosh(f*x + e)^2 + 4*a*b^2 - 3*b^3)*sinh(f*x + e)^4 + 4*(5*b^3*cosh(f*x +
e)^3 + (4*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - b^3 - (4*a*b^2 -
3*b^3)*cosh(f*x + e)^2 + (15*b^3*cosh(f*x + e)^4 - 4*a*b^2 + 3*b^3 + 6*(4*a
*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*b^3*cosh(f*x + e)^5 +
 2*(4*a*b^2 - 3*b^3)*cosh(f*x + e)^3 - (4*a*b^2 - 3*b^3)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2)
 - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/
b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2)
*sqrt((a^2 - a*b)/b^2))/b^2) - ((2*a^2*b - a*b^2)*cosh(f*x + e)^6 + 6*(2*a^
2*b - a*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b - a*b^2)*sinh(f*x + e
)^6 + (8*a^3 - 10*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + (8*a^3 - 10*a^2*b + 3*
a*b^2 + 15*(2*a^2*b - a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(2*a^2
*b - a*b^2)*cosh(f*x + e)^3 + (8*a^3 - 10*a^2*b + 3*a*b^2)*cosh(f*x + e))*s
inh(f*x + e)^3 - 2*a^2*b + a*b^2 - (8*a^3 - 10*a^2*b + 3*a*b^2)*cosh(f*x +
e)^2 + (15*(2*a^2*b - a*b^2)*cosh(f*x + e)^4 - 8*a^3 + 10*a^2*b - 3*a*b^2 +
 6*(8*a^3 - 10*a^2*b + 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(2*
a^2*b - a*b^2)*cosh(f*x + e)^5 + 2*(8*a^3 - 10*a^2*b + 3*a*b^2)*cosh(f*x +
e)^3 - (8*a^3 - 10*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e) + 2*((a*b^
2 - 2*b^3)*cosh(f*x + e)^6 + 6*(a*b^2 - 2*b^3)*cosh(f*x + e)*sinh(f*x + e)^
5 + (a*b^2 - 2*b^3)*sinh(f*x + e)^6 + (4*a^2*b - 11*a*b^2 + 6*b^3)*cosh(f*x
 + e)^4 + (4*a^2*b - 11*a*b^2 + 6*b^3 + 15*(a*b^2 - 2*b^3)*cosh(f*x + e)^2)
*sinh(f*x + e)^4 + 4*(5*(a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (4*a^2*b - 11*a*b
^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - a*b^2 + 2*b^3 - (4*a^2*b - 11*
a*b^2 + 6*b^3)*cosh(f*x + e)^2 + (15*(a*b^2 - 2*b^3)*cosh(f*x + e)^4 - 4*a^
2*b + 11*a*b^2 - 6*b^3 + 6*(4*a^2*b - 11*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*si
nh(f*x + e)^2 + 2*(3*(a*b^2 - 2*b^3)*cosh(f*x + e)^5 + 2*(4*a^2*b - 11*a*b^
2 + 6*b^3)*cosh(f*x + e)^3 - (4*a^2*b - 11*a*b^2 + 6*b^3)*cosh(f*x + e))*si
nh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b)/b^2)
 - 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)
/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b - b^2
)*sqrt((a^2 - a*b)/b^2))/b^2) - sqrt(2)*(b^3*cosh(f*x + e)^5 + 5*b^3*cosh(f
*x + e)*sinh(f*x + e)^4 + b^3*sinh(f*x + e)^5 + (3*a*b^2 - 2*b^3)*cosh(f*x
+ e)^3 + (10*b^3*cosh(f*x + e)^2 + 3*a*b^2 - 2*b^3)*sinh(f*x + e)^3 + (10*b
^3*cosh(f*x + e)^3 + 3*(3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^2 - (
a*b^2 - b^3)*cosh(f*x + e) + (5*b^3*cosh(f*x + e)^4 - a*b^2 + b^3 + 3*(3*a*
b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2)))/(a^2*b^3*f*cosh(f*x + e)^6 + 6*a^2*b^3*f*cosh(f*x + e)*
sinh(f*x + e)^5 + a^2*b^3*f*sinh(f*x + e)^6 - a^2*b^3*f + (4*a^3*b^2 - 3*a^
2*b^3)*f*cosh(f*x + e)^4 + (15*a^2*b^3*f*cosh(f*x + e)^2 + (4*a^3*b^2 - 3*a
```

```
^2*b^3)*f)*sinh(f*x + e)^4 - (4*a^3*b^2 - 3*a^2*b^3)*f*cosh(f*x + e)^2 + 4*
(5*a^2*b^3*f*cosh(f*x + e)^3 + (4*a^3*b^2 - 3*a^2*b^3)*f*cosh(f*x + e))*sin
h(f*x + e)^3 + (15*a^2*b^3*f*cosh(f*x + e)^4 + 6*(4*a^3*b^2 - 3*a^2*b^3)*f*
cosh(f*x + e)^2 - (4*a^3*b^2 - 3*a^2*b^3)*f)*sinh(f*x + e)^2 + 2*(3*a^2*b^3
*f*cosh(f*x + e)^5 + 2*(4*a^3*b^2 - 3*a^2*b^3)*f*cosh(f*x + e)^3 - (4*a^3*b
^2 - 3*a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{coth}\left(e+f\,x\right)}^2}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.500 $\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=341

$$-\frac{(a-b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}-\frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f}+\frac{(3a-4b)\coth(e+fx}{}$$

```
[Out] -(a-b)*coth(f*x+e)*csch(f*x+e)^2/a/b/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*(7*a-8
*b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/f+1/3*(3*a-4*b)*coth(f*x+e)*c
sch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/b/f-1/3*(7*a-8*b)*(1/(1+sinh(f*x
+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^
2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/f/(sech(f
*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a-4*b)*(1/(1+sinh(f*x+e)^2))^(1
/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(
1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/f/(sech(f*x+e)^2*(a
+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(7*a-8*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x
+e)/a^3/f
```

Rubi [A]

time = 0.28, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = $\{3275, 479, 597, 545, 429, 506, 422\}$

$$\frac{(3a-4b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\mathrm{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3a^3f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(7a-8b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\mathrm{ArcTan}(\sinh(e+fx))\,\middle|\,1-\frac{b}{a}\right)}{3a^3f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f}-\frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f}+\frac{(3a-4b)\coth(e+fx)\mathrm{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2bf}-\frac{(a-b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)*Coth[e + f*x]*Csch[e + f*x]^2)/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) - ((7*a - 8*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f) + (
(3*a - 4*b)*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a
^2*b*f) - ((7*a - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f
*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*
Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f*Sqrt[(Sech[e + f*x]^2*(
a + b*Sinh[e + f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh
[e + f*x])/(3*a^3*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{x^4\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{S}}{}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(3a-4b)\coth(e+fx)\text{csch}^2(e+fx}{3a^2bf}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh}}{3a^3f}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh}}{3a^3f}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh}}{3a^3f}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh}}{3a^3f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.66, size = 214, normalized size = 0.63

$$\frac{-\frac{\left(-8a^2+37ab-24b^2+4\left(4a^2-11ab+8b^2\right)\cosh(2(e+fx))+(7a-8b)b\cosh(4(e+fx))\right)\coth(e+fx)\text{csch}^2(e+fx)}{2\sqrt{2}} - 2ia(7a-8b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) + 8ia(a-b)\sqrt{\frac{2a-b+b\cosh(2(e+fx))}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{6a^3f\sqrt{2a-b+b\cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-1/2*((-8*a^2 + 37*a*b - 24*b^2 + 4*(4*a^2 - 11*a*b + 8*b^2)*Cosh[2*(e + f
*x)] + (7*a - 8*b)*b*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt
[2] - (2*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE
[I*(e + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a
]*EllipticF[I*(e + f*x), b/a])/(6*a^3*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]
)
```

Maple [A]
time = 6.10, size = 522, normalized size = 1.53

method	result
default	$-\frac{7\sqrt{-\frac{b}{a}}\, ab(\sinh^6(fx+e))-8\sqrt{-\frac{b}{a}}\, b^2(\sinh^6(fx+e))-3a^2\sqrt{\frac{a+b(\sinh^2(fx+e))}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh(\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(7*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-8*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6
-3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*sinh(f*x+e)^3+11*((a+b*sinh(f*x+e)^2)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))*b*a*sinh(f*x+e)^3-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*
EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3-7*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*si
nh(f*x+e)^3+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4+3*(-1/a*b)^(1/2)*a*b*sinh(f*
x+e)^4-8*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^
2-4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^2)/a^3/sinh(f*x+e)^3/
(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6862 vs. 2(343) = 686.
time = 0.20, size = 6862, normalized size = 20.12

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(((14*a^2*b^2 - 23*a*b^3 + 8*b^4)*cosh(f*x + e)^10 + 10*(14*a^2*b^2 - 2
3*a*b^3 + 8*b^4)*cosh(f*x + e)*sinh(f*x + e)^9 + (14*a^2*b^2 - 23*a*b^3 + 8
*b^4)*sinh(f*x + e)^10 + (56*a^3*b - 162*a^2*b^2 + 147*a*b^3 - 40*b^4)*cosh
(f*x + e)^8 + (56*a^3*b - 162*a^2*b^2 + 147*a*b^3 - 40*b^4 + 45*(14*a^2*b^2
 - 23*a*b^3 + 8*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(14*a^2*b^2 -
 23*a*b^3 + 8*b^4)*cosh(f*x + e)^3 + (56*a^3*b - 162*a^2*b^2 + 147*a*b^3 -
40*b^4)*cosh(f*x + e))*sinh(f*x + e)^7 - 2*(84*a^3*b - 208*a^2*b^2 + 163*a*
b^3 - 40*b^4)*cosh(f*x + e)^6 + 2*(105*(14*a^2*b^2 - 23*a*b^3 + 8*b^4)*cosh
(f*x + e)^4 - 84*a^3*b + 208*a^2*b^2 - 163*a*b^3 + 40*b^4 + 14*(56*a^3*b -
162*a^2*b^2 + 147*a*b^3 - 40*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*
(14*a^2*b^2 - 23*a*b^3 + 8*b^4)*cosh(f*x + e)^5 + 14*(56*a^3*b - 162*a^2*b^
2 + 147*a*b^3 - 40*b^4)*cosh(f*x + e)^3 - 3*(84*a^3*b - 208*a^2*b^2 + 163*a
*b^3 - 40*b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(84*a^3*b - 208*a^2*b^2 +
 163*a*b^3 - 40*b^4)*cosh(f*x + e)^4 + 2*(105*(14*a^2*b^2 - 23*a*b^3 + 8*b^
4)*cosh(f*x + e)^6 + 35*(56*a^3*b - 162*a^2*b^2 + 147*a*b^3 - 40*b^4)*cosh(
f*x + e)^4 + 84*a^3*b - 208*a^2*b^2 + 163*a*b^3 - 40*b^4 - 15*(84*a^3*b - 2
08*a^2*b^2 + 163*a*b^3 - 40*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 - 14*a^2*
b^2 + 23*a*b^3 - 8*b^4 + 8*(15*(14*a^2*b^2 - 23*a*b^3 + 8*b^4)*cosh(f*x + e
)^7 + 7*(56*a^3*b - 162*a^2*b^2 + 147*a*b^3 - 40*b^4)*cosh(f*x + e)^5 - 5*(
84*a^3*b - 208*a^2*b^2 + 163*a*b^3 - 40*b^4)*cosh(f*x + e)^3 + (84*a^3*b -
208*a^2*b^2 + 163*a*b^3 - 40*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 - (56*a^3*
b - 162*a^2*b^2 + 147*a*b^3 - 40*b^4)*cosh(f*x + e)^2 + (45*(14*a^2*b^2 - 2
3*a*b^3 + 8*b^4)*cosh(f*x + e)^8 + 28*(56*a^3*b - 162*a^2*b^2 + 147*a*b^3 -
 40*b^4)*cosh(f*x + e)^6 - 30*(84*a^3*b - 208*a^2*b^2 + 163*a*b^3 - 40*b^4)
*cosh(f*x + e)^4 - 56*a^3*b + 162*a^2*b^2 - 147*a*b^3 + 40*b^4 + 12*(84*a^3
*b - 208*a^2*b^2 + 163*a*b^3 - 40*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2
*(5*(14*a^2*b^2 - 23*a*b^3 + 8*b^4)*cosh(f*x + e)^9 + 4*(56*a^3*b - 162*a^2
*b^2 + 147*a*b^3 - 40*b^4)*cosh(f*x + e)^7 - 6*(84*a^3*b - 208*a^2*b^2 + 16
3*a*b^3 - 40*b^4)*cosh(f*x + e)^5 + 4*(84*a^3*b - 208*a^2*b^2 + 163*a*b^3 -
 40*b^4)*cosh(f*x + e)^3 - (56*a^3*b - 162*a^2*b^2 + 147*a*b^3 - 40*b^4)*co
sh(f*x + e))*sinh(f*x + e) - 2*((7*a*b^3 - 8*b^4)*cosh(f*x + e)^10 + 10*(7*
a*b^3 - 8*b^4)*cosh(f*x + e)*sinh(f*x + e)^9 + (7*a*b^3 - 8*b^4)*sinh(f*x +
 e)^10 + (28*a^2*b^2 - 67*a*b^3 + 40*b^4)*cosh(f*x + e)^8 + (28*a^2*b^2 - 6
7*a*b^3 + 40*b^4 + 45*(7*a*b^3 - 8*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^8 +
```

```
8*(15*(7*a*b^3 - 8*b^4)*cosh(f*x + e)^3 + (28*a^2*b^2 - 67*a*b^3 + 40*b^4)*
cosh(f*x + e))*sinh(f*x + e)^7 - 2*(42*a^2*b^2 - 83*a*b^3 + 40*b^4)*cosh(f*
x + e)^6 + 2*(105*(7*a*b^3 - 8*b^4)*cosh(f*x + e)^4 - 42*a^2*b^2 + 83*a*b^3
 - 40*b^4 + 14*(28*a^2*b^2 - 67*a*b^3 + 40*b^4)*cosh(f*x + e)^2)*sinh(f*x +
 e)^6 + 4*(63*(7*a*b^3 - 8*b^4)*cosh(f*x + e)^5 + 14*(28*a^2*b^2 - 67*a*b^3
 + 40*b^4)*cosh(f*x + e)^3 - 3*(42*a^2*b^2 - 83*a*b^3 + 40*b^4)*cosh(f*x +
e))*sinh(f*x + e)^5 + 2*(42*a^2*b^2 - 83*a*b^3 + 40*b^4)*cosh(f*x + e)^4 +
2*(105*(7*a*b^3 - 8*b^4)*cosh(f*x + e)^6 + 35*(28*a^2*b^2 - 67*a*b^3 + 40*b
^4)*cosh(f*x + e)^4 + 42*a^2*b^2 - 83*a*b^3 + 40*b^4 - 15*(42*a^2*b^2 - 83*
a*b^3 + 40*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 - 7*a*b^3 + 8*b^4 + 8*(15*
(7*a*b^3 - 8*b^4)*cosh(f*x + e)^7 + 7*(28*a^2*b^2 - 67*a*b^3 + 40*b^4)*cosh
(f*x + e)^5 - 5*(42*a^2*b^2 - 83*a*b^3 + 40*b^4)*cosh(f*x + e)^3 + (42*a^2*
b^2 - 83*a*b^3 + 40*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 - (28*a^2*b^2 - 67*
a*b^3 + 40*b^4)*cosh(f*x + e)^2 + (45*(7*a*b^3 - 8*b^4)*cosh(f*x + e)^8 + 2
8*(28*a^2*b^2 - 67*a*b^3 + 40*b^4)*cosh(f*x + e)^6 - 30*(42*a^2*b^2 - 83*a*
b^3 + 40*b^4)*cosh(f*x + e)^4 - 28*a^2*b^2 + 67*a*b^3 - 40*b^4 + 12*(42*a^2
*b^2 - 83*a*b^3 + 40*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(7*a*b^3
 - 8*b^4)*cosh(f*x + e)^9 + 4*(28*a^2*b^2 - 67*a*b^3 + 40*b^4)*cosh(f*x + e)
^7 - 6*(42*a^2*b^2 - 83*a*b^3 + 40*b^4)*cosh(f*x + e)^5 + 4*(42*a^2*b^2 - 8
3*a*b^3 + 40*b^4)*cosh(f*x + e)^3 - (28*a^2*b^2 - 67*a*b^3 + 40*b^4)*cosh(f
*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2
 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2)
 - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(
2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((6*a^3*b - 11*a^2*b^2 + 4*a*b
^3)*cosh(f*x + e)^10 + 10*(6*a^3*b - 11*a^2*b^2 + 4*a*b^3)*cosh(f*x + e)*si
nh(f*x + e)^9 + (6*a^3*b - 11*a^2*b^2 + 4*a*b^3)*sinh(f*x + e)^10 + (24*a^4
 - 74*a^3*b + 71*a^2*b^2 - 20*a*b^3)*cosh(f*x + e)^8 + (24*a^4 - 74*a^3*b +
 71*a^2*b^2 - 20*a*b^3 + 45*(6*a^3*b - 11*a^2*b^2 + 4*a*b^3)*cosh(f*x + e)^
2)*sinh(f*x + e)^8 + 8*(15*(6*a^3*b - 11*a^2*b^2 + 4*a*b^3)*cosh(f*x + e)^3
 + (24*a^4 - 74*a^3*b + 71*a^2*b^2 - 20*a*b^3)*cosh(f*x + e))*sinh(f*x + e)
^7 - 2*(36*a^4 - 96*a^3*b + 79*a^2*b^2 - 20*a*b^3)*cosh(f*x + e)^6 + 2*(105
*(6*a^3*b - 11*a^2*b^2 + 4*a*b^3)*cosh(f*x + e)^4 - 36*a^4 + 96*a^3*b - 79*
a^2*b^2 + 20*a*b^3 + 14*(24*a^4 - 74*a^3*b + 71...
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{4}(e+fx)}{\left(a+b\sinh^{2}(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**4/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.75Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(e+f\,x)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

3.501 $\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=232

$$-\frac{(8a^2+24ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{9/2}f}+\frac{8a^2+24ab+3b^2}{24(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{(8a}{8(a-b)^2f}$$

```
[Out] -1/8*(8*a^2+24*a*b+3*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a
-b)^(9/2)/f+1/24*(8*a^2+24*a*b+3*b^2)/(a-b)^3/f/(a+b*sinh(f*x+e)^2)^(3/2)+1
/8*(8*a-b)*sech(f*x+e)^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/4*sech(f*x+e
)^4/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/8*(8*a^2+24*a*b+3*b^2)/(a-b)^4/f/(a
+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 53, 65, 214}

$$\frac{8a^2+24ab+3b^2}{8f(a-b)^4\sqrt{a+b\sinh^2(e+fx)}}+\frac{8a^2+24ab+3b^2}{24f(a-b)^3\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(8a^2+24ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{9/2}}-\frac{\operatorname{sech}^4(e+fx)}{4f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{(8a-b)\operatorname{sech}^2(e+fx)}{8f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/8*((8*a^2 + 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a -
 b]])/((a - b)^(9/2)*f) + (8*a^2 + 24*a*b + 3*b^2)/(24*(a - b)^3*f*(a + b*S
inh[e + f*x]^2)^(3/2)) + ((8*a - b)*Sech[e + f*x]^2)/(8*(a - b)^2*f*(a + b*
Sinh[e + f*x]^2)^(3/2)) - Sech[e + f*x]^4/(4*(a - b)*f*(a + b*Sinh[e + f*x]
^2)^(3/2)) + (8*a^2 + 24*a*b + 3*b^2)/(8*(a - b)^4*f*Sqrt[a + b*Sinh[e + f*
x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)^3(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{sech}^4(e+fx)}{4(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a-3b)+2(a-b)x}{(1+x)^2(a+bx)^{5/2}}\,dx, x, \text{si}\right.}{4(a-b)f}$$

$$= \frac{(8a-b)\operatorname{sech}^2(e+fx)}{8(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\operatorname{sech}^4(e+fx)}{4(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$= \frac{8a^2+24ab+3b^2}{24(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(8a-b)\operatorname{sech}^2(e+fx)}{8(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/}}$$

$$= \frac{8a^2+24ab+3b^2}{24(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(8a-b)\operatorname{sech}^2(e+fx)}{8(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/}}$$

$$= \frac{8a^2+24ab+3b^2}{24(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(8a-b)\operatorname{sech}^2(e+fx)}{8(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/}}$$

$$= -\frac{(8a^2+24ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{9/2}f} + \frac{8a^2}{24(a-b)^3f\,(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.42, size = 114, normalized size = 0.49

$$\frac{2(8a^2+24ab+3b^2)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sinh^2(e+fx)}{a-b}\right)+3(a-b)(4a+3b+(8a-b)\cosh(2(e+fx)))\operatorname{sech}^4(e+fx)}{48(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*(8*a^2 + 24*a*b + 3*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sinh[e
+ f*x]^2)/(a - b)] + 3*(a - b)*(4*a + 3*b + (8*a - b)*Cosh[2*(e + f*x)])*Se
ch[e + f*x]^4)/(48*(a - b)^3*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.13, size = 213, normalized size = 0.92

method	result
default	$\dfrac{\text{`}\mathit{int/indef0}\text{`}\Bigg(-\dfrac{(\sinh^5(fx+e))\,(b^2(\sinh^4(fx+e))+2ab(\sinh^2(fx+}{(-b^4(\cosh^{18}(fx+e))+(-4a\,b^3+4b^4)(\cosh^{16}(fx+e))+(-6a^2b^2+12a\,b^3-6b^4)(\cosh^{14}(fx+e))+(-4a^3b+12a^2b^2-12a\,b^3+4b^4)(}}{f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 'int/indef0'(-sinh(f*x+e)^5*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)*cos
h(f*x+e)^4/(-b^4*cosh(f*x+e)^18+(-4*a*b^3+4*b^4)*cosh(f*x+e)^16+(-6*a^2*b^2
+12*a*b^3-6*b^4)*cosh(f*x+e)^14+(-4*a^3*b+12*a^2*b^2-12*a*b^3+4*b^4)*cosh(f
*x+e)^12+(-a^4+4*a^3*b-6*a^2*b^2+4*a*b^3-b^4)*cosh(f*x+e)^10)/(a+b*sinh(f*x
+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10051 vs. 2(208) = 416.
time = 1.38, size = 20298, normalized size = 87.49

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2*b^2 + 24*a*b^3 + 3*b^4)*cosh(f*x + e)^16 + 16*(8*a^2*b^2 +
 24*a*b^3 + 3*b^4)*cosh(f*x + e)*sinh(f*x + e)^15 + (8*a^2*b^2 + 24*a*b^3 +
 3*b^4)*sinh(f*x + e)^16 + 8*(8*a^3*b + 24*a^2*b^2 + 3*a*b^3)*cosh(f*x + e)
^14 + 8*(8*a^3*b + 24*a^2*b^2 + 3*a*b^3 + 15*(8*a^2*b^2 + 24*a*b^3 + 3*b^4)
*cosh(f*x + e)^2)*sinh(f*x + e)^14 + 112*(5*(8*a^2*b^2 + 24*a*b^3 + 3*b^4)*
cosh(f*x + e)^3 + (8*a^3*b + 24*a^2*b^2 + 3*a*b^3)*cosh(f*x + e))*sinh(f*x
+ e)^13 + 4*(32*a^4 + 128*a^3*b + 100*a^2*b^2 - 12*a*b^3 - 3*b^4)*cosh(f*x
+ e)^12 + 4*(455*(8*a^2*b^2 + 24*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 32*a^4 +
128*a^3*b + 100*a^2*b^2 - 12*a*b^3 - 3*b^4 + 182*(8*a^3*b + 24*a^2*b^2 + 3*
```

```
a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^12 + 16*(273*(8*a^2*b^2 + 24*a*b^3 +
3*b^4)*cosh(f*x + e)^5 + 182*(8*a^3*b + 24*a^2*b^2 + 3*a*b^3)*cosh(f*x + e)
^3 + 3*(32*a^4 + 128*a^3*b + 100*a^2*b^2 - 12*a*b^3 - 3*b^4)*cosh(f*x + e))
*sinh(f*x + e)^11 + 8*(64*a^4 + 184*a^3*b - 3*a*b^3)*cosh(f*x + e)^10 + 8*(
1001*(8*a^2*b^2 + 24*a*b^ ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(e + fx)}{(a + b\sinh^2(e + fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tanh(e + f*x)**5/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3690 vs. 2(208) = 416.
time = 21.38, size = 3690, normalized size = 15.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*((3*(a^22*b^3*e^(21*e) - 14*a^21*b^4*e^(21*e) + 88*a^20*b^5*e^(21*e) -
320*a^19*b^6*e^(21*e) + 700*a^18*b^7*e^(21*e) - 728*a^17*b^8*e^(21*e) - 728
*a^16*b^9*e^(21*e) + 4576*a^15*b^10*e^(21*e) - 10010*a^14*b^11*e^(21*e) + 1
4300*a^13*b^12*e^(21*e) - 14872*a^12*b^13*e^(21*e) + 11648*a^11*b^14*e^(21*
e) - 6916*a^10*b^15*e^(21*e) + 3080*a^9*b^16*e^(21*e) - 1000*a^8*b^17*e^(21
*e) + 224*a^7*b^18*e^(21*e) - 31*a^6*b^19*e^(21*e) + 2*a^5*b^20*e^(21*e))*e
^(2*f*x)/(a^24*b^2*e^(16*e) - 20*a^23*b^3*e^(16*e) + 190*a^22*b^4*e^(16*e)
- 1140*a^21*b^5*e^(16*e) + 4845*a^20*b^6*e^(16*e) - 15504*a^19*b^7*e^(16*e)
 + 38760*a^18*b^8*e^(16*e) - 77520*a^17*b^9*e^(16*e) + 125970*a^16*b^10*e^(
16*e) - 167960*a^15*b^11*e^(16*e) + 184756*a^14*b^12*e^(16*e) - 167960*a^13
*b^13*e^(16*e) + 125970*a^12*b^14*e^(16*e) - 77520*a^11*b^15*e^(16*e) + 387
60*a^10*b^16*e^(16*e) - 15504*a^9*b^17*e^(16*e) + 4845*a^8*b^18*e^(16*e) -
1140*a^7*b^19*e^(16*e) + 190*a^6*b^20*e^(16*e) - 20*a^5*b^21*e^(16*e) + a^4
*b^22*e^(16*e)) + 2*(8*a^23*b^2*e^(19*e) - 121*a^22*b^3*e^(19*e) + 842*a^21
*b^4*e^(19*e) - 3544*a^20*b^5*e^(19*e) + 9920*a^19*b^6*e^(19*e) - 18844*a^1
8*b^7*e^(19*e) + 22568*a^17*b^8*e^(19*e) - 9256*a^16*b^9*e^(19*e) - 25168*a
^15*b^10*e^(19*e) + 67210*a^14*b^11*e^(19*e) - 93236*a^13*b^12*e^(19*e) + 8
9752*a^12*b^13*e^(19*e) - 64064*a^11*b^14*e^(19*e) + 34468*a^10*b^15*e^(19*
e) - 13880*a^9*b^16*e^(19*e) + 4072*a^8*b^17*e^(19*e) - 824*a^7*b^18*e^(19*
```

```
e) + 103*a^6*b^19*e^(19*e) - 6*a^5*b^20*e^(19*e))/(a^24*b^2*e^(16*e) - 20*a
^23*b^3*e^(16*e) + 190*a^22*b^4*e^(16*e) - 1140*a^21*b^5*e^(16*e) + 4845*a^
20*b^6*e^(16*e) - 15504*a^19*b^7*e^(16*e) + 38760*a^18*b^8*e^(16*e) - 77520
*a^17*b^9*e^(16*e) + 125970*a^16*b^10*e^(16*e) - 167960*a^15*b^11*e^(16*e)
 + 184756*a^14*b^12*e^(16*e) - 167960*a^13*b^13*e^(16*e) + 125970*a^12*b^14*
e^(16*e) - 77520*a^11*b^15*e^(16*e) + 38760*a^10*b^16*e^(16*e) - 15504*a^9*
b^17*e^(16*e) + 4845*a^8*b^18*e^(16*e) - 1140*a^7*b^19*e^(16*e) + 190*a^6*b
^20*e^(16*e) - 20*a^5*b^21*e^(16*e) + a^4*b^22*e^(16*e)))*e^(2*f*x) + 3*(a^
22*b^3*e^(17*e) - 14*a^21*b^4*e^(17*e) + 88*a^20*b^5*e^(17*e) - 320*a^19*b^
6*e^(17*e) + 700*a^18*b^7*e^(17*e) - 728*a^17*b^8*e^(17*e) - 728*a^16*b^9*e
^(17*e) + 4576*a^15*b^10*e^(17*e) - 10010*a^14*b^11*e^(17*e) + 14300*a^13*b
^12*e^(17*e) - 14872*a^12*b^13*e^(17*e) + 11648*a^11*b^14*e^(17*e) - 6916*a
^10*b^15*e^(17*e) + 3080*a^9*b^16*e^(17*e) - 1000*a^8*b^17*e^(17*e) + 224*a
^7*b^18*e^(17*e) - 31*a^6*b^19*e^(17*e) + 2*a^5*b^20*e^(17*e))/(a^24*b^2*e^
(16*e) - 20*a^23*b^3*e^(16*e) + 190*a^22*b^4*e^(16*e) - 1140*a^21*b^5*e^(16
*e) + 4845*a^20*b^6*e^(16*e) - 15504*a^19*b^7*e^(16*e) + 38760*a^18*b^8*e^(
16*e) - 77520*a^17*b^9*e^(16*e) + 125970*a^16*b^10*e^(16*e) - 167960*a^15*b
^11*e^(16*e) + 184756*a^14*b^12*e^(16*e) - 167960*a^13*b^13*e^(16*e) + 1259
70*a^12*b^14*e^(16*e) - 77520*a^11*b^15*e^(16*e) + 38760*a^10*b^16*e^(16*e)
 - 15504*a^9*b^17*e^(16*e) + 4845*a^8*b^18*e^(16*e) - 1140*a^7*b^19*e^(16*e
) + 190*a^6*b^20*e^(16*e) - 20*a^5*b^21*e^(16*e) + a^4*b^22*e^(16*e)))*e^(f
*x)/((b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)^(3
/2)*f) + 1/12*(15*(3*a^2*e^e + 4*a*b*e^e)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b) + sqrt(b))/sqrt(a - b))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqr
t(a - b)) - 24*(a^2*e^e + 2*a*b*e^e)*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqr
t(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(
-b))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqrt(-b)) - 2*(21*(sqrt(b
)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b))^7*a^2*e^e + 12*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a*b*e^e + 243*(
sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*
b*e^(2*f*x + 2*e) + b))^6*a^2*sqrt(b)*e^e - 12*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a
*b^(3/2)*e^e + 436*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*
e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^3*e^e + 117*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^5*a^2*b*e^e + 396*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*
e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^2*e^e - 256*(sqr
t(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e
^(2*f*x + 2*e) + b))^5*b^3*e^e + 1796*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^3*sqrt(b
)*e^e + 363*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*
x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2)*e^e - 1644*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
```

```
 2*e) + b))^4*a*b^(5/2)*e^e + 640*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*b^(7/2)*e^e +
1840*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x...
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

3.502 $\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=163

$$-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{7/2}f}+\frac{2a+3b}{6(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\operatorname{sech}^2(e+f}{2(a-b)f\left(a+b\sinh^2(\right.}$$

```
[Out] -1/2*(2*a+3*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(7/2)/f
+1/6*(2*a+3*b)/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/2*sech(f*x+e)^2/(a-b)/
f/(a+b*sinh(f*x+e)^2)^(3/2)+1/2*(2*a+3*b)/(a-b)^3/f/(a+b*sinh(f*x+e)^2)^(1/
2)
```

Rubi [A]
time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 53, 65, 214}

$$\frac{2a+3b}{2f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}+\frac{2a+3b}{6f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{7/2}}+\frac{\operatorname{sech}^2(e+fx)}{2f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/2*((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/((a - b
)^(7/2)*f) + (2*a + 3*b)/(6*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + Se
ch[e + f*x]^2/(2*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*a + 3*b)/(2*
(a - b)^3*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x)^2(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}}\,dx, x,\right.}{4(a-b)f}$$

$$= \frac{2a+3b}{6(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} +$$

$$= \frac{2a+3b}{6(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} +$$

$$= \frac{2a+3b}{6(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} +$$

$$= -\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{7/2}f} + \frac{2a+3b}{6(a-b)^2f\left(a+b\sinh^2(e\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 82, normalized size = 0.50

$$\frac{(2a+3b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sinh^2(e+fx)}{a-b}\right)+3(a-b)\text{sech}^2(e+fx)}{6(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*a + 3*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sinh[e + f*x]^2)/(a -
b)] + 3*(a - b)*Sech[e + f*x]^2)/(6*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/
2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.08, size = 213, normalized size = 1.31

method	result
default	$\frac{\text{`}int/indef0\text{`}\Bigg(-\frac{(\sinh^3(fx+e))(b^2(\sinh^4(fx+e))+2ab(\sinh^2(fx}{(-b^4(\cosh^{14}(fx+e))+(-4a\,b^3+4b^4)(\cosh^{12}(fx+e))+(-6a^2b^2+12a\,b^3-6b^4)(\cosh^{10}(fx+e))+(-4a^3b+12a^2b^2-12a\,b^3+4b^4}}{f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(-sinh(f*x+e)^3*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)*cos
h(f*x+e)^2/(-b^4*cosh(f*x+e)^14+(-4*a*b^3+4*b^4)*cosh(f*x+e)^12+(-6*a^2*b^2
+12*a*b^3-6*b^4)*cosh(f*x+e)^10+(-4*a^3*b+12*a^2*b^2-12*a*b^3+4*b^4)*cosh(f
*x+e)^8+(-a^4+4*a^3*b-6*a^2*b^2+4*a*b^3-b^4)*cosh(f*x+e)^6)/(a+b*sinh(f*x+e
)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5155 vs. 2(143) = 286.
time = 1.01, size = 10506, normalized size = 64.45

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*a*b^2 + 3*b^3)*cosh(f*x + e)^12 + 12*(2*a*b^2 + 3*b^3)*cosh(f
*x + e)*sinh(f*x + e)^11 + (2*a*b^2 + 3*b^3)*sinh(f*x + e)^12 + 2*(8*a^2*b
+ 10*a*b^2 - 3*b^3)*cosh(f*x + e)^10 + 2*(8*a^2*b + 10*a*b^2 - 3*b^3 + 33*(
2*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(2*a*b^2 + 3*b^
3)*cosh(f*x + e)^3 + (8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x +
 e)^9 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^8 + (495*(2*a*b
^2 + 3*b^3)*cosh(f*x + e)^4 + 32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3 + 90*(8*a
^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(2*a*b^2
+ 3*b^3)*cosh(f*x + e)^5 + 30*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^3
```

```
+ (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + 4*
(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 4*(231*(2*a*b^2 +
3*b^3)*cosh(f*x + e)^6 + 105*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^4 +
 16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3 + 7*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*
b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(2*a*b^2 + 3*b^3)*cosh(f*x +
e)^7 + 63*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 7*(32*a^3 + 48*a^2
*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + 3*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3
*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3
)*cosh(f*x + e)^4 + (495*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^8 + 420*(8*a^2*b +
 10*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 70*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^
3)*cosh(f*x + e)^4 + 32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3 + 60*(16*a^3 + 16*
a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(2*a*b^2
 + 3*b^3)*cosh(f*x + e)^9 + 60*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^7
 + 14*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 20*(16*a^3 +
16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + (32*a^3 + 48*a^2*b - 2*a*b^2
 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 2*a*b^2 + 3*b^3 + 2*(8*a^2*b + 1
0*a*b^2 - 3*b^3)*cosh(f*x + e)^2 + 2*(33*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^10
 + 45*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^8 + 14*(32*a^3 + 48*a^2*b
- 2*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 30*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b
^3)*cosh(f*x + e)^4 + 8*a^2*b + 10*a*b^2 - 3*b^3 + 3*(32*a^3 + 48*a^2*b - 2
*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(2*a*b^2 + 3*b^3)*c
osh(f*x + e)^11 + 5*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^9 + 2*(32*a^
3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^7 + 6*(16*a^3 + 16*a^2*b - 10
*a*b^2 + 3*b^3)*cosh(f*x + e)^5 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cos
h(f*x + e)^3 + (8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e))*s
qrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*s
inh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4
*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 +
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e
)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*co
sh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*s
inh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*si
nh(f*x + e) + 1)) - 4*sqrt(2)*(3*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^9
+ 27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)*sinh(f*x + e)^8 + 3*(2*a^2*b +
 a*b^2 - 3*b^3)*sinh(f*x + e)^9 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*co
sh(f*x + e)^7 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3 + 27*(2*a^2*b + a*b^2
 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^7 + 28*(9*(2*a^2*b + a*b^2 - 3*b^3
)*cosh(f*x + e)^3 + (8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e))*sin
h(f*x + e)^6 + 2*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 2
*(189*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 56*a^3 - 70*a^2*b + 17*a*
b^2 - 3*b^3 + 42*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh
(f*x + e)^5 + 2*(189*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 70*(8*a^3
+ 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + 5*(56*a^3 - 70*a^2*b + 17*a
*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(8*a^3 + 2*a^2*b - 13*a*b^
```

```
2 + 3*b^3)*cosh(f*x + e)^3 + 4*(63*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^
6 + 35*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + 8*a^3 + 2*a^2
*b - 13*a*b^2 + 3*b^3 + 5*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x +
 e)^2)*sinh(f*x + e)^3 + 4*(27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^7 +
21*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^5 + 5*(56*a^3 - 70*a^
2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + 3*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3
*b^3)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x
 + e) + (27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^8 + 28*(8*a^3 + 2*a^2*b
 - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 10*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3
*b^3)*cosh(f*x + e)^4 + 6*a^2*b + 3*a*b^2 - 9*b^3 + 12*(8*a^3 + 2*a^2*b - 1
3*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2)))/((a^4*b^2 - 4*a^3*b^3 +...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1981 vs. $2(143) = 286$.
time = 7.36, size = 1981, normalized size = 12.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*((3*(a^18*b^3*e^(21*e) - 12*a^17*b^4*e^(21*e) + 65*a^16*b^5*e^(21*e) -
208*a^15*b^6*e^(21*e) + 429*a^14*b^7*e^(21*e) - 572*a^13*b^8*e^(21*e) + 429
*a^12*b^9*e^(21*e) - 429*a^10*b^11*e^(21*e) + 572*a^9*b^12*e^(21*e) - 429*a
^8*b^13*e^(21*e) + 208*a^7*b^14*e^(21*e) - 65*a^6*b^15*e^(21*e) + 12*a^5*b^
16*e^(21*e) - a^4*b^17*e^(21*e))*e^(2*f*x)/(a^20*b^2*e^(16*e) - 16*a^19*b^3
*e^(16*e) + 120*a^18*b^4*e^(16*e) - 560*a^17*b^5*e^(16*e) + 1820*a^16*b^6*e
^(16*e) - 4368*a^15*b^7*e^(16*e) + 8008*a^14*b^8*e^(16*e) - 11440*a^13*b^9*
e^(16*e) + 12870*a^12*b^10*e^(16*e) - 11440*a^11*b^11*e^(16*e) + 8008*a^10*
b^12*e^(16*e) - 4368*a^9*b^13*e^(16*e) + 1820*a^8*b^14*e^(16*e) - 560*a^7*b
^15*e^(16*e) + 120*a^6*b^16*e^(16*e) - 16*a^5*b^17*e^(16*e) + a^4*b^18*e^(1
6*e)) + 2*(8*a^19*b^2*e^(19*e) - 103*a^18*b^3*e^(19*e) + 608*a^17*b^4*e^(19
```

```
*e) - 2171*a^16*b^5*e^(19*e) + 5200*a^15*b^6*e^(19*e) - 8723*a^14*b^7*e^(19
*e) + 10296*a^13*b^8*e^(19*e) - 8151*a^12*b^9*e^(19*e) + 3432*a^11*b^10*e^(
19*e) + 715*a^10*b^11*e^(19*e) - 2288*a^9*b^12*e^(19*e) + 1807*a^8*b^13*e^(
19*e) - 832*a^7*b^14*e^(19*e) + 239*a^6*b^15*e^(19*e) - 40*a^5*b^16*e^(19*e
) + 3*a^4*b^17*e^(19*e))/(a^20*b^2*e^(16*e) - 16*a^19*b^3*e^(16*e) + 120*a^
18*b^4*e^(16*e) - 560*a^17*b^5*e^(16*e) + 1820*a^16*b^6*e^(16*e) - 4368*a^1
5*b^7*e^(16*e) + 8008*a^14*b^8*e^(16*e) - 11440*a^13*b^9*e^(16*e) + 12870*a
^12*b^10*e^(16*e) - 11440*a^11*b^11*e^(16*e) + 8008*a^10*b^12*e^(16*e) - 43
68*a^9*b^13*e^(16*e) + 1820*a^8*b^14*e^(16*e) - 560*a^7*b^15*e^(16*e) + 120
*a^6*b^16*e^(16*e) - 16*a^5*b^17*e^(16*e) + a^4*b^18*e^(16*e)))*e^(2*f*x) +
 3*(a^18*b^3*e^(17*e) - 12*a^17*b^4*e^(17*e) + 65*a^16*b^5*e^(17*e) - 208*a
^15*b^6*e^(17*e) + 429*a^14*b^7*e^(17*e) - 572*a^13*b^8*e^(17*e) + 429*a^12
*b^9*e^(17*e) - 429*a^10*b^11*e^(17*e) + 572*a^9*b^12*e^(17*e) - 429*a^8*b^
13*e^(17*e) + 208*a^7*b^14*e^(17*e) - 65*a^6*b^15*e^(17*e) + 12*a^5*b^16*e^
(17*e) - a^4*b^17*e^(17*e))/(a^20*b^2*e^(16*e) - 16*a^19*b^3*e^(16*e) + 120
*a^18*b^4*e^(16*e) - 560*a^17*b^5*e^(16*e) + 1820*a^16*b^6*e^(16*e) - 4368*
a^15*b^7*e^(16*e) + 8008*a^14*b^8*e^(16*e) - 11440*a^13*b^9*e^(16*e) + 1287
0*a^12*b^10*e^(16*e) - 11440*a^11*b^11*e^(16*e) + 8008*a^10*b^12*e^(16*e) -
 4368*a^9*b^13*e^(16*e) + 1820*a^8*b^14*e^(16*e) - 560*a^7*b^15*e^(16*e) +
120*a^6*b^16*e^(16*e) - 16*a^5*b^17*e^(16*e) + a^4*b^18*e^(16*e)))*e^(f*x)/
((b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)^(3/2)*
f) + ((3*a*e^e + 2*b*e^e)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sq
rt(a - b))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a - b)) - 2*(a*e^e + b*e^e
)*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
+ 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*sqrt(-b)) - 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*e^e + 7*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))^2*a*sqrt(b)*e^e - 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(3/2)*e^e + 12*(sqrt(
b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(
2*f*x + 2*e) + b))*a^2*e^e - 17*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b*e^e + 8*(sqrt(
b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(
2*f*x + 2*e) + b))*b^2*e^e - 4*a^2*sqrt(b)*e^e + 9*a*b^(3/2)*e^e - 4*b^(5/2
)*e^e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3*b)^2))/f^2
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(e+f\,x)^3}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.503 $\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f}+\frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{1}{(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+1/3/(a-b)/f/(
a+b*sinh(f*x+e)^2)^(3/2)+1/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3273, 53, 65, 214}

$$\frac{1}{f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{1}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f)) + 1/(
3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + 1/((a - b)^2*f*Sqrt[a + b*Sinh
[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \sinh^2(e\right.}{2(a-b)f}$$

$$= \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + $$

$$= \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + $$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.10, size = 60, normalized size = 0.61

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\frac{b\cosh^2(e+fx)}{a-b}\right)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Cosh[e + f*x]^2)/(a - b)]/(3*(a - b
)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.54, size = 173, normalized size = 1.75

method	result
default	$\frac{\text{`}\mathit{int/indef0}\text{`}\left(-\frac{\sinh(fx+e)\left(b^2\left(\sinh^4(fx+e)\right)+2ab\left(\sinh^2(fx+e)\right)+a^2\right)}{\left(-b^4\left(\sinh^{10}(fx+e)\right)+\left(-4a\,b^3-b^4\right)\left(\sinh^8(fx+e)\right)+\left(-6a^2b^2-4a\,b^3\right)\left(\sinh^6(fx+e)\right)+\left(-4a^3b-6a^2b^2\right)\left(\sinh^4(fx+e)\right)+\left(-a^4-4\right.\right.}\right.}{f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] `int/indef0`(-sinh(f*x+e)*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/(-b^4
*sinh(f*x+e)^10+(-4*a*b^3-b^4)*sinh(f*x+e)^8+(-6*a^2*b^2-4*a*b^3)*sinh(f*x+
e)^6+(-4*a^3*b-6*a^2*b^2)*sinh(f*x+e)^4+(-a^4-4*a^3*b)*sinh(f*x+e)^2-a^4)/(
a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2002 vs. 2(87) = 174.
time = 0.64, size = 4200, normalized size = 42.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cosh(f*x + e)^8 + 8*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*si
nh(f*x + e)^8 + 4*(2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2
```

```
+ 2*a*b - b^2)*sinh(f*x + e)^6 + 8*(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)
*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4
 + 2*(35*b^2*cosh(f*x + e)^4 + 30*(2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8
*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)
*cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 +
 4*(2*a*b - b^2)*cosh(f*x + e)^2 + 4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b
^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b
^2)*sinh(f*x + e)^2 + b^2 + 8*(b^2*cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f
*x + e)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*
x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x +
e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*
(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a - b)*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x +
 e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)
/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(
3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x +
e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*(3*(a*b - b^2)*cosh(f
*x + e)^5 + 15*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^4 + 3*(a*b - b^2)*si
nh(f*x + e)^5 + 2*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^3 + 2*(15*(a*b - b
^2)*cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*sinh(f*x + e)^3 + 6*(5*(a*b -
 b^2)*cosh(f*x + e)^3 + (8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e))*sinh(f*x +
e)^2 + 3*(a*b - b^2)*cosh(f*x + e) + 3*(5*(a*b - b^2)*cosh(f*x + e)^4 + 2*(
8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^2 + a*b - b^2)*sinh(f*x + e))*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4
- b^5)*f*cosh(f*x + e)^8 + 8*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f
*x + e)*sinh(f*x + e)^7 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*sinh(f*x
+ e)^8 + 4*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e
)^6 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^2 + (2*a^4
*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*sinh(f*x + e)^6 + 2*(8*a^5 -
 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)^4 +
 8*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^3 + 3*(2*a^4*b
- 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 +
 2*(35*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^4 + 30*(2*a^4*
b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^2 + (8*a^5 - 32*
a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f)*sinh(f*x + e)^4 + 4*
(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^2 + 8*(7*
(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^5 + 10*(2*a^4*b - 7*a
^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (8*a^5 - 32*a^4*b +
 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e))*sinh(f*x + e)
^3 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^6 + 15*(2*a
^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^4 + 3*(8*a^5
- 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)^2
+ (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*sinh(f*x + e)^2 + (a
```

```
^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f + 8*((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 -
 b^5)*f*cosh(f*x + e)^7 + 3*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^
5)*f*cosh(f*x + e)^5 + (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b
^4 - 3*b^5)*f*cosh(f*x + e)^3 + (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4
+ b^5)*f*cosh(f*x + e))*sinh(f*x + e)), -1/3*(3*(b^2*cosh(f*x + e)^8 + 8*b^
2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*sinh(f*x + e)^8 + 4*(2*a*b - b^2)*cos
h(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^6 + 8*
(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2
*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*b^2*cosh(f*x + e)^4 + 30*(
2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(
7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)*cosh(f*x + e)^3 + (8*a^2 - 8*a*b +
 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a*b - b^2)*cosh(f*x + e)^2 +
4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 8*
a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 8*(b^2*
cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f*x + e)^5 + (8*a^2 - 8*a*b + 3*b^2)
*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)
*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*(3*(a*
b - b^2)*cosh(f*x + e)^5 + 15*(a*b - b^2)*cosh(...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(e + fx\right)}{\left(a + b\sinh^{2}\left(e + fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tanh(e + f*x)/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 1.19Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(e+f\,x)}{{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.504 $\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3/a/f/(a+b*sinh(f*x
+e)^2)^(3/2)+1/a^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3273, 53, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
 b*Sinh[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}\\ &= \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2af}\\ &= \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}} +\\ &= \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}} +\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} +\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 49, normalized size = 0.59

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\frac{b\sinh^2(e+fx)}{a}\right)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sinh[e + f*x]^2)/a]/(3*a*f*(a + b*S
inh[e + f*x]^2)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.53, size = 65, normalized size = 0.78

method	result	size
default	$\frac{\text{‘}\mathit{int/indef0}\text{‘}\left(\frac{1}{\left(b^{2}\left(\sinh^{4}(fx+e)\right)+2ab\left(\sinh^{2}(fx+e)\right)+a^{2}\right)\sinh(fx+e)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	65
risch	Expression too large to display	47171

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(1/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/sinh(f*x+e)/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1441 vs. $2(71) = 142$.
time = 0.50, size = 3084, normalized size = 37.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cosh(f*x + e)^8 + 8*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*si
nh(f*x + e)^8 + 4*(2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2
+ 2*a*b - b^2)*sinh(f*x + e)^6 + 8*(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)
*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4
 + 2*(35*b^2*cosh(f*x + e)^4 + 30*(2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8
*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)
```

```
*cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 +
 4*(2*a*b - b^2)*cosh(f*x + e)^2 + 4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b
^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b
^2)*sinh(f*x + e)^2 + b^2 + 8*(b^2*cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f
*x + e)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*
x + e))*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*c
osh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(
f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*
cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e
)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e
)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*
x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*(3*a*b*cosh(f*x + e)^5 + 15*a*b*cos
h(f*x + e)*sinh(f*x + e)^4 + 3*a*b*sinh(f*x + e)^5 + 2*(8*a^2 - 3*a*b)*cosh
(f*x + e)^3 + 2*(15*a*b*cosh(f*x + e)^2 + 8*a^2 - 3*a*b)*sinh(f*x + e)^3 +
3*a*b*cosh(f*x + e) + 6*(5*a*b*cosh(f*x + e)^3 + (8*a^2 - 3*a*b)*cosh(f*x +
 e))*sinh(f*x + e)^2 + 3*(5*a*b*cosh(f*x + e)^4 + 2*(8*a^2 - 3*a*b)*cosh(f*
x + e)^2 + a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/(a^3*b^2*f*cosh(f*x + e)^8 + 8*a^3*b^2*f*cosh(f*x + e)*sinh(f*x + e)^
7 + a^3*b^2*f*sinh(f*x + e)^8 + 4*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^6 + 4
*(7*a^3*b^2*f*cosh(f*x + e)^2 + (2*a^4*b - a^3*b^2)*f)*sinh(f*x + e)^6 + a^
3*b^2*f + 2*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^4 + 8*(7*a^3*b^2*
f*cosh(f*x + e)^3 + 3*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5
+ 2*(35*a^3*b^2*f*cosh(f*x + e)^4 + 30*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^
2 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f)*sinh(f*x + e)^4 + 4*(2*a^4*b - a^3*b^2
)*f*cosh(f*x + e)^2 + 8*(7*a^3*b^2*f*cosh(f*x + e)^5 + 10*(2*a^4*b - a^3*b^
2)*f*cosh(f*x + e)^3 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e))*sinh(
f*x + e)^3 + 4*(7*a^3*b^2*f*cosh(f*x + e)^6 + 15*(2*a^4*b - a^3*b^2)*f*cosh
(f*x + e)^4 + 3*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^2 + (2*a^4*b
- a^3*b^2)*f)*sinh(f*x + e)^2 + 8*(a^3*b^2*f*cosh(f*x + e)^7 + 3*(2*a^4*b -
 a^3*b^2)*f*cosh(f*x + e)^5 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)
^3 + (2*a^4*b - a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)), 1/3*(3*(b^2*cosh(
f*x + e)^8 + 8*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*sinh(f*x + e)^8 + 4*
(2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2 + 2*a*b - b^2)*sin
h(f*x + e)^6 + 8*(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f*x + e))*si
nh(f*x + e)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*b^2*cosh(
f*x + e)^4 + 30*(2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh
(f*x + e)^4 + 8*(7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)*cosh(f*x + e)^3 +
 (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a*b - b^2)*c
osh(f*x + e)^2 + 4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b^2)*cosh(f*x + e)^
4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^
2 + b^2 + 8*(b^2*cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f*x + e)^5 + (8*a^2
 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
```

```
 e))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*sqrt(2)*(3*a*b*cos
h(f*x + e)^5 + 15*a*b*cosh(f*x + e)*sinh(f*x + e)^4 + 3*a*b*sinh(f*x + e)^5
 + 2*(8*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(15*a*b*cosh(f*x + e)^2 + 8*a^2 -
3*a*b)*sinh(f*x + e)^3 + 3*a*b*cosh(f*x + e) + 6*(5*a*b*cosh(f*x + e)^3 + (
8*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(5*a*b*cosh(f*x + e)^4 +
2*(8*a^2 - 3*a*b)*cosh(f*x + e)^2 + a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2)))/(a^3*b^2*f*cosh(f*x + e)^8 + 8*a^3*b^2*f*cos
h(f*x + e)*sinh(f*x + e)^7 + a^3*b^2*f*sinh(f*x + e)^8 + 4*(2*a^4*b - a^3*b
^2)*f*cosh(f*x + e)^6 + 4*(7*a^3*b^2*f*cosh(f*x + e)^2 + (2*a^4*b - a^3*b^2
)*f)*sinh(f*x + e)^6 + a^3*b^2*f + 2*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f
*x + e)^4 + 8*(7*a^3*b^2*f*cosh(f*x + e)^3 + 3*(2*a^4*b - a^3*b^2)*f*cosh(f
*x + e))*sinh(f*x + e)^5 + 2*(35*a^3*b^2*f*cosh(f*x + e)^4 + 30*(2*a^4*b -
a^3*b^2)*f*cosh(f*x + e)^2 + (8*a^5 - 8*a^4*b +...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(e+f\,x)}{{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.505 $\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}+\frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\operatorname{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3}}$$

```
[Out] -1/2*(2*a-5*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2)/f+1/6*(2*
a-5*b)/a^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/2*csch(f*x+e)^2/a/f/(a+b*sinh(f*x+
e)^2)^(3/2)+1/2*(2*a-5*b)/a^3/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 53, 65, 214}

$$-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}+\frac{2a-5b}{2a^3f\sqrt{a+b\sinh^2(e+fx)}}+\frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\operatorname{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/2*((2*a - 5*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(a^(7/2)*f)
 + (2*a - 5*b)/(6*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - Csch[e + f*x]^2/(2
*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*a - 5*b)/(2*a^3*f*Sqrt[a + b*Sinh[
e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx=\frac{\text{Subst}\left(\int \frac{1+x}{x^2(a+bx)^{5/2}}\,dx,x,\sinh^2(e+fx)\right)}{2f}$$

$$=-\frac{\text{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{(2a-5b)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx,x,\sinh^2(\right.}{4af}$$

$$=\frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\text{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{(2a-5b)}{}$$

$$=\frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\text{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{}{2a^3f\sqrt{a}}$$

$$=\frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\text{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{}{2a^3f\sqrt{a}}$$

$$=-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}+\frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+f\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.21, size = 69, normalized size = 0.48

$$-\frac{3a\text{csch}^2(e+fx)+(-2a+5b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\frac{b\sinh^2(e+fx)}{a}\right)}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*(3*a*Csch[e + f*x]^2 + (-2*a + 5*b)*Hypergeometric2F1[-3/2, 1, -1/2, 1
 + (b*Sinh[e + f*x]^2)/a])/(a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.14, size = 73, normalized size = 0.51

method	result	size

default	$\frac{\text{`}int/indef0\text{`}\left(\frac{\cosh^2(fx+e)}{\left(b^2\left(\sinh^4(fx+e)\right)+2ab\left(\sinh^2(fx+e)\right)+a^2\right)\sinh(fx+e)^3\sqrt{a+b\left(\sinh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$	73
risch	Expression too large to display	309511

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 'int/indef0'(cosh(f*x+e)^2/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/sinh
(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3696 vs. $2(123) = 246$.
time = 0.66, size = 7594, normalized size = 53.10

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*a*b^2 - 5*b^3)*cosh(f*x + e)^12 + 12*(2*a*b^2 - 5*b^3)*cosh(f
*x + e)*sinh(f*x + e)^11 + (2*a*b^2 - 5*b^3)*sinh(f*x + e)^12 + 2*(8*a^2*b
- 26*a*b^2 + 15*b^3)*cosh(f*x + e)^10 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3 + 33
*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(2*a*b^2 - 5*
b^3)*cosh(f*x + e)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e))*sinh(f*
x + e)^9 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^8 + (495
*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^4 + 32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^
3 + 90*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(
99*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^5 + 30*(8*a^2*b - 26*a*b^2 + 15*b^3)*cos
h(f*x + e)^3 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e))*sin
h(f*x + e)^7 - 4*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e)^6 +
4*(231*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^6 + 105*(8*a^2*b - 26*a*b^2 + 15*b^3
)*cosh(f*x + e)^4 - 16*a^3 + 64*a^2*b - 70*a*b^2 + 25*b^3 + 7*(32*a^3 - 144
```

```
*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(2*a*
b^2 - 5*b^3)*cosh(f*x + e)^7 + 63*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x +
e)^5 + 7*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^3 - 3*(16*
a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (32*a^
3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^4 + (495*(2*a*b^2 - 5*b^3
)*cosh(f*x + e)^8 + 420*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^6 + 70*
(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^4 + 32*a^3 - 144*a^
2*b + 190*a*b^2 - 75*b^3 - 60*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(
f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^9 + 60*
(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^7 + 14*(32*a^3 - 144*a^2*b + 19
0*a*b^2 - 75*b^3)*cosh(f*x + e)^5 - 20*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b
^3)*cosh(f*x + e)^3 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x +
e))*sinh(f*x + e)^3 + 2*a*b^2 - 5*b^3 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3)*cos
h(f*x + e)^2 + 2*(33*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^10 + 45*(8*a^2*b - 26*
a*b^2 + 15*b^3)*cosh(f*x + e)^8 + 14*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b
^3)*cosh(f*x + e)^6 - 30*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x +
 e)^4 + 8*a^2*b - 26*a*b^2 + 15*b^3 + 3*(32*a^3 - 144*a^2*b + 190*a*b^2 - 7
5*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(2*a*b^2 - 5*b^3)*cosh(f*x +
 e)^11 + 5*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^9 + 2*(32*a^3 - 144*
a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^7 - 6*(16*a^3 - 64*a^2*b + 70*a*b
^2 - 25*b^3)*cosh(f*x + e)^5 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*co
sh(f*x + e)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e))*sinh(f*x + e))
*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sin
h(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a -
 b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*
x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a
 - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*
sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)
^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e)
+ 1)) - 4*sqrt(2)*(3*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^9 + 27*(2*a^2*b - 5*
a*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + 3*(2*a^2*b - 5*a*b^2)*sinh(f*x + e)^
9 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^7 + 4*(8*a^3 - 26*a^2*b +
 15*a*b^2 + 27*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^7 + 28*(9
*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^3 + (8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f
*x + e))*sinh(f*x + e)^6 - 2*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e)^5
 + 2*(189*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^4 - 56*a^3 + 98*a^2*b - 45*a*b^
2 + 42*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(
189*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^5 + 70*(8*a^3 - 26*a^2*b + 15*a*b^2)*
cosh(f*x + e)^3 - 5*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e))*sinh(f*x
+ e)^4 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 + 4*(63*(2*a^2*b -
 5*a*b^2)*cosh(f*x + e)^6 + 35*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^
4 + 8*a^3 - 26*a^2*b + 15*a*b^2 - 5*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x
 + e)^2)*sinh(f*x + e)^3 + 4*(27*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^7 + 21*(
8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^5 - 5*(56*a^3 - 98*a^2*b + 45*a*
```

```
b^2)*cosh(f*x + e)^3 + 3*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e))*sinh(
f*x + e)^2 + 3*(2*a^2*b - 5*a*b^2)*cosh(f*x + e) + (27*(2*a^2*b - 5*a*b^2)*
cosh(f*x + e)^8 + 28*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^6 - 10*(56
*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e)^4 + 6*a^2*b - 15*a*b^2 + 12*(8*a^
3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/(a^4*b^2*f*cosh(f*x + e)^12 + 12*a^4*b^2*f*
cosh(f*x + e)*sinh(f*x + e)^11 + a^4*b^2*f*sinh(f*x + e)^12 + 2*(4*a^5*b -
3*a^4*b^2)*f*cosh(f*x + e)^10 + 2*(33*a^4*b^2*f...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 2.1Error: Bad Argume
nt Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{coth}(e+f\,x)^3}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.506 $\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=208

$$-\frac{(8a^2-40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(8a-7b)\mathrm{csc}}{8a^2f\left(a+b\,\mathrm{sin}\right.}$$

```
[Out] -1/8*(8*a^2-40*a*b+35*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(9/
2)/f+1/24*(8*a^2-40*a*b+35*b^2)/a^3/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/8*(8*a-7*
b)*csch(f*x+e)^2/a^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/4*csch(f*x+e)^4/a/f/(a+b
*sinh(f*x+e)^2)^(3/2)+1/8*(8*a^2-40*a*b+35*b^2)/a^4/f/(a+b*sinh(f*x+e)^2)^(
1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 53, 65, 214}

$$-\frac{(8a-7b)\operatorname{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(8a^2-40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{8a^2-40ab+35b^2}{8a^4f\sqrt{a+b\sinh^2(e+fx)}}+\frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\operatorname{csch}^4(e+fx)}{4af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/8*((8*a^2 - 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]
])/(a^(9/2)*f) + (8*a^2 - 40*a*b + 35*b^2)/(24*a^3*f*(a + b*Sinh[e + f*x]^2
)^(3/2)) - ((8*a - 7*b)*Csch[e + f*x]^2)/(8*a^2*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - Csch[e + f*x]^4/(4*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (8*a^2 - 40
*a*b + 35*b^2)/(8*a^4*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{csch}^4(e+fx)}{4af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\Big(\int \frac{\frac{1}{2}(8a-7b)+2ax}{x^2(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx}{4af}$$

$$= -\frac{(8a-7b)\operatorname{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\operatorname{csch}^4(e+fx)}{4af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(8a^2 -}{}$$

$$= \frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(8a-7b)\operatorname{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{}{4af\,(a}$$

$$= \frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(8a-7b)\operatorname{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{}{4af\,(a}$$

$$= \frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(8a-7b)\operatorname{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{}{4af\,(a}$$

$$= -\frac{(8a^2-40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f} + \frac{8a^2-4}{24a^3f\,(a+b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.30, size = 117, normalized size = 0.56

$$-\frac{\operatorname{csch}^2(e+fx)\left(3a\operatorname{csch}^2(e+fx)\left(8a-7b+2a\operatorname{csch}^2(e+fx)\right)+\left(-8a^2+40ab-35b^2\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\frac{b\sinh^2(e+fx)}{a}\right)\right)}{24a^3f\left(b+a\operatorname{csch}^2(e+fx)\right)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/24*(Csch[e + f*x]^2*(3*a*Csch[e + f*x]^2*(8*a - 7*b + 2*a*Csch[e + f*x]^
2) + (-8*a^2 + 40*a*b - 35*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sin
h[e + f*x]^2)/a]))/(a^3*f*(b + a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^
2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.03, size = 73, normalized size = 0.35

method	result	size
default	$\frac{\text{‘}\mathit{int/indef0}\text{‘}\left(\frac{\cosh^{4}(fx+e)}{\left(b^{2}\left(\sinh^{4}(fx+e)\right)+2ab\left(\sinh^{2}(fx+e)\right)+a^{2}\right)\sinh(fx+e)^{5}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh(fx+e)\right)}{f}$	73
risch	Expression too large to display	2628058

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ‘int/indef0‘(cosh(f*x+e)^4/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/sinh
(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7450 vs. 2(184) = 368.
time = 1.03, size = 15102, normalized size = 72.61

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2*b^2 - 40*a*b^3 + 35*b^4)*cosh(f*x + e)^16 + 16*(8*a^2*b^2
- 40*a*b^3 + 35*b^4)*cosh(f*x + e)*sinh(f*x + e)^15 + (8*a^2*b^2 - 40*a*b^3
 + 35*b^4)*sinh(f*x + e)^16 + 8*(8*a^3*b - 48*a^2*b^2 + 75*a*b^3 - 35*b^4)*
cosh(f*x + e)^14 + 8*(8*a^3*b - 48*a^2*b^2 + 75*a*b^3 - 35*b^4 + 15*(8*a^2*
b^2 - 40*a*b^3 + 35*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^14 + 112*(5*(8*a^2*
b^2 - 40*a*b^3 + 35*b^4)*cosh(f*x + e)^3 + (8*a^3*b - 48*a^2*b^2 + 75*a*b^3
 - 35*b^4)*cosh(f*x + e))*sinh(f*x + e)^13 + 4*(32*a^4 - 256*a^3*b + 676*a^
2*b^2 - 700*a*b^3 + 245*b^4)*cosh(f*x + e)^12 + 4*(455*(8*a^2*b^2 - 40*a*b^
3 + 35*b^4)*cosh(f*x + e)^4 + 32*a^4 - 256*a^3*b + 676*a^2*b^2 - 700*a*b^3
+ 245*b^4 + 182*(8*a^3*b - 48*a^2*b^2 + 75*a*b^3 - 35*b^4)*cosh(f*x + e)^2)
```

```
*sinh(f*x + e)^12 + 16*(273*(8*a^2*b^2 - 40*a*b^3 + 35*b^4)*cosh(f*x + e)^5
 + 182*(8*a^3*b - 48*a^2*b^2 + 75*a*b^3 - 35*b^4)*cosh(f*x + e)^3 + 3*(32*a
^4 - 256*a^3*b + 676*a^2*b^2 - 700*a*b^3 + 245*b^4)*cosh(f*x + e))*sinh(f*x
 + e)^11 - 8*(64*a^4 - 44 ...
```

Sympy **[F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 2.99Error: Bad Argum
ent Type
```

Mupad **[F(-1)]**
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] \text{Hanged}
```

3.507 $\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=333

$$-\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{8\sqrt{a}\,\sqrt{b}\,(a+b)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3(a-b)^4f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\,\sqrt{a+b\sinh^2(e+fx)}}-$$

```
[Out] -1/3*b*(5*a+3*b)*cosh(f*x+e)*sinh(f*x+e)/(a-b)^3/f/(a+b*sinh(f*x+e)^2)^(3/2
)-8/3*(a+b)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/
a)^(1/2)*EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),
(1-a/b)^(1/2))*a^(1/2)*b^(1/2)/(a-b)^4/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^
2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(3*a+b)*(a+3*b)*(1/(1+sinh(f*x+e)^2
))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1
/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^4/f/(sech(
f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/3*(2*a+b)*tanh(f*x+e)/(a-b)^2/f/(a+
b*sinh(f*x+e)^2)^(3/2)+1/3*sech(f*x+e)^2*tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+
e)^2)^(3/2)
```

Rubi [A]
time = 0.29, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3275, 481, 541, 539, 429, 422}

$$-\frac{8\sqrt{a}\,\sqrt{b}\,(a+b)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3f(a-b)^4\sqrt{a+b\sinh^2(e+fx)}\,\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{(3a+b)(a+3b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{3af(a-b)^4\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(2a+b)\tanh(e+fx)}{3f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{b(5a+3b)\sinh(e+fx)\cosh(e+fx)}{3f(a-b)^3\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\tanh(e+fx)\text{sech}^2(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*(b*(5*a + 3*b)*Cosh[e + f*x]*Sinh[e + f*x])/((a - b)^3*f*(a + b*Sinh[e
 + f*x]^2)^(3/2)) - (8*Sqrt[a]*Sqrt[b]*(a + b)*Cosh[e + f*x]*EllipticE[ArcT
an[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^4*f*Sqrt[(a*Cosh[
e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a +
 b)*(a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*a*(a - b)^4*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[
e + f*x]^2))/a]) - (2*(2*a + b)*Tanh[e + f*x])/(3*(a - b)^2*f*(a + b*Sinh[e
 + f*x]^2)^(3/2)) + (Sech[e + f*x]^2*Tanh[e + f*x])/(3*(a - b)*f*(a + b*Sin
h[e + f*x]^2)^(3/2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))]]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^4}{(1+x^2)^{5/2}(a+bx^2)^{5/2}}\,dx, x, \sinh(e+\right.}{f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Su}}{3(}$$

$$= -\frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)}$$

$$= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)}$$

$$= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{8\sqrt{a}\sqrt{b}\,(a+b)\cosh(e+fx)E\left(\right.}{3(a-b)^4f\sqrt{\frac{a\cosh^2(e+f}{a+b\sinh^2(e+}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.78, size = 252, normalized size = 0.76

$$-\frac{i\left(2ab\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}\left(8a(a+b)E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\left(-5a^2+2ab+3b^2\right)F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)\right)-i\sqrt{2}\,b(2a(a-b)b\sinh(2(e+fx))+4b(a+b)(2a-b+b\cosh(2(e+fx)))\sinh(2(e+fx))+4(a+b)(2a-b+b\cosh(2(e+fx)))^2\tanh(e+fx)-(a-b)(2a-b+b\cosh(2(e+fx)))^2\operatorname{sech}^2(e+fx)\tanh(e+fx))\right)}{6(a-b)^4bf(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-1/6*I)*(2*a*b*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*(8*a*(a + b)*Ell
ipticE[I*(e + f*x), b/a] + (-5*a^2 + 2*a*b + 3*b^2)*EllipticF[I*(e + f*x),
b/a]) - I*Sqrt[2]*b*(2*a*(a - b)*b*Sinh[2*(e + f*x)] + 4*b*(a + b)*(2*a - b
 + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)] + 4*(a + b)*(2*a - b + b*Cosh[2*(
e + f*x)])^2*Tanh[e + f*x] - (a - b)*(2*a - b + b*Cosh[2*(e + f*x)])^2*Sech
[e + f*x]^2*Tanh[e + f*x])))/((a - b)^4*b*f*(2*a - b + b*Cosh[2*(e + f*x)])
^(3/2))
```

Maple [A]
time = 2.66, size = 663, normalized size = 1.99

method	result
default	$-\frac{\left(8\sqrt{-\frac{b}{a}}\,a\,b^{2}+8\sqrt{-\frac{b}{a}}\,b^{3}\right)\left(\cosh^{6}(fx+e)\right)\sinh(fx+e)+\left(13\sqrt{-\frac{b}{a}}\,a^{2}b-2\sqrt{-\frac{b}{a}}\,a\,b^{2}-11\sqrt{-\frac{b}{a}}\,b^{3}\right)\left(\cosh^{4}(fx+e)\right)\mathrm{s}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((8*(-1/a*b)^(1/2)*a*b^2+8*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^6*sinh(f*x+
e)+(13*(-1/a*b)^(1/2)*a^2*b-2*(-1/a*b)^(1/2)*a*b^2-11*(-1/a*b)^(1/2)*b^3)*c
osh(f*x+e)^4*sinh(f*x+e)-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^
(1/2)*b*(3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+2*Elliptic
F(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-5*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*b^2+8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*b+8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e
)^4+(4*(-1/a*b)^(1/2)*a^3-6*(-1/a*b)^(1/2)*a^2*b+2*(-1/a*b)^(1/2)*b^3)*cosh
(f*x+e)^2*sinh(f*x+e)-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/
2)*(3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3-EllipticF(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-7*EllipticF(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))*a*b^2+5*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))
*b^3+8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-8*EllipticE(
sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)*cosh(f*x+e)^2+(-(-1/a*b)^(1/2)
*a^3+3*(-1/a*b)^(1/2)*a^2*b-3*(-1/a*b)^(1/2)*a*b^2+(-1/a*b)^(1/2)*b^3)*sinh
(f*x+e))/(a+b*sinh(f*x+e)^2)^(3/2)/(-1/a*b)^(1/2)/cosh(f*x+e)^3/(a-b)^4/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 15718 vs. $2(333) = 666$.
time = 0.55, size = 15718, normalized size = 47.20

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(4*((2*a^2*b^3 + a*b^4 - b^5)*cosh(f*x + e)^14 + 14*(2*a^2*b^3 + a*b^4
- b^5)*cosh(f*x + e)*sinh(f*x + e)^13 + (2*a^2*b^3 + a*b^4 - b^5)*sinh(f*x
+ e)^14 + (16*a^3*b^2 + 6*a^2*b^3 - 9*a*b^4 + b^5)*cosh(f*x + e)^12 + (16*a
^3*b^2 + 6*a^2*b^3 - 9*a*b^4 + b^5 + 91*(2*a^2*b^3 + a*b^4 - b^5)*cosh(f*x
+ e)^2)*sinh(f*x + e)^12 + 4*(91*(2*a^2*b^3 + a*b^4 - b^5)*cosh(f*x + e)^3
+ 3*(16*a^3*b^2 + 6*a^2*b^3 - 9*a*b^4 + b^5)*cosh(f*x + e))*sinh(f*x + e)^1
1 + (32*a^4*b + 32*a^3*b^2 - 14*a^2*b^3 - 11*a*b^4 + 3*b^5)*cosh(f*x + e)^1
0 + (32*a^4*b + 32*a^3*b^2 - 14*a^2*b^3 - 11*a*b^4 + 3*b^5 + 1001*(2*a^2*b^
3 + a*b^4 - b^5)*cosh(f*x + e)^4 + 66*(16*a^3*b^2 + 6*a^2*b^3 - 9*a*b^4 + b
^5)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 2*(1001*(2*a^2*b^3 + a*b^4 - b^5)*c
osh(f*x + e)^5 + 110*(16*a^3*b^2 + 6*a^2*b^3 - 9*a*b^4 + b^5)*cosh(f*x + e)
^3 + 5*(32*a^4*b + 32*a^3*b^2 - 14*a^2*b^3 - 11*a*b^4 + 3*b^5)*cosh(f*x + e
))*sinh(f*x + e)^9 + (96*a^4*b + 16*a^3*b^2 - 58*a^2*b^3 + 19*a*b^4 - 3*b^5
)*cosh(f*x + e)^8 + (3003 ...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tanh(e + f*x)**4/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 2.65Error: Bad Argum
ent Type
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(e+f\,x)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.508 $\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{4b\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\sqrt{b}\,(7a+b)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3\sqrt{a}\,(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\,\sqrt{a+b\sinh^2(e+fx)}}+\frac{(3a}{}$$

```
[Out] -4/3*b*cosh(f*x+e)*sinh(f*x+e)/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/3*(7*a
+b)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/2)
*EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^
(1/2))*b^(1/2)/(a-b)^3/f/a^(1/2)/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2
)/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(3*a+5*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+si
nh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1
/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^3/f/(sech(f*x+e)^2*(a+b*
sinh(f*x+e)^2)/a)^(1/2)-tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.20, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3275, 482, 541, 539, 429, 422}

$$-\frac{\sqrt{b}\,(7a+b)\cosh(e+fx)E\left(\text{ArcTan}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3\sqrt{a}\,f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\,\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{(3a+5b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\,|1-\frac{b}{a}\right)}{3af(a-b)^3\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\tanh(e+fx)}{f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{4b\sinh(e+fx)\cosh(e+fx)}{3f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (-4*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (Sqrt[b]*(7*a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e +
 f*x])/Sqrt[a]], 1 - a/b])/(3*Sqrt[a]*(a - b)^3*f*Sqrt[(a*Cosh[e + f*x]^2)/
(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + 5*b)*Ellipt
icF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^
2])/(3*a*(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - T
anh[e + f*x]/((a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\Big(\int \frac{x^2}{(1+x^2)^{3/2}(a+bx^2)^{5/2}}\,dx, x, \sinh(e}{f}$$

$$= -\frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right) \mathrm{S}}{}$$

$$= -\frac{4b\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$= -\frac{4b\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$= -\frac{4b\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b}\,(7a+b)\cosh(e+fx)E\Big(\tan}{3\sqrt{a}\,(a-b)^3f\sqrt{\frac{a\cosh^2(e+f}{a+b\sinh^2(e+}}}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 2.14, size = 215, normalized size = 0.78

$$\frac{-2ia^2(7a+b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+8ia^2(a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-\frac{(24a^3-4a^2b+5ab^2-b^3+4a(11a-3b)b\cosh(2(e+fx))+b^2(7a+b)\cosh(4(e+fx)))\tanh(e+fx)}{\sqrt{2}}}{6a(a-b)^3f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-2*I)*a^2*(7*a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] + (8*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3
/2)*EllipticF[I*(e + f*x), b/a] - ((24*a^3 - 4*a^2*b + 5*a*b^2 - b^3 + 4*a*
(11*a - 3*b)*b*Cosh[2*(e + f*x)] + b^2*(7*a + b)*Cosh[4*(e + f*x)])*Tanh[e
+ f*x])/Sqrt[2])/(6*a*(a - b)^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

<u>**Maple**</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(342) = 684$.
time = 2.19, size = 799, normalized size = 2.92

method	result

default	$\frac{-7\sqrt{-\frac{b}{a}}\, a\, b^2 (\sinh^5(fx+e))-\sqrt{-\frac{b}{a}}\, b^3 (\sinh^5(fx+e))+3\sqrt{\frac{a+b(\sinh^2(fx+e))}{a}}\, \sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\, \mathrm{EllipticF}\Bigg(\sinh(fx+}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-7*(-1/a*b)^(1/2)*a*b^2*sinh(f*x+e)^5-(-1/a*b)^(1/2)*b^3*sinh(f*x+e)^5
+3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b*sinh(f*x+e)^2-2*((a+b*sinh(f*x+e)^2)/a)
^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*b^2*sinh(f*x+e)^2-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)
*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3*sinh(f*x+e)^2+7*((a+
b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a*b^2*sinh(f*x+e)^2+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(c
osh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3*s
inh(f*x+e)^2-11*(-1/a*b)^(1/2)*a^2*b*sinh(f*x+e)^3-4*(-1/a*b)^(1/2)*a*b^2*s
inh(f*x+e)^3-(-1/a*b)^(1/2)*b^3*sinh(f*x+e)^3+3*((a+b*sinh(f*x+e)^2)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+7*(
(a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2-3*(-1/a*b)
^(1/2)*a^3*sinh(f*x+e)-5*sinh(f*x+e)*b*a^2*(-1/a*b)^(1/2))/(-1/a*b)^(1/2)/(
a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^3/a/cosh(f*x+e)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8226 vs. 2(280) = 560.
time = 0.29, size = 8226, normalized size = 30.02

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(((14*a^2*b^3 - 5*a*b^4 - b^5)*cosh(f*x + e)^10 + 10*(14*a^2*b^3 - 5*a*
b^4 - b^5)*cosh(f*x + e)*sinh(f*x + e)^9 + (14*a^2*b^3 - 5*a*b^4 - b^5)*sin
h(f*x + e)^10 + (112*a^3*b^2 - 82*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(f*x + e)^
8 + (112*a^3*b^2 - 82*a^2*b^3 + 7*a*b^4 + 3*b^5 + 45*(14*a^2*b^3 - 5*a*b^4
- b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(14*a^2*b^3 - 5*a*b^4 - b^5
)*cosh(f*x + e)^3 + (112*a^3*b^2 - 82*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(f*x +
 e))*sinh(f*x + e)^7 + 2*(112*a^4*b - 96*a^3*b^2 + 26*a^2*b^3 - a*b^4 - b^5
)*cosh(f*x + e)^6 + 2*(112*a^4*b - 96*a^3*b^2 + 26*a^2*b^3 - a*b^4 - b^5 +
105*(14*a^2*b^3 - 5*a*b^4 - b^5)*cosh(f*x + e)^4 + 14*(112*a^3*b^2 - 82*a^2
*b^3 + 7*a*b^4 + 3*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(14*a^2*b^
3 - 5*a*b^4 - b^5)*cosh(f*x + e)^5 + 14*(112*a^3*b^2 - 82*a^2*b^3 + 7*a*b^4
 + 3*b^5)*cosh(f*x + e)^3 + 3*(112*a^4*b - 96*a^3*b^2 + 26*a^2*b^3 - a*b^4
- b^5)*cosh(f*x + e))*sinh(f*x + e)^5 + 14*a^2*b^3 - 5*a*b^4 - b^5 + 2*(112
*a^4*b - 96*a^3*b^2 + 26*a^2*b^3 - a*b^4 - b^5)*cosh(f*x + e)^4 + 2*(105*(1
4*a^2*b^3 - 5*a*b^4 - b^5)*cosh(f*x + e)^6 + 112*a^4*b - 96*a^3*b^2 + 26*a^
2*b^3 - a*b^4 - b^5 + 35*(112*a^3*b^2 - 82*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(
f*x + e)^4 + 15*(112*a^4*b - 96*a^3*b^2 + 26*a^2*b^3 - a*b^4 - b^5)*cosh(f*
x + e)^2)*sinh(f*x + e)^4 + 8*(15*(14*a^2*b^3 - 5*a*b^4 - b^5)*cosh(f*x + e
)^7 + 7*(112*a^3*b^2 - 82*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(f*x + e)^5 + 5*(1
12*a^4*b - 96*a^3*b^2 + 26*a^2*b^3 - a*b^4 - b^5)*cosh(f*x + e)^3 + (112*a^
4*b - 96*a^3*b^2 + 26*a^2*b^3 - a*b^4 - b^5)*cosh(f*x + e))*sinh(f*x + e)^3
 + (112*a^3*b^2 - 82*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(f*x + e)^2 + (45*(14*a
^2*b^3 - 5*a*b^4 - b^5)*cosh(f*x + e)^8 + 28*(112*a^3*b^2 - 82*a^2*b^3 + 7*
a*b^4 + 3*b^5)*cosh(f*x + e)^6 + 112*a^3*b^2 - 82*a^2*b^3 + 7*a*b^4 + 3*b^5
 + 30*(112*a^4*b - 96*a^3*b^2 + 26*a^2*b^3 - a*b^4 - b^5)*cosh(f*x + e)^4 +
 12*(112*a^4*b - 96*a^3*b^2 + 26*a^2*b^3 - a*b^4 - b^5)*cosh(f*x + e)^2)*si
nh(f*x + e)^2 + 2*(5*(14*a^2*b^3 - 5*a*b^4 - b^5)*cosh(f*x + e)^9 + 4*(112*
a^3*b^2 - 82*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(f*x + e)^7 + 6*(112*a^4*b - 96
*a^3*b^2 + 26*a^2*b^3 - a*b^4 - b^5)*cosh(f*x + e)^5 + 4*(112*a^4*b - 96*a^
3*b^2 + 26*a^2*b^3 - a*b^4 - b^5)*cosh(f*x + e)^3 + (112*a^3*b^2 - 82*a^2*b
^3 + 7*a*b^4 + 3*b^5)*cosh(f*x + e))*sinh(f*x + e) - 2*((7*a*b^4 + b^5)*cos
h(f*x + e)^10 + 10*(7*a*b^4 + b^5)*cosh(f*x + e)*sinh(f*x + e)^9 + (7*a*b^4
 + b^5)*sinh(f*x + e)^10 + (56*a^2*b^3 - 13*a*b^4 - 3*b^5)*cosh(f*x + e)^8
+ (56*a^2*b^3 - 13*a*b^4 - 3*b^5 + 45*(7*a*b^4 + b^5)*cosh(f*x + e)^2)*sinh
(f*x + e)^8 + 8*(15*(7*a*b^4 + b^5)*cosh(f*x + e)^3 + (56*a^2*b^3 - 13*a*b^
4 - 3*b^5)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(56*a^3*b^2 - 20*a^2*b^3 + 3*
a*b^4 + b^5)*cosh(f*x + e)^6 + 2*(56*a^3*b^2 - 20*a^2*b^3 + 3*a*b^4 + b^5 +
 105*(7*a*b^4 + b^5)*cosh(f*x + e)^4 + 14*(56*a^2*b^3 - 13*a*b^4 - 3*b^5)*c
osh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(7*a*b^4 + b^5)*cosh(f*x + e)^5 + 1
4*(56*a^2*b^3 - 13*a*b^4 - 3*b^5)*cosh(f*x + e)^3 + 3*(56*a^3*b^2 - 20*a^2*
b^3 + 3*a*b^4 + b^5)*cosh(f*x + e))*sinh(f*x + e)^5 + 7*a*b^4 + b^5 + 2*(56
```

```
*a^3*b^2 - 20*a^2*b^3 + 3*a*b^4 + b^5)*cosh(f*x + e)^4 + 2*(105*(7*a*b^4 +
b^5)*cosh(f*x + e)^6 + 56*a^3*b^2 - 20*a^2*b^3 + 3*a*b^4 + b^5 + 35*(56*a^2
*b^3 - 13*a*b^4 - 3*b^5)*cosh(f*x + e)^4 + 15*(56*a^3*b^2 - 20*a^2*b^3 + 3*
a*b^4 + b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(15*(7*a*b^4 + b^5)*cosh(
f*x + e)^7 + 7*(56*a^2*b^3 - 13*a*b^4 - 3*b^5)*cosh(f*x + e)^5 + 5*(56*a^3*
b^2 - 20*a^2*b^3 + 3*a*b^4 + b^5)*cosh(f*x + e)^3 + (56*a^3*b^2 - 20*a^2*b^
3 + 3*a*b^4 + b^5)*cosh(f*x + e))*sinh(f*x + e)^3 + (56*a^2*b^3 - 13*a*b^4
- 3*b^5)*cosh(f*x + e)^2 + (45*(7*a*b^4 + b^5)*cosh(f*x + e)^8 + 28*(56*a^2
*b^3 - 13*a*b^4 - 3*b^5)*cosh(f*x + e)^6 + 56*a^2*b^3 - 13*a*b^4 - 3*b^5 +
30*(56*a^3*b^2 - 20*a^2*b^3 + 3*a*b^4 + b^5)*cosh(f*x + e)^4 + 12*(56*a^3*b
^2 - 20*a^2*b^3 + 3*a*b^4 + b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(7
*a*b^4 + b^5)*cosh(f*x + e)^9 + 4*(56*a^2*b^3 - 13*a*b^4 - 3*b^5)*cosh(f*x
+ e)^7 + 6*(56*a^3*b^2 - 20*a^2*b^3 + 3*a*b^4 + b^5)*cosh(f*x + e)^5 + 4*(5
6*a^3*b^2 - 20*a^2*b^3 + 3*a*b^4 + b^5)*cosh(f*x + e)^3 + (56*a^2*b^3 - 13*
a*b^4 - 3*b^5)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)
*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*
sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^
2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((6*a^3*b
^2 + 7*a^2*b^3 - 5*a*b^4)*cosh(f*x + e)^10 + 10*(6*a^3*b^2 + 7*a^2*b^3 - 5*
a*b^4)*cosh(f*x + e)*sinh(f*x + e)^9 + (6*a^3*b^2 + 7*a^2*b^3 - 5*a*b^4)*si
nh(f*x + e)^10 + (48*a^4*b + 38*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4)*cosh(f*x +
 e)^8 + (48*a^4*b + 38*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4 + 45*(6*a^3*b^2 + 7*
a^2*b^3 - 5*a*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(6*a^3*b^2 + 7*
a^2*b^3 - 5*a*b^4)*cosh(f*x + e)^3 + (48*a^4*b + 38*a^3*b^2 - 61*a^2*b^3 +
15*a*b^4)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(48*a^5 + 32*a^4*b - 62*a^3*b^
2 + 27*a^2*b^3 - 5*a*b^4)*cosh(f*x + e)^6 + 2*(...
```

Sympy **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tanh(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

Giac **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 1.86Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(e+f\,x)^2}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.509 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a}}{3a^2(a-b)^2f\sqrt{1+\frac{b\sin}{}}}$$

```
[Out] -1/3*b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(2*a
-b)*b*cosh(f*x+e)*sinh(f*x+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-2/3*I
*(2*a-b)*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(
b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)^2/f/(1+b*sinh(f*x+e)^2/a)^(
1/2)+1/3*I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x)
,(b/a)^(1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/
2)
```

Rubi [A]

time = 0.20, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3263, 3252, 3251, 3257, 3256, 3262, 3261}

$$-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] -1/3*(b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt
[a + b*Sinh[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]
*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)^2*f*Sqrt[1 + (b*Sinh[e + f*x]^2)
/a]) + ((I/3)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/
(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a+2b+b\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx}{3a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx}}$$

Mathematica [A]

time = 1.09, size = 190, normalized size = 0.76

$$\frac{-2ia^2(2a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+ia^2(a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)+\sqrt{2}\,b(-5a^2+5ab-b^2+b(-2a+b)\cosh(2(e+fx)))\sinh(2(e+fx))}{3a^2(a-b)^2f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] ((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*
EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a + b
)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b*Cos
h[2*(e + f*x)])^(3/2))
```

Maple [A]

time = 1.76, size = 406, normalized size = 1.62

method	result
default	$\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(-\frac{\sinh(fx+e)\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+\right.\right.}}{3ab(a-b)\left(\sinh^2(fx+e)+\frac{a}{b}\right)^2}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/a/b/(a-b)*sinh(f*x+e)*((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+a/b)^2-2/3*b*cosh(f*x+e
)^2/a^2/(a-b)^2*sinh(f*x+e)*(-b+2*a)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1
/2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*Ellip
ticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2/3*b*(-b+2*a)/a^2/(a-b)^2/(-1
/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(
f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))))/cosh(f*x+e)/(a+b
*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5442 vs. $2(259) = 518$.
time = 0.19, size = 5442, normalized size = 21.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(((4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^8 + 8*(4*a^2*b^3 - 4*a*b^4
+ b^5)*cosh(f*x + e)*sinh(f*x + e)^7 + (4*a^2*b^3 - 4*a*b^4 + b^5)*sinh(f*x
```

```
 + e)^8 + 4*(8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e)^6 + 4*(8
*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5 + 7*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(
f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e
)^3 + 3*(8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e))*sinh(f*x +
e)^5 + 4*a^2*b^3 - 4*a*b^4 + b^5 + 2*(32*a^4*b - 64*a^3*b^2 + 52*a^2*b^3 -
20*a*b^4 + 3*b^5)*cosh(f*x + e)^4 + 2*(32*a^4*b - 64*a^3*b^2 + 52*a^2*b^3 -
 20*a*b^4 + 3*b^5 + 35*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^4 + 30*(8*
a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*
(7*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^5 + 10*(8*a^3*b^2 - 12*a^2*b^3
 + 6*a*b^4 - b^5)*cosh(f*x + e)^3 + (32*a^4*b - 64*a^3*b^2 + 52*a^2*b^3 - 2
0*a*b^4 + 3*b^5)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(8*a^3*b^2 - 12*a^2*b^3
 + 6*a*b^4 - b^5)*cosh(f*x + e)^2 + 4*(7*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f
*x + e)^6 + 8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5 + 15*(8*a^3*b^2 - 12*a^2
*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e)^4 + 3*(32*a^4*b - 64*a^3*b^2 + 52*a^2*b
^3 - 20*a*b^4 + 3*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((4*a^2*b^3 - 4
*a*b^4 + b^5)*cosh(f*x + e)^7 + 3*(8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*
cosh(f*x + e)^5 + (32*a^4*b - 64*a^3*b^2 + 52*a^2*b^3 - 20*a*b^4 + 3*b^5)*c
osh(f*x + e)^3 + (8*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 - b^5)*cosh(f*x + e))*si
nh(f*x + e) - 2*((2*a*b^4 - b^5)*cosh(f*x + e)^8 + 8*(2*a*b^4 - b^5)*cosh(f
*x + e)*sinh(f*x + e)^7 + (2*a*b^4 - b^5)*sinh(f*x + e)^8 + 4*(4*a^2*b^3 -
4*a*b^4 + b^5)*cosh(f*x + e)^6 + 4*(4*a^2*b^3 - 4*a*b^4 + b^5 + 7*(2*a*b^4
- b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(2*a*b^4 - b^5)*cosh(f*x + e
)^3 + 3*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*a*b^
4 - b^5 + 2*(16*a^3*b^2 - 24*a^2*b^3 + 14*a*b^4 - 3*b^5)*cosh(f*x + e)^4 +
2*(16*a^3*b^2 - 24*a^2*b^3 + 14*a*b^4 - 3*b^5 + 35*(2*a*b^4 - b^5)*cosh(f*x
 + e)^4 + 30*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^4 +
 8*(7*(2*a*b^4 - b^5)*cosh(f*x + e)^5 + 10*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh
(f*x + e)^3 + (16*a^3*b^2 - 24*a^2*b^3 + 14*a*b^4 - 3*b^5)*cosh(f*x + e))*s
inh(f*x + e)^3 + 4*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^2 + 4*(7*(2*a*
b^4 - b^5)*cosh(f*x + e)^6 + 4*a^2*b^3 - 4*a*b^4 + b^5 + 15*(4*a^2*b^3 - 4*
a*b^4 + b^5)*cosh(f*x + e)^4 + 3*(16*a^3*b^2 - 24*a^2*b^3 + 14*a*b^4 - 3*b^
5)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((2*a*b^4 - b^5)*cosh(f*x + e)^7 +
3*(4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x + e)^5 + (16*a^3*b^2 - 24*a^2*b^3 +
14*a*b^4 - 3*b^5)*cosh(f*x + e)^3 + (4*a^2*b^3 - 4*a*b^4 + b^5)*cosh(f*x +
e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 - a*b
)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*
a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^2 - 8*a*b + b^2 + 4*(2*a*b
 - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - ((6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh
(f*x + e)^8 + 8*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)*sinh(f*x + e)
^7 + (6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*sinh(f*x + e)^8 + 4*(12*a^4*b - 16*a^3
*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^6 + 4*(12*a^4*b - 16*a^3*b^2 + 7*a^
2*b^3 - a*b^4 + 7*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^2)*sinh(f*x
 + e)^6 + 8*(7*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^3 + 3*(12*a^4*
b - 16*a^3*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*a^3*
b^2 - 5*a^2*b^3 + a*b^4 + 2*(48*a^5 - 88*a^4*b + 66*a^3*b^2 - 23*a^2*b^3 +
```

```
3*a*b^4)*cosh(f*x + e)^4 + 2*(48*a^5 - 88*a^4*b + 66*a^3*b^2 - 23*a^2*b^3 +
 3*a*b^4 + 35*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^4 + 30*(12*a^4*
b - 16*a^3*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(7
*(6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^5 + 10*(12*a^4*b - 16*a^3*b^
2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^3 + (48*a^5 - 88*a^4*b + 66*a^3*b^2 -
23*a^2*b^3 + 3*a*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(12*a^4*b - 16*a^3
*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^2 + 4*(7*(6*a^3*b^2 - 5*a^2*b^3 + a
*b^4)*cosh(f*x + e)^6 + 12*a^4*b - 16*a^3*b^2 + 7*a^2*b^3 - a*b^4 + 15*(12*
a^4*b - 16*a^3*b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^4 + 3*(48*a^5 - 88*a^
4*b + 66*a^3*b^2 - 23*a^2*b^3 + 3*a*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 +
 8*((6*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(f*x + e)^7 + 3*(12*a^4*b - 16*a^3*
b^2 + 7*a^2*b^3 - a*b^4)*cosh(f*x + e)^5 + (48*a^5 - 88*a^4*b + 66*a^3*b^2
- 23*a^2*b^3 + 3*a*b^4)*cosh(f*x + e)^3 + (12*a^4*b - 16*a^3*b^2 + 7*a^2*b^
3 - a*b^4)*cosh(f*x + e))*sinh(f*x + e) + 2*((3*a^2*b^3 - 5*a*b^4 + 2*b^5)*
cosh(f*x + e)^8 + 8*(3*a^2*b^3 - 5*a*b^4 + 2*b^5)*cosh(f*x + e)*sinh(f*x +
e)^7 + (3*a^2*b^3 - 5*a*b^4 + 2*b^5)*sinh(f*x + e)^8 + 4*(6*a^3*b^2 - 13*a^
2*b^3 + 9*a*b^4 - 2*b^5)*cosh(f*x + e)^6 + 4*(6*a^3*b^2 - 13*a^2*b^3 + 9*a*
b^4 - 2*b^5 + 7*(3*a^2*b^3 - 5*a*b^4 + 2*b^5)*cosh(f*x + e)^2)*sinh(f*x + e
)^6 + 8*(7*(3*a^2*b^3 - 5*a*b^4 + 2*b^5)*cosh(f*x + e)^3 + 3*(6*a^3*b^2 - 1
3*a^2*b^3 + 9*a*b^4 - 2*b^5)*cosh(f*x + e))*sin...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(-5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.49Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(1/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.510 $\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=351

$$\frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e}}{3a^3(a-b)f}$$

```
[Out] 1/3*coth(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/3*(3*a-4*b)*coth(f*x+e)/a^2
/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*(7*a-8*b)*coth(f*x+e)*(a+b*sinh(f*x+
e)^2)^(1/2)/a^3/(a-b)/f-1/3*(7*a-8*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f
*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))
*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh
(f*x+e)^2)/a)^(1/2)+1/3*(3*a-4*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sec
h(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x
+e)^2)/a)^(1/2)+1/3*(7*a-8*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a^3/(a-
b)/f
```

Rubi [A]
time = 0.28, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 480, 593, 597, 545, 429, 506, 422}

$$\frac{(3a-4b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3a^3f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(7a-8b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ArcTan}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3a^3f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)}-\frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)}+\frac{(3a-4b)\coth(e+fx)}{3a^2f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Coth[e + f*x]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((3*a - 4*b)*Coth[e +
 f*x])/(3*a^2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((7*a - 8*b)*Coth[e
+ f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)*f) - ((7*a - 8*b)*Ellipt
icE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^
2])/(3*a^3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (
(3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
+ b*Sinh[e + f*x]^2])/(3*a^3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3
*a^3*(a - b)*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2(a+bx^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\Big(\int}{3a}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\Big(\sqrt{c}}{}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a -}{}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a -}{}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a -}{}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a -}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.28, size = 226, normalized size = 0.64

$$\frac{-\frac{(24a^3-68a^2b+69ab^2-24b^3+4b(11a^2-19ab+8b^2)\cosh(2(e+fx))+(7a-8b)b^2\cosh(4(e+fx)))\coth(e+fx)}{\sqrt{2}} - 2ia^2(7a-8b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right) + 8ia^2(a-b)\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{6a^3(a-b)f(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(((24*a^3 - 68*a^2*b + 69*a*b^2 - 24*b^3 + 4*b*(11*a^2 - 19*a*b + 8*b^2)*
Cosh[2*(e + f*x)] + (7*a - 8*b)*b^2*Cosh[4*(e + f*x)])*Coth[e + f*x])/Sqrt[
2]) - (2*I)*a^2*(7*a - 8*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*Ellip
ticE[I*(e + f*x), b/a] + (8*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])
```

```
/a)^(3/2)*EllipticF[I*(e + f*x), b/a])/(6*a^3*(a - b)*f*(2*a - b + b*Cosh[2
*(e + f*x)])^(3/2))
```

Maple [A]
time = 2.15, size = 642, normalized size = 1.83

method	result
default	$-\frac{\left(7\sqrt{-\frac{b}{a}}\,a\,b^{2}-8\sqrt{-\frac{b}{a}}\,b^{3}\right)\left(\cosh^{6}(fx+e)\right)+\left(11\sqrt{-\frac{b}{a}}\,a^{2}b-26\sqrt{-\frac{b}{a}}\,a\,b^{2}+16\sqrt{-\frac{b}{a}}\,b^{3}\right)\left(\cosh^{4}(fx+e)\right)-\sqrt{\frac{b(\cosh}{}}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((7*(-1/a*b)^(1/2)*a*b^2-8*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^6+(11*(-1/a
*b)^(1/2)*a^2*b-26*(-1/a*b)^(1/2)*a*b^2+16*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^
4-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(3*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-11*EllipticF(sinh(f*x+e)*(-1/a*b)
^(1/2),(a/b)^(1/2))*a*b+8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))
*b^2+7*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-8*EllipticE(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+(3*(-1
/a*b)^(1/2)*a^3-14*(-1/a*b)^(1/2)*a^2*b+19*(-1/a*b)^(1/2)*a*b^2-8*(-1/a*b)^
(1/2)*b^3)*cosh(f*x+e)^2-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^
(1/2)*(3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3-14*EllipticF
(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+19*EllipticF(sinh(f*x+e)*(-1
/a*b)^(1/2),(a/b)^(1/2))*a*b^2-8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))*b^3+7*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-15*El
lipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+8*EllipticE(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)*sinh(f*x+e))/(-1/a*b)^(1/2)/(a-b)/a^3/(
a+b*sinh(f*x+e)^2)^(3/2)/sinh(f*x+e)/cosh(f*x+e)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7847 vs. $2(351) = 702$.
time = 0.26, size = 7847, normalized size = 22.36

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] 1/3*(((14*a^2*b^3 - 23*a*b^4 + 8*b^5)*cosh(f*x + e)^10 + 10*(14*a^2*b^3 - 2
3*a*b^4 + 8*b^5)*cosh(f*x + e)*sinh(f*x + e)^9 + (14*a^2*b^3 - 23*a*b^4 + 8
*b^5)*sinh(f*x + e)^10 + (112*a^3*b^2 - 254*a^2*b^3 + 179*a*b^4 - 40*b^5)*c
osh(f*x + e)^8 + (112*a^3*b^2 - 254*a^2*b^3 + 179*a*b^4 - 40*b^5 + 45*(14*a
^2*b^3 - 23*a*b^4 + 8*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(14*a^2
*b^3 - 23*a*b^4 + 8*b^5)*cosh(f*x + e)^3 + (112*a^3*b^2 - 254*a^2*b^3 + 179
*a*b^4 - 40*b^5)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(112*a^4*b - 352*a^3*b^
2 + 410*a^2*b^3 - 211*a*b^4 + 40*b^5)*cosh(f*x + e)^6 + 2*(112*a^4*b - 352*
a^3*b^2 + 410*a^2*b^3 - 211*a*b^4 + 40*b^5 + 105*(14*a^2*b^3 - 23*a*b^4 + 8
*b^5)*cosh(f*x + e)^4 + 14*(112*a^3*b^2 - 254*a^2*b^3 + 179*a*b^4 - 40*b^5)
*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(14*a^2*b^3 - 23*a*b^4 + 8*b^5)*c
osh(f*x + e)^5 + 14*(112*a^3*b^2 - 254*a^2*b^3 + 179*a*b^4 - 40*b^5)*cosh(f
*x + e)^3 + 3*(112*a^4*b - 352*a^3*b^2 + 410*a^2*b^3 - 211*a*b^4 + 40*b^5)*
cosh(f*x + e))*sinh(f*x + e)^5 - 14*a^2*b^3 + 23*a*b^4 - 8*b^5 - 2*(112*a^4
*b - 352*a^3*b^2 + 410*a^2*b^3 - 211*a*b^4 + 40*b^5)*cosh(f*x + e)^4 + 2*(1
05*(14*a^2*b^3 - 23*a*b^4 + 8*b^5)*cosh(f*x + e)^6 - 112*a^4*b + 352*a^3*b^
2 - 410*a^2*b^3 + 211*a*b^4 - 40*b^5 + 35*(112*a^3*b^2 - 254*a^2*b^3 + 179*
a*b^4 - 40*b^5)*cosh(f*x + e)^4 + 15*(112*a^4*b - 352*a^3*b^2 + 410*a^2*b^3
 - 211*a*b^4 + 40*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(15*(14*a^2*b^3
 - 23*a*b^4 + 8*b^5)*cosh(f*x + e)^7 + 7*(112*a^3*b^2 - 254*a^2*b^3 + 179*a
*b^4 - 40*b^5)*cosh(f*x + e)^5 + 5*(112*a^4*b - 352*a^3*b^2 + 410*a^2*b^3 -
 211*a*b^4 + 40*b^5)*cosh(f*x + e)^3 - (112*a^4*b - 352*a^3*b^2 + 410*a^2*b
^3 - 211*a*b^4 + 40*b^5)*cosh(f*x + e))*sinh(f*x + e)^3 - (112*a^3*b^2 - 25
4*a^2*b^3 + 179*a*b^4 - 40*b^5)*cosh(f*x + e)^2 + (45*(14*a^2*b^3 - 23*a*b^
4 + 8*b^5)*cosh(f*x + e)^8 + 28*(112*a^3*b^2 - 254*a^2*b^3 + 179*a*b^4 - 40
*b^5)*cosh(f*x + e)^6 - 112*a^3*b^2 + 254*a^2*b^3 - 179*a*b^4 + 40*b^5 + 30
*(112*a^4*b - 352*a^3*b^2 + 410*a^2*b^3 - 211*a*b^4 + 40*b^5)*cosh(f*x + e)
^4 - 12*(112*a^4*b - 352*a^3*b^2 + 410*a^2*b^3 - 211*a*b^4 + 40*b^5)*cosh(f
*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(14*a^2*b^3 - 23*a*b^4 + 8*b^5)*cosh(f*x
+ e)^9 + 4*(112*a^3*b^2 - 254*a^2*b^3 + 179*a*b^4 - 40*b^5)*cosh(f*x + e)^7
 + 6*(112*a^4*b - 352*a^3*b^2 + 410*a^2*b^3 - 211*a*b^4 + 40*b^5)*cosh(f*x
+ e)^5 - 4*(112*a^4*b - 352*a^3*b^2 + 410*a^2*b^3 - 211*a*b^4 + 40*b^5)*cos
h(f*x + e)^3 - (112*a^3*b^2 - 254*a^2*b^3 + 179*a*b^4 - 40*b^5)*cosh(f*x +
e))*sinh(f*x + e) - 2*((7*a*b^4 - 8*b^5)*cosh(f*x + e)^10 + 10*(7*a*b^4 - 8
*b^5)*cosh(f*x + e)*sinh(f*x + e)^9 + (7*a*b^4 - 8*b^5)*sinh(f*x + e)^10 +
(56*a^2*b^3 - 99*a*b^4 + 40*b^5)*cosh(f*x + e)^8 + (56*a^2*b^3 - 99*a*b^4 +
 40*b^5 + 45*(7*a*b^4 - 8*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(7*
a*b^4 - 8*b^5)*cosh(f*x + e)^3 + (56*a^2*b^3 - 99*a*b^4 + 40*b^5)*cosh(f*x
+ e))*sinh(f*x + e)^7 + 2*(56*a^3*b^2 - 148*a^2*b^3 + 131*a*b^4 - 40*b^5)*c
osh(f*x + e)^6 + 2*(56*a^3*b^2 - 148*a^2*b^3 + 131*a*b^4 - 40*b^5 + 105*(7*
a*b^4 - 8*b^5)*cosh(f*x + e)^4 + 14*(56*a^2*b^3 - 99*a*b^4 + 40*b^5)*cosh(f
```

```
*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(7*a*b^4 - 8*b^5)*cosh(f*x + e)^5 + 14*(
56*a^2*b^3 - 99*a*b^4 + 40*b^5)*cosh(f*x + e)^3 + 3*(56*a^3*b^2 - 148*a^2*b
^3 + 131*a*b^4 - 40*b^5)*cosh(f*x + e))*sinh(f*x + e)^5 - 7*a*b^4 + 8*b^5 -
 2*(56*a^3*b^2 - 148*a^2*b^3 + 131*a*b^4 - 40*b^5)*cosh(f*x + e)^4 + 2*(105
*(7*a*b^4 - 8*b^5)*cosh(f*x + e)^6 - 56*a^3*b^2 + 148*a^2*b^3 - 131*a*b^4 +
 40*b^5 + 35*(56*a^2*b^3 - 99*a*b^4 + 40*b^5)*cosh(f*x + e)^4 + 15*(56*a^3*
b^2 - 148*a^2*b^3 + 131*a*b^4 - 40*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^4 +
8*(15*(7*a*b^4 - 8*b^5)*cosh(f*x + e)^7 + 7*(56*a^2*b^3 - 99*a*b^4 + 40*b^5
)*cosh(f*x + e)^5 + 5*(56*a^3*b^2 - 148*a^2*b^3 + 131*a*b^4 - 40*b^5)*cosh(
f*x + e)^3 - (56*a^3*b^2 - 148*a^2*b^3 + 131*a*b^4 - 40*b^5)*cosh(f*x + e))
*sinh(f*x + e)^3 - (56*a^2*b^3 - 99*a*b^4 + 40*b^5)*cosh(f*x + e)^2 + (45*(
7*a*b^4 - 8*b^5)*cosh(f*x + e)^8 + 28*(56*a^2*b^3 - 99*a*b^4 + 40*b^5)*cosh
(f*x + e)^6 - 56*a^2*b^3 + 99*a*b^4 - 40*b^5 + 30*(56*a^3*b^2 - 148*a^2*b^3
 + 131*a*b^4 - 40*b^5)*cosh(f*x + e)^4 - 12*(56*a^3*b^2 - 148*a^2*b^3 + 131
*a*b^4 - 40*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(7*a*b^4 - 8*b^5)*
cosh(f*x + e)^9 + 4*(56*a^2*b^3 - 99*a*b^4 + 40*b^5)*cosh(f*x + e)^7 + 6*(5
6*a^3*b^2 - 148*a^2*b^3 + 131*a*b^4 - 40*b^5)*cosh(f*x + e)^5 - 4*(56*a^3*b
^2 - 148*a^2*b^3 + 131*a*b^4 - 40*b^5)*cosh(f*x + e)^3 - (56*a^2*b^3 - 99*a
*b^4 + 40*b^5)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - a*b)/b^2))*sqrt(b)
*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*
sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)*(cosh(f*x + e) + sinh(f*x + e))), (8*a^
2 - 8*a*b + b^2 + 4*(2*a*b - b^2)*sqrt((a^2 - a*b)/b^2))/b^2) - 2*((6*a^3*b
^2 - 11*a^2*b^3 + 4*a*b^4)*cosh(f*x + e)^10 + 10*(6*a^3*b^2 - 11*a^2*b^3 +
4*a*b^4)*cosh(f*x + e)*sinh(f*x + e)^9 + (6*a^3*b^2 - 11*a^2*b^3 + 4*a*b^4)
*sinh(f*x + e)^10 + (48*a^4*b - 118*a^3*b^2 + 87*a^2*b^3 - 20*a*b^4)*cosh(f
*x + e)^8 + (48*a^4*b - 118*a^3*b^2 + 87*a^2*b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.87Error: Bad Argum
ent Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(e+f\,x)^2}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.511 $\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=385

$$-\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{8(a-2b)\coth(e+fx)\sqrt{a+}}{3a^4f}$$

```
[Out] -1/3*(a-b)*coth(f*x+e)*csch(f*x+e)^2/a/b/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(a
-3*b)*coth(f*x+e)*csch(f*x+e)^2/a^2/b/f/(a+b*sinh(f*x+e)^2)^(1/2)-8/3*(a-2*
b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^4/f+1/3*(3*a-8*b)*coth(f*x+e)*cs
ch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/b/f-8/3*(a-2*b)*(1/(1+sinh(f*x+e)
^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^
(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^4/f/(sech(f*x+
e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a-8*b)*(1/(1+sinh(f*x+e)^2))^(1/2)
*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b
/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^4/f/(sech(f*x+e)^2*(a+b*
sinh(f*x+e)^2)/a)^(1/2)+8/3*(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a
^4/f
```

Rubi [A]
time = 0.38, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 479, 593, 597, 545, 429, 506, 422}

$$\frac{(3a-8b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\operatorname{ArcTan}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{3a^4f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{8(a-2b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\operatorname{ArcTan}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{3a^4f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{8(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^4f}-\frac{8(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^4f}+\frac{(3a-8b)\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3bf}-\frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*((a - b)*Coth[e + f*x]*Csch[e + f*x]^2)/(a*b*f*(a + b*Sinh[e + f*x]^2)
^(3/2)) - (2*(a - 3*b)*Coth[e + f*x]*Csch[e + f*x]^2)/(3*a^2*b*f*Sqrt[a + b
*Sinh[e + f*x]^2]) - (8*(a - 2*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]
)/(3*a^4*f) + ((3*a - 8*b)*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e
+ f*x]^2])/(3*a^3*b*f) - (8*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 -
b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f*Sqrt[(Sech[e + f*x
]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a - 8*b)*EllipticF[ArcTan[Sinh[e + f
*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f*Sqrt[(Se
ch[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e
 + f*x]^2]*Tanh[e + f*x])/(3*a^4*f)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4(a+bx^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{S}}{}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\text{csch}^2(e+fx}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\text{csch}^2(e+fx}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\text{csch}^2(e+fx}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\text{csch}^2(e+fx}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\text{csch}^2(e+fx}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\text{csch}^2(e+fx}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 2.39, size = 247, normalized size = 0.64

$$-\frac{i\left(\frac{ib\left(8a^3-63a^2b+92ab^2-40b^3-2\left(8a^3-38a^2b+63ab^2-30b^3\right)\cosh(2(e+fx))-b\left(13a^2-36ab+24b^2\right)\cosh(4(e+fx))-2ab^2\cosh(6(e+fx))+4b^3\cosh(6(e+fx))\right)\coth(e+fx)\text{csch}^2(e+fx)}{\sqrt{2}}+2a^2b\left(\frac{2a-b+b\cosh(2(e+fx))}{a}\right)^{3/2}\left(8(a-2b)E\left(i(e+fx)\left|\frac{b}{a}\right.\right)+(-5a+8b)F\left(i(e+fx)\left|\frac{b}{a}\right.\right)\right)\right)}{6a^4bf(2a-b+b\cosh(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-1/6*I)*((I*b*(8*a^3 - 63*a^2*b + 92*a*b^2 - 40*b^3 - 2*(8*a^3 - 38*a^2*b
 + 63*a*b^2 - 30*b^3)*Cosh[2*(e + f*x)] - b*(13*a^2 - 36*a*b + 24*b^2)*Cosh
[4*(e + f*x)] - 2*a*b^2*Cosh[6*(e + f*x)] + 4*b^3*Cosh[6*(e + f*x)])*Coth[e
 + f*x]*Csch[e + f*x]^2)/Sqrt[2] + 2*a^2*b*((2*a - b + b*Cosh[2*(e + f*x)])
/a)^(3/2)*(8*(a - 2*b)*EllipticE[I*(e + f*x), b/a] + (-5*a + 8*b)*EllipticF
[I*(e + f*x), b/a])))/(a^4*b*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 922 vs. $2(433) = 866$.
time = 2.94, size = 923, normalized size = 2.40

method	result	size
default	Expression too large to display	923
risch	Expression too large to display	1124572

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(8*(-1/a*b)^(1/2)*a*b^2*sinh(f*x+e)^8-16*(-1/a*b)^(1/2)*b^3*sinh(f*x+e
)^8-3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b*sinh(f*x+e)^5+16*((a+b*sinh(f*x+e)^2
)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))*a*b^2*sinh(f*x+e)^5-16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3*sinh(f*x+e)^5
-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2*sinh(f*x+e)^5+16*((a+b*sinh(f*x+e)^2)/a
)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*b^3*sinh(f*x+e)^5+13*(-1/a*b)^(1/2)*a^2*b*sinh(f*x+e)^6-16*(-1/a*b)^(1
/2)*a*b^2*sinh(f*x+e)^6-16*(-1/a*b)^(1/2)*b^3*sinh(f*x+e)^6-3*((a+b*sinh(f*
x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*a^3*sinh(f*x+e)^3+16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b*sinh(f*
x+e)^3-16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sin
h(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2*sinh(f*x+e)^3-8*((a+b*sinh(f*x+e
)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a
/b)^(1/2))*a^2*b*sinh(f*x+e)^3+16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2*sinh(f*x
+e)^3+4*(-1/a*b)^(1/2)*a^3*sinh(f*x+e)^4+7*(-1/a*b)^(1/2)*a^2*b*sinh(f*x+e)
^4-24*(-1/a*b)^(1/2)*a*b^2*sinh(f*x+e)^4+5*(-1/a*b)^(1/2)*a^3*sinh(f*x+e)^2
-6*(-1/a*b)^(1/2)*a^2*b*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^3)/a^4/(-1/a*b)^(1/2
)/(a+b*sinh(f*x+e)^2)^(3/2)/sinh(f*x+e)^3/cosh(f*x+e)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 13823 vs. $2(381) = 762$.
time = 0.41, size = 13823, normalized size = 35.90

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(4*((2*a^2*b^3 - 5*a*b^4 + 2*b^5)*cosh(f*x + e)^14 + 14*(2*a^2*b^3 - 5*
a*b^4 + 2*b^5)*cosh(f*x + e)*sinh(f*x + e)^13 + (2*a^2*b^3 - 5*a*b^4 + 2*b^
5)*sinh(f*x + e)^14 + (16*a^3*b^2 - 54*a^2*b^3 + 51*a*b^4 - 14*b^5)*cosh(f*
x + e)^12 + (16*a^3*b^2 - 54*a^2*b^3 + 51*a*b^4 - 14*b^5 + 91*(2*a^2*b^3 -
5*a*b^4 + 2*b^5)*cosh(f*x + e)^2)*sinh(f*x + e)^12 + 4*(91*(2*a^2*b^3 - 5*a
*b^4 + 2*b^5)*cosh(f*x + e)^3 + 3*(16*a^3*b^2 - 54*a^2*b^3 + 51*a*b^4 - 14*
b^5)*cosh(f*x + e))*sinh(f*x + e)^11 + (32*a^4*b - 160*a^3*b^2 + 274*a^2*b^
3 - 185*a*b^4 + 42*b^5)*cosh(f*x + e)^10 + (32*a^4*b - 160*a^3*b^2 + 274*a^
2*b^3 - 185*a*b^4 + 42*b^5 + 1001*(2*a^2*b^3 - 5*a*b^4 + 2*b^5)*cosh(f*x +
e)^4 + 66*(16*a^3*b^2 - 54*a^2*b^3 + 51*a*b^4 - 14*b^5)*cosh(f*x + e)^2)*si
nh(f*x + e)^10 + 2*(1001*(2*a^2*b^3 - 5*a*b^4 + 2*b^5)*cosh(f*x + e)^5 + 11
0*(16*a^3*b^2 - 54*a^2*b^3 + 51*a*b^4 - 14*b^5)*cosh(f*x + e)^3 + 5*(32*a^4
*b - 160*a^3*b^2 + 274*a^2*b^3 - 185*a*b^4 + 42*b^5)*cosh(f*x + e))*sinh(f*
x + e)^9 - (96*a^4*b - 40 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.24Error: Bad Argum
ent Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(e+f\,x)^4}{{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

3.512 $\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx$

Optimal. Leaf size=122

$$\frac{F_1\left(\frac{1+m}{2};\frac{1+m}{2},-p;\frac{3+m}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\cosh^2(e+fx)^{\frac{1+m}{2}}\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e}{a}\right.}{df(1+m)}$$

```
[Out] AppellF1(1/2+1/2*m,1/2+1/2*m,-p,3/2+1/2*m,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a
)*(cosh(f*x+e)^2)^(1/2+1/2*m)*(a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^(1+m)/d
/f/(1+m)/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3276, 525, 524}

$$\frac{\cosh^2(e+fx)^{\frac{m+1}{2}}(d\tanh(e+fx))^{m+1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+1}{2},-p;\frac{m+3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{df(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, -Sinh[e + f*x]^2, -((b*Sinh[
e + f*x]^2)/a)]*(Cosh[e + f*x]^2)^((1 + m)/2)*(a + b*Sinh[e + f*x]^2)^p*(d*
Tanh[e + f*x])^(1 + m))/(d*f*(1 + m)*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3276

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[f
```

```
f*(d*Tan[e + f*x])^(m + 1)*((Cos[e + f*x]^2)^((m + 1)/2)/(d*f*Sin[e + f*x]^
(m + 1))), Subst[Int[(ff*x)^m*((a + b*ff^2*x^2)^p/(1 - ff^2*x^2)^((m + 1)/2
)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !In
tegerQ[m]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx = \frac{\Big(i\cosh^2(e+fx)^{\frac{1+m}{2}}(i\sinh(e+fx))^{-1-m}(d\tanh(e+fx}{}$$

$$= \frac{\Big(i\cosh^2(e+fx)^{\frac{1+m}{2}}(i\sinh(e+fx))^{-1-m}\,\big(a+b\sinh^2(e}{}$$

$$= \frac{F_1\Big(\frac{1+m}{2};\frac{1+m}{2},-p;\frac{3+m}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\Big)\,\mathrm{co}}{}$$

Mathematica [F]
time = 7.84, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m,x]
[Out] Integrate[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m, x]
```

Maple [F]
time = 2.73, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sinh^2(fx+e)\right)\right)^p (d\tanh(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x)
[Out] int((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)
```

Fricas [**F**]
time = 0.56, size = 27, normalized size = 0.22

$$\text{integral}\Big(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}(d\tanh\left(fx+e\right))^{m},x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**p*(d*tanh(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\mathrm{tanh}\left(e+f\,x\right)\right)}^m\,{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tanh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int((d*tanh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p, x)
```

3.513 $\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^3(c+dx)\,dx$

Optimal. Leaf size=110

$$-\frac{(a-b(1+p))\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sinh^2(c+dx)}{a-b}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2(a-b)^2d(1+p)}+\frac{\operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+\right.}{2(a-b)d}$$

```
[Out] -1/2*(a-b*(1+p))*hypergeom([1, 1+p],[2+p],(a+b*sinh(d*x+c)^2)/(a-b))*(a+b*s
inh(d*x+c)^2)^(1+p)/(a-b)^2/d/(1+p)+1/2*sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^(
1+p)/(a-b)/d
```

Rubi [A]
time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3273, 79, 70}

$$\frac{\operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{p+1}}{2d(a-b)}-\frac{(a-b(p+1))\left(a+b\sinh^2(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)+a}{a-b}\right)}{2d(p+1)(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^3,x]
```

```
[Out] -1/2*((a - b*(1 + p))*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*
x]^2)/(a - b)]*(a + b*Sinh[c + d*x]^2)^(1 + p))/((a - b)^2*d*(1 + p)) + (Se
ch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)*d)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
```

```
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int (a+b\sinh^2(c+dx))^p \tanh^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^p}{(1+x)^2}\,dx, x, \sinh^2(c+dx)\right)}{2d}$$

$$= \frac{\operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2(a-b)d} - \frac{(a-b(1+p))\text{Subst}\Big(}{2}$$

$$= -\frac{(a-b(1+p))\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sinh^2(c+dx)}{a-b}\right)\left(a+b\sinh^2(\right.}{2(a-b)^2d(1+p)}$$

Mathematica [A]
time = 0.20, size = 90, normalized size = 0.82

$$\frac{\left((-a+b+bp)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sinh^2(c+dx)}{a-b}\right)+(a-b)(1+p)\operatorname{sech}^2(c+dx)\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2(a-b)^2d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^3,x]
```

```
[Out] (((-a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)
/(a - b)] + (a - b)*(1 + p)*Sech[c + d*x]^2)*(a + b*Sinh[c + d*x]^2)^(1 + p
))/(2*(a - b)^2*d*(1 + p))
```

Maple [F]
time = 2.10, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sinh^2\left(dx+c\right)\right)\right)^p \left(\tanh^3\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x)
```

```
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.44, size = 25, normalized size = 0.23

$$\text{integral}\left(\left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\tanh\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(c+d\,x)^3\,\left(b\sinh(c+d\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(tanh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^p, x)
```

3.514 $\int \left(a+b\sinh^2(c+dx)\right)^p\tanh(c+dx)\,dx$

Optimal. Leaf size=63

$$-\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sinh^2(c+dx)}{a-b}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2(a-b)d(1+p)}$$

```
[Out] -1/2*hypergeom([1, 1+p],[2+p],(a+b*sinh(d*x+c)^2)/(a-b))*(a+b*sinh(d*x+c)^2
)^(1+p)/(a-b)/d/(1+p)
```

Rubi [A]

time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3273, 70}

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)+a}{a-b}\right)}{2d(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x],x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)/(a - b)]*(
a + b*Sinh[c + d*x]^2)^(1 + p))/((a - b)*d*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^p\tanh(c+dx)\,dx=\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{1+x}\,dx,x,\sinh^2(c+dx)\right)}{2d}$$
$$=-\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sinh^2(c+dx)}{a-b}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2(a-b)d(1+p)}$$

Mathematica [A]
time = 0.05, size = 65, normalized size = 1.03

$$-\frac{\left(a-b+b\cosh^2(c+dx)\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\cosh^2(c+dx)}{a-b}\right)}{2(a-b)d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x],x]
[Out] -1/2*((a - b + b*Cosh[c + d*x]^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p
, 1 + (b*Cosh[c + d*x]^2)/(a - b)])/((a - b)*d*(1 + p))
```

Maple [F]
time = 1.02, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sinh^2\left(dx+c\right)\right)\right)^p\tanh\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x)
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)
```

Fricas [F]
time = 0.41, size = 23, normalized size = 0.37

$$\text{integral}\left(\left(b\sinh\left(dx+c\right)^2+a\right)^p\tanh\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^{2}\left(c + dx\right)\right)^{p}\tanh\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)**p*tanh(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tanh(c + d\,x)\,\left(b\sinh(c + d\,x)^{2} + a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(tanh(c + d*x)*(a + b*sinh(c + d*x)^2)^p, x)
```

3.515 $\int \coth(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=54

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sinh^2(c+dx)}{a}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad(1+p)}$$

```
[Out] -1/2*hypergeom([1, 1+p],[2+p],1+b*sinh(d*x+c)^2/a)*(a+b*sinh(d*x+c)^2)^(1+p
)/a/d/(1+p)
```

Rubi [A]

time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3273, 67}

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*
Sinh[c + d*x]^2)^(1 + p))/(a*d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \coth(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx&=\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{x}\,dx,x,\sinh^2(c+dx)\right)}{2d}\\&=-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sinh^2(c+dx)}{a}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad(1+p)}\end{aligned}$$

Mathematica [**A**]
time = 0.07, size = 54, normalized size = 1.00

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sinh^2(c+dx)}{a}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*
Sinh[c + d*x]^2)^(1 + p))/(a*d*(1 + p))
```

Maple [**F**]
time = 1.20, size = 0, normalized size = 0.00

$$\int \coth\left(dx+c\right)\left(a+b\left(\sinh^2\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x)
```

```
[Out] int(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)
```

Fricas [**F**]
time = 0.37, size = 23, normalized size = 0.43

$$\text{integral}\left(\left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(c+d\,x)\,\left(b\,\sinh(c+d\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(coth(c + d*x)*(a + b*sinh(c + d*x)^2)^p, x)
```

3.516 $\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=94

$$-\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad}-\frac{(a+bp)\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\sinh^2(c+dx)}{a}\right)\left(a+b\sinh^2(c+dx)\right)}{2a^2d(1+p)}$$

```
[Out] -1/2*csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^(1+p)/a/d-1/2*(b*p+a)*hypergeom([1,
1+p],[2+p],1+b*sinh(d*x+c)^2/a)*(a+b*sinh(d*x+c)^2)^(1+p)/a^2/d/(1+p)
```

Rubi [A]

time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3273, 79, 67}

$$-\frac{(a+bp)\left(a+b\sinh^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{2a^2d(p+1)}-\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{p+1}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -1/2*(Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/(a*d) - ((a + b*p)*H
ypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh[c
+ d*x]^2)^(1 + p))/(2*a^2*d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
```

```
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x)(a+bx)^p}{x^2}\,dx, x, \sinh^2(c+dx)\right)}{2d}\\ &= -\frac{\text{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad}+\frac{(a+bp)\text{Subst}\left(\int\right.}{}\\ &= -\frac{\text{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad}-\frac{(a+bp)\,{}_2F_1\left(1,1\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 71, normalized size = 0.76

$$-\frac{\left(a\,\text{csch}^2(c+dx)+\frac{(a+bp)\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\sinh^2(c+dx)}{a}\right)}{1+p}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -1/2*((a*Csch[c + d*x]^2 + ((a + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1
+ (b*Sinh[c + d*x]^2)/a])/(1 + p))*(a + b*Sinh[c + d*x]^2)^(1 + p))/(a^2*d)
```

Maple [F]
time = 1.54, size = 0, normalized size = 0.00

$$\int \left(\coth^3\left(dx+c\right)\right)\left(a+b\left(\sinh^2\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x)
```

```
[Out] int(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.50, size = 25, normalized size = 0.27

$$\mathrm{integral}\left(\left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\coth\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sinh(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coth}(c+d\,x)^3\,{\left(b\,{\mathrm{sinh}(c+d\,x)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(coth(c + d*x)^3*(a + b*sinh(c + d*x)^2)^p, x)
```

3.517 $\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\,\sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\Bigl(1+b}{5d}$$

```
[Out] 1/5*AppellF1(5/2,5/2,-p,7/2,-sinh(d*x+c)^2,-b*sinh(d*x+c)^2/a)*sinh(d*x+c)^
4*(a+b*sinh(d*x+c)^2)^p*(cosh(d*x+c)^2)^(1/2)*tanh(d*x+c)/d/((1+b*sinh(d*x+
c)^2/a)^p)
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3275, 525, 524}

$$\frac{\sinh^4(c+dx)\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4,x]
```

```
[Out] (AppellF1[5/2, 5/2, -p, 7/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sq
rt[Cosh[c + d*x]^2]*Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]
)/(5*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx = \frac{\left(\sqrt{\cosh^2(c+dx)}\,\operatorname{sech}(c+dx)\right)\operatorname{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^p}{\left(1+x^2\right)^{5/2}}\,dx, x, \sinh}{d}$$

$$= \frac{\left(\sqrt{\cosh^2(c+dx)}\,\operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\frac{b\sin}{}\right.\right.}{d}$$

$$= \frac{F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}}{}$$

Mathematica [F]
time = 27.71, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4,x]
```

```
[Out] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4, x]
```

Maple [F]
time = 1.33, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sinh^2\left(dx+c\right)\right)\right)^p \left(\tanh^4\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x)
```

```
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

Fricas [**F**]
time = 0.47, size = 25, normalized size = 0.24

$$\text{integral}\Big(\left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\tanh\left(dx+c\right)^{4},x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(c+d\,x)^4\,\left(b\sinh(c+d\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(tanh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^p, x)
```

3.518 $\int \left(a+b\sinh^2(c+dx)\right)^p\tanh^2(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\,\sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\right.}{3d}$$

```
[Out] 1/3*AppellF1(3/2,3/2,-p,5/2,-sinh(d*x+c)^2,-b*sinh(d*x+c)^2/a)*sinh(d*x+c)^
2*(a+b*sinh(d*x+c)^2)^p*(cosh(d*x+c)^2)^(1/2)*tanh(d*x+c)/d/((1+b*sinh(d*x+
c)^2/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3275, 525, 524}

$$\frac{\sinh^2(c+dx)\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2,x]
```

```
[Out] (AppellF1[3/2, 3/2, -p, 5/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sq
rt[Cosh[c + d*x]^2]*Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]
)/(3*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^2(c+dx)\,dx = \frac{\left(\sqrt{\cosh^2(c+dx)}\,\operatorname{sech}(c+dx)\right)\operatorname{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^p}{\left(1+x^2\right)^{3/2}}\,dx,x,\sin}{d}$$

$$= \frac{\left(\sqrt{\cosh^2(c+dx)}\,\operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\frac{b}{}\right.\right.}{d}$$

$$= \frac{F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx}}{}$$

Mathematica [F]
time = 4.02, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^2(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2,x]
[Out] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2, x]
```

Maple [F]
time = 1.28, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sinh^2\left(dx+c\right)\right)\right)^p\left(\tanh^2\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x)
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

Fricas [**F**]
time = 0.44, size = 25, normalized size = 0.24

$$\mathrm{integral}\left(\left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\tanh\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{tanh}\left(c+d\,x\right)^2\,{\left(b\,{\mathrm{sinh}\left(c+d\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(tanh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^p, x)
```

3.519 $\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=99

$$-\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\,\text{csch}(c+dx)\text{sech}(c+dx)\,\big(a+b\sinh^2(}{d}$$

```
[Out] -AppellF1(-1/2,-1/2,-p,1/2,-sinh(d*x+c)^2,-b*sinh(d*x+c)^2/a)*csch(d*x+c)*s
ech(d*x+c)*(a+b*sinh(d*x+c)^2)^p*(cosh(d*x+c)^2)^(1/2)/d/((1+b*sinh(d*x+c)^
2/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3275, 525, 524}

$$-\frac{\sqrt{\cosh^2(c+dx)}\,\text{csch}(c+dx)\text{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)
]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]*Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)
^p)/(d*(1 + (b*Sinh[c + d*x]^2)/a)^p))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(c+dx)}\,\operatorname{sech}(c+dx)\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+x^2}\,(a+bx^2)^p}{x^2}\,dx\right.}{d}$$

$$= \frac{\left(\sqrt{\cosh^2(c+dx)}\,\operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\frac{b\sin}{}\right.\right.}{d}$$

$$= -\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c}}{}$$

Mathematica [F]
time = 4.02, size = 0, normalized size = 0.00

$$\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p,x]
[Out] Integrate[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p, x]
```

Maple [F]
time = 1.33, size = 0, normalized size = 0.00

$$\int \left(\coth^2(dx+c)\right)\left(a+b\left(\sinh^2(dx+c)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x)
[Out] int(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

Fricas [**F**]
time = 0.41, size = 25, normalized size = 0.25

$$\mathrm{integral}\left(\left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\coth\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sinh(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coth}(c+d\,x)^2\,{\left(b\,\mathrm{sinh}(c+d\,x)^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(coth(c + d*x)^2*(a + b*sinh(c + d*x)^2)^p, x)
```

3.520 $\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=103

$$-\frac{F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\,\text{csch}^3(c+dx)\text{sech}(c+dx)\left(a+b\sinh\right.}{3d}$$

```
[Out] -1/3*AppellF1(-3/2,-3/2,-p,-1/2,-sinh(d*x+c)^2,-b*sinh(d*x+c)^2/a)*csch(d*x
+c)^3*sech(d*x+c)*(a+b*sinh(d*x+c)^2)^p*(cosh(d*x+c)^2)^(1/2)/d/((1+b*sinh(
d*x+c)^2/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3275, 525, 524}

$$-\frac{\sqrt{\cosh^2(c+dx)}\,\text{csch}^3(c+dx)\text{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -1/3*(AppellF1[-3/2, -3/2, -p, -1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2
)/a)]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]^3*Sech[c + d*x]*(a + b*Sinh[c + d
*x]^2)^p)/(d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(c+dx)}\,\operatorname{sech}(c+dx)\right)\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^p}{x^4}\,dx}{d}$$

$$= \frac{\left(\sqrt{\cosh^2(c+dx)}\,\operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\right.\right.}{}$$

$$= -\frac{F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2}}{}$$

Mathematica [F]
time = 23.97, size = 0, normalized size = 0.00

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] Integrate[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p, x]
```

Maple [F]
time = 1.43, size = 0, normalized size = 0.00

$$\int \left(\coth^4(dx+c)\right)\left(a+b\left(\sinh^2(dx+c)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x)
```

```
[Out] int(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

Fricas [**F**]
time = 0.43, size = 25, normalized size = 0.24

$$\mathrm{integral}\Big(\left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\coth\left(dx+c\right)^{4},x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*sinh(d*x+c)**2)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coth}(c+d\,x)^4\,{\left(b\,\mathrm{sinh}(c+d\,x)^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(coth(c + d*x)^4*(a + b*sinh(c + d*x)^2)^p, x)
```

3.521 $\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx$

Optimal. Leaf size=152

$$\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sinh(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\right.}{}$$

```
[Out] -1/2*csch(x)^2/a+ln(sinh(x))/a-1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*sinh(x))/a^(5
/3)+1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sinh(x)+b^(2/3)*sinh(x)^2)/a^(5/
3)-1/3*ln(a+b*sinh(x)^3)/a+1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*sinh(x
))/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {3309, 1848, 1885, 12, 206, 31, 648, 631, 210, 642, 266}

$$\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sinh(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sinh(x)+b^{2/3}\sinh^2(x)\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}-\frac{\log\left(a+b\sinh^3(x)\right)}{3a}-\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Sinh[x]^3),x]
```

```
[Out] (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sinh[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(5/3)) - Csch[x]^2/(2*a) + Log[Sinh[x]]/a - (b^(2/3)*Log[a^(1/3) + b^(1/3
)*Sinh[x]])/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sinh[x] +
b^(2/3)*Sinh[x]^2])/(6*a^(5/3)) - Log[a + b*Sinh[x]^3]/(3*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
```

```
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx = \text{Subst}\left(\int \frac{1+x^2}{x^3\,(a+bx^3)}\,dx, x, \sinh(x)\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{ax^3}+\frac{1}{ax}+\frac{-b-bx^2}{a\,(a+bx^3)}\right)dx, x, \sinh(x)\right)$$

$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}+\frac{\text{Subst}\left(\int \frac{-b-bx^2}{a+bx^3}\,dx, x, \sinh(x)\right)}{a}$$

$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{\text{Subst}\left(\int \frac{b}{a+bx^3}\,dx, x, \sinh(x)\right)}{a}-\frac{b\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \right.}{a}$$

$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{\log\left(a+b\sinh^3(x)\right)}{3a}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sinh(x)\right)}{a}$$

$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{\log\left(a+b\sinh^3(x)\right)}{3a}-\frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \sinh\right.}{3a^{5/3}}$$

$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sinh(x)\right)}{3a^{5/3}}-\frac{\log\left(a+b\sinh^3(x)\right)}{3a}+$$

$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sinh(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{\right.}{}$$

$$= \frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sinh(x)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sinh(\right.}{3a^{5/3}}$$

Mathematica [A]
time = 0.25, size = 136, normalized size = 0.89

$$-\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{\left(a^{2/3}+(-1)^{2/3}b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,\sinh(x)\right)+\left(a^{2/3}+b^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sinh(x)\right)+\left(a^{2/3}-\sqrt[3]{-1}\,b^{2/3}\right)\log\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,\sinh(x)\right)}{3a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Sinh[x]^3),x]
```

```
[Out] -1/2*Csch[x]^2/a + Log[Sinh[x]]/a - ((a^(2/3) + (-1)^(2/3)*b^(2/3))*Log[-((
-1)^(2/3)*a^(1/3)) - b^(1/3)*Sinh[x]] + (a^(2/3) + b^(2/3))*Log[a^(1/3) + b
^(1/3)*Sinh[x]] + (a^(2/3) - (-1)^(1/3)*b^(2/3))*Log[a^(1/3) + (-1)^(2/3)*b
^(1/3)*Sinh[x]])/(3*a^(5/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.02, size = 132, normalized size = 0.87

method	result
risch	$-\frac{2\,e^{2x}}{(e^{2x}-1)^2 a}+\frac{\ln(e^{2x}-1)}{a}+\Bigg(\sum_{\_R=\mathrm{RootOf}(27a^5\_Z^3+27a^4\_Z^2+9\_Z a^3+a^2+b^2)}\_R\ln\Big(e^{2x}+\Big(-\frac{6a^2\_R}{b}-\frac{2a}{b}\Big)e^x$
default	$-\frac{\tanh^2(\frac{x}{2})}{8a}-\frac{1}{8a\tanh(\frac{x}{2})^2}+\frac{\ln(\tanh(\frac{x}{2}))}{a}+\frac{\sum_{\_R=\mathrm{RootOf}(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a)}\frac{(-\_R^5 a-\_R^4 b+2\_R^3 a+4\_}{\_R^5 a-2\_R^3}}{3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sinh(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*tanh(1/2*x)^2/a-1/8/a/tanh(1/2*x)^2+1/a*ln(tanh(1/2*x))+1/3/a*sum((-_R
^5*a-_R^4*b+2*_R^3*a+4*_R^2*b-_R*a+b)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(ta
nh(1/2*x)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="maxima")
```

```
[Out] 2*b*(x/(a*b) - integrate((b*e^(5*x) - 3*b*e^(3*x) + 8*a*e^(2*x) + 3*b*e^x)*
e^x/(b*e^(6*x) - 3*b*e^(4*x) + 8*a*e^(3*x) + 3*b*e^(2*x) - b), x)/(a*b)) -
6*b*integrate(e^(4*x)/(b*e^(6*x) - 3*b*e^(4*x) + 8*a*e^(3*x) + 3*b*e^(2*x)
- b), x)/a - 2*(x*e^(4*x) - (2*x - 1)*e^(2*x) + x)/(a*e^(4*x) - 2*a*e^(2*x)
 + a) + log(e^x + 1)/a + log(e^x - 1)/a + 8*integrate(e^(3*x)/(b*e^(6*x) -
3*b*e^(4*x) + 8*a*e^(3*x) + 3*b*e^(2*x) - b), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.19, size = 1115, normalized size = 7.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(1/3)*(a*e^(4*x) - 2*a*e^(2*x) + a)*sqrt((((1/2)^(1/3)*(I*sqr
t(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a +
 4)/a^2)*arctan(-1/8*(2*sqrt(1/3)*sqrt(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3
+ b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^4*e^(2*x) + b^2*e^(4*x) + 2*a
*b*e^(3*x) - 2*a*b*e^x - (a^2*b*e^(3*x) + 4*a^3*e^(2*x) - a^2*b*e^x)*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a) + b
^2 + 2*(2*a^2 - b^2)*e^(2*x))*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^
5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a^3 - 2*a^2)*sqrt((((1/2)^(1/3)*(I*sqrt(3
) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1
/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a + 4)
/a^2) + sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b
^2)/a^5)^(1/3) + 2/a)^2*a^5*e^x + 4*a^2*b*e^(2*x) + 4*a^3*e^x - 4*a^2*b - 2
*(a^3*b*e^(2*x) + 2*a^4*e^x - a^3*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 +
b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)
*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2))
*e^(-x)/b^2) + 2*(a*e^(4*x) - 2*a*e^(2*x) + a)*((1/2)^(1/3)*(I*sqrt(3) + 1)
*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*log(((1/2)^(1/3)*(I*sqrt(
3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a^2*e^x + b*e^(2*x
) - 2*a*e^x - b) - ((a*e^(4*x) - 2*a*e^(2*x) + a)*((1/2)^(1/3)*(I*sqrt(3) +
 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a) - 6*e^(4*x) + 12*e^(2*
x) - 6)*log(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5
)^(1/3) + 2/a)^2*a^4*e^(2*x) + b^2*e^(4*x) + 2*a*b*e^(3*x) - 2*a*b*e^x - (a
^2*b*e^(3*x) + 4*a^3*e^(2*x) - a^2*b*e^x)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a
^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a) + b^2 + 2*(2*a^2 - b^2)*e^(2*x
)) - 12*(e^(4*x) - 2*e^(2*x) + 1)*log(e^(2*x) - 1) + 24*e^(2*x))/(a*e^(4*x)
 - 2*a*e^(2*x) + a)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sinh(x)**3),x)
```

```
[Out] Integral(coth(x)**3/(a + b*sinh(x)**3), x)
```

Giac [A]

time = 0.43, size = 209, normalized size = 1.38

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-e^{(-x)}+e^{x}\right|\right)}{3\,a^{2}}-\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)^{3}+8\,a\right|\right)}{3\,a}+\frac{\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{a}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}-e^{(-x)}+e^{x}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(\left(e^{(-x)}-e^{x}\right)^{2}-2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(e^{(-x)}-e^{x}\right)+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{2}}-\frac{3\left(e^{(-x)}-e^{x}\right)^{2}+4}{2\,a\left(e^{(-x)}-e^{x}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="giac")
```

```
[Out] 1/3*b*(-a/b)^(1/3)*log(abs(-2*(-a/b)^(1/3) - e^(-x) + e^x))/a^2 - 1/3*log(a
bs(-b*(e^(-x) - e^x)^3 + 8*a))/a + log(abs(-e^(-x) + e^x))/a - 1/3*sqrt(3)*
(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) - e^(-x) + e^x)/(-a/b)^(1/3
))/a^2 - 1/6*(-a*b^2)^(1/3)*log((e^(-x) - e^x)^2 - 2*(-a/b)^(1/3)*(e^(-x) -
 e^x) + 4*(-a/b)^(2/3))/a^2 - 1/2*(3*(e^(-x) - e^x)^2 + 4)/(a*(e^(-x) - e^x
)^2)
```

Mupad [B]
time = 0.92, size = 1129, normalized size = 7.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b*sinh(x)^3),x)
```

```
[Out] 2/(a - a*exp(2*x)) - 2/(a - 2*a*exp(2*x) + a*exp(4*x)) + symsum(log((503316
48*a^6*exp(2*x) + 786432*b^6*exp(2*x) - 452984832*root(27*a^5*z^3 + 27*a^4*
z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^7 - 50331648*a^6 - 786432*b^6 - 13589544
96*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^2*a^8 - 135895
4496*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a^9 - 5059
3792*a^2*b^4 - 102498304*a^4*b^2 + 1358954496*root(27*a^5*z^3 + 27*a^4*z^2
+ 9*a^3*z + b^2 + a^2, z, k)^2*a^8*exp(2*x) + 1358954496*root(27*a^5*z^3 +
27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a^9*exp(2*x) + 50593792*a^2*b^4*e
xp(2*x) + 102498304*a^4*b^2*exp(2*x) - 7602176*root(27*a^5*z^3 + 27*a^4*z^2
 + 9*a^3*z + b^2 + a^2, z, k)*a^3*b^4 - 465305600*root(27*a^5*z^3 + 27*a^4*
z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^5*b^2 + 524288*a*b^5*exp(x) - 24379392*r
oot(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^2*a^4*b^4 - 138333
3888*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^2*a^6*b^2 -
18874368*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a^5*b^
4 - 1370750976*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*
a^7*b^2 + 452984832*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z,
k)*a^7*exp(2*x) + 5242880*a^3*b^3*exp(x) - 524288*root(27*a^5*z^3 + 27*a^4*
z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^2*b^5*exp(x) + 8912896*root(27*a^5*z^3 +
 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^4*b^3*exp(x) + 7602176*root(27*a
^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^3*b^4*exp(2*x) + 4653056
00*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^5*b^2*exp(2*
x) - 14155776*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a
^6*b^3*exp(x) + 24379392*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2
, z, k)^2*a^4*b^4*exp(2*x) + 1383333888*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^
3*z + b^2 + a^2, z, k)^2*a^6*b^2*exp(2*x) + 18874368*root(27*a^5*z^3 + 27*a
^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a^5*b^4*exp(2*x) + 1370750976*root(27
*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a^7*b^2*exp(2*x))/(a^6
*b^6))*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k), k, 1, 3)
+ log(3221225472*a^6*exp(2*x) + 786432*b^6*exp(2*x) - 3221225472*a^6 - 7864
```

```
32*b^6 - 101449728*a^2*b^4 - 3321888768*a^4*b^2 + 101449728*a^2*b^4*exp(2*x
) + 3321888768*a^4*b^2*exp(2*x))/a
```

3.522 $\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3309, 272, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Sinh[x]^3],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^3}}\,dx, x, \sinh(x)\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^3(x)\right)\\ &= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^3(x)}\right)}{3b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sinh[x]^3],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Maple [A]
time = 6.38, size = 21, normalized size = 0.75

method	result	size

derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sinh^{3}\left(x\right)\right)}}{\sqrt{a}}\right)}{3\sqrt{a}}$	21
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sinh^{3}\left(x\right)\right)}}{\sqrt{a}}\right)}{3\sqrt{a}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x)^3 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(Expression(Integer)),failed) can
not be coerced to mode SparseUnivariatePolynomial(Expression(Integer))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(x\right)}{\sqrt{a+b\sinh^{3}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)**3)**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*sinh(x)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x)^3 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\coth(x)}{\sqrt{b\sinh\left(x\right)^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*sinh(x)^3)^(1/2),x)
```

```
[Out] int(coth(x)/(a + b*sinh(x)^3)^(1/2), x)
```

3.523 $\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx$

Optimal. Leaf size=45

$$-\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)+\frac{2}{3}\sqrt{a+b\sinh^3(x)}$$

```
[Out] -2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*sinh(x)^3)^(1/
2)
```

Rubi [A]

time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3309, 272, 52, 65, 214}

$$\frac{2}{3}\sqrt{a+b\sinh^3(x)}-\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Sinh[x]^3],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sinh[
x]^3])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x}\,dx, x, \sinh(x)\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sinh^3(x)\right)\\
&= \frac{2}{3}\sqrt{a+b\sinh^3(x)} + \frac{1}{3}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^3(x)\right)\\
&= \frac{2}{3}\sqrt{a+b\sinh^3(x)} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^3(x)}\right)}{3b}\\
&= -\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right) + \frac{2}{3}\sqrt{a+b\sinh^3(x)}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$-\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right) + \frac{2}{3}\sqrt{a+b\sinh^3(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sinh[x]^3],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sinh[
x]^3])/3
```

Maple [A]
time = 5.72, size = 34, normalized size = 0.76

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sinh^{3}\left(x\right)\right)}}{\sqrt{a}}\right)\sqrt{a}}{3}+\frac{2\sqrt{a+b\left(\sinh^{3}\left(x\right)\right)}}{3}$	34
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sinh^{3}\left(x\right)\right)}}{\sqrt{a}}\right)\sqrt{a}}{3}+\frac{2\sqrt{a+b\left(\sinh^{3}\left(x\right)\right)}}{3}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sinh(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*sinh(x)^3)^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x)^3 + a)*coth(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(33) = 66$.
time = 1.51, size = 1663, normalized size = 36.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*(cosh(x) + sinh(x))*log(-(b^2*cosh(x)^12 + 12*b^2*cosh(x)*sin
h(x)^11 + b^2*sinh(x)^12 - 6*b^2*cosh(x)^10 + 64*a*b*cosh(x)^9 + 6*(11*b^2*
cosh(x)^2 - b^2)*sinh(x)^10 + 15*b^2*cosh(x)^8 + 4*(55*b^2*cosh(x)^3 - 15*b
^2*cosh(x) + 16*a*b)*sinh(x)^9 - 192*a*b*cosh(x)^7 + 3*(165*b^2*cosh(x)^4 -
 90*b^2*cosh(x)^2 + 192*a*b*cosh(x) + 5*b^2)*sinh(x)^8 + 24*(33*b^2*cosh(x)
^5 - 30*b^2*cosh(x)^3 + 96*a*b*cosh(x)^2 + 5*b^2*cosh(x) - 8*a*b)*sinh(x)^7
 + 192*a*b*cosh(x)^5 + 4*(128*a^2 - 5*b^2)*cosh(x)^6 + 4*(231*b^2*cosh(x)^6
```

```
 - 315*b^2*cosh(x)^4 + 1344*a*b*cosh(x)^3 + 105*b^2*cosh(x)^2 - 336*a*b*cos
h(x) + 128*a^2 - 5*b^2)*sinh(x)^6 + 15*b^2*cosh(x)^4 + 24*(33*b^2*cosh(x)^7
 - 63*b^2*cosh(x)^5 + 336*a*b*cosh(x)^4 + 35*b^2*cosh(x)^3 - 168*a*b*cosh(x
)^2 + 8*a*b + (128*a^2 - 5*b^2)*cosh(x))*sinh(x)^5 - 64*a*b*cosh(x)^3 + 3*(
165*b^2*cosh(x)^8 - 420*b^2*cosh(x)^6 + 2688*a*b*cosh(x)^5 + 350*b^2*cosh(x
)^4 - 2240*a*b*cosh(x)^3 + 320*a*b*cosh(x) + 20*(128*a^2 - 5*b^2)*cosh(x)^2
 + 5*b^2)*sinh(x)^4 - 6*b^2*cosh(x)^2 + 4*(55*b^2*cosh(x)^9 - 180*b^2*cosh(
x)^7 + 1344*a*b*cosh(x)^6 + 210*b^2*cosh(x)^5 - 1680*a*b*cosh(x)^4 + 480*a*
b*cosh(x)^2 + 20*(128*a^2 - 5*b^2)*cosh(x)^3 + 15*b^2*cosh(x) - 16*a*b)*sin
h(x)^3 + 6*(11*b^2*cosh(x)^10 - 45*b^2*cosh(x)^8 + 384*a*b*cosh(x)^7 + 70*b
^2*cosh(x)^6 - 672*a*b*cosh(x)^5 + 320*a*b*cosh(x)^3 + 10*(128*a^2 - 5*b^2)
*cosh(x)^4 + 15*b^2*cosh(x)^2 - 32*a*b*cosh(x) - b^2)*sinh(x)^2 + b^2 - 16*
(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 - 3*b*cosh(x)^6 + (28*b*
cosh(x)^2 - 3*b)*sinh(x)^6 + 16*a*cosh(x)^5 + 2*(28*b*cosh(x)^3 - 9*b*cosh(
x) + 8*a)*sinh(x)^5 + 3*b*cosh(x)^4 + (70*b*cosh(x)^4 - 45*b*cosh(x)^2 + 80
*a*cosh(x) + 3*b)*sinh(x)^4 + 4*(14*b*cosh(x)^5 - 15*b*cosh(x)^3 + 40*a*cos
h(x)^2 + 3*b*cosh(x))*sinh(x)^3 - b*cosh(x)^2 + (28*b*cosh(x)^6 - 45*b*cosh
(x)^4 + 160*a*cosh(x)^3 + 18*b*cosh(x)^2 - b)*sinh(x)^2 + 2*(4*b*cosh(x)^7
 - 9*b*cosh(x)^5 + 40*a*cosh(x)^4 + 6*b*cosh(x)^3 - b*cosh(x))*sinh(x))*sqrt
(a)*sqrt((b*sinh(x)^3 + 3*(b*cosh(x)^2 - b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 12*(b^2*cosh(x)^11 - 5*b^2*cosh(x)^9 + 48*a*
b*cosh(x)^8 + 10*b^2*cosh(x)^7 - 112*a*b*cosh(x)^6 + 80*a*b*cosh(x)^4 + 2*(
128*a^2 - 5*b^2)*cosh(x)^5 + 5*b^2*cosh(x)^3 - 16*a*b*cosh(x)^2 - b^2*cosh(
x))*sinh(x))/(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(
x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 - 3*cosh(x))*sinh(x)
^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33
*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 - 315*c
osh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 + 24*(33*cosh(x)^7 -
 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 84
*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20
*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*si
nh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 1
5*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 1
0*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*sqrt(
(b*sinh(x)^3 + 3*(b*cosh(x)^2 - b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)))/(cosh(x) + sinh(x)), 1/3*(sqrt(-a)*(cosh(x) + sinh(x))
*arctan(8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(-a)*sqrt((b*sinh
(x)^3 + 3*(b*cosh(x)^2 - b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2))/(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 - 3*b*cosh(
x)^4 + 3*(5*b*cosh(x)^2 - b)*sinh(x)^4 + 16*a*cosh(x)^3 + 4*(5*b*cosh(x)^3
- 3*b*cosh(x) + 4*a)*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 - 6*b*cos
h(x)^2 + 16*a*cosh(x) + b)*sinh(x)^2 + 6*(b*cosh(x)^5 - 2*b*cosh(x)^3 + 8*a
*cosh(x)^2 + b*cosh(x))*sinh(x) - b)) + sqrt((b*sinh(x)^3 + 3*(b*cosh(x)^2
- b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)
+ sinh(x))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^3(x)}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(x)**3)*coth(x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^3 + a)*coth(x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x)\,\sqrt{b\sinh(x)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b*sinh(x)^3)^(1/2),x)
```

```
[Out] int(coth(x)*(a + b*sinh(x)^3)^(1/2), x)
```

3.524 $\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

```
[Out] -2*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3309, 272, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Sinh[x]^n],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
```

```
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, \sinh(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^n(x)\right)}{n}\\ &= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^n(x)}\right)}{bn}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 29, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sinh[x]^n],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [**A**]
time = 7.47, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sinh^{n}\left(x\right)\right)}}{\sqrt{a}}\right)}{n\sqrt{a}}$	24
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sinh^{n}\left(x\right)\right)}}{\sqrt{a}}\right)}{n\sqrt{a}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x)^n + a), x)
```

Fricas [A]
time = 0.55, size = 113, normalized size = 3.90

$$\left[\frac{\log\left(\frac{b\cosh(n\log(\sinh(x)))+b\sinh(n\log(\sinh(x)))-2\sqrt{b\cosh\left(n\log\left(\sinh\left(x\right)\right)\right)+b\sinh\left(n\log\left(\sinh\left(x\right)\right)\right)+a}\sqrt{a}+2a}{\cosh(n\log(\sinh(x)))+\sinh(n\log(\sinh(x)))}\right)}{\sqrt{a}\,n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{b\cosh\left(n\log\left(\sinh\left(x\right)\right)\right)+b\sinh\left(n\log\left(\sinh\left(x\right)\right)\right)+a}\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) - 2*sqrt(b*cosh(n*log
(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*log(sinh(x)
)) + sinh(n*log(sinh(x)))))/(sqrt(a)*n), 2*sqrt(-a)*arctan(sqrt(b*cosh(n*lo
g(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(-a)/a)/(a*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(x\right)}{\sqrt{a+b\sinh^{n}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)**n)**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*sinh(x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x)^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*sinh(x)^n)^(1/2),x)
```

```
[Out] int(coth(x)/(a + b*sinh(x)^n)^(1/2), x)
```

3.525 $\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx$

Optimal. Leaf size=47

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n}+\frac{2\sqrt{a+b\sinh^n(x)}}{n}$$

```
[Out] -2*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*sinh(x)^n)^(1/2)
/n
```

Rubi [A]
time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3309, 272, 52, 65, 214}

$$\frac{2\sqrt{a+b\sinh^n(x)}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Sinh[x]^n],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/n + (2*Sqrt[a + b*Sinh[
x]^n])/n
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx, x, \sinh(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sinh^n(x)\right)}{n}\\ &= \frac{2\sqrt{a+b\sinh^n(x)}}{n} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^n(x)\right)}{n}\\ &= \frac{2\sqrt{a+b\sinh^n(x)}}{n} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^n(x)}\right)}{bn}\\ &= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n} + \frac{2\sqrt{a+b\sinh^n(x)}}{n}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 45, normalized size = 0.96

$$\frac{-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\sinh^n(x)}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sinh[x]^n],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]^n
])/n
```

Maple [A]
time = 1.45, size = 38, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\left(\sinh^{n}\left(x\right)\right)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(\sinh^{n}\left(x\right)\right)}}{\sqrt{a}}\right)}{n}$	38
default	$\frac{2\sqrt{a+b\left(\sinh^{n}\left(x\right)\right)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(\sinh^{n}\left(x\right)\right)}}{\sqrt{a}}\right)}{n}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sinh(x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2*(a+b*sinh(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x)^n + a)*coth(x), x)
```

Fricas [A]
time = 0.43, size = 156, normalized size = 3.32

$$\left[\frac{\sqrt{a}\log\left(\frac{b\cosh(n\log(\sinh(x)))+b\sinh(n\log(\sinh(x)))-2\sqrt{b\cosh\left(n\log\left(\sinh\left(x\right)\right)\right)+b\sinh\left(n\log\left(\sinh\left(x\right)\right)\right)+a}\,\sqrt{a}+2a}{\cosh(n\log(\sinh(x)))+\sinh(n\log(\sinh(x)))}\right)+2\sqrt{b\cosh\left(n\log\left(\sinh\left(x\right)\right)\right)+b\sinh\left(n\log\left(\sinh\left(x\right)\right)\right)+a}}{n}, \frac{2\left(\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\cosh\left(n\log\left(\sinh\left(x\right)\right)\right)+b\sinh\left(n\log\left(\sinh\left(x\right)\right)\right)+a}\,\sqrt{-a}}{a}\right)+\sqrt{b\cosh\left(n\log\left(\sinh\left(x\right)\right)\right)+b\sinh\left(n\log\left(\sinh\left(x\right)\right)\right)+a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) - 2*sqrt(b*c
osh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*lo
g(sinh(x))) + sinh(n*log(sinh(x))))) + 2*sqrt(b*cosh(n*log(sinh(x))) + b*si
nh(n*log(sinh(x))) + a))/n, 2*(sqrt(-a)*arctan(sqrt(b*cosh(n*log(sinh(x)))
+ b*sinh(n*log(sinh(x))) + a)*sqrt(-a)/a) + sqrt(b*cosh(n*log(sinh(x))) + b
*sinh(n*log(sinh(x))) + a))/n]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^{n}(x)}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(x)**n)*coth(x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^n + a)*coth(x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x)\,\sqrt{a+b\sinh(x)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b*sinh(x)^n)^(1/2),x)
```

```
[Out] int(coth(x)*(a + b*sinh(x)^n)^(1/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of r
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
    print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```